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決策球模式及其運用

Decision Ball Models and Applications

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Abstract

Many decision-making or choice problems in Marketing incorporate preferences. How to assist decision makers in understanding the decision context and improving inconsistencies in judgments are two important issues in ranking choices. This study develops a decision-making framework based on the screening, ordering, and choosing phases. Two optimization models and a Decision Ball model are proposed to assist decision makers in improving inconsistencies and observing relationships among alternatives. By examining a Decision Ball, a decision maker can observe ranks of and similarities among alternatives, and iteratively adjust preferences and improve inconsistencies thus to achieve a more consistent and informed decision.

Key words: Decision Ball; Ranking; Inconsistency; Decision-making

摘要

在行銷偏好的選擇上有很多的決策擬定跟選擇的問題。如何去幫助決策者瞭解決策問題的內容及改善不一致性是選擇性排序的兩個主要議題。本研究以檢視、排序及選擇等步驟發展了一套決策制定的架構方法，其架構包含有兩個最佳化模型及一個決策球模型。目的在提供使用者改善偏好的不一致性及檢視方案選擇之間的關係。利用決策球，決策者可以看出決策方案之間的相似度及優先順序，並且透過不斷的調整喜好程度來改善決策過中出現的偏好不一致，使得決策者能夠獲得一個更好的決策資訊。

關鍵字：決策球、排序、不一致性、決策擬定

Part I

1. Introduction

Many decision-making or choice problems in Marketing incorporate preferences (Liechty et al, 2005; Horsky et al. 2006; Gilbride and Allenby, 2006). Keeney (2002) identified 12 important mistakes frequently made that limit one's ability in making good value judgments, in which "not understanding the decision context" and "failure to use consistency checks in assessing value trade-offs" are two critical mistakes. Hence, how to assist decision makers in understanding the decision context and adjusting inconsistencies in judgments are two important issues in ranking choices.

There is evidence that decision makers' preferences are often influenced by the visual background information (e.g., Simonson and Tversky 1992; Tversky and Simonson, 1993; Seiford and Zhu, 2003). From marketing it is known from consumer choice theories that context impacts the choices consumers make (Seiford and Zhu, 2003). For example, a product may appear attractive against a background of less attractive alternatives and unattractive when compared to more attractive alternatives (Simonson and Tversky, 1992). Visual representations can simplify and aggregate complex information into meaningful pattern, assist people in comprehending their environment, and allow for simultaneous perception of parts as well as a perception of interrelations between parts (Maruyama, 1986; Meyer, 1991; Sullivan, 1998). Hence, how to provide visual aids to help decision makers make a more informed decision is the first issue addressed by this study.

Ranking alternatives incorporating preferences is a popular issue in decision-making. One common format for expressing preferences is to use pairwise comparisons, which forces one to make a direct choice of one object over another when compariing two objects, rather than requiring one to comparing all objects simultaneously (Cook et al., 2005). For example, in sports competitions, such as tennis, football and baseball, pairwise rankings are the typical input (Hochbaum and Levin, 2006). Several methods have been proposed (e.g., Saaty, 1980; Jensen, 1984; Genest and Rivest, 1994) to rank alternatives in pairwise comparisons fashion. However, inconsistencies are not unexpected, as making value judgments is difficult (Keeney, 2002). The ranks different methods yield do not vary much when the decision makers' preferences are consistent. But, if a preference matrix is highly inconsistent, different ranking methods may produce wildly different priorities and rankings. Hence, how to help the decision makers to detect and improve those inconsistencies thus to make a more reliable decision is the second issue addressed here.

Multicriteria decision makers tend to use screening, ordering and choosing phases to find a preference (Brugha, 2004). They tend to make little effort in the first phase as they screen out clearly unwanted alternatives, use somewhat more effort in the second phase as they try to put a preference order on the remaining alternatives, and reach the highest effort in the final phase when making a choice between a few close alternatives.

This study develops a decision-making framework based on these three phases. Preferences in pairwise comparison fashion are adopted in the choosing phase. Two optimization models and a Decision Ball model are proposed to assist decision makers in improving inconsistency and observing relationships among alternatives. By examining Decision Balls, a decision maker can iteratively adjust preferences and improve inconsistencies thus to achieve a more consistent decision. The proposed approach can be extensively applied in Marketing. Possible applications are the selection of promotion plans, decisions regarding product sourcing, choice of marketing channels, evaluation of advertising strategy, research of customer behavior …etc.

The reasons why this study uses a sphere model instead of a traditional 2-dimensional plane or a 3-dimensional cube model are described as follows. A 2-dimensional plane model cannot depict three points that do not obey the triangular inequality (i.e. the total length of any two edges must be larger than the length of the third edge) neither can it display four points that are not on the same plane. For instance, as illustrated in Figure 1, consider three points, Q_1, Q_2, Q_3, where the distance between Q_1Q_2, Q_2Q_3, and Q_1Q_3 are 3, 1, and 6, respectively, as shown in Figure 1(b). It is impossible to show their relationships by three line segments on a 2-dimensional plane, as shown in Figure 1(a). If there are four points, Q_1, Q_2, Q_3, and Q_4, which are not on the same plane, as shown in Figure 1(c), it is impossible to present these four points on a 2-dimensional plane too. In addition, a sphere model is also easier for a decision maker to observe than a 3-dimensional cube model because the former exhibits alternatives on the surface of a sphere rather than inside the cube.

This paper is organized as follows. Section 2 reviews the relevant literature. Section 3 sets the three-phase decision making framework, including the screening, ordering and choosing phases. Section 4 proposes a weight-approximation model and a Decision Ball model to support a decision maker to filter out poor alternatives in the ordering phase. Section 5 develops an optimization model which can assist a decision maker in improving inconsistencies in preferences, and provides three methods to allow a decision maker to iteratively adjust his preferences in the choosing phase. Sections 4 and 5 form the main theoretical part of this paper; therefore, readers only interested in the application of proposed approach can skip these two sections. Section 6 uses an example to demonstrate the whole decision process.

2. Relevant Literature

Several visualization approaches have been developed to provide visual aids to support decision-making process. For instance, Li (1999) used deduction graphs to treat decision problems associated with expanding competence sets. Jank and Kannan (2005) proposed a spatial multinomial model of customer choice to assist firms in understanding how their online customers' preferences and choices vary across geographical markets. Kiang (2001) extended a self-organizing map (SOM) (Kohonen, 1995) network to classify decision groups by neural network techniques. Many studies (Kruskal, 1964; Borg and Groenen, 1997; Cox and Cox, 2000) adopted Multidimensional scaling (MDS), which is widely used in Marketing, to provide a visual representation of similarities among a set of alternatives. For instance, Desarbo and Jedidi (1995) proposed a new MDS method to spatially represent preference intensity collected over consumers' consideration sets. However, most of conventional visualization approaches are incapable of detecting and improving the decision makers' inconsistent preferences. Gower (1977), Genest and Zhang (1996) proposed a powerful graphical tool, the so-called Gower Plot, to detect the inconsistencies in decision maker's preferences on a 2-dimensional plane. Nevertheless, the Gower plots do not provide suggestions about how to improve those inconsistencies either.

A pairwise-comparison ranking problem can be provided with magnitude of the degree of preference, intensity ranking; or in terms of ordinal preferences only, preference ranking. These are sometimes referred to also as cardinal versus ordinal preference (Hochbaum and Levin, 2006). Many studies (Saaty, 1980; Saaty and Vargas, 1984; Hochbaum and Levin, 2006; etc.) use multicriteria decision making approaches to find a consistent ranking at minimum error. However, conventional eigenvalue approaches cannot treat preference matrix with incomplete judgments. And, most of them focus on adjusting cardinal or ordinal inconsistencies instead of adjusting both cardinal and ordinal inconsistencies simultaneously. Li and Ma (2006)(2007) developed goal programming models which can treat incomplete judgments and improve cardinal and ordinal inconsistencies simultaneously. However, the ranks of and similarities among alternatives can be displayed.

This study cannot only improve cardinal and ordinal inconsistencies simultaneously but provide visual aids to decision makers. They can observe ranks of and similarities among alternatives, and iteratively adjust their preferences to achieve a more consistent decision.

3. Setting the Decision-Making Framework

The proposed decision-making framework is illustrated by the screening, ordering, and choosing phases as listed below:

(i) The screening phase: the decision maker tries to screen out clearly unwanted alternatives. The decision maker specifies upper and/or lower bounds of attributes to screen out poor alternatives.

(ii) The ordering phase: the decision maker tries to put a preference order on the remaining alternatives.

- The decision maker roughly specifies partial order of alternatives.
- An optimization model and a Decision Ball model are developed to assist decision maker in calculating and viewing ranks of and similarities among alternatives.
- The decision maker filters out poor alternatives according to the information displayed on the Decision Ball.

(iii) The choosing phase: the decision maker tries to make a final choice among a few alternatives. There are four steps in this phase, including specifying pairwise-comparison preferences, detecting and improving inconsistencies, adjusting preferences, and determining the best alternatives.

- Specifying pairwise-comparison preferences. Decision maker has to make more sophisticated comparisons for the remaining alternatives in this phase. Pairwise-comparison fashion, like analytical hierarchy process (AHP; Saaty, 1980), is adopted here because it is good for choosing phase (Brugha, 2004).
- Detecting and improving inconsistencies. Because inconsistent preferences may result in unreliable rank order, significant inconsistencies should be modified to obtain a more consistent solution. An optimization model is proposed to assist decision maker in detecting and improving inconsistencies. After inconsistencies have been reduced, the ranks of and similarities among alternatives are calculated and displayed on a Decision Ball.
- Adjusting preferences. According to the information displayed on the Decision Ball, the decision maker can iteratively adjust his preferences and see the corresponding changes on the Decision Ball.
- Determining the best alternatives. Decision maker makes the final choice with the assistance of the Decision Ball.

The detailed explanations about the ordering and choosing phases are illustrated in the following two sections.

4. The models for ordering phase

Consider a set of alternatives $\mathbf{A} = \{A_1, A_2, \ldots, A_n\}$ for solving a choice problem, where the decision maker selects m criteria to fulfill. The values of criteria $c_1, \ldots, c_m$ for alternative A_i are expressed as $c_{i,k}$, for $k = 1,\ldots, m$. All criterion values are assumed to be continuous data. Denote C $= [c_{i,k}]_{n\times m}$ as the criterion matrix of the decision problem. Denote $\underline{c_k}$ and $\overline{c_k}$ as the lower and upper bounds of the criterion value of c_k, respectively. The value of $\underline{c_k}$ and $\overline{c_k}$ can be either given by the decision maker directly or calculated by the minimum and maximum raw criterion value of c_k. The score function in this study is assumed to be in an additive form because it is the most commonly used form in practice and more understandable for the decision maker (Belton and Stewart, 2002). Denote S_i as the score value of an alternative A_i. An additive score function of an alternative A_i ($c_{i,1}$, $c_{i,2}$, …, $c_{i,m}$) is defined as below:

$$S_i(\mathbf{w}) = \sum_{k=1}^{m} w_k \frac{c_{i,k} - \underline{c_k}}{\overline{c_k} - \underline{c_k}}, \tag{1}$$

where (i) w_k is the weight of criterion k, $w_k \geq 0, \ \forall k$ and $\sum_{k=1}^{m} w_k = 1$. $\mathbf{w} = (w_1, w_2, ..., w_m)$ is a weight vector, (ii) $0 \leq S_i(\mathbf{w}) \leq 1$. In order to make sure that all weights of criteria and scores of alternatives are positive, a criterion $c_{i,k}$ with cost feature (i.e., a DM likes to keep it as small as possible) is transferred from $c_{i,k}$ to ($\overline{c_k} - c_{i,k}$) in advance.

