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決策球模式及其運用

Decision Ball Models and Applications

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Abstract

Many decision-making or choice problems in Marketing incorporate preferences. How to assist decision makers in understanding the decision context and improving inconsistencies in judgments are two important issues in ranking choices. This study develops a decision-making framework based on the screening, ordering, and choosing phases. Two optimization models and a Decision Ball model are proposed to assist decision makers in improving inconsistencies and observing relationships among alternatives. By examining a Decision Ball, a decision maker can observe ranks of and similarities among alternatives, and iteratively adjust preferences and improve inconsistencies thus to achieve a more consistent and informed decision.

***Key words*:** Decision Ball; Visualization; Ranking; Inconsistency; Decision-making

1. Introduction

Many decision-making or choice problems in Marketing incorporate preferences (Liechty et al, 2005; Horsky et al. 2006; Gilbride and Allenby, 2006). Keeney (2002) identified 12 important mistakes frequently made that limit one's ability in making good value judgments, in which "not understanding the decision context" and "failure to use consistency checks in assessing value trade-offs" are two critical mistakes. Hence, how to assist decision makers in understanding the decision context and adjusting inconsistencies in judgments are two important issues in ranking choices.

There is evidence that decision makers' preferences are often influenced by the visual background information (e.g., Simonson and Tversky 1992; Tversky and Simonson, 1993; Seiford and Zhu, 2003). From marketing it is known from consumer choice theories that context impacts the choices consumers make (Seiford and Zhu, 2003). For example, a product may appear attractive against a background of less attractive alternatives and unattractive when compared to more attractive alternatives (Simonson and Tversky, 1992). Visual representations can simplify and aggregate complex information into meaningful pattern, assist people in comprehending their environment, and allow for simultaneous perception of parts as well as a perception of interrelations between parts (Maruyama, 1986; Meyer, 1991; Sullivan, 1998). Hence, how to provide visual aids to help decision makers make a more informed decision is the first issue addressed by this study.

Ranking alternatives incorporating preferences is a popular issue in decision-making. One common format for expressing preferences is to use pairwise comparisons, which forces one to make a direct choice of one object over another when compariing two objects, rather than requiring one to comparing all objects simultaneously (Cook et al., 2005). For example, in sports competitions, such as tennis, football and baseball, pairwise rankings are the typical input (Hochbaum and Levin, 2006). Several methods have been proposed (e.g., Saaty, 1980; Jensen, 1984; Genest and Rivest, 1994) to rank alternatives in pairwise comparisons fashion. However, inconsistencies are not unexpected, as making value judgments is difficult (Keeney, 2002). The ranks different methods yield do not vary much when the decision makers' preferences are consistent. But, if a preference matrix is highly inconsistent, different ranking methods may produce wildly different priorities and rankings. Hence, how to help the decision makers to detect and improve those inconsistencies thus to make a more reliable decision is the second issue addressed here.

Multicriteria decision makers tend to use screening, ordering and choosing phases to find a preference (Brugha, 2004). They tend to make little effort in the first phase as they screen out clearly unwanted alternatives, use somewhat more effort in the second phase as they try to put a preference order on the remaining alternatives, and reach the highest effort in the final phase when making a choice between a few close alternatives.

This study develops a decision-making framework based on these three phases. Preferences in pairwise comparison fashion are adopted in the choosing phase. Two optimization models and a Decision Ball model are proposed to assist decision makers in improving inconsistency and observing relationships among alternatives. By examining Decision Balls, a decision maker can iteratively adjust preferences and improve inconsistencies thus to achieve a more consistent decision. The proposed approach can be extensively applied in Marketing. Possible applications are the selection of promotion plans, decisions regarding product sourcing, choice of marketing channels, evaluation of advertising strategy, research of customer behavior …etc.

The reasons why this study uses a sphere model instead of a traditional 2-dimensional plane or a 3-dimensional cube model are described as follows. A 2-dimensional plane model cannot depict three points that do not obey the triangular inequality (i.e. the total length of any two edges must be larger than the length of the third edge) neither can it display four points that are not on the same plane. For instance, as illustrated in Figure 1, consider three points, Q_1, Q_2, Q_3, where the distance between Q_1Q_2, Q_2Q_3, and Q_1Q_3 are 3, 1, and 6, respectively, as shown in Figure 1(b). It is impossible to show their relationships by three line segments on a 2-dimensional plane, as shown in Figure 1(a). If there are four points, Q_1, Q_2, Q_3, and Q_4, which are not on the same plane, as shown in Figure 1(c), it is impossible to present these four points on a 2-dimensional plane too. In addition, a sphere model is also easier for a decision maker to observe than a 3-dimensional cube model because the former exhibits alternatives on the surface of a sphere rather than inside the cube.

This paper is organized as follows. Section 2 reviews the relevant literature. Section 3 sets the three-phase decision making framework, including the screening, ordering and choosing phases. Section 4 proposes a weight-approximation model and a Decision Ball model to support a decision maker to filter out poor alternatives in the ordering phase. Section 5 develops an optimization model which can assist a decision maker in improving inconsistencies in preferences, and provides three methods to allow a decision maker to iteratively adjust his preferences in the choosing phase. Sections 4 and 5 form the main theoretical part of this paper; therefore, readers only interested in the application of proposed approach can skip these two sections. Section 6 uses an example to demonstrate the whole decision process.

2. Relevant Literature

Several visualization approaches have been developed to provide visual aids to support decision-making process. For instance, Li (1999) used deduction graphs to treat decision problems associated with expanding competence sets. Jank and Kannan (2005) proposed a spatial multinomial model of customer choice to assist firms in understanding how their online customers' preferences and choices vary across geographical markets. Kiang (2001) extended a self-organizing map (SOM) (Kohonen, 1995) network to classify decision groups by neural network techniques. Many studies (Kruskal, 1964; Borg and Groenen, 1997; Cox and Cox, 2000) adopted Multidimensional scaling (MDS), which is widely used in Marketing, to provide a visual representation of similarities among a set of alternatives. For instance, Desarbo and Jedidi (1995) proposed a new MDS method to spatially represent preference intensity collected over consumers' consideration sets. However, most of conventional visualization approaches are incapable of detecting and improving the decision makers' inconsistent preferences. Gower (1977), Genest and Zhang (1996) proposed a powerful graphical tool, the so-called Gower Plot, to detect the inconsistencies in decision maker's preferences on a 2-dimensional plane. Nevertheless, the Gower plots do not provide suggestions about how to improve those inconsistencies either.

