行政院國家科學委員會補助專題研究計畫

□期中進度報告
■期末報告

多輸入多輸出放大傳遞合作式系統聯合前置編碼器設計
Joint Source/Relay Precoders Design in Amplify-and-Forward MIMO Relay Systems

計畫類別：■個別型計畫　□整合型計畫
計畫編號：NSC 98－2221－E－009－046－MY3
執行期間： 98 年 8 月 1 日至 101 年 7 月 31 日

執行機構及系所：國立交通大學電機系

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本計畫除繳交成果報告外，另須繳交以下出國報告：
□赴國外移地研究心得報告
□赴大陸地區移地研究心得報告
■出席國際學術會議心得報告及發表之論文
□國際合作研究計畫國外研究報告

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中 華 民 國 101 年 11 月 29 日

附件二

行政院國家科學委員會專題研究計畫成果報告

Robust MMSE Transceiver Design in Amplify-and-Forward MIMO Relay System with Tomlison-Harashima Source Precoding

計畫編號：NSC 98-2221-E-009-046
執行計畫：100年8月1日至101年7月31日
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中文摘要—我們探討在放大傳遞式多輸入多輸出中繼系統中，針對訊源端採用 **Tomlinson-Harashima** 前置編碼時，如何根據最小均方誤差的準則進行傳收機的設計。首先，我們考慮在訊源端額外使用一個么正前置編碼來簡化我們的設計問題。接著，藉由 primal decomposition 的技巧，我們可以將原本的聯合前置編碼器設計化簡為只需針對中繼端前置編碼器進行設計。然而此時問題求解依舊相當困難與複雜。因此，我們再透過將原本的目標函數取下界值來當作新的目標函數。如此可將問題轉化為一個 convex 最佳化的問題，進而求得解析解。透過模擬結果，我們可以發現我們所提出的方法可以有效地改善系統的效能。

關鍵詞 —放大傳遞、多輸入多輸出、**Tomlinson-Harashima** 前置編碼、最小均方誤差。

Abstract—**In this paper, we propose a robust transceiver design for an amplify-and-forward (AF) multiple-input multiple output (MIMO) relay systems where a Tomlinson-Harashima source precoder (THSP), a linear relay precoder, and a minimum-mean-squared-error (MMSE) receiver are taken into design. Since two precoders and imperfect channel state information (CSI) are involved, the transceiver design is much difficult. To overcome the difficulty, we first propose cascading an additional unitary precoder after THSP. The unitary precoder cannot only simplify the optimization but also improve the performance of MMSE receiver. We then adopt the primal decomposition transferring the original two-precoder optimization into a single relay precoder optimization. However, the problem is still unsolvable. By technically proposing a lower bounded objective function, we can further transfer the problem into a convex optimization. A closed-form solution can then be obtained by Karush-Kuhn-Tucker (KKT) conditions. Simulations show that the proposed transceiver can significantly outperform the existing linear transceivers with perfect or imperfect CSIs.**

Index Terms – **Amplify-and-forward (AF), multiple-input multiple-output (MIMO), Tomlinson-Harashima precoding (THP), minimum-mean-squared-error (MMSE).**

I. INTRODUCTION

Current research on amplify-and-forward (AF) multiple-input multiple-output (MIMO) cooperative networks, namely MIMO relay system, mainly focuses on linear transceiver designs, either for boosting capacity [1]-[2], or for improving link reliability [3]-[5]. Most of these proposals, however, only consider relay precoders [1]-[4]. Some of them even neglect the direct (or, source-to-destination) link so as to simplify the design [1], [3], [4]. Recently, a joint source/relay precoders design method was proposed in [5] and it is shown that the system performance can be significantly improved. However, the precoders considered in previous works are all linear. In this paper, we consider a nonlinear precoding scheme in which a Tomlinson-Harashima precoder (THP) is used at the source and a linear precoder at the relay, and an MMSE received is used in the destination. THP is a well known precoding scheme and has been shown to have a better performance than the linear ones [6], [7].

Since the MMSE is a complicated function of the source and relay precoders, the design is difficult. In addition, the problem is non-convex. Using the primal decomposition method [8], we first decompose the original optimization problem into a master and a subproblem optimization problems. We then propose an upper bound for the relay power constraint. Using the upper bound, the optimal THP precoder in the subproblem can be derived as a function of the relay precoder, and the problem can be reduced to the THP precoding design in the conventional MIMO system [7]. The solution is readily obtained. The cost function in the master problem then becomes a function of the relay precoder only. Due to the nonlinearity in the cost function, the optimum relay precoder is still difficult to find. We then propose some approximations, valid in high signal-to-noise (SNR) environments, to translate the master problem into a scalar-valued concave optimization problem. As a result, the closed-form solution of the relay precoder can be obtained by the KKT conditions. Simulations show that the proposed method can significantly outperform existing non-precoded and precoded systems, either in terms of MSE or bit-error-rate (BER).

II. SYSTEM MODEL

A. System Model

We consider a three-node AF MIMO relay precoding system in which N, R, and M antennas are placed at the source, the relay and the destination, respectively, as shown in Fig. 1. As we can see from the figure, two precoders are included – a THP source precoder and a linear relay precoder $\mathbf{F}_R$. Also, a linear MMSE receiver, **G,** is applied at the destination. Here, we consider the general two-phase transmission protocol [1]-[5]. In the first phase, the source signal $\mathbf{s} \in \mathbb{C}^{N\times 1}$ is fed into the nonlinear THP in which a successive cancellation operation characterized by a backward squared matrix **B** and a modulo operation MOD$_m$(•). The source signals $\mathbf{s} = [s_1, \cdots, s_N]^T$ are modulated by *m*-QAM where the real and image parts of s_k as the set $\{\pm 1, \cdots, \pm(m-1)\}$. The feedback matrix **B** has a lower triangular structure and the diagonal elements are all zeros. The modulo operation acts over the real and image parts of the inputs, respectively, is expressed as follows:

$$\mathrm{MOD}_m(x) = x - 2\sqrt{m} \cdot \left\lfloor \frac{x + \sqrt{m}}{2\sqrt{m}} \right\rfloor, \tag{1}$$