Following the score function, the dissimilarity function of reflecting the dissimilarity between alternatives A_i and A_j is defined as

$$\delta_{i,j}(\mathbf{w}) = \sum_{k=1}^{m} w_k \frac{| c_{i,k} - c_{j,k} |}{\overline{c_k} - \underline{c_k}}, \tag{2}$$

where $0 \leq \delta_{i,j}(\mathbf{w}) \leq 1$ and $\delta_{i,j}(\mathbf{w}) = \delta_{j,i}(\mathbf{w})$. Clearly, if $c_{i,k} = c_{j,k}$ for all k then $\delta_{i,j}(\mathbf{w}) = 0$.

In the ordering phase, a decision maker has to roughly specify partial order of alternatives. If the decision maker prefers A_i to A_j, denoted as $A_i \succ A_j$, score of A_i should be higher than that of A_j ($S_i > S_j$). However, there may be some inconsistent preferences. For instance, a decision maker may specify $A_i \succ A_j$, $A_j \succ A_k$ and $A_k \succ A_i$. A binary variable $t_{i,j}$ is used to record the inconsistent relationship between A_i and A_j: if $A_i \succ A_j$ and $S_i > S_j$, then $t_{i,j}$ =0; otherwise, $t_{i,j}$ = 1. A weight approximation model for ordering phase is developed as follows:

Model 1 (Weight approximation model for ordering phase)

$$\underset{\{w_k\}}{Min} \quad \sum_{i=1}^{n}\sum_{j=1}^{n} t_{i,j}$$

$$s.t. \quad S_i(\mathbf{w}) = \sum_{k=1}^{m} w_k \frac{c_{i,k} - \underline{c_k}}{\overline{c_k} - \underline{c_k}}, \ \forall i, \tag{3}$$

$$\sum_{k=1}^{m} w_k = 1, \tag{4}$$

$$S_i \geq S_j + \varepsilon - Mt_{i,j}, \quad \forall A_i \succ A_j, \tag{5}$$

$$\underline{w_k} \leq w_k \leq \overline{w_k}, \; w_k \geq 0, \forall k, \tag{6}$$

$$u_{i,j} \in \{0,1\}, \text{ } M \text{ is a large value, } \varepsilon \text{ is a tolerable error.} \tag{7}$$

The objective of Model 1 is to minimize the sum of $t_{i,j}$. Expressions (3) and (4) are from the definition of an additive score function (1). Expression (5) indicates that if $A_i \succ A_j$ and $S_i \geq S_j + \varepsilon$, then $t_{i,j} = 0$; otherwise, $t_{i,j} = 1$, where ε and M are a computational precision and a large value which can be normally set as 10^{-6} and 10^{6}, respectively. Denote $\underline{w_k}$ and $\overline{w_k}$ as the lower and upper bound of w_k, which could be set by the decision maker as in Expression (6). From (1) and (2), the score S_i of alternative A_i and dissimilarity $\delta_{i,j}$ between alternative A_i and A_j can be calculated based on the results of Model 1.

A Decision Ball model is then constructed to display all alternatives A_i in A = $\{A_1, A_2, \ldots, A_n\}$ on the surface of a hemisphere. A non-metric multidimensional scaling technique is adopted here to provide a visual representation of the dissimilarities among alternatives. The arc length between two alternatives is used to represent the dissimilarity between them, e.g., the larger the difference, the longer the arc length. However, because the arc length is monotonically related to the Euclidean distance between two points and both approximation methods make little difference to the resulting configuration (Cox and Cox, 1991), the Euclidean distance is used here for simplification.

In addition, the alternative with a higher score is designed to be closer to the North Pole so that alternatives will be located on the concentric circles in the order of score from top view. For the purpose of comparison, we define an ideal alternative A_*, where $A_* = A_*(\overline{c_1}, \overline{c_2}, \ldots, \overline{c_m})$ and $S_* = 1$. A_* is designed to be located at the north pole with coordinate (x_*, y_*, z_*) = (0, 1, 0).

The following propositions are deduced:

<u>Proposition 1</u> The relationship between $\delta_{i,*}(\mathbf{w})$ (the dissimilarity between A_i and A_*)) and $S_i(\mathbf{w})$ is expressed as $\delta_{i,*}(\mathbf{w}) = 1 - S_i(\mathbf{w})$.

<Proof> $\delta_{i,*}(\mathbf{w}) = \sum_{k=1}^{m} w_k \frac{|c_{i,k} - \overline{c_k}|}{\overline{c_k} - \underline{c_k}} = \sum_{k=1}^{m} w_k \frac{(\overline{c_k} - \underline{c_k}) - (c_{i,k} - \underline{c_k})}{\overline{c_k} - \underline{c_k}}$

$$= (\sum_{k=1}^{m} w_k \frac{(\overline{c_k} - \underline{c_k})}{\overline{c_k} - \underline{c_k}} - \sum_{k=1}^{m} w_k \frac{(c_{i,k} - \underline{c_k})}{\overline{c_k} - \underline{c_k}}) = 1 - S_i(\mathbf{w})$$

Denote $d_{i,j}$ as the Euclidean distance between A_i and A_j . Let $d_{i,j} = \sqrt{2}\delta_{i,j}$, such that if $\delta_{i,j}$ = 0 then $d_{i,j}$ = 0 and if $\delta_{i,j}$ = 1 then $d_{i,j} = \sqrt{2}$, where $\sqrt{2}$ is used because the distance between the north pole and equator is $\sqrt{2}$ when radius = 1. Denote the coordinates of an alternative A_i on a ball as (x_i, y_i, z_i). The relationship between y_i and S_i is expressed as

Proposition 2 $y_i = 2S_i - S_i^2$.

<Proof> Since $d_{i,*}^2 = (x_i - 0)^2 + (y_i - 1)^2 + (z_i - 0)^2 = 2\delta_{i,*}^2 = 2(1 - S_i)^2$,

it is clear $y_i = 2S_i - S_i^2$. Clearly, if S_i = 1 then y_i = 1; if S_i = 0, then y_i = 0.

Based on the non-metric multidimensional scaling technique, denote $\hat{d}_{i,j}$ as a monotonic transformation of $\delta_{i,j}$ satisfying following condition: if $\delta_{i,j} < \delta_{p,q}$, then $\hat{d}_{i,j} < \hat{d}_{p,q}$. The coordinate ($x_i$, y_i, z_i) of alternative A_i all i can be calculated by the following Decision Ball model:

Model 2 (A Decision Ball Model)

$$\underset{\{x_i, y_i, z_i\}}{Min} \quad \sum_{i=1}^{n} \sum_{j>i}^{n} (d_{i,j} - \hat{d}_{i,j})^2$$

s.t.

$$y_i = 2S_i - S_i^2, \ \forall i, \tag{8}$$

$$\hat{d}_{i,j} \le \hat{d}_{p,q} - \varepsilon, \ \forall \delta_{i,j} < \delta_{p,q}, \tag{9}$$

$$d_{i,j}^2 = (x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2, \ \forall i, j, \tag{10}$$

$$x_i^2 + y_i^2 + z_i^2 = 1, \quad \forall i, \tag{11}$$

$$-1 \le x_i, z_i \le 1, \ 0 \le y_i \le 1, \ \forall i, \ \varepsilon \text{ is a tolerable error.} \tag{12}$$

The objective of Model 2 is to minimize the sum of squared differences between $d_{i,j}$ and $\hat{d}_{i,j}$. Expression (8) is from Proposition 2, where the alternative with a higher score is designed to be closer to the North Pole. Expression (9) is the monotonic transformation from $\delta_{i,j}$ to $\hat{d}_{i,j}$.

All alternatives are graphed on the surface of the northern hemisphere (11)(12).

Model 2 is a nonlinear model, which can be solved by some commercialized optimization software, such as Global Solver of Lingo 9.0, to obtain an optimum solution. One restriction of this model is the running time that may considerably increase when the number of alternatives becomes large because the time complexity of Model 2 is n^2. This model has good performance when the number of alternatives less than 10. However, in this case of alternatives more than 10, some classification techniques, like k-means (MacQueen,1967) for instance, can be used to reduce the solving time by dividing alternatives into several groups. The coordinates of group centers are calculated first. Then, these group centers are treated as anchor points. The coordinates of alternatives can be obtained by calculating dissimilarity between alternatives and anchor points. Thus, all alternatives can be displayed on the Decision Ball within tolerable time.

According to the information displayed on the Decision Ball, the decision maker can select better alternatives into the next phase.

5. The models for choosing phase

In this phase, the decision maker has to make more sophisticated comparisons for the remaining alternatives. Pairwise comparisons are adopted here (Brugha, 2004). For some i and j pairs, assume a decision maker can specify $p_{i,j}$, the ratio of the score of A_i to that of A_j, which is expressed as

$$p_{i,j} = \frac{S_i}{S_j} \times e_{i,j} , \tag{13}$$

where S_i is the score of A_i and $e_{i,j}$ is a multiplicative term accounting for inconsistencies, as illustrated in the Analytic Hierarchy Process (AHP) (Saaty, 1980). It is assumed that $p_{i,j} = 1/p_{j,i}$. If the decision maker cannot specify the ratio for a specific pair i and j then $p_{i,j} = \phi$. Denote **P** $=[p_{i,j}]_{n \times n}$ as a $n \times n$ preference matrix. **P** is incomplete if there is any $p_{i,j} = \phi$. **P** is perfectly consistent if $e_{i,j} = 1$ for all i, j (i.e. $p_{i,j} = S_i/S_j$ for all i, j). **P** is ordinally inconsistent (intransitive) if for some $i, j, k \in \{1, 2, 3, \ldots, n\}$ there exists $p_{i,j} > 1$, $p_{j,k} > 1$, but $p_{i,k} < 1$. **P** is cardinally inconsistent if for some $i, j, k \in \{1, 2, 3, \ldots, n\}$ there exists $p_{i,k} \neq p_{i,j} \times p_{j,k}$ (Genest and Zhang, 1996).

If **P** is complete and ordinal consistent, all A_i can be ranked immediately. However, if there is ordinal or highly cardinal inconsistency, these inconsistencies should be improved before ranking because significant inconsistencies may result in unreliable rank order.

An optimization model, developed by a goal-programming optimization technique, is developed to assist decision maker in detecting and improving inconsistencies. In order to reduce the ordinal inconsistency, a binary variable $u_{i,j}$ is used to record if the preference $p_{i,j}$, specified by the decision maker, is suggested to be reversed or not. If $p_{i,j}$ is suggested to be reversed, then $u_{i,j}$ = 1; otherwise, $u_{i,j}$ = 0. A variable $\alpha_{i,j}$, defined as the difference between $p_{i,j}$ and S_i/S_j, is used to indicate the degree of cardinal inconsistency of $p_{i,j}$: the larger the value of $\alpha_{i,j}$, the higher the cardinal inconsistency. The inconsistencies improving model is formulated as below:

Model 3 (Inconsistencies improving model)

$$\operatorname*{Min}_{\{w_k\}} \quad M \times Obj1 + Obj2$$

$$Obj1 = \sum_{i=1}^{n}\sum_{j>i}^{n} u_{i,j}$$

$$Obj2 = \sum_{i=1}^{n}\sum_{j>i}^{n} \alpha_{i,j}$$

s.t. $$\left(\frac{S_i}{S_j} - 1\right) \times (p_{i,j} - 1) + M \times u_{i,j} \geq \varepsilon, \quad \text{for all } i, j \text{ where } p_{i,j} \neq \phi \text{ and } p_{i,j} \neq 1, \tag{14}$$

$$-\left|S_i - S_j\right| + M \times u_{i,j} \geq 0, \quad \text{for all } i,j \text{ where } p_{i,j} = 1, \tag{15}$$

$$\left|\frac{S_i}{S_j} - p_{i,j}\right| \leq \alpha_{i,j}, \ \forall i, j, \tag{16}$$

$$S_i(\mathbf{w}) = \sum_{k=1}^{m} w_k \frac{c_{i,k} - \underline{c_k}}{\overline{c_k} - \underline{c_k}}, \quad \forall i, \tag{17}$$

$$\sum_{k=1}^{m} w_k = 1, \tag{18}$$

$$\underline{w_k} \leq w_k \leq \overline{w_k}, \qquad w_k \geq 0, \forall k, \tag{19}$$

$$u_{i,j} \in \{0,1\}, M \text{ is a large value, } \varepsilon \text{ is a tolerable error.} \tag{20}$$