A pairwise-comparison ranking problem can be provided with magnitude of the degree of preference, intensity ranking; or in terms of ordinal preferences only, preference ranking. These are sometimes referred to also as cardinal versus ordinal preference (Hochbaum and Levin, 2006). Many studies (Saaty, 1980; Saaty and Vargas, 1984; Hochbaum and Levin, 2006; etc.) use multicriteria decision making approaches to find a consistent ranking at minimum error. However, conventional eigenvalue approaches cannot treat preference matrix with incomplete judgments. And, most of them focus on adjusting cardinal or ordinal inconsistencies instead of adjusting both cardinal and ordinal inconsistencies simultaneously. Li and Ma (2006)(2007) developed goal programming models which can treat incomplete judgments and improve cardinal and ordinal inconsistencies simultaneously. However, the ranks of and similarities among alternatives can be displayed.

This study cannot only improve cardinal and ordinal inconsistencies simultaneously but provide visual aids to decision makers. They can observe ranks of and similarities among alternatives, and iteratively adjust their preferences to achieve a more consistent decision.

3. Setting the Decision-Making Framework

The proposed decision-making framework is illustrated by the screening, ordering, and choosing phases as listed below:

(i) The screening phase: the decision maker tries to screen out clearly unwanted alternatives. The decision maker specifies upper and/or lower bounds of attributes to screen out poor alternatives.

(ii) The ordering phase: the decision maker tries to put a preference order on the remaining alternatives.

- The decision maker roughly specifies partial order of alternatives.
- An optimization model and a Decision Ball model are developed to assist decision maker in calculating and viewing ranks of and similarities among alternatives.
- The decision maker filters out poor alternatives according to the information displayed on the Decision Ball.

(iii) The choosing phase: the decision maker tries to make a final choice among a few alternatives. There are four steps in this phase, including specifying pairwise-comparison preferences, detecting and improving inconsistencies, adjusting preferences, and determining the best alternatives.

- Specifying pairwise-comparison preferences. Decision maker has to make more sophisticated comparisons for the remaining alternatives in this phase. Pairwise-comparison fashion, like analytical hierarchy process (AHP; Saaty, 1980), is adopted here because it is good for choosing phase (Brugha, 2004).
- Detecting and improving inconsistencies. Because inconsistent preferences may result in unreliable rank order, significant inconsistencies should be modified to obtain a more consistent solution. An optimization model is proposed to assist decision maker in detecting and improving inconsistencies. After inconsistencies have been reduced, the ranks of and similarities among alternatives are calculated and displayed on a Decision Ball.
- Adjusting preferences. According to the information displayed on the Decision Ball, the decision maker can iteratively adjust his preferences and see the corresponding changes on the Decision Ball.
- Determining the best alternatives. Decision maker makes the final choice with the assistance of the Decision Ball.

The detailed explanations about the ordering and choosing phases are illustrated in the following two sections.

4. The models for ordering phase

Consider a set of alternatives $\mathbf{A} = \{A_1, A_2, \ldots, A_n\}$ for solving a choice problem, where the decision maker selects m criteria to fulfill. The values of criteria $c_1, \ldots, c_m$ for alternative A_i are expressed as $c_{i,k}$, for $k = 1,\ldots, m$. All criterion values are assumed to be continuous data. Denote C = $[c_{i,k}]_{n\times m}$ as the criterion matrix of the decision problem. Denote $\underline{c_k}$ and $\overline{c_k}$ as the lower and upper bounds of the criterion value of c_k, respectively. The value of $\underline{c_k}$ and $\overline{c_k}$ can be either given by the decision maker directly or calculated by the minimum and maximum raw criterion value of c_k. The score function in this study is assumed to be in an additive form because it is the most commonly used form in practice and more understandable for the decision maker (Belton and Stewart, 2002). Denote S_i as the score value of an alternative A_i. An additive score function of an alternative A_i ($c_{i,1}$, $c_{i,2}$, …, $c_{i,m}$) is defined as below:

$$S_i(\mathbf{w}) = \sum_{k=1}^{m} w_k \frac{c_{i,k} - \underline{c_k}}{\overline{c_k} - \underline{c_k}}, \tag{1}$$

where (i) w_k is the weight of criterion k, $w_k \geq 0,\ \forall k$ and $\sum_{k=1}^{m} w_k = 1$. $\mathbf{w} = (w_1, w_2, ..., w_m)$ is a weight vector, (ii) $0 \leq S_i(\mathbf{w}) \leq 1$. In order to make sure that all weights of criteria and scores of alternatives are positive, a criterion $c_{i,k}$ with cost feature (i.e., a DM likes to keep it as small as possible) is transferred from $c_{i,k}$ to ($\overline{c_k} - c_{i,k}$) in advance.

Following the score function, the dissimilarity function of reflecting the dissimilarity between alternatives A_i and A_j is defined as

$$\delta_{i,j}(\mathbf{w}) = \sum_{k=1}^{m} w_k \frac{|c_{i,k} - c_{j,k}|}{\overline{c_k} - \underline{c_k}}, \tag{2}$$

where $0 \leq \delta_{i,j}(\mathbf{w}) \leq 1$ and $\delta_{i,j}(\mathbf{w}) = \delta_{j,i}(\mathbf{w})$. Clearly, if $c_{i,k} = c_{j,k}$ for all k then $\delta_{i,j}(\mathbf{w}) = 0$.

