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中文摘要

吾人考慮感測器警報機率分佈改變偵測的問題。在無參數改變偵測的架構下，吾人基於 Rao test 發表一套演算法。吾人也將感測器分群並且估計各群的期望值來做檢測。我們獲得了理論上的效能。我們所提出方法的複雜度為線性，適用於多感測器的情況之下。吾人也考慮在感測器與資料融合中心的連線上有干擾情況時的效能增強。

Abstract

The problem of detecting changes in the distribution of alarmed sensors is considered. Under a nonparametric change detection framework, we present an algorithm based on the Rao test. We also partition sensors into small groups and estimate their mean to perform detection. Theoretical performance guarantees are obtained. Our approach has linear complexity, which is suitable to large number of sensors. We also enhance change detection performance for sensors-to-fusion links with interference.

I. Rao Test

In this part, we will introduce the method of the Rao test. The Rao test [11] has the asymptotic detection performance as the generalized likelihood ratio test. For finite data records, there is no guarantee that the performance will be the same. The main benefit is that this asymptotically equivalent statistic may be easier to compute. This is especially true of the Rao test for which it is not necessary to determine the maximum likelihood estimator for $\mathcal{H}_1$, but only the maximum likelihood estimator for $\mathcal{H}_0$ to be found. The PDF is denoted $p(\mathbf{x};\theta)$. The hypothesis test is

$$\begin{aligned} \mathcal{H}_0 &: \theta = \theta_0 \\ \mathcal{H}_1 &: \theta \neq \theta_0. \end{aligned} \tag{3.1}$$

The Rao test just only needs to know θ_0, and is particularly suitable for the considered scenario. The Rao test decides $\mathcal{H}_1$ if

$$T_R(\mathbf{x}) = \frac{\partial \ln p(\mathbf{x};\theta)^T}{\partial\theta}\Bigg|_{\theta=\theta_0} \mathbf{I}^{-1}(\theta_0) \frac{\partial \ln p(\mathbf{x};\theta)}{\partial\theta}\Bigg|_{\theta=\theta_0} > \gamma, \tag{3.2}$$

where $\mathbf{I}(\theta_0)$ denote the Fisher information matrix, and γ is a threshold. In (3.2) it is implicitly assumed that the PDFs under $\mathcal{H}_0$ and $\mathcal{H}_1$ differ only in the value of θ. The maximum likelihood estimator for $\mathcal{H}_1$ need not to be found for the Rao test. This is advantageous when

A. Rao Test for Independent and Identically Distributed Sensors

In this part, we consider the simple homogeneous case, i.e., the alarming probabilities among sensors are independent and are identically distributed. We also assume for the moment that the channel is errorless (the cross-over probability of each BSC is zero). Under this condition, the number of sensors that change state x between two collections is Bernoulli distributed:

$$p(x;\alpha) = \binom{N}{x}\alpha^x(1-\alpha)^{N-x}, \tag{3.3}$$

where α is the probability of sensor changing state between two collections from (2.2). The composite hypothesis testing:

$$\mathcal{H}_0 : \alpha = \alpha_0$$
$$\mathcal{H}_1 : \alpha \neq \alpha_0.$$

where α_0 denotes the probability of sensor changing state between two collections before change occurs. The Fisher information for a Bernoulli distribution is given by

$$\begin{aligned}
\mathrm{I}(\alpha) &= -E\left[\frac{\partial^2}{\partial\alpha^2}\ln\left(p(x;\alpha)\right)\right] \\
&= -E\left[\frac{\partial^2}{\partial\alpha^2}\ln\left(\alpha^x(1-\alpha)^{s-x}\frac{N!}{x!(N-x)!}\right)\right] \\
&= -E\left[\frac{\partial^2}{\partial\alpha^2}\left[x\ln(\alpha)+(N-x)\ln\left((1-\alpha)\right)\right]\right] \\
&= -E\left[\frac{\partial}{\partial\alpha}\left[\frac{x}{\alpha}-\frac{N-x}{1-\alpha}\right]\right] \\
&= E\left[\left[\frac{x}{\alpha^2}+\frac{N-x}{(1-\alpha)^2}\right]\right] \\
&= \frac{N\alpha}{\alpha^2}+\frac{N(1-\alpha)}{(1-\alpha)^2} \\
&= \frac{N}{\alpha(1-\alpha)},
\end{aligned}$$

and

$$\frac{\partial}{\partial\alpha}\ln\left(p(x;\alpha)\right) = \frac{x}{\alpha}-\frac{N-x}{1-\alpha}.$$

Substituting above result into the formula of Rao test gives

$$T_R(x) = \frac{\alpha_0(1-\alpha_0)}{N}\left(\frac{x}{\alpha_0}-\frac{N-x}{1-\alpha_0}\right)^2 > \gamma. \tag{3.4}$$

Such that

$$(x-N\alpha_0)^2 > \gamma'. \tag{3.5}$$

Thus the Rao test in our case claims change occurs if the squared difference between the measurement x and the mean $N\alpha_0$ exceeds a certain threshold γ'.

