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Frequency estimators for MIMO-OFDM systems

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Abstract Orthogonal frequency division multiplexing (OFDM) systems are known to be sensitive to carrier frequency offset (CFO). This paper is concerned with the CFO estimation for multiple input multiple output (MIMO) systems employing OFDM waveforms. We present two approaches to derive maximum likelihood (ML) pilot-assisted frequency estimators that use either two or multiple identical training symbols. It is shown that the resulting ML frequency estimators are similar to maximum ratio combining versions of Moose estimator and Yu–Su solution, respectively. Numerical examples demonstrate that the proposed frequency estimators are robust against spatial Signal-to-noise ratio (SNR) variation and they yield performance superior to that of the corresponding single-antenna system.

Keywords Orthogonal frequency division multiplexing (OFDM) · Multiple input multiple output (MIMO) · Frequency estimation

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1 Introduction

It is well known that the performance of an orthogonal frequency division multiplexing (OFDM) system is sensitive to the carrier frequency offset (CFO) caused by Doppler shifts or instabilities of and mismatch between transmitter and receiver oscillators [1]. Amongst the numerous proposals for CFO compensation, two are directly related to our work. Moose' [2] maximum likelihood (ML) estimator is based on the observation of two consecutive and identical symbols. Its maximum frequency acquisition range is only $\pm 1/2$ subcarrier spacing because of the modulo 2π ambiguity. Yu and Su (YS) [6] used a transform domain approach to derive an ML estimator based on multiple (>2) identical pilot symbols and presented very efficient alternatives that give mean squared error (MSE) performance close to the Cramér–Rao bound (CRB).

Combining the advantages of OFDM and multiple input multiple output (MIMO) techniques, a variety of (MIMO-OFDM) system architectures and the associated detection techniques have been studied. The related CFO estimation issue, however, does not received much attention so far. Mody and Stüber proposed a CFO estimator based on the autocorrelation of the cyclic prefix [3,4]. The residual integer part of CFO is obtained by increasing the range of CFO estimator [3] or using the cyclic cross-correlation in the frequency domain [4]. The solution of Asai et al. [5] is based on the average of the autocorrelation values derived from two receiving antennas. Both approaches are of heuristic nature and no optimality was claimed. This paper presents two ML fractional CFO estimators for MIMO-OFDM systems. We employ two different approaches to derive the ML solutions for systems using two-symbol and multiple-symbol preamble structures, respectively. The resulting estimators are found to be extended versions of the corresponding ML estimators for the single-antenna scenario, namely, the Moose and the YS algorithms.

The rest of the paper is organized as follows. The following section gives a description of the signal and system model. In Sect. 3, we derive the ML frequency estimator and show that they are extended versions of Moose and YS ML CFO estimators. Section 4 provides some numerical examples and related discussions. Conclusions are then given in Sect. 5.

2 System and signal model

Consider a frequency selective fading channel associated with a MIMO system of M_{T} transmit and M_{R} receive antennas. If the duration of the cyclic prefix (CP), N_g samples, is greater than or equal to the maximum relative delay that includes users' timing ambiguities and the maximum multipath delay, we can express the equivalent received time-domain baseband sequence at the output of the ith receive antenna, after removing the CP part, as

$$y_i[n] = \sum_{j=1}^{M_{\mathrm{T}}} r_{ij}[n] + w_i[n],$$
$$n = 1, 2, \ldots, N, \quad i = 1, 2, \ldots, M_{\mathrm{R}}, \tag{1}$$

where $\{w_i[n]\}$ is a zero-mean complex additive white Gaussian noise (AWGN) sequence and

$$r_{ij}[n] = \frac{1}{N}\sum_{k\in D_j}\sqrt{\frac{E_s}{M_{\mathrm{T}}}}S_j[k]H_{ij}[k]\mathrm{e}^{j2\pi n(k+\epsilon)/N} \tag{2}$$

is the part of the received waveform contributed by the OFDM signal sent by the jth transmit antenna and received by the ith receive antenna. Moreover,