In the ordering phase, a decision maker has to roughly specify partial order of alternatives. If the decision maker prefers A_i to A_j, denoted as $A_i \succ A_j$, score of A_i should be higher than that of A_j ($S_i > S_j$). However, there may be some inconsistent preferences. For instance, a decision maker may specify $A_i \succ A_j$, $A_j \succ A_k$ and $A_k \succ A_i$. A binary variable $t_{i,j}$ is used to record the inconsistent relationship between A_i and A_j: if $A_i \succ A_j$ and $S_i > S_j$, then $t_{i,j}$ =0; otherwise, $t_{i,j}$ = 1. A weight approximation model for ordering phase is developed as follows:

<u>Model 1 (Weight approximation model for ordering phase)</u>

$$\underset{\{w_k\}}{Min} \quad \sum_{i=1}^{n}\sum_{j=1}^{n} t_{i,j}$$

s.t.

$$S_i(\mathbf{w}) = \sum_{k=1}^{m} w_k \frac{c_{i,k} - \underline{c_k}}{\overline{c_k} - \underline{c_k}}, \quad \forall i, \tag{3}$$

$$\sum_{k=1}^{m} w_k = 1, \tag{4}$$

$$S_i \geq S_j + \varepsilon - Mt_{i,j}, \quad \forall A_i \succ A_j, \tag{5}$$

$$\underline{w_k} \le w_k \le \overline{w_k},\ w_k \ge 0, \forall k, \tag{6}$$

$u_{i,j} \in \{0,1\}$, M is a large value, ε is a tolerable error. (7)

The objective of Model 1 is to minimize the sum of $t_{i,j}$. Expressions (3) and (4) are from the definition of an additive score function (1). Expression (5) indicates that if $A_i \succ A_j$ and $S_i \ge S_j + \varepsilon$, then $t_{i,j}$ =0; otherwise, $t_{i,j}$ = 1, where ε and M are a computational precision and a large value which can be normally set as 10^{-6} and 10^6, respectively. Denote $\underline{w_k}$ and $\overline{w_k}$ as the lower and upper bound of w_k, which could be set by the decision maker as in Expression (6). From (1) and (2), the score S_i of alternative A_i and dissimilarity $\delta_{i,j}$ between alternative A_i and A_j can be calculated based on the results of Model 1.

A Decision Ball model is then constructed to display all alternatives A_i in A = $\{A_1, A_2, \ldots, A_n\}$ on the surface of a hemisphere. A non-metric multidimensional scaling technique is adopted here to provide a visual representation of the dissimilarities among alternatives. The arc length between two alternatives is used to represent the dissimilarity between them, e.g., the larger the difference, the longer the arc length. However, because the arc length is monotonically related to the Euclidean distance between two points and both approximation methods make little difference to the resulting configuration (Cox and Cox, 1991), the Euclidean distance is used here for simplification.

In addition, the alternative with a higher score is designed to be closer to the North Pole so that alternatives will be located on the concentric circles in the order of score from top view. For the purpose of comparison, we define an ideal alternative A_*, where $A_* = A_*(\overline{c_1}, \overline{c_2}, \ldots, \overline{c_m})$ and $S_* = 1$. A_* is designed to be located at the north pole with coordinate (x_*, y_*, z_*) = (0, 1, 0).

The following propositions are deduced:

Proposition 1 The relationship between $\delta_{i,*}(\mathbf{w})$ (the dissimilarity between A_i and A_*)) and $S_i(\mathbf{w})$ is expressed as $\delta_{i,*}(\mathbf{w}) = 1 - S_i(\mathbf{w})$.

<Proof> $$\delta_{i,*}(\mathbf{w}) = \sum_{k=1}^{m} w_k \frac{|c_{i,k} - \overline{c_k}|}{\overline{c_k} - \underline{c_k}} = \sum_{k=1}^{m} w_k \frac{(\overline{c_k} - \underline{c_k}) - (c_{i,k} - \underline{c_k})}{\overline{c_k} - \underline{c_k}}$$

$$= \left(\sum_{k=1}^{m} w_k \frac{(\overline{c_k} - \underline{c_k})}{\overline{c_k} - \underline{c_k}} - \sum_{k=1}^{m} w_k \frac{(c_{i,k} - \underline{c_k})}{\overline{c_k} - \underline{c_k}}\right) = 1 - S_i(\mathbf{w})$$

Denote $d_{i,j}$ as the Euclidean distance between A_i and A_j . Let $d_{i,j} = \sqrt{2}\delta_{i,j}$, such that if

$\delta_{i,j}$ = 0 then $d_{i,j}$ = 0 and if $\delta_{i,j}$ = 1 then $d_{i,j} = \sqrt{2}$, where $\sqrt{2}$ is used because the distance between the north pole and equator is $\sqrt{2}$ when radius = 1. Denote the coordinates of an alternative A_i on a ball as (x_i, y_i, z_i). The relationship between y_i and S_i is expressed as

<u>Proposition 2</u> $y_i = 2S_i - S_i^2$.

<Proof> Since $d_{i,*}^2 = (x_i - 0)^2 + (y_i - 1)^2 + (z_i - 0)^2 = 2\delta_{i,*}^2 = 2(1 - S_i)^2$,

it is clear $y_i = 2S_i - S_i^2$. Clearly, if S_i = 1 then y_i = 1; if S_i = 0, then y_i = 0.

