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多屬性多方法潛伏成長模型及其實證應用 Latent Growth Modeling with MTMM and Its Empirical Applications

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一、中英文摘要

本研究計畫為兩年期計畫，分兩部分進行，第一部分為一般多屬性多方法潛伏成長模型之建立，第二部分運用第一部分所提出之模型從事投資人情緒實證研究。第一部分的重點在於成長模型中誤差共變結構之設定方法，已完成並撰文投稿 *Structural Equation Modeling* 期刊，研究進度符合預期。我們示範如何利用 SAS PROC CALIS/TCALIS 進行誤差共變結構之配適，並提出一基於卡方差異檢定之有效方法以發掘 level-1 誤差共變結構，另具體說明如何以 PROC TCALIS 中之 SIMTEST statement 進行誤差穩態檢定，針對外顯變數與潛伏構念之成長模型，皆提供對應的 SAS 語法。

關鍵詞：多屬性多方法，潛伏成長模型，二階成長模型，誤差共變結構，卡方差異檢定，穩態

This project consists of two parts. The first part is to develop a general multitrait-multimethod (MTMM) latent growth model. The second part is to apply the model proposed to an empirical study in investor sentiment. The focus of the first part is how to identify the error covariance structure. The task has been completed, with a paper submitted to *Structural Equation Modeling*. Using SAS PROC CALIS/TCALIS to fit error covariance structures of latent growth models (LGM) has been illustrated. A tutorial on the SAS syntax is provided for both manifest variables and latent constructs in LGM. While the second-level error covariance structure is usually specified as unstructured, an effective approach for identifying the first-level error covariance structure based on the sequential chi-square difference test is proposed and demonstrated. Moreover, how to test for stationarity of an error process by using the SIMTEST statement in PROC TCALIS is specifically addressed.

Keywords: multitrait-multimethod (MTMM), latent growth model, second-order growth model, error covariance structure chi-square difference test, stationarity

二、報告内容

The latent growth model (LGM) plays an important role in repeated-measure analysis over a limited occasions in large sample data (e.g., Meredith & Tisak, 1990; Muthén & Khoo, 1998; Preacher, Wichman, MacCallum, & Briggs, 2008, p.12; Singer & Willett, 2003, p.9). The model can not only characterize intraindividual (within-subject) change over time but also examine interindividual (between-subject) difference by means of a random intercept and random slopes, and is a typical application of the multilevel model (or hierarchical linear model (HLM)). The between-subject errors (representing random effects for the intercept and slopes) and the within-subject errors over time are conventionally referred to as level-2 and level-1 errors, respectively.

LGM is also an application of structural equation modeling (SEM) (e.g., Bauer, 2003; Boolean & Curran, 2006; Curran, 2003; Duncan, Duncan, & Hops, 1996; Mehta & Neal 2005; Meredith & Tisak, 1990; Willet & Sayer, 1994). The structural modeling of multilevel data is a relatively new area of methodological research. SEM and HLM stem from different traditional statistical theory, and each has developed its own terminology and standard ways of framing research questions. However, it has become clear that there exists much overlap between the two methodologies under some circumstances. More specifically, when a two-level data structure arises from the repeated observations of a set of individuals over time (such that time is hierarchically nested within an individual), SEM is analytically equivalent to HLM (e.g., Bauer, 2003; Curran, 2003; MacCallum, Kim, Malarkey, & Kiecolt-Glaser, 1997; Raudenbush, 2001; Rovine & Molenaar, 2000; Willett & Sayer, 1994). Thus, despite the inherent differences between the estimation procedures by SEM and HLM, these two approaches provide analytically identical solutions for LGM. Nevertheless, the measurement model capabilities of SEM possess superiority in examining model fit to the HLM approach. The SEM approach also brings the possibility of modeling of change over time for latent constructs or multivariate data (e.g., Bollen & Curran, 2006, Chap. 7, 8; Chan, 1998; Duncan, Duncan, & Strycker, 2006, Chap. 4; Hancock, Kuo, & Lawrence, 2001; MacCallum, et al., 1997; Rovine & Molenaar, 2000). As a result, SEM has become a more commonly used approach for longitudinal data. Specialized software such as Mplus (Muthén & Muthén, 2007), Mx (Neale, Boker, Xie, & Maes, 2003), AMOS (Arbuckle, 2006), EQS (Bentler & Wu, 2005), Lisrel (Joreskog & Sorbom, 2004), and SAS PROC CALIS (TCALIS in SAS 9.2) (SAS Institute Inc., 2007) are readily available and allow individuals to be measured at unique times, having unequal spacing between assessments.