- $S_j[k]$ represents the symbol carried by the kth subcarrier at the jth transmit antenna.
- $H_{ij}[k]$ is the frequency response at the kth subcarrier for the channel between the ith receive and the jth transmit antennas. The corresponding impulse response is given by $\{h_{ij}[n], n = 0, 1, \ldots, L-1\}$, i.e. $\left\{H_{ij}[k] = \sum_{n=0}^{L-1} h_{ij}[n]\mathrm{e}^{-\frac{j2\pi kn}{N}}\right\}$.
- L is the maximum channel memory of all $M_{\mathrm{T}}M_{\mathrm{R}}$ SISO component channels. Thus not all $h_{ij}[n]$ have nonzero values.
- ϵ denotes the relative CFO of the channel (the ratio of the actual CFO to the intercarrier spacing).
- D_j is the set of modulated subcarrier for the jth transmit antenna.
- E_s is the average energy allocated to the kth subcarrier evenly divided across the transmit antennas.

Using the short form notations

$$\mathbf{y}[n] = \left(y_1[n]\ y_2[n]\ \cdots\ y_{M_{\mathrm{R}}}[n]\right)^T, \tag{3a}$$
$$\mathbf{S}[k] = \left(S_1[k]\ S_2[k]\ \cdots\ S_{M_{\mathrm{T}}}[k]\right)^T, \tag{3b}$$
$$\mathbf{H}[k] = \left[H_{ij}[k]\right], \tag{3c}$$
$$\mathbf{w}[n] = \left(w_1[n]\ w_2[n]\ \cdots\ w_{M_{\mathrm{R}}}[n]\right)^T \tag{3d}$$

we rewrite (1) in matrix form

$$\mathbf{y}[n] = \frac{1}{N}\sum_{k\in D_j}\sqrt{\frac{E_s}{M_{\mathrm{T}}}}\mathbf{H}[k]\mathbf{S}[k]\mathrm{e}^{j2\pi n(k+\epsilon)/N} + \mathbf{w}[n]. \tag{4}$$

Taking N-point DFT on both sides of the above equation leads to

$$\mathbf{Y}[k] = \sqrt{\frac{E_s}{M_{\mathrm{T}}}}\sum_{m\in D_j}\mathbf{H}[m]\mathbf{S}[m] \times\left(\frac{1}{N}\sum_{n=0}^{N-1}\mathrm{e}^{j2\pi(m+\epsilon)n/N}\mathrm{e}^{-j2\pi kn/N}\right) + \mathbf{W}[k], \tag{5}$$

where

$$\mathbf{Y}[k] = \left(Y_1[k]\ Y_2[k]\ \cdots\ Y_{M_{\mathrm{R}}}[k]\right)^T, \tag{6a}$$
$$\mathbf{W}[k] = \left(W_1[k]\ W_2[k]\ \cdots\ W_{M_{\mathrm{T}}}[k]\right)^T, \tag{6b}$$
$$Y_i[k] = \sum_{n=0}^{N-1} y_i[n]\mathrm{e}^{-\frac{j2\pi kn}{N}}, \tag{6c}$$
$$W_i[k] = \sum_{n=0}^{N-1} w_i[n]\mathrm{e}^{-\frac{j2\pi kn}{N}}. \tag{6d}$$

Let ϵ_f and ϵ_i be, respectively, the fractional and integer parts of the CFO so that $\epsilon = \epsilon_f + \epsilon_i$. We rewrite (5) as

$$
\begin{aligned}
\mathbf{Y}[k] = &\sqrt{\frac{E_s}{M_\mathrm{T}}}\mathbf{H}[k-\epsilon_i]\mathbf{S}[k-\epsilon_i] \\
&\times\left(\frac{\frac{1}{N}\sin(\pi\epsilon_f)}{\sin(\pi\epsilon_f/N)}\mathrm{e}^{j\pi\epsilon_f(N-1)/N}\right) \\
&+\sqrt{\frac{E_s}{M_\mathrm{T}}}\sum_{\substack{m\in D_j\\ m\neq k-\epsilon_i}}\mathbf{H}[m]\mathbf{S}[m] \\
&\times\left[\frac{\frac{1}{N}\sin(\pi(\epsilon_f+\epsilon_i))}{\sin(\pi(m-k+\epsilon_f+\epsilon_i)/N)}\right. \\
&\left.\times\mathrm{e}^{j\pi(\epsilon_f+\epsilon_i)(N-1)/N}\mathrm{e}^{-j\pi(m-k)/N}\right]+\mathbf{W}[k].
\end{aligned} \tag{7}
$$