This model tries to improve ordinal and cardinal inconsistencies simultaneously. The first objective ($Obj1$) is to achieve ordinal consistency by minimizing the number of preferences (i.e., $p_{i,j}$) being reversed. Constraint (14) means: when $p_{i,j} \neq \phi$ and $p_{i,j} \neq 1$, $u_{i,j}$ = 0, if (i)

$(\frac{S_i}{S_j} > 1)$ and $(p_{i,j} > 1)$ or (ii) $(\frac{S_i}{S_j} < 1)$ and $(p_{i,j} < 1)$; and otherwise $u_{i,j}$ = 1. A tolerable positive number ε is used to avoid $\frac{S_i}{S_j} = 1$. Constraint (15) means: when $p_{i,j}$ = 1, if $S_i = S_j$; then $u_{i,j}$ = 0; otherwise $u_{i,j}$ = 1. The second objective (*Obj*2) is to reduce cardinal consistency by minimizing the $\alpha_{i,j}$ values, i.e. to minimize the difference between $\frac{S_i}{S_j}$ and $p_{i,j}$. Since ordinal consistency (*Obj*1) is more important than cardinal consistency (*Obj*2), *Obj*1 is multiplied by a large value *M* in the objective function. Constraints (17) and (18) come from Notation 1. Constraint (19) sets the upper and lower bound of weights. An improved complete preference matrix can be obtained as $\mathrm{P}' = [p'_{i,j}]_{n\times n}$, where $p'_{i,j} = \frac{S_i}{S_j}$ if $p_{i,j} = \phi$ or $u_{i,j}$ = 1; otherwise, $p'_{i,j} = p_{i,j}$.

Model 3 is a nonlinear model, which can be converted into the following linear mixed 0-1 program:

$$\underset{\{w_k\}}{\text{Min}} \quad M \times Obj1 + Obj2$$

$$Obj1 = \sum_{i=1}^{n}\sum_{j>i}^{n} u_{i,j}$$

$$Obj2 = \sum_{i=1}^{n}\sum_{j>i}^{n} \alpha_{i,j}$$

s.t. $$(S_i - S_j) \times (p_{i,j} - 1) + M \times u_{i,j} \ge \varepsilon, \quad \text{for all } i, j \text{ where } p_{i,j} \ne \phi \text{ and } p_{i,j} \ne 1, \tag{21}$$

$$-M \times u_{i,j} \le S_i - S_j \le M \times u_{i,j}, \quad \text{for all } i,j \text{ where } p_{i,j} = 1, \tag{22}$$

$$S_j \times p_{i,j} - \alpha_{i,j} \le S_i \le S_j \times p_{i,j} + \alpha_{i,j}, \ \forall i, j, \tag{23}$$

(17) ~ (20),

where (21), (22) and (23) are converted from (14), (15) and (16) respectively.

After the weight vector, (w_1, w_2, …, w_m), is found, $S_i(\mathbf{w}) = \sum_{k=1}^{m} w_k \frac{c_{i,k} - \underline{c_k}}{\overline{c_k} - \underline{c_k}}$ and $\delta_{i,j}(\mathbf{w}) = \sum_{k=1}^{m} w_k \frac{|c_{i,k} - c_{j,k}|}{\overline{c_k} - \underline{c_k}}$ can be calculated. All alternatives are shown on a Decision Ball by

Model 2.

According to the information visualized on the Decision Ball, the decision maker can iteratively adjust his preferences by the following ways:

(i) Adjusting preference order. Since alternative with a higher score is designed to be closer to the North Pole so that a decision maker can see the rank order by the location of alternative: the higher the latitude, the higher the score. If the decision maker would like to adjust a preference order, from $A_1 \prec A_3$ to $A_1 \succ A_3$ for instance, a constraint $S_1 \geq S_3 + \varepsilon$ will be added into Model 3.

(ii) Adjusting dissimilarity. The distance between two alternatives on a Decision Ball implies the dissimilarity between them: the larger the dissimilarity, the longer the distance. Therefore, if a decision maker observes the Decision Ball and decides to adjust the dissimilarity relationship, from $\delta_{1,3}(\mathbf{w}) < \delta_{1,2}(\mathbf{w})$ to $\delta_{1,3}(\mathbf{w}) > \delta_{1,2}(\mathbf{w})$ for example, a constraint $\delta_{1,3}(\mathbf{w}) > \delta_{1,2}(\mathbf{w})$ (i,e. $\sum_{k=1}^{m} w_k \frac{|c_{1,k} - c_{3,k}|}{\overline{c_k} - \underline{c_k}} \geq \sum_{k=1}^{m} w_k \frac{|c_{1,k} - c_{2,k}|}{\overline{c_k} - \underline{c_k}} + \varepsilon$) will be added into Model 3.

(iii) Adjusting preference matrix. A decision maker can choose to adjust the preference matrix directly. The value of $p_{i,j}$ in Model 3 will be modified according to the change in the preference matrix.

Solving Model 3 yields a new set of weights, and an adjusted Decision Ball will be displayed. The decision maker can iteratively adjust his preferences until he feels no adjustments have to be made. A final choice can be made with the assistance of a resulting Decision Ball.

6. Application to choice data: selection of a store location

Example 1 (*Selection of a store location*)

The choice of a store location has a profound effect on the entire business life of a retail operation. Suppose a manager of a convenience store in Taiwan who needs to select a store location from a list of 43 spots $\mathbf{A} = \{A_1, \ldots, A_{43}\}$. The manager sets four criteria to fulfill: (c_1) sufficient space, (c_2) high population density, (c_3) heavy traffic, and (c_4) low cost. Store size is measured in square feet. The number of people who live within a one-mile radius is used to calculate population density. The average number of vehicle traffic passing the spot per hour is adopted to evaluate the volumes of traffic. Cost is measured by monthly rent. The criteria values of 43 candidate locations are listed in the criterion matrix $\mathbf{C_1}$, as shown in Table 1.

The manager would like to rank choices incorporating his personal preferences. The manager can rank these choices by the following three phases:

Phase 1 – the screening phase

The manager tries to screen out clearly unwanted alternatives by setting upper or lower bound of each criterion. He sets the minimum space required to be 800 square feet, the minimum population density to be 700, the minimal traffic to be 400, and the maximum rental fee to be 5000. That is, $\underline{c_1}$ = 800, $\underline{c_2}$ = 700, $\underline{c_3}$ = 400 and $\overline{c_4}$ = 5000. The values of $\overline{c_1}$, $\overline{c_2}$, $\overline{c_3}$ and $\underline{c_4}$ can be set as the maximum values of c_1, c_2, c_3 and minimum value of c_4, i.e. $\overline{c_1}$ = 1500, $\overline{c_2}$ = 1260, $\overline{c_3}$ =780, and $\underline{c_4}$ = 3100. After filtering out alternatives with criterion values exceeding these boundaries, only 23 choices $\{A_3, A_4, A_6, A_7, A_8, A_{11}, A_{13}, A_{15}, A_{17}, A_{18}, A_{21}, A_{23}, A_{24}, A_{25}, A_{26}, A_{29}, A_{31}, A_{32}, A_{34}, A_{37}, A_{40}, A_{42}, A_{43}\}$ are remaining for the next phase.

Phase 2 – the ordering phase

The decision maker roughly specifies partial order of alternatives. He specifies $A_3 \succ A_7$, $A_7 \succ A_{37}, A_{15} \succ A_8, A_{17} \succ A_6, A_{31} \succ A_{25}$ and $A_{42} \succ A_{40.}$ The minimum weight of each criterion is set as $\underline{w_k}$ = 0.01 for all k by the decision maker. Applying Model 1 to these preference relationships yields w = $\{w_1, w_2, w_3, w_4\}$ = {0.21, 0.43, 0.01, 0.35}, $t_{15,8}$ =1, and the rest of $t_{i,j}$ = 0. The objective value is 1. The variable $t_{15,8}$ = 1 indicates the preference relationship $A_{15} \succ A_8$ should be reversed. When checking criterion matrix in Table 1, all criterion values of A_8 are better than or equal to those of A_{15} which makes $A_{15} \succ A_8$ impossible; therefore, the relationship between A_{15} and A_8 is reversed.

The score of alternatives can be calculated according to Expression (1), where S_3 = 0.54, S_4 = 0.10, S_6 = 0.33, S_7 = 0.54, S_8 = 0.71, S_{11} = 0.29, S_{13} = 0.59, S_{15} = 0.36, S_{17} = 0.53, S_{18} = 0.31, S_{21} = 0.30, S_{23} = 0.30, S_{24} = 0.45, S_{25} = 0.22, S_{26} = 0.39, S_{29} = 0.23, S_{31} = 0.22, S_{32} = 0.42, S_{34} = 0.46, S_{37} = 0.39, S_{40} = 0.31, S_{42} = 0.34, S_{43} = 0.24. The dissimilarity between alternatives can also be calculated according to Expression (2).

Applying Model 2 to this example yields coordinates of alternatives. The resulting Decision Ball is displayed in Figure 2. Because the alternative with a higher score is designed to be closer to the North Pole, the order of alternatives can be read by the latitudes of alternative: the higher the latitude, the higher the score. The order of top ten alternatives is $A_8 \succ A_{13} \succ A_3 \succ A_7 \succ A_{17} \succ A_{34} \succ A_{24} \succ A_{32} \succ A_{37} \succ A_{26}$. In addition, the distance between two alternatives represents the dissimilarity between them: the longer the distance, the larger the dissimilarity. For instance, the dissimilarity between A_{26} and A_{37} is smaller than that of between A_{37} and A_7.

Based on the information provided on the Decision Ball, assume the decision maker decides to select the top eight alternatives to make more sophisticated comparisons. That is, only

A_8, A_{13}, A_3, A_7, A_{17}, A_{34}, A_{24} and A_{32} are remaining for the next phase.

Phase 3 – the choosing phase

In the choosing phase, the manager uses pairwise comparisons to express preferences among pairs of choices in preference matrix $\mathbf{R_1}$, as listed in Table 2. Because the manager is unable to make comparison among some spots, the relationships $p_{3,34}$, $p_{7,17}$, $p_{8,24}$, $p_{13,34}$ are left blank, which means $\mathbf{R_1}$ is incomplete. The preference matrix $\mathbf{R_1}$ is ordinally inconsistent because there is an intransitive relationship among A_3, A_8 and A_{32}. That is, A_3 is preferred to A_8 ($p_{3,8} > 1$), and A_8 is preferred to A_{32} ($p_{8,32} > 1$); however, A_{32} is preferred to A_3 ($p_{3,32} < 1$). $\mathbf{R_1}$ is also cardinally inconsistent. For instance, there exists $p_{3,8} = 1.6$, $p_{8,13} = 2.5$; but, $p_{3,13} = 2$ ($1.6 \times 2.5 = 4$, that is $p_{3,8} \times p_{8,13} \neq p_{3,13}$).

Applying Model 3 to the example yields $Obj1 = 1$, $Obj2 = 3.91$, $u_{3,8} = 1$ and the rest of $u_{i,j} = 0$, $(w_1, w_2, w_3, w_4) = (0.04, 0.19, 0.06, 0.71)$, $(S_3, S_7, S_8, S_{13}, S_{17}, S_{24}, S_{32}, S_{34}) = (0.55, 0.55, 0.78, 0.27, 0.39, 0.40, 0.74, 0.51)$. The variable $u_{3,8} = 1$ implies that the value of $p_{3,8}$ is suggested to be changed from $p_{3,8} > 1$ to $p_{3,8} < 1$ (i.e. from $A_3 \succ A_8$ to $A_3 \prec A_8$) to improve ordinal inconsistency. The values of unspecified preferences can be computed as $p_{3,34} = \frac{S_3}{S_{34}} = 1.08$, , $p_{7,17}$, $= 1.41$, $p_{8,24} = 1.93$, and $p_{13,34} = 0.76$. The corresponding Decision Ball is shown in Figure 3. The order of alternatives is $A_8 \succ A_{32} \succ A_3 \succ A_7 \succ A_{34} \succ A_{24} \succ A_{17}$.

