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Physica C 269 (1996) 330–338

PHYSICA C

The interaction between a single vortex and a columnar defect in the superconducting multilayers

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Received 15 May 1996

Abstract

The interaction energy per unit length $U_{\rm int}(r)$ between a single vortex and a columnar defect with radius a in the superconducting multilayers is calculated with the help of London theory. We assume that these superconducting layers are coupled by the magnetic field and the interlayer Josephson coupling can be neglected. We obtain a general expression of $U_{\rm int}(r)$ for any t, where t is half of the spacing between the layers. For $t \to \infty$, our theory can be reduced to the result of a thin film. But when t becomes smaller, $U_{\rm int}(r)$ would tend to exhibit the behavior of 3D superconducting bulk.

PACS: 74.60Ge; 74.80.Dm
Keywords: London penetration depth; Multilayers; Thin films; Type-II superconductors

1. Introduction

For technical applications of superconductors, the enhancement of the critical current density (J_c) in high magnetic fields is one of the most important tasks. To obtain a large J_c it is required to introduce strong pinning centers into a superconductor and to enhance the magnetic-flux pinning strength. One of the promising methods of introducing defects inside a superconductor is based on heavy-ion irradiation techniques [1–5]. These nonsuperconducting defects which interact with the vortices would induce screening currents and lead to the so-called electromagnetic pinning [6].

It is necessary to study the structure and interactions of the vortices with columnar defects in order to understand the effect of columnar defects on enhancing J_c. At first, for understanding the structure of the vortices, some theory has been developed. Clem et al. have calculated the structure of the vortices in layered superconductors [7–9] and Bulaevskii et al. [10] have calculated the Lawrence–Doniach model [11] in the linear approximation. Recently Benkraouda and Clem [12] have shown the instability of a tilted vortex in the layered superconductors. Next, the interaction between a single vortex with a columnar defect in the 3D type-II superconducting bulk has been calculated by Mkrtchyan and Shmidt [13], and their theory has been generalized to a periodical structure of columnar defects [14]. On the other hand, for a 2D superconducting thin film, the

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PII S0921-4534(96)00488-1

interaction of a 2D pancake vortex with a circular defect has been studied by Buzdin and Feinberg [15] in the short range and generalized by Chen and Tseng [16] to all ranges. But, unfortunately, as far as we know, the articles about the interation between vortices and columnar defects in the layered superconductors are still deficient.

In this paper, we generalize the result of the interaction of a single 2D pancake vortex with a circular defect in a superconducting thin film [16] to that in superconducting multilayers. We assume that the layered superconductors are coupled by the magnetic field and the interlayer Josephson coupling could be neglected. We improve the mathematical methods developed in our previous work [16] in order to apply them to layered superconductors. For simplicity, we assume the radius of the columnar defect a is a small quantity. This assumption is very reasonable because $a \sim 100$ Å [1–5]. Under these conditions, we obtain a general expression of the interaction energy per unit length $U_{\text{int}}(r)$ for any layer spacing of $2t$ and the whole range $r > a$. The pinning potential and pinning force per unit length are also obtained. In the limit $t \to \infty$, our theory can be reduced to the result in a thin film [16]. But in the opposite limit, $t \to 0$, we obtain the result appropriate for the 3D superconducting bulk [13]. Therefore our theory can successfully explain how $U_{\text{int}}(r)$ varies from the 2D superconducting thin film to the 3D superconducting bulk.

2. A single vortex in the superconducting multilayers

We first consider that a single vortex lies along the z-axis in an infinite stack of parallel, thin (Josephson decoupled) superconducting layers ($\kappa \gg 1$) in the planes $z = z_m$, where $z_m = (2m+1)t$ and $m = 0, \pm 1, \pm 2, \cdots$. Then the vector potential $\boldsymbol{A}_0$ satisfies the following equation [17]:

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{A}_0 = \frac{4\pi}{c} \boldsymbol{j}_{\text{so}} = \frac{2\delta(z - z_m)}{\Lambda} \left[\frac{\phi_0 \hat{\boldsymbol{e}}_\theta}{2\pi\rho} - \boldsymbol{A}_0 \right], \tag{1}$$

where the screening length $\Lambda = 2\lambda^2/d$ plays the role of an effective penetration depth in the superconducting thin film [18–20]. λ is the London penetration depth and d is the thickness of the superconducting layers. Because the vector potential $\boldsymbol{A}_0$ has cylindrical symmetry and we have even and periodical functions of z [i.e. $\boldsymbol{A}_0(\rho, \theta, z) = \boldsymbol{A}_0(\rho, \theta, -z)$ and $\boldsymbol{A}_0(\rho, \theta, z + 2t) = \boldsymbol{A}_0(\rho, \theta, z)$], the general solution of Eq. (1) has the form [7]

$$\boldsymbol{A}_0(\rho, \theta, z) = \hat{\boldsymbol{e}}_\theta \int_0^\infty \mathrm{d}k a_0(k) J_1(k\rho) \cosh k(z - 2mt) \qquad \text{for } |z - 2mt| \leq t. \tag{2}$$

Substituting Eq. (2) into Eq. (1), we then obtain

$$\int_0^\infty \mathrm{d}k a_0(k)(\Lambda k \sinh kt + \cosh kt) J_1(k\rho) = \frac{\phi_0}{2\pi\rho}, \tag{3}$$

from which the Hankel transform yields [21]

$$a_0(k) = \frac{\phi_0}{2\pi} \frac{1}{\Lambda k \sinh kt + \cosh kt}. \tag{4}$$

From this result we obtain the magnetic field

$$\boldsymbol{H}_0 = \frac{\phi_0}{2\pi} \int_0^\infty \mathrm{d}k \frac{k[-\hat{\boldsymbol{e}}\boldsymbol{\rho} J_1(k\rho) \sinh k(z - 2mt) + \hat{\boldsymbol{e}}_z J_0(k\rho) \cosh k(z - 2mt)]}{\Lambda k \sinh kt + \cosh kt}$$
$$\text{for } |z - 2mt| \leq t. \tag{5}$$

where $J_n(x)$ is the Bessel function.

The supercurrent density in each superconducting layer is given by

$$j_{so}(\rho, \theta, z) = \frac{c\delta(z-z_m)}{2\pi\Lambda}\left[\frac{\phi_0 \hat{e}_\theta}{2\pi\rho} - A_0\right] = \frac{\phi_0 c\delta(z-z_m)\hat{e}_\theta}{4\pi^2}\int_0^\infty \mathrm{d}k \frac{kJ_1(k\rho)\tanh kt}{\Lambda k \tanh kt + 1}. \tag{6}$$

The self-energy of a single vortex per unit length U_{self} is obtained by integrating the supercurrent density j_{so} [22]

$$U_{self} = \frac{\phi_0}{2c}\int j_{so}\cdot \mathrm{d}\sigma/2t \approx \frac{\phi_0^2}{16\pi^2 t}\int_0^{1/\xi}\mathrm{d}k \frac{\tanh kt}{\Lambda k \tanh kt + 1}, \tag{7}$$

where $\xi(T)$ is the Ginzberg–Landau coherence length and $2t$ is the layer spacing. If we substitute $\tanh x \approx x$ for $x \leq 1$ and $\tanh x \approx 1$ for $x > 1$ into Eq. (7), we have

$$U_{self} \approx \frac{\phi_0^2}{16\pi^2 \Lambda t}\left[\ln\left(\frac{\Lambda}{\xi}\right) + \frac{1}{2}\ln\left(\frac{t}{\Lambda + t}\right)\right]. \tag{8}$$

Eq. (8) is an approximate expression of U_{self} for any t. In the following paragraphs we discuss this equation in two limits:
(1) $t \gg \Lambda$;
(2) $t \ll \Lambda$.