It is clear that the transmitted signal **x** is bounded between $-\sqrt{m}$ and $\sqrt{m}$. With **B** and the operation in (1), the elements of ***x*** can be recursively expressed as [7]

$$\mathbf{x}_k = \mathbf{s}_k - \sum_{l=1}^{k-1} \mathbf{B}(k,l)\mathbf{x}_l + \mathbf{e}_k, \tag{2}$$

where $\mathbf{x}_k$ is the *k*th elements of vector **x** and $\mathbf{B}(k,l)$ is the (*k*,*l*) element of matrix **B**; $\mathbf{e} = [\mathbf{e}_1, \cdots, \mathbf{e}_N]^T$ denotes the errors of the modulo operation (the difference of the input and the output). From (2), we can reformulate the transmitted signal **x** after THP with the following matrix form

$$\mathbf{x} = \mathbf{C}^{-1}\mathbf{v}, \tag{3}$$

where $\mathbf{C} = \mathbf{B} + \mathbf{I}_N$ is a lower triangular with ones in its diagonal, and $\mathbf{v} = \mathbf{s} + \mathbf{e}$. The THP precoded **x** is then passed through a prefiltering matrix $\mathbf{F}_S$ and subsequently sent to the relay and the destination simultaneously. The prefilter can provide an additional performance boost as described in conventional MIMO systems [7].

In the second phase, the received signal at the relay is multiplied the relay precoder and then is transmitted to the destination. Therefore, the signal received at the destination in the two consecutive phases can be expressed as a vector form as

$$\mathbf{y}_D := \underbrace{\begin{bmatrix} \mathbf{H}_{SD} \\ \mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR} \end{bmatrix}}_{:=\mathbf{H}} \mathbf{F}_S\mathbf{x} + \underbrace{\begin{bmatrix} \mathbf{n}_{D,1} \\ \mathbf{H}_{RD}\mathbf{F}_R\mathbf{n}_R + \mathbf{n}_{D,2} \end{bmatrix}}_{:=\mathbf{w}}. \tag{4}$$

where **H** and **w** denote the equivalent channel matrix and the equivalent noise vector, respectively. In (4), $\mathbf{x} \in \mathbb{C}^{N\times 1}$ is the THP precoded signal vector (3); $\mathbf{y}_D \in \mathbb{C}^{2M\times 1}$ is the received signal vector at the destination; $\mathbf{H}_{SR} \in \mathbb{C}^{R\times N}$, $\mathbf{H}_{SD} \in \mathbb{C}^{M\times N}$ and $\mathbf{H}_{RD} \in \mathbb{C}^{M\times R}$ are the channel matrices of the source-to-relay, the source-to-destination, and the relay-to-destination links, respectively; $\mathbf{n}_{D,1} \in \mathbb{C}^{M\times 1}$, $\mathbf{n}_R \in \mathbb{C}^{R\times 1}$, and $\mathbf{n}_{D,2} \in \mathbb{C}^{M\times 1}$ are the received noise vectors at the destination and at the relay in the first-phase, and at the destination in the second-phase. Here, we assume that $N \le \min\{R, M\}$ to provide sufficient degree of freedom for signal transmission.

Note that if **v** can be estimated at the destination, **s** can then be recovered by the modulo operation in (1). Thus, the optimum $\mathbf{G} \in \mathbb{C}^{2M\times N}$ can be found by minimizing the MSE defined as

$$J = E\left\{\left\|\mathbf{G}\mathbf{y}_D - \mathbf{v}\right\|^2\right\}. \tag{5}$$

To solve the problem in (5), we assume that the precoded signal $\mathbf{x}_k$'s are statistically independent and they have the zero-mean and the same variance. Let the variance of each element in s be denoted as σ_s^2. We then have $E\left[\mathbf{x}\mathbf{x}^H\right] = \sigma_s^2\mathbf{I}_N$ and $E\left[\mathbf{v}\mathbf{v}^H\right] = \sigma_s^2\mathbf{C}\mathbf{C}^H$. It is noted that the assumption is valid when the QAM size is large ($m \ge 16$) [7]. Then, the optimum solution of (5) can be obtained as [9]

$$\mathbf{G}_{opt} = \sigma_s^2\mathbf{C}\mathbf{F}_S^H\mathbf{H}^H\left(\sigma_s^2\mathbf{H}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}^H + \mathbf{R}_w\right)^{-1}, \tag{6}$$

where $\mathbf{G}_{opt}$ is the optimum **G**, $\mathbf{R}_w = E\left[\mathbf{w}\mathbf{w}^H\right]$ is the covariance matrix of the equivalent noise vector **w**. Note here that **w** is not white. Denote the variance of the noise components at the destination as $\sigma_{n,d}^2$, and that at the relay as $\sigma_{n,r}^2$. Substituting (6) in (5), we can have the MSE matrix

$$\begin{aligned}\mathbf{E} &= \mathbf{C}\left(\sigma_s^{-2}\mathbf{I}_N + \mathbf{F}_S^H\mathbf{H}^H\mathbf{R}_w^{-1}\mathbf{H}\mathbf{F}_S\right)^{-1}\mathbf{C}^H \\ &= \mathbf{C}\left(\sigma_s^{-2}\mathbf{I}_N + \mathbf{F}_S^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{F}_S\right)^{-1}\mathbf{C}^H\end{aligned} \tag{7}$$

and

$$J_{\min} = tr\{\mathbf{E}\}, \tag{8}$$

where

$$\begin{aligned}\tilde{\mathbf{H}} &= \mathbf{R}_w^{-1/2}\mathbf{H} \\ &= \begin{bmatrix} \sigma_{n,d}^{-1}\mathbf{H}_{SD} \\ \left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1/2}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR} \end{bmatrix}\end{aligned} \tag{9}$$

is defined as the equivalent channel matrix after noise whitening. Note that the MSE is contributed by both the direct and relay links. By ignoring the direct link and adopting a single precoder at the relay, the problem is reduced those considered in [3] and [4]. Here, we incorporate the THP as the source precoder and take the direct link into consideration. A significant performance enhancement can then be expected.