Based on the non-metric multidimensional scaling technique, denote $\hat{d}_{i,j}$ as a monotonic transformation of $\delta_{i,j}$ satisfying following condition: if $\delta_{i,j} < \delta_{p,q}$, then $\hat{d}_{i,j} < \hat{d}_{p,q}$. The coordinate ($x_i$, y_i, z_i) of alternative A_i all i can be calculated by the following Decision Ball model:

Model 2 (A Decision Ball Model)

$$\min_{\{x_i, y_i, z_i\}} \sum_{i=1}^{n} \sum_{j>i}^{n} (d_{i,j} - \hat{d}_{i,j})^2$$

s.t. $y_i = 2S_i - S_i^2, \ \forall i,$ (8)

$\hat{d}_{i,j} \le \hat{d}_{p,q} - \varepsilon, \ \forall \delta_{i,j} < \delta_{p,q},$ (9)

$d_{i,j}^2 = (x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2, \ \forall i, j,$ (10)

$x_i^2 + y_i^2 + z_i^2 = 1, \quad \forall i,$ (11)

$-1 \le x_i, z_i \le 1, \ 0 \le y_i \le 1, \ \forall i, \ \varepsilon$ is a tolerable error. (12)

The objective of Model 2 is to minimize the sum of squared differences between $d_{i,j}$ and $\hat{d}_{i,j}$. Expression (8) is from Proposition 2, where the alternative with a higher score is designed to be closer to the North Pole. Expression (9) is the monotonic transformation from $\delta_{i,j}$ to $\hat{d}_{i,j}$. All alternatives are graphed on the surface of the northern hemisphere (11)(12).

Model 2 is a nonlinear model, which can be solved by some commercialized optimization software, such as Global Solver of Lingo 9.0, to obtain an optimum solution. One restriction of this model is the running time that may considerably increase when the number of alternatives becomes large because the time complexity of Model 2 is n^2. This model has good performance when the number of alternatives less than 10. However, in this case of alternatives more than 10, some classification techniques, like k-means (MacQueen,1967) for instance, can be used to reduce the solving time by dividing alternatives into several groups. The coordinates of group centers are calculated first. Then, these group centers are treated as anchor points. The coordinates of alternatives can be obtained by calculating dissimilarity between alternatives and anchor points. Thus, all alternatives can be displayed on the Decision Ball within tolerable time.

According to the information displayed on the Decision Ball, the decision maker can select better alternatives into the next phase.

5. The models for choosing phase

In this phase, the decision maker has to make more sophisticated comparisons for the remaining alternatives. Pairwise comparisons are adopted here (Brugha, 2004). For some i and j pairs, assume a decision maker can specify $p_{i,j}$, the ratio of the score of A_i to that of A_j, which is expressed as

$$p_{i,j} = \frac{S_i}{S_j} \times e_{i,j} , \qquad (13)$$

where S_i is the score of A_i and $e_{i,j}$ is a multiplicative term accounting for inconsistencies, as illustrated in the Analytic Hierarchy Process (AHP) (Saaty, 1980). It is assumed that $p_{i,j} = 1/p_{j,i}$. If the decision maker cannot specify the ratio for a specific pair i and j then $p_{i,j} = \phi$. Denote **P** $=[p_{i,j}]_{n\times n}$ as a $n \times n$ preference matrix. **P** is incomplete if there is any $p_{i,j} = \phi$. **P** is perfectly consistent if $e_{i,j}$ =1 for all i, j (i.e. $p_{i,j} = S_i/S_j$ for all i, j). **P** is ordinally inconsistent (intransitive) if for some $i, j, k \in \{1, 2, 3, \ldots, n\}$ there exists $p_{i,j} > 1$, $p_{j,k} > 1$, but $p_{i,k} < 1$. **P** is cardinally inconsistent if for some $i, j, k \in \{1, 2, 3, \ldots, n\}$ there exists $p_{i,k} \neq p_{i,j} \times p_{j,k}$ (Genest and Zhang, 1996).

If **P** is complete and ordinal consistent, all A_i can be ranked immediately. However, if there is ordinal or highly cardinal inconsistency, these inconsistencies should be improved before ranking because significant inconsistencies may result in unreliable rank order.

An optimization model, developed by a goal-programming optimization technique, is developed to assist decision maker in detecting and improving inconsistencies. In order to reduce the ordinal inconsistency, a binary variable $u_{i,j}$ is used to record if the preference $p_{i,j}$, specified by the decision maker, is suggested to be reversed or not. If $p_{i,j}$ is suggested to be reversed, then $u_{i,j}$

= 1; otherwise, $u_{i,j}$ = 0. A variable $\alpha_{i,j}$, defined as the difference between $p_{i,j}$ and S_i/S_j, is used to indicate the degree of cardinal inconsistency of $p_{i,j}$: the larger the value of $\alpha_{i,j}$, the higher the cardinal inconsistency. The inconsistencies improving model is formulated as below:

Model 3 (Inconsistencies improving model)

$$\operatorname*{Min}_{\{w_k\}} \quad M \times Obj1 + Obj2$$

$$Obj1 = \sum_{i=1}^{n}\sum_{j>i}^{n} u_{i,j}$$

$$Obj2 = \sum_{i=1}^{n}\sum_{j>i}^{n} \alpha_{i,j}$$

s.t. $$\left(\frac{S_i}{S_j}-1\right)\times(p_{i,j}-1)+M\times u_{i,j}\geq\varepsilon, \quad \text{for all } i, j \text{ where } p_{i,j}\neq\phi \text{ and } p_{i,j}\neq 1, \tag{14}$$

$$-\left|S_i - S_j\right| + M\times u_{i,j}\geq 0, \quad \text{for all } i,j \text{ where } p_{i,j}=1, \tag{15}$$

$$\left|\frac{S_i}{S_j}-p_{i,j}\right|\leq\alpha_{i,j}, \ \forall i, j, \tag{16}$$

$$S_i(\mathbf{w})=\sum_{k=1}^{m} w_k \frac{c_{i,k}-\underline{c_k}}{\overline{c_k}-\underline{c_k}}, \quad \forall i, \tag{17}$$

$$\sum_{k=1}^{m} w_k = 1, \tag{18}$$

$$\underline{w_k}\leq w_k\leq\overline{w_k}, \qquad w_k\geq 0, \forall k, \tag{19}$$

$$u_{i,j}\in\{0,1\}, M \text{ is a large value, } \varepsilon \text{ is a tolerable error.} \tag{20}$$

