Research note

Testing process precision for truncated normal distributions

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Abstract

Process precision index C_p has been widely used in the manufacturing industry to provide numerical measures of process precision, which essentially reflects product quality consistency. Precision measures using C_p for normal processes, contaminated normal processes, have been investigated extensively, but are neglected for truncated normal processes. Truncated normal processes are common in the manufacturing industry, particularly, for factories equipped with automatic machines handling fully inspections, and scrap/rework products falling outside the specification limits. If the processes follow the normal distribution, then the inspected products (processes) must follow the truncated normal distribution. In this note, we consider the precision measure for truncated normal processes. We investigate the analytically intractable sampling distribution of the estimated C_p, and obtain a rather accurate approximation. Using the results, we develop a practical testing procedure for practitioners to use in their in-plant applications.
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1. Introduction

Process capability indices, which establish the relationships between the actual process performance and the manufacturing specifications, have been the focus of recent research in quality assurance and process capability analysis. The precision index C_p is the first process capability index appeared in the literature, which is defined in Kane [9] as:

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma},$$

where USL and LSL are the upper and lower specification limits, and σ is the process standard deviation. The index C_p was designed to measure the magnitude of the overall process variation relative to the manufacturing tolerance, which is used for controlled normal processes. Clearly, the index measures the manufacturing precision, which reflects product quality consistency (uniformity), an important criterion for judging manufacturing quality. A small value of C_p implies that the product quality is not consistent causing complaints from the customers not only damaging marketing potentials but also more cost for repair.

The use of the capability indices was first explored within the automotive industry. Ford Motor Company [7] has used C_p to keep track of the process performance and to reduce process variation. Recently, the manufacturing industries have been making an extensive effort to implement statistical process control in their plants and supply bases. Capability indices have received increasing usage not only in capability assessments, but also in the evaluation of purchasing decisions, which are becoming the standard tools for quality report. Proper understanding and use of them are essential for the company to maintain capable product supplies. Process precision measures using C_p for normal or contaminated normal processes based on one single, multiple, (X, R), or (X, S) control chart samples, have been investigated extensively. Examples include Kane [9], Cheng and Spiring [2], Chou and Owen [3], Kirmani et al. [10], Kocherlakota [11], Pearn et al. [13], Pearn et al. [14], Pearn and Wu [15], Pearn et al. [16], and Pearn and Chang [12]. But, research in precision measure using C_p has been neglected for truncated normal processes.

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Truncated normal processes are common in the manufacturing industry, particularly, for manufacturing factories equipped with automated inspection systems handling fully inspections. Products falling outside the manufacturing specification limits must be scrapped or reworked. If the manufacturing process follows the normal distributions, then the inspected products (process) must follow the truncated normal distributions. Clearly, product uniformity would be the primary concern rather than the process yield.

2. Truncated processes

In the manufacturing industry, particularly, for those making electronics accessories such as resistors, inductances, capacitances, smart fuses, electronic system protection components, the factories often set the manufacturing specifications more stringent than what the customers have requested to ensure product reliability and safety. In those cases, automatic inspection machines are often equipped to handle fully inspections. Products falling outside the manufacturing specification limits are scrapped or reworked. In other cases, the factories may have several customers requesting different specifications. Products must be sorted according to the customer's specifications. The factories only ship products to the customer that is within each customer's specifications. If the manufacturing process follows the normal distribution, then the inspected products must follow the truncated normal distribution. Figs. 1 and 2 depict a typical normal and the associated truncated processes.

Due to the truncation, the usual normality characteristic no longer exists for the data, even if the original process is normal, creating a problem when the customer attempts to
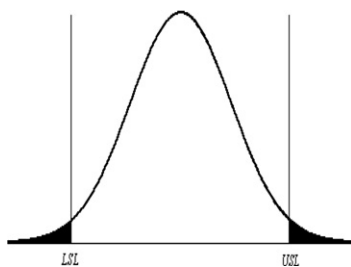

Fig. 1. A normal process.

