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中　華　民　國　101　年　4　月　5　日

中文摘要

1. 在計畫執行期間，我與加拿大多倫多大學晏啟樹博士，中國科學院高能物理研究所黃梅教授以及他們的學生進行合作。我們運用我們通過研究超引力空間下的 D 膜系統，進一步完善了我們之前建立的 hard-wall Dp-Dq gravity/gauge 對偶模型。我們把基本原理方法與現象學方法結合起來，研究第二類超弦理論中的 Dp-Dq 膜系統和強子理論中的 SU(2)_{L} ×SU(2)_{R}對稱模型。通過對比介子譜的 Regge 行為，我們研究了有可能與真實的 QCD 理論對偶的時空背景結構。從基本原理方法出發，我們系統的研究了 Dp-Dq 膜系統。通過對比向量與膺向量介子譜的線性 Regge 行為，我們可以得到最接近真實 QCD 理論的 Dp-Dq 膜系統。從現象學方法出發，我們研究了真空中的手徵對稱性破缺以及在高激發量子態中的手徵對稱性漸進恢復。我們的結果對於尋找與真實的 QCD 理論對偶的弦理論模型以及建立正確的現象學全息模型有理論上的指導作用。我們進一步研究了 Dp-Dq 膜系統標量介子與膠子球的能譜。我們的成果寫在論文[8,10]中。

關鍵字： AdS/QCD，D 膜系統，粒子譜，Regge 表現

2. 在計畫執行期間，我還與國立台灣大學電子物理系教授李仁吉教授合作研究弦理論中的對稱性與高能弦散射振幅之間的關係。我們把我們之前計算任意質量玻色開弦在 Regge 極限下的高能散射振幅的方法進一步推廣，分別應用到閉弦與超弦理論中。我們得到超弦在 Regge 極限下的高能散射振幅同樣可以表示成 Kummer 函數。並且通過一些新的關於 Stirling 數的恒等式，我們可以從 Regge 極限下的高能散射振幅得到在 Gross 極限下的高能散射振幅之間的線性比例常數。在之前研究閉弦高能散射振幅的過程中，我們發現鞍點近似方法並不適用，使得我們只能用傳統方法計算一部份振幅，進而對其他振幅的行為進行猜測。在我們新的工作中，我們發現所有高能散射振幅在 Regge 極限下都是不需要鞍點近似就可以計算的。通過 KLT 公式，我們了計算閉弦在 Regge 極限下的高能散射振幅。再通過應用關於 Stirling 數的恒等式，我們最終得到了所有的在 Gross 極限下的高能散射振幅之間的線性比例常數。我們的成果寫在論文[1, 3-7, 9, 11, 12]中。

關鍵字： 弦散射振幅，高能極限，Regge 表現，對稱關係

3. 在計畫執行期間，我還與中國科學院數學研究所吳小寧教授合作研究黑洞相變問題。研究黑洞相變結構對於理解 AdS/CFT 對偶中的場論相變結構非常重要。我們研究了 Kerr/AdS 黑洞的相結構，仔細分析了 Kerr/AdS 黑洞中的三個不同的相變。我們的成果寫在論文[2]中。

關鍵字： Kerr/AdS 黑洞，相變

英文摘要

1. During the project, I have collaborated with Prof. Qi-Shu Yan in Graduate School of Chinese Academic of Science and Prof. Mei Huang in Institute of High Energy Physics of Chinese Academic of Science as well as their students. By studying the D-brane system under the supergravity background, we improved our hard-wall Dp-Dq gravity/gauge duality model which we constructed before. We combined the top-down and bottem-up methods to study the Dp-Dq branes system in type II superstring theory and its conjectured dual SU(2)_{L} ×SU(2)_{R} hadrons model of QCD. By comparing the Regge behavior of the meson spectrum with our model, we studied some space-time backgrounds which may dual to the realistic QCD theory. Using the top-down method, we systematically studied Dp-Dq branes system. By comparing the Regge behaviors of the spectrum for the vector and pseudovector mesons, we obtained a promising Dp-Dq branes model. By using the bottom-up method, we studied the chiral symmetry spontaneously broken in vacuum and the chiral symmetry restoring in the high exited states. We further studied the spectrum of scalar meson and gluball in our medel. We wrote our result in [8,10].