According to the information observed on the Decision Ball, the decision maker can iteratively adjust his preferences. Suppose he would like to adjust a preference order from $A_7 \succ A_{34}$ to $A_{34} \succ A_7$. A constraint $S_{34} \geq S_7 + \varepsilon$ is added into Model 3. Solving Model 3 yields $Obj1 = 3$, $Obj2 = 3.96$, $u_{3,8} = u_{7,34} = u_{17,24} = 1$ and the rest of $u_{i,j} = 0$, $(w_1, w_2, w_3, w_4) = (0.01, 0.13, 0.17, 0.69)$, $(S_3, S_7, S_8, S_{13}, S_{17}, S_{24}, S_{32}, S_{34}) = (0.53, 0.50, 0.76, 0.27, 0.44, 0.40, 0.71, 0.51)$. In order to satisfy the relationship $A_{34} \succ A_7$, the relationship between A_{17} and A_{24} has to be reversed ($u_{17,24} = 1$). Applying Model 2 to this result yields a new set of coordinates. An adjusted Decision Ball is displayed in Figure 4. On this Decision Ball, the latitude of A_{34} is higher than that of A_7.

By seeing the relationships of alternatives displayed on the Decision Ball in Figure 4, the decision maker would like to adjust some dissimilarity relationships between alternatives. His adjustment is that the dissimilarity between A_3 and A_8 is larger than that of between A_7 and A_8. A constraint $\sum_{k=1}^{m} w_k \frac{|c_{3,k} - c_{8,k}|}{\overline{c_k} - \underline{c_k}} \geq \sum_{k=1}^{m} w_k \frac{|c_{7,k} - c_{8,k}|}{\overline{c_k} - \underline{c_k}} + \varepsilon$ is added into Model 3. Solving Model 3 again yields $Obj1 = 5$, $Obj2 = 4.33$, $u_{3,8} = u_{7,34} = u_{17,24} = u_{3,7} = u_{8,32} = 1$ and the rest of $u_{i,j} = 0$, $(w_1, w_2, w_3, w_4) = (0.01, 0.04, 0.19, 0.76)$, $(S_3, S_7, S_8, S_{13}, S_{17}, S_{24}, S_{32}, S_{34}) = (0.51, 0.53, 0.74, 0.19, 0.39, 0.36, 0.78, 0.53)$. This result shows that in addition to rank reversal of A_3 and A_8, A_7 and A_{34},

A_{17} and A_{24} ($u_{3,8} = u_{7,34} = u_{17,24} = 1$), the relationship between A_3 and A_7, A_8 and A_{32} are suggested to be reversed to satisfy the adjustment of dissimilarity. A corresponding Decision Ball is depicted in **Figure 5.**

Suppose the decision maker stops further adjustment. The decision maker can make a final decision based on the Decision Ball in Figure 5. From the latitude of alternatives, the decision maker can tell the rank of choices as $A_{32} \succ A_8 \succ A_{34} \succ A_7 \succ A_3 \succ A_{17} \succ A_{24} \succ A_{13}$. The best choice is A_{32}. The dissimilarity between alternatives can be read by the distance between them. For instance, the dissimilarity between A_3 and A_{34} is the smallest because the distance between them is the shortest. That is, if A_{32}, A_8 and A_{34} are not available, A_3 as well as A_7 will be a good choice.

It is important to notice that A_3 is more similar to A_{34} than A_7 is but $A_{34} \succ A_7 \succ A_3$. This kind of relationship is possible. For instance, comparing with three alternatives A, B, C with benefit criterion values (5, 5, 5), (4, 4, 6) and (3, 5, 5), given equal weight and $\underline{c_k} = 0$ and $\overline{c_k} = 10$ for $k = 1\ldots3$. The scores of three alternatives are $S_A = 0.5$, $S_B = 0.47$, and $S_C = 0.43$. The dissimilarities between alternatives are $\delta_{A,B} = 0.1$, $\delta_{B,C} = 0.1$ and $\delta_{A,C} = 0.067$. It is obvious that $A \succ B \succ C$ but C is more similar to A than B is because $\delta_{A,C} < \delta_{A,B}$.

Example 1 was solved by Global Solver of Lingo 9.0 [20] on a Pentium 4 personal computer. The running time was less than 3 minimums for three phases totally.

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Part II

1. Research Motivation and Purpose

The research on business school rankings appeared in the late 1970s (Hunger and Wheelen, 1980; Schatz, 1993) and incurred disputes in the early 1980s (Hunger and Wheelen, 1980; Laoria, 1984; Ball and McCulloch, 1984 and 1988; Jimenez, 1985; Mackay-Smith, 1985; Douglas, 1989), but received considerable and substantial attentions from the public until U.S. News & World Report (NWR) published its first America's Best Business School Report in 1987 (Solorzano et al., 1987; Kiechel, 1989)[1]. Following, popular media began to publish their business school rankings such as Business Week (BW) in 1988, Financial Times (FT) in 1995, Economists (Econ) in 1996, Forbes in 1999, and Wall Street Journal in 2000 (Business Week, 2006 and 2009; Financial Times, 2006 and 2009; U.S. News & World Report, 2006, 2007, and 2009; Economist.com, 2009). Aside from BW and Forbes, who rank business schools every other year, prevailing publications release their rankings every year.

Notably, whenever ranking surveys had been published during past decades, criticism in response to each ranking immediately occurred and the controversy (such as considerable variation in the number of surveyed companies, the relative size of the alumni population, respondents' representation, biases related to the locations and classifications of surveyed firms, very different responses/opinions between people in the personnel department and people in line management, the fact that no respondents are familiar with all business schools, and so on) had continued over the years (Jimenez, 1985; Mackay-Smith, 1985; Nehrt, 1987; Ball and McCulloch, 1988; Douglas, 1989; Schatz, 1993; Elsbach and Kramer, 1996; Tracy and Waldfogel, 1997; Dichev, 1999; Corley and Gioia, 2000; Morse and Flanigan, 2006 and 2009; Business Week, 2006 and 2009; Financial Times, 2006 and 2009; ; U.S. News & World Report, 2006, 2007, and 2009; Holbrook, 2007; Brady, 2007; Peters, 2007; Economist.com, 2009). Disputes and criticisms are amplified at each time the press announce their annual rankings to the world.

Although the debates on business school rankings released by current media still exist and publishing these rankings may be considered a way for publishers to sell more magazines, current periodicals have already become powerful references for students, who use them to evaluate which schools to attend. Recruiters, in turn, use them to determine which schools to hire from. Therefore, it is difficult to ignore or dismiss the impact of current media that report business school rankings over decades. Consequently, AACSB, the world largest business-school accrediting association, publically called on media in September 2005 to rethink the way they rank business schools. AACSB contended that the assessment of business schools is a multifaceted concept and varies in

[1] Although U.S. News & World Report published its first version of America's Best Business Schools in 1987, its first America's Best College report was published in 1983. Similarly, other major periodicals usually ranked colleges earlier than started to rank business schools (source: Wikipedia.org).

the overriding vision of what a business education likes to be (Pringle and Michel, 2007). Accordingly, a business school should focus on the improvement and achievement of itself and the peers it likes to be or compare with.

Moreover, upon entering the 2000s, the competitive pressure on the education market quickly and stably increase, and the rankings become widely perceived as the single most useful indicators of a school's ability (or inability) to compete in this market. Unlike 1980s and 1990s, competition among business schools is becoming normal and increasingly coming from worldwide peers instead of within single nation. Consequently, business school rankings have increasingly received prominent concerns not only from faculties, current and prospective students, alumni, but also gradually from school deans, university presidents, and even government leaders. As a result, many business schools recently have greater attention and interests in analyzing current rankings and finding clues to devise their strategies.

Motivated by the aforementioned phenomena, this research aims to propose a novel clustering method that partition business schools with keeping ranking orders produced by current rankings and display significant clustering schools on a three-dimension ball which can help a business school visualize its position for effectively devising development strategy.

2. Overview of Business School Rankings

The first business school ranking appeared in 1977, reported by Carter (Schatz, 1993). For ranking criteria, the Carter Report used the frequency of faculty publications in academic journals, asked the deans of the business schools to vote on the best program, and questioned business school faculty about which schools they thought were the best. In 1979, through collecting opinions from deans at business schools accredited by Association to Advance Collegiate Schools of Business (AACSB) and senior personal executives in industry, Hunger and Wheelen (1980) ranked business schools using four criteria: faculty reputation, academic reputation, student quality, and curriculum.

Later on, Ball and McCulloch (1984) conducted a survey using ten criteria (namely, Faculty Quality, Internationalization, Faculty Research, Reputation, Publications, Competence of Graduates, Graduate Placement, Student Quality, Number of Students, and Foreign Study Internships) to rank business schools by collecting 212 questionnaires from 1286 Academy of International Business members. In the same year, Laoria (1984) ranked business schools in New Jersey by sending questionnaires to 83 business schools in New Jersey and 65 corporations that had headquarters in New Jersey. Brecker & Merryman Inc. (1985) ranked American business schools by surveying executives at 134 national companies of the 250 largest industrial and service firms.

Following these ranking reports, a variety of studies discussing business school rankings appeared. Some studies explore the impact of rankings on business school values (Petrof et al., 1982) when other studies criticize the business school rankings and point out their flaws (Schatz, 1993). Some studies summarize how business school administrators and faculties respond to these rankings (Elsbach and Kramer, 1996), some discuss the business school rankings from a market perspective (Tracy and Waldfogel, 1997), some found certain insights from current ranking systems (Dichev, 1999), some attempted to identify core courses and concentration areas from leading business schools (Segev, 1999), some analyze how many constituencies could influence a business school programs and how the school responds to its constituencies (Trieschmann et al., 2000), and some studies evaluated the program efficiency or performance of business school (Colbert and Gioia, 2000; Walker and Black, 2000).

Additionally, some literature focuses on ranking and remarking certain fields of business schools, including research performance ranking (Baden-Fuller et al, 2000), management information systems ranking (Lee, 2001), technology management ranking (Linton, 2004), and international business orientation ranking (Chan et al., 2005). Other studies examine the relationship between business school rankings and research productivity in prestigious business journals (Christie et al., 2002; Siemens et al., 2005), the relationship between business school rankings and business school dean turnover (Fee et al., 2005), and the changes that business schools make based on business school rankings (Martins, 2005). Finally, some studies analyze the key impacts on the business school community due to reputation rankings released by the media (Zell, 2005), and some try to find indicators of business school quality and build a conceptual framework for quality and ranking relations (Michael, 2005).

Although various business school rankings exist, popular media such as FT and Econ (two most recognized resources in ranking worldwide business schools) and NWR and BW (two oldest resources in ranking American business schools) hold dominant influence to the public (including school deans and corporation recruiters). The criteria with their weights used by NWR and BW are summarized in Table 1, while the criteria used in FT and Econ are listed in Table 2. Tables 1-2 reveal that some criteria were commonly used by NWR, BW, FT, and Econ, while each ranking system has its unique highlights which encourage interested students to consider using its rankings as their reference. That is, one magazine ranking might place more emphasis on certain criteria (goal or mission) while another magazine might choose to emphasize on other area. The aforementioned may help explain why different publications produce different rankings and employ different methodologies to compile their lists. The detail information regarding these rankings is briefed in Appendix 1.