Fig. 2. A truncated normal process.

assess process precision using the defined precision formula with the truncated data. A popular approach for handling the non-normal distribution is to first transform the non-normal data into normal one then apply the existing methods to the transformed data using the z-scores. Existing transformation techniques include Box–Cox power transformation [1], Johnson transformation [8], and Gilchrist quantile transformation [5]. Chou et al. [4] applied the Johnson transformation to convert the truncated normal data into the normal one to measure process capability. Those transformations provide easy ways to deal with non-normal distributions, but lack of theoretical justification. Further, it is difficult to interpret the results since the transformed data may have lost the important characteristics of the original distribution.

3. Distribution of the estimated C_p for truncated processes

For the truncated normal processes, we consider the following natural estimator, where $\widehat{\sigma}^2$ (estimated variance) is defined to be $\sum_{i=1}^{n}(X_i - \overline{X})^2/(n-1)$

$$\widehat{C}_p = \frac{\text{USL} - \text{LSL}}{6\widehat{\sigma}}.$$

If we denote the truncated normal distribution truncated at (a, b), $a < x < b$, as $N(\mu, \varphi^2)|_a^b$, then the probability density function of $N(\mu, \varphi^2)|_a^b$ can be expressed below, where Φ is the cumulated distribution of $N(0,1)$. Exact probability density and the distribution functions of $\widehat{C}_p$ are difficult to find. We will apply the Edgeworth expansion technique [6] to derive a rather accurate approximation

$$f(x) = \frac{1}{\sqrt{2\pi\varphi^2}} \exp\left(-\frac{1}{2\varphi^2}(x-\mu)^2\right) \Big/ \left(\Phi\left(\frac{b-\mu}{\varphi}\right) - \Phi\left(\frac{a-\mu}{\varphi}\right)\right).$$

3.1. Edgeworth approximation

For a random variable Y with zero mean and unit variance, the second-order Edgeworth approximation of the probability density function and the cumulative distribution function of Y, denoted f and F, respectively, can be written as

$$f(y) = \phi(y)\left\{1 + \frac{1}{6}\rho_3 H_3(y) + \frac{1}{24}\rho_4 H_4(y) + \frac{1}{72}\rho_3^2 H_6(y)\right\}, \tag{1}$$

$$F(y) = \Phi(y) - \phi(y)\left\{\frac{1}{6}\rho_3 H_2(y) + \frac{1}{24}\rho_4 H_3(y) + \frac{1}{72}\rho_3^2 H_5(y)\right\}, \tag{2}$$

where ϕ and Φ stand for the probability density function and the cumulative distribution function of $N(0, 1)$, and H_2, H_3, H_4, H_5 and H_6 are the Hermite polynomials

defined below, where $\rho_3 = E(Y^3)$ and $\rho_4 = E(Y^4) - 3$ are the 3rd and 4th order moments of Y, respectively.

$$H_2(y) = y^2 - 1, \quad H_3(y) = y^3 - 3y, \quad H_4(y) = y^4 - 6y^2 + 3,$$
$$H_5(y) = y^5 - 10y^3 + 15y, \quad H_6(y) = y^6 - 15y^4 + 45y^2 - 15.$$

Let $(X_1, X_2, \ldots, X_n)$ be a random sample from $N(0,1)|_a^b$. The second-order Edgeworth approximations of the probability density function and the cumulative distribution function of $\widehat{C}_p$ can be obtained as follows: (i) find the first eight moments of X distributed as $N(0,1)|_a^b$, (ii) let $U = \sum_{i=1}^n (X_i - \overline{X})^2$ and find the first four moments of U, denoted μ_i, $i = 1, \ldots, 4$, (iii) let $Y = (U - \mu_1)/\sigma_1$ where σ_1^2 is the variance of U, and find the parameters ρ_3 and ρ_4 in Eqs. (1) and (2), (iv) find $f_Y(y)$ and $F_Y(y)$ in Eqs. (1) and (2). Since $\widehat{C}_p = (USL - LSL)/6\widehat{\sigma} = \sqrt{n-1}(USL - LSL)/(6\sqrt{Y\sigma_1 + \mu_1})$, then the approximate cumulative distribution function and probability density function of $\widehat{C}_p$ are of the following forms, for $w \geqslant 0$,

$$F_{\widehat{C}_p}(w) = \text{prob}(\widehat{C}_p \leqslant w)$$

$$= \text{prob}\left(\frac{\sqrt{n-1}(USL - LSL)}{6\sqrt{Y\sigma_1 + \mu_1}} < w\right)$$