Keywords： AdS/QCD，D-brane system，spectrum，Regge behavior

2. During the project, I have collaborated with Professor Jen-Chi Lee in the Department of Electrophysics at NCTU on the relations between string high energy scattering amplitudes and symmetries in string theory. We extended our previous study on bosonic strings scattering amplitudes in the Regge limit to closed strings and superstrings. We found that superstring scattering amplitudes in the Regge limit could be expressed in terms of Kummer function as in the bosonic string case. By using some new identities of Stirling number, we reproduced the ratios among the superstring scattering amplitudes in the Gross limit from the amplitudes in the Regge limit. In our previous work, we found that the saddle-point approximation is not available when studying closed string scattering in the Gross limit. We could only calculate some amplitudes and guess the behavior of others. In our new work, we found that all closed string scattering amplitudes could be calculated without the saddle-point approximation in the Regge limit. Using KLT formula, we calculated the closed string scattering amplitudes in the Regge limit. We finally obtained all previously guessed ratios among the closed string amplitudes from the identity of Stirling number. We wrote our results in [1, 3-7, 9, 11, 12].

Keywords: string scattering amplitudes, high energy limits, Regge behavior, symmetry

3. During the project, I collaborated with Professor Xiaoning Wu in the institute of mathematics at Chinese Academic Sciences to study the phase transitions in black holes. It is very important to study the phase structures of black holes to understand the phase structure in the AdS/CFT dual field theory. We studied the phase structure of Kerr/AdS black hole and carefully analyze the three different phase transitions in it. We wrote our result in [2].

Keywords： Kerr/AdS black hole，phase transition

報告內容

AdS/QCD duality:

Quantum Chromodynamics (QCD) has been accepted as the basic theory of describing strong interaction for more than 30 years. However, it is still a challenge to solve QCD in non-perturbative region where gauge interaction is strong. In the early 1970's, string theory was proposed to describe strong interacting particles. Recently, the discovery of the gravity/gauge duality has revived the hope to understand QCD in strongly coupled region using string theory. The gravity/gauge, or anti-de Sitter/conformal field theory (AdS/CFT) correspondence provides a revolutionary method to tackle the problem of strongly coupled gauge theories. The string description of realistic QCD has not been successfully formulated yet. Many efforts are invested in searching for such a realistic description by using the "top-down" approach, i.e. by deriving holographic QCD from string theory, as well as by using the "bottom-up" approach, i.e. by examining possible holographic QCD models from experimental data.

In the "top-down" approach, many string induced models have been seriously studied recently by using the string/gauge duality. Different branes configurations, such as D3-D7, D4-D6 and D4-D8 etc. were considered to describe QCD effectively. In these models, people extensively studied confinement, spontaneous chiral breaking and the thermal effects in QCD. Mesons, baryons and glueballs specturm as well as their decay constants are also calculated and compared with experimental data. However, each model has its own advantage to explain some characters of QCD, but fails in others.

It is an essential and crucial point for the realistic holographic QCD model to reproduce Regge behavior. Regge behavior is a well-known feature of QCD, and it was the commanding evidence for suggesting the string-like structure of hadrons. A general empirical expression for Regge trajectories can be cast as $M^2 = a_{n}n + a_{S}S + b$, where n and S are the quantum number of high radial and spin excitations, respectively. The slope a_{n} and a_{S} have dimension GeV^2 and describe the mass square increase rate in radial excitations and spin excitations, respectively. In principle, a_{n} is not necessarily the same as a_{S}, though $a_{n}=a_{S}$ can be taken as a good approximation by fitting experimental data. The parameter b is the ground state mass square, and it is channel-dependent.

Currently, one of the most successful models derived from string theory is the Sakai-Sugimoto model, which can describe spontaneous chiral symmetry breaking naturally, but fails to generate the linear Regge behavior. On the other hand, in the "bottom-up" approach, many efforts have been paid to generate the linear Regge behavior for meson spectra and for baryon spectra. Karch, Katz, Son and Stephanov (KKSS) found that a z^2 dilaton field correction in the AdS_5 background leads to the linear behavior $M_{n}^2 \propto n$.