Table 1 Criteria used by NWR and BW

NWR			BW		
k	Criteria (c)	Weight (w)	k	Criteria (c)	Weight (w)
1	Peer Assessment Score	0.25	1	Student Survey	0.30
2	Recruiter Assessment Score	0.15	2	Recruiter Survey	0.20
3	Mean Undergraduate GPA	0.075	3	Median Starting salaries	0.10
4	Mean GMAT Score	0.1625	4	MBA feeder school measure	0.10
5	Acceptance Rate	0.0125	5	Average SAT Scores	0.06
6	Mean Starting Salary and Bonus	0.14	6	Ratio of full-time faculty to students	0.06

7	Employment rate at Graduation	0.07	7	Average class size	0.06
8	Employment in 3 Months	0.14	8	Percentage of business majors with internships	0.06
			9	Hours students spend every week on school work	0.06

Table 2 Criteria used by FT and Econ

FT			Econ		
k	Criteria (c)	Weight (w)	k	Criteria (c)	Weight (w)
1	Salary Today	0	1	Diversity of recruiters	0.0875
2	Weighted Salary	0.2	2	Jobs found in 3 Months	0.0875
3	Salary Percentage Increase	0.2	3	Jobs found through the career service	0.0875
4	Value for Money Rank	0.03	4	Student assessment	0.0875
5	Career Progress Rank	0.03	5	Ratio of faculty to students	0.0175
6	Aims Achieved	0.03	6	Faculty with PhD	0.0350
7	Placement Success Rank	0.02	7	Faculty rating by students	0.0350
8	Employment at 3 Months	0.02	8	Average GMAT score	0.0656
9	Alumni Recommendation Rank	0.02	9	Average work experience	0.0021
10	Women Faculty	0.02	10	International diversity score	0.0291
11	Women Students	0.02	11	Women Students	0.0291
12	Women Board	0.01	12	Culture and classmates rating by students	0.0291
13	International Faculty	0.04	13	Program and electives rating by students	0.0021
14	International Students	0.04	14	Overseas exchange programs	0.0021
15	International Board	0.02	15	Number of Languages on offer	0.0021
16	International Mobility Rank	0.06	16	Facilities and services rating by students	0.0021
17	International Experience Rank	0.02	17	Salary increased	0.1250
18	Number of Languages on offer	0.02	18	Leaving Salary	0.1250
19	Faculty with Doctorates	0.05	19	Breadth of alumni network	0.0999
20	FT Doctoral Rank	0.05	20	Internationalism of alumni	0.0999
21	FT Research Rank	0.10	21	Alumni effectiveness	0.0999

Notably, since the current ranking systems originate to offer a student guidance about how to select a business school that best fits his/her needs (Business Week, 2006 and 2009; Financial Times, 2006 and 2009; Economist, 2009; U.S. News & World Report, 2009; Lavelle, 2009), these rankings are designed to make it easier for prospective students to compare institutions on a set of limited measuring criteria rather than offer a business school comprehensive and systematic guidance about how to devise a development strategy that best fits its interests. That is, current rankings do not give schools clear hints on their strategy developments. As a result, the goal of this research is to propose a novel clustering method, called cluster ranking framework, that will help business schools to devise development strategies by visualizing their aspirational schools (the upper cluster schools they wants to upgrade to in future) and competitive schools (the same cluster schools they needs to compete with at present).

3. Cluster Ranking Algorithm

Clustering is one of the oldest and important activities of human beings, and one of most used techniques to partition a set of observations into a set of meaningful groups where observations are similar to each other if they belong to the same group while observations are dissimilar to each other if they belong to different groups. Clustering techniques, first used by Tryon in 1939 (Cooper, 1963; Sokal and Sneath, 1963; Hakimi, 1965; Tryon and Bailey, 1970), have been investigated over decades and applied to a wide variety of fields such as psychology, biology, sociology, ecology, taxonomy, medicine, culture, marketing, economic, and pattern recognition in various sciences (Hartigan, 1975; Can and Ozkarahan, 1984; Aronson and Klein, 1989; Murray, 1999; Farley and Raftery, 2002; Zhang et al., 2005; Gugler and Brunner, 2007; Bar-Yossef et al., 2008; Xu and Wunsch II, 2008; Sharma and Wadhawan, 2009).

Clustering is an unsupervised classification technique, has been studies extensively in statistics, machine learning, and data mining over decades, and can be broadly classified into five approaches: partition-based clustering, hierarchical clustering, density-based clustering, grid-based clustering, and model-based clustering (Zhang et al., 2005; Han and Kamber, 2006). Partition-based clustering is to partition observations into some pre-specified number of clusters and them evaluate them by pre-defined criteria, hierarchical clustering is to partition observations by creating a hierarchical decomposition tree via either agglomerative or divisive approach, density-based clustering considers clusters as regions and partition observations by judging the density function within a specified neighboring scope, grid-based clustering uses a grid data structure to quantize the data space into a finite number of cells on which clustering is then carried out, and model-based clustering is to partition observations by optimizing the fit between the data and the used model.

The literature reveals that the conventional clustering methods are usually simply designed to classify observation into low dimension and cope with linear distance relationships between observations. The reason is heavily due to the curse of dimensionality, introduced by Bellman (1961), which describes the computational complexity is the explosive growth of dimensionality of the observation vector (Murtagh et al., 2000; Donoho, 2000). Comparing to abundant traditional clustering that partitions observations into two-dimension space with the coordinates of x-axis and y-axis, this study presents a cluster ranking framework which is able to cluster schools on a three-dimension space with the coordinates of x-axis, y-axis, and z-axis. Obviously, in one dimension, all observations are clustered very close and not easily to be visualized, meanwhile in two dimension, all clustered observations become more sparse but restricted to deal with linear distance. As shown in Fig. 1, a three-dimension sphere can depict three points that do not obey the triangular inequality, as depicted in Figs. 1(a) and 1(b), and can show four points which not on the same plane, as depicted in Fig. 1(c).

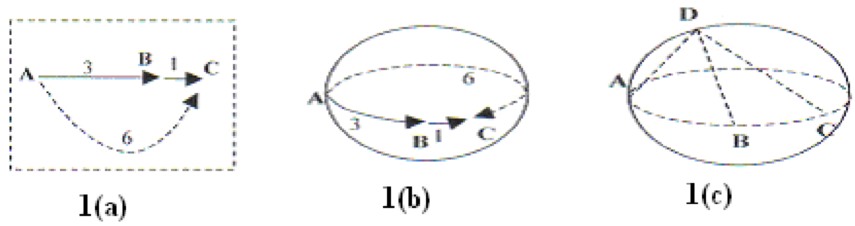

Fig. 1 The advantages of 3-D over 2-D

The ‘trick’ this study use to achieve this breakthrough result is that using two axes to cluster observations while the third axis is predetermined to interpret the rankings of business schools. Accordingly, rather than simply partitioning observations into clusters, this proposed cluster ranking framework is to partition business schools into the three-dimension space positioning by 3 non-parallel vectors of x-axis, y-axis, and z-axis. The initial idea behind using one-axis (the vertical middle axis of the sphere) to interpreting school rankings is that a business school cares not only its rank, but also the “ranking tier” it belongs to. That is, a school needs to focus on the comparison with its aspirational schools (the upper tier schools it wants to be in future) and its competitive schools (the same tier schools it needs to compete with at present). Therefore, this study attempts to provide this kind of “tier” knowledge for each business school, while clustering business schools in terms of their homogeneity (or heterogeneity).

During clustering observations, finding the optimal number of clusters is another challenge work and has been considered as a NP-hard problem (Rinzivillo et al., 2008). Since the literature reveals that k-means algorithm is well-known for its efficiency and widely applied in practice, particularly for observations with numerical attributes (Everitt et al., 2001), the k-means is employed to determine the optimal number of clusters in this study. That is, this work first partitions business schools into k groups where k is initialized as 2, and add 1 to k at each iterative procedure until the within-cluster sum of squares is increasing. That is, clustering observations is terminated when the within-cluster sum of squares is increasing, while clustering observations is continuing as long as the within-cluster sum of squares is decreasing

Given a set of observations (x_1, x_2, …., x_n) where n is the number of observations, the within-cluster sum of squares can be obtained by $\sum_{k=1}^{K}\sum_{X_I \in G_k} \| x_i - \mu_k \|^2$ where K is the total number of cluster, $x_i \in G_k$ means the observation x_i is partitioned into the k’th cluster (group), and μ_k is the mean value of observations in the k’th cluster. Taken the above together, the algorithm of clustering business schools into three-dimension sphere with maintaining their ranking orders by one axis is described as follows.

<u>Cluster Ranking Algorithm</u>

Step 1: To choose criteria (attributes) to rank schools
Step 2: To compute all dissimilarity (similarity) between schools

Step 3: To partition schools into k clusters and obtain the means of k clusters based on k-means algorithms, and the process continues until the minimum of within-cluster sum of squares, computing by $\sum_{k=1}^{K} \sum_{X_I \in G_k} \| x_i - \mu_k \|^2$, is found.

Step 4: To obtain the center of each cluster and allocate each school on the sphere in terms of three-plane coordinates.

4. Proposed Cluster Ranking Method

This section is organized as six subsections. The first subsection formulates the ranking functions, the second one briefs how to calculate the dissimilarity coefficients, the third describes the rule for allocating objects on three-dimension sphere, the fourth introduces the computation of the coordinates of business schools, the fifth describes how to compute centric points for k clusters, and the six subsection is to present the model for computing coordinates of each cluster's group center.

4.1 Ranking business schools

Notably, it has long been recognized that not all attributes contribute equally to valuing objects (DeSarbo et al., 1984; Donoghue, 1990; xxxxx, 200?; Steinley and Brusco, 2008), while no matter which clustering technique is used, an important step is to select attributes to define objects. Following the tables 1-2, NWR, BW, FT and Econ employ two types of data to assess business schools. One is soft data collected from opinion surveys of deans, faculties, students, alumni, and recruiters, and the other is hard data gathered from publicly available resources or provided by the schools (Elsbach and Kramer, 1996; Martins, 2005; Zell, 2005; Lavelle and Lehman, 2006; Joyce, 2006; Milton, 2006; Morse and Flanigan, 2006).

After extensively reviewing the reports released by the NWR, BW, FT, and Econ (Morse and Flanigan, 2006 and 2009; Business Week, 2006 and 2009; Financial Times, 2006 and 2009; U.S. News & World Report, 2006, 2007, and 2009; Economist.com, 2009), this research found that all rankings in the media are subjective and assessed by the combination of different criteria with different weights suggested, modified, and determined by experts and survey project leaders invited by individual media over the time. That is, the weights on criteria every year have been discussed, debated, and adjusted to fit the expected awareness centered on the invited professionals, each media focus, and the public interests. Consequently, following Tables 1 and 2, the ranking functions used by NWR, BW, FT, and Econ can be formulated as the follows:

NWR Ranking Function

$$s_i = \sum_{i=1}^{I} \sum_{k=1}^{8} (w_k c_{ik}) \quad (1)$$

BW Ranking Function

$$s_i = \sum_{i=1}^{I} \sum_{k=1}^{9} (w_k c_{ik}) \quad (2)$$

FT Ranking Function

$$s_i = \sum_{i=1}^{I}\sum_{k=1}^{21}(w_k c_{ik}) \quad (3)$$

Econ Ranking Function

$$s_i = \sum_{i=1}^{I}\sum_{k=1}^{21}(w_k c_{ik}) \quad (4)$$

In (1)-(4), s_i denotes the score of i'th school, I denotes the number of surveyed business schools, w_k represents the weight of k'th criteria, and c_{ik} is the value of k'th criteria of i'th school.