$$= \text{prob}\left(\left(\frac{(n-1)(USL - LSL)^2}{36w^2} - \mu_1\right)\bigg/\sigma_1 < Y\right)$$

$$= 1 - F_Y\left(\left(\frac{(n-1)(USL - LSL)^2}{36w^2} - \mu_1\right)\bigg/\sigma_1\right), \tag{3}$$

$$f_{\widehat{C}_p}(w) = f_Y\left(\left(\frac{(n-1)(USL - LSL)^2}{36w^2} - \mu_1\right)\bigg/\sigma_1\right)$$

$$* \left(\left(\frac{(n-1)(USL - LSL)^2}{18w^3}\right)\bigg/\sigma_1\right). \tag{4}$$

Therefore,

$$F_{\widehat{C}_p}(w) = 1 - \left(\Phi(r) - \phi(r)\left\{\frac{1}{6}\rho_3 H_2(r) + \frac{1}{24}\rho_4 H_3(r) + \frac{1}{72}\rho_3^2 H_5(r)\right\}\right),$$

$$f_{\widehat{C}_p}(w) = \phi(r)\left\{1 + \frac{1}{6}\rho_3 H_3(r) + \frac{1}{24}\rho_4 H_4(r) + \frac{1}{72}\rho_3^2 H_6(r)\right\} * r,$$

where $r = \left(\frac{(n-1)(USL - LSL)^2}{36w^2} - \mu_1\right)\bigg/\sigma_1$.

For random sample $(X_1, X_2, \ldots, X_n)$ taken from $N(\mu, \varphi^2)|_a^b$ with upper specification limit USL and lower specification limit LSL, we have the following propositions.

Propositions

(1) Let $(Z_1, Z_2, \ldots, Z_n)$ be a random sample from $N(0,1)|_c^d$ with $c = (a - \mu)/\varphi$ and $d = (b - \mu)/\varphi$. Also, let $U = \sum_{i=1}^n (X_i - \overline{X})^2$, $V = \sum_{i=1}^n (Z_i - \overline{Z})^2$, $Y = (U - \mu_1)/\sigma_1$ and $Y^* = (V - \mu_2)/\sigma_2$ where $\mu_1 = E(U)$, $\sigma_1^2 = \text{Var}(U)$, $\mu_2 = E(V)$ and $\sigma_2^2 = \text{Var}(V)$. Then we have $\mu_1 = \varphi^2\mu_2$, $\sigma_1 = \varphi^2\sigma_2$ and the distributions of Y and Y^* are the same.

(2) Let $\overline{F}_{\widehat{C}_p}(w)$ and $\overline{f}_{\widehat{C}_p}(w)$ be the second-order Edgeworth approximate cumulative distribution function and probability density function of $\widehat{C}_p$ obtained in Eqs. (3) and (4) for the random sample from $N(0,1)t|_{(a-\mu)/\varphi}^{(b-\mu)/\varphi}$ with upper specification limit $\overline{USL} = (USL - \mu)/\varphi$ and lower specification limit $\overline{LSL} = (LSL - \mu)/\varphi$. Then, the second-order Edgeworth approximate cumulative distribution function and probability density function of $\widehat{C}_p$ are $F_{\widehat{C}_p}(w) = \overline{F}_{\widehat{C}_p}(w)$ and $f_{\widehat{C}_p}(w) = \overline{f}_{\widehat{C}_p}(w)$, respectively:

$$F_{\widehat{C}_p}(w) = 1 - \left(\Phi(r) - \phi(r)\left\{\frac{1}{6}\rho_3 H_2(r)\right.\right.$$
$$\left.\left. + \frac{1}{24}\rho_4 H_3(r) + \frac{1}{72}\rho_3^2 H_5(r)\right\}\right),$$

$$f_{\widehat{C}_p}(w) = \phi(r)\left\{1 + \frac{1}{6}\rho_3 H_3(r) + \frac{1}{24}\rho_4 H_4(r) + \frac{1}{72}\rho_3^2 H_6(r)\right\} * r,$$

