附件一

行政院國家科學委員會補助專題研究計畫 □ 成果報告 ☑期中進度報告

操控半導體電子自旋的動力行為與傳輸的研究

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執行單位：國立交通大學電子物理系

中 華 民 國 97 年 6 月 10 日

行政院國家科學委員會專題研究計劃成果報告

操控半導體電子自旋的動力行為與傳輸的研究: (一)在介觀系統中電偶極感應的自旋共振；(二)在低磁場下的本質霍爾效應的不對稱性質。

A study on the active control and manipulation of spin dynamics and transport in semiconductors: [I] Electric dipole induced spin resonance in mesoscopic system; [II] Asymmetries in intrinsic spin-Hall effect in low in-plane magnetic field; [III] Spin-Hall effects in a Josephson contact.

一、中文摘要:

在本計劃中，我們研究了操控半導體電子自旋的動力行為與傳輸的研究，其中包括(一)在介觀系統中電偶極感應的自旋共振；(二) 在低磁場下的本質霍爾效應的不對稱性質。

(一) 在介觀系統中電偶極感應的自旋共振：

此論文焦點於在介觀系統中電偶極引起的自旋共振現象。進一步來說，量子通道被引進來探討傳輸系統中有可能被自旋軌道耦合效應引發的自旋極化。主要的發現是當由電場提供的能量大小是二分之一Zeeman能隙大小時自旋極化被產生了。此吾考慮了intraband 躍遷由磁場混和了兩個方向的自旋狀態進而造成的，而不是由破壞橫向對稱所造成的。這些結果由為擾理論得到，而主要的效應由第一sideband方法即可得。

Evanescent 模式被發現了很久但直到最近才慢慢受到關注。吾研究了電子在外加磁場下的能譜，能譜也包括了 Evanescent 模式。這像結果事由考慮了 secular equation 和二次方程式。並且在本篇論文裡有關的計算都包含了 Evanescent 模式。

(二) 在低磁場下的本質霍爾效應的不對稱性質：

我們研究了在二維的擴散半導體條狀系統中，在低磁場下塊材的自旋密度以及邊緣的自旋堆積的效應。對於Dresselhaus本質自旋軌道交互作用，我們發現了對於兩個方向的平面磁場:平行於二維條狀系統或橫向於二維條狀系統，我們發現了對稱與不對稱的特性。對於縱向場，自旋密度的垂直分量Sz對於條狀系統顯露了奇宇稱而對磁場則是顯露了偶宇稱關係。對於橫向場，Sz對於空間和場的相關性變成不對稱，對於非零的磁場會有一個非零的塊材自旋密度存在，我們的研究結果顯示可以利用低磁場去偵測材料本身的自旋軌道交互作用的根本來源。

關鍵詞:

自旋軌道交互作用, 自旋霍爾效應, 塊材自旋密度, 自旋堆積, 均勻散射子, 非均勻散射子, 自旋共振, 平面磁場, evanescent 模, 自旋極化, Rashba 自旋軌道交互作用, Dresselhaus 自旋軌道交互作用, Zeeman 能隙, intraband 躍遷

Abstract:

We study the quantum transport in mesoscopic structure: [I] Electric dipole induced spin resonance in mesoscopic system; [II] Asymmetries in intrinsic spin-Hall effect in low in-plane magnetic field; [III] Spin-Hall effects in a Josephson contact

[I] Electric dipole induced spin resonance in mesoscopic system:

This thesis focuses on electric-dipole-induced spin resonance[1] in the mesoscpic system.

Furthermore, a quantum channel is introduced to explore possible spin polarization of the spin-orbital effect from quantum resonance allowed by such transport system. A major finding in this thesis is that generation of polarization occurs when the energy offered by electric field matches the one half of Zeeman gap by considering inelastic scattering process between intraband, different branches in the same subband. The intraband transition is not caused by breaking symmetry in y-direction, whereas a static in-plane magnetic which mix two spin states of σ_x and σ_y. These analytical results are derived with a perturbed method and the dominant effects coming from one side band approach,[2] and the other result of numerical iteration is compared.

And the important of the evanescent modes exists for long time but until recent get more and more attention.[3-5] We study total spectrum of electron states which contain evanescent states in different external field magnitudes. The results are obtained analytically by secular equation and quadratic equation. In this paper, the all calculation include the affect of evanescent modes.

[II] Asymmetries in intrinsic spin-Hall effect in low in-plane magnetic field:

Effects of low in-plane magnetic field on bulk spin densities and edge spin accumulations of a diffusive two dimensional semiconductor strip are studied. Focusing upon the Dresselhaus-type intrinsic spin-orbit interaction (SOI), we look for the symmetry, or asymmetry, characteristics in two magnetic field orientations: along and transverse to the strip. For longitudinal field, the out-of-plane spin density Sz exhibits odd parity across the strip, even parity in the magnetic field, and is edge accumulation. For transverse field, Sz becomes asymmetric in both spatial and field dependences, and has finite bulk values for finite magnetic fields. Our results support utilizing low in-plane magnetic fields for the probing of the underlying SOI.

Keywords:
Spin-orbit interaction, spin-Hall effect, bulk spin densities, spin accumulation, isotropic scatterer, anisotropic scatterer, spin resonance, in-plane magnetic field, evanescent mode, spin polarization, Rashba SOI, Dresselhaus SOI, Zeeman gap, intraband transition, Josephson contact

二、Motivations and goals

[I] Electric dipole induced spin resonance in mesoscopic system:

(1) Electric-dipole-induced spin resonance：

The basic physical concepts of EDSR are analogous to those of nuclear magnetic resonance (NMR). Our fundamental idea of the paper base on the concept that employ an effective magnetic field due to spin orbital interaction(SOI) instead of the external rotating magnetic field.[1]

(2) Energy spectra of Evanescent mode：

The intraband transition, transition between different branches in the same subband should involve evanescent modes. The evanescent mode is important in the transport property and these has been pointed out by other groups also.[3-5] Besides, we will discuss another kind of Evanescent modes outside of the gap which have not been discussed numerously so far. The two kinds of Evanescent modes have different behaviors individually.

[II] Spin Orientation and spin-Hall effect induced by tunneling electrons:

Generation and manipulation of spin densities by electrical means are major goals of semiconductor spintronics that are made possible by spin-orbit interactions (SOI). [1-8] SOIs being considered are either of the intrinsic-type: the Rashba [2,5,7,9-11] and the Dresselhaus SOIs; [4,8,12,13] or of the extrinsic-type: the impurity-induced SOI. [1,3,6,10,14] These SOIs contribute, in an external electric field, to either spin densities in the bulk or spin accumulations at lateral edges, or both. Out-of-plane spin polarization is of particular interest because it permits efficient optical probe by Kerr rotation. The edge spin accumulation, according to the spin-Hall effect (SHE), has an out-of-plane component, and is resulted from a transverse spin current induced by the electric field. [1,4-6] However, for the case of intrinsic SOI, consensus has been reached that the SHE is quenched by background scatterers, be they isotropic or anisotropic, [15] as long as the momentum dependence in the intrinsic SOI is linear. Meanwhile, no out-of-plane bulk spin densities is expected in an electric field. [2,10,16] When applying an in-plane magnetic field to a two-dimensional (2D) systems, one might be led by the in-plane nature of the effective spin-orbit magnetic field, $\mathbf{h}_{\mathbf{eff}} = \langle \mathbf{h}(\mathbf{k}) \rangle \neq 0$, that there were no out-of-plane spin densities. This is shown not to be the case by Engel et al. 11 for a Rashba-type 2D system, where out-of-plane spin densities are found when the external in-plane magnetic field is longitudinal: a configuration studied by recent experiments. [17,18] However, either the scatterer has to be anisotropic or the electron dispersion has to be nonparabolic for the effect to hold. [11] In this work, we have shown that out-of-plane bulk spin density can be generated in another configuration with less restrictive assumptions. The configuration is a Dresselhaus-type 2D system and the external in-plane magnetic field is in a transverse orientation. More importantly, the effect holds for isotropic background scatterers and for parabolic dispersion for electrons. Our calculation has included the cubic Dresselhaus SOI.

[III] Spin-Hall effects in a Josephson contact

The Josephson tunneling through a 2D normal contact with the spin-orbit split conduction band has been studied in the diffusive regime. Linearlized Usadel equations for triplet components of the pairing function revealed a striking similarity to the equations of spin diffusion driven by the electric field in normal metals. We predict that the out-of-plane spin-Hall polarization accumulates towards lateral sample edges and the in-plane polarization is finite throughout the entire normal region. The spin-Hall current is absent in the considered case of the stationary Josephson effect.

三、Results and discussion:

[I] Electric dipole induced spin resonance in mesoscopic system

The Hamiltonian is written as

$$H=\frac{1}{2m}\left[\vec{p}+\frac{e}{c}\vec{A}(t)\theta(\frac{L}{2}-|x|)\right]^2+\left[\Omega\left(\vec{p}+e\vec{A}(t)\theta(\frac{L}{2}-|x|)\right)+\vec{b}_0\right]\cdot\vec{\sigma}+V_c$$

where

$\vec{\Omega}(p)=\alpha\vec{p}\times\hat{e}_z$

$\vec{E}(t)=E_0\cos(wt)\hat{x}$, $\vec{A}=-c\int^t dt'\vec{E}(t')=-\frac{cE_0}{w}\sin(wt)\hat{x}$

$\theta(\frac{L}{2}-|x|)$ is a step function

$\vec{b}_0=\frac{g\mu_0}{2}\vec{B}_0$

α is Rashba constant

V_c comfinement potential $\frac{1}{2}m^*\left(2w_y\right)^2 y^2$

m^* effective mass

The evanescent modes are found that their behavior quite different in weak ($B < B_c$) and strong magnetic field ($B > B_c$). The critical field B_c is given by the Rashba spin-orbit interaction (typically B_c 10mT).

The three following energy dispersion figures corresponding to three magnitude magnetic field .

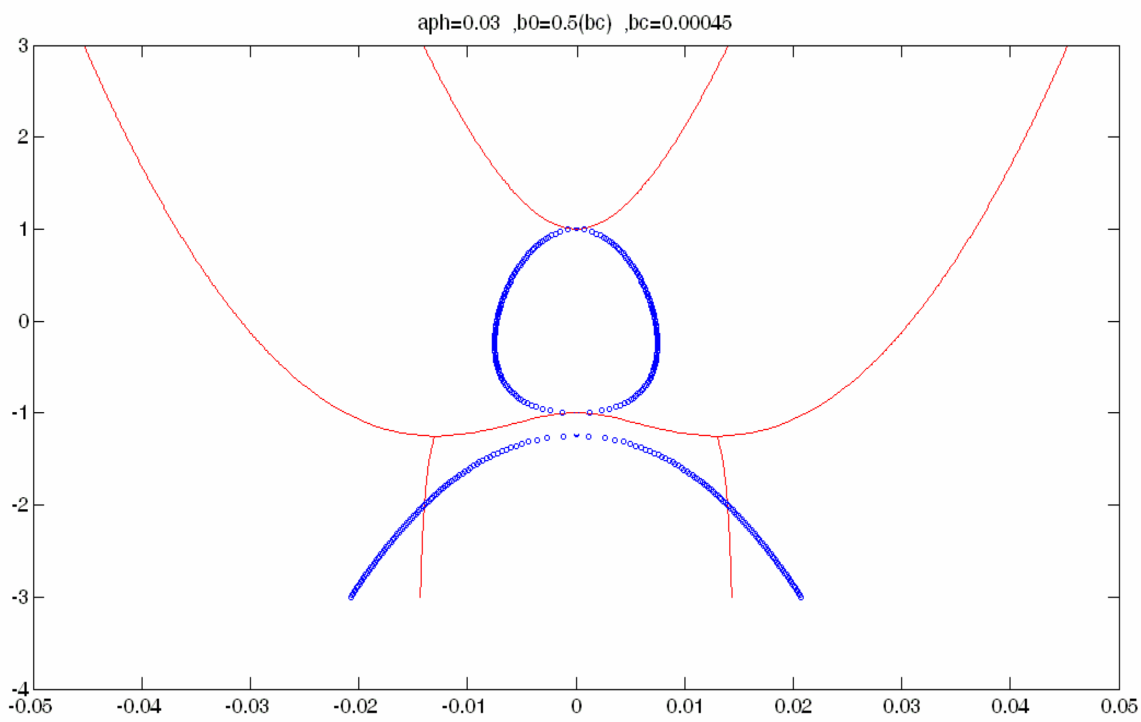

Fig1：X-axis is momentum k （Unit：1.89×10^{8}/m）； Y-axis is Energy $(\varepsilon-0.16)/b_0$ （Unit：59.5mev ） Red curves ： Propagating mode ； Blue curves ： Evanescent mode. When $(\varepsilon-0.16)/b_0$<-1, the two modes couple together. The magnetic energy is small then Rashba energy and the energy dispersion is Rashba-like.