4.2 Calculating the dissimilarity between business schools

After using weighting attributers to rank business schools, the next important step is to generate the dissimilarity matrix between business schools. Accordingly, to calculate the similarity between schools, the dissimilarity function is formulated below:

$$d_{ij} = \sum_{k=1}^{L} w_k \frac{|c_{i,k} - c_{j,k}|}{\bar{c}_k - \underline{c}_k}, \quad (5)$$

where $c_{i,k}$ denotes the value of the attribute k for school I, $\bar{c}_k$ and $\underline{c}_k$ denote the upper and lower bounds of $c_{i,k}$, L is the number of attributes, w_k are weights for attribute k. Accordingly, if $c_{i,k} = c_{j,k}$ for all *k* then $d_{ij} = 0$, and if $c_{i,k} = \bar{c}_k$ and $c_{i,k} = \underline{c}_k$ then $d_{ij} = 1$. Besides, $0 \leq d_{ij} \leq 1$ and $d_{ij} = d_{ji}$.

4.3 Rule for allocating objects on three-dimension sphere

Denote the coordinates of the i'th school as (x_i, y_i, z_i), where $0 \leq x_i \leq 1$, $0 \leq y_i \leq 1$, and $0 \leq z_i \leq 1$. The following proposition is presented.

Proposition 1

The relationship between z_i and s_i (the score of the i'th school) is computed as $z_i=2s_i-s_i^2$

Proof: Let $d_{i,j} = \sqrt{2}\delta_{i,j}$, such that if $\delta_{i,j} = 0$ then $d_{i,j} = 0$ and if $\delta_{i,j} = 1$ then $d_{i,j} = \sqrt{2}$, where $\sqrt{2}$ is used because the distance between the north pole and equator is $\sqrt{2}$ when radius = 1. Since $d_{i,*}^2 = (x_i - 0)^2 + (y_i - 0)^2 + (z_i - 1)^2 = 2\delta_{i,*}^2 = 2(1-s_i)^2$, it is clear that $z_i = 2s_i - s_i^2$.

Accordingly, the rules of allocating objects on spheres with maintaining ranking orders are described below.

(i) For three objectives i, j, and k, if the dissimilarity of i and j points is higher than that of i and k points, then the distance of $\widehat{ij}$ arc is larger than that of $\widehat{ik}$ arc. This relationship can be expressed as:
if $d_{ij} > d_{ik}$ than $(x_i-x_j)^2+(y_i-y_j)^2+(z_i-z_j)^2 > (x_i-x_k)^2+(y_i-y_k)^2+(z_i-z_k)^2$

(ii) The relationship between z_i and s_i (the score of the i'th school) is computed as

$z_i = 2s_i - s_i^2$

4.4 Computing coordinates for business schools

Following the above rules, a point in space can be positioned by 3 non-parallel vectors. Therefore, all coordinates for business schools can be generated by the following model:

<u>Model for computing coordinates for business schools</u>

$$\underset{\{x_i, y_i, z_i\}}{Min} \quad Obj = \sum_{i=1}^{IJ}\sum_{j>i}^{IJ}(q_{ij} - d_{ij})^2 \tag{6}$$

subject to $z_i = e^{0.5*(2s_i - s_i^2)}$, $\forall$ i, j, (7)

$$q_{ij}^2 = (x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2, \ \forall i, j, \tag{8}$$

$$x_i^2 + y_i^2 + z_i^2 \le 1, \quad \forall i, \tag{9}$$

$$-1 \le x_i, y_i \le 1, \ 0 \le z_i \le 1, \ \forall i, \tag{10}$$

where *Obj* is the objective function intending to minimize the sum of difference between q_{ij} and d_{ij}; IJ represents the number of clusters, $q_{i,j}$ denotes the distance between objects i and j, d_{ij} come from dissimilarity matrix generated by Formula (5), and x_i, y_i, and z_i are coordinates of the school i on a sphere. Constraint (7) is to specify the relationship between z_i and s_i based on Proposition 1, which provides a scale adjustment for schools. Constraint (8) is the Euclidian distance between i and j schools. Constraint (9) aims to ensure that all points must allocate on the inside or surface of a sphere. Constraint (10) is to ensure all schools plotted on the northern hemisphere for comparison convenience. Notably, the concept of monotonic increasing function is used to scale z value for all schools in the presented cluster ranking methodology, and s_i is determined by Formula of (1)-(4).

4.5 Computing centric points for k clusters

As we know, to determine the optimal number of clusters is one of the major challenges for clustering, not just for k-means (Xu and Wunsch II, 2008). Since k-means algorithm is well-known for its efficiency and widely applied in practice, particularly for observations with numerical attributes (Everitt et al., 2001), the proposed method partitions schools into k clusters and obtain the means of k clusters based on k-means algorithms, and the iterative process continues until the minimum of within-cluster sum of squares is found. Hence, the model for computing k centric points for k cluster is formulated as follows.

<u>Model for computing k centric points for k clusters</u>

Minimize $\sum\sum(d_{ij} - \hat{d}_{ij})^2$ (11)

Subject to: $z_i = 2s_i - s_i^2$, (12)

$$d_{ij}^2 = (x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2, \tag{13}$$

$$x_i^2 + y_i^2 + z_i^2 \le 1, \quad \forall i, \tag{14}$$

$$\hat{y}_{ij} \le \hat{d}_{pq} - \varepsilon, \tag{15}$$

$$\hat{d}_{\min} = a_{\min} \times t, \tag{16}$$

$$\hat{d}_{\max} = a_{\max} \times t, \tag{17}$$

$$\frac{2+\sqrt{2}}{2} \le t \le 2 \qquad (18)$$

where d_{ij} is dissimilarity degree between school i and school j, x_i, y_i, and z_i are coordinates of the school i on a three-dimension sphere, x-axis is latitude, y-axis is longitude, z-axis is the vertical middle axis of the sphere to reflect school ranks, the constraint (12) is to determine the z_i of each centric point, the constraint (15) is to determine the distance between school i and school j subject to their dissimilarity matrix created by their attributes, the constraint (16) is to determine the smallest value of the distance among clusters, the constraint (17) is to determine the biggest value of the distance among clusters, and the constraint (18) is to set the scope of allowance of the value t.

4.6 Computing coordinates for each cluster center

Following the rules of allocating objects on spheres, the model for computing coordinates of each cluster's group center is presented below:

Computing center's coordinates for each cluster

Minimize $\sum \sum_{ij=G+G_k} (d_{ij} - \hat{d}_{ij})^2$, $\forall$ g, g = 1, 2, …., k (19)

Subject to: $z_i = 2s_i - s_i^2$, (20)

$d_{ij}^2 = (x_i\text{-}x_j)^2 + (y_i\text{-}y_j)^2 + (z_i\text{-}z_j)^2$, (21)

$\hat{d}_{ij} = a_{ijk} \times t$, (22)

$x_i^2 + y_i^2 + z_i^2 \le 1, \quad \forall i$, (23)

$x_i = G_i$, $y_i = G_y$, i = 1, 2, …, k (24)

$$\frac{2+\sqrt{2}}{2} \le t \le 2 \qquad (25)$$

where the constraint (22) is to set the value of $\hat{d}_{ij}$ being the power of t of a_{ijk} which is the dissimilarity coefficient between clusters i and j in the dissimilarity matrix created from the k'th clusters' attributes coming from Model of (11)-(18), and the values of x_i and y_i for center's coordinates of each cluster i is determined by the constraint (24) and the value of z_i is determined by the constraint (20).

5. Numerical Analysis

The first part of this section is to analyze the current business ranking results, while the second part is to analyze the results generated by the proposed cluster ranking method.

5. 1 Analysis of Current Business School Ranking Results

Following Tables 1-2 and Ranking Functions (1)-(4) used by NWR, BW, FT, and Econ, the scores of worldwide business schools can be generated. However, for the purpose of simplifying illustration, this work use top 50 American business schools ranking data during the last five-year period produced by NWR and FT. Since FT ranked worldwide business schools, the rankings listed in the columns of FT are renumbered after removing non-American business schools (as shown in Table 3). Besides, the “i” (from 1 to 50) appearing in the first column of Table 3 is numbered by the order of NWR ranks in 2009. Accordingly, the scores of the top 50 American business schools during 2005-2009, ranking by NWR and FT by their respective ranking function (1) and (3), are displayed in Tables 3 and 4.

Table 3 Top American business schools ranked by NWR and FT during 2008-2009

i	Abbreviation of School Name	2009		2008	
		NWR	FT	NWR	FT
		Rank(Score)	Rank*(Score)	Rank(Score)	Rank* (Score)
1	Harvard University	1 (0.9122)	2 (0.7387)	1 (0.9313)	4 (0.6718)
2	Stanford University	2 (0.8987)	4 (0.6631)	2 (0.9273)	3 (0.6803)
3	University of Penn	3 (0.8293)	1 (0.7912)	3 (0.8604)	1 (0.6955)
4	Northwestern University	3 (0.8293)	10 (0.5854)	4 (0.8503)	10 (0.6211)
5	MIT	5 (0.8108)	5 (0.6542)	4 (0.8503)	5 (0.6544)
6	University of Chicago	5 (0.8108)	7 (0.6157)	4 (0.8503)	6 (0.6517)
7	University of California – Berkeley	7 (0.7890)	16 (0.5172)	7 (0.7958)	15 (0.5846)
8	Dartmouth University	8 (0.7625)	8 (0.6090)	7 (0.7958)	8 (0.6359)
9	Columbia University	9 (0.7270)	3 (0.6812)	9 (0.7626)	2 (0.6887)
10	Yale University	10 (0.7182)	9 (0.5874)	13 (0.6630)	9 (0.6310)
11	NYU	11 (0.6877)	6 (0.6322)	10 (0.7137)	7 (0.6398)
12	Duke University	12 (0.6724)	11 (0.5766)	14 (0.6529)	14 (0.5933)
13	University of M. – Ann Arbor	13 (0.6302)	12 (0.5665)	12 (0.6871)	13 (0.5984)
14	UCLA	14 (0.6278)	15 (0.5215)	11 (0.6981)	11 (0.6153)
15	University of Virginia	15 (0.6268)	14 (0.5403)	14 (0.6529)	16 (0.5831)
16	Carnegie Mellon University	15 (0.6268)	24 (0.4148)	17 (0.6341)	23 (0.5522)
17	Cormell University	17 (0.6138)	17 (0.5002)	14 (0.6529)	17 (0.5802)
18	University of Texas – Austin	18 (0.5465)	23 (0.4207)	18 (0.5798)	43 (0.4365)
19	Georgetown University	19 (0.5392)	18 (0.4886)	22 (0.5208)	19 (0.5737)
20	University of North Carolina	20 (0.5232)	21 (0.4449)	19 (0.5486)	20 (0.5684)
21	University of South California	21 (0.4869)	32 (0.3517)	21 (0.5228)	30 (0.5333)
22	Emory University	22 (0.4761)	13 (0.5478)	24 (0.4818)	12 (0.6077)
23	Indiana U. – Kelley	22 (0.4761)	31 (0.3589)	20 (0.5391)	31 (0.5236)
24	GIT				
25	Washington U. in St. Louis	22 (0.4761)	33 (0.3476)	25 (0.4795)	39 (0.4718)
26	Ohio State University	26 (0.4718)	47 (0.2582)	27 (0.4480)	
27	University of Washington	26 (0.4718)	38 (0.3178)	34 (0.4016)	21 (0.5661)
28	U. of Wisconsin – Madison				
29	Arizona State University	29 (0.4083)	44 (0.2745)		
30	Brigham Young University			29 (0.4441)	48 (0.3862)
31	University of Rochester	29 (0.4083)	22 (0.4287)	25 (0.4795)	22 (0.5618)
32	Purdue University	32 (0.3928)	46 (0.2660)	33 (0.4237)	34 (0.5084)
33	Texas A&M University	33 (0.3648)	30 (0.3726)	29 (0.4401)	32 (0.5196)
34	U. of Minnesota – Twin Cities	33 (0.3648)	42 (0.2935)		
35	University of Notre Dame	33 (0.3648)	45 (0.2682)	34 (0.4016)	45 (0.4176)
36	Vanderbilt University	33 (0.3648)	28 (0.3913)	44 (0.3268)	44 (0.4222)
37	University of. Florida	37 (0.3428)	34 (0.3432)	34 (0.4016)	36 (0.4936)
38	Rice University	38 (0.3303)	25 (0.4025)	40 (0.3507)	35 (0.4988)

39	U. of Illinois – Urbana Champaign	38 (0.3303)	27 (0.3966)	38 (0.3902)	37 (0.4876)
40	Michigan State University	40 (0.3186)	35 (0.3384)	40 (0.3507)	24 (0.5507)
41	Penn State University	40 (0.3186)	37 (0.3219)	40 (0.3507)	28 (0.5436)
42	University of California – Davis			44 (0.3268)	29 (0.5412)
43	U. of Maryland –College Park	40 (0.3186)	20 (0.4467)	39 (0.3574)	18 (0.5765)
44	Boston College	44 (0.3095)	50 (0.2344)	34 (0.4016)	50 (0.3368)
45	University of Iowa	44 (0.3095)	36 (0.3355)	49 (0.2679)	25 (0.5623)
46	Boston University	46 (0.2893)	29 (0.3885)	40 (03507)	40 (0.4667)
47	Southern Methodist University	47(0.2735)	41 (0.2958)		
48	Tulane University				
49	Babson College			48 (0.2986)	49 (0.3411)

* The ranking is renumbered after removing non-American business schools.

