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操控半導體電子自旋的動力行為與傳輸的研究

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行政院國家科學委員會專題研究計劃成果報告

目錄

1. 中、英文摘要及關鍵詞

（一）計劃中文摘要

這份研究計畫主要有下列三個目標:

[I] 對自旋霍爾效應(spin-Hall)中自旋堆疊(spin accumulation)的物理取得更透徹的瞭解，並全面的瞭解到自旋堆疊與同時結合內秉的(intrinsic)與非內秉的(extrinsic)自旋軌道交互作用(SOI: spin-orbit interaction)之間的相互對應關係:

內秉的自旋軌道交互作用包括 Rashba 和 Dresselhaus 兩項,而非內秉的 SOI 主要係由 SOI 的散射效應而來,其散射體如一些重雜質的存在即是。其中關於不同 SOIs 之間如何相互作用影響的物理圖像更是大家特別感興趣與關注的。其他一併加進來研究考慮的相關影響因子有如:二維電子氣(2DEG)的有限厚度效應(finite-thickness effect)，晶格的方向性，和局部散射粒子附近的自旋堆疊效應等。我們希望能透過這些一一的瞭解而藉以再給予直流(dc)電場的環境影響下，能找出”最適化”的條件以達到最大的自旋堆疊效果。

[II] 對直流(dc)磁場和交流(ac)電場與自旋堆疊的現象、機制與影響取得透徹而全面的瞭解:

垂直於的或平行於(in-plane)樣品方向的直流(dc)磁場都是研究的重點。其中垂直於樣品方向的直流磁場乃作為提供操控系統中自旋對稱(spin-symmetry)的額外自由度以獲致最大自旋堆疊的可能。而垂直於樣品方向的直流磁場，如調制在弱場範圍的影響下的迴旋運動情形；與如在強場範圍的影響下產生的 Landau-quantization 效應，則可來用以做為另一種調制觀察自旋堆疊效果的手段。除此以外，於推廣到有線頻率範圍時，自旋動力學中的同調非彈性碰撞過程便自然需要再加以引入進來計算裡。未了能以一種似古典的途徑來加以描述這些效應和過程，我們也將從 Keldysh-Green 函數出發，來推廣目前自旋擴散方程，讓其進一步能兼顧到同時含有 ac 以及 dc 外場的作用。藉此也同時來了解這些效應對於塊材極化與邊緣極化情形之影響。

[III] 探討透過純粹電性的方法來偵測傳輸自旋的現象:

透過利用 SOI 效應，橫向的荷電流自然也可以由自旋電流中加以產生。這也正是所謂的逆向自旋霍爾效應的結果。我們循此觀念將考慮在不同的系統配置中的自旋電流注射的情形:從而分析計算由擴散機制主導範圍及彈道機制主導範圍裡所可能衍生的各種對應的荷電流表現的情形。這些結構中將包含一條開放性的導線，和一條與它相交的導線。外在場的作用影響亦在此放進來考慮。而關於在彈道機制主導的系統中其自旋電流注射的實現則可由我們先前的研究工作中的方法:以交流相變的手指閘極所給出。對於擴散機制所主導的作用範圍裡，自旋擴散方程中的 SOI-coupling 參數自然也隨之需要擴散到兼顧時變性和空間變動性。對此於荷電流，荷電堆疊，與自旋電流的相關性將被更深入完整地探討。這些瞭解都將可用於設計發展以純粹電性的方法來達到自旋電流偵測的目標。

（二）計劃英文摘要

There are three major goals in this study:

To understand in a comprehensive way the relation of the spin accumulation in the spin-Hall configuration to the combined effects of the intrinsic and the extrinsic spin-orbit-interactions (SOI):

Intrinsic SOI includes both the Rashba and the Dresselhaus SOIs, and the extrinsic SOI is resulted from the SOI scatterers such as the heavy impurities. Of particular interest is to obtain physical pictures for the interplay between the various SOIs. Other factors such as the finite thickness effect of the 2DEG, the crystal orientation, and particular spin accumulation around local scatterers will be studied. With these understanding, we are hoping to propose some simple optimal situation for the production of spin accumulation from a given driving dc electric field.

To understand in a comprehensive way the effects of dc magnetic fields and ac electric field on the spin accumulations:

dc magnetic fields, either perpendicular or parallel to the sample, will be incorporated into our study. The in plane magnetic field will introduce additional handle for monitoring the spin symmetry in the system and thus allows an extra degree of freedom to better achieve maximal spin accumulation. The perpendicular magnetic field will introduce cyclotron motion, in the weak field regime, and the Landau quantization, in the strong field regime, for another possible way of tuning the spin accumulation. Furthermore, extending our calculation to the finite frequency regime, which we will study first the case of ac electric field, may introduce additional coherent inelastic processes to the spin dynamics. To describe these processes in a "classical"-like approach, we will extend our derivation of the spin diffusion equation to incorporate both the ac and the dc external fields, starting from the Keldysh Green's function. We will look for both the bulk and the edge polarizations.

To explore pure electrical means of probing the transport of spins:

Making use of the SOI, a spin current can generate charge current in the transverse direction. This is the so-called inverse spin-Hall effect. We will consider various system configurations, which include both the diffusive and the ballistic regimes, into which a spin current is injected and calculate the resulting charge current. These structures include a wire, a wire with an opening, and a cross-wire situation. External field effects will also be taken into consideration. The spin current injection can be generated from an ac biased finger-gate proposed in our recent work for the case of the ballistic regime. For the diffusive regime, we need to extend our spin diffusion equation to the situation when the SOI coupling parameter varies spatially and temporally. The correlation of the resulting charge current, or charge accumulation, with the injected spin current will be studied. These understanding will form a basic for the devising of spin current detection by pure electrical measurements.

（三）關鍵詞、Keywords

自旋霍爾效應、自旋堆疊、非內秉、內秉、自旋軌道交互作用、Rashba、Dresselhaus、二維電子氣、直流磁場、自旋對稱、Keldysh-Green 函數

Spin-Hall effect, spin accumulation, extrinsic, intrinsic, spin-orbit interaction, Rashba, Dresselhaus, 2 dimensional electron gas, dc magnetic field, spin symmetry, Keldysh-Green's function.

2. 報告內容

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Spin generation in a Rashba-type diffusive electron system by nonuniform driving field

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We show that the Rashba spin-orbit interaction contributes to edge spin accumulation S_z in a diffusive regime when the driving field is nonuniform. Specifically, we solve the case of nonuniform driving field in the vicinity of a circular void locating in a two-dimensional electron system and we identify the key physical process leading to the edge spin accumulation. The void has radius R_0 in the range of spin-relaxation length l_{so} and is far from both source and drain electrodes. The key physical process we find is originated from the nonuniform in-plane spin polarizations. Their subsequent diffusive contribution to spin current provides the impetus for the edge spin accumulation S_z at the void boundary. The edge spin accumulation is proportional to the Rashba coupling constant α and is in a spin-dipole form oriented transversely to the driving field. We expect similar spin accumulation to occur if the void is at the sample edge.

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I. INTRODUCTION

A major goal for the semiconductor spintronics is to generate and to manipulate spin polarization by mere electrical means. Spin-orbit interaction (SOI) provides the key leverage and spin-Hall effect (SHE) (Refs. 1–15) provides the key paradigm, where it is possible for a uniform driving electric field to induce bulk spin polarization and spin current and, in turn, out-of-plane spin accumulations S_z at lateral edges. The Rashba SOI (RSOI) (Ref. 16) is of particular interest because of its gate-tuning capability. However, background scatterers lead to a complete quenching of the RSOI's contribution to the edge spin accumulation S_z, a direct consequence of its linear dependence on the electron momentum k.[17,18] It is legitimate then to find ways to restore the RSOI's contribution to the edge spin accumulation. Our interest here is in the diffusive regime, when the spin-relaxation length $l_{so} \gg l_e$, the mean-free path. Even though the spin accumulation is finite in the mesoscopic ballistic regime ($l_{so} < l_e$, and $L < l_\phi$),[19–21] with L the sample size and l_ϕ the phase coherent length, it is still important to see whether the RSOI alone can contribute to SHE in the impurity-dominate regime.

Indeed, RSOI was found by Mishchenko *et al.*[18] to give rise to edge spin accumulation S_z near electrodes even though its contribution to bulk spin current vanishes. The edge spin accumulation is concentrated at the two ends of an electrode-sample interface, covering a region of size l_{so}. This finding was identified to arise from a nonzero spin current I_y^z flowing along the sample-electrode interface, in direction $\hat{y}$.[18] This nonzero spin current was understood from the way the spin current vanishes in the bulk, when an exact cancellation occurs between two terms, one related to the spin polarization and the other related to the driving field.[18] This exact cancellation no longer holds at the sample-electrode interface, when the driving field has reached its bulk value but the spin polarization has not. Similar result was also obtained by Raimondi *et al.*,[22] where spin-density spatial profiles at the sample corners were obtained. Yet, it would be more desirable that we can find schemes and identify physical processes for the restoring of the RSOI-induced edge spin accumulation at locations other than the sample-electrode interfaces and according to our specification.

In this work, we turn to nonuniform driving field for the restoration of the RSOI-induced spin accumulation. The effect of nonuniform driving field on spin accumulation is also interesting in its own right. Earlier study considered nonuniform driving field in systems in the presence of "extrinsic" SOI, that is, SOI due to SOI impurities.[23] Here, instead, we consider nonuniform driving field in the vicinity of a circular void located in a diffusive RSOI-type two-dimensional electron gas (2DEG). We obtain spin accumulation in the vicinity of the void. This problem allows us to identify the key physical process for the spin accumulation and also sheds light on the case if the void were to form at a lateral edge. The radius R_0 of the void is of the order of l_{so}.

Most important is our finding that the main physical process is in marked contrast to the conventional one. While the conventional one is associated with the nonvanishing of the out-of-plane spin current I_n^z,[18] the key process we find is associated with the in-plane spin currents, I_n^x or I_n^y, and with the way they vanishes at the void boundary. Here, $\hat{n}$ denotes the flow direction normal to the void boundary. The in-plane spin currents consist of two terms, a diffusive term and a term related to the spin accumulation S_z, which are given by

$$I_\rho^i = -2D\frac{\partial}{\partial \rho}S_i - R^{izi}S_z\hat{\rho}\cdot\hat{i}, \qquad (1)$$

where $\boldsymbol{\rho}$ is the position vector measured from the center of the void, $i \in \{x, y\}$, and D is the diffusion constant.[24] R^{ijk} denotes the precession of S_j into S_i when it flows along $\hat{k}$. The factor $\hat{\rho}\cdot\hat{i}$ denotes the projection of the flow. The RSOI governs the symmetry of R^{ijk} such that $R^{izj}=0$ for $i \neq j$.