B. Rao Test for Independent and Different Distributed Sensors

In this part, we consider the inhomogeneous case in which the alarming probabilities of sensors are distinct. This case arises, e.g., when the alarming probabilities of sensors are identical but the BSC's assume different cross-over probabilities across the sensor-to-fusion links. We do not care the changes of probability distribution in each sensor, but in the area. Hence, we combine sensors in the same area to perform detection. By the central limit theorem, when the number of sensors is large and sensors are independent, the probability distribution of the number of sensors alarming looks like the Normal distribution. So, the PDF of number of sensor alarming is denoted by $p(x; u, \sigma^2)$, where p is the Normal distribution, u is the mean, and σ^2 is variance. Consider the composite hypothesis problem

$$\begin{aligned} \mathcal{H}_0 &: u = u_0, \sigma^2 = \sigma_0^2 \\ \mathcal{H}_1 &: u \neq u_0, \sigma^2 \neq \sigma_0^2. \end{aligned} \tag{3.6}$$

This is a two-parameters composite hypothesis problem. The Normal distribution, $N(u, \sigma^2)$ belongs to the exponential family and its log-likelihood function $l(\theta \mid x)$ is

$$-\frac{1}{2}\ln(2\pi\sigma^2) - \frac{(x-u)^2}{2\sigma^2} \tag{3.7}$$

where $\theta = \left(u, \sigma^2\right)$. The Fisher information matrix $\mathbf{I} = -E\left[\frac{\partial U}{\partial \theta}\right]$, where U is given by

$$\left(\frac{\partial l}{\partial u}, \frac{\partial l}{\partial \sigma^2}\right) = \left(\frac{x-u}{\sigma^2}, \frac{(x-u)^2}{2\sigma^4} - \frac{1}{2\sigma^2}\right). \tag{3.8}$$

Taking the derivative with respect to θ, we have

$$\frac{\partial U}{\partial \theta} = \begin{pmatrix} \frac{\partial U_1}{\partial u} & \frac{\partial U_2}{\partial u} \\ \frac{\partial U_1}{\partial \sigma^2} & \frac{\partial U_2}{\partial \sigma^2} \end{pmatrix} = \begin{pmatrix} \frac{-1}{\sigma^2} & -\frac{x-u}{\sigma^4} \\ -\frac{x-u}{\sigma^4} & \frac{1}{2\sigma^4} - \frac{(x-u)^2}{\sigma^6} \end{pmatrix}. \tag{3.9}$$

So, the Fisher information matrix $\mathbf{I}$ is

$$-E\left[\frac{\partial U}{\partial \theta}\right] = \frac{1}{2\sigma^4}\begin{pmatrix} 2\sigma^2 & 0 \\ 0 & 1 \end{pmatrix}. \tag{3.10}$$

The Rao test decides $\mathcal{H}_1$, if

$$T_R(\mathbf{x}) = \frac{\partial \ln p(\mathbf{x};\theta)^T}{\partial\theta}\Bigg|_{\theta = \left(m_0,\sigma_0^2\right)} \mathbf{I}^{-1}(\theta_0)\frac{\partial \ln p(\mathbf{x};\theta)}{\partial\theta}\Bigg|_{\theta = \left(m_0,\sigma_0^2\right)}$$

$$= 2\sigma_0^4\begin{bmatrix}\dfrac{x-u_0}{\sigma_0^2} & \dfrac{\left(x-u_0\right)^2}{\sigma_0^4}-\dfrac{1}{2\sigma_0^2}\end{bmatrix}\begin{bmatrix}\dfrac{1}{2\sigma_0^2} & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\dfrac{x-u_0}{\sigma_0^2}\\ \dfrac{\left(x-u_0\right)^2}{\sigma_0^4}-\dfrac{1}{2\sigma_0^2}\end{bmatrix} \quad (3.11)$$

$$= \frac{\left(x-u_0\right)^4}{2\sigma_0^2}+\frac{1}{2} > \gamma$$

The statistic (3.10) is forth moment of $x-u_0$. Compare (3.10) with (3.4), (3.10) is the square of (3.4).

II. Kullback Leibler Distance

In probability theory and information theory, the Kullback Leibler distance is a measure of the distance between two distributions. In statistics, it arises as an expected logarithm of the likelihood ratio. The Kullback Leibler distance $D(p\,||\,q)$ is a measure of the inefficiency of incorrectly taking the distribution as *q* when the true distribution is instead *p*. For example, if we knew the true distribution of the random variable, then we could construct a code with average description length $H(p)$. If, instead, we use the code for a distribution *q*, we would need $H(p)+D(p\,||\,q)$ bits on the average to describe the random variable.

