行政院國家科學委員會專題研究計畫成果報告

多率訊號處理於轉化多工器上的應用
Multirate Signal Processing for Transmultiplexing Systems
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1 中文摘要

離散多調調變系統(DMT)或轉化多工器在高速資料傳輸的各類應用佔有重要地位,如區域網路資料傳輸.一般收發器設計均以DFT矩陣爲主.我們將證明,當次頻道的數目較少時最佳化系統的成效顯著比DFT收發器好.但是當次頻道的數目多時, DFT收發器爲漸進式最佳化系統.

關鍵詞：轉化多工器,離散多調調變系統(DMT),最佳化DMT,漸進式最佳化

Abstract. The DMT (discrete multitone modulation) technique has been widely applied to high speed data transmission. It is known that the DMT system with ideal filters can achieve within 8 to 9 dB of the channel capacity of ADSL. The DFT based DMT system is proposed as a practical DMT implementation but its optimality is never asserted. In this project we show that the DFT based DMT systems are asymptotically optimal although they are not optimal for finite number of channels. The DFT based DMT system and the DMT system with ideal filters achieve the same bound. However, for a modest number of channels the optimal transceiver can provide substantial gain over the DFT based system as will be demonstrated by examples.

Keywords: transmultiplexer, discrete multitone modulation (DMT), asymptotically optimal

2 緣由與目的

Recently there has been great interest in applying the discrete multitone modulation (DMT) technique to high speed data transmission over channels such as ADSL and HDSL [1]. Fig. 1 shows an M-channel DMT system over a channel $C(z)$ with additive noise $e(n)$. The channel is divided into M subchannels using the transmitting filters $F_k(z)$ and receiving filters $H_k(z)$. The input is parsed and coded as modulation symbols, e.g., QAM (quadrature amplitude modulation). With judicious power and bit allocation, DMT can provide significant gain over channels. In [2], Kalet shows that the DMT system with ideal filters can achieve within 8 to 9 dB of the channel capacity of ADSL.

In the widely used DFT based DMT system, the transmitting and receiving filters are DFT filters. For a given probability of error and transmission power, bits can be allocated among the subchannels to achieve maximum

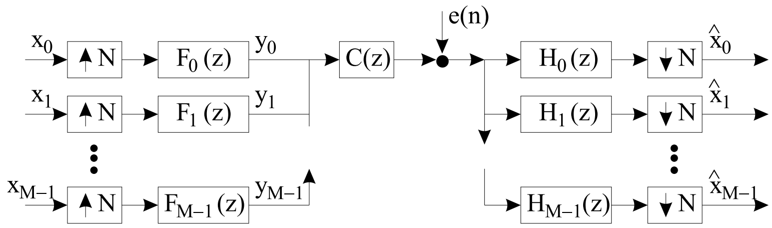

圖 1: An M-channel DMT system over channel $C(z)$.

total bit rate $R_{b,max}$. Very high speed data transmission can be achieved using DFT based DMT system at a relatively low cost [1]. In the DMT system the bit rate $R_{b,max}$ depends on the choice of the transmitting and receiving filters. The use of more general orthogonal transmitting filters instead of DFT filters is proposed in [3]. From the view point of multidimentional signal constellations it is shown that, for AWGN channels the optimal transmitting and receiving filters are eigen vectors associated with the channel. However in ADSL or HDSL applications, the channel noise is often the colored NEXT noise due to cross talk [2]. For channels with general colored noise source, the optimal transceiver is derived in [4].

The DFT based DMT system is proposed as a practical DMT implementation but its optimality is not asserted. In this project we show that the DFT based DMT systems are asymptotically optimal. The performance of the DFT based DMT systems becomes close to that of optimal DMT systems when the channel number M is sufficiently large. Furthermore the asymptotic performance of these two systems is the same as that of the DMT system with ideal filters in [2]. Although the DFT based DMT system is asymptotic optimal, the optimal transceiver provides significant gain over the DFT based system for a modest number of channels. An example with NEXT noise source will be given to demonstrate this.

3 結果與討論：

Consider the system model of an M-channel DMT transceiver over a channel $C(z)$ with additive noise $e(n)$ in Fig. 1. Suppose the channel $C(z)$ is an FIR filter with order L, which is a reasonable assumption after channel equalization. In practice, to cancel ISI (inter-symbol interference) some degree of redundancy is introduced and the interpolation ratio $N > M$. Usually we have $N = M + L$. The length of the transmitting and receiving filters is also N.