Since errors within the same subject are often correlated across time (e.g., Preacher et al., 2008, p.63; Sayer & Cumsille, 2001), not modeling autocorrelated errors present in longitudinal data or misspecifying error covariance structure has a substantial impact on the inferences for model parameters (e.g., Ferron, Dailey, & Yi, 2002; Kwok, West, & Green, 2007; Murphy & Pituch, 2009; Sivo, Fan, & Witt, 2005). A variety of processes underlying level-1 errors can be used (e.g., Newsom, 2002; Singer & Willett, 2003, Chap. 7; Wolfinger, 1996). The HLM software (Raudenbush, Bryk, & Congdon, 2005) provides the first-order autoregressive (AR(1)) option. The MIXED procedure in SAS/STAT (SAS Institute Inc., 2007) contains many types of level-1 error processes such as unstructured (UN), compound symmetry (CS), AR(1), first-order autoregressive moving average (ARMA(1,1)), Toeplitz (TOEP), Toeplitz with q bands (TOEP(q)), etc. The processes are either stationary or nonstationary. The most general covariance structure is the so-called unstructured model, indicating that no structure is imposed and covariance estimates are to be determined by the data. However, the use of UN is likely to inflate type I error rates for the tests of the fixed effects (Kwok, West, & Green, 2007; Murphy & Pituch, 2009). Moreover, the number of parameters to be estimated becomes excessive and can cause convergence problems, especially when series length is long. Autocorrelations among level-1 errors are generally regarded as a nuisance. Although the error covariance structure is not of theoretical interest, failing to add its specification to the growth model would bias parameter estimates of primary interest (Sivo & Fan, 2008). While its specification is difficult to determine based on theory, how to conduct an effective specification search becomes needed (Kwok, West, & Green, 2007), and will be discussed in this study.

The growth curve ARMA(p, q) model has been proposed to filter out the effects of error autocorrelation on parameter estimates (e.g., Sivo et al., 2005; Sivo & Fan, 2008). The AR part is specified to represent the current value of a time series as a function of previous values of the same time series, and the MA part is specified to represent the current value of a time series as a linear function of the current and previous disturbances, which are independent and identically distributed. Note that the growth curve ARMA(1, 1) model differs from the model specified by using the REPEATED statement with TYPE=ARMA(1,1) in SAS PROC MIXED. The within-subject error covariance matrix (ECM) with an ARMA(1,1) structure can be well captured by the latter only. In fact, PROC MIXED, based on the HLM approach, contains more than 30 different types of level-1 preprogrammed structures. Although it is a powerful tool, how to identify an appropriate structure is not documented. Besides, PROC MIXED cannot handle higher-order latent growth models. To improve, use PROC CALIS/

TCALIS, which is based on the SEM approach. How modeling longitudinal change can be effectively done under a broad class of error covariance structures based on the SEM framework is rarely seen, and will be specifically addressed in this study. The purpose of this article is to give a tutorial on the syntax using PROC CALIS/TCALIS for handling longitudinal change in a manifest variable and that in a latent construct, known as a second-order growth model (Hancock, Kuo, & Lawrence, 2001). For the purpose of verification, we use the data generated from a given model so that the syntax can be checked for correctness by comparing if parameter estimates obtained are close to the population values. In addition, PROC MIXED will be used with respect to the manifest variable to further confirm the results obtained. Two sample SAS programs will be provided. One is for modeling longitudinal change in a manifest variable, and the other for modeling the change in a latent construct. In this study, an effective approach for identifying the first-level error covariance structure based on the sequential chi-square difference test will be proposed. Moreover, how to test if an error process is stationary by using PROC TCALIS will be demonstrated.

LATENT GROWTH MODEL

In this section, we briefly introduce the LGM with a variety of level-1 error covariance structures through a typical example depicted in Figure 1. In the figure, y_1 – y_4 denote repeated-measures on four occasions and X a level-2 predictor. η_{α_i} is the random intercept representing the initial status for individual i , η_{β_i} is the random slope showing the individual's linear rate of change per unit increase in time. The first-level model can be written as

$$\mathbf{y} = \mathbf{\Lambda}_y^* \boldsymbol{\eta} + \boldsymbol{\varepsilon}, \quad (1)$$

where $\mathbf{y} = [y_1 \ y_2 \ y_3 \ y_4]'$, $\mathbf{\Lambda}_y^* = \begin{bmatrix} 1 & 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ T_1 - 1 & T_2 - 1 & T_3 - 1 & T_4 - 1 \end{bmatrix}$, $\boldsymbol{\eta} = [\eta_{\alpha} \ \eta_{\beta}]'$, and $\boldsymbol{\varepsilon} = [\varepsilon_1 \ \varepsilon_2 \ \varepsilon_3 \ \varepsilon_4]'$. λ_t is the measurement time points ($T_t = 1, 2, 3, 4$), and $\boldsymbol{\varepsilon}$ denotes level-1 errors. The double-headed curved arrows presented in Figure 1 indicate that ε_t are correlated. The factor loading associated with initial status are all fixed at 1, whereas those associated with the slope are set at the value λ_t to reflect the particular time point t for individual i . A common coding of λ_t for different time points is to set $\lambda_1 = 0$ for baseline

and $\lambda_t = T_t - 1$ for the follow-ups. For this model, subject i 's growth trajectory is a straight line, $\eta_{\alpha_i} + \lambda_t \eta_{\beta_i}$, $\lambda_t = 0, 1, 2, 3$.