The Kullback Leibler distance between two probability mass functions *p(x)* and *q(x)* is defined as

$$\begin{aligned} D(p\,||\,q) &= \sum_{x\in\mathcal{X}} p(x)\log\frac{p(x)}{q(x)} \\ &= E_p\left[\log\frac{p(x)}{q(x)}\right]. \end{aligned} \quad (3.12)$$

The Kullback Leibler distance is always non-negative, it is zero if only if *p*=*q* [12]. However, it is not a true distance measure between distributions because it is not symmetric and does not satisfy the triangle inequality,

$$D(p\,||\,q) \neq D(q\,||\,p). \quad (3.13)$$

Nonetheless, it is often useful to think of Kullback Leibler distance as a distance between two distributions. The Kullback Leibler distance remains well-defined for continuous distributions, and furthermore is invariant under parameter transformations.

A. Kullback Leibler Distance for Independent and Identically Distributed Sensors

In the Section 3.1, we discuss the detection problem from the viewpoint of the Rao test. In this section, we will consider this problem from another viewpoint, the Kullback Leibler distance. First, we discuss the homogeneous case in which the alarming probabilities of sensors are distinct. We also assume that the error probability in the binary symmetric channel is equal to zero. Under above condition, the number of sensors that change state between two collections is a Bernoulli distribution. We assume that *p* and *q* are the alarming probabilities before and after change occurs, and then we have:

$$p(x;\alpha) = \binom{N}{x} \alpha^x (1-\alpha)^{N-x} \tag{3.14}$$

and

$$q(x;\alpha') = \binom{N}{x} (\alpha')^x (1-\alpha')^{N-x}, \tag{3.15}$$

where N is the number of sensors, x is number of alarming sensors, α is from (2.2), $\alpha' = \alpha + \Delta$, and $-\alpha \le \Delta \le 1-\alpha$. Because sensors have the same probability distribution, we remove the index word i of α . The Kullback Leibler distance $D(p \,||\, q)$ between *p* and *q* is

$$\begin{aligned} D(p \,||\, q) &= \sum_{x=0}^{N} \binom{N}{x} \alpha^x (1-\alpha)^{N-x} \log\left(\frac{\alpha^x (1-\alpha)^{N-x}}{(\alpha')^x (1-\alpha')^{N-x}}\right) \\ &= \sum_{x=0}^{N} \binom{N}{x} \alpha^x (1-\alpha)^{N-x} \log\left(\frac{\alpha^x (1-\alpha)^{N-x}}{(\alpha+\Delta)^x (1-(\alpha+\Delta))^{N-x}}\right) \\ &= \sum_{x=0}^{N} \binom{N}{x} \alpha^x (1-\alpha)^{N-x} \log\left(\frac{1}{(1+\frac{\Delta}{\alpha})^x (1-\frac{\Delta}{1-\alpha})^{N-x}}\right). \end{aligned} \tag{3.16}$$

We declare the probability distribution changed if the Kullback Leibler distance $D(p \,||\, q)$ is larger than λ . Otherwise, we declare the probability distribution non-changed. This composite hypothesis test can be written as

$$\begin{gathered} H_0 : D(p \,||\, q) \le \lambda \\ \text{versus} \\ H_1 : D(p \,||\, q) > \lambda. \end{gathered} \tag{3.17}$$

Actually, this composite hypothesis problem (3.16) can be rewritten as

$$\begin{gathered} H_0 : \varepsilon \le \Delta \le \eta \\ \text{versus} \\ H_1 : \Delta < \varepsilon \ or \ \Delta > \eta. \end{gathered} \tag{3.18}$$

Hence, we decide H_1 if

$$\begin{gathered} x - N\alpha < \varepsilon \\ \text{or} \\ x - N\alpha > \eta. \end{gathered} \tag{3.19}$$

Compare (3.18) with (3.5), and we find that (3.5) is equivalent to (3.18) when $-\varepsilon = \eta$.

However, from (3.16) and (3.17), the values of ε and η in (3.18) is difficultly to obtain. We have to use numerical method to get them. This is an obstruction for us to use the composite testing from the viewpoint of the Kullback Leibler distance.

B. Kullback Leibler Distance for Independent and Different Distributed Sensors

For the inhomogeneous case the central limit theorem implies that, when the number of sensors is large and sensors are independent, the probability distribution of the number of sensors alarming likes the Normal distribution. So, the PDFs of non-changed *p* and changed *q* are approximately given by

$$p(x) = \frac{1}{\sqrt{2\pi\sigma_0^2}} \exp\left[-\frac{(x-u_0)^2}{2\sigma_0^2}\right]$$
$$\text{and} \tag{3.20}$$
$$q(x) = \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left[-\frac{(x-u_1)^2}{2\sigma_1^2}\right].$$