Our scheme of S_z generation is made possible by Eq. (1), the boundary condition $I^i_\rho=0$, at $\rho=R_0$, and the presence of a radially nonuniform in-plane spin polarization S_i. The spin polarization $\boldsymbol{S}_\parallel=\boldsymbol{S}^{Ed}_\parallel+\Delta\boldsymbol{S}_\parallel$ we obtain in this work has

$$\boldsymbol{S}^{Ed}_\parallel=-N_0\alpha\tau e/\hbar\hat{z}\times\boldsymbol{E}(\boldsymbol{\rho}), \tag{2}$$

which radial dependence is acquired from the driving field $\boldsymbol{E}$. Outside the void, $\boldsymbol{E}(\boldsymbol{\rho})=E_0\hat{x}-E_0(R_0/\rho)^2(\cos 2\phi\hat{x}+\sin 2\phi\hat{y})$ where $E_0\hat{x}$ is the uniform field far away from the void and $\boldsymbol{\rho}\cdot\hat{x}=\cos\phi$. The driving electric field $\boldsymbol{E}=-\nabla\varphi(\boldsymbol{\rho})=\sigma_0\boldsymbol{j}$ has to satisfy the steady state condition $\nabla\cdot\boldsymbol{j}=0$ and the boundary condition $j_\rho=0$. Here σ_0 is the electric conductivity, $\boldsymbol{j}$ is the electric current density, N_0 is the energy density per spin, α is the RSOI coupling constant, $e>0$, and τ is the mean-free time. The term $\boldsymbol{S}^{Ed}_\parallel$ is an Edelstein-like spin polarization[3] which we have obtained for the case of nonuniform driving fields. That this term of $\boldsymbol{S}_\parallel$ alone fails to satisfy the boundary condition Eq. (1), because of its radial dependence, has prompted the generation of $\Delta\boldsymbol{S}_\parallel$ and $\boldsymbol{S}_z$.

The aforementioned key physical process associated with the spin current is meant to establish a boundary condition for the spin-diffusion equation. We have used the conventional form of the spin current operator $J^i_l=(1/4)(V_l\sigma_i+\sigma_iV_l)$, where spin unit of $\hbar$ is implied, and the kinetic velocity operator $V_l=(1/i\hbar)[\hat{x}_l,H]$. This is appropriate for hard wall boundary.[24–26] As the boundary condition is applied to a region much shorter in distance than l_{so} from the boundary, the effect of spin torque[27,28] here should be of secondary importance. In Sec. II we present the spin-diffusion equation for nonuniform driving fields, and the analytical solutions for spin densities around a circular void. In Sec. III we present our numerical results and discussion. Finally, in Sec. IV, we will present our conclusion.

II. THEORY

The derivation of the spin-diffusion equation (SDE) by the Keldysh nonequilibrium Green's function method[13,24] is extended to the case when the driving field is nonuniform. With the RSOI Hamiltonian $\mathbf{H}_{so}=\mathbf{h_k}\cdot\boldsymbol{\sigma}$ and $\mathbf{h}_k=-\alpha\hat{z}\times\mathbf{k}$, where $\boldsymbol{\sigma}$, and $\mathbf{h_k}$ are, respectively, the Pauli's matrix vector and the SOI-effective magnetic field, the SDE is given by

$$D\nabla^2S_x-\frac{\Gamma^{xx}}{\hbar^2}S_x+\frac{R^{xzx}}{\hbar}\frac{\partial}{\partial x}S_z-\frac{\boldsymbol{M}^{x0}\cdot\boldsymbol{\nabla}}{2\hbar^3}D^0_0=0,$$

$$D\nabla^2S_y-\frac{\Gamma^{yy}}{\hbar^2}S_y+\frac{R^{yzy}}{\hbar}\frac{\partial}{\partial y}S_z-\frac{\boldsymbol{M}^{y0}\cdot\boldsymbol{\nabla}}{2\hbar^3}D^0_0=0,$$

$$D\nabla^2S_z-\frac{\Gamma^{zz}}{\hbar^2}S_z+\frac{R^{zxx}}{\hbar}\frac{\partial}{\partial x}S_x+\frac{R^{zyy}}{\hbar}\frac{\partial}{\partial y}S_y=0, \tag{3}$$

where the spin density S_i is in units of $\hbar$, and $D=v_F^2\tau/2$.

Even though the form of the SDE in Eq. (3) is essentially the same as that for the uniform driving field,[24] the spin-charge coupling term, through $\boldsymbol{\nabla}D^0_0$, becomes position dependent. To get at this Eq. (3), we have performed a systematic scrutiny on possible additional terms in it that are up to appropriate orders, as will be detailed in the following. The spin-charge coupling terms, given by $-\boldsymbol{M}^{i0}\cdot\boldsymbol{\nabla}D^0_0$, have $\boldsymbol{M}^{i0}=4\tau^2\overline{h^3_k\frac{\partial n^i_k}{\partial\mathbf{k}}}=-2\tau^2h_F^2\alpha(\hat{i}\times\hat{z})$ where $D^0_0=2N_0e\varphi(\boldsymbol{\rho})$ is the *effective* local equilibrium density. The overline denotes angular average over the Fermi surface, $\varphi(\rho)=-E_0(\rho+R_0^2/\rho)\cos\phi$ for $\rho\geq R_0$ and $\boldsymbol{n}_\mathbf{k}=\boldsymbol{h}_\mathbf{k}/h_k$. The Edelstein-like spin polarization $\boldsymbol{S}^{Ed}_\parallel$ [Eq. (2)] is solved directly from Eq. (3).

The D'yakonov-Perel' (DP) spin-relaxation rates, given by $\Gamma^{il}=4\tau\overline{h^2_k(\delta^{il}-n^i_kn^l_k)}$,[30] have $\Gamma^{xx}=\Gamma^{yy}=\Gamma^{zz}/2=2h_F^2\tau$ for RSOI. Spin precession arising from diffusive flow is characterized by $R^{ilm}=4\tau\Sigma_n\epsilon^{iln}\overline{h^n_kv^m_k}$, where ϵ^{iln} is the Levi-Civita symbol, and we have $R^{zii}=-R^{izi}=-2h_Fv_F\tau$ for RSOI and for $i=(x,y)$.[24] Since $k_Fl_e\gg 1$, with l_e the mean-free path, the charge neutrality is maintained by the condition of zero charge density throughout due to screening effect. Within the linear response to the driving electric field, the effect of the screening potential on the spin accumulation can be neglected.

A brief note on the systematic scrutiny of the possible additional terms in Eq. (3) is in order here. The spin-charge coupling term in Eq. (3) is resulted from $\Psi^{i0}D^0_0$,[29] which lowest order in RSOI and first order in spatial gradient is given by the expansion of Ψ^{i0} to the order h_F^3q. This is appropriate for uniform driving field because $\boldsymbol{\nabla}\varphi$ would become position independent. We take caution here, for the case of nonuniform driving fields, to check for additional terms of higher order in q that could have arisen from $\Psi^{i0}D^0_0$. Here,[29]

$$\Psi^{il}=\frac{\Gamma}{2\pi N_0}\sum_{p'}\mathrm{Tr}[\tau^jG^{r(0)}(\boldsymbol{p}',\omega+\omega')\tau^jG^{a(0)}(\boldsymbol{p}'-\boldsymbol{q},\omega')], \tag{4}$$

where $\Gamma=1/2\tau$, $G^{r/a(0)}$ are retarded (advanced) Green's functions averaged over impurity configuration, $\tau^{j=0}=1$, and $\tau^{j=x,y,z}=\sigma_{x,y,z}$. To identify additional expansion terms in Ψ^{i0} for nonuniform driving fields, we note first of all that $\boldsymbol{S}^{Ed}_\parallel$ is of order $h_Fq\varphi$. If $\boldsymbol{S}^{Ed}_\parallel$ is to satisfy the SDE, all the terms in Eq. (3) involving S_i will have to be replaced by $S_i-S^{Ed}_{\parallel,i}$. This implies, according to Eq. (3), that terms of order $h_Fq^3\varphi$ and $h_F^2q^2\varphi$ will be needed, and thus we should look for terms of the same order in the expansion of the spin-charge coupling $\Psi^{i0}D^0_0$. The above two orders can also be identified based on symmetry argument, that the combined power in h_F and q must be even and that they are the lowest RSOI contributions to the respective q orders. Starting from

$$\Psi^{i0}(\omega=0,\omega',\boldsymbol{q}) = \frac{\Gamma}{2\pi N_0}\sum_p \mathrm{Tr}\left[\tau^i \frac{\left(\omega'-\varepsilon_p-\boldsymbol{q}\cdot\frac{\partial\varepsilon_p}{\partial \boldsymbol{p}}+i\Gamma\right)+\left(h_p^i+\boldsymbol{q}\cdot\frac{\partial h_p^i}{\partial \boldsymbol{p}}\right)\sigma^i}{\left(\omega'-\varepsilon_p-\boldsymbol{q}\cdot\frac{\partial\varepsilon_p}{\partial \boldsymbol{p}}+i\Gamma\right)^2}\left(\frac{1}{\omega'-\varepsilon_p-i\Gamma}+\frac{h_p^i\sigma^i}{(\omega'-\varepsilon_p-i\Gamma)^2}\right)\right], \quad (5)$$

we expand it, for instance, up to the order $h_F q^3$, and obtain

$$\Psi^{i0}(h_F q^3) = 2N_0\int d\varepsilon \times\left[\frac{\overline{(\boldsymbol{q}\cdot\boldsymbol{v}_p)^3 h_p^i}}{(\omega'-\varepsilon+i\Gamma)^4(\omega'-\varepsilon+i\Gamma)^2} + \frac{\overline{3(\boldsymbol{q}\cdot\boldsymbol{v}_p)^2\boldsymbol{q}\cdot\frac{\partial h_p^i}{\partial \boldsymbol{p}}}}{(\omega'-\varepsilon+i\Gamma)^4(\omega'-\varepsilon-i\Gamma)}\right], \quad (6)$$

where $\boldsymbol{v}_p=\boldsymbol{p}/m^*$. The angular averages in Eq. (6) over the Fermi surface give rise to $\boldsymbol{q}$ dependences of the form $q^2 q_{j=x,y}$, which will not contribute to Eq. (3) because $\nabla^2\varphi=0$. Following similar procedure, $\Psi^{i0}(h_F^2 q^2)$ is found to be identically zero. Thus Eq. (3) is the SDE for the case of nonuniform driving field.

As has been explained in the previous section, $\boldsymbol{S}_\parallel^{Ed}$ alone cannot satisfy the boundary condition $I_\rho^{i=x,y}=0$. Thus in the end we expect to have an additional $\Delta\boldsymbol{S}$ so that $\boldsymbol{S}=\boldsymbol{S}_\parallel^{Ed}+\Delta\boldsymbol{S}$. On the other hand, the contribution from $\boldsymbol{S}_\parallel^{Ed}$ to I_ρ^z is found to vanish already. The generation of ΔS_z thus does not fall into the conventional scheme that spin accumulation S_z is caused by I_n^z near the sample boundary. Our major task in the following is to calculate $\Delta\boldsymbol{S}$.