where $r = \left(\frac{(n-1)(USL - LSL)^2}{36w^2\sigma^2} - \mu_1\right)\bigg/\sigma_1$.
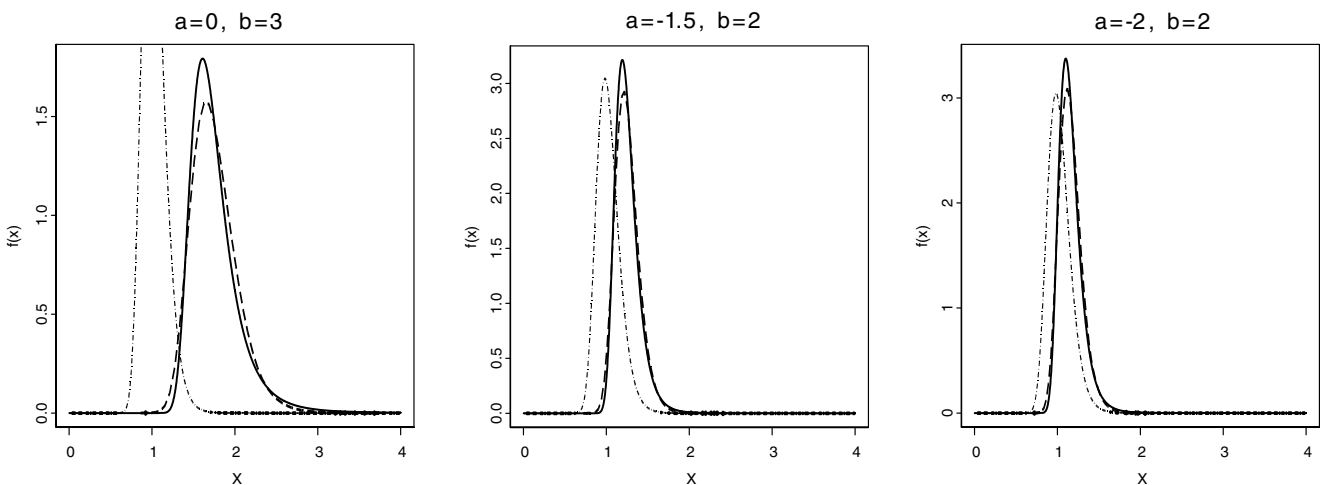

Fig. 3. Pdf plots for $(a, b) = (0, 3), (1.5, 2), (2, 2)$, from left to right, with $n = 30$.
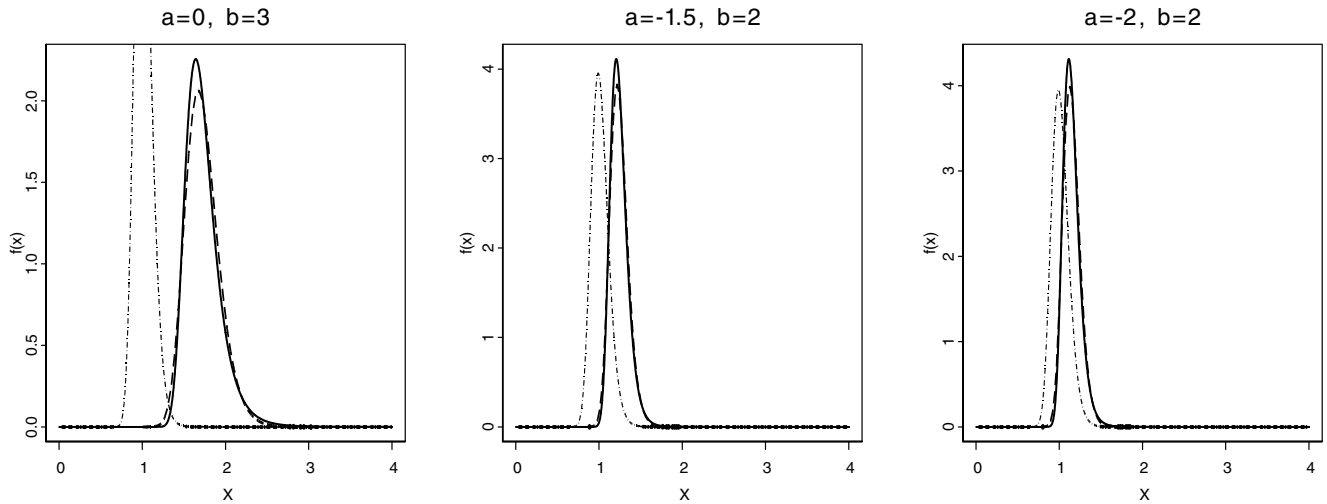
Fig. 4. Pdf plots for $(a, b) = (0, 3)$, $(1.5, 2)$, $(2, 2)$, from left to right, with $n = 50$.