B. Problem Formulation

From the MMSE criterion in (7)-(8), we now can formulate our joint design problem as:

$$\min_{\mathbf{C},\mathbf{F}_S,\mathbf{F}_R} tr\left\{\underbrace{\mathbf{C}\left(\sigma_s^{-2}\mathbf{I}_N+\mathbf{F}_S^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{F}_S\right)^{-1}\mathbf{C}^H}_{:=\mathbf{E}}\right\} \quad s.t$$
$$\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}=\sigma_{n,d}^{-2}\mathbf{H}_{SD}^H\mathbf{H}_{SD}+$$
$$\mathbf{H}_{SR}^H\mathbf{F}_R^H\mathbf{H}_{RD}^H\left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H+\sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}.$$
$$tr\left\{E\left[\mathbf{F}_S\mathbf{x}\mathbf{x}^H\mathbf{F}_S^H\right]\right\}=\sigma_s^2 tr\left\{\mathbf{F}_S\mathbf{F}_S^H\right\}\le P_{S,T},$$
$$tr\left\{\mathbf{F}_R\left(\sigma_{n,r}^2\mathbf{I}_R+\sigma_s^2\mathbf{H}_{SR}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}_{SR}^H\right)\mathbf{F}_R^H\right\}\le P_{R,T} \tag{10}$$

where the inequalities in (10) indicate the transmitted power constraints at source and relay (the maximal available power is $P_{S,T}$ and $P_{R,T}$, respectively). Taking a close look at (10), we can observe that the cost function and the power constraints are nonlinear functions of $\mathbf{F}_S$ and $\mathbf{F}_R$. Moreover, (10) is not a convex optimization problem. As a result, it is difficult to solve the problem, directly. In the next section, we propose a new approach to seek for a suboptimal solution.

III. Joint Source/Relay Precoder Design

A. Proposed Approach

We resort to the primal decomposition method [8] translating (10) into a subproblem and a master problem. The subproblem is first optimized for the source precoder, and subsequently the master problem is optimized for the relay precoder. To proceed, we reformulate (10) as

$$\min_{\mathbf{C},\mathbf{F}_S,\mathbf{F}_R} tr\{\mathbf{E}\}=\min_{\mathbf{F}_R}\min_{\mathbf{C},\mathbf{F}_S} tr\{\mathbf{E}\}$$
$$s.t.$$
$$\mathbf{E}=\mathbf{C}\left(\sigma_s^{-2}\mathbf{I}_N+\mathbf{F}_S^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{F}_S\right)^{-1}\mathbf{C}^H$$
$$\tilde{\mathbf{H}}^H\tilde{\mathbf{H}} \text{ in } (10),$$
$$\sigma_s^2 tr\left\{\mathbf{F}_S\mathbf{F}_S^H\right\}\le P_{S,T}$$
$$tr\left\{\mathbf{F}_R\left(\sigma_{n,r}^2\mathbf{I}_R+\sigma_s^2\mathbf{H}_{SR}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}_{SR}^H\right)\mathbf{F}_R^H\right\}\le P_{R,T}. \tag{11}$$

In the subproblem, the relay precoder $\mathbf{F}_R$ is assumed to be given. Then, the optimum $\mathbf{C}$ and $\mathbf{F}_S$ can first be derived as a function of $\mathbf{F}_R$. Therefore, the joint precoders design is reduced to the master optimization problem in which the optimum relay precoder remains to be determined.

B. Proposed Subproblem Optimization

As we can see from (11), both the power constraints are functions of the source prefiltering $\mathbf{F}_S$. To facilitate the subproblem optimization, we first propose the following upper bound for the relay power constraint.

$$tr\left\{\mathbf{F}_R\left(\sigma_{n,r}^2\mathbf{I}_R+\sigma_s^2\mathbf{H}_{SR}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}_{SR}^H\right)\mathbf{F}_R^H\right\}$$
$$=\sigma_{n,r}^2 tr\left\{\mathbf{F}_R\mathbf{F}_R^H\right\}+\sigma_s^2 tr\left\{\mathbf{F}_R\mathbf{H}_{SR}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}_{SR}^H\mathbf{F}_R^H\right\}$$
$$\le\sigma_{n,r}^2\left\|\mathbf{F}_R\right\|_2^2+\sigma_s^2\left\|\mathbf{F}_R\mathbf{H}_{SR}\right\|_2^2\left\|\mathbf{F}_S\right\|_2^2 \tag{12}$$
$$\le\sigma_{n,r}^2\left\|\mathbf{F}_R\right\|_2^2+P_{S,T}\left\|\mathbf{F}_R\mathbf{H}_{SR}\right\|_2^2\le P_{R,T}, \tag{13}$$

where the inequality in (12) follows from the sub-multiplicative property of the matrix norm [10], and the inequality (13) follows from $\sigma_s^2 tr\left\{\mathbf{F}_S\mathbf{F}_S^H\right\}=\sigma_s^2\left\|\mathbf{F}_S\right\|_2^2\le P_{S,T}$ in (11).