2.1. $t \gg \Lambda$

From Eqs. (2–7) we have the vector potential

$$A_0(\rho, \theta, z) = \frac{\phi_0 \hat{e}_\theta}{2\pi}\int_0^\infty \mathrm{d}k \frac{e^{-k|z-z_m|}J_1(k\rho)}{\Lambda k + 1}, \qquad \text{for } |z - z_m| \leq t, \tag{9}$$

the supercurrent density

$$j_{so}(\rho, \theta, z) = \frac{\phi_0 c\delta(z-z_m)\hat{e}_\theta}{8\pi\Lambda^2}\left[H_1\left(\frac{\rho}{\Lambda}\right) - Y_1\left(\frac{\rho}{\Lambda}\right) - \frac{2}{\pi}\right], \tag{10}$$

where $H_n(x)$ is the sturve function and $Y_n(x)$ is the Neumann function. The self-energy of a single vortex per unit length is

$$U_{self} = \frac{\phi_0^2}{16\pi^2 \Lambda t}\ln\left(\frac{\Lambda}{\xi}\right). \tag{11}$$

2.2. $t \ll \Lambda$

We obtain the vector potential

$$A_0(\rho, \theta, z) = \frac{\phi_0 \hat{e}_\theta}{2\pi\lambda_\parallel}\left\{\frac{\lambda_\parallel}{\rho} - K_1\left(\frac{\rho}{\lambda_\parallel}\right) + \frac{(z-2mt)^2}{2\lambda_\parallel^2}K_1\left(\frac{\rho}{\lambda_\parallel}\right) - \frac{t^2}{6\lambda_\parallel^2}\left[\frac{\rho}{\lambda_\parallel}K_0\left(\frac{\rho}{\lambda_\parallel}\right) + K_1\left(\frac{\rho}{\lambda_\parallel}\right)\right]\right\}$$

$$\text{for } |z - 2mt| \leq t, \tag{12}$$

the magnetic field

$$H_0(\rho, \theta, z) = -\frac{\phi_0(z-2mt)\hat{e}_\rho}{4\pi\lambda_\parallel^3}K_1\left(\frac{\rho}{\lambda_\parallel}\right) + \frac{\phi_0\hat{e}_z}{2\pi\lambda_\parallel^2}\left\{K_0\left(\frac{\rho}{\lambda_\parallel}\right) - \frac{(z-2mt)^2}{2\lambda_\parallel^2}K_0\left(\frac{\rho}{\lambda_\parallel}\right)\right.$$
$$\left. - \frac{t^2}{6\lambda_\parallel^2}\left[K_0\left(\frac{\rho}{\lambda_\parallel}\right) - \frac{\rho}{\lambda_\parallel}K_1\left(\frac{\rho}{\lambda_\parallel}\right)\right]\right\} \quad \text{for } |z-2mt| \le t, \tag{13}$$

the supercurrent density

$$j_{so}(\rho, \theta, z) = \frac{\phi_0 ct\delta(z-z_m)}{4\pi^2\lambda_\parallel^3}\hat{e}_\theta\left\{K_1\left(\frac{\rho}{\lambda_\parallel}\right) + \frac{t^2}{3\lambda_\parallel^2}\left[\frac{\rho}{2\lambda_\parallel}K_0\left(\frac{\rho}{\lambda_\parallel}\right) - K_1\left(\frac{\rho}{\lambda_\parallel}\right)\right]\right\}. \tag{14}$$

Therefore, the self-energy of a single vortex per unit length is

$$U_{self} = \frac{\phi_0^2}{16\pi^2\lambda_\parallel^2}\left(1 - \frac{t^2}{3\lambda_\parallel^2}\right)\ln\left(\frac{\lambda_\parallel}{\xi}\right), \tag{15}$$

where $\lambda_\parallel = \sqrt{\Lambda t}$ is the effective penetration depth [7] in the superconducting multilayers for $t \ll \Lambda$. $K_n(x)$ is the modified Bessel function.

As expected, in the limit $t \gg \Lambda$, the vector potential A_0 is reduced to the result appropriate for an isolated thin film as indicated in Eq. (9), but in the opposite limit, $t \ll \Lambda$, we obtain the result appropriate for the 3D superconducting bulk as shown in Eq. (12) in which $\lambda_\parallel$ is the effective penetration depth. Moreover, we obtain the corrective terms in the order of $t/\Lambda = t^2/\lambda_\parallel^2$ in Eqs. (12–15) for $t \ll \Lambda$. Therefore Eqs. (4–7) are valid for a superconductor with any layer spacing.