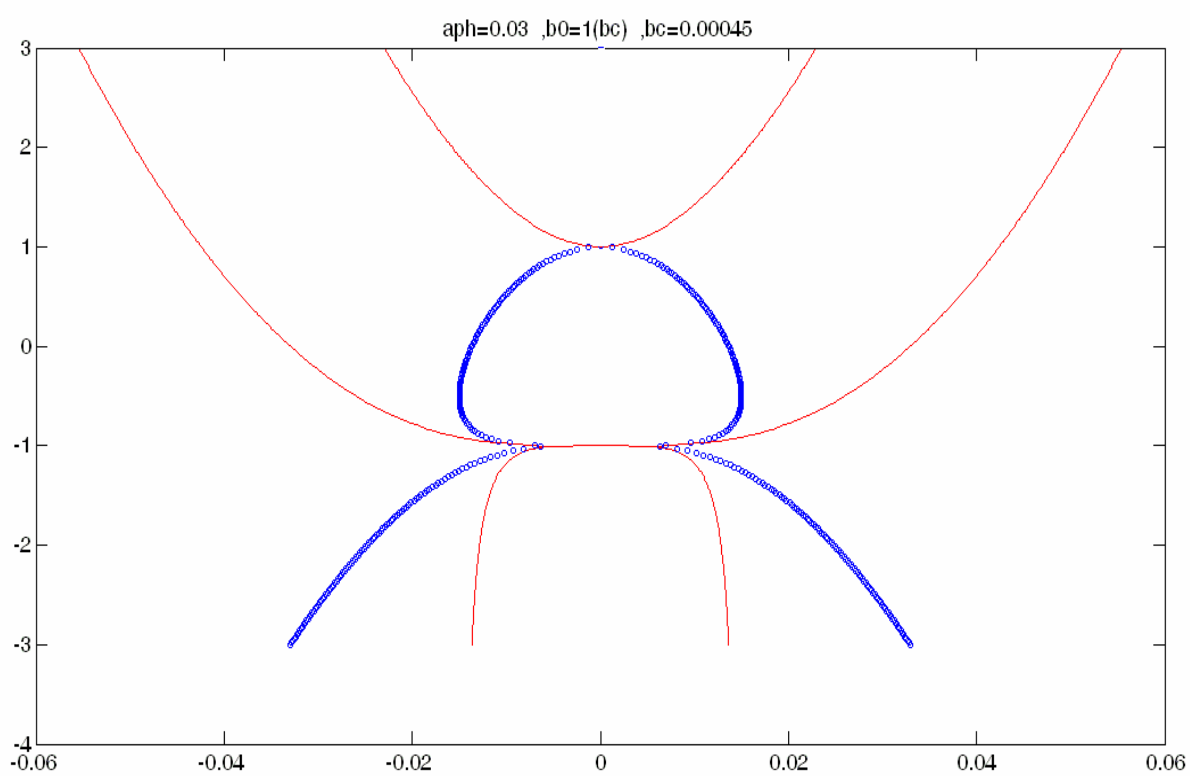

Fig2：X-axis is momentum k （Unit：1.89×10^{8}/m）； Y-axis is Energy $(\varepsilon-0.16)/b_0$ （Unit：59.5mev ） Red curves ： Propagating mode ； Blue curves ： Evanescent mode.When $(\varepsilon-0.16)/b_0$<-1, the two modes couple together. The magnetic energy is equal to Rashba energy and the energy dispersion bottom is flat.

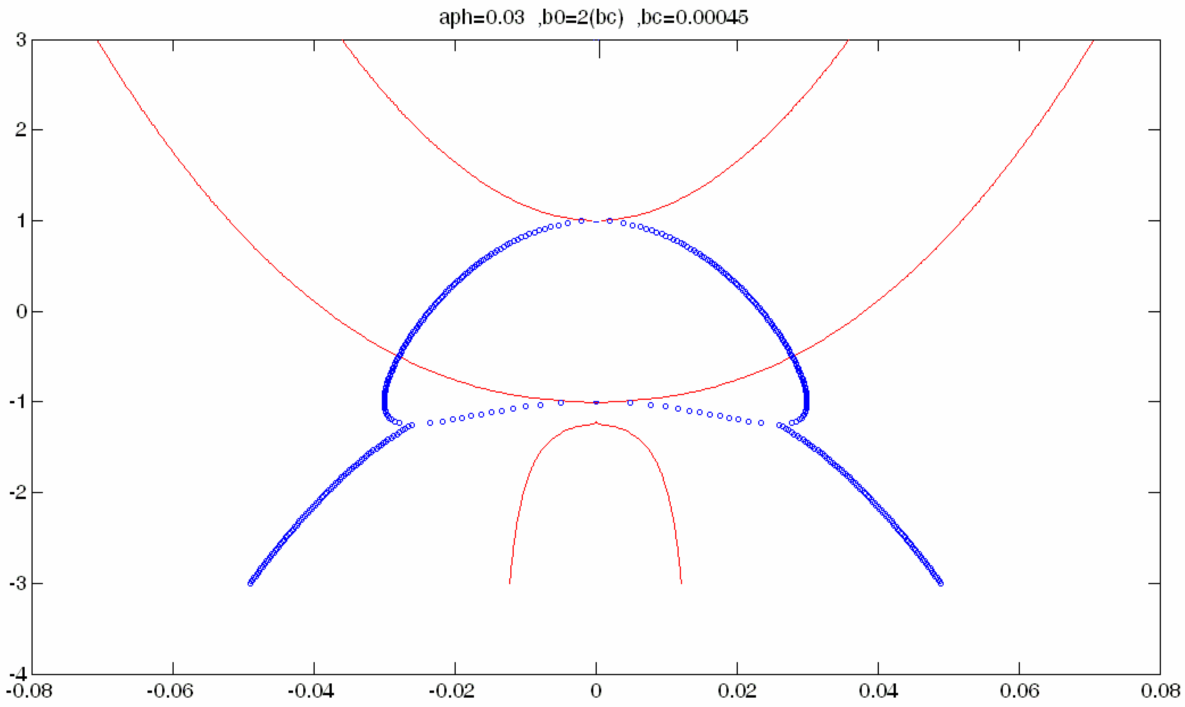

Fig3：X-axis is momentum k （Unit：1.89×10^{8}/m）； Y-axis is Energy $(\varepsilon-0.16)/b_0$（Unit：59.5mev） Red curves：Propagating mode；Blue curves：Evanescent mode. When $(\varepsilon-0.16)/b_0$<-1, the two modes couple together. The magnetic energy is lager then Rashba energy and the energy dispersion is Zeeman-like.

We also studied spin density variation in position with different frequency. And the different frequency cause first side band have different evanescent mode which affect strongly on spin density.

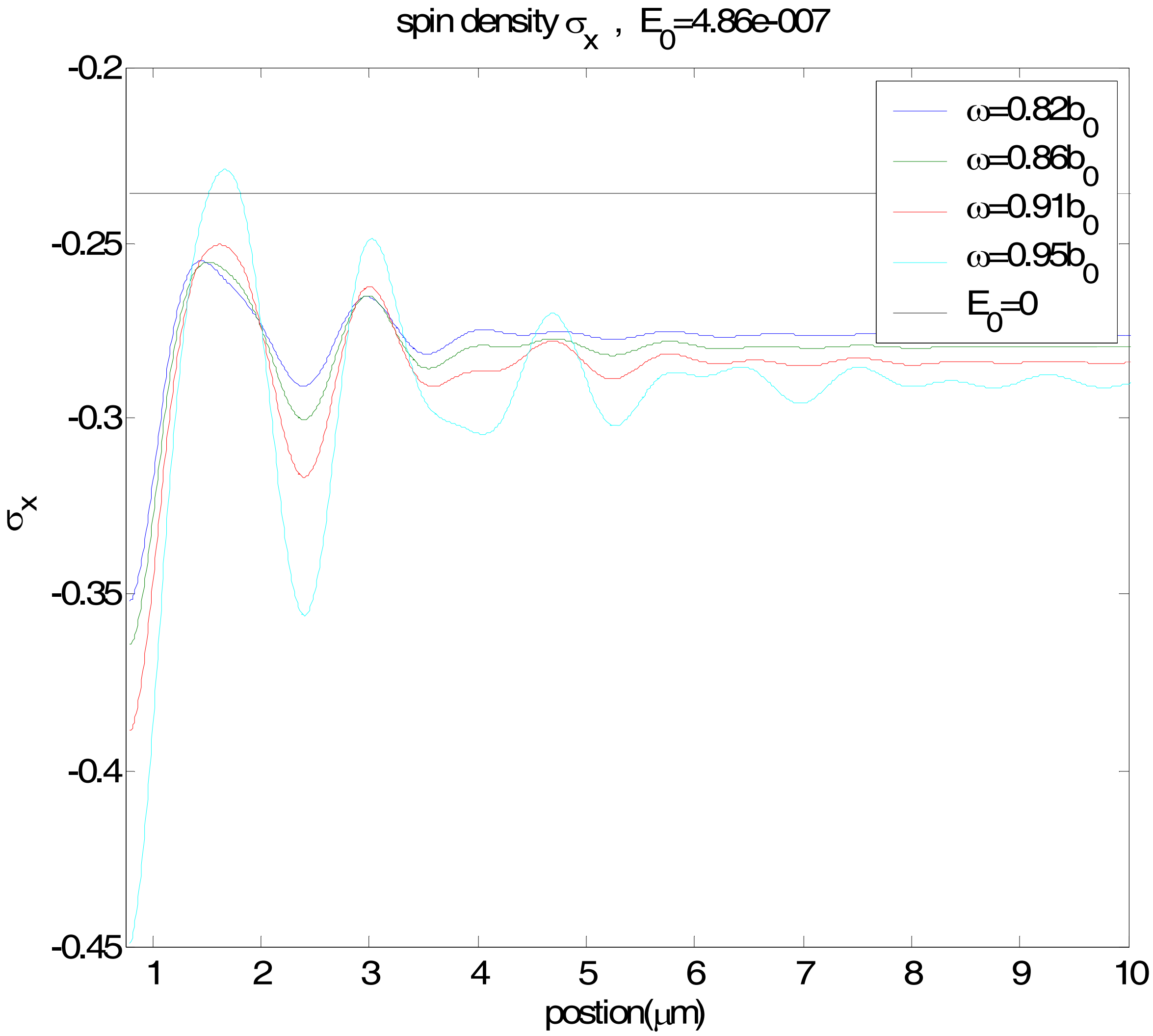

Fig4：The energy locates the center of Zeeman gap. The fluctuation of σ_x spin density has faster saturation when the first side band locates at the lager evanescent mode, see the relation of the side band and evanescent mode in fig1. And the spin density becomes larger when we turn on the electric field and tune up the frequency. Electric field E_0 is in unit of 0.338 KV/cm.

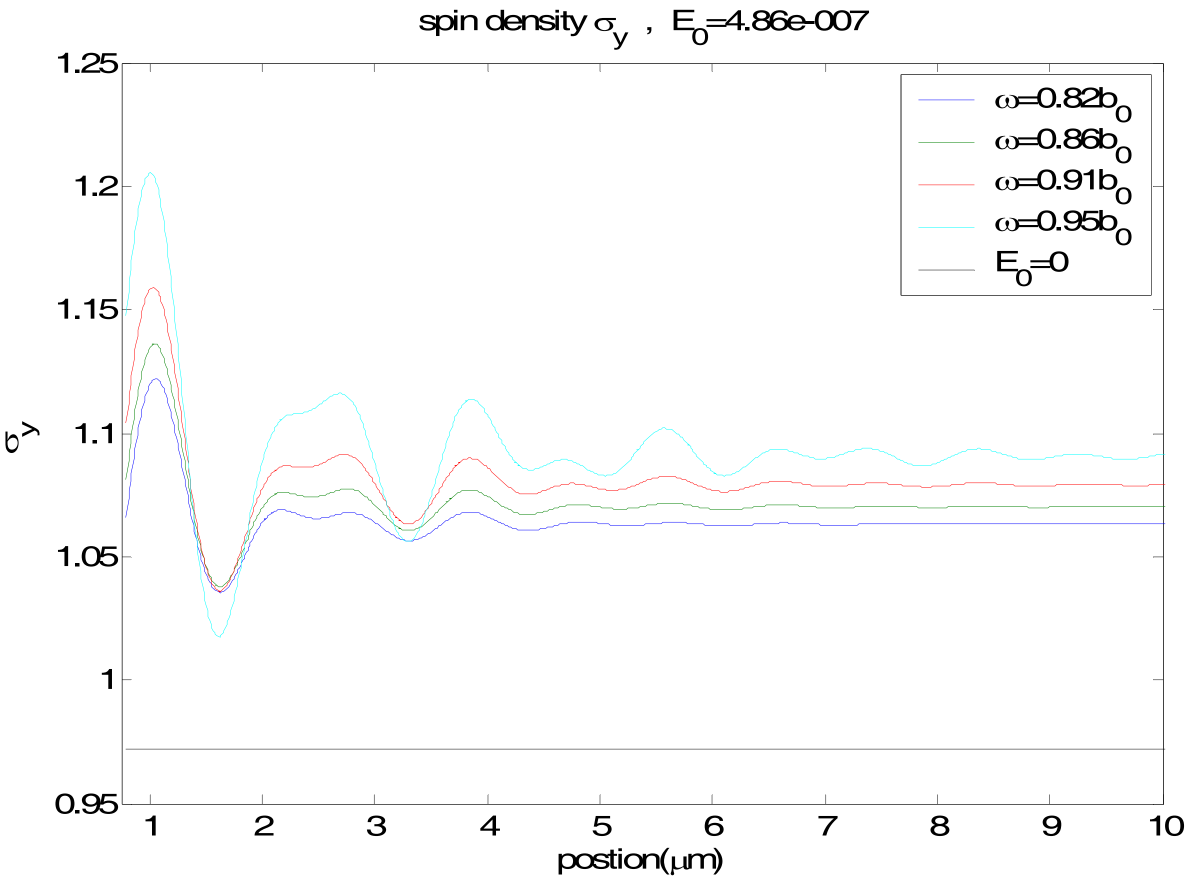

Fig5：The energy locates the center of Zeeman gap. The fluctuation of σ_x spin density has faster saturation when the first side band locates at the lager evanescent mode, see the relation of the side band and evanescent mode in fig1. And the spin density becomes larger when we turn on the electric field and tune up the frequency. Electric field E_0 is in unit of 0.338 KV/cm.