The Kullback Leibler distance $D(p\,||\,q)$ between *p* and *q* is

$$\begin{aligned} D(p\,||\,q) &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_0^2}} \exp\left[-\frac{(x-u_0)^2}{2\sigma_0^2}\right] \log\left(\sqrt{\frac{\sigma_1^2}{\sigma_0^2}} \frac{\exp\left[-\frac{(x-u_0)^2}{2\sigma_0^2}\right]}{\exp\left[-\frac{(x-u_1)^2}{2\sigma_1^2}\right]}\right) dx \\ &= \log\sqrt{\frac{\sigma_0^2 + \Delta_{\sigma^2}}{\sigma_0^2}} \\ &\quad + \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_0^2}} \exp\left[-\frac{(x-u_0)^2}{2\sigma_0^2}\right] \left[\frac{\sigma_0^2\Delta_u^2 - 2\sigma_0^2\Delta_u(x-u_0) - \Delta_{\sigma^2}(x-u_0)^2}{2\sigma_0^2\left(\sigma_0^2 + \Delta_{\sigma^2}\right)}\right] dx, \end{aligned} \tag{3.21}$$

where $\Delta_u = u_1 - u_0$, and $\Delta_{\sigma^2} = \sigma_1^2 - \sigma_0^2$. This case with two variables is difficult to analyze, and we cannot get the variance from only one data sample. Hence, we only consider the variance invariant case. The Kullback Leibler distance $D(p\,||\,q)$ between *p* and *q* is

$$\begin{aligned} D(p\,||\,q) &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_0^2}} \exp\left[-\frac{(x-u_0)^2}{2\sigma_0^2}\right] \log\left(\frac{\exp\left[-\frac{(x-u_0)^2}{2\sigma_0^2}\right]}{\exp\left[-\frac{(x-u_1)^2}{2\sigma_1^2}\right]}\right) dx \\ &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-u_0)^2}{2\sigma^2}\right] \frac{-\Delta_u(2x - 2u_0 - \Delta_u)}{2\sigma^2} dx. \end{aligned} \tag{3.22}$$

We declare the probability distribution changes if the Kullback Leibler distance $D(p\,||\,q)$ is larger than λ. Otherwise, we declare the probability distribution does not change. The composite hypothesis test can be written as

$$H_0 : D(p \,||\, q) \leq \lambda$$
$$\text{versus} \tag{3.23}$$
$$H_1 : D(p \,||\, q) > \lambda.$$

Also, this composite hypothesis problem (3.22) can be rewritten as

$$H_0 : \varepsilon \leq \Delta_u \leq \eta$$
$$\text{versus} \tag{3.24}$$
$$H_1 : \Delta_u < \varepsilon \ or \ \Delta_u > \eta.$$

Hence, we decide H_1 if

$$x - u_0 < \varepsilon,$$
$$\text{or} \tag{3.25}$$
$$x - u_0 > \eta.$$

III. Algorithm of Nonparametric Low Complexity Change Detection in Wireless Sensor Networks

In this section, we propose our nonparametric low complexity change detection algorithm based on the results of sections 3.1 and 3.2. We propose to partition sensors into several small groups according to their position. If we find that the probability distribution of *at least one among these groups changes*, we declare that the two probability distributions are different. We also have to estimate a parameter in our algorithm. We calculate the lower bound of the number of the data samples for estimation. Then, we discuss the performance of our algorithm and simulations.

A. Algorithm

We are interested in the change in the geographical distribution of alarmed sensors. Hence, we propose to partition sensors into several small groups according to their positions to detect geographical distribution change. If we find that the probability distribution of *at least one among these groups changes*, we declare that the two probability distributions are different.

In (3.5) and (3.11), we derive the results of composite hypothesis testing by the Rao test. Alternatively, in (3.19) and (3.25), we got the results of composite hypothesis testing based on the Kullback Leibler distance. However, we have to use numerical methods to find the values of ε and η in (3.19) and (3.25). This increases the difficulty of implementing our algorithm. From (3.5) and (3.11), we propose that if the composite hypothesis testing is:

$$\begin{aligned} \mathcal{H}_0 &: m^{(j)} = m_0^{(j)}, \text{ for all } j = 1, \ldots, K \\ \mathcal{H}_1 &: m^{(j)} \neq m_0^{(j)}, \text{ for some } j. \end{aligned} \tag{3.26}$$

where $m_0^{(j)}$ is the mean of distribution of jth partition under $\mathcal{H}_0$ and K is the number of partitions, we decide $\mathcal{H}_1$ if

$$\left|x^{(j)} - m_0^{(j)}\right| > r^{(j)}, \quad \text{for some } j, \tag{3.27}$$

where $x^{(j)}$ is the number of sensor changing state and $r^{(j)}$ is a threshold in the *j*th partition. We assume that the PDF under $\mathcal{H}_0$ does not change during a periods of *T* time instants (we call *T* "training number"). Since we do not know the PDF under $\mathcal{H}_0$, we have to estimate the mean $m_0^{(j)}$. We estimate $m_0^{(j)}$ as

$$\hat{m}_0^{(j)} = \frac{1}{T}\sum_{t=0}^{T-1} x^{(j)}[t]. \tag{3.28}$$

This estimator is a minimum variance unbiased estimator for both homogeneous and inhomogeneous cases, and we will proof it later. Then, we design the thresholds of partitions by Neyman-Pearson's criterion that maximizes the detection probability subject to that the false alarm probability is not larger than the threshold.