3. A single vortex in the superconducting multilayers with a columnar defect

We next consider a columnar defect with radius a in an infinite stack of parallel, thin (Josephson decoupled) superconducting layers in the planes $z = z_m$, and a single vortex is located at a distance r from the center of the defect. Both columnar defects and vortices are aligned along the z-axis (as shown in Fig. 1). For simplicity we assume $a \ll \min(\Lambda, \lambda_\parallel)$ in this paper. Then the vector potential A satisfies the equations [16]

$$\nabla\times\nabla\times A = \frac{4\pi}{c}j_s = \frac{2\delta(z-z_m)}{\Lambda}\left[\frac{\rho_0\hat{e}_\phi}{2\pi R} - A\right] \quad \text{for } \rho > a,$$
$$\nabla\times\nabla\times A = 0 \quad \text{for } \rho < a, \tag{16}$$

where the center of the columnar defect is located at (0, 0, z) which corresponds to the z-axis of the cylindrical coordinates (ρ, θ, z), and the single vortex is located at (r, 0, z) which corresponds to the z-axis of the other cylindrical coordinates (R, ϕ, z). In order to solve Eq. (16), we would like to generalize the mathematical methods applicable in a thin film [16] to the multilayers. The solution of Eq. (16) includes two parts and can be written as $A(\rho, \theta, z) = A_0 + A_H$, where A_0 is the vector potential of the single vortex and has been solved and given in Section 2. A_H is the homogeneous solution of Eq. (16), and is caused by the screening current of the columnar defect. On the basis of Eqs. (2) and (4) in Section 2, we have

$$A_0(R, \phi, z) = \frac{\phi_0\hat{e}_\phi}{2\pi}\int_0^\infty dk\frac{J_1(kR)\cosh k(z-2mt)}{\Lambda k\sinh kt + \cosh kt} \quad \text{for } |z-2mt| \le t. \tag{17}$$

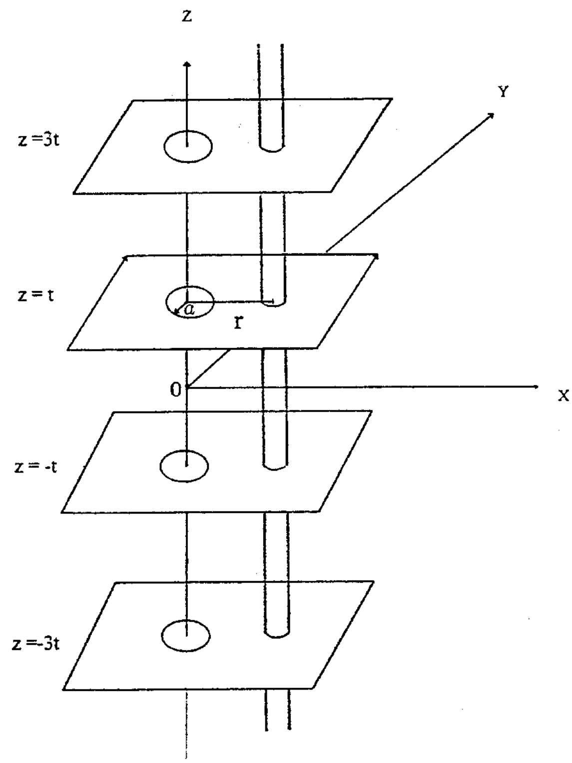

Fig. 1. A columnar defect with radius a lies in the Josephson decoupled superconducting multilayers in the planes $z = z_m = (2m+1)t$, and a single vortex is located at a distance r from the center of the defect.

Now we transform the coordinates from (R, ϕ, z) to (ρ, θ, z) in Eq. (17). By the summation theorem, the Bessel function $J_0(kR)$ can be expanded in the following form [23]

$$J_0(kR) = J_0(k\rho)J_0(kr) + 2\sum_{n=1}^{\infty} J_n(k\rho)J_n(kr)\cos n\theta. \tag{18}$$