[II] Asymmetries in intrinsic spin-Hall effect in low in-plane magnetic field (see Appendix A, preparing to submit to Physical Review B)

[III] Spin-Hall effects in a Josephson contact

The distinct feature of this work is that the predicted effect here occurs as a equilibrium result. This is in sharp contrast with the spin-Hall effect, where non-equilibrium spin polarization in the bulk or the non-equilibrium spin accumulation at edges are generated by a driven electric field or an influx of charge current. This work has been submitted to Physical Review Letters (arXiv:0801.4419v1 for an earlier draft in archive, also see Appendix B).

Appendix A:

Asymmetries in intrinsic spin-Hall effect in low in-plane magnetic field

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Effects of low in-plane magnetic field on bulk spin densities and edge spin accumulations of a diffusive two dimensional semiconductor strip are studied. Focusing upon the Dresselhaus-type intrinsic spin-orbit interaction (SOI), we look for the symmetry, or asymmetry, characteristics in two magnetic field orientations: along and transverse to the strip. For longitudinal field, the out-of-plane spin density S_z exhibits odd parity across the strip, even parity in the magnetic field, and is edge accumulation. For transverse field, S_z becomes asymmetric in both spatial and field dependences, and has finite bulk values for finite magnetic fields. Our results support utilizing low in-plane magnetic fields for the probing of the underlying SOI.

PACS numbers: 72.25.Dc, 71.70.Ej, 73.40.Lq

I. INTRODUCTION

Generation and manipulation of spin densities by electrical means are major goals of semiconductor spintronics that are made possible by spin-orbit interactions (SOI).[1–8] SOIs being considered are either of the intrinsic-type: the Rashba[2,5,7,9–11] and the Dresselhaus SOIs;[4,8,12,13] or of the extrinsic-type: the impurity-induced SOI.[1,3,6,10,14] These SOIs contribute, in an external electric field, to either spin densities in the bulk or spin accumulations at lateral edges, or both. Out-of-plane spin polarization is of particular interest because it permits efficient optical probe by Kerr rotation. The edge spin accumulation, according to the spin-Hall effect (SHE), has an out-of-plane component, and is resulted from a transverse spin current induced by the electric field.[1,4–6] However, for the case of intrinsic SOI, consensus has been reached that the SHE is quenched by background scatterers, be they isotropic or anisotropic,[15] as long as the momentum dependence in the intrinsic SOI is linear. Meanwhile, no out-of-plane bulk spin densities is expected in an electric field.[2,10,16] When applying an in-plane magnetic field to a two-dimensional (2D) systems, one might be led by the in-plane nature of the effective spin-orbit magnetic field, $\mathbf{h}_{\rm eff} = \langle \mathbf{h}(\mathbf{k}) \rangle \neq 0$, that there were no out-of-plane spin densities. This is shown not to be the case by Engel *et al.*[11] for a Rashba-type 2D system, where out-of-plane spin densities are found when the external in-plane magnetic field is longitudinal: a configuration studied by recent experiments.[17,18] However, either the scatterer has to be anisotropic or the electron dispersion has to be nonparabolic for the effect to hold.[11] In this work, we have shown that out-of-plane bulk spin density can be generated in another configuration with less restrictive assumptions. The configuration is a Dresselhaus-type 2D system and the external in-plane magnetic field is in a transverse orientation. More importantly, the effect holds for isotropic background scatterers and for parabolic dispersion for electrons. Our calculation has included the cubic Dresselhaus SOI.

Another major focus of this work is upon the symmetries, or asymmetries, in the bulk spin densities and the edge spin accumulations with respect to either the magnetic field or the transverse locations. Reference on in-plane magnetic fields and their findings. Proposing the weak in-plane magnetic field characteristics as a useful probe for the SOI mechanism, without the need to fabricate sample of different crystal orientations.

A thorough study on the low magnetic field characterization of the spin densities and accumulations is important for the exploration of Different SOI may be invoked for different spin-control functionalities because Studies show that the spin densities or spin accumulations are intricately dependent on the form of the SOI and on the effects from background scatterers. As the Rashba and the extrinsic SOI do not depend on the crystal orientation but the Dresselhaus SOI does depend on the crystal orientation, to distinguish the SOI might require preparation of samples with different crystal orientation, as have been done by Awschalom. Thus, the pivotal importance of the SOIs to the spintronics demands a full-scale development of diagnostic tools for the identification of the SOI mechanism in a sample.

Recent years, the great potential of the spintronics attracts a lot of studies in manipulation of the electron spin because the spintronics provides a novel way to combine the charge dynamics and the spin degree of freedom in the application of semiconductor devices. One of key issues is to control the electron spin by the spin-orbit interaction (SOI) in the semiconductor, which plays an important role of coupling the electron orbital motion and the spin degree of freedom. The strength of SOI is much larger in the semiconductor than in the vacuum.[21] One new phenomenon is SHE which refers to the generation of a spin current transverse to a charge current in non-magnetic systems in the presence of SOI. The SHE can be understand that the electron spin encounters a transverse force which is induced by a longitudinal driving electric field.[22]

The *intrinsic* and *extrinsic* SHE can generate spin current transverse to an applied electric field due to different origins of spin-orbit coupling in the semiconductor. The intrinsic SHE[23] is come from the spin-split band via either *Rashba* or *Dresselhaus*[24] SOI in the structure in-

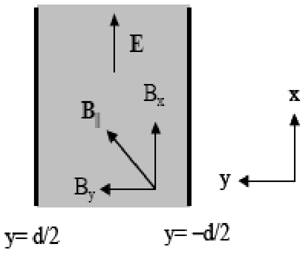

FIG. 1: Top-view schematic illustration of the 2D strip. The 2D strip has width d and its boundaries are at $y = \pm d/2$. The longitudinal electric field **E** and an in-plane magnetic field $\mathbf{B}_\parallel$ are applied on the 2D trip.

version asymmetry (SIA) and bulk inversion asymmetry (BIA) semiconductor, respectively. However, the extrinsic SHE is due to impurity scattering in the skew-scattering processes, which induce the spin-dependent propagation of the electron.[25,26] The intrinsic SHE has been experimentally demonstrated for p-doped two-dimensional electron gas (2DEG).[27] The extrinsic SHE also have been performed in several experiments.[32,33]

It is interesting to study the SHE in the presence of an external magnetic field coupling to the spin dynamics. The spin transport and relaxation of the intrinsic SHE with a perpendicular magnetic field have been studied in the diffusion approximation for *Rashba* SOI.[28,29] Recently, Rashba *et al.* studied the time-dependent electric field with a static in-plane magnetic field to produce a z-component spin accumulation via either non-parabolic band or the anisotropic scatterer.[30] Lin *et al.* studied the spin current and spin-Hall conductivity for short-range and remote impurities in the case of the intrinsic SHE with an in-plane magnetic field[31]. As known, the spin current is not conserved and its definition still remains an issue.[36] However, the spin accumulation can be measured directly by Kerr rotation technique.[32] Therefore, it is interesting to study the the behavior of the spin accumulation versus an in-plane magnetic field near the boundaries. The symmetric property of spin accumulations have been observed experimentally when an in-plane magnetic field normal to the electric field is applied with the same magnitude but in the opposite direction.[32–34] This symmetric spin accumulation is explained as the extrinsic SHE in the presence of an in-plane magnetic field.[34] As know, the extrinsic SHE produces the zero bulk spin density S_z which is perpendicular to 2DEG due to the spin-dependent distribution being proportional to linear electron momentum.[35] Therefore, the lowest-order spin accumulation S_z is expected up to the second order of the in-plane magnetic field. In this paper, we focus on the behavior of the spin accumulations near boundaries for intrinsic SHE in the presence of an in-plane magnetic field.

The in-plane magnetic field $\mathbf{B}_{||}$ is acted on the 2D strip with a driving electric field **E** in the Fig. 1. In the case of the intrinsic SHE with an in-plane magnetic field, the diffusion equation can be employed to investigate the spin accumulation and spin current with boundaries. The spin accumulation which is generated by the intrinsic SHE can be affected by applying a weak in-plane magnetic field $\mathbf{B}_{||}$ ($\omega_c\tau \ll 1$, ω_c is cyclotron frequency due to $\mathbf{B}_{||}$ and τ is the momentum relaxation time) in the diffusion region. In the absence of the external magnetic field, it is known that the intrinsic SHE vanishes for arbitrary weak disorder in linear *Rashba* and *Dresselhau* cases.[37–39] However, the intrinsic SHE can produce a finite spin accumulation in the cubic *Dresselhau* case.[40] The applied in-plane magnetic field induces two effects which are the spin precession about the magnetic field and the spin-charge coupling effect via the magnetic field coupling to SOI. The spin precession can be interpreted by a spin processing about the magnetic field with cyclotron frequency ω_c. In the weak magnetic field limit, the electron spin can encounter many scattering events within a precession period such that the spin would be relaxed via impurity scattering processes. Besides, the direction and magnitude of an effective magnetic field $\mathbf{h_p}$ induced by SOI is related to the electron momentum. In weak SOI case, the spin-split energy due to SOI is much smaller than the disorder energy. This result causes the mechanism of the spin relaxation, called D'yakonov-Perel' (DP) relaxation.[41] Below, we will extend non-equilibrium Green's function method in diffusive region. In Section II, the spin density and spin current will be calculated in the diffusion approximation in the presence of the in-plane magnetic field. In Section III, we have studied the spin accumulation by an applied in-plane magnetic field with either *Rashba* or *Dresselhau* SOI. The conclusion would be presented in Section IV.

II. FORMULATION

A. Spin Density

We use the Keldysh Green's function technique to derive the diffusion equation of charge and spin densities, which is equivalent to the Boltzmann kinetic equation.[42] By averaging all elastic and spin-independent impurities in the method of nonequilibrium Green's function has been calculated in our previous work.[43] It is know that the spin accumulation is induced by applying a uniform electric field to a homogeneous 2DEG with SOI in the diffusion regime due to the intrinsic SHE. This method can be generalized in the case of applying in-plane magnetic in 2DEG. The SOI term can be expressed as $h_{so} = \mathbf{h_p}\cdot\boldsymbol{\sigma}$ where $\mathbf{h_p}$ denotes the momentum-dependent effective magnetic field due to SOI and $\boldsymbol{\sigma}\equiv(\sigma^x,\sigma^y,\sigma^z)$ is Pauli matrix vector. The effective SOI field have specific forms of $(h_p^x, h_p^y) = (\alpha k_y, -\alpha k_x)$ for *Rashba* SOI[?] and $(h_p^x, h_p^y) = (\beta k_x(k_y^2-\kappa^2), \beta k_y(\kappa^2-k_x^2))$ for cubic *Dresselhaus* SOI.[24] The spin-orbit coupling constants are α and β, and κ is the average of wave function in the direction perpendicular to 2DEG. Both of the in-plane magnetic field $\mathbf{B}_{\|}$ and the driving electric field $\mathbf{E}$ are applied parallel to 2D strip. One can combine the in-plane magnetic field with effective SOI field together into the form $\mathbf{H}_B \cdot \boldsymbol{\sigma} = (\mathbf{h_p} + \widetilde{\mathbf{B}}_{\|}) \cdot \boldsymbol{\sigma}$. The magnetic energy is defined by $\widetilde{\mathbf{B}}_{\|} = g^* \mu_B \mathbf{B}_{\|}/2$, where g^* is the effective g factor and μ_B is the Bohr magneton. In the weak magnetic field case, assuming $E_F \gg h_{so} > h_B$ is valid and the expansion of the exact Green's function only need to expand up to linear order of $\widetilde{\mathbf{B}}_{\|}$. It is the good approximation to treat the external electric field as a perturbation such that it is expressed in the four vector of potentials in form of $H' = \sum_i \Phi_i(\mathbf{r}, t)\, \tau^i$, where 2×2 matrices $\tau^0 = 1, \tau^{x,y,z} = \sigma_{x,y,z}$. H' is calculated throughout the linear response theory by Kubo formula. It is convenient to introduce four vector of densities $D_i(\mathbf{r}, t)$, whose index $i = 0$ is referring to the charge and $i = x, y, z$ are referring to the spin. The unit of one spin is taken by $\hbar/2$ and spin densities are $S_{x,y,z} = (\hbar/2) D_{x,y,z}$. The four densities is expressed by using nonequilibrium Green's function method

$$\begin{aligned} \mathbf{D}_i(\mathbf{r}, t) &= \left\langle T_l [\hat{\mathbf{D}}_i(\mathbf{r}, t)\, S_l(-\infty_{down}, -\infty_{up})] \right\rangle \\ &= -i Tr[\tau^i G^{-+}(\mathbf{r}, \mathbf{r}, t, t)], \end{aligned} \tag{1}$$

where T_l is time-loop order operator and G^{-+} is the Keldysh Green's function in the matrix form. The angular brackets denote the average over random distribution of impurities. The upper time-loop branch $(+)$ denotes the time order evolution and the lower one $(-)$ denotes anti-time order evolution. The Green's function G^{-+} is described the time loop branch from $-$ to $+$. In stationary state, the system response depends only on time difference such that one gets the density in the Fourier space ω, ω'

$$\begin{aligned} \mathbf{D}_i(\mathbf{r}, \omega) &= \int d^2 r' \sum_j \Pi_{ij}(\mathbf{r}, \mathbf{r}', \omega + \omega')\, \Phi_j(\mathbf{r}', \omega) \\ &+ \mathbf{D}_i^0(\mathbf{r}, \omega). \end{aligned} \tag{2}$$