Using the upper bound in (13), we see that the transmission power at the relay is not function of $\mathbf{F}_S$. Thus, it is not necessary to consider the relay power constraint in the derivation of $\mathbf{F}_S$. As a result, we can write the subproblem problem as

$$\min_{\mathbf{C}(\mathbf{F}_R),\mathbf{F}_S(\mathbf{F}_R)} tr(\mathbf{E})$$
$$s.t.$$
$$\mathbf{E}=\mathbf{C}\left(\sigma_s^{-2}\mathbf{I}_N+\mathbf{F}_S^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{F}_S\right)^{-1}\mathbf{C}^H. \tag{14}$$
$$\tilde{\mathbf{H}}^H\tilde{\mathbf{H}} \text{ in } (10),$$
$$\sigma_s^2 tr\left\{\mathbf{F}_S\mathbf{F}_S^H\right\}\le P_{S,T}$$

It is simple to show that (14) can be seen as the THP design in a conventional MIMO system. The solution of (14) has been considered in [7]. It has been shown that the solutions can be expressed as [7]:

$$\mathbf{C}_{opt}=\mathbf{D}\mathbf{L}^{-1}; \tag{15}$$
$$\mathbf{F}_{S,opt}=\mathbf{V}_{\tilde{\mathbf{H}}}\Theta\mathbf{U}_S, \tag{16}$$

where

$$\mathbf{L}\mathbf{L}^H=\left(\sigma_s^{-2}\mathbf{I}_N+\mathbf{F}_S^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{F}_S\right)^{-1}. \tag{17}$$

is the Cholesky factorization of $\left(\sigma_s^{-2}\mathbf{I}_N+\mathbf{F}_S^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{F}_S\right)^{-1}$; $\mathbf{D}$ is a diagonal matrix that scales the elements on the diagonal of $\mathbf{C}$ to unity; $\mathbf{V}_{\tilde{\mathbf{H}}}\in\mathbb{C}^{N\times N}$ is the left singular matrices of $\tilde{\mathbf{H}}$; $\Theta\in\mathbb{R}^{N\times N}$ is a diagonal matrix with the ith diagonal element θ_i and $\mathbf{U}_S\in\mathbb{C}^{N\times N}$ is an unitary matrix. Both $\Theta\in\mathbb{R}^{N\times N}$ and $\mathbf{U}_S\in\mathbb{C}^{N\times N}$ needs to be further specified. Substituting (16) into (17), we have

$$\mathbf{L}\mathbf{L}^H=\mathbf{U}_S^H\underbrace{\left(\sigma_s^{-2}\mathbf{I}_N+\Theta^H\Lambda\Theta\right)^{-1}}_{:=\tilde{\mathbf{D}}}\mathbf{U}_S. \tag{18}$$

where $\Lambda=daig\left\{\lambda_{\tilde{\mathbf{H}},1},\cdots,\lambda_{\tilde{\mathbf{H}},N}\right\}$ is the eigenvalues of $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$. It is simple to see that that $\tilde{\mathbf{D}}$ is diagonal matrix here. Applying geometric mean decomposition (GMD) on $\tilde{\mathbf{D}}$, we can express $\tilde{\mathbf{D}}$ as

$$\tilde{\mathbf{D}}^{1/2}=\mathbf{Q}\mathbf{R}\mathbf{P}^H, \tag{19}$$

where $\mathbf{Q}$, $\mathbf{P}$ are unitary matrix and $\mathbf{R}$ is upper triangular matrix with equal diagonal elements. Letting $\mathbf{U}_S$=$\mathbf{P}$ and substituting (15), (16) in (8), we then have the result MSE as

$$J_{\min} = \sum_{k=1}^{N} \mathbf{L}(k,k)^2 = N\prod_{k=1}^{N}\left(\frac{1}{\lambda_{\tilde{\mathbf{H}},k}\theta_k^2 + \sigma_s^{-2}}\right)^{1/N}. \quad (20)$$

Now, the problem becomes the minimization of (20).

By taking *ln* operation to (20) and neglecting *N* and $(\bullet)^{1/N}$, the subproblem optimization is then reformulated as

$$\begin{aligned}&\min_{p_{s,k}} \sum_{k=1}^{N} \ln\frac{1}{\lambda_{\tilde{\mathbf{H}},k}p_{s,k} + \sigma_s^{-2}}\\ &s.t.\\ &\tilde{\mathbf{H}}^H\tilde{\mathbf{H}} \ \text{ in } \ (10) \qquad\qquad (21)\\ &\sigma_s^2 tr\left(\mathbf{F}_S\mathbf{F}_S^H\right) = \sigma_s^2\sum_{k=1}^{N} p_{s,k} \le P_{S,T}\\ &p_{s,k} = \theta_k^2, \quad p_{s,k} \ge 0, \quad k = 1,\cdots,N,\end{aligned}$$

The problem is then a scalar-valued optimization problem and $p_{s,k}$, $k = 1,\cdots,N$ can be solved with a standard convex optimization method. The optimal solutions for $p_{s,k}$, $k = 1,\cdots,N$, derived from the KKT conditions [8], are given by

$$p_{s,k} = \left[v - \frac{1}{\sigma_s^2\lambda_{\tilde{\mathbf{H}},k}}\right]^+, \quad (22)$$

where $[y]^+ = \max(0,y)$ and v is the water-level chosen to satisfy the power constraint in (21). Observing (22), we can find that the source prefilter is a function of $\lambda_{\tilde{\mathbf{H}},k}$, $k = 1,\cdots,N$, which is the eigenvalues of $\tilde{\mathbf{H}}$. From (9), we see that $\tilde{\mathbf{H}}$ is a function of the relay precoder. Substituting (22) into (21), the cost function of the subproblem becomes that of the master problem from which we can derive the relay precoder. This will be described in the next subsection.

C. Proposed MasterProblem Optimization

Observing (15)-(17) and (22), we see that $\mathbf{C}$ and $\mathbf{F}_S$ are functions of $\mathbf{F}_R$. Also, these functions involve matrix inversion and eigen-decomposition; they are highly nonlinear. To solve such a problem is still difficult. In what follows, we propose some approximations such that a feasible solution for $\mathbf{F}_R$ can be solved in the master problem.