This model tries to improve ordinal and cardinal inconsistencies simultaneously. The first objective ($Obj1$) is to achieve ordinal consistency by minimizing the number of preferences (i.e., $p_{i,j}$) being reversed. Constraint (14) means: when $p_{i,j}\neq\phi$ and $p_{i,j}\neq 1$, $u_{i,j}$ = 0, if (i) $(\frac{S_i}{S_j}>1)$ and $(p_{i,j}>1)$ or (ii) $(\frac{S_i}{S_j}<1)$ and $(p_{i,j}<1)$; and otherwise $u_{i,j}$ = 1. A tolerable positive number ε is used to avoid $\frac{S_i}{S_j}=1$. Constraint (15) means: when $p_{i,j}$ = 1, if $S_i = S_j$; then

$u_{i,j}$ = 0; otherwise $u_{i,j}$ = 1. The second objective (*Obj*2) is to reduce cardinal consistency by minimizing the $\alpha_{i,j}$ values, i.e. to minimize the difference between $\frac{S_i}{S_j}$ and $p_{i,j}$. Since ordinal consistency (*Obj*1) is more important than cardinal consistency (*Obj*2), *Obj*1 is multiplied by a large value *M* in the objective function. Constraints (17) and (18) come from Notation 1. Constraint (19) sets the upper and lower bound of weights. An improved complete preference matrix can be obtained as $\mathrm{P}' = [p'_{i,j}]_{n\times n}$, where $p'_{i,j} = \frac{S_i}{S_j}$ if $p_{i,j} = \phi$ or $u_{i,j}$ = 1; otherwise, $p'_{i,j} = p_{i,j}$.

Model 3 is a nonlinear model, which can be converted into the following linear mixed 0-1 program:

$$\underset{\{w_k\}}{\text{Min}} \quad M \times Obj1 + Obj2$$

$$Obj1 = \sum_{i=1}^{n}\sum_{j>i}^{n} u_{i,j}$$

$$Obj2 = \sum_{i=1}^{n}\sum_{j>i}^{n} \alpha_{i,j}$$

s.t. $(S_i - S_j)\times(p_{i,j}-1) + M\times u_{i,j} \geq \varepsilon$, for all i, j where $p_{i,j} \neq \phi$ and $p_{i,j} \neq 1$, (21)

$-M\times u_{i,j} \leq S_i - S_j \leq M\times u_{i,j}$, for all i,j where $p_{i,j} = 1$, (22)

$S_j \times p_{i,j} - \alpha_{i,j} \leq S_i \leq S_j \times p_{i,j} + \alpha_{i,j}$, $\forall i, j$, (23)

(17) ~ (20),

where (21), (22) and (23) are converted from (14), (15) and (16) respectively.

After the weight vector, (w_1, w_2, …, w_m), is found, $S_i(\mathbf{w}) = \sum_{k=1}^{m} w_k \frac{c_{i,k} - \underline{c_k}}{\overline{c_k} - \underline{c_k}}$ and $\delta_{i,j}(\mathbf{w}) = \sum_{k=1}^{m} w_k \frac{|c_{i,k} - c_{j,k}|}{\overline{c_k} - \underline{c_k}}$ can be calculated. All alternatives are shown on a Decision Ball by Model 2.

According to the information visualized on the Decision Ball, the decision maker can iteratively adjust his preferences by the following ways:

(i) Adjusting preference order. Since alternative with a higher score is designed to be closer to the North Pole so that a decision maker can see the rank order by the location of alternative: the higher the latitude, the higher the score. If the decision maker would like to adjust a preference order, from $A_1 \prec A_3$ to $A_1 \succ A_3$ for instance, a constraint $S_1 \geq S_3 + \varepsilon$ will be added into Model 3.

(ii) Adjusting dissimilarity. The distance between two alternatives on a Decision Ball implies the dissimilarity between them: the larger the dissimilarity, the longer the distance. Therefore, if a decision maker observes the Decision Ball and decides to adjust the dissimilarity relationship, from $\delta_{1,3}(\mathbf{w}) < \delta_{1,2}(\mathbf{w})$ to $\delta_{1,3}(\mathbf{w}) > \delta_{1,2}(\mathbf{w})$ for example, a constraint $\delta_{1,3}(\mathbf{w}) > \delta_{1,2}(\mathbf{w})$ (i,e. $\sum_{k=1}^{m} w_k \frac{|c_{1,k} - c_{3,k}|}{\overline{c_k} - \underline{c_k}} \geq \sum_{k=1}^{m} w_k \frac{|c_{1,k} - c_{2,k}|}{\overline{c_k} - \underline{c_k}} + \varepsilon$) will be added into Model 3.

(iii) Adjusting preference matrix. A decision maker can choose to adjust the preference matrix directly. The value of $p_{i,j}$ in Model 3 will be modified according to the change in the preference matrix.

Solving Model 3 yields a new set of weights, and an adjusted Decision Ball will be displayed. The decision maker can iteratively adjust his preferences until he feels no adjustments have to be made. A final choice can be made with the assistance of a resulting Decision Ball.

6. Application to choice data: selection of a store location

Example 1 (*Selection of a store location*)

The choice of a store location has a profound effect on the entire business life of a retail operation. Suppose a manager of a convenience store in Taiwan who needs to select a store location from a list of 43 spots $\mathbf{A} = \{A_1, \ldots, A_{43}\}$. The manager sets four criteria to fulfill: (c_1) sufficient space, (c_2) high population density, (c_3) heavy traffic, and (c_4) low cost. Store size is measured in square feet. The number of people who live within a one-mile radius is used to calculate population density. The average number of vehicle traffic passing the spot per hour is adopted to evaluate the volumes of traffic. Cost is measured by monthly rent. The criteria values of 43 candidate locations are listed in the criterion matrix $\mathbf{C_1}$, as shown in Table 1.