4. Accuracy of the approximation

To investigate the accuracy of the proposed Edgeworth approximation, we perform some simulation study and compare our Edgeworth approximation with the simulated exact distribution. The comparisons are shown in Figs. 3 and 4, which include the plots of two probability density function curves with sample sizes $n = 30$, 50, and the plot of the χ^2-approximation to the probability density function for reference, where the curve in (___) is the Edgeworth approximation, the curve in (- -) is the simulated exact distributions, and the curve in ($-\cdots$) is the χ^2-approximation based on 10^6 replications,. The χ^2-approximation is obtained from treating the truncated $N(0,1)|_a^b$ as $N(0,1)$. It can be seen that as the sample size n reaches 50, the Edgeworth approximation and the simulated exact distributions are almost indistinguishable. In fact, even with $n = 30$ the Edgeworth approximation is quite satisfactory for practical purposes.

Fig. 6. Plots of c_0 versus μ for $C_p = 1.25$, $\alpha = 0.05$, $n = 30$, 50, 70, 100, 200, 300 (top to bottom in plot).

Fig. 5. Plots of c_0 versus μ for $C_p = 1.00$, $\alpha = 0.05$, $n = 30$, 50, 70, 100, 200, 300 (top to bottom in plot).
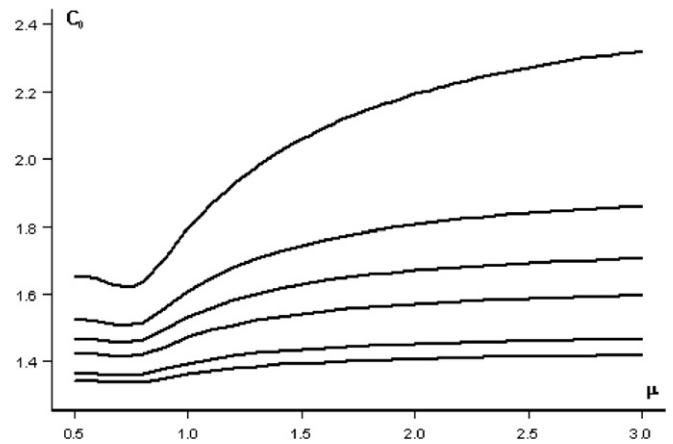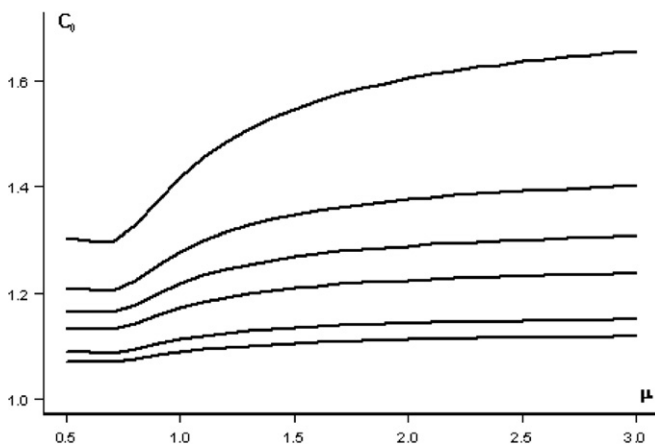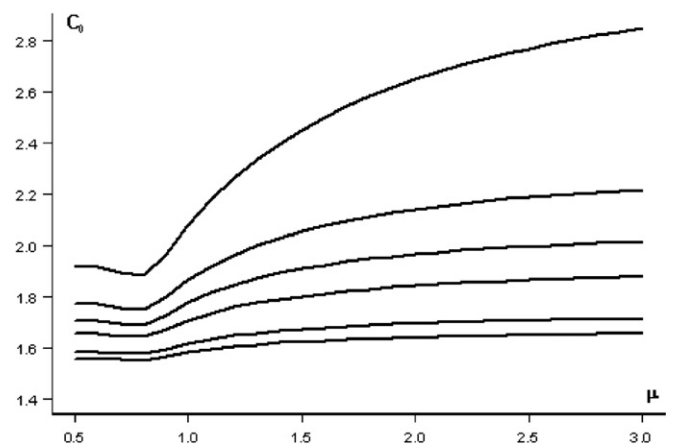