Putting the coordinates in units of $l_{so}=\sqrt{D\tau_{so}}$, with $\tau_{so}=2\hbar^2/(h_F^2\tau)$, the SDE for $\Delta\boldsymbol{S}$ is given by

$$\nabla^2\Delta S_x - 4\Delta S_x + 4\frac{\partial}{\partial x}\Delta S_z = 0,$$

$$\nabla^2\Delta S_y - 4\Delta S_y + 4\frac{\partial}{\partial y}\Delta S_z = 0,$$

$$\nabla^2\Delta S_z - 8\Delta S_z - 4\frac{\partial}{\partial x}\Delta S_x - 4\frac{\partial}{\partial y}\Delta S_y = 0. \quad (7)$$

Modes of solution of Eq. (7) have the form $\Delta S_j^{(q)}=\Sigma_m a_j^{(q)} e^{im(\delta+\phi)} H_m^{(1)}(\gamma_q\rho)$, where $H_m^{(1)}(z)$ is the Hankel function of the first kind and the index q denotes the q-th mode. Substituting into Eq. (7) we obtain

$$\begin{bmatrix} (-\gamma^2-4) & 0 & 4i\gamma\sin\delta \\ 0 & (-\gamma^2-4) & 4i\gamma\cos\delta \\ -4i\gamma\sin\delta & -4i\gamma\cos\delta & (-\gamma^2-8)\end{bmatrix}\begin{bmatrix} a_x^{(q)} \\ a_y^{(q)} \\ a_z^{(q)}\end{bmatrix}=0. \quad (8)$$

The asymptotic behavior required of $\Delta S_j^{(q)}$ leads to $\mathrm{Im}\,\gamma>0$. Thus $\gamma_1=2i$, $\gamma_2=\sqrt{2+2i\sqrt{7}}$, and $\gamma_3=-\gamma_2^*$. We also have $(a_x^{(1)},a_y^{(1)},a_z^{(1)})=a_x^{(1)}(1,-\tan\delta,0)$, $(a_x^{(2)},a_y^{(2)},a_z^{(2)})=a_z^{(2)}(2ig_2\sin\delta,2ig_2\cos\delta,1)$, and $(a_x^{(3)},a_y^{(3)},a_z^{(3)})=a_z^{(3)}(2ig_2\sin\delta,2ig_2\cos\delta,1)^*$, for $q=1$, 2, and 3, respectively. Here $g_2=\gamma_2/(\gamma_2^2+4)$. As δ takes on continuous values, there are effective infinite solutions per q-mode. In terms of these modes ΔS_j is expanded in the form

$$\Delta S_j = \int_0^{2\pi} d\delta \sum_{q=1}^{3}\sum_m a_j^{(q)}(\delta) H_m^{(1)}(\gamma_q\rho) e^{im(\delta+\phi)}. \quad (9)$$

The condition that $\Delta\boldsymbol{S}$ is real requires $a_x^{(1)}$ to be pure imaginary and $a_z^{(2)}=-a_z^{(3)*}$.

The boundary condition for the nonuniform driving field is established by applying to the spin current expression similar procedure that we have applied to Eq. (3). The spin-current expression is found to resemble the uniform driving field case,[24] albeit now that $\boldsymbol{\nabla}\varphi$ becomes position dependent. We have

$$I_j^i = -2D\nabla_j S_i - R^{ixj}S_x - R^{iyj}S_y - R^{izj}S_z + \sum_{l=x,y} 4\tau^2\epsilon^{xyi} v_F^j\left(\mathbf{h_p}\times\frac{\partial \mathbf{h_p}}{\partial k_l}\right)_z eN_0\nabla_l\varphi(\mathbf{r}), \quad (10)$$

where the last term is the explicit contribution from the driving field and is nonzero for I_j^z only. The boundary condition $I_\rho^i(\rho=\rho_0)=0$ becomes

$$-\nabla_\rho\Delta S_x - 2\cos\phi\Delta S_z - 2\alpha\tilde{E}/\rho\sin 2\phi|_{\rho=\rho_0}=0,$$

$$-\nabla_\rho\Delta S_y - 2\sin\phi\Delta S_z + 2\alpha\tilde{E}/\rho\cos 2\phi|_{\rho=\rho_0}=0,$$

$$-\nabla_\rho\Delta S_z + 2\cos\phi\Delta S_x + 2\sin\phi\Delta S_y|_{\rho=\rho_0}=0, \quad (11)$$

where $\nabla_\rho\equiv\partial/\partial\rho$, $\rho_0=R_0/l_{so}$, and $\tilde{E}=eE_0N_0\tau/\hbar$. We note that the $\tilde{E}$ terms in Eq. (11) originate from the spin current due to $\boldsymbol{S}_\parallel^{Ed}$, which are the driving terms here. We solve Eqs. (9) and (11) for $a_j^q(\delta)$ by a direct numerical approach and by an analytical approach. Excellent matching is obtained between the two approaches. The analytical approach is facilitated by the assumed forms $a_x^{(1)}=it_x\sin 2\delta$ and $a_z^{(2)}=t_z\cos\delta$, where t_x is real and t_z is complex. The former is guided by the observation, from Eq. (11), that ΔS_x depends on ϕ as $\sin 2\phi$. Substituting these forms into Eqs. (9) and (11), and after some algebra, gives

$$-\gamma_1 H_2^{(1)\prime}(z_1)t_x + i\,\mathrm{Im}[t_z X] = i\alpha\tilde{E}/(\pi\rho_0),$$

$$-i\gamma_1 H_1^{(1)}(z_1)t_x + 2\,\mathrm{Im}[t_z Y] = 0,$$

$$\frac{4}{z_1}H_1^{(1)}(z_1)t_x + i\,\mathrm{Im}[t_z Z] = 0, \qquad (12)$$

where $z_1=\gamma_1\rho_0$, $f'(z)\equiv df/dz$, $X=2H_1^{(1)}(z_1)-2g_2\gamma_2 H_2^{(1)\prime}(z_2)$, $Y=(g_2\gamma_2-1)H_1^{(1)}(z_2)$, $Z=2(\gamma_2-4g_2)H_1^{(1)\prime}(z_2)$, and $z_2=\gamma_2\rho_0$. Equation (12) allows us to solve for t_x and t_z analytically, which are proportional to $\alpha\tilde{E}$. Explicit expressions of t_x and t_z are

$$t_x = -\frac{4\alpha\tilde{E}}{\pi}\frac{\mathrm{Im}[YZ^*]}{8H_1^{(1)}(z_1)\mathrm{Im}[XY^*] + \gamma_1 z_1 H_1^{(1)}(z_1)\mathrm{Im}[ZX^*] + 2\gamma_1 z_1 H_2^{(1)\prime}(z_1)\mathrm{Im}[ZY^*]}, \qquad (13)$$

and

$$t_z = \frac{\alpha\tilde{E}}{\pi\rho_0}\frac{H_1^{(1)}(z_1)\mathrm{Im}[8Y^* - \gamma_1 z_1 Z^*]}{8H_1^{(1)}(z_1)\mathrm{Im}[XY^*] + \gamma_1 z_1 H_1^{(1)}(z_1)\mathrm{Im}[ZX^*] + 2\gamma_1 z_1 H_2^{(1)\prime}(z_1)\mathrm{Im}[ZY^*]}.$$

The spin densities ΔS_i are then obtained to give

$$\Delta S_x = 2\pi\{-it_x H_2^{(1)}(\gamma_1\rho) + 2\,\mathrm{Im}[t_z g_2 H_2^{(1)}(\gamma_2\rho)]\}\sin 2\phi,$$

$$\Delta S_y = 2\pi\{-it_x H_0^{(1)}(\gamma_1\rho) - 2\,\mathrm{Im}[t_z g_2 H_0^{(1)}(\gamma_2\rho)]\} - \Delta S_x \cot 2\phi,$$

$$\Delta S_z = -4\pi\,\mathrm{Im}[t_z H_1^{(1)}(\gamma_2\rho)]\sin\phi. \qquad (14)$$

This and Eq. (2) together are our main results. In particular, $\Delta S_z \neq 0$ confirms that RSOI's contribution to spin accumulation can be restored in a nonuniform driving field. The parity in ϕ of ΔS_i is consistent with that implied in Eq. (11), which is determined by the $\tilde{E}$ terms. The spin accumulation, given in its entirety by ΔS_z, is in a dipole distribution which orients transversely to the driving field $E_0\hat{x}$. Furthermore, Eq. (14) shows that γ_1 and γ_2 contribute to, respectively, decaying and oscillatory behavior in ΔS_i.

III. NUMERICAL RESULTS

Figure 1 presents the spin accumulation S_z in the vicinity of the circular void. We use for our numerical results material parameters that are consistent with GaAs: effective mass $m^*=0.067m_0$ with m_0 the free-electron mass; electron density $n_e=1\times10^{12}$ cm^{-2}; electron mean free path $l_e=0.43$ μm; radius of the circular hole $R_0=0.5l_{so}$; and Rashba coupling constant $\alpha=0.3\times10^{-12}$ eV m.[31,32] The spin-relaxation length is $l_{so}=3.76$ μm and the driving field is $E_0=40$ mV/μm. As shown in Fig. 1, the core of the spin accumulation consists of two spin pockets of opposite spin and of largest spin density magnitude at $\phi=\pm\pi/2$. The spin pockets have radial thickness of about $0.3l_{so}\sim1.1$ μm. In the outer region, spin densities of opposite signs and of smaller magnitudes are dispersed to a wider spatial extent, in the form of two curved spin clouds. The spin cloud center is located about one l_{so} from the void boundary at $\phi=\pm\pi/2$. Both the spin pocket thickness and the spin cloud distance from the void boundary are not sensitive to the void radius R_0.

This spin accumulation can be probed optically by Kerr rotation. To simulate the case of an optical probe scanning along the $\phi=\pi/2$ direction, we calculate the net number of out-of-plane electron spin within the probe area which center is located at a distance d from the void center. For simplicity, we take the probe area to be the same as that of the void. The result is presented in Fig. 2, where we have included several R_0 cases, from $R_0=0.5l_{so}$ up to $R_0=1.2l_{so}$. Distinct contributions from the spin pocket and the spin cloud can be identified. The former are negative minima around $d\approx0.5l_{so}$ and

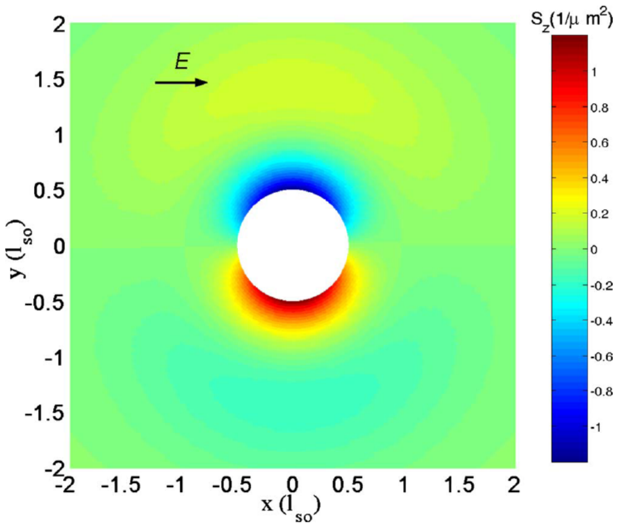

FIG. 1. (Color online) Spin accumulation S_z in the vicinity of a circular void (white circle). S_z is in unit of $1/\mu$m^2, void radius $R_0=0.5l_{so}$, and $l_{so}=3.76$ μm. Dark arrow indicates the driving field direction.