Table 4 Top American business schools ranked by NWR and FT during 2005-2007

i	Abbreviation of School Name	2007		2006		2005	
		NWR	FT	NWR	FT	NWR	FT
		Rank(Score)	Rank*(Score)	Rank(Score)	Rank* (Score)	Rank(Score)	Rank* (Score)
1	Harvard	1 (0.9465)	3 (0.5863)	1 (0.9517)	2 (0.7320)	1 (0.9400)	1 (0.7655)
2	Stanford	2 (0.9357)	3 (0.5863)	2 (0.9308)	3 (0.7288)	2 (0.9156)	4 (0.6917)
3	U. of Penn	3 (0.8823)	1 (0.5946)	3 (0.9227)	1 (0.7599)	2 (0.9038)	1 (0.7655)
4	Northwestern	5 (0.8430)	11 (0.4973)	4 (0.8447)	11 (0.6168)	4 (0.8481)	9 (0.5893)
5	MIT	4 (0.8597)	9 (0.5218)	4 (0.8447)	8 (0.6725)	4 (0.8481)	10 (0.5661)
6	Chicago	5 (0.8430)	5 (0.5796)	6 (0.8360)	5 (0.6820)	8 (0.7642)	5 (0.6793)
7	UC – Berkeley	8 (0.8084)	14 (0.4802)	7 (0.7846)	11 (0.6343)	6 (0.7880)	10 (0.5661)
8	Dartmouth	7 (0.8300)	7 (0.5535)	9 (0.7680)	7 (0.6728)	6 (0.7880)	6 (0.6773)
9	Columbia	9 (0.7372)	2 (0.5902)	7 (0.7846)	4 (0.7184)	9 (0.7529)	3 (0.7381)
10	Yale	14 (0.6633)	8 (0.5391)	15 (0.6162)	9 (0.6502)	15 (0.5947)	7 (0.6514)
11	NYU	10 (0.7143)	6 (0.5668)	13 (0.6676)	6 (0.6793)	13 (0.7086)	7 (0.6514)
12	Duke	12 (0.6673)	13 (0.4819)	11 (0.6997)	13 (0.5547)	11 (0.7154)	14 (0.5485)
13	U. of M. – Ann Arbor	11 (0.6689)	11 (0.4973)	11 (0.6997)	10 (0.6378)	10 (0.7205)	12 (0.5625)
14	UCLA	16 (0.6469)	10 (0.4301)	10 (0.7133)	13 (0.6098)	11 (0.7154)	17 (0.4737)
15	U. of Virginia	12 (0.6673)	15 (0.4726)	13 (0.6676)	12 (0.5703)	14 (0.5947)	15 (0.5148)
16	Carnegie Mellon	17 (0.6356)	24 (0.4158)	16 (0.5953)	23 (0.5075)	17 (0.5705)	22 (0.4503)
17	Cormell	14 (0.6633)	16 (0.4698)	16 (0.5953)	18 (0.5303)	15 (0.5947)	16 (0.4896)
18	U. of Texas – Austin	18 (0.5706)	35 (0.3509)	18 (0.5402)	36 (0.4299)	18 (0.4751)	34 (0.3483)
19	Georgetown	25 (0.4211)	20 (0.4322)	34 (0.3784)	19 (0.5141)	27 (0.3652)	22 (0.4503)
20	U. of North Carolina	18 (0.5706)	18 (0.4467)	20 (0.4914)	16 (0.5379)	21 (0.4491)	13 (0.5597)
21	U. of South California	21 (0.4976)	40 (0.3217)	29 (0.4142)	29 (0.4577)	26 (0.3744)	25 (0.4407)
22	Emory	20 (0.5369)	19 (0.4359)	18 (0.5402)	24 (0.5065)	18 (0.4751)	18 (0.4670)
23	Indiana U. – Kelley	24 (0.4301)	37 (0.3497)				
24	G. I. T.	25 (0.4211)	49 (0.2134)	34 (0.3784)	44 (0.3979)	32 (0.3356)	44 (0.2348)
25	Washington U. in St. Louis	29 (0.4165)	39 (0.3232)	26 (0.4187)	28 (0.4643)	32 (0.3356)	37 (0.3210)
26	Ohio State	22 (0.4819)	46 (0.2418)	22 (0.4650)	39 (0.4118)	21 (0.4491)	42 (0.2630)
27	U. of Washington	29 (0.4165)	31 (0.3814)	29 (0.4142)	35 (0.4340)		
28	U. of Wisconsin – Madison	29 (0.4165)	47 (0.2225)	31 (0.3921)	44 (0.3979)		
29	Arizona State	41 (0.3408)	31 (0.3814)	34 (0.3784)	38 (0.4158)	31 (0.3610)	37 (0.3210)
30	Brigham Young	41 (0.3408)	31 (0.3814)	34 (0.3784)	26 (0.4870)	40 (0.2726)	30 (0.4037)
31	U. of Rochester	36 (0.3769)	21 (0.4234)	26 (0.4187)	22 (0.5080)	23 (0.4267)	18 (0.4670)
32	Purdue	22 (0.4819)	29 (0.3910)	21 (0.4697)	41 (0.4052)		
33	Texas A&M	29 (0.4165)	43 (0.2671)	31 (0.3921)	47 (0.3878)	32 (0.3356)	45 (0.2051)

34	U. of Minnesota – Twin Cities	25 (0.4211)	26 (0.4086)	23 (0.4562)	32 (0.4401)	23 (0.4267)	32 (0.4029)
35	U. of Notre Dame	39 (0.3424)	37 (0.3497)	31 (0.3921)	33 (0.4359)	32 (0.3356)	26 (0.4248)
36	Vanderbilt U.	34 (0.3815)	34 (0.3737)	49 (0.2398)	34 (0.4356)	45 (0.2565)	21 (0.4529)
37	U. of. Florida			41 (0.3291)	48 (0.3866)		
38	Rice U.	48 (0.2531)	28 (0.4049)	44 (0.3105)	37 (0.4220)	49 (0.2169)	28 (0.4162)
39	U. of I.– Urbana Champaign	37 (0.3722)	21 (0.4234)	28 (0.4130)	21 (0.5171)	27 (0.3652)	29 (0.4151)
40	Michigan State	29 (0.4165)	21 (0.4234)	23 (0.4562)	17 (0.5359)	32 (0.3356)	30 (0.4037)
41	Penn State	34 (0.3815)	25 (0.4122)	38 (0.3357)	25 (0.4987)	37 (0.3289)	33 (0.3911)
42	UC – Davis	46 (0.2985)	41 (0.3129)	46 (0.2945)	43 (0.4008)	42 (0.2714)	41 (0.2525)
43	U. of Maryland -College Park	25 (0.4211)	17 (0.4547)	38 (0.3357)	20 (0.5244)	27 (0.3652)	20 (0.4618)
44	Boston College	39 (0.3424)	30 (0.3826)	41 (0.3291)	27 (0.4667)		
45	U. of Iowa	50 (0.2160)	26 (0.4086)			37 (0.3289)	22 (0.4503)
46	Boston U.	41 (0.3408)	35 (0.3509)	44 (0.3105)	31 (0.4489)	48 (0.2344)	27 (0.4190)
47	Southern Methodist			41 (0.3291)	40 (0.4078)		
48	Tulane	45 (0.3114)	48 (0.2178)			45 (0.2565)	39 (0.3050)
49	Babson College	41 (0.3408)	45 (0.2585)	49 (0.2398)	41 (0.4052)		

* The ranking is renumbered after removing non-American business schools.

Since the top 50 American business schools in each press (i.e., NWR and FT) are not same, for the purpose of demonstration convenience, Tables 3 and 4 only shows the schools ranked within the top 50 schools in both FT and NWR from 2005-2009. For example, NWR consecutively ranked Georgia Institute of Technology (GIT) and University of Wisconsin (Madison) in the Top 30, but these schools were consecutive ranked out of Top 50 by FT. Similarly, University of Arizona, Thunderbird University, University of South Carolina, and George Washington University were consecutively ranked in Top 50 in FT, but these four universities never appeared in the Top 50 of NWR rankings. These findings also confirm that <u>ranking inconsistency does exist among dominant ranking systems.</u>

In contrast, both tables also indicate that forty American business schools were consecutively ranked in top 50 in both FT and NWR ranking systems during 2005-2009. Looking at specific schools during 2008-2009, both FT and NWR consecutively ranked University of Pennsylvania superior to Massachusetts Institute of Technology (MIT), MIT superior to University of California (Berkeley), University of California (Berkeley) superior to Carnegie Mellon University, Carnegie Mellon University superior to University of South California, University of South California superior to Washington University (in St. Louis), and Washington University (in St. Louis) superior to University of North Dame. Many other consistent ranking orders during 2005-2009 can be also found in Tables 3-4. These findings reveal that <u>common superior ranking relations (or called inferior ranking relations) do exist in dominant ranking systems.</u>

Tables 3-4 also indicate that even within the same ranking system, some schools' ranks changed significantly between years. For example, in only one year, Purdue University fell from 22 in 2007 down to 33 in 2008 under the same ranking system (NWR). Take the University of Texas (Austin) as another example: the University of Texas (Austin) was ranked 43 in 2008 and up to 23 in 2009 under the same ranking system (FT). How did a university regress or advance so quickly within one year? Particularly, during the same years 2008-2009, the University of Texas (Austin) was ranked at the same position (the 18th place) in the NWR ranking system. Similarly, when NWR ranked Purdue University as the 32-33th place in 2008-2009, Purdue University fell from 34 in 2008 down to 46 in 2009 under the FT ranking system. Accordingly, this study re-computed the business schools rankings based on Functions (1) and (3) by randomly selecting ten schools from Tables 3-4, meanwhile a 5% change was made in the weight on Salary Percentage Increase for FT and a 5% percent change in the weight on Mean Starting Salary and Bonus for NWR. In doing so, this research found the average difference in rank change was 11.3 places, which reveals that a little change in a school's input data may cause a big difference in its score and rank. That is, school rank is sensitive to collected data.

Looking at FT and NWR rankings during 2005-2007, this work found that Indiana University was not ranked in the Top 50 by FT in 2005-2006 and the University of Washington was not ranked in the Top 50 by FT in 2005, but these two schools were continuously ranked in the top 50 by NWR during 2005-2009. The University of California (Irvine) was only ranked in the top 50 by NWR in 2008, but this school was consecutively ranked in the top 50 by FT during 2005-2009. Besides, some schools' ranks such as Harvard University, University of California (Berkeley), and University of Texas (Austin) in NWR were always better than them in FT during 2005-2009. Similarly, during 2005-2009, some schools' ranks such as University of Pennsylvania, Yale University, New York University (NYU), and Rice University in FT were always better than them in NWR. According to the above observation, this research substituted a NWR system's criterion (i.e., Average Starting Salary) by a FT system's criterion (e.g., Aims Achieved), and then re-computed the business schools via Functions (1) and (3). Such a doing, the paper found that around 63% of school ranks across 2005-2009 were altered, which reveals that one magazine ranking might place more emphasis on certain area while another magazine might choose to emphasize on other area. That is, the ranks of schools heavily depend on from which angles to view the schools.