Fig. 7. Plots of c_0 versus μ for $C_p = 1.45$, $\alpha = 0.05$, $n = 30$, 50, 70, 100, 200, 300 (top to bottom in plot).
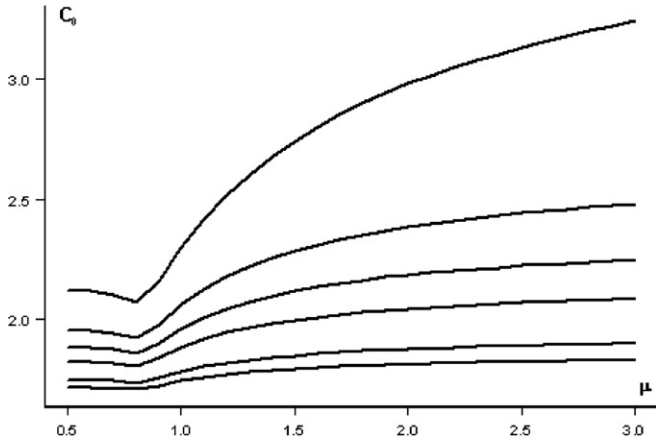
Fig. 8. Plots of c_0 versus μ for $C_p = 1.6$, $\alpha = 0.05$, $n = 30$, 50, 70, 100, 200, 300 (top to bottom in plot).

5. Precision testing with critical values

It should be noted that the Edgeworth approximation involves the unknown parameter μ, which must be estimated in real applications. The estimation of μ certainly introduces additional sampling errors while calculating the critical values for capability testing purpose, which makes our approach less reliable. To examine the behavior of the Edgeworth approximation against the parameter μ, we calculate the critical values for various values of μ. The results are displayed in Figs. 5–8. For process following $N(\mu, \varphi^2)|_0^1$, Figs. 5–8 plot c_0 against μ for $n = 30$, 50, 70, 100, 200, 300 (top–bottom), and $C = 1.0$, 1.25, 1.45, 1.6 with $\alpha = 0.05$. The results are consistent for all α we examined. The results show that the critical values increase as

the process mean μ increases, but converges to a constant as μ increases to infinity.

For a fixed C_p, we can show that as μ goes to infinity, the truncated distribution converges to the one with the following probability density function: It is not difficult to see by mathematical calculation that the limiting distribution

Table 1
Critical values for $C_p = 1.0$, 1.25 with $n = 30(5)250$

n	(C_p, α)					
	(1.0, 0.01)	(1.0, 0.025)	(1.0, 0.05)	(1.25, 0.01)	(1.25, 0.025)	(1.25, 0.05)
30	2.794517	1.760149	1.626345	4.316283	3.16443	2.580338
40	2.36794	1.695656	1.544734	3.649149	2.718395	2.149509
50	1.939296	1.618963	1.44188	3.090662	2.300438	1.955648
60	1.739607	1.514651	1.378022	2.567752	2.091684	1.842099
70	1.621796	1.446652	1.334188	2.304392	1.963231	1.766068
80	1.541955	1.39775	1.301488	2.143279	1.875859	1.711496
90	1.484196	1.360873	1.27625	2.03464	1.812773	1.670765
100	1.440103	1.331843	1.256012	1.953766	1.763693	1.638086
110	1.404932	1.308206	1.239234	1.890281	1.723718	1.610926
120	1.376705	1.288818	1.225325	1.841312	1.692069	1.589104
130	1.353181	1.27245	1.213478	1.798154	1.663592	1.569188
140	1.332651	1.257978	1.202907	1.763002	1.639908	1.552421
150	1.315972	1.246108	1.194168	1.735264	1.620952	1.538862
160	1.300378	1.234907	1.185857	1.710594	1.603974	1.526613
170	1.287472	1.225556	1.178895	1.689546	1.589249	1.515966
180	1.275465	1.216804	1.172343	1.668838	1.574641	1.505294
190	1.264806	1.208968	1.166471	1.653491	1.563791	1.497254
200	1.255798	1.202312	1.161419	1.637769	1.552555	1.488986
210	1.246387	1.195365	1.156154	1.620875	1.540307	1.479918
220	1.238516	1.189506	1.151701	1.610325	1.532668	1.474234
230	1.231934	1.184568	1.147925	1.598137	1.523803	1.467589
240	1.22506	1.179396	1.143997	1.587806	1.51628	1.461932
250	1.218624	1.174554	1.140269	1.578308	1.509238	1.456618