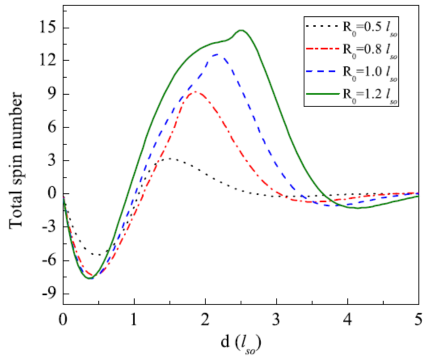

FIG. 2. (Color online) Net number of out-of-plane electron spin within a circular area the same size as the void. The center is shifted by a distance d from the void center along $\phi=\pi/2$.

the latter are positive peaks around $d\approx R_0+l_{so}$. That the negative minima are essentially unshifted reflects the insensitivity of the core radial thickness to the void radius R_0. Additional peak for the $R_0=1.2l_{so}$ curve at $d\approx 2R_0$ corresponds to the situation when the probe area moves out of the spin pocket.

The spin accumulation in a spin-dipole form oriented transversely to the driving field is a generic feature signifying the redistribution of spin rather than the net transport of spin. It has been found in the vicinity of a non-SOI elastic scatterer in a RSOI 2DEG,[33,34] and in the vicinity of a mesoscopic cylindrical barrier in a 2DEG with the barrier profile providing the SOI.[35] Both objects are of sizes much less than l_e. Of course, the physical mechanisms leading to all the above spin-dipole forms are entirely different. Furthermore, here we demonstrate that such spin-dipole feature can exist in the neighborhood of a much larger object $R_0\approx l_{so}$, is robust against background scatterers and is within reach of present measurement technology.

Finally, we note that the spin accumulation features we obtain above are relevant to the case when the circular void is located at a sample edge: an edge-semicircular void (ESV) as shown in Fig. 3. The nonuniform driving field $\boldsymbol{E}(\boldsymbol{\rho})$ for the circular void satisfies also the additional boundary condition $j_y=0$ imposed by the ESV case at the sample edge, $\phi=(0,\pi)$. Thus the same $\boldsymbol{E}(\boldsymbol{\rho})$ holds in the two cases. However, to satisfy the additional boundary condition for spin current at the sample edge, a further additional spin accumulation $\Delta\boldsymbol{S}_{\rm ESV}$ is needed, leading to the total spin accumulation $\boldsymbol{S}=\boldsymbol{S}_{\parallel}^{Ed}+\Delta\boldsymbol{S}+\boldsymbol{S}_{\rm ESV}$. The imposing of the spin current boundary condition in this case is much more complicated, particularly for the spin accumulation near the two corners of the ESV structure, but we find that the spin pocket and the spin cloud features in Fig. 1 remains essentially intact except for near corner regions of the ESV structure.[36]

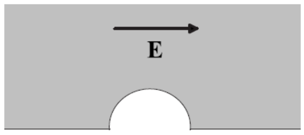

FIG. 3. Patterning the sample edge with a semicircular void

IV. CONCLUSIONS

In conclusions, we have demonstrated that nonuniform driving field can give rise to spin accumulation in a diffusive Rashba-type 2DEG. The nonuniform driving field can be realized by patterning the sample such as with a circular void in the sample or with a semicircular void at the sample edge. The physical process is identified to be associated with spin current for the in-plane spin at the boundary. Our proposed scheme of restoring the RSOI contribution to gate-tunable spin accumulation is relatively simple, and we hope that this will draw experimental effort in the near future.

ACKNOWLEDGMENTS

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Rashba-type spin accumulation near a void at a system edge

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We show that contributions to spin accumulation S_z from the Rashba spin-orbit interaction can be turned on locally by the positioning of a void (edge-void) at a system edge. The two-dimensional electron system is in the diffusive regime and the edge-void is semicircular in shape with a radius $R_0 \approx l_{so}$, the spin-relaxation length. Nonuniform driving field in the vicinity of the void provides the essential condition, and diffusive contributions to the spin currents from nonuniform in-plane spin polarizations provide the primary impetus for the spin accumulation. The conditions that the spin currents are zero at both the system edge and the void boundary mandate a self-consistent procedure for the determination of the spin accumulation. Physical mechanisms leading to spin accumulations at the void boundary and at the void-edge corners are identified within our semi-analytical approach. These physical mechanisms are expected to remain intact for edge-voids of general shapes.

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I. INTRODUCTION

All-electrical generations and manipulations of spin polarization are the main goals of semiconductor spintronics. Rashba spin-orbit interaction (RSOI) has been the key knob for achieving this goal due to its gate-tunning capability.[1] However, in the diffusive regime ($l_{so} \gg l_e$), the RSOI's contribution to the edge spin accumulation S_z is completely quenched due to the linear-k dependence of the SOI.[2–6] The edge spin accumulation is an essential feature of spin-Hall effect (SHE).[7–22] Here k, l_{so}, and l_e are, respectively, the electron momentum, spin-relaxation length, and mean-free path. Though RSOI contributes to SHE in the mesoscopic ballistic regime ($l_{so} < l_e$, and the system dimension $L < l_\phi$, the phase coherent length), it remains important to seek for ways to restore the RSOI contribution to SHE in the diffusive regime.

Effects of RSOI on SHE in the diffusive regime have been obtained at two corners of an electrode-sample interface,[4,23] and in its competing interplay with the cubic-k Dresselhaus SOI.[24] However, the former has the spin accumulation restricted to within a l_{so} region about the interface corners, whereas the latter has the RSOI restricted to suppressing the spin accumulations due to the cubic-k DSOI.[18,19] Seeking for more flexible ways of RSOI's contribution has prompted a recent study on a nonuniform field SHE.[25] A void in the bulk of a Rashba-type two-dimensional electron system (2DES) and its surrounding nonuniform driving field were found to generate a spin accumulation S_z.[25] The underlying physical process is different from the conventional one. While the conventional one is associated with a finite out-of-plane spin current (SC) I_ν^z, the key process in Ref.[25] is associated with an in-plane SC I_ν^μ, and with its vanishing at the void boundary. Here, $\mu, \nu \in \{x, y\}$, and superscript (subscript) denotes spin (flow) direction.

In this work, we show that positioning a void (edge-void, see Fig. 1) at a system edge can locally turn on the RSOI's contribution to SHE at the edge. The physical processes are more complicated than that for a void (bulk-void) in the 2DES bulk. A physical picture is presented below, and it starts from the finding in Ref.[25] that the nonuniform driving field $\boldsymbol{E}(\boldsymbol{\rho})$ gives rise to an Edelstein-like spin polarization[25]

$$\boldsymbol{S}_\parallel^{Ed} = -N_0 \alpha \tau e/\hbar \, \hat{z} \times \boldsymbol{E}(\boldsymbol{\rho}), \tag{1}$$

which spin is in-plane and spatial variation is from the driving field. Here N_0, α, τ, e are the energy density per spin, RSOI coupling contant, mean-free time, and charge magnitude, respectively. Already $\boldsymbol{S}_\parallel^{Ed}$ satisfies the spin diffusion equation, but the boundary condition for the SC has not. The SC, $I_n^i(\boldsymbol{S}, J_n^E)$, contains terms related to the spin polarization $\boldsymbol{S}$ and to the direct field-driving term J_n^E, and is given by[19,25,26]

$$\begin{aligned} I_n^\nu &= -2D\nabla_n S_\nu - R^{\nu z \nu} S_z \, (\hat{\nu}\cdot\hat{n}), \\ I_n^z &= -2D\nabla_n S_z - \sum_{\nu=x,y} R^{z\nu\nu} S_\nu \, (\hat{\nu}\cdot\hat{n}) + J_n^E, \end{aligned} \tag{2}$$

where the gradient terms correspond to diffusive contributions with D the diffusion constant, the $R^{\nu lm}$ denotes the precession of S_l into S_ν when the flow is along $\hat{m}$, and J_n^E corresponds to the direct effect of the driving field. Setting $\boldsymbol{S} = \boldsymbol{S}_\parallel^{Ed}$ leads to zero I_n^z but nonzero I_n^ν. That the former SC is zero reflects the quenching of the conventional RSOI's contribution to SHE, but the nonzero value for the latter SC shows that diffusive contributions from in-plane spin polarization provide the primary impetus for the SHE. A nonuniform driving field opens up this unconventional contribution of RSOI to SHE. The condition $I_n^{\nu,z} = 0$ at the void boundary ($\hat{n}$ normal to the boundary) generates an additional $\Delta\boldsymbol{S}$ including, most importantly, a nonzero spin accumulation $\Delta\boldsymbol{S}_z$, as is evident from the I_n^ν expression in Eq. (2). For the bulk-void, the void boundary is self-explanatory, while for the edge-void, the boundaries include both the void boundary and the system edge. The conditions that the spin currents are zero at both the void boundary and the system edge mandates a self-consistent procedure for the determination of the spin accumulation.

The SC given by Eq. (1) has $D = v_F^2\tau/2$, where v_F is the Fermi velocity, and $R^{ilm} = 4\tau\sum_n \epsilon^{iln}\overline{h_k^n v_k^m}$, where ϵ^{iln} is the Levi-Civita symbol, $\boldsymbol{h_k}$ the effective RSOI field, and the overline denotes angular average over Fermi surface. The direct field-driving term in the SC is given by $J_n^E = \sum_\nu \overline{4\tau^2 v_k^n(\boldsymbol{h}_k \times \frac{\partial \boldsymbol{h_k}}{\partial k_l})_z}\, eN_0\nabla_l\varphi(\boldsymbol{r})$, where the nonuniform driving field $\boldsymbol{E} = -\nabla\varphi(\rho) = \sigma_0\boldsymbol{j}$ has the electric current density $\boldsymbol{j}$ satisfing the steady state condition $\nabla\cdot\boldsymbol{j} = 0$ and the boundary condition $j_n = 0$ for $\hat{n}$ normal to the boundary. Here σ_0 is the electric conductivity and $\boldsymbol{E} = E_0\hat{x} - E_0(R_0/\rho)^2(\cos 2\phi\hat{x} + \sin 2\phi\hat{y})$ outside the circular edge-void (EV), with ρ and ϕ being the coordinates originated from the EV center.