 Insert Figure 1 about here

The second-level model can be written as

$$\boldsymbol{\eta} = \boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_x \mathbf{x} + \boldsymbol{\zeta}_\eta, \quad (2)$$

where $\boldsymbol{\Gamma}_0 = [\gamma_{00} \ \gamma_{01}]'$, $\boldsymbol{\Gamma}_x = [\gamma_{10} \ \gamma_{11}]'$, $\mathbf{x} = [X]$, $\boldsymbol{\zeta}_\eta = [\zeta_{\eta_\alpha} \ \zeta_{\eta_\beta}]$. Both growth factors (intercept and slope), η_α and η_β , are influenced by a subject-level predictor X . γ_{00} and γ_{10} denote, respectively, the intercept and slope of the regression of η_α on X , γ_{01} and γ_{11} are those of η_β on X , and ζ_{η_α} and ζ_{η_β} are level-2 errors. It is assumed that $\boldsymbol{\zeta}_\eta$ and $\boldsymbol{\varepsilon}$ are uncorrelated. The models can be rewritten in combined form as

$$\mathbf{y} = \boldsymbol{\Lambda}_y^* (\boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_x \mathbf{x}) + \boldsymbol{\Lambda}_y^* \boldsymbol{\zeta}_\eta + \boldsymbol{\varepsilon}, \quad (3)$$

based on which the mean vector $\boldsymbol{\mu}$ and the covariance matrix $\boldsymbol{\Sigma}$ of the manifest variables y_1 – y_4 and X can be expressed as functions of the model parameters as follows: (Bollen & Curran, 2006, p.134-135):

$$\boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}_y \\ \boldsymbol{\mu}_x \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Lambda}_y^* (\boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_x \boldsymbol{\mu}_x) \\ \boldsymbol{\mu}_x \end{bmatrix}, \quad (4)$$

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Lambda}_y^* (\boldsymbol{\Gamma}_x \boldsymbol{\Sigma}_{xx} \boldsymbol{\Gamma}_x' + \boldsymbol{\Psi}_{\zeta_\eta}) \boldsymbol{\Lambda}_y^{*'} + \boldsymbol{\Theta}_\varepsilon & \boldsymbol{\Lambda}_y^* \boldsymbol{\Gamma}_x \boldsymbol{\Sigma}_{xx} \\ \boldsymbol{\Sigma}_{xx} \boldsymbol{\Gamma}_x' \boldsymbol{\Lambda}_y^{*'} & \boldsymbol{\Sigma}_{xx} \end{bmatrix}, \quad (5)$$

where $\boldsymbol{\Theta}_\varepsilon$ and $\boldsymbol{\Psi}_{\zeta_\eta}$ denote the variance-covariance matrices of $\boldsymbol{\varepsilon}$ and $\boldsymbol{\zeta}_\eta$, respectively, and $\boldsymbol{\mu}_x$ and $\boldsymbol{\Sigma}_{xx}$ denote, respectively the mean vector and the variance-covariance matrix of predictors ($\boldsymbol{\mu}_x = \mu_X$ and $\boldsymbol{\Sigma}_{xx} = \sigma_X^2$ for this model since there is only one predictor).

The first-level errors, ε_1 , ε_2 , ε_3 , and ε_4 , are assumed to be normally distributed with zero means. The general form of the ECM is unstructured, and is given by

$$\Theta_{\varepsilon} = \begin{bmatrix} \sigma_{\varepsilon_1}^2 & & & \\ \sigma_{\varepsilon_2\varepsilon_1} & \sigma_{\varepsilon_2}^2 & & \\ \sigma_{\varepsilon_3\varepsilon_1} & \sigma_{\varepsilon_3\varepsilon_2} & \sigma_{\varepsilon_3}^2 & \\ \sigma_{\varepsilon_4\varepsilon_1} & \sigma_{\varepsilon_4\varepsilon_2} & \sigma_{\varepsilon_4\varepsilon_3} & \sigma_{\varepsilon_4}^2 \end{bmatrix}. \quad (6)$$

The corresponding option given in SAS PROC MIXED is TYPE=UN. Other types of ECM, with less parameters (parsimonious) may be desirable. The chi-square test resulting from PROC CALIS (not available by PROC MIXED) can be used to help determine an appropriate type. The second-level errors ζ_{η_α} and ζ_{η_β} are assumed to be normally distributed with zero means. Their covariance matrix is usually specified as unstructured (Murphy & Pituch, 2009):

$$\Psi_{\zeta_\eta} = \begin{bmatrix} \sigma_{\zeta_{\eta_\alpha}}^2 & \sigma_{\zeta_{\eta_\alpha}\zeta_{\eta_\beta}} \\ \sigma_{\zeta_{\eta_\alpha}\zeta_{\eta_\beta}} & \sigma_{\zeta_{\eta_\beta}}^2 \end{bmatrix}. \quad (7)$$