The manager would like to rank choices incorporating his personal preferences. The manager can rank these choices by the following three phases:

Phase 1 – the screening phase

The manager tries to screen out clearly unwanted alternatives by setting upper or lower bound of each criterion. He sets the minimum space required to be 800 square feet, the minimum population density to be 700, the minimal traffic to be 400, and the maximum rental fee to be

5000. That is, $\underline{c_1}$ = 800, $\underline{c_2}$ = 700, $\underline{c_3}$ = 400 and $\overline{c_4}$ = 5000. The values of $\overline{c_1}$, $\overline{c_2}$, $\overline{c_3}$ and $\underline{c_4}$ can be set as the maximum values of c_1, c_2, c_3 and minimum value of c_4, i.e. $\overline{c_1}$ = 1500, $\overline{c_2}$ = 1260, $\overline{c_3}$ =780, and $\underline{c_4}$ = 3100. After filtering out alternatives with criterion values exceeding these boundaries, only 23 choices {A_3, A_4, A_6, A_7, A_8, A_{11}, A_{13}, A_{15}, A_{17}, A_{18}, A_{21}, A_{23}, A_{24}, A_{25}, A_{26}, A_{29}, A_{31}, A_{32}, A_{34}, A_{37}, A_{40}, A_{42}, A_{43}} are remaining for the next phase.

Phase 2 – the ordering phase

The decision maker roughly specifies partial order of alternatives. He specifies $A_3 \succ A_7$, $A_7 \succ A_{37}$, $A_{15} \succ A_8$, $A_{17} \succ A_6$, $A_{31} \succ A_{25}$ and $A_{42} \succ A_{40}$. The minimum weight of each criterion is set as $\underline{w_k}$ = 0.01 for all k by the decision maker. Applying Model 1 to these preference relationships yields w = {w_1, w_2, w_3, w_4} = {0.21, 0.43, 0.01, 0.35}, $t_{15,8}$ =1, and the rest of $t_{i,j}$ = 0. The objective value is 1. The variable $t_{15,8}$ = 1 indicates the preference relationship $A_{15} \succ A_8$ should be reversed. When checking criterion matrix in Table 1, all criterion values of A_8 are better than or equal to those of A_{15} which makes $A_{15} \succ A_8$ impossible; therefore, the relationship between A_{15} and A_8 is reversed.

The score of alternatives can be calculated according to Expression (1), where S_3 = 0.54, S_4 = 0.10, S_6 = 0.33, S_7 = 0.54, S_8 = 0.71, S_{11} = 0.29, S_{13} = 0.59, S_{15} = 0.36, S_{17} = 0.53, S_{18} = 0.31, S_{21} = 0.30, S_{23} = 0.30, S_{24} = 0.45, S_{25} = 0.22, S_{26} = 0.39, S_{29} = 0.23, S_{31} = 0.22, S_{32} = 0.42, S_{34} = 0.46, S_{37} = 0.39, S_{40} = 0.31, S_{42} = 0.34, S_{43} = 0.24. The dissimilarity between alternatives can also be calculated according to Expression (2).

Applying Model 2 to this example yields coordinates of alternatives. The resulting Decision Ball is displayed in Figure 2. Because the alternative with a higher score is designed to be closer to the North Pole, the order of alternatives can be read by the latitudes of alternative: the higher the latitude, the higher the score. The order of top ten alternatives is $A_8 \succ A_{13} \succ A_3 \succ A_7 \succ A_{17} \succ A_{34} \succ A_{24} \succ A_{32} \succ A_{37} \succ A_{26}$. In addition, the distance between two alternatives represents the dissimilarity between them: the longer the distance, the larger the dissimilarity. For instance, the dissimilarity between A_{26} and A_{37} is smaller than that of between A_{37} and A_7.

Based on the information provided on the Decision Ball, assume the decision maker decides to select the top eight alternatives to make more sophisticated comparisons. That is, only A_8, A_{13}, A_3, A_7, A_{17}, A_{34}, A_{24} and A_{32} are remaining for the next phase.

Phase 3 – the choosing phase

In the choosing phase, the manager uses pairwise comparisons to express preferences among pairs of choices in preference matrix $\mathbf{R_1}$, as listed in Table 2. Because the manager is unable to make comparison among some spots, the relationships $p_{3,34}$, $p_{7,17}$, $p_{8,24}$, $p_{13,34}$ are left blank, which means $\mathbf{R_1}$ is incomplete. The preference matrix $\mathbf{R_1}$ is ordinally inconsistent because there is an intransitive relationship among A_3, A_8 and A_{32}. That is, A_3 is preferred to A_8 ($p_{3,8} > 1$), and A_8 is preferred to A_{32} ($p_{8,32} > 1$); however, A_{32} is preferred to A_3 ($p_{3,32} < 1$). $\mathbf{R_1}$ is also cardinally inconsistent. For instance, there exists $p_{3,8} = 1.6$, $p_{8,13} = 2.5$; but, $p_{3,13} = 2$ ($1.6 \times 2.5 = 4$, that is $p_{3,8} \times p_{8,13} \neq p_{3,13}$).

Applying Model 3 to the example yields $Obj1 = 1$, $Obj2 = 3.91$, $u_{3,8} = 1$ and the rest of $u_{i,j} = 0$, $(w_1, w_2, w_3, w_4) = (0.04, 0.19, 0.06, 0.71)$, $(S_3, S_7, S_8, S_{13}, S_{17}, S_{24}, S_{32}, S_{34}) = (0.55, 0.55, 0.78, 0.27, 0.39, 0.40, 0.74, 0.51)$. The variable $u_{3,8} = 1$ implies that the value of $p_{3,8}$ is suggested to be changed from $p_{3,8} > 1$ to $p_{3,8} < 1$ (i.e. from $A_3 \succ A_8$ to $A_3 \prec A_8$) to improve ordinal inconsistency. The values of unspecified preferences can be computed as $p_{3,34} = \frac{S_3}{S_{34}} = 1.08$, , $p_{7,17}$, $= 1.41$, $p_{8,24} = 1.93$, and $p_{13,34} = 0.76$. The corresponding Decision Ball is shown in Figure 3. The order of alternatives is $A_8 \succ A_{32} \succ A_3 \succ A_7 \succ A_{34} \succ A_{24} \succ A_{17}$.