Taking the partial derivatives $\partial/\partial\rho$ and $\partial/\partial\theta$ on both sides of Eq. (18), we have

$$\left(\frac{\rho - r\cos\theta}{R}\right)J_1(kR) = J_1(k\rho)J_0(kr) + \sum_{n=1}^{\infty}\left[J_{n+1}(k\rho) - J_{n-1}(k\rho)\right]J_n(kr)\cos n\theta. \tag{19}$$

$$\frac{r\sin\theta}{R}J_1(kR) = \sum_{n=1}^{\infty}\left[J_{n+1}(k\rho) + J_{n-1}(k\rho)\right]J_n(kr)\sin n\theta. \tag{20}$$

The angular unit vector in (R, ϕ, z), $\hat{e}_\phi$ can be expressed as

$$\hat{e}_\phi = \frac{-r\sin\theta\hat{e}_\rho + (\rho - r\sin\theta)\hat{e}_\theta}{R}, \tag{21}$$

From Eqs. (19–21), we have

$$J_1(kR)\hat{e}_\phi = \tfrac{1}{2}J_0(kr)L_0(\rho,\theta;k) + \sum_{n=1}^{\infty} J_n(kr)L_n(\rho,\theta;k), \tag{22}$$

where

$$L_n(\rho, \theta; k) = -\hat{e}_\phi[J_{n+1}(k\rho) + J_{n-1}(k\rho)]\sin n\theta + e_\theta[J_{n+1}(k\rho) - J_{n-1}(k\rho)]\cos n\theta,$$
$$n = 0, 1, 2, \ldots \tag{23}$$

Substituting Eq. (22) into Eq. (17), we have

$$A_0(\rho, \theta, z) = \frac{\phi_0}{2\pi}\int_0^\infty \mathrm{d}k \frac{\cosh k(z-2mt)}{\Lambda k \sinh kt + \cosh kt}\left[\frac{1}{2}J_0(kr)L_0(\rho, \theta; k) + \sum_{n=1}^{\infty} J_n(kr)L_n(\rho, \theta; k)\right]$$
$$\text{for } |z - 2mt| \le t. \tag{24}$$

From Eq. (24) we find that $A' \equiv \phi_0\hat{e}_\phi/2\pi R - A_0$ can be expanded in a series of the orthogonal functions $U_n(\rho, \theta, z; r)$, where $U_n(\rho, \theta, z; r)$ is the solution of the equation $\nabla\times\nabla U_n = -2\delta(z)U_n/\Lambda$ and can be written as

$$U_n(\rho, \theta, z; r) = \frac{\phi_0}{2\pi}\int_0^\infty\left[1 - \frac{\cosh k(z-2mt)}{\Lambda k \sinh kt + \cosh kt}\right]J_n(kr)L_n(\rho, \theta; k). \tag{25}$$

In Eq. (25) r is regarded as a parameter. Then we can construct the homogeneous solution of Eq. (16), A_H, in the region of interest $\rho > a$ with the expansions of $U_n(\rho, \theta, z; a)$ in a series as follows:

$$A_H(\rho, \theta, z) = \frac{\alpha_0(r)}{2}U_0(\rho, \theta, z; a) + \sum_{n=1}^{\infty}\alpha_n(r)U_n(\rho, \theta, z; a) \qquad \text{for } \rho > a \text{ and } |z-2mt| \le t. \tag{26}$$

The supercurrent density j_s is given by

$$\begin{aligned} j_s(\rho, \theta, z) &= \frac{c\delta(z-z_m)}{2\pi\pi\Lambda}\left[\frac{\phi_0\hat{e}_\phi}{2\pi R} - A\right] \\ &= \frac{\phi_0 c\delta(z-z_m)}{4\pi^2}\int_0^\infty \mathrm{d}k\frac{k\tanh kt}{\Lambda k\tanh kt + 1}\left\{\frac{1}{2}[J_0(kr) - \alpha_0(r)J_0(ka)]L_0(\rho, \theta; k)\right. \\ &\left.\quad + \sum_{n=1}^{\infty}[J_n(kr) - \alpha_n(r)J_n(ka)]L_n(\rho, \theta; k)\right\} \qquad \text{for } \rho > a, \end{aligned} \tag{27}$$

where $\alpha_n(r)$, can be determined by boundary conditions.