Table 2
Critical values for $C_p = 1.45$, 1.6 with $n = 30(5)250$

n	(C_p, α)					
	(1.45, 0.01)	(1.45, 0.025)	(1.45, 0.05)	(1.6, 0.01)	(1.6, 0.025)	(1.6, 0.05)
30	8.572409	5.343088	3.294039	11.446713	6.224281	3.821585
40	6.363363	3.536507	2.639193	8.38591	4.129412	2.987645
50	4.222455	2.860835	2.363412	5.095149	3.26171	2.65657
60	3.283793	2.556249	2.208911	3.781049	2.881282	2.470017
70	2.874418	2.377558	2.108354	3.274906	2.671544	2.354434
80	2.633257	2.256377	2.03555	2.975242	2.526123	2.268466
90	2.474914	2.169447	1.981041	2.787863	2.425927	2.206393
100	2.362937	2.104317	1.938902	2.648507	2.346255	2.155433
110	2.276493	2.051991	1.904171	2.54817	2.286325	2.115986
120	2.212996	2.012272	1.877286	2.469746	2.237993	2.083496
130	2.151299	1.972697	1.850139	2.404124	2.196408	2.055109
140	2.106501	1.943379	1.829735	2.35162	2.162315	2.031463
150	2.068041	1.91761	1.811594	2.299781	2.128026	2.007449
160	2.034336	1.894918	1.795439	2.265739	2.105156	1.991337
170	2.006867	1.876097	1.781945	2.237792	2.086106	1.977691
180	1.984408	1.860485	1.770746	2.204655	2.063282	1.961463
190	1.960649	1.843902	1.758678	2.179068	2.045644	1.948685
200	1.936284	1.826875	1.746182	2.151657	2.026502	1.934587
210	1.920153	1.815351	1.737809	2.134823	2.014623	1.925914
220	1.905415	1.804889	1.730021	2.114113	1.99992	1.915076
230	1.889552	1.793463	1.72151	2.091025	1.983284	1.90293
240	1.877048	1.784395	1.714845	2.079924	1.975328	1.897004
250	1.863506	1.774557	1.707539	2.062491	1.962687	1.887491

of $N(\mu, c^2\mu)|_0^1$, say X, as $\mu \to \infty$, has the following density function:

$$\frac{\exp\left(\frac{x}{c^2}\right)}{c^2\left(\exp\left(\frac{1}{c^2}\right) - 1\right)},$$

where the constant c satisfies

$$\left(\frac{1}{6C_p}\right)^2 = \frac{c^2 \exp\left(\frac{1}{c^2}\right)}{c^2\left(\exp\left(\frac{1}{c^2}\right) - 1\right)} - 2c^2 \frac{(c^2 - c^4)\exp\left(\frac{1}{c^2}\right) + c^4}{c^2\left(\exp\left(\frac{1}{c^2}\right) - 1\right)}$$
$$- \left(\frac{(c^2 - c^4)\exp\left(\frac{1}{c^2}\right) + c^4}{c^2\left(\exp\left(\frac{1}{c^2}\right) - 1\right)}\right)^2.$$

Consequently, to eliminate the need for further estimating the parameter μ we could calculate the critical values using the obtained asymptotic distribution. The critical value can be solved as the following for given C_p, n and α − risk.

$$c_0 = \frac{1}{6}(1/\hat{\sigma})_{n, 1-\alpha}$$

where $(1/\hat{\sigma})_{n, 1-\alpha}$ is the $(1 - \alpha)$th quantile of $\frac{1}{\hat{\sigma}}$.

This approach ensures that the Type-I error will be always no greater than the designated value α, hence the result is more reliable. Tables 1 and 2 summarize the critical values c_0 for $C = 1$, 1.25, 1.45, and 1.6, with sample sizes $n = 30(5)250$, and α − risk = 0.01, 0.025, 0.05.