Types of Level-1 Error Covariance Structures and SAS Statements

Any type of ECM, Θ_{ε} or Ψ_{ζ_η} , can be expressed as a set of linear and/or nonlinear constraints. The SAS statements by PROC CALIS for specifying six types of the first-level error covariance structure are summarized in Table 1. They include the first-order autoregressive (AR(1)), the first-order autoregressive moving-average (ARMA(1,1)), the second-order autoregressive (AR(2)), heterogeneous ARH(1), heterogeneous Toeplitz (TOEPH), and unstructured (UN), commonly considered in time series analysis and among which there exist nested relationships (Kwok, West, & Green, 2007). Stationarity is assumed for the first three processes, and should be examined to justify the use of the corresponding ECM.

Insert Table 1 about here

Statements by SAS PROC MIXED. PROC MIXED provides a REPEATED statement, in which many types of the first-level error covariance structure can be specified through the TYPE= option (e.g., Singer, 1998).

Statements by SAS PROC CALIS. The STD, COV, and PARAMETERS statements in PROC CALIS can be used together to specify any type of ECM (Θ_ε or Ψ_{ζ_η}). The PARAMETERS statement defines additional parameters that are not specified in the models, and uses both the original and additional parameters for modeling ECM. In other words, each specific type of ECM is composed of functions of the original and additional parameters.

Example 1: Θ_ε Resulting from ARMA(1,1)

The ECM resulting from the stationary ARMA(1,1) process, defined as $\varepsilon_t = \phi_1 \varepsilon_{t-1} + v_t - \theta_1 v_{t-1}$, where ϕ_1 denotes the autoregressive parameter, θ_1 the moving average parameter, and v_t a white noise process (independent and identically distributed disturbances) (Box, Jenkins, & Reinsel, 1994, p.77), is given by

$$\Theta_\varepsilon = \sigma_\varepsilon^2 \begin{bmatrix} 1 & & & \\ \rho_1 & 1 & & \\ \rho_2 & \rho_1 & 1 & \\ \rho_3 & \rho_2 & \rho_1 & 1 \end{bmatrix}, \quad (8)$$

where $\rho_1 = \frac{(\phi_1 - \theta_1)(1 - \phi_1 \theta_1)}{(1 - 2\phi_1 \theta_1 + \theta_1^2)}$, $\rho_k = \phi_1 \rho_{k-1}$, $k = 2, 3$, with the constraints of $|\phi_1| < 1$ and $|\rho_1| < 1$. Program 1 in Appendix 1 illustrates how to use SAS PROC CALIS for modeling LGM with an ARMA(1,1) level-1 ECM and an unstructured level-2 ECM for four equally spaced time points. Under the assumption of stationarity for level-1 errors, their variances are equal, the autocovariances at lag 1 are equal, and the autocovariances at lag 2 are equal as well. Level-2 error variances/covariances are unstructured, as shown in Equation 7. Therefore, the STD and COV statements for specifying error variances and pairwise covariances are given as follows:

*****;

STD

E1=VARE, E2=VARE, E3=VARE, E4=VARE, D0=VARD0, D1=VARD1;

COV

E1 E2=COV_lag1, E2 E3=COV_lag1, E3 E4=COV_lag1,

E1 E3=COV_lag2, E2 E4=COV_lag2,

E1 E4=COV_lag3,

D0 D1=COVD0D1;

*****;

In the STD statement, VARE represents the estimate of the common variance σ_ε^2 of the four level-1 errors $\varepsilon_1 - \varepsilon_4$ (denoted by E1-E4), and VARD0 and VARD1 the estimates of the variances, $\sigma_{\zeta_{\eta_\alpha}}^2$ and $\sigma_{\zeta_{\eta_\beta}}^2$, of the two level-2 errors ζ_{η_α} and ζ_{η_β} (denoted by D0 and D1).

In the COV statement, COV_lag1 and COV_lag2 represent, respectively, the common level-1 error autocovariance estimates at lag 1 and lag 2. COV_lag3 is the estimate of the error autocovariance at lag 3. CD0D1 is the estimate of $\sigma_{\zeta_{\eta_\alpha} \zeta_{\eta_\beta}}$. Parameters ρ_1 and ϕ_1 defined in

Equation 8 and their relationships with the autocovariances can be specified by using the following PARAMETERS statement. Starting values for estimating ρ_1 and ϕ_1 are needed and should appear in the parenthesis. Population parameters are used as starting values to achieve convergence more efficiently for simulation studies (Paxton, Curran, Bollen, Kirby, & Chen, 2001).