Because the radial component of j_s vanishes at $\rho = a^+ = a + \xi(T)$, $\alpha_n(r)$ $(n \neq 0)$ can be determined as

$$\alpha_n(r) = \frac{\displaystyle\int_0^\infty \mathrm{d}k\frac{J_n(ka)J_n(kr)\tanh kt}{\Lambda k\tanh kt + 1}}{\displaystyle\int_0^\infty \mathrm{d}k\frac{J_n(ka)J_n(ka^+)\tanh kt}{\Lambda k\tanh kt + 1}}, \qquad n = 1, 2, 3, \cdots \text{ for } r > a; \tag{28}$$

$\alpha_0(r)$ can be determined by the condition that the total magnetic flux trapped in the superconducting multilayers (including the defect) is equal to ϕ_0. The z component of the magnetic field $H_z(\rho < a, z = z_m)$ is distributed uniformly and the radial and angular components of $H(\rho < a, z = z_m)$ vanish because $\nabla\times H = 4\pi j_s/c = 0$ for $\rho < a$ and $z = z_m$. By integrating Eq. (16) along the circular contour of the defect in $z = z_m$ just as was done in Ref. [13], we have

$$\oint_{\rho=a} A\cdot \mathrm{d}l = \pi a^2 H_z(\rho < a, z = z_m). \tag{29}$$

Substituting Eqs. (24) and (26) into Eq. (29), we obtain

$$\alpha_0(r)=\frac{\displaystyle\int_0^\infty \mathrm{d}k\frac{aJ_0(kr)}{\Lambda k\tanh\,kt+1}\left[\tfrac{1}{2}kaJ_0(ka)-J_1(ka)\right]}{\displaystyle\int_0^\infty \mathrm{d}k\frac{aJ_0(ka^+)}{\Lambda k\tanh\,kt+1}\left[\tfrac{1}{2}kaJ_0(ka)-J_1(ka)\right]+1}\quad \text{for } r>a. \tag{30}$$

The free energy $U(r)$ per unit length is given by

$$U(r)=\frac{\phi_0}{4ct}\int j_s\cdot\,\mathrm{d}\boldsymbol{\sigma}=U_{\mathrm{self}}+U_{\mathrm{int}}(r), \tag{31}$$

where the self-energy of a single vortex per unit length U_{self} is given in Eq. (7), and the interaction energy per unit length $U_{\mathrm{int}}(r)$ between single vortex and columnar defect in the superconducting multilayers is given as

$$U_{\mathrm{int}}(r)=-\frac{\phi_0^2}{16\pi^2 t}\int_0^\infty \mathrm{d}k\frac{\tanh\,kt}{\Lambda k\tanh\,kt+1}\left[\alpha_0(r)J_0(ka)J_0(kr)+2\sum_{n=1}^{\infty}\alpha_n(r)J_n(ka)J_n(kr)\right]. \tag{32}$$

Because we assumed $a \ll \min(\Lambda,\ \lambda_\parallel)$ previously, the first term in the right-hand side of Eq. (32) is much smaller than the second term, and therefore it can be neglected. We consequently get the general expression of $U_{\mathrm{int}}(r)$ for any t:

$$U_{\mathrm{int}}(r)=-\frac{\phi_0^2}{8\pi^2 t}\sum_{n=1}^{\infty}\alpha_n(r)\int_0^\infty \mathrm{d}k\frac{J_n(ka)J_n(kr)\tanh\,kt}{\Lambda k\tanh\,kt+1}, \tag{33}$$

where $\alpha_n(r)$ is given in Eq. (28). We can examine Eq. (33) in the two limits $t\to\infty$ and $t\to 0$. In the limit $t\to\infty$, Eq. (33) reduces to the results of the superconducting thin film which we had been obtained in Ref. [16]. In the opposite limit $t\to 0$, Eq. (33) then approaches the results of Mkrtchyan and Shmidt [13] appropriate for the 3D superconducting bulk with effective penetration depth $\lambda_\parallel=\sqrt{\Lambda t}$. Since the high-$T_c$ superconductor possesses a multilayer structure, and the spacing between the CuO_2 layers is very short (~ 10 Å), we are interested in the limit of this small layer spacing. Taking the limit $t \ll \Lambda$, we can calculate Eq. (33) up to the order of t/Λ ($t^2/\lambda_\parallel^2$) as in the following:

$$U_{\mathrm{int}}(r)=-\frac{\phi_0^2}{4\pi^2\lambda_\parallel^2}\sum_{n=1}^{\infty}\frac{(a/2\lambda_\parallel)^{2n}}{n!(n-1)!}K_n^2\left(\frac{r}{\lambda_\parallel}\right)\left[1-\frac{t^2}{3\lambda_\parallel^2}+\frac{t^2 r}{3\lambda_\parallel^3}\frac{K_{n-1}(r/\lambda_\parallel)}{K_n(r/\lambda_\parallel)}\right]$$
$$\text{for } a\ll\lambda_\parallel \text{ and } t\ll\Lambda, \tag{34}$$

where the first term of the right-hand side in Eq. (34) is equal to the result of Mkrtchyan and Shmidt [13], and the other terms in Eq. (34) are the correction up to the order of $t^2/\lambda_\parallel^2$.

Taking the two limits $r \ll \lambda_\parallel$ and $r \gg \lambda_\parallel$, Eq. (34) can be rewritten as

$$U_{\mathrm{int}}(r)=\begin{cases}\left(\dfrac{\phi_0}{4\pi\lambda_\parallel}\right)^2\left(1-\dfrac{t^2}{3\lambda_\parallel^2}\right)\ln\left(1-\dfrac{a^2}{r^2}\right), & \text{for } a<r\ll\lambda_\parallel,\\[2ex] -\dfrac{\phi_0^2 a^2}{32\pi^2\lambda_\parallel^3 r}\mathrm{e}^{-2r/\lambda_\parallel}\left[1-\dfrac{t^2}{3\lambda_\parallel^2}+\dfrac{t^2 r}{3\lambda_\parallel^3}\right] & \text{for } a\ll\lambda_\parallel\ll r.\end{cases} \tag{35}$$

The pinning potential per unit length for the single vortex, $U_{\rm pin}(a)$, is defined by

$$U_{\rm pin}(a) = U_{\rm int}(r = a^+)$$
$$= -\left(\frac{\phi_0}{4\pi\lambda_{\|}}\right)^2 \left(1 - \frac{t^2}{3\lambda_{\|}^2}\right) \ln\left(\frac{\alpha}{\xi}\right) \qquad \text{for } a \ll \lambda_{\|} \text{ and } t \ll \Lambda. \tag{36}$$

The pinning force per unit length is

$$f_{\rm p} = -\nabla U_{\rm int}(r)\,|_{r=a^+}$$
$$= -\left(\frac{\phi_0}{4\pi\lambda_{\|}}\right)^2 \left(1 - \frac{t^2}{3\lambda_{\|}^2}\right) \frac{\hat{e}\rho}{\xi} \qquad \text{for } a \ll \lambda_{\|} \text{ and } t \ll \Lambda. \tag{37}$$

4. Conclusions

In this paper we generalize the mathematical methods which we had developed to apply in the superconducting thin film [16] to the Josephson decoupled superconducting multilayers. The interaction energy per unit length, $U_{\rm int}(r)$, between a single vortex and a columnar defect is consequently derived for the whole range $r > a$, where a is the radius of the columnar defect and we assume a is small for simplicity.

It is interesting that our theory cannot only reduce to the results of an isolated thin film which we had derived in Ref. [16] in the limit $t \to \infty$, but also our theory corresponds to the result of Mkrtchyan and Shmidt [13] in the limit $t \to 0$, where t is half of the spacing between the layers. For $r \gg \Lambda$ the interaction energy per unit length $U_{\rm int}(r)$ exhibits 2D behavior. But $U_{\rm int}(r)$ more likely exhibits the behavior of the 3D superconducting bulk for smaller t. Therefore our theory can be applied to superconducting multilayers with any layer spacing.

Finally we calculate $U_{\rm int}(r)$ up to the order of t/Λ in the limit $t \ll \Lambda$ (the condition of high-T_c superconductors). Under this condition we also calculate the pinning potential and pinning force per unit length in our system and we believe that this work is useful to realize the pinning mechanism of high-T_c superconductors.

Acknowledgement

The authors gratefully acknowledge the financial support from the National Science Council of Taiwan under contract no. NSC85-2112-M009-037.

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