According to the information observed on the Decision Ball, the decision maker can iteratively adjust his preferences. Suppose he would like to adjust a preference order from $A_7 \succ A_{34}$ to $A_{34} \succ A_7$. A constraint $S_{34} \geq S_7 + \varepsilon$ is added into Model 3. Solving Model 3 yields $Obj1 = 3$, $Obj2 = 3.96$, $u_{3,8} = u_{7,34} = u_{17,24} = 1$ and the rest of $u_{i,j} = 0$, $(w_1, w_2, w_3, w_4) = (0.01, 0.13, 0.17, 0.69)$, $(S_3, S_7, S_8, S_{13}, S_{17}, S_{24}, S_{32}, S_{34}) = (0.53, 0.50, 0.76, 0.27, 0.44, 0.40, 0.71, 0.51)$. In order to satisfy the relationship $A_{34} \succ A_7$, the relationship between A_{17} and A_{24} has to be reversed ($u_{17,24} = 1$). Applying Model 2 to this result yields a new set of coordinates. An adjusted Decision Ball is displayed in Figure 4. On this Decision Ball, the latitude of A_{34} is higher than that of A_7.

By seeing the relationships of alternatives displayed on the Decision Ball in Figure 4, the decision maker would like to adjust some dissimilarity relationships between alternatives. His adjustment is that the dissimilarity between A_3 and A_8 is larger than that of between A_7 and A_8. A constraint $\sum_{k=1}^{m} w_k \frac{|c_{3,k} - c_{8,k}|}{\overline{c_k} - \underline{c_k}} \geq \sum_{k=1}^{m} w_k \frac{|c_{7,k} - c_{8,k}|}{\overline{c_k} - \underline{c_k}} + \varepsilon$ is added into Model 3. Solving Model 3 again yields $Obj1 = 5$, $Obj2 = 4.33$, $u_{3,8} = u_{7,34} = u_{17,24} = u_{3,7} = u_{8,32} = 1$ and the rest of $u_{i,j} = 0$, $(w_1, w_2, w_3, w_4) = (0.01, 0.04, 0.19, 0.76)$, $(S_3, S_7, S_8, S_{13}, S_{17}, S_{24}, S_{32}, S_{34}) = (0.51, 0.53, 0.74, 0.19, 0.39, 0.36, 0.78, 0.53)$. This result shows that in addition to rank reversal of A_3 and A_8, A_7 and A_{34}, A_{17} and A_{24} ($u_{3,8} = u_{7,34} = u_{17,24} = 1$), the relationship between A_3 and A_7, A_8 and A_{32} are suggested to be reversed to satisfy the adjustment of dissimilarity. A corresponding Decision Ball is depicted in **Figure 5.**

Suppose the decision maker stops further adjustment. The decision maker can make a final decision based on the Decision Ball in Figure 5. From the latitude of alternatives, the decision maker can tell the rank of choices as $A_{32} \succ A_8 \succ A_{34} \succ A_7 \succ A_3 \succ A_{17} \succ A_{24} \succ A_{13}$. The best choice is A_{32}. The dissimilarity between alternatives can be read by the distance between them. For instance, the dissimilarity between A_3 and A_{34} is the smallest because the distance between them is the shortest. That is, if A_{32} , A_8 and A_{34} are not available, A_3 as well as A_7 will be a good choice.

It is important to notice that A_3 is more similar to A_{34} than A_7 is but $A_{34} \succ A_7 \succ A_3$. This kind of relationship is possible. For instance, comparing with three alternatives A, B, C with benefit criterion values (5, 5, 5), (4, 4, 6) and (3, 5, 5), given equal weight and $\underline{c_k}$ = 0 and $\overline{c_k}$ =10 for k = 1…3. The scores of three alternatives are S_A = 0.5, S_B = 0.47, and S_C = 0.43. The dissimilarities between alternatives are $\delta_{A,B}$ =0.1, $\delta_{B,\mathrm{C}}$ =0.1 and $\delta_{A,\mathrm{C}}$ =0.067. It is obvious that $A \succ B \succ C$ but C is more similar to A than B is because $\delta_{A,\mathrm{C}} < \delta_{A,B}$.

Example 1 was solved by Global Solver of Lingo 9.0 [20] on a Pentium 4 personal computer. The running time was less than 3 minimums for three phases totally.

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Table 1 Criterion Matrix C_1 of Example 1

Alternative	C_1 Store Size	C_2 Population	C_3 Traffic	C_4 Rental Fee
A_1	1600	580	320	3200
A_2	390	680	450	2900
A_3	850	1140	550	4000
A_4	1000	750	440	5000
A_5	900	840	450	5500
A_6	1000	900	500	4400
A_7	1500	840	450	3800
A_8	800	1260	600	3500
A_9	755	700	400	1800
A_{10}	1400	600	500	4800
A_{11}	1100	720	480	4000
A_{12}	700	800	450	4800
A_{13}	1300	1250	650	4950
A_{14}	1250	1500	800	6800
A_{15}	800	900	420	3900
A_{16}	820	500	450	3200
A_{17}	1000	1200	780	4600
A_{18}	1300	720	420	4200
A_{19}	950	700	330	3500
A_{20}	1550	550	390	4100
A_{21}	850	780	480	3800