6. A light emitting diodes example

Consider a Light Emitting Diodes (LEDs) manufacturing process. The application of LEDs is expanding rapidly including color displays, traffic signals, roadway signs, airport signaling and lighting. As various LED applications are developed, accurate specifications of LED characteristics become increasingly important. However, serious discrepancy in measurement is gathered from different LED manufacturers and users. A transfer of photometric scales from traditional luminous intensity standard lamps to LEDs is not a trivial task causing large uncertainties. The temperature-dependent characteristics and a large variety of optical designs of LEDs make it even more difficult to reproduce measurements. In order to solve this problem, the factory has been requested to provide calibrated standard LEDs for luminous intensity and luminous flux to improve the accuracy of measurement at industry level.

A photometric technique has been developed to determine the effective reference plane of a photometer with an uncertainty of 0.2 mm, using a photometric bench and a stable integrating sphere source instead of a tungsten filament lamp. With this method, any photometer head with unknown reference plane position can be calibrated for LED measurements at any distances longer than 10 cm within an uncertainty of less than 1%. The alignment of LEDs is still a major uncertainty component for luminous intensity. As described above LEDs generally do not follow

the inverse-square law, so setting the distances accurately is critical to achieve reproducible results. One method of setting the alignment is permanently mounting an LED in a mount that has a reference surface. The distance from the tip of the LED to the reference surface can be measured accurately. The angular alignment will not change because the reference surface will align the LED with the apparatus. A good method is aligning the bare LEDs optically. Using a fixed telescope, a point in space is defined along the detector axis. The detector is on a translational stage with an optical encoder. We have established a capability for calibrating the luminous intensity of LEDs and built a tentative measurement set up for LED measurements in the photometric bench to make the calibration service available for submitted LEDs. The measurement of LED luminous intensity currently has an overall uncertainty of 1.5% for LEDs with a special fixture, and 3% for normal bare LEDs with no alignment aids. A dedicated small photometric bench for LED measurements is to be built. Long-term stability and temperature dependence of these LEDs will be studied and standard LEDs for luminous intensity are to be developed. LEDs are unique light sources and are very different from traditional lamps in terms of physical size, flux level, spectrum and spatial distribution. The transfer of photometric scales from luminous intensity standard lamps to LEDs has not been trivial and large discrepancies among companies have been measured. The factory has established two measurement conditions for single element LEDs with diameters less than 10 mm. These two measurement techniques compare LED luminous intensities without strictly using point source conditions. The factory has started research programs to establish appropriate measurement methods and calibration standards for all photometric quantities of LEDs. In particular, the measurement of luminous intensity of LED sources will be focused in our study. We investigated a particular model of the LED product with the upper and the lower specification limits of luminous intensity are set to USL = 90 mcd, LSL = 40 mcd. The precision requirement is set to $C_p \geqslant 1.33$.

A total of 120 observations were collected from a stable process in the factory are displayed in Table 3. It can be examined that the data is collected from the truncated normally process. To determine whether the process precision is satisfactory, we calculate $\widehat{C}_p = 1.7008$. Since the 95%

Table 3
A total of 120 observations

68	61	61	66	64	63	66	61	66	64	58	61
86	70	56	64	63	64	63	59	64	61	66	64
63	61	63	60	69	65	69	59	69	53	57	63
68	66	67	60	78	64	65	65	92	62	59	58
63	64	76	35	66	91	63	62	57	66	67	67
59	59	59	59	55	63	60	64	57	62	59	62
64	62	63	60	57	63	62	59	66	64	68	68
62	62	66	67	65	66	65	68	57	59	64	63
61	62	46	64	62	52	71	64	66	62	59	63
69	59	62	58	62	62	63	72	60	60	69	64

and 99% confidence lower bound are 1.4935 and 1.4246 exceed 1.33, we can conclude that the process precision is satisfactory.

7. Conclusion

Process precision index C_p has been widely used in the manufacturing industry to provide numerical measures on process precision (product quality consistency). Precision measures using C_p for normal processes have been investigated extensively, but is neglected for truncated normal processes. In this short note we considered the precision measure using the index C_p for truncated normal processes. We investigated the analytically intractable sampling distribution of the estimated C_p, and obtained a rather accurate approximation. The approximation is used to develop a practical procedure for testing process precision. Practitioners can use the proposed procedure on their in-plant applications to obtain reliable decisions.

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