The first term on the right hand side of the above equation indicates that, besides suffering from multiplicative distortion caused by the channel response, the kth subcarrier signal is circularly shifted and amplitude-reduced. The second term represents the inter-carrier interference (ICI). It is clear that the presence of a fractional CFO ϵ_f causes reduction of the desired subcarrier's amplitude and induces ICI. If ϵ_f is perfectly compensated for, the integer CFO ϵ_i, if exists, will result in a circular shift of the desired output, causing decision errors.

3 Maximum likelihood frequency estimator

3.1 Estimator based on two pilot symbols

We consider a pilot structure similar to that proposed by Moose [2] and let $\mathcal{D}$ be the set of modulated subcarrier (indexes) that bear a training sequence on the even frequencies and zeros on the odd frequencies. The resulting received time-domain training sequence has two identical halves

$$
\mathbf{r}[n] = \frac{1}{N}\sum_{k\in\mathcal{D}_e}\sqrt{\frac{E_s}{M_\mathrm{T}}}\mathbf{H}[k]\mathbf{S}[k]\mathrm{e}^{j2\pi n(k+\epsilon)/N}, \tag{8a}
$$

$$
\begin{aligned}
\mathbf{r}[n+N/2] &= \frac{1}{N}\sum_{k\in\mathcal{D}_e}\sqrt{\frac{E_s}{M_\mathrm{T}}}\mathbf{H}[k]\mathbf{S}[k]\mathrm{e}^{j2\pi(n+\frac{N}{2})(k+\epsilon)/N} \\
&= \mathbf{r}[n]\mathrm{e}^{j2\pi\epsilon/2},
\end{aligned} \tag{8b}
$$

where $n = 1, 2, \ldots, N/2$, $\mathbf{r}[n] = (r_1[n]\, r_2[n] \cdots r_{M_\mathrm{R}}[n])^T$ and $\mathcal{D}_e$ is the subset of even numbers in $\mathcal{D}$.

Taking into account AWGN, we obtain

$$
\mathbf{y}[n] = \mathbf{r}[n] + \mathbf{w}[n], \tag{9a}
$$

$$
\mathbf{y}[n+N/2] = \mathbf{r}[n]\mathrm{e}^{j2\pi\epsilon/2} + \mathbf{w}[n+N/2], \tag{9b}
$$

where $\mathbf{w}[n] = (w_1[n]\, w_2[n] \cdots w_{M_\mathrm{R}}[n])^T$. Define the abbreviated notations

$$
\bar{\mathbf{y}}_1[i] = (y_i[1]\, y_i[2], \cdots y_i[N/2]) \tag{10a}
$$

$$
\bar{\mathbf{y}}_2[i] = (y_i[N/2+1]\, y_i[N/2+2] \cdots y_i[N]), \tag{10b}
$$

$$
\bar{\mathbf{r}}_1[i] = (r_i[1]\, r_i[2] \cdots r_i[N/2]), \tag{10c}
$$

$$
\bar{\mathbf{r}}_2[i] = (r_i[N/2+1]\, r_i[N/2+2] \cdots r_i[N]) \tag{10d}
$$

and

$$
\overline{\mathbf{w}}_1[i] = (w_i[1]\, w_i[2], \cdots w_i[N/2]) \tag{11a}
$$

$$
\overline{\mathbf{w}}_2[i] = (w_i[N/2+1]\, w_i[N/2+2] \cdots w_i[N]), \tag{11b}
$$

where the subscripts on the right-hand sides of the above equations indicate either the first or the second half of a time-domain OFDM frame and the indexes within the brackets denote from which receive antenna the time domain sample is derived. Equation 9a and 9b then have the equivalent expressions

$$
\bar{\mathbf{y}}_1[i] = \bar{\mathbf{r}}_1[i] + \overline{\mathbf{w}}_1[i], \tag{12a}
$$

$$
\bar{\mathbf{y}}_2[i] = \bar{\mathbf{r}}_1[i]\mathrm{e}^{j2\pi\epsilon/2} + \overline{\mathbf{w}}_2[i], \quad i = 1, 2, \cdots, M_\mathrm{R}. \tag{12b}
$$