Here, it is convenient to express all coordinate-dependent quantities in the momentum representation for a homogeneous 2DEG system. The momentum conservation is obeyed for an electron collides with the random elastic impurity. The most important physical mechanism occurs near the Fermi energy E_F such that the energy can be treated as $\omega' \approx E_F$. Applying the relation of $f_{FD}(\omega') - f_{FD}(\omega + \omega') \approx \omega (d f_{FD}(\omega')/d\omega')$, the retarded and advanced Green's function $G^r(\mathbf{p_1}, \mathbf{k_1} + \mathbf{q}, \omega + \omega')$ and $G^a(\mathbf{k_1}, \mathbf{p_1} - \mathbf{q}, \omega)$ can be employed to calculate the response functions in momentum space

$$\begin{aligned} &\Pi_{ij}(\mathbf{q}, \omega) = i\omega \sum_{\mathbf{p_1 k_1}} \int \frac{d\omega'}{2\pi} \frac{d f_{FD}(\omega')}{d\omega'} \\ &\times \langle Tr[G^a(\mathbf{k_1}, \mathbf{p_1} - \mathbf{q}, \omega)\tau^i G^r(\mathbf{p_1}, \mathbf{k_1} + \mathbf{q}, \omega + \omega')\tau^j] \rangle, \end{aligned} \tag{3}$$

, where $f_{FD}(\omega')$ is the Fermi-Dirac distribution function at energy ω'. The brackets in Eq. (1) denote averaging over the random distribution of impurities in the 2DEG. For $\omega \ll E_F$, the relation $f_{FD}(\omega' + \omega) \approx f_{FD}(\omega')$ is assumed and one can obtain the local equilibrium densities

$$\begin{aligned} &\mathbf{D}_i^0(\mathbf{q}, \omega) = i \sum_{\mathbf{p_1 k_1 q'}} \int \frac{d\omega'}{2\pi} f_{FD}(\omega') \sum_j \Phi_j(\mathbf{q}', \omega) \\ &\times \langle Tr[G^r(\mathbf{p_1}, \mathbf{k_1} - \mathbf{q}, \omega')\tau^i G^r(\mathbf{k_1}, \mathbf{p_1} + \mathbf{q}', \omega')\tau^j \\ &- G^a(\mathbf{p_1}, \mathbf{k_1} - \mathbf{q}, \omega')\tau^i G^a(\mathbf{k_1}, \mathbf{p_1} + \mathbf{q}', \omega')\tau^j] \rangle, \end{aligned} \tag{4}$$

which are associated with four vector of potentials $\Phi_j(\mathbf{q}', \omega)$. Assuming each random impurity potential $V_{sc}(\mathbf{r})$ is delta-profile correlation so that the pair correlation $\langle V_{sc}(\mathbf{r}) V_{sc}(\mathbf{r}') \rangle = \Gamma \delta(\mathbf{r} - \mathbf{r}')/\pi N_F$, where $\Gamma = 1/2\tau$ is characterized by the mean elastic scattering time τ. Assuming the semiclassical approximation $E_F \tau \gg 1$ is valid, the standard perturbation theory can be employed. The unperturbed average Green's functions are given by 2×2 matrix form

$$\begin{aligned} G^{r(0)}(\mathbf{p}, \omega) &= (G^{a(0)}(\mathbf{p}, \omega))^{\dagger} \\ &= 1/(\omega - E_{\mathbf{p}} - \mathbf{H}_B \cdot \boldsymbol{\sigma} \pm i\Gamma), \end{aligned} \tag{5}$$

where $E_{\mathbf{p}} = p^2/2m^*$. The local equilibrium densities $\mathbf{D}_i^0$ are calculated up to the lowest order expansion of the average Green's function by setting $\mathbf{H}_B \approx 0$ in Eq. (5). Eventually, we obtain the local equilibrium densities $\mathbf{D}_i^0(\mathbf{q}, \omega) = -2N_0 \Phi_i(\mathbf{q}, \omega)$ by setting $\mathbf{q} = 0$ in the average Green's function. The N_0 is the electron density of state at Fermi energy E_F. The nth higher order term of average Green's functions produce the order of power $1/(E_F)^n$ for $\mathbf{q} = 0$ and it is the small correlation to the $\mathbf{D}_i^0$. Obviously, it is good enough to estimate the $\mathbf{D}_i^0$ up to the lowest order approximation in the average Green's function.

In the presence of SOI, the spin would be relaxed due to DP-relaxation mechanism in the disorder system after an electron spin travels the characteristic distance, so-called spin relaxation length l_{so}. In the diffusion limit, $l_{so} \gg l_{mean}$ is valid such that one electron spin can be scattered by several impurities before it is relaxed completely. The most important goal is to obtain the response function from Eq. (3) by calculating the mean products of the retarded and advanced Green's functions in the diffusive regime. Only the pair of retarded and advanced Green's functions of Eq. (3) carrying close enough momenta have to be taken into count in the ladder series. Redefining $\mathbf{k_1} = \mathbf{p}$ and $\mathbf{p_1} = \mathbf{p}' - \mathbf{q}$, each matrix element of the impurity averaging can be evaluated in ladder expansions as following

$$\begin{aligned} &\sum_{\mathbf{pp}'} \tau^i_{\mu\alpha} \tau^j_{\beta\nu} \left\langle G^r_{\alpha\beta}(\mathbf{p}, \mathbf{p}', \omega + \omega')\, G^a_{\nu\mu}(\mathbf{p}' - \mathbf{q}, \mathbf{p} - \mathbf{q}, \omega) \right\rangle \\ &= \sum_{\mathbf{p}} \tau^i_{\mu\alpha} \tau^j_{\beta\nu} \{ G^{r(0)}_{\alpha\beta}(\mathbf{p}, \omega + \omega')\, G^{a(0)}_{\nu\mu}(\mathbf{p} - \mathbf{q}, \omega)\, \delta_{\mathbf{pp}'} \\ &+ \Psi^{\alpha\gamma}_{\mu\lambda}(\omega, \omega', q)\, G^{r(0)}_{\gamma\beta}(\mathbf{p}, \omega + \omega')\, G^{a(0)}_{\nu\lambda}(\mathbf{p} - \mathbf{q}, \omega) \\ &+ \Psi^{\alpha\gamma'}_{\mu\lambda'} \Psi^{\gamma'\gamma}_{\lambda'\lambda} G^{r(0)}_{\gamma\beta}(\mathbf{p}, \omega + \omega')\, G^{a(0)}_{\nu\lambda'}(\mathbf{p} - \mathbf{q}, \omega) + \ldots \}, \end{aligned} \tag{6}$$

where the simplified notation $\Psi^{\alpha\gamma}_{\mu\lambda}(\omega,\omega',q) \equiv (c_i |V_{sc}|^2/V)\sum_{\mathbf{p}'} G^{r(0)}_{\alpha\gamma}(\mathbf{p}',\omega+\omega')\, G^{a(0)}_{\lambda\mu}(\mathbf{p}'-\mathbf{q},\omega)$, where c_i is the impurity concentration and V is the volume of system . All the repeated indices have to be summed and V_{sc} is the strength of the impurity. From Eq. (3), the response function is expressed in the form of

$$\Pi_{ij}(\mathbf{q},\omega) = \frac{i\omega}{2\pi}\sum_j \int d\omega' \frac{df_{FD}}{d\omega'}\left(\frac{\pi N_0}{\Gamma}\right)\tau^i_{\mu\alpha}\tau^j_{\beta\nu} \times \Psi^{\alpha\gamma}_{\mu\lambda}(\omega,\omega',\mathbf{q})[(1-\Psi(\omega,\omega',\mathbf{q}))^{-1}]^{\gamma\beta}_{\lambda\nu}, \quad (7)$$

where $1^{\gamma\beta}_{\lambda\nu} \equiv \delta_{\gamma\beta}\delta_{\lambda\nu}$ and $\Gamma/(\pi N_0) = c_i|V_{sc}|^2/V$. From Eq. (2), Eq. (3) and Eq. (7), the four densities is given by

$$D_i(\mathbf{q},\omega) - D^0_i(\mathbf{q},\omega) = \Pi_{ij}(\mathbf{q},\omega)\Phi_j(\mathbf{q},\omega). \quad (8)$$

The four components tensor in Eq. (7) can be transformed into two components vector form via the equality $\Psi^{\gamma\gamma'}_{\lambda\lambda'} = (1/2)\sum_{ij}\tau^i_{\lambda\gamma}\Psi^{ij}\tau^j_{\gamma'\lambda'}$, where $i,j = 0$, x, y and z. Immediately, we can obtain

$$\tau^i_{\mu\alpha}\tau^j_{\beta\nu}\left[\Psi(1-\Psi)^{-1}\right]^{\alpha\beta}_{\mu\nu} = 2\sum_{ij}\Psi^{il}\left[(1-\Psi)^{-1}\right]^{lj},$$

and the diffusion mechanism is decided by

$$\Psi^{il} = \frac{\Gamma}{2\pi N_0}\sum_{\mathbf{p}'} Tr\left[\tau^i G^{r(0)}(\mathbf{p}',\omega+\omega')\tau^l G^{a(0)}(\mathbf{p}'-\mathbf{q},\omega')\right]. \quad (9)$$

The Eq. (8) can be expressed in matrix form

$$(1-\Psi)^{il}(\mathbf{D}_l - \mathbf{D}^0_l) = i\omega\tau\Psi^{il}\mathbf{D}^0_l, \quad (10)$$

via the relation $\int d\omega'(df_{FD}/d\omega') = -\delta(\omega'-E_F)$ at zero temperature. For the case of a static homogeneous electric field, the left-hand side of Eq. (10) is equal to zero by setting $\omega = 0$ and the charge density $D_0 = 0$ due to charge neutrality.

First, Ψ^{il} can be calculated in the absence of the SOI and external magnetic field, saying $\mathbf{H}_B = 0$, to easily obtain

$$\Psi^{il}(\mathbf{q},\omega)|_{\mathbf{H}_B=0} = \left(1 + i\omega\tau - D\tau\mathbf{q}^2\right)\delta_{il}, \quad (11)$$

where the diffusion constant is $D = v_F^2\tau/2$. Under the consideration of the static and homogeneous electric field, the time component of the electric field $\omega = 0$ is assumed in the below. The standard perturbation theory is employed to expand Eq. (9) respecting to the small parameters of $\mathbf{H}_B$ and $\mathbf{q}$. Considering $H_{so} > \widetilde{B}_{\parallel}$ in the weak magnetic field case, the most important effects of the magnetic field come from the contribution of the linear $\widetilde{B}_{\parallel}$ term. It is known that $\mathbf{h_k} = -\mathbf{h_{-k}}$ is odd parity and $\widetilde{B}_{\parallel}$ is even parity respecting to the electron momentum $\mathbf{k} \rightarrow -\mathbf{k}$. The Eq. (9) is given by

$$\Psi^{il}(\mathbf{q}=0,\omega=0)|_{\widetilde{B}_{\parallel}} = \tau R^{ilm}_B, \quad (12)$$

where $R^{ilm}_B \equiv -\sum_m 2\varepsilon^{ilm}\widetilde{B}_m$ and $m =$x, y denotes the x, y component of the inplane magnetic field. This term is simply related to linear magnetic field term without coupling to SOI. It can be interpreted that the electron spin processes along the axis of an external magnetic field $\mathbf{B}_{\parallel}$ in semiclassical picture. However, the travelling direction of an electron would be changed by random impurities in diffusion region such that the spin precession can be randomized due to elastic scattering processes.

The lowest order expansion of $\mathbf{h_k}$ in Ψ_{il} is zero for $\mathbf{q} = 0$ because the angular averaging integration contains odd parity in the momentum $\mathbf{k}$. It has to be expanded in small $\mathbf{q}$ to get the finite result of

$$\Psi^{il}(\mathbf{q},\omega=0)|_{\mathbf{h_p}} = \tau R^{ilm} iq_m, \quad (13)$$

where $R^{ilm} \equiv 4\tau\sum_n \varepsilon^{iln}\overline{h^n_k v^m_F}$. The overline denotes the angular average over Fermi surface and v^m_F is the m component of Fermi velocity. This term is associated with spin precession due to the SOI.