There are five steps in our algorithm:

Step1. Partition sensors into *K* groups according to their position.

Step2. Record the number of sensor changing state in each partition.

Step3. During *T* pairs of consecutive two time instants, we estimate the mean in each partition by summing average (3.28).

Step4. According to Neyman-Person test, we set threshold in (3.27) for every partition.

Step5. In every two time instants, repeat the test of (3.27).

B. Lower Bound on Training Number

From previous section, we have designed the algorithm of change detection. In our algorithm, we have to know the original probability distribution via the observe sample sequence (a "training" process). We address the problem: how large the number of data samples is needed for guaranteeing the estimation accuracy to be within a prescribed level? This is an important issue in our research. We have to do a tradeoff between the detection performance and the calculation complexity. First, for the homogeneous case consider the set of observations

$$x[t] = c(t;\alpha), t = 0,1,\ldots,T-1, \tag{3.29}$$

where $c(t;\alpha) \sim$ Binormail distribution $p(x;\alpha) = \binom{N}{x}\alpha^x(1-\alpha)^{N-x}$ and N is the number of sensors. Then, the estimator

$$\hat{\alpha} = \frac{1}{NT}\sum_{t=0}^{T-1} x[t] \tag{3.30}$$

from Section 3.3 is minimum variance unbiased estimator[15]. Because

$$
\begin{aligned}
I(\alpha) &= -E\left[\frac{\partial^2 \ln p(\mathbf{x};\alpha)}{\partial\alpha^2}\right] \\
&= -TE\left[\frac{\partial^2}{\partial\alpha^2}\left[\ln\binom{N}{x} + x[t]\ln\alpha + (N - x[t])\ln(1-\alpha)\right]\right] \\
&= \frac{NT}{\alpha(1-\alpha)}
\end{aligned}
$$

and

$$\frac{\partial \ln p(x;\alpha)}{\partial\alpha} = I(\alpha)(\hat{\alpha} - \alpha).$$

A good approximation for the Binomial distribution is the Normal distribution when $N\alpha(1-\alpha) \gg 1$. In our problem, N is usually large. Hence, the Normal distribution is a good approximation for the Binomial distribution. The probability distribution of $\hat{\alpha}$ is approximated

$$\hat{\alpha} = \frac{1}{NT}\sum_{t=0}^{T-1} x[t] \sim N\left(\alpha, \frac{\alpha(1-\alpha)}{NT}\right). \tag{3.31}$$

We want to know how large T can guarantee the probability that the deviation of the estimated mean from the true mean $N|\hat{\alpha} - \alpha|$ is less than D to be greater than ε. Toward a solution we note that

$$
\begin{aligned}
& erf\left(\frac{|\hat{\alpha}-\alpha|}{\sqrt{2I(\alpha)}}\right) \geq \varepsilon, \\
& \frac{(\hat{\alpha}-\alpha)^2\, TN}{2\alpha(1-\alpha)} \geq [erfinv(\varepsilon)]^2, \\
& T \geq \frac{N\,(erfinv(\varepsilon))^2}{2D^2} (\text{by } \alpha(1-\alpha) \leq \frac{1}{4}),
\end{aligned} \tag{3.32}
$$

where $erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$ is the error function and $erfinv$ is the inverse error function.

Now, we discuss the general inhomogeneous case. Consider the observation

$$x[t] = c(t;m,\sigma^2), t = 0,1,\ldots,T-1, \tag{3.33}$$

where $c(t;m,\sigma^2) \sim$ Normal distribution $p(x;m,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{(x-m)^2}{2\sigma^2}\right]$. The estimator

$$\hat{m} = \frac{1}{T}\sum_{t=0}^{T-1} x[t] \tag{3.34}$$

is the minimum variance unbiased estimator. Because

$$
\begin{aligned}
p(\mathbf{x};m) &= \prod_{t=0}^{T-1} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x[t]-m)^2}{2\sigma^2}\right] \\
&= \frac{1}{\left(2\pi\sigma^2\right)^{\frac{N}{2}}} \exp\left[-\frac{1}{2\sigma^2}\sum_{t=0}^{T-1}(x[t]-m)^2\right], \\
I(\alpha) &= -E\left[\frac{\partial^2 \ln p(\mathbf{x};m)}{\partial m^2}\right] \\
&= \frac{T}{\sigma^2},
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \ln p(\mathbf{x};m)}{\partial m} &= \frac{\partial}{\partial m}\left[-\ln\left[\left(2\pi\sigma^2\right)^{\frac{N}{2}}\right] - \frac{1}{2\sigma^2}\sum_{t=0}^{T-1}(x[t]-m)^2\right]. \\
&= \frac{1}{\sigma^2}\sum_{t=0}^{T-1}(x[t]-m) \\
&= \frac{N}{\sigma^2}(\hat{m}-m)
\end{aligned}
\tag{3.35}
$$