The ML estimator of the parameter ϵ, given the received vector $(\bar{\mathbf{y}}_1[i],\ \bar{\mathbf{y}}_2[i])$, is obtained by maximizing the likelihood function

$$
f(\bar{\mathbf{y}}_1[i], \bar{\mathbf{y}}_2[i]|\epsilon) = f(\bar{\mathbf{y}}_2[i]|\bar{\mathbf{y}}_1[i], \epsilon) f(\bar{\mathbf{y}}_1[i]|\epsilon), \tag{13}
$$

where, for simplicity, we have denoted various conditional probability density functions by the generic functional expression, $f(\cdot|\cdot)$. As one can see from (8) and (12) that ϵ provides no information about $\bar{\mathbf{y}}_1[i]$ unless $\mathbf{H}[k]$ and $\mathbf{S}[k]$ is also given, i.e., $f(\bar{\mathbf{y}}_1[i]|\epsilon) = f(\bar{\mathbf{y}}_1[i])$, the ML estimator of ϵ is given by

$$
\begin{aligned}
\widehat{\epsilon} &= \arg\max_{\epsilon}\left[f(\bar{\mathbf{y}}_2[i]|\bar{\mathbf{y}}_1[i], \epsilon) f\left(\bar{\mathbf{y}}_1[i]|\epsilon\right)\right] \\
&= \arg\ \max_{\epsilon}\left[f\left(\bar{\mathbf{y}}_2[i]|\bar{\mathbf{y}}_1[i], \epsilon\right)\right].
\end{aligned} \tag{14}
$$

Since

$$
\begin{aligned}
\bar{\mathbf{y}}_2[i] &= \left(\bar{\mathbf{y}}_1[i] - \overline{\mathbf{w}}_1[i]\right)\mathrm{e}^{j2\pi\epsilon/2} + \overline{\mathbf{w}}_2[i] \\
&= \bar{\mathbf{y}}_1[i]\mathrm{e}^{j2\pi\epsilon/2} + \left(\overline{\mathbf{w}}_2[i] - \overline{\mathbf{w}}_1[i]\mathrm{e}^{j2\pi\epsilon/2}\right)
\end{aligned} \tag{15}
$$

and $\overline{\mathbf{w}}_1[i]$, $\overline{\mathbf{w}}_2[i]$ are independent temporally white Gaussian vector with zero mean and variance $\sigma_{n_i}^2\mathbf{I}$, where $\mathbf{I}$ is the identity matrix, the multivariate Gaussian vector $\overline{\mathbf{y}}_2[i]$ has conditional mean $\overline{\mathbf{y}}_1[i]\mathrm{e}^{j2\pi\epsilon/2}$ and covariance matrix

$$E\left[\left(\overline{\mathbf{w}}_2[i]-\overline{\mathbf{w}}_1[i]\mathrm{e}^{j2\pi\epsilon/2}\right)\left(\overline{\mathbf{w}}_2[i]-\overline{\mathbf{w}}_1[i]\mathrm{e}^{j2\pi\epsilon/2}\right)^H\right] = 2\sigma_{n_i}^2\mathbf{I}. \quad (16)$$

Then, given the received vectors $\overline{\mathbf{y}}_i[n]$, $i = 1, 2$, $n = 1, 2, \ldots, M_\mathrm{R}$, the likelihood function becomes