In this work, we calculate the spin polarization $\boldsymbol{S}$ in the vicinity of the EV. Our result is in the form $\boldsymbol{S} = \boldsymbol{S}^B + \Delta\boldsymbol{S}^{EV}$. The first term $\boldsymbol{S}^B = \boldsymbol{S}_\parallel^{Ed} + \Delta\boldsymbol{S}^B$ is the spin polarization for a bulk circular void,[25] where $I_n^i(\boldsymbol{S}^B, J_n^E) = 0$ at the void boundary. J_n^E's sole contribution to SC is in $i = z$ and is exactly canceled by that from $\boldsymbol{S}_\parallel^{Ed}$. Along the seemingly symmetry axis ($\phi = 0, \pi$, or $y = 0$) of the bulk void, we find that the SCs $I_n^x(\boldsymbol{S}^B, 0)$ and $I_n^z(\Delta\boldsymbol{S}^B, 0)$ are nonzero for $\hat{n} = \hat{y}$. This hidden asymmetry in SC is revealed when the circular void is positioned at a system edge, and is exhibited via its generation of an additional $\Delta\boldsymbol{S}^{EV}$.

The calculation of $\Delta\boldsymbol{S}^{EV}$ is carried out in a two-step procedure. The first step produces $\Delta\boldsymbol{S}^{E1}$ which, together with $\boldsymbol{S}^B$, has $I_n^i(\boldsymbol{S}^B + \Delta\boldsymbol{S}^{E1}, J_n^E) = 0$ at the sample edge ($y = 0$). This step is solved analytically and $\Delta\boldsymbol{S}^{E1}$ is found to have already incorporated an essential part of the spin accumulation at the sample edge, especially at the corners junctioning the edge and the void boundary. The second, and final, step is to find $\Delta\boldsymbol{S}^{E2}$ such that, with $\Delta\boldsymbol{S}^{EV} = \Delta\boldsymbol{S}^{E1} + \Delta\boldsymbol{S}^{E2}$, $\boldsymbol{S}$ satisfies the SC boundary conditions at both the void boundary and the sample edge. All the $\Delta\boldsymbol{S}$'s above for each step are solutions to the spin diffusion equation, Eq. (3), but each has to satisfy a designated SC boundary condition and each is driven by a designated SC source term. Nonzero SC at the boundary or edge in a step of our calculation will be treated as a SC source term for the determination of $\Delta\boldsymbol{S}$ in the next step. This semi-analytical approach reveals a clear physical picture for the mechanisms of spin accumulation formation around a semi-circular EV. We expect, however, that these mechanisms to remain intact for EVs of general shapes.

We note, in passing, that the SC is used for the establishment of a boundary condition for the spin-diffusion equation. A conventional form of the spin current operator $J_l^i = (1/4)(V_l\sigma_i + \sigma_i V_l)$ is appropriate for hard wall boundary,[19,27,28] where the kinetic velocity $V_l = (1/i\hbar)[\hat{x}, H]$, and spin unit of $\hbar$ is implied. As the boundary condition is applied to a region much shorter in distance than $l_{\rm so}$ from the boundary, the effect of spin torque[29,30] should be of secondary importance here.

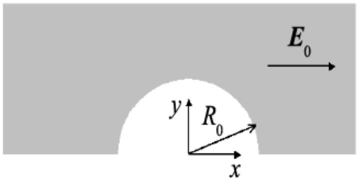

FIG. 1: An edge-void of radius R_0 is positioned at a system edge. Asymptotic driving field is $\boldsymbol{E}_0$ and the origin of the coordinate coincides with the void center.

In Sec. II, we present the theory and model for the nonuniform driving field effects in the vicinity of an EV. In Sec. III we present our numerical results and discussion. Finally, in Sec. IV, we will present our conclusion.

II. MODEL AND THEORY

The spin diffusion equation (SDE) for the case of nonuniform driving field has been derived in Ref. [25]. For the case of RSOI, the Hamiltonian $H_{\rm so} = \boldsymbol{h}_k\cdot\boldsymbol{\sigma}$ has the effective SOI field $\boldsymbol{h}_k = -\alpha\hat{z}\times\boldsymbol{k}$, where $\boldsymbol{\sigma}$ denotes the Pauli matrices. The SDE is given by

$$\begin{aligned} D\nabla^2 S_\nu - \frac{\Gamma^{\nu\nu}}{\hbar^2}S_\nu + \frac{R^{\nu z\nu}}{\hbar}\nabla_\nu S_z - \frac{\boldsymbol{M}^{\nu 0}\cdot\boldsymbol{\nabla}}{2\hbar^3}D_0^0 = 0, \\ D\nabla^2 S_z - \frac{\Gamma^{zz}}{\hbar^2}S_z - \frac{R^{zxx}}{\hbar}\nabla_x S_x - \frac{R^{zyy}}{\hbar}\nabla_y S_y = 0, \end{aligned} \tag{3}$$

where $\boldsymbol{S}$ is in unit of $\hbar$.

The spin-charge coupling term is $-\boldsymbol{M}^{\nu 0}\boldsymbol{\nabla}D_0^0$ where $\boldsymbol{M}^{\nu 0} = 4\tau^2\overline{h_k^3\frac{\partial n_k^\nu}{\partial\boldsymbol{k}}}$, and $\boldsymbol{\nabla}D_0^0$ becomes position dependent in a nonuniform driving field. Here, $D_0^0 = 2N_0e\varphi(\boldsymbol{\rho})$, $\hat{n}_k = \boldsymbol{h}_k/h_k$, and $R^{z\nu\nu} = -R^{\nu z\nu} = -2h_F v_F\tau$ for RSOI. Furthermore, $\Gamma^{il} = 4\tau\overline{h_k^2(\delta^{il} - n_k^i n_k^l)}$ is the D'yakonov-Perel' (DP) spin relaxation rates for which $\Gamma^{xx} = \Gamma^{yy} = \Gamma^{zz}/2 = 2h_F^2\tau$ in the RSOI case.[31] The boundary condition for the SDE, as mentioned above, is given by $I_n^i = 0$ for $\hat{n}$ normal to either the system edge or the EV boundary.

Our main goal is to solve for $\Delta\boldsymbol{S}^{EV}$. Adding this term to $\boldsymbol{S}^B$, the spin polarization for a bulk circular void, will give us the total spin polarization $\boldsymbol{S}$. The expression for $\boldsymbol{S}^B$ has been obtained analytically[25] but it is too lengthy and is not presented here. An essential part of $\Delta\boldsymbol{S}^{EV}$, namely, $\Delta\boldsymbol{S}^{E1}$, is the spin accumulation at the system edge, and it can be captured by first applying our SC boundary condition to the edge, at $y = 0$ and for $\hat{n} = \hat{y}$. The remaining part of $\Delta\boldsymbol{S}^{EV}$, namely, $\Delta\boldsymbol{S}^{E2}$, is obtained by imposing the SC boundary conditions at both the EV boundary and the sample edge. Thus $\Delta\boldsymbol{S}^{EV} = \sum_{j=1,2}\Delta\boldsymbol{S}^{Ej}$.

To address the SC boundary condition at the system edge, we start from the SDE for $\Delta\boldsymbol{S}^{Ej}$ ($j = 1, 2$), which is obtained from Eq. (3), given by

$$\begin{aligned}
&\nabla^2 \Delta S_x^{Ej} - 4\Delta S_x^{Ej} + 4\nabla_x \Delta S_z^{Ej} = 0, \\
&\nabla^2 \Delta S_y^{Ej} - 4\Delta S_y^{Ej} + 4\nabla_y \Delta S_z^{Ej} = 0, \\
&\nabla^2 \Delta S_z^{Ej} - 8\Delta S_z^{Ej} - 4\nabla_x \Delta S_x^{Ej} - 4\nabla_y \Delta S_y^{Ej} = 0.
\end{aligned} \tag{4}$$

This equation has adopted a length unit $l_{\rm so} = \sqrt{D\tau_{\rm so}}$, where $\tau_{\rm so} = 2\hbar^2/(h_F^2\tau)$ and h_F is the RSOI field at the Fermi surface. A Fourier transform solution to this SDE with respect to x is facilitated by writing $\Delta\boldsymbol{S}^{Ej}$ in the form

$$\Delta S_i^{Ej} = \int dk \sum_{q=1,2,3} \eta_q^{(j)}(k)\, a_{iq}\, e^{ikx} e^{-\beta_q y}, \tag{5}$$

where index q denotes the $q-$th mode of solution for the SDE and $e^{-\beta_q y}$ indicates that $\Delta\boldsymbol{S}^{Ej}$ is localized near the $y = 0$ edge. The eigen-modes, given by a_{iq}, depend on k and the amplitude for each such mode is attributed to $\eta_q^{(j)}(k)$.

Substituting $\Delta\boldsymbol{S}^{Ej}$ into Eq. (4), we obtain

$$\begin{bmatrix} -k^2+\beta_q^2-4 & 0 & 4ik \\ 0 & -k^2+\beta_q^2-4 & -4\beta_q \\ -4ik & 4\beta_q & -k^2+\beta_q^2-8 \end{bmatrix} \begin{bmatrix} a_{xq} \\ a_{yq} \\ a_{zq} \end{bmatrix} = 0, \tag{6}$$

where $\beta_1 = \sqrt{k^2+4}$, $\beta_2 = \sqrt{k^2-2+2i\sqrt{7}}$, and $\beta_3 = \beta_2^*$. The eigen-modes are $(a_{i1}) = (1, g_1, 0)$, $(a_{i2}) = (g_2, g_3, 1)$, and $(a_{i3}) = (-g_2^*, g_3^*, 1)$ with $g_1 = ik/\sqrt{k^2+4}$, $g_2 = ik(3+i\sqrt{7})/8$, and $g_3 = ig_2\sqrt{k^2-2+2i\sqrt{7}}/k$. The fact that $\Delta\boldsymbol{S}^{Ej}$ are real requires $\eta_1^{(j)}$ to be pure imaginary and $\eta_2^{(j)}(k) = (\eta_3^{(j)}(-k))^*$. The amplitudes $\eta_q^{(j)}(k)$ will be fixed by the SC boundary conditions.