So, the probability distribution of $\hat{m}$ can be written

$$
\hat{m} = \frac{1}{T}\sum_{t=0}^{T-1} x[t] \sim N\left(m, \frac{\sigma^2}{T}\right). \tag{3.36}
$$

From (3.7),

$$
\begin{aligned}
& erf\left(\frac{|\hat{m}-m|\sqrt{T}}{\sqrt{2\sigma^2}}\right) \ge \varepsilon, \\
& \frac{(\hat{m}-m)^2 T}{2\sigma^2} \ge [erfinv(\varepsilon)]^2, \\
& T \ge \frac{2\sigma^2 \left(erfinv(\varepsilon)\right)^2}{D^2}, \\
& T \ge \frac{N\left(erfinv(\varepsilon)\right)^2}{2D^2}, \ by\ \sigma^2 \le \frac{N}{4}.
\end{aligned}
\tag{3.37}
$$

We can find that (3.7) is the same with (3.12). Hence, we get a lower bound in the common use.

C. Performance

In this section, we present the detection probability P_D and false alarm probability P_F for our estimator. From (3.27), the detection probability $P_D^{(j)}$ and false alarm probability $P_F^{(j)}$ in the *j*th partition are

$$P_D^{(j)} = \Pr\left\{ | x^{(j)} - m_0^{(j)} |> r^{(j)} \mid \mathcal{H}_1 \right\}$$

and (3.38)

$$P_F^{(j)} = \Pr\left\{ | x^{(j)} - m_0^{(j)} |> r^{(j)} \mid \mathcal{H}_0 \right\}.$$

In the homogeneous case, the PDF $P_j(x^{(j)})$ of number $x^{(j)}$of sensor changing state in *j*th partition is the Binomial distribution $B(S, \alpha)$, where S is the number of sensors, α is probability of sensor changing state. The detection probability $P_D^{(j)}$ and false alarm probability $P_F^{(j)}$ can be determined by

$$\begin{aligned} P_D^{(j)} &= \Pr\left\{ | x^{(j)} - m_0^{(j)} |> r^{(j)} \mid \mathcal{H}_1 \right\} \\ &= \Pr\left\{ | x^{(j)} - S^{(j)}\alpha_0^{(j)} |> r^{(j)} \mid \mathcal{H}_1 \right\} \\ &= 1 - \sum_{x=\left\lceil S^{(j)}\alpha_0^{(j)} - r^{(j)} \right\rceil}^{\left\lfloor S^{(j)}\alpha_0^{(j)} + r^{(j)} \right\rfloor} P\left(x; \mathcal{H}_1\right), \end{aligned}$$

and (3.39)

$$\begin{aligned} P_F^{(j)} &= \Pr\left\{ | x^{(j)} - m_0^{(j)} |> r^{(j)} \mid \mathcal{H}_0 \right\} \\ &= \Pr\left\{ | x^{(j)} - S^{(j)}\alpha_0^{(j)} |> r^{(j)} \mid \mathcal{H}_0 \right\} \\ &= 1 - \sum_{x=\left\lceil S^{(j)}\alpha_0^{(j)} - r^{(j)} \right\rceil}^{\left\lfloor S^{(j)}\alpha_0^{(j)} + r^{(j)} \right\rfloor} P\left(x; \mathcal{H}_o\right). \end{aligned}$$

where $\lfloor\ \rfloor$ is the floor function and $\lceil\ \rceil$ is the ceil function. When $N\alpha(1-\alpha) \gg 1$, the Normal distribution is a good approximation for the Binomial distribution. Then (3.39) can be approximated by

$$P_F^{(j)} = 2\left(1 - \Phi\left\{ \frac{r^{(j)} + S\alpha_0^{(j)}}{\sqrt{S\alpha_0^{(j)}(1-\alpha_0^{(j)})}} \right\}\right),$$

and (3.40)

$$P_D^{(j)} = 1 - \left\{ \Phi\left\{ \frac{r^{(j)} + S(\alpha_0^{(j)} - \alpha_1^{(j)})}{\sqrt{S\alpha_1^{(j)}(1-\alpha_1^{(j)})}} \right\} - \Phi\left\{ \frac{-r^{(j)} + S(\alpha_0^{(j)} - \alpha_1^{(j)})}{\sqrt{S\alpha_1^{(j)}(1-\alpha_1^{(j)})}} \right\} \right\},$$

where Φ is the CDF of the Normal distribution $N(0,1)$ and α_1 denotes the alarming probability after change occurs. From (3.40), when the number of sensors tends to infinity, the false alarm probability approaches zero and the detection probability is approaches unity. So, if we want to improve the detection performance, we have to increase the number of sensors in a partition. However, if we increase the number of sensors in a partition, the number of partitions is decreased (since the total number of sensors is fixed) and this may incur geographical detection performance degradation. There is thus a tradeoff between the number of partitions and the achievable performance.