$$\begin{aligned}\Lambda(\epsilon) &= f\left(\overline{\mathbf{y}}_1[1]\cdots\overline{\mathbf{y}}_1[M_\mathrm{R}],\ \overline{\mathbf{y}}_2[1]\cdots\overline{\mathbf{y}}_2[M_\mathrm{R}]\,\middle|\,\overline{\mathbf{y}}_1[1]\ \cdots\overline{\mathbf{y}}_1[M_\mathrm{R}],\epsilon\right)\\ &\propto \exp\left\{\sum_{i=1}^{M_\mathrm{R}}\frac{1}{\sigma_{n_i}^2}\Re\{\overline{\mathbf{y}}_2[i]\overline{\mathbf{y}}_1^H[i]\mathrm{e}^{-j2\pi\epsilon/2}\}\right\}\\ &\propto \exp\left[\Re\left\{\left(\sum_{i=1}^{M_\mathrm{R}}\gamma_i\sum_{n=1}^{N/2}y_i^*[n]y_i[n+N/2]\right)\times\mathrm{e}^{-j2\pi\epsilon/2}\right\}\right],\end{aligned} \quad (17)$$

where in the last two equations we have dropped terms that are not dependent on ϵ, $\gamma_i = (\sigma_{n_i}^2)^{-1}$ and $\Re\{x\}$ denotes the real part of the complex number x. the ML estimator of ϵ is given by

$$\widehat{\epsilon} = \arg\max_{\epsilon}\Lambda(\epsilon) = \frac{1}{\pi}\mathrm{Arg}\left(\sum_{i=1}^{M_\mathrm{R}}\gamma_i\sum_{n=1}^{N/2}y_i^*[n]y_i[n+N/2]\right), \quad (18)$$

where $\mathrm{Arg}(x)$ denotes the principal argument of the complex number x. In general, the ML estimator for two identical halves pilot symbols of length N_W and N_D-spaced is given by

$$\widehat{\epsilon} = \frac{N}{2\pi N_D}\mathrm{Arg}\left(\sum_{i=1}^{M_\mathrm{R}}\gamma_i\sum_{n=1}^{N_W}y_i^*[n]y_i[n+N_D]\right). \quad (19)$$

Note that the correlation $\sum_{n=1}^{N/2}y_i^*[n]y_i[n+N/2] = C_{y_i}$ is weighted by the reciprocal of the noise sample's variance and the product $\gamma_i C_{y_i}$ is unitless and is proportional to the signal-to-noise ratio (SNR) at the ith receive antenna. This weighting strategy is similar to the maximum ratio combining (MRC) rule used in a diversity receiver for combating frequency-selective fading. When equal gain weighting $\gamma_i = 1$ is used, the estimator is just the multiple antenna extension of the original Moose estimator, averaging all two symbol correlation values over all receiving antennas. We refer to this estimator as the weighted Moose (WM) algorithm. The acquisition range of the WM algorithm is $\pm\frac{N}{2N_D}$ subcarrier spacings. For a system that uses arbitrary even number (say $2k$) of identical training symbols, we can also apply the WM algorithm by regarding the first half (k symbols) as the first training symbol and the second half as the second one.

3.2 Estimator based on multiple short symbols

Now let us consider a MIMO-OFDM system that uses multiple $(K+1)$ identical pilot symbols. For this case, we can use a procedure similar to that presented in the previous section to derive the corresponding ML estimator. An alternative but more efficient approach for the case of multiple pilot symbols is to apply the transform domain method of [6]. The mth sample of the kth (time-domain) pilot symbol received by the ith receive antenna, $y_i(k,m)$, can be represented as

$$y_i(k,m) = x_i(k,m) + w_i(k,m), \quad k = 1,\ldots K, \quad m = 1,\ldots,\ldots M, \quad (20)$$

where $x_i(k,m)$ and $w_i(k,m)$ are the corresponding signal and noise components. The latter are uncorrelated circularly symmetric Gaussian random variables with zero mean and variance $\sigma_{w,i}^2 = E\{|w_i(k,m)|^2\}$. Define

$$Y_i(m) = [y_i(1,m)\ \cdots\ y_i(K,m)]^T, \quad (21a)$$

$$A(\epsilon) = \left[1\ \mathrm{e}^{j2\pi\epsilon M/N}\ \cdots\ \mathrm{e}^{j2\pi\epsilon(K-1)M/N}\right]^T, \quad (21b)$$

$$W_i(m) = [w_i(1,m)\ \cdots\ w_i(K,m)]^T, \quad (21c)$$

where $(\cdot)^T$ denote the matrix transpose.