In our work, by combing the "top-down" method and "bottum-up" method, i.e. by matching Dp-Dq system in type II superstring theory with the Regge trajectories of meson spectra in the hidden local symmetry model with group $SU(2)_{L}\times SU(2)_{R}$, we investigated the possible background metric structure dual to realistic QCD. From "top-down" approach, we systematically studied the Dp-Dq system, by matching the linear Regge trajectories of vector and axial-vector meson spectra, we can know which Dp-Dq system is more close to the realistic holographic QCD model. From "bottom-up" approach, we explored the realization of the chiral symmetry breaking in the vacuum and asymptotic chiral symmetry restoration in high excitation states. Our results can shed some lights on the correct prescription of the string theory dual to the realistic QCD and is useful to construct a correct phenomenological holographic model.

The glueball spectrum has attracted much attention more than three decades. In recent years, there have been intense studies on scalar mesons and scalar glueballs and their mixing. Study particles like glueballs where the gauge field plays a more important dynamical role than in the standard hadrons, offers a good opportunity of understanding the nonperturbative aspects of QCD. The complexity of determing the glueball states lies in that gluonic bound states always mix with $\bar{q}q$ states. For example, one has to distinguish the lightest scalar glueball state among other scalar mesons observed in the energy range below 2GeV. Though the pseudoscalar, vector and axial-vector, and tensor mesons with light quarks have been reasonably well known in terms of their SU(3) classification and quark content, the scalar meson sector, on the other hand, is much less understood in this regard. There are 19 states which are more than twice the usual $\bar{q}q$ nonet as in other sectors.

Despite of extensive study from both experimental side and theoretical side, no conclusive answer has been obtained on scalar mesons and scalar glueballs. One possible scenario is: The lightest scalars σ, κ, f0, a0 below 1GeV make a full SU(3) flavor nonet. The inversion of the κ and f0 or a0 mass ordering, suggests that these mesons are not naive $\bar{q}q$ states, one natural explanation for this inverted mass spectrum is that these mesons are diquark and antidiquark bound states, or tetraquark states. Above 1GeV, the nonet $\bar{q}q$ mesons are made of an octet with largely unbroken SU(3) symmetry and a fairly good singlet which is f0(1370). The other left scalar meson f0(1710) is identified as an almost pure scalar glueball with a ∼ 10% mixture of $\bar{q}q$, which is supported from lattice calculation and experimental observation of the copious f0(1710) production in radiative J/ψ decays.

Recently, the discovery of the gravity/gauge duality, or anti-de Sitter/conformal field theory (AdS/CFT) correspondence provides a revolutionary method to tackle the problem of strongly coupled gauge theories. Many efforts have been invested in examining meson spectra, baryon spectra as well as in the glueball sector. It is widely expected that this new analytical approach can shed some light on our understanding of the nonperturbative aspects of QCD.

The string description of realistic QCD has not been successfully formulated yet. By using AdS/CFT correspondence to study non-conformal field theory like QCD, the usual way of breaking conformal symmetry is by introducing a hard infrared (IR) cut-off, i.e. the hard-wall AdS5 model or introducing a smooth cut-off through a dilaton background field, i.e. the soft-wall AdS5 model. One can extend the AdS/CFT correspondence to a more general case, and expect the realistic QCD is dual to a non-conformal Dp brane system, like the D4 − D8/$\bar{\text{D}}$8 system, i.e. the Sakai-Sugimoto model. We have investigated the general embedding Dp − Dq systems, where the Nc background Dp-brane describes the effects of pure QCD theory, while the Nf probe Dq-brane is to accommodate the fundamental flavors.

We investigated the scalar meson and glueball spectra in the general embedding Dp − Dq systems, and study which Dp − Dq system is more close to the dual theory of realistic QCD. Our finding is that in the D4 − D6 and D4 − D8 hard wall models, the predicted masses of the $\bar{q}q$ scalar meson f0 and the scalar glueball are consistent with their experimental or lattice results, which indicates that D4 − D6 and D4 − D8 hard-wall models are favorite candidates of the realistic holographic QCD model.

Phase structure of black holes

During the last few decades, black hole thermodynamics has been playing the role of a "thinking experiment" to understand quantum gravity. The discovery of Hawking radiation shows that the analogy between black hole mechanical laws and the laws of thermodynamics is physically meaningful. Based on this analogy, Davis pioneered to consider the phase transition of RN black holes. Hawking and Page later investigated the phase transition of Schwarzschild-AdS black holes. Following their path-breaking research, many works have been done along this direction and rich phase structures have been discovered. The later established AdS/CFT duality further inspired people to focus on the asymptotically anti-de Sitter (AdS) black holes. Critical phenomena were discovered in asymptotically AdS black holes.