Polyphase representation. The DMT system can be redrawn as in Fig. 2 using polyphase decomposition. The transmitter $\mathbf{G}$ is an $N \times M$ constant matrix; the kth column of $\mathbf{G}$ contains the coefficients of the transmitting filter $F_k(z)$. The receiver $\mathbf{S}$ is an $M \times N$ constant matrix; the kth row of $\mathbf{S}$ contains the coefficients of the receiving filter $H_k(z)$. The matrix $\mathbf{C}(z)$ is an $N \times N$ pseudo circulant matrix with the first column given by

$$\begin{pmatrix} c_0 & c_1 & \cdots & c_L & 0 & \cdots & 0 \end{pmatrix}^T$$

where $\{c_n\}_{n=0}^{L}$ is the channel impulse response. The condition for zero ISI is $\mathbf{SC}(z)\mathbf{G} = \mathbf{I}$.

Using singular value decomposition, we can decompose $\mathbf{C}_0$ as,

$$\mathbf{C}_0 = \underbrace{[\mathbf{U}_0 \mathbf{U}_1]}_{\mathbf{U}} \begin{pmatrix} \mathbf{\Lambda} \\ \mathbf{0} \end{pmatrix}_{N\times M} \mathbf{V}^T = \mathbf{U}_0 \mathbf{\Lambda} \mathbf{V}^T, \quad (1)$$

where $\mathbf{U}$ and $\mathbf{V}$ are $N \times N$ and $M \times M$ unitary matrices. Consider the case that the trans-

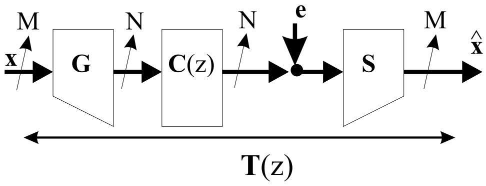

圖 2: The polyphase representation of the DMT system.

mitter is a unitary transformation followed by padding of L zeros, in particular $\mathbf{G} = \begin{pmatrix} \mathbf{G}_0 \\ \mathbf{0} \end{pmatrix}$, where $\mathbf{G}_0$ is an arbitrary $M \times M$ unitary matrix. For zero ISI, we can choose $\mathbf{S} = \mathbf{G}_0^T \mathbf{V} \mathbf{\Lambda}^{-1} \mathbf{U}_0^T$. When the transmitter is chosen as $\mathbf{G}_0 = \mathbf{V}^T$, the receiver is $\mathbf{S} = \mathbf{\Lambda}^{-1}\mathbf{U}_0^T$. This becomes the DMT system developed in [3].

Transmission Power For a given average bit rate R_b, the design of the transmitter and receiver affects the required transmission power. Let $\mathbf{R}_N$ be the $N \times N$ autocorrelation matrix of the channel noise process $e(n)$. The $M \times 1$ output noise vector of the receiver has autocorrelation function given by

$$\widehat{\mathbf{R}} = \mathbf{S}\mathbf{R}_N\mathbf{S}^T.$$

Let the number of bits allocated to the k-th channel be b_k, then the average bit rate is $R_b = \frac{1}{N}\sum_{k=0}^{M-1} b_k$. The actual bit rate is $\frac{1}{T}R_b$, where T is the sampling period of the system. Let $P(R_b, P_e, M)$ be the transmission power required for the M channel transceiver to achieve an average bit rate of R_b and probability of error P_e. With optimal bit allocation, the transmission power for the given transceiver is minimized and is equal to [4]

$$P(R_b, P_e, M) = c2^{2R_bN/M}(\Pi_{k=0}^{M-1}[\mathbf{S}\mathbf{R}_N\mathbf{S}^T]_{kk})^{1/M}, \quad (2)$$

where the constant c depends on the given probability of symbol error P_e and the modulation scheme.

In the DFT based DMT system, the receiver is $\mathbf{S} = \mathbf{\Gamma}^{-1}[\mathbf{0}\ \mathbf{W}]$. In this case we can verify that transmission power is

$$\begin{aligned} & P_{DFT}(R_b, P_e, M) \\ = \ & c2^{2R_bN/M}\frac{\left(\Pi_{k=0}^{M-1}[\mathbf{W}\mathbf{R}_M\mathbf{W}^\dagger]_{kk}\right)^{1/M}}{\det(\mathbf{\Gamma}^\dagger\mathbf{\Gamma})^{1/M}}. \end{aligned}$$