Following an approach similar to [6] we obtain the joint log-likelihood function

$$\Lambda(\epsilon, x_i(1,m)) = \sum_{i=1}^{M_\mathrm{R}}1/\sigma_{n_i}^2\sum_{m=1}^{M}||Y_i(m) - A(\epsilon)x_i(1,m)||^2, \quad (22)$$

where we have used the fact

$$x_i(k,m) = x_i(1,m)\mathrm{e}^{j2\pi(k-1)M\epsilon/N}. \quad (23)$$

For a given $A(\epsilon)$, setting $\nabla_{x_i(1,m)}||Y_i(m) - A(\epsilon)\, x_i(1,m)||^2 = 0$, we obtain the conditional ML estimator, $\hat{x}_i(1,m) = x_{\mathrm{LS}_i}(1,m) = A^+(\epsilon)Y_i(m)$, where

$A^{+}(\epsilon) = A(\epsilon)^{H}/K$ and H denotes the Hermitian operation. By substituting the least-square solution, $x_{\mathrm{LS}_i}(1, m)$, for $x_i(1, m)$, we obtain

$$\Lambda(\epsilon) = \sum_{i=1}^{M_{\mathrm{R}}} \frac{1}{\sigma_{n_i}^2} \sum_{m=1}^{M} ||P_A^{\perp} Y_i(m)||^2 = (M_{\mathrm{R}} M)\mathrm{tr}(P_A^{\perp} \hat{R}_{YY}), \tag{24}$$

where $tr(\cdot)$ denotes the trace of a matrix, $P_A^{\perp} \stackrel{\mathrm{def}}{=} I - A(\epsilon)A^{+}(\epsilon)$ and

$$\hat{R}_{YY} \stackrel{\mathrm{def}}{=} \frac{1}{M_{\mathrm{R}} M} \sum_{i=1}^{M_{\mathrm{R}}} \gamma_i \sum_{m=1}^{M} Y_i(m) Y_i^{\mathrm{H}}(m) \tag{25}$$

The above equation indicates that, like the 2-pilot case, the elements in the correlation matrix are also maximum ratio combinations of correlation values computed in each component antenna. The desired frequency estimator is then given by

$$\hat{\epsilon} = \arg\{\min_{\epsilon} \mathrm{tr}(P_A^{\perp} \hat{R}_{YY})\} = \arg\{\max_{\epsilon} \mathrm{tr}(P_A \hat{R}_{YY})\} = \arg\{\max_{\epsilon} A^{\mathrm{H}} \hat{R}_{YY} A\}. \tag{26}$$

This ML solution is similar to that derived in [6] with an *extended* definition (MR combined) of the correlation matrix $\hat{R}_{YY}$. It can be proved that using this extended correlation matrix and the transform domain approach of [6] we obtain an ML algorithm similar to that of [6]. For convenience of reference, we summarize the resulting ML estimation procedure as following.

1. Collect K received symbols from all receive antennas and construct the sample correlation matrix $\hat{R}_{YY}$ according to (25).
2. Calculate the coefficients of $F(z)$ based on $\hat{R}_{YY}$, where $z = \mathrm{e}^{j2\pi\epsilon M/N}$ and

$$F(z) = \sum_{n=1}^{K-1} n s(n) z^n, \tag{27a}$$

$$s(n) = \sum_{(i,j)\in \digamma_n} \hat{R}_{YY}(i, j), \tag{27b}$$

$$\digamma_n = \{(i, j) | 1 \le i, j \le K, j - i = n\}, \tag{27c}$$

3. Find the nonzero unit-magnitude roots of $F(z) - F^{*}(z) = 0$.
4. The CFO estimator is obtained by

$$\hat{\epsilon} = \frac{N}{j2\pi M} \ln \hat{z}, \tag{28a}$$

$$\hat{z} = \arg\{\max_{z} \Lambda(z)\}, \tag{28b}$$

where

$$\Lambda(z) = A(z)^{H} \hat{R}_{YY} A(z), \tag{29a}$$

$$A(z) = \left[1 \; z \; z^2 \; \cdots \; z^{K-1}\right]^{T}. \tag{29b}$$

This algorithm will be referred to as the extended YS (EYS) algorithm whose frequency acquisition range is $\pm\frac{N}{2M}$ subcarrier spacings.