Firstly, we assume that the channel quality is good enough such that (22) can be approximated as

$$p_{s,k} = v - \frac{1}{\sigma_s^2\lambda_{\tilde{\mathbf{H}},k}}. \quad (23)$$

Thus, the water-level can be derived by the power constraint in (21) given as

$$v = \frac{P_{S,T} + \sum_{k=1}^{N}\frac{1}{\lambda_{\tilde{\mathbf{H}},k}}}{\sigma_s^2 N}. \quad (24)$$

Substituting (24) into (22), and then the cost function in (21), we then have the following upper bound

$$\begin{aligned}&\ln\prod_{k=1}^{N}\frac{1}{\lambda_{\tilde{\mathbf{H}},k}p_{s,k} + \sigma_s^{-2}} = \ln\prod_{k=1}^{N}\frac{1}{\lambda_{\tilde{\mathbf{H}},k}v} =\\ &\ln\frac{\sigma_s^2 N}{P_{S,T} + \sum_{k=1}^{L}\frac{1}{\lambda_{\tilde{\mathbf{H}},k}}}\prod_{k=1}^{N}\frac{1}{\lambda_{\tilde{\mathbf{H}},k}} \le \ln\frac{\sigma_s^2 N}{P_{S,T}}\prod_{k=1}^{N}\frac{1}{\lambda_{\tilde{\mathbf{H}},k}}.\end{aligned} \quad (25)$$

The upper bound in (25) can be achieved if $\lambda_{\tilde{\mathbf{H}},k} \to \infty$, for all k. We can further consider the following equivalence in the upper bound of (25):

$$\min_{\mathbf{F}_R} \ln\frac{\sigma_s^2 N}{P_{S,T}}\prod_{k=1}^{N}\frac{1}{\lambda_{\tilde{\mathbf{H}},k}} = \max_{\mathbf{F}_R} \det\left(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right), \quad (26)$$

where the equality is obtained form the property that $\det\left(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right) = \prod_{k=1}^{N}\lambda_{\tilde{\mathbf{H}},k}$.

With (26) and the power constraint (13), we can now formulate the master optimization problem as

$$\begin{aligned}&\max_{\mathbf{F}_R} \det\left(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right)\\ &s.t.\\ &\tilde{\mathbf{H}}^H\tilde{\mathbf{H}} \ \text{ in } \ (10) \qquad\qquad (27)\\ &\sigma_{n,r}^2\left\|\mathbf{F}_R\right\|_2^2 + P_{S,T}\left\|\mathbf{F}_R\mathbf{H}_{SR}\right\|_2^2 \le P_{R,T}.\end{aligned}$$

To solve (27), we use the Hardamard inequality, described in the following Lemma.

***Lemma 1* [*10*]**: Let $\mathbf{M} \in \mathbb{C}^{N\times N}$ be a positive definite matrix, then

$$\det(\mathbf{M}) \le \prod_{i=1}^{N}\mathbf{M}_{(i,i)}, \quad (28)$$

where $\mathbf{M}_{(i,i)}$ denotes the ith diagonal element of $\mathbf{M}$. The equality in (28) holds when $\mathbf{M}$ is a diagonal matrix. If we let $\mathbf{M} = \tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$, it turns out that when $\mathbf{M}$ is diagonalized, the cost function in (27) is maximized. Unfortunately, from (10) we can see that $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$ is a summation of two separated matrices and one of them dose not depend on $\mathbf{F}_R$, and the diagonalization cannot be directly conducted. The following lemma suggests a feasible way to overcome the problem.

***Lemma 2* [*10*]:** Let $\mathbf{A} \in \mathbb{C}^{N\times N}$ be a positive matrix and $\mathbf{B} \in \mathbb{C}^{N\times N}$, then

$$\det(\mathbf{A}+\mathbf{B}) = \det(\mathbf{A})\det\left(\mathbf{I}_N + \mathbf{A}^{-1/2}\mathbf{B}\mathbf{A}^{-1/2}\right). \quad (29)$$

Form (29), we let $\mathbf{B} = \mathbf{H}_{SR}^{H}\mathbf{F}_{R}^{H}\mathbf{H}_{RD}^{H} \times \left(\sigma_{n,r}^{2}\mathbf{H}_{RD}\mathbf{F}_{R}\mathbf{F}_{R}^{H}\mathbf{H}_{RD}^{H} + \sigma_{n,d}^{2}\mathbf{I}_{M}\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_{R}\mathbf{H}_{SR}$ and $\mathbf{A} = \sigma_{n,d}^{-2}\mathbf{H}_{SD}^{H}\mathbf{H}_{SD}$, we have the following equivalence

$$\begin{aligned}&\arg\max_{\mathbf{F}_R} \det\left(\tilde{\mathbf{H}}^{H}\tilde{\mathbf{H}}\right) \\ &= \arg\max_{\mathbf{F}_R} \det\Big(\mathbf{I}_{N} + \sigma_{n,d}^{2}\mathbf{H}_{SR}^{\prime H}\mathbf{F}_{R}^{H}\mathbf{H}_{RD}^{H} \\ &\left(\sigma_{n,r}^{2}\mathbf{H}_{RD}\mathbf{F}_{R}\mathbf{F}_{R}^{H}\mathbf{H}_{RD}^{H} + \sigma_{n,d}^{2}\mathbf{I}_{M}\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_{R}\mathbf{H}_{SR}^{\prime}\Big),\end{aligned} \tag{30}$$

where $\mathbf{H}_{SR}^{\prime} := \mathbf{H}_{SR}\left(\mathbf{H}_{SD}^{H}\mathbf{H}_{SD}\right)^{-\frac{1}{2}}$ and $\det(\mathbf{A})$ are ignored since they are not functions of $\mathbf{F}_{R}$. Equation (30) provides a feasible way to diagonalize the cost function. The optimization problem in (27) can now be reformulated as

$$\begin{aligned}&\max_{\mathbf{F}_R} \ \det\left(\mathbf{M}^{\prime}\right) \\ &s.t. \\ &\mathbf{M}^{\prime} = \Big(\mathbf{I}_{N} + \sigma_{n,d}^{2}\mathbf{H}_{SR}^{\prime H}\mathbf{F}_{R}^{H}\mathbf{H}_{RD}^{H} \\ &\left(\sigma_{n,r}^{2}\mathbf{H}_{RD}\mathbf{F}_{R}\mathbf{F}_{R}^{H}\mathbf{H}_{RD}^{H} + \sigma_{n,d}^{2}\mathbf{I}_{M}\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_{R}\mathbf{H}_{SR}^{\prime}\Big) \\ &\mathbf{M}^{\prime} \text{ is diagonal} \\ &\sigma_{n,r}^{2}\left\|\mathbf{F}_{R}\right\|_{2}^{2} + P_{S,T}\left\|\mathbf{F}_{R}\mathbf{H}_{SR}\right\|_{2}^{2} \leq P_{R,T}.\end{aligned} \tag{31}$$