The boundary condition $I_n^i(\boldsymbol{S}^B + \Delta\boldsymbol{S}^{E1}, J_n^E) = 0$ at $y = 0$ and $\hat{n} = \hat{y}$ is obtained from Eq. (2), given by

$$\begin{aligned}
&\left[-\frac{\partial}{\partial y}\left(\Delta S_x^{E1} + \Delta S_x^B\right) + 2\alpha\widetilde{E}\frac{R_0^2}{x^3}\right]_{y=0} = 0, \\
&\left[-\frac{\partial}{\partial y}\left(\Delta S_y^{E1} + \Delta S_y^B\right) - 2\left(\Delta S_z^{E1} + \Delta S_z^B\right)\right]_{y=0} = 0, \\
&\left[-\frac{\partial}{\partial y}\left(\Delta S_z^{E1} + \Delta S_z^B\right) + 2\left(\Delta S_y^{E1} + \Delta S_y^B\right)\right]_{y=0} = 0,
\end{aligned} \tag{7}$$

where $\widetilde{E} = eE_0N_0\tau/\hbar$, and the term involving $\widetilde{E}$ is due to $\boldsymbol{S}_\parallel^{Ed}$. Note that if the terms in Eq. (7) that involve $\Delta\boldsymbol{S}^B$ and $\widetilde{E}$ were to add up to zero for their respective equations, then $\Delta\boldsymbol{S}^{E1}$ would be obviously zero. However, the contrary turns out to be the case here. Therefore, by moving these $\Delta\boldsymbol{S}^B$ and $\widetilde{E}$ terms to the right-hand-side of Eq. (7), they become the SC sources $f_i^{(1)}$ for the $\Delta\boldsymbol{S}^{E1}$ generation, as is given by

$$\begin{aligned}
&\left[-\frac{\partial}{\partial y}\Delta S_x^{E1}\right]_{y=0} = f_x^{(1)}, \\
&\left[-\frac{\partial}{\partial y}\Delta S_y^{E1} - 2\Delta S_z^{E1}\right]_{y=0} = f_y^{(1)}, \\
&\left[-\frac{\partial}{\partial y}\Delta S_z^{E1} + 2\Delta S_y^{E1}\right]_{y=0} = f_z^{(1)},
\end{aligned} \tag{8}$$

where $f_i^{(1)} = -I_y^i(\boldsymbol{S}_\parallel^{Ed} + \Delta\boldsymbol{S}^B, J^E)|_{y=0}$ for $|x| \geq R_0$ and $f_i^{(1)} = 0$ for $|x| < R_0$. Analytical forms of $f_i^{(1)}$ are obtained to be

$$\begin{aligned}
f_x^{(1)}(x) &= \frac{4\pi}{x}\left\{-X_2 + 2\text{Im}\left[gZ_2\right]\right\} - 2\alpha\widetilde{E}\frac{R_0^2}{x^3}, \\
f_y^{(1)}(x) &= 0, \\
f_z^{(1)}(x) &= -\frac{4\pi}{|x|}\text{Im}\left[Z_1\right] + 4\pi\left\{X_0 + 2\text{Im}\left[gZ_0\right]\right\} \\
&\quad + 4\pi\left\{-X_2 + 2\text{Im}\left[gZ_2\right]\right\},
\end{aligned} \tag{9}$$

where $g = \gamma_2/(\gamma_2^2+4)$, $X_m = it_xH_m^{(1)}(\gamma_1|x|)$, and $Z_m = t_zH_m^{(1)}(\gamma_2|x|)$. Here $H_m^{(1)}(z)$ denotes the Hankel function of the first kind, and the constants $\gamma_1 = 2i$ and $\gamma_2 = \sqrt{2+2i\sqrt{7}}$.[25] Explicit expressions for t_x and t_z are not shown here but they were defined for the expression of $\boldsymbol{S}^B$ in a bulk circular void.[25]

It is worth mentioning that $f_i^{(1)} \neq 0$ reflects an unexpected asymmetry. The symmetric structure of a bulk circular void seems to suggest that all currents, including charge and spin, flowing normally to the symmetry axis ($\phi = 0, \pi$) must be zero. This is indeed the case for the charge current, but is otherwise for the spin current. The reason is related to the fact that $\boldsymbol{S}_x^B$ and $\boldsymbol{S}_z^B$ are odd in y while that for $\boldsymbol{S}_y^B$ is even.[25] As a consequence, according to Eq. (2), there are diffusive contributions to $f_x^{(1)}$ and $f_z^{(1)}$ but not to $f_y^{(1)}$. Furthermore, there is additional contribution to $f_z^{(1)}$ through the spin precession of $\boldsymbol{S}_y^B$. Thus we have nonzero $f_i^{(1)}$ except for $i = y$, as is shown in Fig. 2(a).

The amplitudes $\eta_q^{(1)}(k)$ for the $q-$modes is determined by Fourier transforming Eq. (8), via the integral $\frac{1}{2\pi}\int dx e^{-i\kappa x}$, to obtain

$$\begin{bmatrix} \beta_1 & \beta_2 g_2 & -\beta_3 g_2^* \\ \beta_1 g_1 & \beta_2 g_3 - 2 & \beta_3 g_3^* - 2 \\ 2g_1 & \beta_2 + 2g_3 & \beta_3 + 2g_3^* \end{bmatrix} \begin{bmatrix} \eta_1^{(1)} \\ \eta_2^{(1)} \\ \eta_3^{(1)} \end{bmatrix} = \begin{bmatrix} \tilde{f}_x^{(1)} \\ 0 \\ \tilde{f}_z^{(1)} \end{bmatrix}, \tag{10}$$

and from it, the $\eta_q^{(1)}(k)$. Here $\tilde{f}_i^{(1)}(\kappa)$ is the Fourier transform of $f_i^{(1)}$.

The amplitudes $\eta_q^{(2)}(k)$ for $\Delta\boldsymbol{S}^{E2}$ can be calculated similarly. However, we need to address both the SC boundary conditions at the EV boundary and the system edge. The construction of $\Delta\boldsymbol{S}^{E1}$ has removed any further

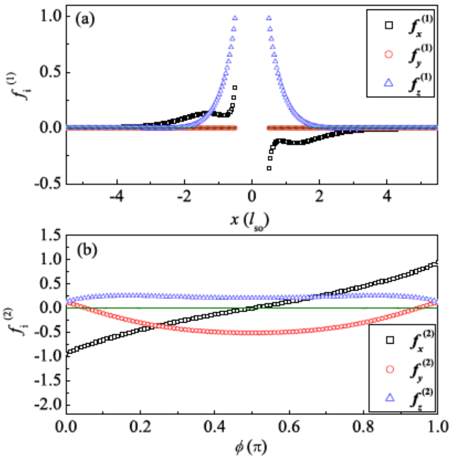

FIG. 2: Spin current source terms $f_i^{(j)}$ for $j = 1, 2$ are plotted, respectively, in (a), (b), and with abscissas x and ϕ. In (a), empty symbols denote $\boldsymbol{f}_i^{(1)}$, and solid lines denote SC I_y^i at $y = 0$ when $\Delta\boldsymbol{S}^{E1}$ is included. In (b), open symbols denote $\boldsymbol{f}_i^{(2)}$, and solid lines denote SC I_n^i, for $\hat{n} = \hat{\rho}$ and at $\rho = R_0$, when the total $\boldsymbol{S}$ is used.

need of introducing SC source at the system edge. Yet additional spin polarization is generated because the SC from $\Delta\boldsymbol{S}^{E1}$ does not satisfy the boundary condition at the EV boundary. In effect, this gives rise to a SC source at the EV boundary that, in turn, generates $\Delta\boldsymbol{S}^{E2}$. In this work, we find out that it is convenient to replace the SC source $f_i^{(2)}$ at the EV boundary by an auxiliary SC source $f_i^{\rm aux}$ at $y = 0$ but located outside the system edge, for $|x| < R_0$. This approach is in line with the concept of introducing image charges for the electrostatic problem where the image charges that help satisfying the boundary condition for the electrostatic potential must locate outside the region of interest. Taking into account the parity of $\boldsymbol{S}^B$ with respect to x, the auxiliary SC sources $f_i^{\rm aux}(|x| < R_0)$ are written in the form

$$
\begin{aligned}
f_x^{\rm aux}(x) &= f_{x0}^{\rm aux} + \sum_{n=1,2,\ldots} A_{x,n}\sin\left(\frac{n\pi x}{R_0}\right),\\
f_y^{\rm aux}(x) &= f_{y0}^{\rm aux} + \sum_{n=1,2,\ldots} A_{y,n}\cos\left(\frac{(2n-1)\pi x}{2R_0}\right),\\
f_z^{\rm aux}(x) &= f_{z0}^{\rm aux} + \sum_{n=1,2,\ldots} A_{z,n}\cos\left(\frac{(2n-1)\pi x}{2R_0}\right),
\end{aligned}
\tag{11}
$$

where $f_{i0}^{\rm aux} = (f_x^{(1)}(R_0)\,x/R_0,\,0,\,f_z^{(1)}(R_0))$ is to make sure that $f_i^{\rm aux}$ connects to $f_i^{(1)}$ continuously to avoid the Gibbs phenomenon in the Fourier transformation. With the Fourier transformed auxiliary SC sources $\tilde{f}_i^{\rm aux}$ substituting into the right-hand side of Eq. (10), the column vector on the left-hand side of the equation becomes $\eta_i^{(2)}$. Thus $\eta_i^{(2)}$ and $\Delta\boldsymbol{S}^{E2}$ are expressed in terms of $A_{j,n}$. These $A_{j,n}$ coefficients will then be fixed by the SC boundary condition at the EV boundary, $I_n^i(\rho = R_0) = 0$, which is obtained from Eq. (2), given by

$$
\begin{aligned}
&\left[-\frac{\partial}{\partial\rho}\Delta S_x^{EV} - 2\cos\phi\,\Delta S_z^{EV}\right]_{\rho=R_0} = 0,\\
&\left[-\frac{\partial}{\partial\rho}\Delta S_y^{EV} - 2\sin\phi\,\Delta S_z^{EV}\right]_{\rho=R_0} = 0,\\
&\left[-\frac{\partial}{\partial\rho}\Delta S_z^{EV} + 2\cos\phi\,\Delta S_x^{EV} + 2\sin\phi\,\Delta S_y^{EV}\right]_{\rho=R_0} = 0,
\end{aligned}
\tag{12}
$$

where $\phi \in (0, \pi)$. Similar to Eq. (10), the contributions from $\Delta\boldsymbol{S}^{E1}$ in Eq. (12) will become the SC source $f_i^{(2)}$, when it is moved to the right-hand side of the equation. Solving the equation by direct discretisation lead us to the total spin polarization $\boldsymbol{S} = \boldsymbol{S}^B + \Delta\boldsymbol{S}^{EV}$.

III. NUMERICAL RESULTS

Numerical examples presented in this section are organized as follows. Figure 2 illustrates the effectiveness of our approach. Fig. 3 presents our main results, that spin accumulation $\boldsymbol{S}_z$ does occur near an EV due to RSOI. Finally, Fig. 3 presents the distribution of the spin accumulation as probed by a scanning optical beam. We have assumed material parameters that are consistent with GaAs. Specifically, the effective mass $m^* = 0.067 m_0$, with m_0 the free-electron mass; electron density $n_e = 1\times10^{12}$ cm^{-2}; electron mean free path $l_e = 0.43$ μm; the Rashba coupling constant $\alpha = 0.3\times10^{-12}$ eV m,[32,33] and the spin-relaxation length $l_{so} = 3.76$ μm. Furthermore, the driving field $E_0 = 40$ mV/μm, and the EV structure radius $R_0 = 0.5 l_{so}$.

The effectiveness of our self-consistent procedure is illustrated in Fig. 2. We plot the SC I_n^i (solid curves) at the system edge ($y = 0$) and at the EV boundary ($\rho = R_0$) in, respectively, Figs. 2(a) and 2(b). For comparison, we plot the SC source terms $f_i^{(1)}$ and $f_i^{(2)}$ (open symbols) in Figs. 2(a) and 2(b), respectively. Our result that all the six solid curves, three each for I_y^i and I_ρ^i, overlap on the zero abscissas shows that the SC boundary conditions are satisfied remarkably. Furthermore, the symmetries of the SC source are consistent with that derived from $\boldsymbol{S}^B$, as have been discussed in the previous section for the case of $f_i^{(1)}$.