4 Simulation results and discussion

The computer simulation results reported in this section are obtained by using a pilot format the same as the IEEE 802.11a standard with a sample interval of 50 ns. The frequency-selective fading channel has a exponentially decaying power delay profile whose 16 paths and a rms delay spread of 50ns. The complex Gaussian distributed path amplitudes are normalized such that the sum of the average power is unity. The DFT size is $N = 64$. The SNR, defined as the ratio of the received signal power (from all M_{T} transmitters) to the noise power at the ith receive antenna, is assumed to be the same for each receive antenna for Figs. 1–2. For the WM algorithm, the training part consists of two identical halves with length $N_W = 32$. Fig. 1 shows the MSE, $E[(\hat{\epsilon} - \epsilon)^2]$, performance of the WM CFO estimator for different number of transmit and receive antennas. Obviously, the MSE performance improves as the number of receive antennas, M_{R}, increases. In fact if one follows the analysis of [7], one can show that the corresponding CRB has a similar trend and is given by

$$\mathrm{CRB}_{\mathrm{MIMO}} = \frac{3(N/M)^3(\mathrm{SNR})^{-1}}{2\pi^2 N K(K^2 - 1) M_{\mathrm{R}}}. \tag{30}$$

Figure 2 shows the performance of EYS and WM estimators when four short training symbols are used. The WM estimator regards the first two training symbols as one period so that two training symbols with length $N_w = 32$ are used in computing the CFO estimator. The acquisition range of the WM estimator is ± 1 subcarrier spacings while that of the EYS estimator is ± 2 subcarrier spacings. Because the EYS estimator use all second-order information of the training symbols, it outperforms the WM estimator.

In the remaining two figures we examine the effect of the weighting schemes in combining time-correlations from various receive antennas; see (18), (19) or (25), (26). Figure 3 shows the performance of the WM CFO estimator using two weighting schemes. We plot

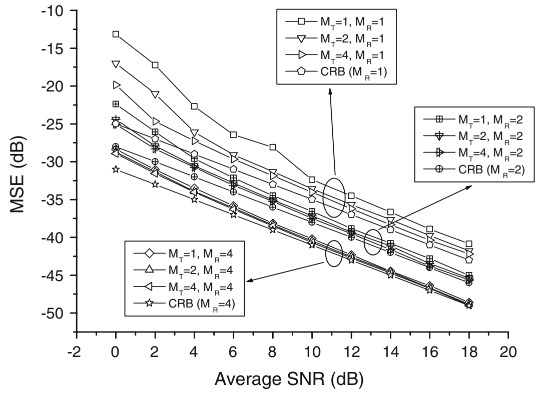

Fig. 1 MSE performance of the weighted Moose estimator for systems using two identical training symbols; true CFO = 0.7 subcarrier spacings

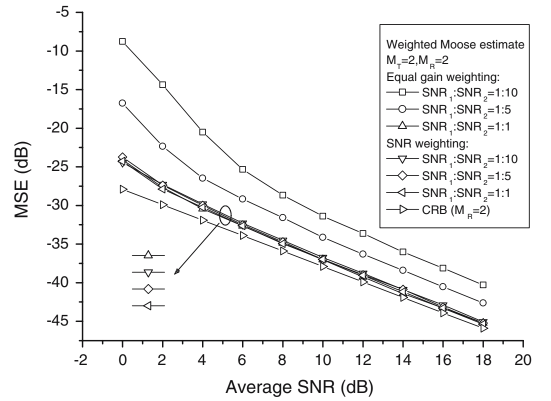

Fig. 3 The effect of the weighting strategy on the MSE performance of the WM algorithm for systems using two identical training symbols; true CFO = 0.7 subcarrier spacings