From (2) we see that the transmission power can be further minimized by optimizing the transceiver. Using the optimal transceiver, the minimum transmission power is [4]

$$P_{opt}(R_b, P_e, M) = c2^{2R_bN/M}\frac{\left(\det(\mathbf{U}_0^T\mathbf{R}_N\mathbf{U}_0)\right)^{1/M}}{\det(\mathbf{\Lambda}^2)^{1/M}}.$$

Asymptotic Performance We will show that the DFT based DMT systems are asymptotically optimal although they are not optimal for finite number of channels. For a given error probability and bit rate, we will show that the power required in DFT based DMT system approaches that of the optimal system for large M. In particular,

$$\begin{aligned} & \lim_{M\to\infty} P_{opt}(R_b, P_e, M) \\ = \ & \lim_{M\to\infty} P_{DFT}(R_b, P_e, M) \\ = \ & c2^{2R_b} exp\left(\int_{-\pi}^{\pi} \ln\frac{S_{ee}(e^{j\omega})}{|C(e^{j\omega})|^2}\frac{d\omega}{2\pi}\right). \quad (3) \end{aligned}$$

Note that this is the same bound achieved by the DMT system with ideal filters as derived in [2]. The proof can be done in two steps.

Step 1: Using the distribution of eigenvalues for Toeplitz matrices [5], we are able to show that

$$\begin{aligned} & \lim_{M\to\infty}\det(\mathbf{\Lambda}^2)^{1/M} = \lim_{M\to\infty}\det(\mathbf{\Gamma}^\dagger\mathbf{\Gamma})^{1/M} \\ & = exp\left(\int_{-\pi}^{\pi}\ln|C(e^{j\omega})|^2\frac{d\omega}{2\pi}\right), \quad (4) \end{aligned}$$

where $C(e^{j\omega})$ is the Fourier transform of c_n.

Step 2: Using properties of positive definite matrices, we can show that

$$\begin{aligned} & \lim_{M\to\infty}\left(\det(\mathbf{U}_0^T\mathbf{R}_N\mathbf{U}_0)\right)^{1/M} \\ = \ & exp\left(\int_{-\pi}^{\pi}\ln S_{ee}(e^{j\omega})\frac{d\omega}{2\pi}\right). \quad (5) \end{aligned}$$

On the other hand, properties of Toeplitz matrices give us [6],

$$\lim_{M\to\infty} \left(\Pi_{k=0}^{M-1}[\mathbf{W}\mathbf{R}_M\mathbf{W}^\dagger]_{kk}\right)^{1/M} = exp\left(\int_{-\pi}^{\pi} \ln S_{ee}(e^{j\omega})\frac{d\omega}{2\pi}\right). \qquad (6)$$

With (4)-(6), we can establish (3).

Example. Suppose the channel $C(z)$ is an FIR filter of order 1 and $C(z) = 1+0.5z^{-1}$. For the same probability of error and same bit rate, Fig. 3 shows $\frac{\mathcal{P}_{opt}(R_b,P_e,M)}{\mathcal{P}_{DFT}(R_b,P_e,M)}$, the ratio of power needed in optimal system over the power needed in the DFT-based system. We plot the ratio as a function of M for two different noise sources, the AWGN and NEXT noise source, which is colored channel noise due to cross talk [2]. From Fig. 3 we see that, for both noise sources the ratio $\frac{\mathcal{P}_{opt}(R_b,P_e,M)}{\mathcal{P}_{DFT}(R_b,P_e,M)}$ approaches unity as the channel number M increases. But for the NEXT noise channel, the ratio approaches unitary only for very large M. We can see that for a modest number of channel the optimal system provides substantial gain.

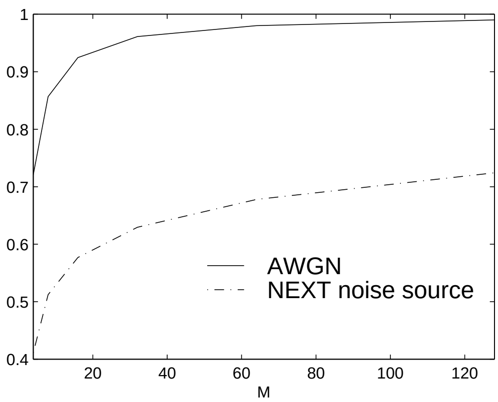

圖 3: The ratio of the power needed in the optimal DMT system over the power needed in DFT based system for the same probability of error and the same bit rate.

4 計畫成果自評：

In this project we show that the DFT based DMT systems are asymptotically optimal although they are not optimal for finite number of channels. The results show that for a modest number of subchannels the optimal transceiver can provide substantial gain over the DFT based system. For large number of subchannels, DFT based DMT systems are just as good.

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