*****;

PARAMETERS

PHI1 RHO1 (0.6 0.7);

COV_lag1=RHO1*VARE;

COV_lag2=PHI1* COV_lag1; /* i.e., COV_lag2=PHI1*RHO1* VARE; */

COV_lag3=PHI1* COV_lag2; /* i.e., COV_lag3=(PHI1**2)*RHO1*VARE; */

*****;

‘COV_lag1=RHO*VARE’ corresponds to the requirement that the common autocovariance at lag 1 be equal to $\sigma_\varepsilon^2 \rho_1$. The syntax corresponding to the requirements for the common autocovariances at lag 2 ($=\sigma_\varepsilon^2 \phi_1 \rho_1$) and lag 3 ($=\sigma_\varepsilon^2 \phi_1 \rho_2 = \sigma_\varepsilon^2 \phi_1^2 \rho_1$) is given in a similar way.

The constraints of $|\phi_1| < 1$ and $|\rho_1| < 1$ are specified by the following BOUNDS statement:

*****;

BOUNDS

-1. < PHI1 < 1., -1. < RHO1 < 1.;

*****;

Example 2: Θ_ε Resulting from TOEPH

The ECM resulting from heterogeneous Toeplitz is given by

$$\Theta_\varepsilon = \begin{bmatrix} \sigma_{\varepsilon_1}^2 & & & & \\ \sigma_{\varepsilon_2} \sigma_{\varepsilon_1} \rho_1 & \sigma_{\varepsilon_2}^2 & & & \\ \sigma_{\varepsilon_3} \sigma_{\varepsilon_1} \rho_2 & \sigma_{\varepsilon_3} \sigma_{\varepsilon_2} \rho_1 & \sigma_{\varepsilon_3}^2 & & \\ \sigma_{\varepsilon_4} \sigma_{\varepsilon_1} \rho_3 & \sigma_{\varepsilon_4} \sigma_{\varepsilon_2} \rho_2 & \sigma_{\varepsilon_4} \sigma_{\varepsilon_3} \rho_1 & \sigma_{\varepsilon_4}^2 & \end{bmatrix}, \quad (9)$$

where σ_{ε_t} denotes the standard deviation for ε_t , $t = 1, 2, 3, 4$, and ρ_j the autocorrelation at lag j , $j = 1, 2, 3$, with the constraints of $|\rho_j| < 1$, $j = 1, 2, 3$. The level-1 error variances are allowed to be unequal but the autocorrelations at the same lag are equal. The STD and COV statements used for specifying estimates of error variances and pairwise covariances are given as follows:

*****;

STD

E1=VARE1, E2=VARE2, E3=VARE3, E4=VARE4,

D0=VARD0, D1=VARD1;

COV

E1 E2=COVE1E2, E1 E3=COVE1E3, E1 E4=COVE1E4,

E2 E3=COVE2E3, E2 E4=COVE2E4, E3 E4=COVE3E4,

D0 D1=COVD0D1;

*****;

VARE1–VARE4 represent the estimates of the four level-1 error variances, and VARD0 and VARD1 those of the two level-2 error variances. COVE1E2–COVE3E4 represent the corresponding level-1 error autocovariance estimates, and COVD0D1 the level-2 error autocovariance estimate. Since $\sigma_{\varepsilon_t \varepsilon_{t'}} = \sigma_{\varepsilon_t} \sigma_{\varepsilon_{t'}} \rho_{\varepsilon_t \varepsilon_{t'}}$ and the autocorrelations at the same lag are

constrained to be equal, the following PARAMETERS statement needs to be added:

```
*****
PARAMETERS
  RHO1 RHO2 RHO3 (0.7 0.6 0.7);
  COVE1E2=SQRT(VARE1)*SQRT(VARE2)*RHO1;
  COVE1E3=SQRT(VARE1)*SQRT(VARE3)*RHO2;
  COVE1E4=SQRT(VARE1)*SQRT(VARE4)*RHO3;
  COVE2E3=SQRT(VARE2)*SQRT(VARE3)*RHO1;
  COVE2E4=SQRT(VARE2)*SQRT(VARE4)*RHO2;
  COVE3E4=SQRT(VARE3)*SQRT(VARE4)*RHO1;
```

The constraints of $|\rho_j| < 1$ ($j = 1, 2, 3$) can be handled by using the BOUNDS statement given below.

```
*****
BOUNDS
  -1. < RHO1 < 1. , -1.< RHO2 <1. , -1.< RHO3 < 1.;
```

The LINEQS statement used for this example is the same as that given in Example 1. Although there are many defaulted error covariance structures in PROC MIXED, modification is not allowed for any of them. In contrast, PROC CALIS can be used to specify any error covariance structure to meet researchers' need. For example, the AR(2) process, given by $\varepsilon_t = \phi_1 \varepsilon_{t-1} + \phi_2 \varepsilon_{t-2} + \nu_t$, where ϕ_1 and ϕ_2 are autoregressive parameters and ν_t a white noise process (Box, Jenkins, & Reinsel, 1994, p.54), leads to the following level-1 ECM:

$$\sigma_\varepsilon^2 \begin{bmatrix} 1 & & & \\ \rho_1 & 1 & & \\ \rho_2 & \rho_1 & 1 & \\ \rho_3 & \rho_2 & \rho_1 & 1 \end{bmatrix}, \quad (10)$$

where $\rho_0 = 1$, $\rho_1 = \phi_1 / (1 - \phi_2)$, and $\rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2}$, $k = 2, 3$, with the constraints of $|\phi_2| < 1$, $\phi_2 + \phi_1 < 1$, and $\phi_2 - \phi_1 < 1$. It is not possible to model AR(2) for level-1 errors by using PROC MIXED, but the task can be done by using PROC CALIS, with the statements shown in Table 1. Note that the last two constraints are specified by using the LINCON

statement. PROC CALIS is certainly more flexible than PROC MIXED.