There exists certain structure for the relay precoder such that the diagonalization can be achieved. Consider following singular value decomposition (SVD):

$$\mathbf{H}_{RD} = \mathbf{U}_{rd}\Sigma_{rd}\mathbf{V}_{rd}^{H}; \tag{32}$$

$$\mathbf{H}_{SR}^{\prime} = \mathbf{U}_{sr}^{\prime}\Sigma_{sr}^{\prime}\mathbf{V}_{sr}^{\prime H}, \tag{33}$$

where $\mathbf{U}_{rd} \in \mathbb{C}^{M\times M}$ and $\mathbf{U}_{sr}^{\prime} \in \mathbb{C}^{R\times R}$ are left singular matrices of $\mathbf{H}_{RD}$ and $\mathbf{H}_{SR}^{\prime}$, respectively; $\Sigma_{rd} \in \mathbb{R}^{M\times R}$ and $\Sigma_{sr}^{\prime} \in \mathbb{R}^{R\times N}$ are the diagonal singular value matrices of $\mathbf{H}_{RD}$ and $\mathbf{H}_{SR}^{\prime}$, respectively; $\mathbf{V}_{rd}^{H} \in \mathbb{C}^{R\times R}$ and $\mathbf{V}_{sr}^{\prime H} \in \mathbb{C}^{N\times N}$ are the right singular matrices of $\mathbf{H}_{RD}$ and $\mathbf{H}_{SR}^{\prime}$, respectively. We found that if the optimal $\mathbf{F}_{R}$ have the following structure, a full diagonalization of $\mathbf{M}^{\prime}$ can be achieved:

$$\mathbf{F}_{R,opt} = \mathbf{V}_{rd}\Sigma_{r}\mathbf{U}_{sr}^{\prime H}, \tag{34}$$

where Σ_{r} is a diagonal matrix with ith diagonal element $\sigma_{r,i}$, yet to be determined. Let $\sigma_{rd,i}$ and $\sigma_{sr,i}^{\prime}$ be the ith diagonal element of Σ_{rd} and $\Sigma_{sr}^{\prime}$, respectively. Substituting (32), (33) and (34) into (31) and taking the *ln* operation to the cost function, we can then rewrite (31) as

$$\begin{aligned}&\max_{p_{r,i},\ 1\leq i\leq N} \sum_{i=1}^{N}\ln\left(1 + \frac{p_{r,i}\sigma_{n,d}^{2}\sigma_{rd,i}^{2}\sigma_{sr,i}^{\prime 2}}{p_{r,i}\sigma_{n,r}^{2}\sigma_{rd,i}^{2} + \sigma_{n,d}^{2}}\right) \\ &s.t. \\ &\sum_{i=1}^{N} p_{r,i}\left(P_{S,T}\sigma_{sr,i}^{\prime 2}\mathbf{D}_{sr}^{\prime}(i,i) + \sigma_{n,r}^{2}\right) \leq P_{R,T}, p_{r,i} \geq 0,\end{aligned} \tag{35}$$

where $p_{r,i} = \sigma_{r,i}^{2}$ and $\mathbf{D}_{sr}^{\prime} = \mathbf{V}_{sr}^{\prime}\left(\mathbf{H}_{SD}^{H}\mathbf{H}_{SD}\right)\mathbf{V}_{sr}^{\prime H}$ with $\mathbf{D}_{sr}^{\prime}(i,i)$ being the ith diagonal element of $\mathbf{D}_{sr}^{\prime}$. The cost function now is simplified to a function of scalar parameters. Since the cost function and the inequalities are all concave for $p_{r,i} \geq 0$ [8], (35) is a standard concave optimization problem. As a result, the optimal solutions $p_{r,i}$, $i = 1,\cdots,N$, can be solved by means of KKT conditions given by

$$p_{r,i} = \left[\sqrt{\frac{\mu}{\sigma_{rd,i}^{2}\left(P_{S,T}\sigma_{sr,i}^{\prime 2}\mathbf{D}_{sr}^{\prime}(i,i) + \sigma_{n,r}^{2}\right)\left(\sigma_{n,r}^{2}\sigma_{n,d}^{-2}\sigma_{sr,i}^{\prime -2} + 1\right)} + \frac{\frac{1}{4}\frac{\sigma_{n,d}^{4}}{\sigma_{n,r}^{4}}}{\sigma_{rd,i}^{4}\left(\frac{\sigma_{n,r}^{2}}{\sigma_{rd,i}^{2}\sigma_{sr,i}^{\prime 2}} + 1\right)^{2}}} - \frac{1 + \frac{1}{2}\frac{\sigma_{n,d}^{2}\sigma_{sr,i}^{\prime 2}}{\sigma_{n,r}^{2}}}{\sigma_{rd,i}^{2}\left(\frac{\sigma_{n,r}^{2}}{\sigma_{n,d}^{2}} + \sigma_{sr,i}^{\prime 2}\right)}\right]^{+}, \tag{36}$$

where μ is chosen to satisfy the power constraint in (35). Substituting (36) into (34), we can finally obtain the optimum relay precoder. With the relay precoder, $\tilde{\mathbf{H}}$ in (9) can be obtained. Subsequently, the source prefilter can be derived by substituting (22) into (16) and $\mathbf{C}$ can be obtained by (15).