Expansion of Ψ^{il} in the second order $\mathbf{h_k}$ and $\mathbf{q} = 0$ gives rise to

$$\Psi^{il}(\mathbf{q}=0,\omega=0)|_{\mathbf{h}^2_\mathbf{k}} = -\tau\Gamma^{il}, \quad (14)$$

where $\Gamma^{il} \equiv 4\tau\overline{h^2_k(\delta^{il} - n_k{}^i n_k{}^l)}$ for $i,l \neq 0$ and the unit vector $\mathbf{n_k} \equiv \mathbf{h_k}/h_k$. Its physical interpretation is recognized as the DP relaxation. The physical meaning of the DP relaxation can be understood in the following discussion. In the interval between collisions, the spin of each electron precesses about an effective magnetic field which is related to the electron momentum in the SOI system. Consequently, the direction of electron momentum will be changed via collide with the random elastic impurities and leads to the change of the precession axis. If the time between collisions is much less than the precession period, then the electron spin will not be able to follow the change of the precession axis. Such that the electron spin precession would be relaxed after collisions.

The SHE is strongly related to the spin-charge coupling terms which induce the spin-Hall current moving normal to the driving electric field $\mathbf{E}$. The spin-charge coupling terms can be calculated in higher order expansion of Ψ^{il} in the general form of

$$\Psi^{i0}(\mathbf{q},\omega=0)|_{\text{spin-charge}} = \mathrm{M}^{i0} + \mathrm{M}^{i0}_B \quad (15)$$

, where the $\mathbf{q}$-dependent operators are defined by

$$\begin{cases} \mathrm{M}^{i0} = 4\tau^3 i\mathbf{q}\overline{h^3_k\frac{\partial n^i_\mathbf{k}}{\partial\mathbf{k}}} \\ \mathrm{M}^{i0}_B = 2\tau^2(-i\mathbf{q})\left(\widetilde{B}_x\overline{\frac{\partial h^y_\mathbf{k}}{\partial\mathbf{k}}} - \widetilde{B}_y\overline{\frac{\partial h^x_\mathbf{k}}{\partial\mathbf{k}}}\right)\delta_{iz} \end{cases} \quad (16)$$

The first term M^{i0} is original spin-charge coupling term in the absence of in-plane magnetic field, which couples spin and charge together due to SOI for $i =$ x, y, or z. It is worth to notice that the second term denoted by M^{z0}_B connects the charge and z component spin in the

SOI background through $\widetilde{\mathbf{B}}_{\|}$. This term is a new contribution in the diffusion equation with a external magnetic field and it causes the bulk value of spin density varying by $\widetilde{\mathbf{B}}_{\|}$. Finally, the diffusion propagator D^{il} is defined by $-(1-\Psi)/\tau$ in the dc limit ($\omega = 0$) and it can be transformed into the real space representation by replacing $i\mathbf{q}$ into the spatial differential operator $\bigtriangledown$

$$\begin{aligned} D^{il} &= -D\bigtriangledown^2 + R_B^{ilm} - \Gamma^{il} + R^{ilm}\bigtriangledown_m \\ &+ \left(\mathrm{M}^{i0} + \mathrm{M}_B^{z0}\right)|_{i\mathbf{q}\to\bigtriangledown}. \end{aligned} \tag{17}$$

Form Eqs. (12) and (18) by using $S_i = D_i/2$ and $\omega = 0$, the diffusion equations for Rashba SOI are given by

$$\begin{aligned} (D\nabla_i^2 - \frac{\Gamma^{ii}}{\hbar^2})S_i &+ \sum_{j=x,y,z}\frac{R^{ijy}}{\hbar}\{[\mathbf{\nabla}'\mathbf{S}\cdot\hat{z}] + [\mathbf{\nabla}'\times(\mathbf{S}\times\hat{z})]\}_i \\ &+ \frac{2}{\hbar}(\widetilde{\mathbf{B}}_{\|}\times\mathbf{S})_i + (\frac{\Gamma^{ii}}{\hbar^2}\delta_{iy} - \frac{2}{\hbar}\widetilde{B}_x\delta_{iz})S_y^b = 0 \end{aligned} \tag{18}$$

where $\hbar$ is restored for physical unit and $\mathbf{\nabla}' \equiv \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y}$ and i =x, y, and z.

$$\begin{cases} D\frac{\partial^2}{\partial y^2}S_x - \frac{\Gamma^{xx}}{\hbar^2}S_x + 2\widetilde{B}_y S_z = 0 \\ D\frac{\partial^2}{\partial y^2}S_y + \frac{R^{yzy}}{\hbar}\frac{\partial}{\partial y}S_z - \frac{2}{\hbar}\widetilde{B}_x S_z - \frac{\Gamma^{yy}}{\hbar^2}(S_y - S_y^b) = 0 \\ D\frac{\partial^2}{\partial y^2}S_z + \frac{R^{zyy}}{\hbar}\frac{\partial}{\partial y}S_y - \frac{\Gamma^{zz}}{\hbar^2}S_z - \frac{2}{\hbar}\widetilde{B}_y S_x + \frac{2}{\hbar}\widetilde{B}_x(S_y - S_y^b) = 0 \end{cases} \tag{19}$$

The bulk spin density is $S_y^b = -\tau\alpha N_0 eE\delta_{jy}$ and it is as same as the result in zero magnetic field. It is noticeable that a homogeneous electric field $\mathbf{E}$ is applied along x direction leading to S_y^b due to a shift of Fermi sphere. The spin precession term $R^{yzy} = -R^{zyy} = 2\tau h_{k_F} v_F$ and the DP relaxation term $\Gamma_{xx} = \Gamma_{yy} = \Gamma_{zz}/2 = 2\tau h_{k_F}^2$ can be calculated from the definition in Eqs. (13) and (14). Since the bulk solution of the spin density is spatial-independent, one can drops the derivative terms respecting to coordinate y. The majority of electrons are driven by $\mathbf{E}$ with the drift velocity $\mathbf{v_d}$ toward $-x$ direction and the effective SOI field $\mathbf{h_{so}}$ is lying in y direction normal to $\mathbf{v_d}$ for *Rashba* SOI case. Such that the bulk is naturally revealed y-polarized spin density S_y^b in *Rashba* SOI case. The magnetic field appearing in diffusion equations doesn't change the solutions of spin densities. It is easily to solve the spin densities $S_x = S_z = 0$ and $S_y = S_y^b$. Even in the presence of external magnetic field, again, there is no spin accumulation in diffusion region for *Rashba* SOI case.

For *Dresselhaus* SOI case, the diffusion equations with the external magnetic field are given by

$$\begin{aligned} (D\nabla_i^2 - \frac{\Gamma^{ii}}{\hbar^2})S_i \; &+ \sum_{j,l=x,y,z}\frac{R^{ijl}}{\hbar}(\mathbf{\nabla}'\times S_j\hat{j})_i + \frac{2}{\hbar}(\widetilde{\mathbf{B}}_{\|}\times\mathbf{S})_i \\ &- \frac{C_1}{\hbar^2}\delta_{ix} - \frac{\widetilde{B}_y}{\hbar}C_2\delta_{iz} = 0, \end{aligned} \tag{20}$$

where $\hbar$ is restored for physical unit and i = x, y, and z.

$$\begin{cases} D\frac{\partial^2}{\partial y^2}S_x + \frac{R^{xzy}}{\hbar}\frac{\partial}{\partial y}S_z - \frac{\Gamma^{xx}}{\hbar^2}S_x + \frac{2}{\hbar}\widetilde{B}_y S_z - \frac{C_1}{\hbar^2} = 0 \\ D\frac{\partial^2}{\partial y^2}S_y - \frac{\Gamma^{yy}}{\hbar^2}S_y - \frac{2}{\hbar}\widetilde{B}_x S_z = 0 \\ D\frac{\partial^2}{\partial y^2}S_z - \frac{\Gamma^{zz}}{\hbar^2}S_z + \frac{R^{zxy}}{\hbar}\frac{\partial}{\partial y}S_x - \frac{2}{\hbar}\widetilde{B}_y S_x + \frac{2}{\hbar}\widetilde{B}_x S_y \\ -\frac{\widetilde{B}_y}{\hbar}C_2 = 0 \end{cases} \tag{21}$$

The notations are $R^{zxy} = -R^{xzy} = \frac{\beta\tau k_F^4}{m^*}(2C^2 - 1/2) = 2D/l_{so}$ and $\Gamma_{xx} = \Gamma_{yy} = \Gamma_{zz}/2 = \beta^2\tau k_F^6(1/4 - C^2 + 2C^4)$, where $C \equiv \kappa/k_F$. The spin-charge coupling terms $C_1 \equiv M^{x0}D_0^0/2$ is related to M^{i0} and $C_2 \equiv \tau\overline{(\partial h_k^x/\partial k_x)}(\partial D_0^0/\partial x)$ is related to M_B^{i0}. $D_0^0 = -2N_0 eEx$ (e>0) is related to the electric field. The DP relaxation terms have the relation $\Gamma^{xx} = \Gamma^{yy} = \Gamma^{zz}/2$. The bulk solutions of spin densities are given by

$$\begin{cases} S_z^b = \Lambda_y\left(-\frac{1}{2}C_2 + \frac{C_1}{\Gamma^{xx}}\right)/\left(1 + 2\Lambda_x^2 + 2\Lambda_y^2\right) \\ S_y^b = -2\Lambda_x S_z^b \\ S_x^b = 2\Lambda_y S_z^b - \frac{C_1}{\Gamma^{xx}}. \end{cases} \tag{22}$$

, where the ratio parameter is defined by $\Lambda_i \equiv \widetilde{B}_i/\Gamma^{xx}$. All bulk spin densities S_i^b are the function of the electric field E and Λ_i. When the in-plane magnetic field is turn off ($\Lambda_i = 0$), these bulk spin densities become $S_y^{b(0)} = S_z^{b(0)} = 0$ and $S_x^{b(0)} = -C_1/\Gamma^{xx}$ is finite value, independent of the external magnetic field.[43] All bulk spin densities are coming from the spin-charge coupling terms in Eq. (15) and they vanish as $E = 0$. For *Dresselhaus* SOI, the electric field $E\hat{x}$ produces the bulk spin density $S_x^{b(0)}$ in the zero magnetic field case and S_y^b, S_z^b are induced by $S_x^b(0)$ through $\widetilde{B}_{\|}$. In the semiclassical picture and diffusion region of $l_{so} \gg l_{mean}$, the external magnetic field $\widetilde{B}_{\|}$ can make $S_x^{b(0)}$ flipping to contribute the bulk spin densities S_z^b and S_y^b. For $S_z^b(E)$, the electric field terms produce the x-component bulk spin density and the y-component external magnetic field can make S_x^b flipping to contribute $S_z^b(E)$. For $S_y^b(E)$, it is coming from the flip of $S_z^b \to S_y^b$ via the x-component external magnetic field. For $S_x^b(E)$, the first term describes the flip of $S_z^b \to S_x^b$ via the y-component external magnetic field and the second term is the contribution coming from the driving electric field. It is clear that the bulk spin densities without the external magnetic field would be modified by $\widetilde{B}_{\|}$ in *Dresselhaus* SOI case.

B. Spin Current

The spin current operator are defined by $J_l^i \equiv (1/2)\left(V_l\sigma_i + \sigma_i V_l\right)$ and each spin unit is $\hbar/2$. The velocity operator is given by

$$V_l \equiv \frac{k_l}{m*} + \frac{\partial\mathbf{h_k}\cdot\boldsymbol{\sigma}}{\partial k_l}, \tag{23}$$

where $m*$ denotes the effective mass of electron. The first term in right-hand side of Eq. (20) is classical kinetic term and the second term is spin-dependent velocity due to SOI. The spin current J_l^i stands for the electron moving with the velocity $v_l = (k_l/m^*)$ and spin state σ_i. After some algebra, one can obtain the expression for spin current densities

$$\begin{aligned} I_l^i(\mathbf{q},\omega) = i\omega \int \tfrac{d\omega'}{2\pi}\tfrac{dN_F}{d\omega'}\times \\ \sum_{\mathbf{k},\mathbf{k}'}\langle\left(v_l\sigma_i + \tfrac{\partial h_{\mathbf{k}}^i}{\partial k^l}\right)G^r\left(\mathbf{k}+\tfrac{\mathbf{q}}{2},\mathbf{k}'+\tfrac{\mathbf{q}}{2},\omega+\omega'\right) \\ \times\tau^j G^a\left(\mathbf{k}'-\tfrac{\mathbf{q}}{2},\mathbf{k}-\tfrac{\mathbf{q}}{2},\omega'\right)\rangle\Phi_j(\mathbf{q},\omega) \end{aligned} \quad (24)$$

, where the spin indices i = x, y and z; j = 0, x, y and z. In the dc limit ($\omega = 0$) and at zero temperature ($\omega' = E_F$), the spin current densities can be simplified in the form of

$$I_l^i = \frac{1}{m^*}\left[X_l^{ij'}D_{j'} - X_l^{i0}D_0^0 + Y_l^{ij'}D_{j'} - Y_l^{i0}D_0^0\right] \quad (25)$$