Alternative	C_1 Store Size	C_2 Population	C_3 Traffic	C_4 Rental Fee
A_{23}	960	750	650	3900
A_{24}	860	1100	550	4350
A_{25}	866	810	550	4400
A_{26}	1058	750	450	3500
A_{27}	998	1100	750	5200
A_{28}	665	900	650	3900
A_{29}	1055	800	450	4600
A_{30}	1008	900	650	5100
A_{31}	1100	850	520	4950
A_{32}	885	720	420	3100
A_{33}	750	780	185	2800
A_{34}	1205	880	580	3950
A_{35}	1900	400	280	3000
A_{36}	680	1500	950	5200
A_{37}	920	780	480	3400
A_{38}	1204	1200	550	5300
A_{39}	580	1000	850	5500
A_{40}	850	960	520	4500
A_{41}	565	665	380	2500
A_{42}	980	920	650	4400
A_{43}	810	810	520	4200

Table 2 Preference matrix R_1 of Example 1

$p_{i,j}$	A_3	A_7	A_8	A_{13}	A_{17}	A_{24}	A_{32}	A_{34}
A_3		1.4	1.6	2	1.2	2	0.5	
A_7			0.5	1.5		2	0.5	2
A_8				2.5	2		1.2	1.5
A_{13}					0.6	0.6	0.8	
A_{17}						0.5	0.5	0.7
A_{24}								0.5
A_{32}								2
A_{34}								

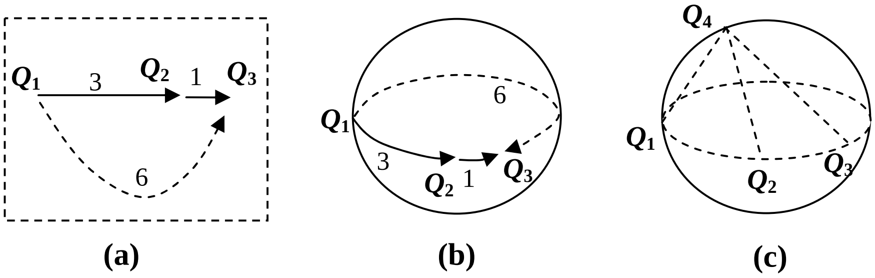

Figure 1 Advantage of a sphere model (a) Display line segments on a 2-D plane (b) Display curves on a sphere (c) Display four points that are not on the same plane

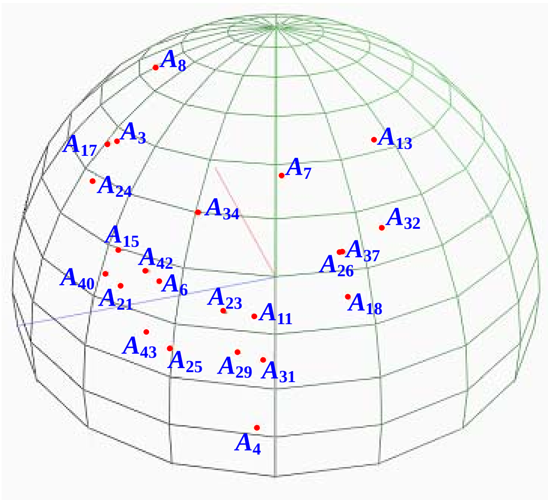

Figure 2 The resulting Decision Ball after Phase 2

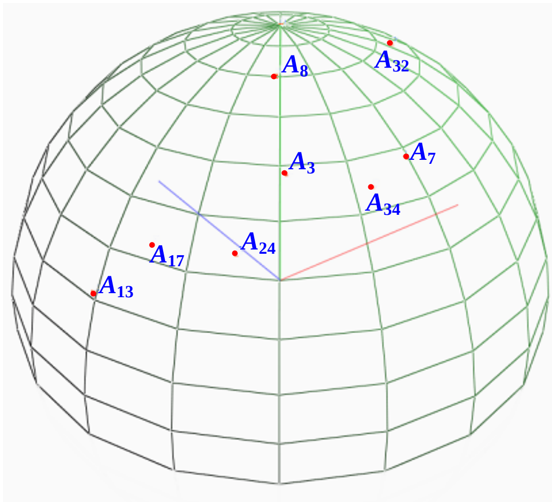

Figure 3 The resulting Decision Ball after Phase 3

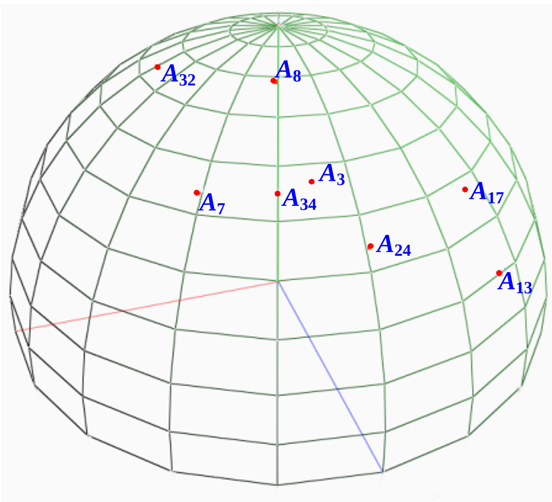

Figure 4 The resulting Decision Ball after adjusting $A_{34} \succ A_7$

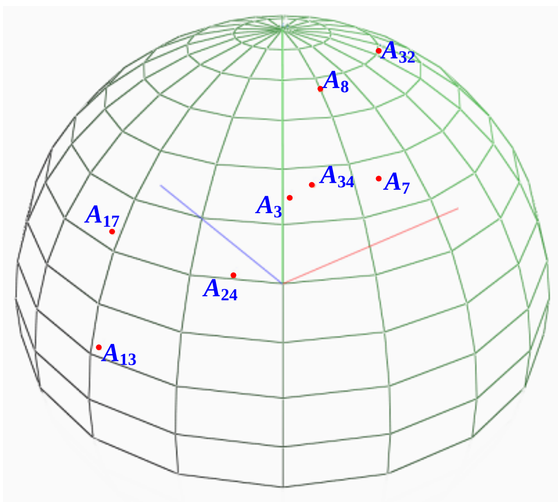

Figure 5 The resulting Decision Ball after adjusting $\delta_{3,8} > \delta_{7,8}$