In 1999, Chamblin et al. studied the phase structures of RN-AdS black hole. They identified a critical point in RN-AdS black hole by considering the divergence of heat capacity. Near this critical point, the behavior of isotherms are similar to that of van der Waals liquid/gas system. However, Wu showed that the critical exponents of RN-AdS black hole are different from that of the van der Waals case in 2000. A detailed investigation of the phase structure of Kerr-AdS black hole is needed to be compared to the previous results. Finding the critical phenomena will help us to achieve the ultimate goal of finding a microscopic description of the black hole phase structure.

The phase structure of Kerr-AdS black holes is also related to the holographic superconductors and their rotating extension. Sonner studied the superconducting phase transition on the boundary of Kerr-Newman-AdS black hole. The phase structure of the background field may affect some properties of the rotating holographic superconductor. And the knowledge of phase transitions of Kerr-AdS and RN-AdS black holes could be essential to fully understand the holographic superconductors.

We studied the phase structure of Kerr-AdS black hole. Rich phase structure were discovered at three diverse critical temperatures, and this multi-critical phenomenon in Kerr-AdS black hole has not been carefully discussed in the previous literatures. We plotted the isotherm to describe the three critical temperatures. We detailed discussed the critical behavior of each isotherm near the three critical temperature T_{L}, T_{c1} and T_{c2}, respectively. We determined the asymptotic value of the angular momentum, which is important to understand the thermal stability of the Kerr-AdS black hole. At a certain temperature T_{c1}, we discovered van der Waals-like phase transition. Unlike the case of RN-AdS black hole, the critical exponents of Kerr-AdS black hole are found identical to the van der Waals liquid/gas system and the Weiss ferromagnet. It provides a strong evidence that Kerr-AdS black hole system belongs to the universality class which contains these two systems. We also discuss the scaling symmetry of the free energy near this critical point.

String scattering amplitudes and symmetries in string theory

High energy, fixed angle limit of string scattering amplitudes had been used to probe the fundamental spacetime symmetry of string theory. In this approach, one needs to calculate infinite number of massive string scattering amplitudes. By taking high energy limit of the calculation, a lot of mathematical simplifications result and many interesting characteristics of high energy behavior of the theory can be obtained. There are two fundamental regimes of high energy string scattering amplitudes. These are the fixed angle regime or Gross regime (GR), and the fixed momentum transfer regime or Regge regime (RR). These two regimes represent two different high energy perturbation expansions of the scattering amplitudes, and contain complementary information of the theory. The high energy string scattering amplitudes in the GR were recently intensively reinvestigated for massive string states at arbitrary mass levels. See also the developments in. An infinite number of linear relations, or stringy symmetries, among string scattering amplitudes of different string states were obtained. Moreover, these linear relations can be solved for each fixed mass level, and ratios among the amplitudes with different spins can be obtained. An important new ingredient of these calculations is the decoupling of zero-norm states (ZNS) in the old covariant first quantized (OCFQ) string spectrum.

Another fundamental regime of high energy string scattering amplitudes is in the RR. See also. An interesting breakthrough of the subject was made in 2008 through the calculation of high energy string scattering amplitudes for arbitrary mass levels in the RR. It turns out that both the saddle-point method and the method of decoupling of high energy ZNS adopted in the calculation of GR do not apply to the case of RR. However, a direct calculation to get the complete form of the amplitudes is achievable and the general formula for the high energy scattering amplitudes for each fixed mass level in the RR can be written down explicitly. It was found that the number of high energy scattering amplitudes for each fixed mass level in the RR is much more numerous than that of GR calculated previously. In contrast to the case of scatterings in the GR, there is no linear relation among scatterings in the RR. Moreover, it was discovered that the leading order amplitudes at each fixed mass level in the RR can be expressed in terms of the Kummer function of the second kind. Furthermore, for those leading order high energy amplitudes in the RR with the same type of spin as those of GR, we can extract from them the ratios in the GR by using this Kummer function. Mathematically, the proof was based on a set of summation algorithm for Stirling number identities derived by Mkauers in 2007.