SAS statements by PROC CALIS for modeling other types of stationary and nonstationary level-1 error processes such as those presented in Appendix 3 can be obtained in a similar way.

AN EFFECTIVE APPROACH FOR IDENTIFYING THE FIRST-LEVEL ERROR COVARIANCE STRUCTURE

By following the sequential chi-square difference test (SCDT) given by Anderson and Gerbing (1988), an effective approach is proposed to identify the first-level error covariance structure as follows:

Stage 1: Testing for stationarity of the first-level error process.

The conditions of stationarity include the equality of error variances and the equality of error autocovariances at any lag (Box, Jenkins, & Reinsel, 1994, p. 24-26). For example, for the model shown in Figure 1, the null hypothesis of stationarity is given by H_{01} : $\sigma_{\varepsilon_1}^2 = \sigma_{\varepsilon_2}^2 = \sigma_{\varepsilon_3}^2 = \sigma_{\varepsilon_4}^2$, $\sigma_{\varepsilon_2\varepsilon_1} = \sigma_{\varepsilon_3\varepsilon_2} = \sigma_{\varepsilon_4\varepsilon_3}$, $\sigma_{\varepsilon_3\varepsilon_1} = \sigma_{\varepsilon_4\varepsilon_2}$. To test H_{01} , we use the SIMTEST statement in PROC TCALIS, in addition to the STD and COV statements given in Example 2, as follows:

```
*****;
SIMTEST  ERR_STATIONARY_TEST = [VAREQ_1 VAREQ_2 VAREQ_3
                                COVLag1EQ_1 COVLag1EQ_2 COVLag2EQ];
VAREQ_1=VARE1-VARE2;
VAREQ_2=VARE1-VARE3;
VAREQ_3=VARE1-VARE4;
COVLag1EQ_1=COVE1E2-COVE2E3;
COVLag1EQ_2=COVE1E2-COVE3E4;
COVLag2EQ=COVE1E3-COVE2E4;
*****;
```

The null hypothesis given above can be reexpressed as H_{01} : $\sigma_{\varepsilon_1}^2 - \sigma_{\varepsilon_2}^2 = 0$, $\sigma_{\varepsilon_1}^2 - \sigma_{\varepsilon_3}^2 = 0$, $\sigma_{\varepsilon_1}^2 - \sigma_{\varepsilon_4}^2 = 0$, $\sigma_{\varepsilon_2\varepsilon_1} - \sigma_{\varepsilon_3\varepsilon_2} = 0$, $\sigma_{\varepsilon_2\varepsilon_1} - \sigma_{\varepsilon_4\varepsilon_3} = 0$, $\sigma_{\varepsilon_3\varepsilon_1} - \sigma_{\varepsilon_4\varepsilon_2} = 0$, in which six functions of the model parameters, named VAREQ_1, VAREQ_2, VAREQ_3, COVLag1EQ_1, COVLag1EQ_2, and COVLag2EQ, respectively, are all equal to zero. To conduct a simultaneous test for the six functions by using the SIMTEST statement, we first assign a

name, ERR_STATIONARY_TEST, for the test, and then put the function names inside a pair of brackets '[' and ']'. The six functions are defined in the SAS programming statements, and follow after the SIMTEST statement. VAREQ_1, VAREQ_2, and VAREQ_3 represent the differences between $\sigma_{\varepsilon_1}^2$ and each of the other error variances. COVLag1EQ_1 and COVLag1EQ_2 represent, respectively, $\sigma_{\varepsilon_2\varepsilon_1} - \sigma_{\varepsilon_3\varepsilon_2}$ and $\sigma_{\varepsilon_2\varepsilon_1} - \sigma_{\varepsilon_4\varepsilon_3}$, the two pairwise comparisons among the three error covariances at lag 1. COVLag2EQ is the contrast between the two error covariances at lag 2. The SIMTEST statement would lead to a chi-square test (with 6 degrees of freedom) for examining stationarity.

If stationarity is supported, then go to Stage 2A; otherwise go to Stage 2B.

Stage 2A: Identifying the type of stationary process using SCDT.