, where the index j' = x, y and z. The operators are defined by

$$\begin{aligned} X_l^{ij} \equiv \left(\tfrac{\Gamma}{2\pi N_0}\right)\sum_{\mathbf{k}} k_l Tr[\tau^i G^{r(0)}\left(\mathbf{k}+\tfrac{\mathbf{q}}{2},\omega+E_F\right) \\ \times\tau^j G^{a(0)}\left(\mathbf{k}-\tfrac{\mathbf{q}}{2},E_F\right)] \end{aligned} \quad (26)$$

and

$$\begin{aligned} Y_l^{ij} \equiv \left(\tfrac{\Gamma}{2\pi N_0}\right)\sum_{\mathbf{k}} \tfrac{\partial h_{\mathbf{k}}^i}{\partial k_l} Tr[G^{r(0)}\left(\mathbf{k}+\tfrac{\mathbf{q}}{2},\omega+E_F\right) \\ \times\tau^j G^{a(0)}\left(\mathbf{k}-\tfrac{\mathbf{q}}{2},E_F\right)]. \end{aligned} \quad (27)$$

For the SHE, it is most important to study spin currents flowing along y direction when a static electric field is applied along x axis. To obtain the spin current densities I_i^y has to calculate X_y^{ij} and Y_y^{ij}

$$\begin{aligned} X_y^{ij} = -m^*\overline{\left(iq_y D\delta_{ij} + \tfrac{1}{2}R^{ijy}(\delta_{iz}+\delta_{jz})\right)} \\ -2iq_x m^*\tau^2 v_{F,y}\left(\mathbf{h}_{\mathbf{k}}\times\tfrac{\partial\mathbf{h}_{\mathbf{k}}}{\partial k_x}\right)_z\delta_{iz}\delta_{j0} - \overline{\tfrac{\partial h_{\mathbf{k}}^i}{\partial k_y}}\delta_{j0} \end{aligned} \quad (28)$$

and

$$Y_y^{ij} = \overline{\frac{\partial h_{\mathbf{k}}^i}{\partial k_y}}\delta_{j0}. \quad (29)$$

It is found that the last term of X_y^{ij} is exactly cancelled out the contribution of Y_y^{ij}. By restoring $\hbar$ into physical unit, Eq. (24), Eq. (27) and Eq. (28) give us spin current density expressions

$$I_y^i(\mathbf{r}) = -2D\frac{\partial S_i}{\partial y} - \frac{R^{ijy}}{\hbar}(S_j - S_j^b) + \frac{I_{sH}}{\hbar}\delta_{iz}, \quad (30)$$

which are associated with spin densities S_i. The first term of I_y^i describes the normal diffusion process of S_i along y direction and the second term is contributed from spin precession due to SOI. The total spin-Hall current is defined by I_{sH}. The first term I_{sH} is the spin-Hall current term in the absence of external magnetic field. The additional term I_{sH}^B is totally contributed from the external magnetic field. Naturally, these two terms are proportional to the linear electric field $\mathbf{E}$ because the origin of SHE is coming from spin-charge coupling by SOI. Their expressions are given by

$$I_{sH} = -R^{zjy}S_j^b + 4\tau^2 eEN_0\overline{v_{F,y}\left(\frac{\partial\mathbf{h}_{\mathbf{k}}}{\partial k_x}\times\mathbf{h}_{\mathbf{k}}\right)_z}. \quad (31)$$

For *Rashba* SOI case, it is easily to check that I_{sH} vanishes without an external magnetic field $\widetilde{B}_{\|}$. Furthermore, the bulk spin density S_y^b is equal to $S_y^{b(0)}$ such that the total spin-Hall current I_{sH} is still zero even in the presence of external magnetic field. For *Dresselhau* SOI case, I_{sH} is finite even without $\widetilde{B}_{\|}$. However, I_{sH}^B is dependent on $\widetilde{B}_{\|}$ and can modulate I_{sH} by tuning either the strength or the direction of $\widetilde{B}_{\|}$.

In the cases of a 2D strip, the hard-wall boundary conditions $I_y^i(y = \pm d/2) = 0$ are imposed. The boundary conditions indicate that both of the spin and charge current cannot penetrate the edges. The solutions of spin densities can be obtained by solving Eq. (17), (18) with the imposed boundary conditions. For *Rashba* SOI case, the spin densities $S_{x,z} = 0$ and $S_y = S_y^{b(0)}$ are analytically solved for both cases of the zero and finite in-plane magnetic field. For *Dresselhaus* SOI case, the spin density has form of $S_i = \sum_j A_{ij}e^{i\lambda_j y}$ for indices $j = 1 \sim 6$, $i = x, y$, and z. One can solve A_{ij} and λ_j by using the Eq.(19) and boundary conditions.

III. THE SPIN DENSITIES OF A 2D STRIP WITH IN-PLANE MAGNETIC FIELD

In this section, the spin accumulation behavior has been investigated for the case of the 2D strip in the presence of an in-plane magnetic. The driving electric field $E\hat{x}$ is applied along the x axis of the 2D strip and the transverse direction is in the y axis with boundaries at $y = \pm d/2$. For the *Rashba* SOI, there is no spin accumulations in the diffusion region without the in-plane magnetic field on the 2D strip[43]. The diffusion equation and boundary conditions of the spin density on a 2D strip with the in-plane magnetic field have been calculated in sec. II. According to Eqs. (18) and (28), it is easily to obtain zero spin accumulation corresponding to the case of *Rashba* SOI with an in-plane magnetic field. However, for the case of *Dresselhaus* SOI on the 2D strip, the spin-Hall current I_y^i survives after averaging over all impurities without the external magnetic field. Furthermore, it is remarkable the behavior of the spin accumulation due to the in-plane magnetic. As following, we will focus on the case of *Dresselhaus* SOI because there is no accumulation for the case of *Rashba* SOI with either zero magnetic field or an in-plane magnetic field. It is known that the spin density S_z is exhibited the anti-symmetric behavior but S_x is exhibited the symmetric behavior in the case of $\mathbf{B}_{\|} = 0$. For $\mathbf{B}_{\|} \neq 0$ case, the symmetric properties of spin accumulations are determined by not only the magnitude but also the direction of $\mathbf{B}_{\parallel}$. The hard-wall boundary conditions requires spin-Hall current $I_y^i = 0$ at the boundaries. The nonzero spin-Hall currents are compensated by the spin density accumulations near the boundaries to achieving $I_y^i = 0$ in *Dresselhaus* SOI case. Below, we studied the spin density accumulations for several different values of $\mathbf{B}_{\parallel}$ on a 2D strip and the spin density accumulations at a fixed boundary $y = -d/2$ with scanning inplane magnetic field from $-B_i$ to B_i on one-boundary case.

In our numerical result, the effective mass of GaAs is $0.067m_0$ and m_0 is the free electron mass. The electric field $E = 25mV/\mu m$ along x axis[32]. It is convenient to define a unit electron density $n_0 = 10^{15}(1/m^2)$ such that the units of the Fermi wave vector and the Fermi velocity are $k_{F0} = \sqrt{2\pi n_0} = 7.92 \times 10^7$ $(1/m)$ and v_{F0} =1.36 $\times$ 10^5 (m/s), respectively. The typical mean free path is $l_{mean} = 1\ \mu m$ (actually, l_{mean} becomes smaller in more disorder system) so that the unit scattering time $\tau_0 = 7.3 \times 10^{-12}(s)$ is given by $l_{mean} = v_{F0}\tau_0$. The *Dresselhaus* SOI constant is β = 27.5(eV Å^3) for GaAs[21] and the DP relaxation energy is given by $\Gamma^{xx} = 0.0042(C^4 - C^2/2 + 1/8)$ (meV). The unit of quantum well thickness is $\mathrm{w}_0 = 1 \times 10^{-8}$ (m) such that the nth subband energy are $\varepsilon_z^n = \frac{\hbar^2}{2m^*}(n\pi/\mathrm{w}_0)^2$. The effective g-factor of GaAs is $g^* = 0.44$ and the unit thickness of GaAs quantum well is w_0 = 100Å. The magnetic field energy $\widetilde{B}_{\parallel}$ is equal to 0.013 (meV) corresponding to $B_{\parallel} = 1\ Tesla$. The electron density is $n = n^* n_0$ and quantum well thickness is $w = w^* w_0$, where n^*, w* are dimensionless numbers. The Fermi wave vector is $k_F = k_{F0}\sqrt{n^*}$ and the parameter denotes $C = C_0/\sqrt{X}$, where $X = nw^2$ and $C_0 = \kappa_0/k_{F0}$. Therefore, the spin relaxation length becomes $l_{so} = \frac{\hbar^2}{m^*\beta k_{F0}^2}[\frac{X}{w^{*2}}(2C_0^2\frac{1}{X} - \frac{1}{2})]^{-1}$ and $l_{so} \gg l_{mean}$ is required in the diffusive regime. On the other hand, the electron energy is required to be lower than the second subband of a hard-wall confinement in z direction corresponding to $k_F^2 + (\pi/w)^2 < (2\pi/w)^2$.

The Fig. 2 (a)-(c) show that spin densities S_i as a function of y for B_x = -300 (black-triangular), 0 (blue-solid) and 300 (red-dashed) (mT), respectively. We choose the parameter $X = 22$ corresponding to the thickness of GaAs quantum well w = 300Å and the electron density $n = 2.4 \times 10^{15}(1/m^2)$. The spin relaxation length is $l_{so} = 2.9\mu m$. When the longitudinal magnetic field B_x is applied, the spin densities can be calculated from Eq. (21) with boundary conditions $I_y^i(y = \pm d/2) = 0$. According to Eq. (21) with $\mathbf{B}_{\parallel} = B_x$, it is easily to obtain that S_x is even parity in y and S_y, S_z are odd parity in y corresponding to Fig. 2(a), (b), and (c), respectively. It is worth to note that S_y is absence for $B = 0$ and become odd parity in y by applying B_x. As such, S_z is flipped by B_x leading the variation of S_y in Fig. 2 (b) for the $B_x = \pm 300mT$. The bulk spin densities S_x^b is finite and $S_y^b = S_z^b = 0$ satisfying Eq. (22). The corresponding bulk spin densities are demonstrated far away from edges

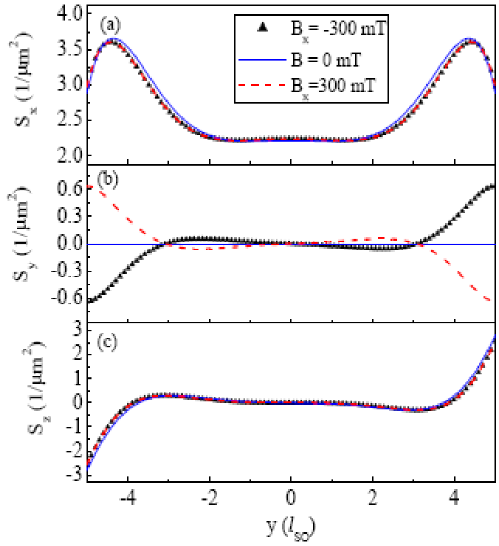

FIG. 2: Spin densities S_i are plotted as a function of y in unit of l_{so} corresponding to $X = 22$, where w = 300Å and $n = 2.4 \times 10^{15}(1/m^2)$. Each panel shows the different curves with parameters $B_x = -300mT$ (black-triangular), $\Lambda_x = 0$ (blue-solid) and $B_x = 300mT$ (red-dashed). Spin densities S_x, S_y and S_z in the unit of $1/\mu m^2$ are shown in Fig. 2 (a), (b) and (c), respectively.

$y = \pm d/2$. When the magnetic field direction is reversed $(B_x \to -B_x)$, Eq. (21) reveal the parity in B_x for a fixed y. It is found that S_x, S_z are the even parity of B_x and S_y is odd parity in B_x. Accordingly, spin densities of the $\pm B_x$ coincide with each other for both cases of S_x and S_z in Fig. 2 (a) and (c).

When $\mathbf{B}_{\parallel} = B_y$ is applied, the spin densities S_i exhibit asymmetric behaviors in y. Immediately, $S_y = 0$ is straightforwardly obtained from the second equation of Eq. (21). From Eq. (21), both of driving terms C_1 and $B_y C_2/\hbar$ exist such that S_x and S_z exhibit asymmetric properties for finite B_y in Fig. 3 (a) and (b), respectively. The black-triangular, blue-solid and red-dashed curves are referred to $B_y = -300mT$, $B_y = 0mT$ and $B_y = 300mT$, respectively. By applying B_y, our calculation points out that $S_z(y)$ is asymmetric for intrinsic SHE. Recently, extrinsic SHE experiments show the symmetric behavior of S_z versus B_y[32,34]. It is important that S_z can be a diagnostic tool to identify the origin of SOI in the sample by applying B_y.