IV. SMULATIONS

We consider an AF MIMO relay system with $N=R=M=4$. The elements of each channel matrix are assumed to be i.i.d. complex Gaussian random variables with zero-mean and unity variance. Let SNR_{sr}, SNR_{rd}, and SNR_{sd} denote, respectively, the SNR per receive antenna of the source-to-relay, the relay-to-destination, and the source-to-destination links. Here, we let $SNR_{sr} = 15$ dB, $SNR_{rd} = 10$ dB and vary SNR_{sd}. Also, we use 16-QAM for each transmitted symbols. Fig. 2 and Fig. 3 show the MSE and BER performances comparison, respectively, for (a) an un-precoded system with the MMSE receiver, (b) an MIMO relay system with the optimum relay precoder in [3], (c) an MIMO relay system with the source/relay precoders in [5], and (d) an MIMO relay system with the proposed source/relay precoders. Note that optimum relay precoder in [3] only considers the relay link. For better performance, we further include the direct link when implementing the MMSE receiver. As we can see, the proposed method significantly outperforms other methods. Although two precoders are used in [5], the performance is limited. This is because both precoders are linear.

V. Conclusions

In this paper, we consider a precoding scheme in AF MIMO relay systems. In this scheme, a THP source precoder is used at the source, a linear relay precoder at the relay, and an MMSE receiver is used the destination. Since MSE is a complicated function of the source and relay precoders, a direct minimization is difficult. To solve the problem we propose to design the precoders using the primal decomposition method. Using this approach, the original problem can be formulated as a relay precoder design problem called the master problem, and a source precoder design problem called the subproblem. With some approximations and manipulations, both problems can be translated into standard scalar-valued convex optimization problems. Finally, using the KKT conditions, we obtain the closed-form solutions of the precoders. Simulations show the proposed method significantly outperforms the existing un-precoded and precoded systems.

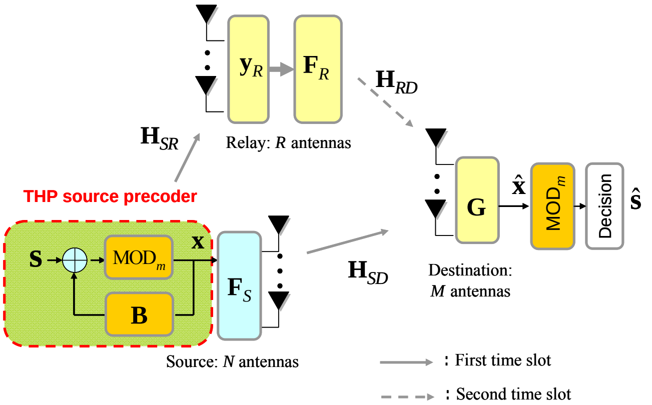

Figure 1. Three node AF MIMO relay system with THP source precoder and linear relay precoder.

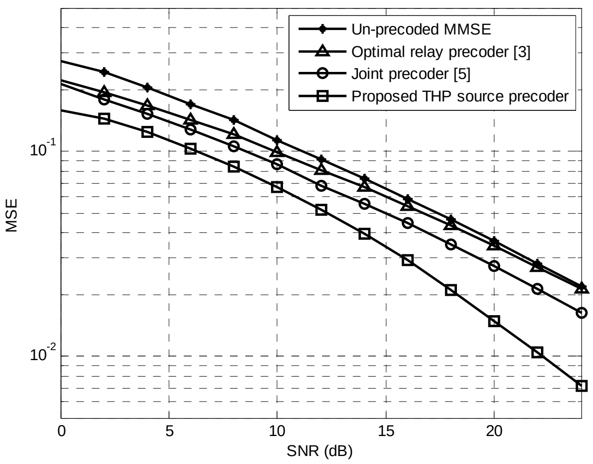

Figure 2. MSE performance comparison of the proposed precoders method and the other schemes.

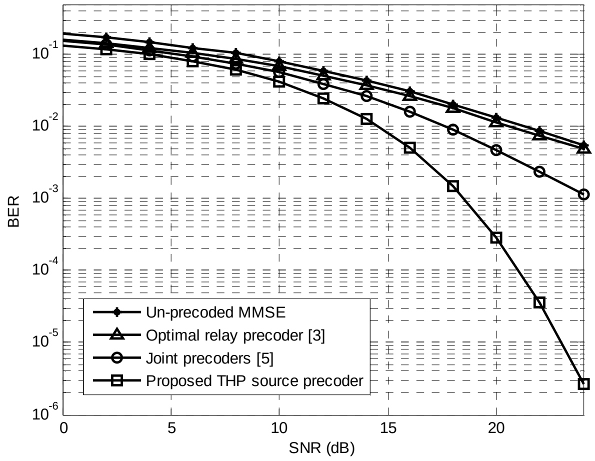

Figure 3. BER performance comparison of the proposed precoders method and the other schemes.

References

[1] X. Tang and Y. Hua, "Optimal design of non-regenerative MIMO wireless relays," *IEEE Trans. Wireless Communications*, vol. 6, no. 4, pp. 1398-1407, April 2007.

[2] O. Munoz-Medina, J. Vidal, and A. Agustin, "Linear transceiver design in nonregenerative relays with channel state information," *IEEE Trans. Signal Processing*, vol. 55, no. 6, pp. 2593-2604, June 2007.

[3] W. Guan and H. Luo, "Joint MMSE transceiver design in non-regenerative MIMO relay systems," *IEEE Trans. Communication Letters*, vol. 12, no. 7, pp. 517-519, July 2008.

[4] A. S. Behbahani, R. Merched, and A. M. Eltawl, "Optimzations of a MIMO relay networks," *IEEE Trans. Signal Processing*, vol. 56, no. 10, pp. 5062-5073, Oct. 2008.

[5] F. S. Tseng, W. R. Wu, and J. Y. Wu, "Joint source/relay precoder design in nonregenrative cooperative systems using an MMSE criterion," *IEEE Trans. Wireless Comm.*, to appear.