In the inhomogeneous case, the PDF $P_j(x^{(j)})$ of number $x^{(j)}$ of sensor changing state in *j*th partition is the Normal distribution $N(m,\sigma^2)$. The false alarm probability and detection probability can be written as

$$P_F^{(j)} = 2\left(1-\Phi\left\{\frac{r^{(j)}+m_0^{(j)}}{\sigma_0^{(j)}}\right\}\right),$$

and (3.41)

$$P_D^{(j)} = 1-\left\{\Phi\left\{\frac{r^{(j)}+m_0^{(j)}-m_1^{(j)}}{\sigma_1^{(j)}}\right\}-\Phi\left\{\frac{-r^{(j)}+m_0^{(j)}-m_1^{(j)}}{\sigma_1^{(j)}}\right\}\right\}.$$

From above description, we present the detection probability and the false alarm probability in each partition. Now, we will discuss the *total* detection probability P_D and the false alarm probability P_F. From (3.26), we have P_F:

$$\begin{aligned} P_F &= \Pr\{H_1 \mid H_0\} \\ &= 1-\Pr\{H_0 \mid H_0\} \\ &= 1-\prod_{j=1}^{K}\left(1-P_F^{(j)}\right). \end{aligned} \tag{3.42}$$

We also assume A_j is the *j*th combination of partition with distribution change, and the set of total combinations is $A=\{A_1, A_{2,\ldots,}A_j\}$. We have P_D

$$P_D = 1-\sum_{A_j\in A}\Pr\{A_j\}\prod_{a\in A_j}\left(1-P_D^{(a)}\right)\prod_{b\in\Omega-A_j}\left(1-P_D^{(b)}\right). \tag{3.43}$$

IV. Computer Simulation

In this part, we show our simulation result by the Matlab. In Figure 3.1, we set the number of sensors to be 4096. The (x,y) is the coordinate of sensor, $x\in\{0,1,\ldots,61\}$ and $y\in\{0,1,\ldots,61\}$. Before change occurs, sensors have identical alarming probability equal to 0.8. The change occurs in sensors position at $x\in\{0,1,\ldots,31\}$ and $y\in\{0,1,\ldots,31\}$ with alarming probability decreases to 0.7. Use 100,000 Monte Carlo runs. As our description in Section (3.3.3), if we want to improve the detection performance, we have to increase the number of sensors in a partition.

In Figure 3.2, we set the number of sensors to be 3600. We partition sensors in to nine groups. Before change occurs, sensors have identical alarming probability equal to 0.8. The change occurs in two partitions, with alarming probability decreases to 0.7. We simulate the training number from 11 to 24. Use 50,000 Monte Carlo runs. We have a tradeoff between performance and complexity. In this case, we find that we have performance similar to the optimal optimum as the training number equals 24.