We fit the model with stationary level-1 error processes sequentially, starting from TOEP(1), the most constrained one (the most parsimonious stationary one as well), followed by the order of AR(1), (ARMA(1,1) or AR(2)), and TOEP, according to the degree of constraint. TOEP, the least constrained (the least parsimonious) structure, is actually the saturated stationary one. At each step, we first test if the model fit using the current structure, denoted by M_T , is significantly different from the model fit using TOEP, denoted by M_S , with the chi-square difference test. The difference of the chi-square statistic for M_T and that for M_S is distributed as a chi-square distribution with degrees of freedom equaling the difference of the number of parameters within the two ECM. The significant result indicates that the current M_T possesses significantly worse model fit, and we enter the next step by updating M_T with a less constrained structure. The test is continued until the fit does not show significant difference (denoted as $M_T - M_S = 0$), and M_T is the error covariance structure we need.

Stage 2B: Identifying the type of nonstationary process using SCDT.

The SCDT is conducted for nonstationary processes in a similar way as Stage 2A, following the order of UN(1), ARH(1), TOEPH, and UN. UN(1) is the most parsimonious of all. UN is the saturated structure.

The reasons to recommend TOEP(1), AR(1), (ARMA(1,1) or AR(2)), and TOEP in Stage 2A, and UN(1), ARH(1), TOEPH, and UN in Stage 2B are simply because they are nested and easy to interpret. More detailed structures may also be used if they can meet the same conditions. A flow chart for identifying the first-level error covariance structure is given in Figure 3.

Illustrations

Two illustrations are given. The first one is based on the data generated from the linear growth model shown in Figure 1 with the TOEPH level-1 error structure and the UN level-2 error structure. Population parameters and the sample covariance matrix of y_1 - y_4 and X are given in Table 2. The sample size of 300 was used (Muthén & Muthén, 2002).

Insert Table 2 about here

Following the approach for identifying the first-level error covariance structure, we first test for stationarity. Since stationarity was rejected ($\chi^2_{df=6} = 123.93, p < .0001$), we proceeded with a specification search for a nonstationary process. Error covariance structures were fit sequentially. We started by fitting UN(1). Since the model fit between UN(1) and UN was significant (chi-square difference = 15.903 with $df = 6, p = .014$), UN(1) should not be adopted. Then we fit ARH(1), a less constrained one. Since the model fit between ARH(1) and UN became insignificant (chi-square difference = .441 with $df = 5, p = .994$), the sequential search was terminated by choosing ARH(1) as the first-level error covariance structure. The improvement of ARH(1) over UN(1) can be verified by the significant results between UN(1) and ARH(1) (chi-square difference = 15.462 with $df = 1, p < .0001$). Although the final structure identified, ARH(1), is not the one specified in the population model (TOEPH), their performance in model fit was insignificant (chi-square difference = .441 with $df = 2, p = .802$). ARH(1) is preferred since it is more parsimonious (with two less parameters). The results of SCDT are summarized in Table 3. The parameter estimates of the final model (with ARH(1)) are also reported in the same table by using both PROC CALIS (the SEM approach) and PROC MIXED (the HLM approach) for the purpose of verification. It appears that parameter estimates resulting from the two approaches are pretty close. The model fit was satisfactory ($\chi^2_{df=6} = 1.483, p = .961, AGFI = .996, RMSEA = .000$). However, the test for stationarity and the sequential chi-square difference tests that can be implemented by PROC CALIS cannot work under PROC MIXED. Moreover, while the estimates of Θ_ε and $\Psi_{\zeta_{\eta}}$ could be obtained by both PROC MIXED and PROC CALIS, the significance test for error covariances can be achieved by PROC CALIS only. Two approaches yield about the

same results when PROC MIXED works. PROC CALIS supplies richer information in model fit and parameter inference.

Insert Table 3 about here

The second illustration is based on the data given in Singer and Willett (2003, Chap. 7). 35 people completed an inventory measuring their performance on a timed task called “opposite’s naming” on each of four days, spaced exactly one week apart. At wave 1, they also completed a standardized instrument assessing general cognitive skill, used as a subject-level predictor. The growth model used is exactly the one shown in Figure 1. Since stationarity was not rejected ($\chi^2_{df=6} = 3.302, p = .77$), the covariance structure to be identified should come from the pool of stationary processes. Error covariance structures were fit sequentially, by using TOEP(1) in the first step. The results are summarized in Table 4. Since the model fit between TOEP(1) and TOEP was insignificant (chi-square difference = 2.801 with $df = 4, p = .592$), no further examination was needed, and we selected TOEP(1) to be the first-level error covariance structure. The level-1 errors were uncorrelated and identically distributed in this case. The model fit was good ($\chi^2_{df=10} = 6.68, p = .755, AGFI = .987, RMSEA = .000$). The parameter estimates by fitting TOEP(1) are also given in the same table by using both PROC CALIS and PROC MIXED. Again, the results are close to each other.