Our main results are presented in Fig. 3. The spatial distribution of out-of-plane spin densities $\boldsymbol{S}_z^B$, $\Delta\boldsymbol{S}_z^{E1}$, and $\Delta\boldsymbol{S}_z^{E2}$ are shown, respectively, in Figs. 3(a), 3(b), and 3(c). The spin accumulation $\boldsymbol{S}_z$, given by the sum of these three out-of-plane spin densities, is denoted by Fig. 3(d). It is clearly shown that RSOI's contribution to spin accumulation $\boldsymbol{S}_z$ can be turned on locally by an edge-void due to the nonuniform driving field. Basically, Figs. 3(a)-(c) provide a pictorial way of viewing the formation of the spin accumulation. Fig. 3(a) shows the spin density ΔS_z^B had the void been in the bulk of the 2DEG.[25] The spin density is centered along the $\phi = \pi/2$ direction and is separated into two regions of opposite spin polarization: a core region and an outer region. The core

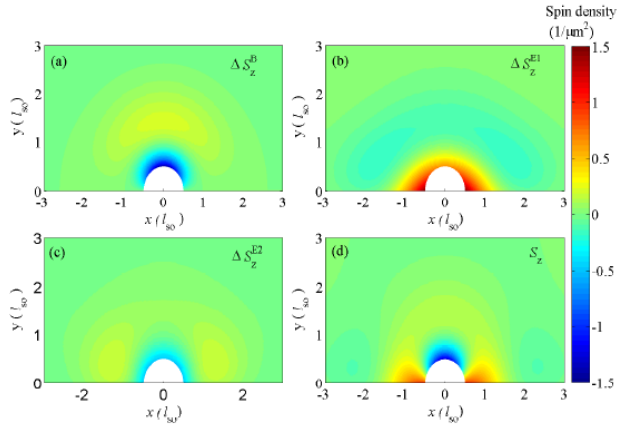

FIG. 3: Out-of-plane spin densities $\Delta \boldsymbol{S}_z^B$, $\Delta \boldsymbol{S}_z^{E1}$, $\Delta \boldsymbol{S}_z^{E2}$, and spin accumulation S_z are plotted in (a), (b), (c), and (d), respectively. Fig. 3(d) is the sum of (a), (b), and (c). The external electric field $\boldsymbol{E}$ is applied along $\hat{x}$ and the EV structure has a radius $R_0 = 0.5\, l_{\rm so}$.

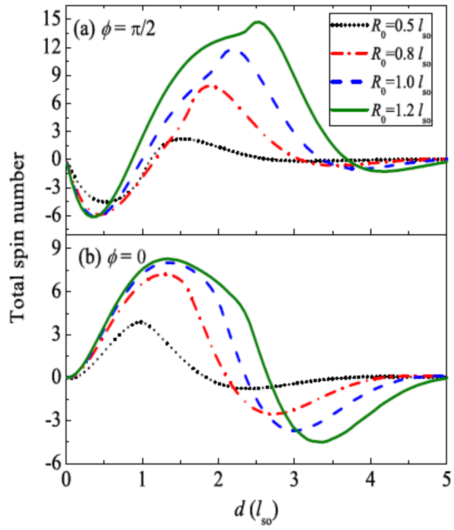

FIG. 4: Net number of out-of-plane electron spin, from $\boldsymbol{S}_z$, within a circular probe area of the same radius as the EV structure. The probe center is shifted by a distance d from the EV center (a) along $\hat{y}$ ($\phi = \pi/2$), and (b) along edge ($\hat{x}$, or $\phi = 0$).

region is concentrated along the void boundary and has a radial thickness of about $0.3\, l_{\rm so} \sim 1.1\, \mu m$. The outer region has a much wider spatial extent, in the form of a curved spin cloud, and having its center located about a distance of one $l_{\rm so}$ from the void boundary. This spin density is driven by an in-plane SC of diffusive origin. The SC boundary condition is satisfied at the void boundary but not at the system edge. The residual SC at the system edge becomes SC source terms $f_i^{(1)}$, shown in Fig. 2(a), that drives the generation of $\Delta \boldsymbol{S}_z^{E1}$ in Fig. 3(b). The spin density $\Delta \boldsymbol{S}_z^{E1}$ is concentrated mostly at the two corners of the EV with a range of about $0.5\, l_{\rm so}$ and with spin polarization opposite to that of the core spin density in Fig. 3(a). $\Delta \boldsymbol{S}_z^{E1}$ also contains a wide outer region of compensating spin cloud with much smaller spin density magnitude. The SC boundary condition, however, is not satisfied at the void boundary. Again, residual SC gives rise to SC source terms $f_i^{(2)}$ that drives the generation of spin density $\Delta \boldsymbol{S}_z^{E2}$. The spin density $\Delta \boldsymbol{S}_z^{E2}$, in general, brings about correction of even smaller magnitude, including a void-boundary distribution that reinforces the core region in Fig. 3(a). Since $\Delta \boldsymbol{S}_z^{E2}$ and $f_i^{(2)}$ together have satisfied the SC boundary condition at both the system edge and the void boundary, $\Delta \boldsymbol{S}^{E2}$ is the results from our self-consistent procedure. it is numerically more stable and efficient. The total spin accumulation $\boldsymbol{S}_z$, as shown in Fig. 3(d).

The spin accumulation S_z can be optically probed by Kerr rotation. To simulate the optical prob by scanning along the cases of $\phi = \pi/2$ and $\phi = 0$. The net number of out-of-plane spin within a circular probe area with a radius R_0 is located at a distance d from the EV center. The results is plotted in Fig. 4 for $R_0 = 0.5 l_{so}$ to $R_0 = 1.2 l_{so}$. For $\phi = \pi/2$ case, the contribution of spin packet and spin cloud can be identified. The spin pocket are negative minima around $d \approx l_{so}$ and the spin cloud are positive peaks around $d \approx R_0 + l_{so}$. We notice that the negative minima essentially unshifted indicating the insensitivity of radial thickness of the core spin pocket to different R_0. Additional peak for the $R_0 = 1.2 l_{so}$ curve at $d \approx 2R_0$ corresponding to the situation when the probe area moves out of the core spin pocket. It is found that the net spin number features in Fig. 4 are essentially the same as the case of a circular void in the bulk. The core spin pocket at $\phi = \pi/2$ is quite robust against the EV boundary due to the nonuniform driving field. Furthermore, the net spin number is plotted in Fig. 4(b) when the probe area is scanning along radial direction at $\phi = 0$ for $R_0 = 0.5 l_{so}$ to $1.2 l_{so}$. The maxima occurring around $d \approx 1 l_{so} \sim 1.3 l_{so}$ indicating that the core spin pockets remain the similar core radial thickness at $\phi = 0(\pi)$. The minima occurring around $2R_0 + 0.8 l_{so}$ when the probe area moves out the region of the core spin-pocket for $\phi = 0$ in Fig. 4(b).

IV. CONCLUSIONS

In conclusions, we have studied that the nonuniform driving field against the EV boundaries can generate the bulk-like spin accumulation at $\phi = pi/2$ and simultaneously generate edge-like spin accumulations at $\phi = 0(\pi)$ in a diffusive RSOI EV structure. Our new finding is that edge-like spin accumulations have comparable magnitudes and opposite signs respecting to bulk-like spin accumulation. The advantage of edge-like spin accumulations provides the tunable capability of spin accumulation location and it has the potential to be a nonmagnetic spin-injection source. Our proposed scheme of restoration RSOI spin accumulation in EV structure is hopeful to achieve spintronics by fully electric means. We hope that this will draw experimental effort in the near future.

Acknowledgments

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國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

■ 達成目標

□ 未達成目標（請說明，以 100 字為限）

□ 實驗失敗

□ 因故實驗中斷

□ 其他原因

說明：

The major goals of the project are to study the spin accumulations due to the combined effects of various spin-orbit interactions (SOI), the effects of magnetic field, and the electrical means of manipulation of these spin polarizations. In these research topics, we have obtained solid results and understanding. The research content matches nicely the original proposal.

2. 研究成果在學術期刊發表或申請專利等情形：

論文：■已發表 ■未發表之文稿 ■撰寫中 □無

專利：□已獲得 □申請中 ■無

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其他：（以 100 字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限）

Results obtained in this project have been published in leading SCI journals. These results are of academic values to the field of spintronics in semiconductors, and may be of importance for technological exploration. Specifically, we pointed out an experimental diagnostic scheme for probing of the major ingredient SOI in a system, proposed two ways for the restoration of Rashba SOI's contribution to spin accumulation (either by way of the competing interplay between the Rashba SOI and the Dresselhaus SOI or by way of the nonuniform driving field), and suggested an electrical means of detection of spin current. Besides, our research has established a theoretical framework for further exploration of spin accumulations driven by an external electric field. Therefore, future citation to published work supported by this project is expected to be reasonably high.

國科會補助專題研究計畫項下赴國外(或大陸地區)出差或研習心得報告

日期：2011 年 1 月 26 日

計畫編號	NSC 96 － 2112 － M － 009 － 038 － MY3		
計畫名稱	操控半導體電子自旋的動力行為與傳輸的研究		
出國人員姓名	朱仲夏	服務機構及職稱	國立交通大學電子物理系
出國時間	2009 年 8 月 26 日至 2009 年 8 月 29 日	出國地點	杭州 中國
會議名稱	（中文） 第十屆全國磁學理論會議(杭州,2009) （英文） 10th Conference on Theory of Magnetism		
發表論文題目	（中文）半導體中自旋累積：自旋軌道作用競爭效應與非均勻驅動場效應 （英文）Spin accumulations in semiconductors: Competing spin-orbit interactions effect and Non-uniform driving field effect		

一、 國外(大陸)研究過程

The meeting date was from 26th August, 2009 to 29th August 2009. It was an annual meeting organized by the Chinese Physical Society Magnetism Sub-committee. The number of attendants was about 150. Participants included researchers from Singapore, Taiwan, Hong Kong, and from major domestic Institutes and universities. There were two parallel sessions and talks covered research results from both experimental and theoretical endeavors. I was invited to give an invited talk in the conference. Apart from airport pickup, I had to pay for my own air ticket, hotel, and conference registration. The registration covered all meals in the hotel, which also is the venue for the conference.

I left Taoyuan Airport on 26th August at 8:15 a.m. and flew to Hong Kong International Airport for transit to Hanghou. The 11:35 a.m. flight from Hong Kong arrived Hangzhou at 13:35. The conference program that day was registration and reception. During the reception dinner, I met Professors S.Q. Shen (Hong Kong University) and Y.Q. Li (Zejiang University), whom I knew already. I also came to know Professors D.S. Wang (Academician), M.W. Wu (Chinese Academy of Science, CAS), and many others.