This new development of high energy behavior of string theory enables one to expressed the ratios of string theory in terms of Kummer function and thus may shed light on a deeper understanding of algebraic structure of stringy symmetries. Mathematically, the realization of the Stirling number identities by string theory brings an interesting bridge between string theory and combinatoric number theory. It is thus important to probe the structure of more high energy string scattering amplitudes in this context, and relate it to the Kummer function and more Stirling number identities.

We calculated four classes of high energy massive string scattering amplitudes of fermionic string theory in the Regge regime (RR). We show that, as in the case of bosonic string, the leading order amplitudes in the RR can be expressed in terms of the Kummer function of the second kind. Based on the summation algorithm of a set of extended Stirling number identities (among them, one remains to be proved mathematically), we showed that all four ratios calculated previously among scattering amplitudes in the Gross Regime (GR) can be extracted from this Kummer function in the RR. We point out that we failed to prove one of the Stirling number identity we used in the text. This identity is taken as an identity predicted by string theory. We also provided some numerical evidences in the appendix to support our prediction. Hopefully, someone can give a rigorous proof of it in the near future. Finally, we conjectured and gave evidences that the existence of these four GR ratios in the RR persists to all subleading orders in the Regge expansion of all four high energy string scattering amplitudes for the even mass level with $(N+1)=((M_{2}\,3)/2)=$ odd. For the even mass levels with $(N+1)=((M_{2}\,3)/2)=$ even, the existence of the GR ratios will be terminated and shows up only in the first $((N+1)/2)+1$ terms in the Regge expansion of the amplitudes.

Although the saddle-point method was developed to calculate the general formula for tree-level high-energy open string scattering amplitudes of four arbitrary string states, it was soon realized that the saddle-point method was applicable to (t,u) channel only but not (s,t) channel. It was also pointed out that, through the observation of the KLT formula, this difficulty is associated with the lack of saddle-point in the integration regime for the closed string calculation. To calculate the complete high energy closed string scattering amplitudes in the fixed angle regime, one relies on calculation based on the method of decoupling of zero-norm states in the spectrum. With this new input, the complete ratios among high energy closed string scattering amplitudes at each fixed mass level can be determined. One can now calculate only high energy amplitude corresponding to the highest spin state at each mass level in the spectrum, and the complete closed string scattering amplitudes can then be obtained.

We used another method to calculate the closed string ratios in the fixed angle regime mentioned above. We calculated the complete closed string scattering amplitudes in the Regge regime. It turned out that both the saddle-point method and the method of decoupling of zero-norm states adopted in the calculation of fixed angle regime do not apply to the case of Regge regime. However a direct calculation is manageable. The calculation will be based on the KLT formula and the open string (s,t) channel scattering amplitudes in the Regge regime calculated previously. By using a set of Stirling number identities developed in combinatoric number theory, one can then extract the ratios in the fixed angle regime from Regge closed string scattering amplitudes.

we calculate the general high-energy scattering amplitudes of arbitrary higher spin massive closed string states scattered from D-particle in the RR. We assumed that the mass of the D-particle is infinitely heavy and so does not recoil. In contrast to the case of scatterings in the GR, we found that there is no linear relation among string D-particle scatterings in the RR. However, as in the case of Regge string-string scattering amplitude calculation, we can extract the infinite fixed angle ratios of string D-particle scatterings from these Regge string D-particle scattering amplitudes by using a set of identities we proved recentl in [4].

The fixed angle ratios calculated by this indirect method from the Regge calculation are for the most general high-energy vertex rather than only a subset of ratios obtained directly from the fixed angle calculation previously. More importantly, we discovered that the amplitudes of closed string D-particle scatterings can not be factorized and thus are different from amplitudes for the high-energy closed string-string scattering calculated previously. Amplitudes for the high-energy closed string-string scattering can be factorized into two open string scattering amplitudes by using a calculation based on the KLT formula. Presumably, this non-factorization is due to the non-existence of a KLT-like formula for the string D-brane scattering amplitudes. There is no physical picture for open string D-particle tree scattering amplitudes and thus no factorizaion for closed string D-particle scatterings into two channels of open string D-particle scatterings. However, we discovered that in spite of the non-factorizability of the closed string D-particle scattering amplitudes, the complete ratios derived for the fixed angle regime are found to be factorized. These ratios are consistent with the decoupling of high-energy zero norm states calculated previously

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