Following the above discussion, this research subsequently decreased 0.01 of the weights of the top half of criterion except for criterion 1 because its weight is zero (which means the criterion 1 not used to assess the schools in FT system) as shown in Table 2, and increased 0.01 of the weights of the bottom half of criterion. After using Functions (1) and (3) to re-calculate the business schools scores, this work found that approximate 54% of school ranks across 2005-2009 are changed. This phenomenon is similarly occurred in NWR system. Nevertheless, empirical results generated by Functions of (1) and (3) reveal that any little changes on the weights of specific criteria will significantly alter the ranks of business schools. As a result, school rank is sensitive to weights on evaluation criteria because the weights on criteria play critical roles of generating the scores for specific business schools.

Although slight changes in criterion selection or weights on criteria, the school ranks are significantly impacted, this research also noticed that approximate 80% of schools in the top tier (ranked 1-15) and around 60% of schools in the second tier (ranked 16-30) or in the third tier (ranked 30-49) stay in the same tier via the weights or criteria simulation.

Taking the data in 2007 listed in Table 4 for instance, after examining results of the weights or criteria simulation, 14 of the top 15 schools were same in NWR and FT, and only one debate is to take Cornell or UCLA into the Top 15. For the schools belonging to ranked 16-30 or 30-49, after examining results of the weights or criteria simulation in NWR and FT, 14 schools of 33 schools were changed between the second tier and third tier. Consequently, change in ranking place is easier than in ranking tier when few changes occur in the school data, which reveals that the rank tier of a school is more stable than the rank position of a school.

5.2 Analysis of Cluster Ranking Business Schools

Building in the above, the proposed Cluster Ranking Framework works in five phases: ranking business schools, generating dissimilarity matrix, determining the number of cluster by k-means algorithm, calculating the coordinates of business schools and centers of k clusters, and plotting business schools on three-dimension sphere with keeping their ranking orders.

5.1 Ranking business schools

Without losing the generality, this study randomly NWR and FT as resource data from prevailing business school ranking systems such as NWR, FT, BW, and Econo. The reason may be attributed to NWR is the oldest medium in ranking American business schools and FT is the first medium in ranking worldwide business schools. Accordingly, the top American business schools' rankings during the last five-year are summarized as Tables 3 and 4, based on NWR and FT.

5.2 Generating dissimilarity matrix

Take ranking data of NWR and FT during 2005-2009 (shown in Tables 3-4), the similarity coefficients between business schools can be calculated by Formula of (5). The dissimilarity matrixes among top American business schools are then formed as Tables 5 and 6 for NWR and FT systems, respectively. The value of dissimilarity coefficient between schools represents the degree of dissimilarity. That is, the smaller the value of dissimilarity coefficient, the more similar two business schools are. For example, d_{12} = 0.0646 and d_{13} = 0.1310 express that the dissimilarity degree between the schools 1 and 3 is double that between the schools 1 and 2.

Table 5 Part of Dissimilarity Matrix for NWR system at 2009

i j						……………………… ……..	**7**	**8**	**9**
	0	0.0646	0.1310	0.1610	0.1582	………………………	0.6579	0.6486	0.5781

	0 0.1287 0.1494 0.1193 …………………………… 0.6463 0.6530
	0.5697
	0 0.1438 0.1476 …………………………… 0.6324 0.6350
	0.5697
	0 0.0608 …………………………… 0.5331 0.5238
	0.4566
	0 …………………………… 0.5427 0.5495
	0.4661
47	………
	…………
48	…………
	………… 0 0.1627 0.2519
49	0
	0.2724
	0

Table 6 Part of Dissimilarity Matrix a for FT system at 2009

i						…………………………………….	**7**	**8**	**9**
	0	0.065	0.1373	0.1662	0.1553	………………………	0.6495	0.6461	0.5754
		0	0.1315	0.1488	0.1198	………………	0.6347	0.6410	0.5676
			0	0.1479	0.1440	………………	0.6354	0.6289	0.5536
				0	0.0663	………………	0.5266	0.5159	0.453
					0	………………	0.5341	0.5274	0.4361
						……			

	……… ……….
	……. 0 0.1823 0.2429 0 0.235 0

5.3 Determine the optimal number of cluster

Following k-means algorithms, the following table is generated.

Table 7 The total variance within-cluster observations

Number of Cluster	NWR	FT
	the within-cluster sum of squares	the within-cluster sum of squares
2	252.745	324.76
3	212.47	289.45
4	178.56	254.13
5	145.67	206.76
6	167.43	233.54
7	192.33	267.77

5.4 Calculating the coordinates of business schools and centers of k clusters

Following the cluster ranking framework, the coordinates of business schools are computed, where the x-axis and y-axis are used to gauge the differences between schools while z-axis (the vertical middle axis of the sphere) is to reflect school ranks.

5.5 Calculating the coordinates of business schools and centers of k clusters

After computing the coordinates for business schools and centroid for five clusters, all business schools are displayed in the following Figs 2-6 and Figs. 7-11 for NWR and FT, respectively.

Take Figs 2-6 and 7-11

6. Conclusion and Future Research Discussion

Since the assessment of business school is a multifaceted concept shaped by a wider range of elements (Rogers, 1988; Schatz, 1993; Elsbach and Kramer, 1996; Tracy and Waldfogel, 1997; Trieschmann et al., 2000; AACSB, 2002, 2005, and 2007; Fee et al., 2005), it is better to cover a wide range of elements such as course content, teaching methods, faculty and research features, school environment and mission, and so forth to evaluate the homogeneity (heterogeneity) between business schools. However, it is difficult for a single research or a school affording such a huge cost to obtain all data from worldwide business schools. Since prevailing ranking systems have put large manpower and budget to gather a large scale of data and create highly visible metrics, this study select attributes offered and used by current ranking systems to assess schools. However, in the future study, if one can get more data in terms of characteristics of business schools to cluster schools, the clustered information will be enriched.

Appendix 1

NWR started to rank business schools in 1987. The methodology adopted by NWR is first to standardize collected data under each criterion by weighting the standardized scores, secondly to rescale the scores so that the top school received 100 and others received their percentages of the top scores (NWR, April, 22 2009). Since late 1990s, NWR assessed business schools from three aspects of Quality Assessment (weighted by 0.40), Place Success (weighted by 0.35), and Student Selectivity (weighted by 0.25). Under Quality Assessment, there are two criteria named Peer Assessment and Recruiter Assessment; Under Place Success, there are three criteria labeled Mean Starting Salary and Bonus, Employment rate at Graduation, and Employment in 3 Months; Under Student Selectivity, there are three criteria called Mean Undergraduate GPA, Mean

GMAT Score, and Acceptance Rate. In 2009, the Peer Assessment Scores for each school come from 381 business school deans via sending the survey to 426 deans, revealing 89.43% of the response rate.

BW began to rank business schools in 1988. The methodology adopted by BW centers on business school's customer satisfaction and concerns, and considers business school's customers are graduation students and corporate recruiters who hire these students. Hence, in contrast to NWR collecting data from students, recruiters, and school deans, BW gathered data from graduation students and corporate recruiters (BW, October 23, 2006). Since 2000, BW scored business schools from aspects of Student Survey (weighted by 0.3), Recruiter Survey (weighted by 0.2), Average Starting Salaries (weighted by 0.1), the MBA Feeder School Measure (weighted by 0.1), and Academic Quality (weighted by 0.3). Under Academic Quality, there are five criteria named Average SAT Scores, Ratio of Full-Time Faculty to Students, Average Class Size, Percentage of Business Majors with Internships, and Hours Students Spend Every Week on School Work. Regarding Student Survey, BW sent a 50-question survey from the quality of teaching to recreational facilities to more than 85,000 business school graduates, and the response rate of graduate survey is 27%. Regarding Recruiter Survey, BW polled 580 corporate recruiters asking them to rate which programs turn out the best graduates, which schools have the most innovative curriculums, and which schools have most effective career services, and the response rate of recruiter survey is about 33%.

FT conducted business school ranking in 1995. In contrast to other media, FT survey data is audited and provided by a Swiss-based global professional service firm named KPMG (Financial Times, 2006). In contrast to NWR and BW, the methodology originally set by FT is to rank global business schools and highlight on strong international orientation, high research reputation, alumni satisfaction, and gender diversity on faculty. Accordingly, FT employs 21 criteria and associated weights to rank business schools as shown in Table 2. Noteworthy, where FT Doctoral Rank is rated by number of doctoral graduates taking up a faculty position at one of the top 50 business schools, while FT Research Rank is assessed by faculty publications in 40 international journals, points are accrued by the business school at which the author is presently employed, and adjustment is made for faculty size. Although the FT rankings are mostly global in its scope, its global view may be heavily from European and English-speaking nations. Which may explain why salary-related criteria occupy 40% of the weight, and the research reputation is only evaluated by a selected group of 40 English language journals (10% of the weight). Due to cultural biases embedded in the ranking methodology, FT rankings are dominated by English-speaking business schools (Financial Times, December 23, 2009).

Econ is another main resource in global business school ranking. Comparing with FT, Econ takes a two-stage survey and uses data over a three-year period to give students and schools a more rounded picture. To quantify and score the business schools, Econ ranking is made up of four categories: Open New Career Opportunities (weighted by 35%), Personal Development and Educational Experience (weighted by 35%), Increase in Salary (weighted by 20%), and Potential to Network (weighted by 10%), and each of the categories is made up of individual criteria. For example, Open New Career

Opportunities contain four criteria named Diversity of Recruiters, Jobs Found in 3 Months, Jobs Found through the Career Service, and Student Assessment. Notably, unlike other rankings, Econ does not include any "equal" schools (even the difference between schools might be very slight), and gives each business school a unique score known to statisticians as a z-score (Economist, September 25, 2009).

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2010 年出國報告心得（出國類別：研習）

會議主題：Asia Summer Institute in Behavioral Economics

出國成果報告書

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出國地點	Singapore-NUS	出國經費	國科會

一、目的：

With my advisor Prof. Li, I try to enhance the algorithm of branch-and-bound and let the bound of problems be as tight as possible. Therefore, the resolving time can be accelerated and the solutions will be guaranteed to reach optimum.

For this summer institute in behavior economics, I got

1. Some behavioral economics models have been used in different areas (i.e., price reactions for the marketing and forecasting). In my study, I try to enhance the whole model to reach global optimum with approximate algorithm.
2. Clearly defining an optimal method for solving management science problems such as behavior economics, psychology, social preference and risk management.
3. Finally, develop and formulate strong models for the management science problem and solve it by proposed algorithm to reach the global optimal solution.

二、預期成果

1. 提升學生的能見度
2. 獲取不同的應用議題
3. 尋合適合的研究夥伴

4. 與各國著名學者互動，掌握研究發展趨勢。

二、行程

1. 開會時間：99 年 7 月 26-99 年 8 月 6
2. 開會地點：Singapore-國立新加坡大學

三、心得

1. 參與國際重要學術會議研討，不僅可提升我國的能見度，亦能增加本校的知名度、更能提昇博士生的競爭力。
2. 參與各國學者互動的機會，增加研究主題的靈感並能掌握國際研究趨勢。
3. 參與國際重要學術會議有助於推動跨國合作研究計畫。
4. 最後，學生正積極發展幾何規劃方法之應用性議題，利用最佳化技術對管理的解析，學生藉這次的參與的機會，多方面嘗試結合最佳化方法，以期許能夠全方法解決管理性的議題。