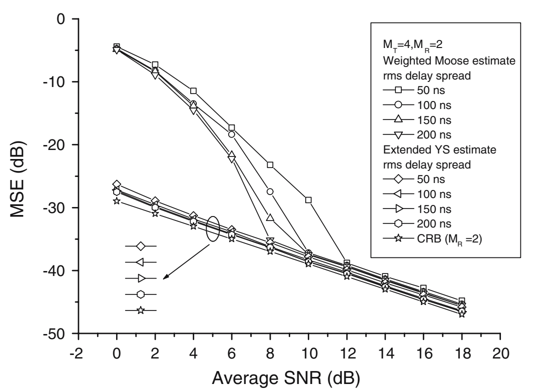

Fig. 2 MSE performance of the EYS algorithm for systems using four short training symbols; true CFO = 0.93 subcarrier spacings

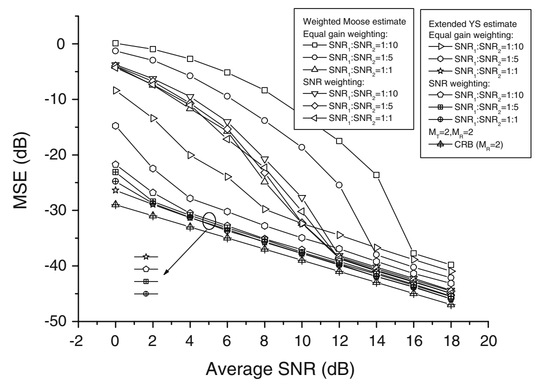

Fig. 4 The effect of the weighting strategy on the MSE performance of the EYS algorithm for systems using four identical training symbols; true CFO = 0.93 subcarrier spacings

the frequency estimator's MSE performance when the SNR for each receive antenna is assumed to be perfectly known. The importance of a proper weighting is clearly demonstrated. Performance of both frequency estimators using the optimal weighting is insensitive to the SNR variation across antennas. Similar observation is obtained in Fig. 4 which shows the performance of the EYS and WM estimators using either equal gain or optimal combining for different SNRs.

For reference purpose, we also present the CRB curves in each figures. It is found that, for a fixed number of receive antennas N_R, when the number of transmit antennas increases the corresponding MSE performance becomes closer to the CRB for M_R, especially at high SNRs.

5 Conclusion

We have shown that the ML frequency estimator for MIMO-OFDM systems with multiple (> 2) identical training symbols is equivalent to an extended version of the YS algorithm. For the special case when only two (short) training symbols are available the ML solution is a weighted combination of the Moose estimators for every receive antennas. Our derivation is based on the assumption that the length of cyclic prefix is greater than or equal to the maximum delay that accounts for the all subchannels of the MIMO channel. When the SNRs across different receive antennas are not identical, the frequency estimators use weighted combinations of time correlations values from various antennas. With an MRC-like combining scheme, the performance of both ML frequency estimators is insensitive to spatial SNR variation and improves as the number of transmit/receive antennas increases. In other words, the presence of multiple antennas not only promises great capacity enhancement but entails performance improvement for the associated frequency synchronization subsystem.

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References

1. Pollet, T., Van Bladel, M., & Moeneclaey, M. (1995). BER sensitivity of OFDM systems to carrier frequency offset and Wiener phase noise. *IEEE Transactions on Communications, 43*, 191–193.
2. Moose, P. H. (1994). A technique for orthogonal frequency division multiplexing frequency offset correction. *IEEE Transaction Communications, 42*, 2908–2914.
3. Mody, A. N., & Stuber, G.L. (2001). Synchronization for MIMO OFDM systems. *IEEE Global Telecommunication, 1*, 25–29.
4. Mody, A. N., & Stuber, G. L. (2002). Receiver implementation for a MIMO OFDM system. *IEEE Global Telecommunication, 1*, 17–21.
5. Asai, Y., Kurosaki, S., Sugiyama, T., & Umehira, M. (2002). Precise AFC scheme for performance improvement of SDM-COFDM. *IEEE Vehicular Technology, 3*, 24–28.
6. Yu, J.-H., & Su, Y. T. (2004). Pilot-assisted maximum likelyhood frequency-offset estimation for OFDM systems. *IEEE Transactions on Communications, 52*, 1997–2008.
7. Stoica, P., & Nehorai, A. (1989). MUSIC, maximum likelihood, and Cramer-Rao bound. *IEEE Transactions on Acoustics Speech, Signal Processing, 37*, 720V741.

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