[6] R. F. H. Fischer, C. Windpassinger, A.Lampe, and J. H. Huber, "Space-time transmission using Tomlinson-Harashima precoding," in *Proc. 4th ITC Conf. on Source and Channel Coding*, Berlin, 2002.

[7] A. A. D'Amico and M. Morelli, "Joint Tx-Rx MMSE Design for MIMO Multicarrier Systems with Tomlinson-Harashima Precoding," *IEEE Trans. Wireless Communications*, vol. 7, no. 8, pp. 3118-3127, Aug. 2008.

[8] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press 2004.

[9] H. Sampath, P. Stoica, and A. Paulraj, "Generalized linear precoder and decoder design for MIMO channels using the weighted MMSE criterion," *IEEE Trans. Commun.*, vol. 49, no. 12, pp. 2198-2206, Dec. 2001.

[10] D.S. Bernstein, *Matrix Mathematics*, Princeton University Press 2005.

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估 ■ 達成目標 □ 未達成目標（請說明，以 100 字為限） □ 實驗失敗 □ 因故實驗中斷 □ 其他原因 說明：
2. 研究成果在學術期刊發表或申請專利等情形： 論文：■已發表 □未發表之文稿 □撰寫中 □無 專利：□已獲得 □申請中 □無 技轉：□已技轉 □洽談中 □無 其他：（以 100 字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限）

1. **學術成就**：本計畫之研究成果堪稱豐碩，目前已刊登的期刊論文以及會議論文共計七篇。其中，國際期刊如下：

[1] Fan-Shuo Tseng, Wen-Rong Wu, and Jwo-Yuh Wu, "Joint source/relay precoder design in nonregenerative cooperative systems using an MMSE criterion," *IEEE Trans. Wireless Commun*., vol. 8, no. 10, pp. 4928-4933, Oct. 2009.

[2] Fan-Shuo Tseng and Wen-Rong Wu, "Linear MMSE transceiver design in amplify-and-forward MIMO relay systems," *IEEE Trans. Vehicular Technology*, vol. 59, no. 2, pp. 754-765, Feb. 2010.

[3] Fan-Shuo Tseng and Wen-Rong Wu, “Nonlinear Transceiver Designs in MIMO Amplify-and-Forward Relay Systems”, *IEEE Trans. Vehicular Technology*, vol 60, pp. 528-538, Feb.2011.

[4] Fan-Shuo Tseng and Wen-Rong Wu, “Joint Tomlinson-Harashima source and linear relay precoder design in amplify-and-forward MIMO relay system via MMSE criterion,” *IEEE Trans. Vehicular Technology*, vol 60, pp. 11687-1698, May.2011

[5] Fan-Shuo Tseng and Wen-Rong Wu, “Robust Tomlinson-Harashima Source and Linear Relay Precoders Designs in Amplify-and-Forward MIMO Relay Systems”, *IEEE Trans. Commun.*, vol. 60, no.4, pp.1124-1137, Apr. 2012.

另有國際會議論文共三篇:

[1] Fan-Shuo Tseng and Wen-Rong Wu,“Joint source/relay precoders design in amplify-and-forward relay systems: A geometric mean decomposition approach,” in *Proc. IEEE ICASSP* 2009.

[2] Fan-Shuo Tseng, Wen-Rong Wu and Jwo-Yuh Wu,“Joint source/relay precoder design in amplify-and -forward relay systems using and MMSE criterion,” in *Proc. IEEE WCNC* 2009.

[3] Fan-Shuo Tseng, Ming-Yao Chang, and Wen-Rong Wu,“Robust MMSE transceiver design in amplify-and-forward MIMOrelay system with Tomlinson-Harashima source precoding,” in *Proc. IEEE WCNC* 2012.

2. **技術創新**：本計劃在第一年首先探討探討針對線性接收機(MMSE)提出其所對

應之聯合前置編碼。在第二年我們則是探討針對QR-SIC與MMSE-SIC非線性接收機中如何設計前置編碼器。同時，我們亦探討在放大傳遞式中繼系統訊源端採用非線性前置編碼器(THP)是否可進一步提昇效能的可行性。在第三年，我們則是考慮在固定接收端之QoS之下，如何設計聯合前置編碼。同時，我們也延伸第二年的成果，針對非線性訊源端前置編碼，發展出robust的聯合前置編碼演算法。這三年中的成果一共涵蓋多種不同形式的接收機以及前置編碼方式。在技術創新的部分，我們的研究課題在現有文獻均尚未被解決。因此，我們的研究成果極具創新價值。

3. **<u>學術價值</u>**：有別於現有文獻中之傳統系統模型，我們考慮當direct link存在時，如何利用最佳化的數學工具以及近似的技巧，進行前述各種不同類型傳收機所對應之前置編碼器。從引用率我們可以發現，我們所發表的期刊論文已被其他相關研究論文所引用。因此，我們的研究成果在此課題中具有領先的指標地位。此外，在我們的研究內容中，我們不斷地利用矩陣特性來大幅簡化原先的問題。此一技巧也提供其他研究者在面臨類似問題時有所參考。由於在未來的通訊系統中，中繼器的使用將逐漸扮演重要角色，我們所提出的演算法多具有（解析解）低複雜度的特性。因此，對於未來通訊技術的提升具有相當程度的影響。

4. **<u>未來學術發展之可能性</u>**：在我們的討論中，我們目前僅考慮一個中繼器的使用。然而，許多文獻已開始針對多個中繼器的通訊環境進行探討。再者，現實環境中多個中繼器的使用尚屬合理。因此，我們也將積極討論針對多個中繼器的環境如何設計前置編碼器。另外，本計畫中的非線性接收機僅著墨於連續干擾消除接收機。事實上，最大似然(ML)接收機在近幾年由於硬體技術成長，也逐漸在實作上採用。然而，針對ML接收機的前置編碼設計相較困難

許多，目前現有文獻均僅有次佳解，且效能與真正的最佳解的差距有待考驗。因此，我們將基於目前的成果，試圖探討如何將成果延伸至ML接收機中最佳的前置編碼器設計。