We also study spin accumulation $S_i^{\pm}$ at $y = \pm d/2$ by varying B_x in Fig. 4(a)-(b) and B_y in Fig. 4(c)-(d). The edge spin densities $S_i^{\pm}$ are plotted in Figs. 4 for $i = x$, y and z corresponding to dashed, dash-dotted, and solid curves, respectively. In the case of $\mathbf{B}_{\parallel} = B_x$ by changing $y \to -y$, it can be found out the $S_y^{\pm}$ and $S_z^{\pm}$ are odd parity in y in Eq. (21). Therefore, $S_x^{\pm}$ have to satisfy the even parity such that three equations become consistent in Eq. (21) corresponding to boundary conditions. For a

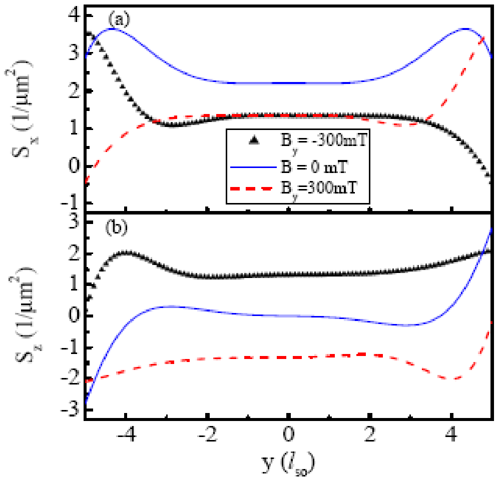

FIG. 3: Spin densities are plotted as a function of y in unit of l_{so} corresponding to $X = 22$, where $w = 300\text{Å}$ and $n = 2.4 \times 10^{15}(1/m^2)$. Each panel shows the different curves with parameters $B_y = -300mT$ (black-triangular), $B_y = 0mT$ (blue-solid) and $B_y = 300mT$ (red-dashed). Spin densities S_x and S_z in unit of $1/\mu m^2$ are shown in Fig. 3 (a) and (b), respectively. The S_y is zero in these cases.

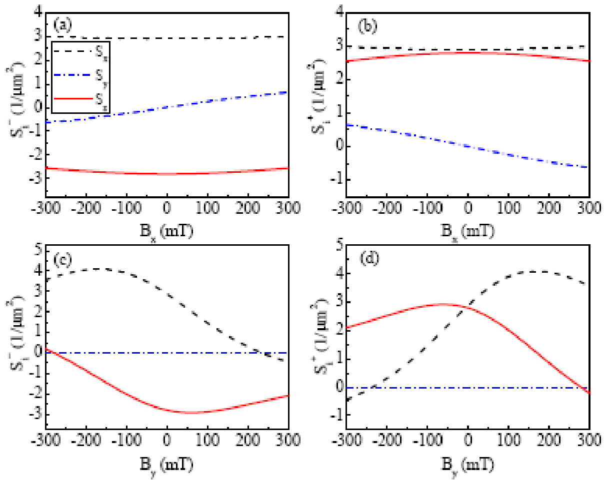

FIG. 4: The spin densities $S_i^{\pm}$ is plotted as a function of in-plane magnetic field B_x and B_y for i=x, y, and z. The spin densities $S_i^{\pm}$ at the right (left) edge $y = -d/2$ $(d/2)$ is labelled by $-$ $(+)$ in the upper index.

fixed B_x, these parities obey the $S^+_{y(z)}(y) = -S^-_{y(z)}(-y)$ and $S^+_x(y) = S^-_x(-y)$ and are shown in Fig. 4 (a) and (b). For $B_x \to -B_x$, Eq. (21) also shows the characteristics of $S^{\pm}_{x(z)}(B_x) = S^{\pm}_{x(z)}(-B_x)$ and $S^{\pm}_y(B_x) = -S^{\pm}_y(-B_x)$ in Fig. 4 (a) and (b). For $\mathbf{B}_{||} = B_y$ by changing $y \to -y$, the $S^{\pm}_y = 0$ is easily to calculated from the second equation of Eq. (21). In this case, $S^{\pm}_{x(z)}$ show asymmetric behaviors

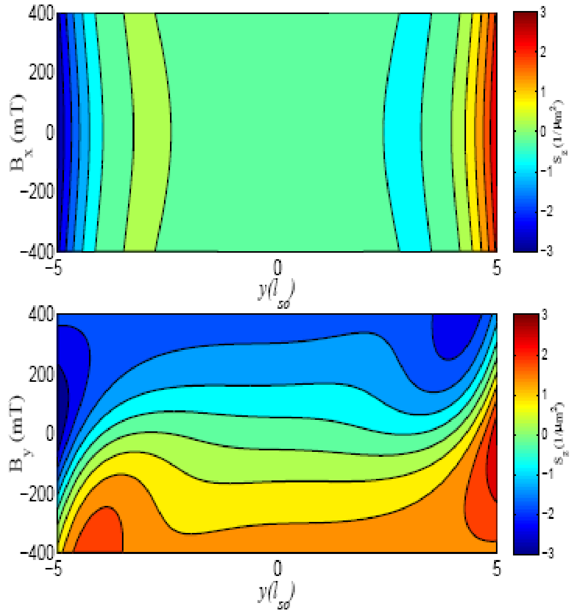

FIG. 5: The contour feature of the spin density S_z is plotted as a function of y versus the (a) longitudinal magnetic field B_x and (b) transverse magnetic field B_y.

for $B_y \to -B_y$ in Fig. 4 (c) and (d). If the B_y and coordinate y are reversed at the same time, it is found out that $S_z(B_y, y) = -S_z(-B_y, -y)$ and $S_x(B_y, y) = S_x(-B_y, -y)$ are agreed with Eq. (21).

More clearly, Fig. 5 (a) and (b) present the contour plot of spin densities S_z versus y with varying B_x and B_y, respectively. In Fig. 5 (a), spin S_z demonstrate the anti-symmetric accumulations in transverse coordinate y by varying B_x from $400mT$ to $-400mT$. It shows that the spin density S_z is odd function in y by varying B_x. However, the Fig. 5 (b) reveals the asymmetric behavior of S_z in transverse coordinate y by varying B_y. For intrinsic SHE, the bulk solution of S^b_z is proportional to linear B_y leading to this asymmetry characteristic.

IV. CONCLUSION

In conclusion, we have studied the spatial distribution of the spin density S_i in the presence of an in-plane magnetic field for either *Rashba* or *Dresselhaus* SOI cases. In the weak magnetic field limit, the diffusion equation is proportional to linear magnetic field. For *Rashba* SOI case, the in-plane magnetic field doesn't affect the spatial distribution of the spin density in space. For *Dresselhau* SOI case, the spatial distribution of spin density shows symmetric or asymmetric properties depending on the direction of the in-plane magnetic field. These results lead to a diagnostic tool for the identification of the origin of SOI in the system.

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Appendix B:

Spin-Hall effects in a Josephson contact

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The Josephson tunneling through a 2D normal contact with the spin-orbit split conduction band has been studied in the diffusive regime. Linearized Usadel equations for triplet components of the pairing function revealed a striking similarity to the equations of spin diffusion driven by the electric field in normal metals. Consequently, we predict that the out-of-plane spin-Hall polarization accumulates towards lateral sample edges and the in-plane polarization is finite throughout the entire normal region. At the same time, the spin-Hall current is absent in the considered case of the stationary Josephson effect.

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In connection with various spintronic applications, much interest have been attracted recently to spin-orbit interaction (SOI) effects on electron transport in normal metals and semiconductors. This interaction gives rise to fundamental transport phenomena, such as the spin-Hall effect (SHE) (for a review see [1]) and electric spin orientation [1, 2]. These effects represent a direct manifestation of the spin-orbit coupling between spin and charge degrees of freedom in electron transport. At the same time, spin-orbit effects were also discussed for superconductors. Some works dealt with SFS junctions [3] (F stands for ferromagnet), others considered SNS [4], SN [6] systems, or bulk superconductors [5, 6]. As was pointed out in Ref. [5, 6], SOI leads to admixture of triplet components to the pairing function. This sort of singlet-triplet coupling looks similar to the spin-charge coupling in normal systems. Therefore, one would expect that phenomena closely related to SHE could manifest themselves in superconductors. At the first sight on this problem it becomes clear that, at least in the case of zero voltage across the junction, the spin-Hall current can not be generated as a linear response to the superconducting current. The reason is that these currents have opposite parities with respect to the time inversion, while they must be equal in the stationary nondissipative superconducting transport. On the other hand, besides the spin currents, in normal systems SHE leads to spin accumulation near sample edges. Therefore, it is interesting to find out, if similar accumulation of magnetization takes place in superconducting systems. It should be noted that, despite formal similarities, such a magnetization is fundamentally distinct from that induced by the normal SHE, since it is not subject to the energy dissipation accompanying spin diffusion and relaxation in normal systems.

We will consider SHE and the electric spin orientation for a Josephson tunneling through a 2D normal contact (see Fig 1). The SOI there is represented by the Hamiltonian $H_{so} = \boldsymbol{\sigma} \cdot \mathbf{h}_{\boldsymbol{k}}$, where $\boldsymbol{\sigma}$ is a vector consisting of Pauli matrices. The spin-orbit field $\mathbf{h}_{\boldsymbol{k}}$, which is a function of the electron wave vector k, can be given, for example, by Rashba [7], or Dresselhaus [8] SOI, as well as by their combination. In this case the vector $\mathbf{h}_{\boldsymbol{k}}$ lies in the plane of the 2D system. The electron transport through the contact will be treated within the diffusion approximation, so that the length of the junction L, the electron coherence length L_c and the spin precession length $L_{so} = v_F/h$, where v_F is the Fermi velocity and h is the angular averaged spin orbit field, are assumed to be much larger than the electron mean free path l. The electric voltage across the junction is set to zero. Hence, the supercurrent is provided by the phase difference between two electrodes. The analysis of such a problem will be performed within a standard semiclassical treatment of Gor'kov's equations in the diffusion approximation (for a review see [9]). Our goal is to derive linearized Usadel type equations and calculate the spin density induced by SHE.

As far as the thermal equilibrium state is considered, all observables of interest can be expressed via retarded and advanced Green functions. The corresponding Gor'kov's equations in the Nambu representation have the form

$$\left(i\frac{\partial}{\partial t} - \check{H} - \check{\Sigma}^{r/a}\right)\check{G}^{r/a}(X,X') = \delta(X - X'), \quad (1)$$

where r, a denote retarded or advanced functions, $X =$

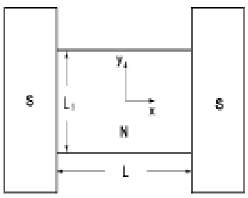

FIG. 1: Josephson contact with S and N denoting superconducting and normal regions.

t, r and

$$\check{H} = \frac{\tau_3}{2m^*}\hat{k}^2 - \tau_3\mu + \boldsymbol{\sigma}\cdot \mathrm{h}_{\hat{\mathrm{k}}}\,, \qquad (2)$$

with the momentum operator $\hat{\mathrm{k}} = -i\partial/\partial r$ and the chemical potential μ. After averaging of initial Green functions over random positions of short-range impurities, the self-energy in (1) takes the form [10]

$$\check{\Sigma}^{r/a}(X,X') = \frac{\tau_3}{2\tau\pi N_F}\check{G}^{r/a}(t,t',r,r)\tau_3\delta(r-r')\,, \quad (3)$$

where τ is the elastic scattering time. Unperturbed Green functions are easily obtained from Eq.(1). In the momentum representation and after the time Fourier transform they can be written as

$$\check{G}^{0r/a}(\omega,k) = (\omega - \tau_3 E_k - \boldsymbol{\sigma}\cdot \mathrm{h}_{\boldsymbol{k}} \pm i\Gamma)^{-1}\,, \qquad (4)$$

where $E_k = (k^2/2m^*) - \mu$. Below we will perform calculations for retarded functions and drop the labels r, a.

Proximity to superconducting contacts results in an admixture to the Green functions of anomalous (proportional to τ_1 and τ_2) components. Also, these functions become inhomogeneous in space. In order to calculate them, we will follow a well known procedure in the framework of the semiclassical approximation [11]. First, we perform the Fourier transform with respect to $X - X'$ introducing, accordingly, the frequency and wave vector variables, ω and k. The center of mass variables will be remained intact and denoted as r. Since the problem is stationary, the corresponding center of mass time variable is absent. Taking into account that variations of G in the scale of the Fermi wave-length are small, Eq. (1) should be expanded in terms of gradients $\partial/\partial r$. The next step is to simplify the self-energy part of Eq. (1) keeping there only terms linear in the anomalous part. Such a linearization can be done if the transparency of the SN contact is small, or the leads are close to the superconducting critical temperature. Taking the sum of Eq. (1) and its conjugate one, and making use of the fact that $\mathrm{h}_{\boldsymbol{k}}$ is an odd function of k, for the anomalous part G_{12} we obtain the equation

$$\begin{aligned}(2\omega - v\cdot\hat{\mathrm{q}} + \frac{i}{\tau})G_{12} \;&-\; \{\mathrm{h}_{\boldsymbol{k}}\cdot\boldsymbol{\sigma}, G_{12}\} - \\ \frac{1}{2}\left[\delta\mathrm{h}_{\boldsymbol{k},\hat{\mathrm{q}}}\cdot\boldsymbol{\sigma}, G_{12}\right] \;&=\; I_{sc}\,, \end{aligned}\qquad (5)$$

where $\delta\mathrm{h}_{\boldsymbol{k},\hat{\mathrm{q}}} = (\hat{\mathrm{q}}\cdot\nabla_k)\mathrm{h}_{\boldsymbol{k}}$ with $\hat{\mathrm{q}} = -i\partial/\partial\mathbf{r}$, and

$$I_{sc} = -\frac{1}{2\tau\pi N_F}\left(G^0_{11}g_{12} + g_{12}G^0_{22}\right)\,. \qquad (6)$$

The lower labels in G^0 denote the matrix elements in the Nambu space and $g_{12} = \sum_{\boldsymbol{k}} G_{12}$. Usually, the quantum kinetic equation, such as (5), can be reduced to the Eilenberger [12] equation by integration with respect to the electron energy. In our case this procedure is not convenient because of electron energy spin splitting. Instead, within the diffusion approximation, from Eq. (5) we will express G_{12} in terms of g_{12}, and taking its sum over k obtain the closed diffusion equation for g_{12}. Before doing this, we transform the 2×2 matrix $G_{12,\alpha\beta}$ to the conventional pairing function $F_{\alpha\overline{\beta}} \equiv G_{12,\alpha\beta}$, where $\overline{\beta}$ denotes the spin projection opposite to β. Further, it is convenient to decompose F into triplet F_1, F_{-1}, F_0 and singlet F_s components as

$$\begin{aligned} F_0 \;&=\; \frac{F_{12}+F_{21}}{\sqrt{2}}\,, \quad F_s = \frac{F_{12}-F_{21}}{\sqrt{2}} \\ F_1 \;&=\; F_{11}\,, \qquad F_{-1} = F_{22}. \end{aligned}\qquad (7)$$

The corresponding density function $f = \sum_{\boldsymbol{k}} F$ will also be represented in a similar way. After this transformation, it is easy to see that the last term in the l.h.s. of (5) is responsible for a coupling between the singlet and triplet components of the pairing function. Besides, the singlet-triplet coupling also originates from the spin dependent parts of G^0_{11} and G^0_{22} in Eq. (6). Due to such coupling, the triplet component of F is generated within the junction between two singlet superconductors.