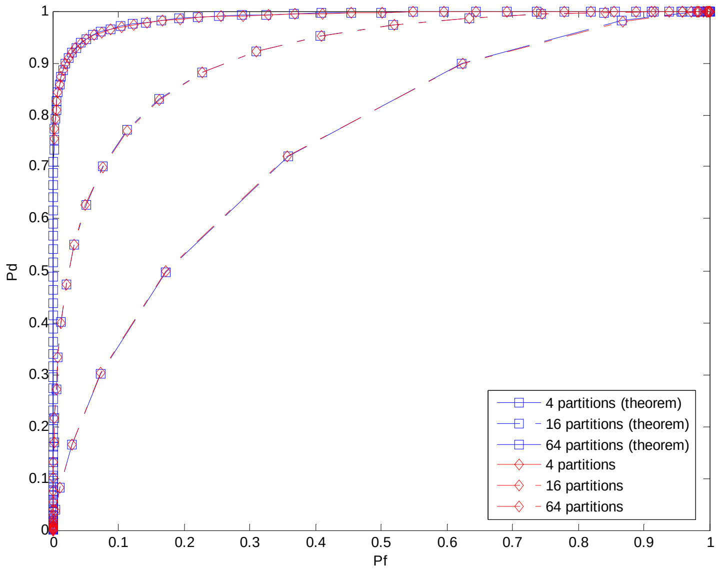

Figure 0.1: Receiver operating characteristics for different partition number

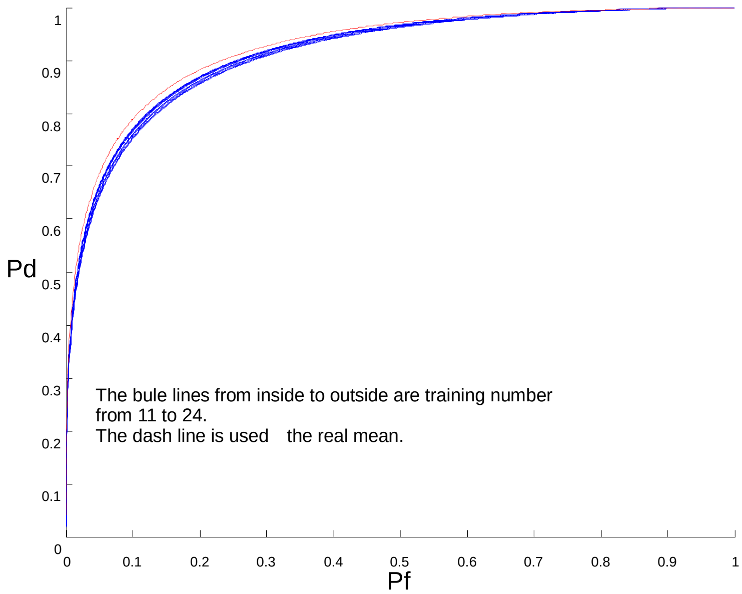

Figure 0.2: Receiver operating characteristics for different training number

References

[11] S. M. Kay, *Fundamentals statistical signal processing estimation theory*, Vol.2, New Jersey: Prentice Hall, 1993.

[12] T. M. Cover and J. A. Thomas, *Elements of information theory*, John Wiley & Sons, 1991.

[15] S. M. Kay, *Fundamentals statistical signal processing estimation theory*, Vol.1, New Jersey: Prentice Hall, 1993.

計畫成果自評

一、研究內容與原計畫相符程度

大部份相符。

二、達成預期目標情況

1. 完成演算法模擬平台，將偵測器分成幾個群組，使用 Monte Carlo 的方法以決定最佳的 training number。
2. 建立二元對稱通道(BSC)通訊聯結，使用 Rayleigh Fading Channel，並考慮感測器與資料融合中心的連線上有干擾情況下，藉由模擬來驗證所採用的可適性方法的效能。
3. 完成演算法複雜度的分析，並將採用的演算法的複雜度與其他方法做比較。除了一般的情況，另外也將加入了一些極端條件的情形於模擬驗證中。
4. 計畫成果將發表於國際期刊或國際會議。

三、研究成果之學術或應用價值

應用於無線感測器網路之低複雜度變化檢測。

四、是否適合在學術期刊發表或申請專利

適合在學術期刊發表。

五、主要發現或其他有關價值

無線感測器網路起源於軍事上的用途，藉由感測器節點的佈建可以偵查戰場上所需的敵方資訊而不需要人力去觀察，進而提升安全性和局勢的掌握。隨著科技的進步，積體電路、微電機、無線技術的發展成果使得感測器節點更能實現低功率、低成本、多功能的要求，也因此在近幾年間無線感測器網路的運用更加普及於商業上、家庭上或環境應用之中，例如對聲音、溫度、震動、污染情況、壓力的監控等等。基於 Rao test 的演算法，配合將感測器分群並且估計各群的期望值來做檢測，可以有效降低複雜度。

可供推廣之研發成果資料表

□ 可申請專利　　■ 可技術移轉　　　　　　　　　　日期：98 年 5 月 26 日

國科會補助計畫	計畫名稱：應用於無線感測器網路之低複雜度及分散式變化檢測 計畫主持人：李大嵩 教授 計畫編號：NSC 97-2221-E-009-056-MY2　　學門領域：通訊
技術/創作名稱	應用於無線感測器網路之低複雜度變化檢測
發明人/創作人	羅楚威、吳卓諭、李大嵩
技術說明	中文： 吾人考慮感測器警報機率分佈改變偵測的問題。在無參數改變偵測的架構下，吾人基於 Rao test 發表一套演算法。吾人也將感測器分群並且估計各群的期望值來做檢測。我們獲得了理論上的效能。我們所提出方法的複雜度為線性，適用於多感測器的情況之下。吾人也考慮在感測器與資料融合中心的連線上有干擾情況時的效能增強。
	英文： The problem of detecting changes in the distribution of alarmed sensors is considered. Under a nonparametric change detection framework, we present an algorithm based on the Rao test. We also partition sensors into small groups and estimate their mean to perform detection. Theoretical performance guarantees are obtained. Our approach has linear complexity, which is suitable to large number of sensors. We also enhance change detection performance for sensors-to-fusion links with interference.
可利用之產業 及 可開發之產品	可利用之產業： 1. 智慧居家、生理訊號監測與環境安全等產業 可開發之產品： 1. 無線感測器網路節點及閘道
技術特點	基於 Rao test 發表一套演算法，也將感測器分群並且估計各群的期望值來做檢測。
推廣及運用的價值	無線感測器網路節點常需要大量的佈建，才能有效提升準確性。尤其是災害預警系統的應用，成功關鍵在於能夠以更低的成本提供更廣的覆蓋率。因此低複雜度變化檢測將有助於降低感測器的成本，促成產業的發展。

※　1. 每項研發成果請填寫一式二份，一份隨成果報告送繳本會，一份送　貴單位研發成果推廣單位（如技術移轉中心）。

※　**2. 本項研發成果若尚未申請專利，請勿揭露可申請專利之主要內容。**

※　3. 本表若不敷使用，請自行影印使用。