Sessions of the conference started at 8:30 a.m. on the 27th of August. I had the honor to give the first talk of the conference. The title of my talk was: "Spin accumulations in semiconductors: Competing spin-orbit interactions effect and Non-uniform driving field effect". This talk had attracted question from Professor X.R. Wang (Hong Kong University of Science and Technology) and response from Professor K. Chang (CAS). It was also well received, as I knew it later from nice comments from other participants. I attended the talks of Professors Q.F. Sun (CAS), H.G. Luo (Lanzhou University), J. Wang, K. Chang (CAS), X. Tao (CAS), and D.S. Wang (CAS). The conference ended on 29th August (Sat.) noon, where I left the conference center for Taipei in the middle of the last talk.

二、 研究成果

Content of my talk includes work published in two papers:

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Competing interplay between Rashba and cubic-*k* Dresselhaus spin-orbit interactions in spin-Hall effect

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Focusing on the interplay between the Rashba and cubic-*k* Dresselhaus spin-orbit interactions (SOI), we calculate the spin accumulation S_z and the spin polarizations S_i^B at, respectively, the lateral edges and in the bulk of the two-dimensional electron gas. Their dependences on both the ratio between the Rashba and the Dresselhaus SOI coupling constants and the electron densities are studied systematically. Strong competition features in S_z are found. In the Dresselhaus-dominated regime S_z changes sign when the electron density is large enough. In the Rashba-dominated regime S_z is essentially suppressed. Most surprising is our finding that the Rashba-dominated regime occurs when $\alpha \approx 2\tilde{\beta}$, where α and $\tilde{\beta}$ are the Rashba and the effective linear-*k* Dresselhaus SOI coupling constants, respectively. For the spin polarizations S_i^B, the Rashba-dominated regime occurs when $\alpha \geq \tilde{\beta}$. Our results point out that decreasing $|\alpha|$ leads to the restoration of the spin accumulation S_z.

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I. INTRODUCTION

Spin-orbit interaction (SOI) provides the key leverage for the recent strive for all electrical generations and manipulations of spin densities in semiconductors.[1–9] Intrinsic SOIs, such as the Rashba SOI (RSOI) (Refs. 3, 7, and 9–12) and the Dresselhaus SOIs (DSOIs),[6,13,14,23] are of particular interest. It is due to their tunability, gate tuning for the RSOI and either sample thickness or electron-density tuning for the DSOI, and to their physical origins, being independent of disorder that requires the presence of SOI impurities. Yet the ever present background scatterers do play a subtle role in the intrinsic spin-Hall effect.[15] In spin-Hall effect (SHE), an external electric field induces a transverse spin current and, in turn, an out-of-plane spin accumulation S_z at lateral edges.[1,5–9] For intrinsic SOIs, the background scatterers lead to a complete quenching of the edge spin accumulation S_z when the SOI depends only linearly on the electron momentum k,[15] but S_z maintains finite and dependent on the momentum relaxation time τ when the SOI has a cubic-k dependence.[14–17] Thus, separately considered, the RSOI does not contribute to edge spin accumulation S_z while the cubic-k DSOI does. For a more realistic situation, when the two SOIs coexist in a sample, RSOI could exert its effect on the edge spin accumulation S_z, but that would have to be mediated through the cubic-k DSOI. It is of great interest to see whether this effect would be reinforcing or competing for S_z.

Thus, in this work, we focus upon the interplay between the RSOI and the cubic-k DSOIs in their combined, or competing, effects on both the edge spin accumulation S_z and the bulk spin density S_i^B. Bulk spin density S_i^B, formed in an external electric field, is another important physical quantity of interest that is closely related to the intrinsic SOIs. The subscript i denotes the vector component of spin. The effect of the background scatterers on S_i^B is less subtle than that on S_z:S_i^B, remains finite for all intrinsic SOIs and depends on τ also.[3] Intuitively, up to leading order in the SOI coupling constant one might expect this S_i^B feature to arise from a SOI-effective magnetic field.[4] It turns out to be the case when there is only one dominated SOI and the SOI depends on k linearly. Take, for instance, a Rashba-type two-dimensional electron gas (2DEG) in the diffusive regime, the k-dependent effective magnetic field becomes $\langle \mathbf{h_k} \rangle = -\alpha \mathbf{z} \times \langle \mathbf{k} \rangle$ when $\langle \mathbf{k} \rangle$ is averaged over the electron distribution given by a shifted Fermi sphere $f(\epsilon,\mathbf{k}) = f_0(\epsilon,\mathbf{k}) - \frac{\tau e \hbar \mathbf{k}\cdot\mathbf{E}}{m^*}\delta(\epsilon_F - \epsilon)$, where α, f_0, and m^* are, respectively, the Rashba coupling constant, Fermi-Dirac distribution, and electron mass and for $e > 0$. With $\langle \mathbf{h_k} \rangle = \alpha \tau e/\hbar \hat{\mathbf{z}} \times \mathbf{E}$, the bulk spin density, in units of $\hbar$, is given by $\mathbf{S}^B = -N_0 \alpha \tau e/\hbar \hat{\mathbf{z}} \times \mathbf{E}$, which was first obtained by Edelstein.[3] In the above expression the density of states per spin is denoted by N_0. Beyond leading order or linear k dependence in the SOIs, or for the coexistence of different types of SOIs, the derivation of S_i^B becomes more involved. In this work, we calculate the S_i^B within a spin-diffusion equation approach and perform a systematic study on the competing interplay between the RSOI and the cubic-k SOIs.

Interplay between the RSOI and the linear-k DSOI in a sample has attracted much attention lately.[18–29] Earlier work studied the effect of $\alpha = \tilde{\beta}$, where $\tilde{\beta}$ is the effective linear-k DSOI coupling constant, on the magnetoconductivity.[18] More recent work on the same $\alpha = \tilde{\beta}$ regime pointed out that the spin becomes a good quantum number, independent of k, and has a long relaxation time.[19] The D'yakonov-Perel' (DP) mechanism[1] for spin relaxation is suppressed. This finding led to proposals for spintronic transistor that would manipulate polarized spin transport in the diffusive regime.[19,20] It was later shown, within the same $\alpha/\tilde{\beta} = 1$ regime, that the Fermi circles of opposite spins are connected by a wave vector $\mathbf{Q}$ that depends only on the SOI constant and the effective mass.[24] This leads to the persistent spin-helix state.[24] Since the ratio $\alpha/\tilde{\beta}$ is important for the development of spintronics, and the transport is anisotropic when both α and $\tilde{\beta}$ are

Spin generation in a Rashba-type diffusive electron system by nonuniform driving field

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We show that the Rashba spin-orbit interaction contributes to edge spin accumulation S_z in a diffusive regime when the driving field is nonuniform. Specifically, we solve the case of nonuniform driving field in the vicinity of a circular void locating in a two-dimensional electron system and we identify the key physical process leading to the edge spin accumulation. The void has radius R_0 in the range of spin-relaxation length l_{so} and is far from both source and drain electrodes. The key physical process we find is originated from the nonuniform in-plane spin polarizations. Their subsequent diffusive contribution to spin current provides the impetus for the edge spin accumulation S_z at the void boundary. The edge spin accumulation is proportional to the Rashba coupling constant α and is in a spin-dipole form oriented transversely to the driving field. We expect similar spin accumulation to occur if the void is at the sample edge.

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I. INTRODUCTION

A major goal for the semiconductor spintronics is to generate and to manipulate spin polarization by mere electrical means. Spin-orbit interaction (SOI) provides the key leverage and spin-Hall effect (SHE) (Refs. 1–15) provides the key paradigm, where it is possible for a uniform driving electric field to induce bulk spin polarization and spin current and, in turn, out-of-plane spin accumulations S_z at lateral edges. The Rashba SOI (RSOI) (Ref. 16) is of particular interest because of its gate-tuning capability. However, background scatterers lead to a complete quenching of the RSOI's contribution to the edge spin accumulation S_z, a direct consequence of its linear dependence on the electron momentum k.[17,18] It is legitimate then to find ways to restore the RSOI's contribution to the edge spin accumulation. Our interest here is in the diffusive regime, when the spin-relaxation length $l_{so} \gg l_e$, the mean-free path. Even though the spin accumulation is finite in the mesoscopic ballistic regime ($l_{so} < l_e$, and $L < l_\phi$),[19–21] with L the sample size and l_ϕ the phase coherent length, it is still important to see whether the RSOI alone can contribute to SHE in the impurity-dominate regime.

Indeed, RSOI was found by Mishchenko *et al.*[18] to give rise to edge spin accumulation S_z near electrodes even though its contribution to bulk spin current vanishes. The edge spin accumulation is concentrated at the two ends of an electrode-sample interface, covering a region of size l_{so}. This finding was identified to arise from a nonzero spin current I_y^z flowing along the sample-electrode interface, in direction $\hat{y}$.[18] This nonzero spin current was understood from the way the spin current vanishes in the bulk, when an exact cancellation occurs between two terms, one related to the spin polarization and the other related to the driving field.[18] This exact cancellation no longer holds at the sample-electrode interface, when the driving field has reached its bulk value but the spin polarization has not. Similar result was also obtained by Raimondi *et al.*,[22] where spin-density spatial profiles at the sample corners were obtained. Yet, it would be more desirable that we can find schemes and identify physical processes for the restoring of the RSOI-induced edge spin accumulation at locations other than the sample-electrode interfaces and according to our specification.

In this work, we turn to nonuniform driving field for the restoration of the RSOI-induced spin accumulation. The effect of nonuniform driving field on spin accumulation is also interesting in its own right. Earlier study considered nonuniform driving field in systems in the presence of "extrinsic" SOI, that is, SOI due to SOI impurities.[23] Here, instead, we consider nonuniform driving field in the vicinity of a circular void located in a diffusive RSOI-type two-dimensional electron gas (2DEG). We obtain spin accumulation in the vicinity of the void. This problem allows us to identify the key physical process for the spin accumulation and also sheds light on the case if the void were to form at a lateral edge. The radius R_0 of the void is of the order of l_{so}.

Most important is our finding that the main physical process is in marked contrast to the conventional one. While the conventional one is associated with the nonvanishing of the out-of-plane spin current I_n^z,[18] the key process we find is associated with the in-plane spin currents, I_n^x or I_n^y, and with the way they vanishes at the void boundary. Here, $\hat{n}$ denotes the flow direction normal to the void boundary. The in-plane spin currents consist of two terms, a diffusive term and a term related to the spin accumulation S_z, which are given by

$$I_\rho^i = -2D\frac{\partial}{\partial\rho}S_i - R^{izi}S_z\hat{\rho}\cdot\hat{i}, \tag{1}$$

where $\boldsymbol{\rho}$ is the position vector measured from the center of the void, $i \in \{x,y\}$, and D is the diffusion constant.[24] R^{ijk} denotes the precession of S_j into S_i when it flows along $\hat{k}$. The factor $\hat{\rho}\cdot\hat{i}$ denotes the projection of the flow. The RSOI governs the symmetry of R^{ijk} such that $R^{izj}=0$ for $i \neq j$.

In attending this conference, I had the opportunity to know in person quite a number of active physicists, listened to and learned from their recent work, and might establish possible collaborations in the future. For example, the second renormalization of tensor network state method presented by Professor X. Tao had been inspiring for me, particularly when I am considering topological insulator characteristics for different lattices recently.

三、建議

四、其他