For simplicity, when deriving the diffusion equation, let us assume that SOI is strong enough, so that $L_{so} \ll L_c$. Further, considering I_{sc} together with the last term in the l.h.s. of (5) as sources, we resolve Eq.(5) performing expansion in $(v\cdot\hat{\mathrm{q}})\tau$ and $\mathrm{h}_{\boldsymbol{k}}\tau$ up to the second order. Finally, we obtain the following diffusion equation for the triplet pairing function $f_m = (i/\pi N_F)\sum_{\boldsymbol{k}} F_m$, $(m = 0, 1, -1)$:

$$2i\omega f = \tau\langle\left(-iv\cdot\frac{\partial}{\partial r} + 2\mathrm{J}\cdot\mathrm{h}_{\boldsymbol{k}}\right)^2\rangle f + \mathrm{M}f_s\,, \qquad (8)$$

where J is the vector of 3×3 angular moment operators and $\langle ...\rangle$ denotes the angular averaging over the Fermi surface. The triplet-singlet coupling is given by

$$\mathrm{M}_0 = 0\,,\mathrm{M}_{\pm 1} = \frac{4\tau^2}{\sqrt{2}}\langle \mathrm{h}^{\mp}_{\boldsymbol{k}}(\mathrm{h}_{\boldsymbol{k}}\times\delta\mathrm{h}_{\boldsymbol{k},\hat{\mathrm{q}}})\rangle f_s\,, \qquad (9)$$

with $\mathrm{h}^{\mp}_{\boldsymbol{k}} = \mathrm{h}^x_{\boldsymbol{k}} \mp i\mathrm{h}^y_{\boldsymbol{k}}$. The singlet f_s satisfies the usual Usadel equation [3] with an additional term which is Hermitian conjugate to M. Since this term is small, we will neglect a corresponding correction to f_s in Eq.(8). Hence, f_s is given by the well known unperturbed solution in the SNS contact. Since it varies within the scale $L_c \gg L_{so}$, we neglected all contributions to M with higher powers of gradients, as well as terms proportional to $\omega \sim D/L_c^2$, where D is the diffusion constant.

Without the last term in the r.h.s., Eq.(8) formally coincides with the spin diffusion equation for 2DEG in a zero electric field [13]. The spin diffusion equation in the presence of the electric field has been derived in Ref. [14] for the case of the Rashba SOI, and for a general SOI in

[15]. After a linear transformation [13] to spin density variables Eq.(8) will also coincide with these equations, if, apart from a constant factor, f_s is formally identified with the electric field potential. Hence, a coupling of the spin to the electric field in normal spin transport appears to be very similar to the singlet-triplet coupling in Eq.(8).

Let us consider an example of the Rashba SOI. In this case $h^x_{\mathbf{k}} = \alpha k_y$ and $h^y_{\mathbf{k}} = -\alpha k_x$. For a homogeneous in y-direction case all functions depend only on x and we get $f_0 = 0, f_1 = f_{-1}$, with f_1 satisfying the equation

$$D\frac{\partial^2}{\partial x^2} f_1 - \Gamma_{so} f_1 = i\frac{\alpha\tau\Gamma_{so}}{\sqrt{2}}\frac{\partial}{\partial x} f_s \,, \qquad (10)$$

where $\Gamma_{so} = 2\tau\alpha^2 k_F^2$ is the D'yakonov-Perel' spin relaxation time [16]. The small l.h.s. of Eq.(8) has been neglected in (10). Boundary conditions at $x = \pm L/2$ can be written in a way similar to a singlet SN interface [17]. At least in the linearized approximation the boundary conditions contain only characteristics of one-particle transmission. Therefore, they can be easily generalized to the case of a triplet pairing. Following calculations of Ref. [17] we obtain

$$(f_{1S} - g f_{1N})|_{x=\mp L/2} = \pm b\frac{\partial f_{1N}}{\partial x}|_{x=\mp L/2} \,, \qquad (11)$$

where the labels S and N denote superconductor and normal sides of SN contacts at $x = \pm L/2$, and $g = |\omega|/\sqrt{(\omega + i0^+)^2 - |\Delta|^2}$ is a DOS factor for a superconductor. The characteristic length b depends on the SN barrier transmittance. For our choice of parameters $b \gg L_{so}$. The same equation (11) takes place for f_s. At the low SN barrier transmission one may use the so called rigid boundary conditions and set $f_{1S} = 0$. At the same time, the singlet paring function $f_{sS}|_{\mp L/2} = g\Delta\exp(\pm i\phi)/\omega$. Neglecting the third derivative of f_s, the solution of Eq. (10) can be written as

$$f_1 = -i\frac{\alpha\tau}{\sqrt{2}}\frac{\partial}{\partial x} f_s + \psi(x) \,, \qquad (12)$$

where $\psi(x)$ is a linear combination of $\exp(\pm kx)$, with $k = \sqrt{D/\Gamma} \equiv 1/L_{so}$. It is easy to see from (11) that at $kb \gg 1$ the first term dominates in Eq.(12). Therefore, ψ will be neglected below.

Our next step is to calculate the spin polarization density associated with triplet components of the pairing function. This polarization is given by

$$S^t(r) = \frac{i}{2}\sum_{\mathbf{k}}\int\frac{d\omega}{2\pi} n_F(\omega) \times Tr[\sigma^t(G^r_{\mathbf{k}11}(\omega, r) - G^a_{\mathbf{k}11}(\omega, r)] \,, \qquad (13)$$

where n_F is the equilibrium Fermi distribution function. It is easy to see that the nonzero value of Eq. (13) is provided by triplet components of anomalous Green functions which contribute to G_{11} with a correction term $\propto f^2$. Up to the leading second order with respect to f_s and keeping only the linear terms of the triplet f_m $(m = 1, -1, 0)$, for the retarded function we obtain from Eqs. (1-4)

$$\sum_{\mathbf{k}} Tr[\sigma^t G^{r/a}_{\mathbf{k}11}] = \frac{\mp 1}{\pi N_F}[\frac{i\delta^{tz}}{2}(f_0^{r/a} f_s^{+r/a} - f_s^{r/a} f_0^{+r/a}) + \frac{1}{\sqrt{2}}(f_t^{r/a} f_s^{+r/a} + f_s^{r/a} f_t^{+r/a})] \,, \qquad (14)$$

where $f_y = (f_1 + f_{-1})/2$ and $f_x = -i(f_1 - f_{-1})/2$. The conjugate functions $f^+(\omega) = -f^*(-\omega)$.

In the case of Rashba SOI $f_x = f_0 = 0$ and $f_y = f_1$. The latter is given by Eq. (12). Then, from (14) it immediately follows that only the y-projection of the spin density is finite. Using the relations $f^a_s(\omega) = f^r_s(-\omega)$ and $f^a_m(\omega) = -f^r_m(-\omega)$ $(m = 1, -1, 0)$, we arrive to the spin polarization

$$S^y(x) = eN_F\alpha\tau\frac{J(x)}{\sigma_{dc}} \,, \qquad (15)$$

where σ_{dc} is the dc conductivity of the normal metal and J is the Josephson current density

$$J = \frac{eD}{4\pi^2 N_F}\int d\omega n_F(\omega)[(f^r_s\frac{\partial f^{+r}_s}{\partial x} - \frac{\partial f^r_s}{\partial x} f^{+r}_s) - (r \rightleftarrows a)] \,. \qquad (16)$$

The spin polarization (15) coincides with polarization induced in normal metals by the electric field E [2], if the Josephson current is substituted for the normal dissipative dc current $J_{dc} = \sigma_{dc}E$. It is easy to check that this analogy takes place also for the Dresselhaus SOI, with a little more complicated expression for $S^t(x)$ [15]. An important distinction from the electric spin orientation in normal metals is that due to the charge neutrality, $J_{dc} = const$ in the x direction, while the supercurrent varies inside the contact. Similar effect has been predicted by Edelstein [6] for bulk superconductors and at NS boundary, providing the supercurrent flows along the SN interface.

Let us now check, if the analogy with the electric spin orientation extends to the spin-Hall effect. Hence, our goal is to calculate J^z_y, which is the y projection of a spin flux polarized in the z-direction. The corresponding spin current operator can be written as $J^z_y = \{\sigma_z, v_y\}/2$, where the velocity $v_y = k_y/m^* + \partial(\sigma \cdot h_{\mathbf{k}})/\partial k_y$. Since it has been assumed that $h_z = 0$, one gets $J^z_y = \sigma_z k_y/m^*$. The spin-Hall current J_{sH}, in its turn, can be derived from Eq.(13), with σ^t substituted for J^z_y. Keeping the same leading terms as in calculation of the spin density, we arrive to $J_{sH} = 0$. This result does not depend on whether $h_{\mathbf{k}}$ is given by the Rashba or Dresselhaus interactions. That is very distinct from the normal spin-Hall effect, where in the diffusive regime the spin-Hall conductance is zero for the Rashba SOI, but finite for the cubic

Dresselhaus interaction [18]. In general, as it was discussed above, the zero value of J_{sH} in superconducting transport follows from the time inversion symmetry.

Besides J_{sH}, in normal systems the DC current together with SOI gives rise to accumulation of the z-component of spin at the lateral edges of the sample [15, 19, 20]. In the case of the Josephson junction the z-projection of the spin density is given by Eq.(13) and the first term in Eq. (14). Hence, it is proportional to the f_0 component of the pairing function which, in its turn, can be found from Eq.(8). For simplicity, let us consider hard wall boundaries of 2DEG at $y = \pm L_y/2$. In this case one can borrow the boundary conditions for Eq. (8) from Ref. [15, 20]. In normal systems these conditions correspond to the vanishing spin current at $y = \pm L_y/2$. In our case similar equations can be written for triplet "currents" $j = \sum_k vF$. We thus have $j^y|_{y=\pm L_y/2} = 0$, where the 0 triplet component is given by

$$j_0^y = -D\frac{\partial f_0}{\partial y} - 2i\tau\langle(v_y[\mathbf{h}_{\mathbf{k}} \times (\sqrt{2}f + \tau\delta\mathbf{h}_{\mathbf{k},\hat{\mathbf{q}}}f_s)]\rangle . \quad (17)$$

The first term in this equation is the diffusive current, the first term in the brackets is determined by the spin precession in the effective spin-orbit field, and the last term looks as the spin-Hall current in the normal spin transport. As far as f_s is treated a slowly varying function of x, thus allowing one to ignore its higher gradients together with edge terms like ψ in (12), the analysis of Eq. (8) with the above boundary conditions is the same, as for SHE in normal systems. Henceforth, following Ref.[15, 20] one may conclude that $f_0 = 0$ for Rashba SOI, but f_0 is finite in the case of the cubic Dresselhaus interaction. From Eqs.(13) and (14) it is immediately seen that in the former case $S^z = 0$. For the Dresselhaus SOI the solution of Eq. (8) has the form $f_0 = \chi(y)(\partial f_s/\partial x)$, where χ is a real odd function of y. Then, Eqs.(13),(14) and (16) give $S^z = -eN_F\chi(y)(J/\sigma_{dc})$. The function χ, in its turn, have been calculated in Ref. [15].

In conclusion, the spin-Hall effect induced by a supercurrent across an SNS junction has been studied in the diffusive regime for a relatively strong ($L_{so} \gg L_c$) SOI in the 2D junction and for low conducting SN barriers. We found out that, although the spin-Hall current is forbidden by the time inversion symmetry, in the case of cubic Dresselhaus SOI the out-of-plane magnetization is accumulated near sample edges at $y = \pm L_y/2$, in a very close analogy to SHE in normal systems. Also, similar to the electric spin orientation, the spin polarization parallel to 2DEG is finite throughout the entire N-region.

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