Insert Table 4 about here

SECOND-ORDER LATENT GROWTH MODELS

When the growth trajectory is to be analyzed for a latent construct, measured with multiple indicators, only the SEM approach can work, based on the second-order LGM (e.g. Bollen & Curran, 2006, Chap. 8; Hancock, Kuo, & Lawrence, 2001; Preacher et al., 2008, Chap. 3; Sayer & Cumsille, 2001). In Figure 2, latent construct F is measured with three indicators, y_1 – y_3 , which are observed at four occasions. The constructs at the four occasions, denoted by F_1 – F_4 , are termed the first-order factors, and the growth factors (i.e., intercept and slope),

denoted by η_α and η_β , are termed the second-order factors. There exist a predictor construct, ζ , for the growth factors, and it is measured with three indicators, x_1-x_3 . The errors associated with the indicators y_1-y_3 , ε_{jt} , $j = 1, 2, 3$; $t = 1, 2, 3, 4$, and those associated with the first-order factors, $\zeta_{F_1} - \zeta_{F_4}$, are level-1 errors. The errors associated with the second-order factors, ζ_{η_α} and ζ_{η_β} , and those associated with the indicators x_1-x_3 , $\delta_1-\delta_3$, are level-2 errors. The level-1 errors associated with the same indicator at different points in time ($\varepsilon_{j1} - \varepsilon_{j4}$, $j = 1, 2, 3$) are serially correlated. They are linked in the figure by three solid lines with four arrowheads, one line for each indicator. $\zeta_{F_1} - \zeta_{F_4}$ are serially correlated as well (Sivo, 2001). They are linked by a dash line with four arrowheads. Their covariance structures need to be well identified, and can be done by using the approach proposed in the previous section. Other errors are assumed to be uncorrelated.

 Insert Figure 2 about here

The second-order LGM pictorially presented in Figure 2 can be represented in matrix form as follows:

$$\begin{aligned}
 \mathbf{y} &= \Lambda_y \mathbf{F} + \boldsymbol{\varepsilon}, \\
 \mathbf{x} &= \Lambda_x \zeta + \boldsymbol{\delta}, \\
 \mathbf{F} &= \Lambda_y^* \boldsymbol{\eta} + \boldsymbol{\zeta}_F, \\
 \boldsymbol{\eta} &= \boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_\xi \zeta + \boldsymbol{\zeta}_\eta,
 \end{aligned} \tag{11}$$

where $\mathbf{y} = [y_{11} \ y_{21} \ y_{31} \ y_{12} \ y_{22} \ y_{32} \ y_{13} \ y_{23} \ y_{33} \ y_{14} \ y_{24} \ y_{34}]'$, $\mathbf{x} = [x_1 \ x_2 \ x_3]'$, $\mathbf{F} = [F_1 \ F_2 \ F_3 \ F_4]'$, $\boldsymbol{\eta} = [\eta_\alpha \ \eta_\beta]'$, $\boldsymbol{\varepsilon} = [\varepsilon_{11} \ \varepsilon_{21} \ \varepsilon_{31} \ \varepsilon_{12} \ \varepsilon_{22} \ \varepsilon_{32} \ \varepsilon_{13} \ \varepsilon_{23} \ \varepsilon_{33} \ \varepsilon_{14} \ \varepsilon_{24} \ \varepsilon_{34}]'$, $\boldsymbol{\delta} = [\delta_1 \ \delta_2 \ \delta_3]'$, $\boldsymbol{\zeta}_F = [\zeta_{F_1} \ \zeta_{F_2} \ \zeta_{F_3} \ \zeta_{F_4}]'$, and $\boldsymbol{\zeta}_\eta = [\zeta_{\eta_\alpha} \ \zeta_{\eta_\beta}]'$. Λ_y and Λ_x denote the loading matrices in the measurement model, Λ_y^* denotes the loading matrix in the growth model, and $\boldsymbol{\Gamma}_0$ and $\boldsymbol{\Gamma}_\xi$ denote, respectively, the vector of intercepts and slopes of the regressions of the growth factors $\boldsymbol{\eta}$ on the predictor construct ζ . They are specifically shown in Table 5. It is assumed that $E(\boldsymbol{\varepsilon}) = E(\boldsymbol{\delta}) = E(\boldsymbol{\zeta}_F) = E(\boldsymbol{\zeta}_\eta) = \mathbf{0}$, $Cov(\zeta, \boldsymbol{\delta}) = Cov(\zeta, \boldsymbol{\zeta}_\eta) = Cov(\zeta, \boldsymbol{\zeta}_F) = Cov(\zeta, \boldsymbol{\varepsilon}) = \mathbf{0}$, and $Cov(\boldsymbol{\varepsilon}, \boldsymbol{\delta}) = Cov(\boldsymbol{\varepsilon}, \boldsymbol{\zeta}_F) = Cov(\boldsymbol{\varepsilon}, \boldsymbol{\zeta}_\eta) = Cov(\boldsymbol{\delta}, \boldsymbol{\zeta}_F) = Cov(\boldsymbol{\delta}, \boldsymbol{\zeta}_\eta) = Cov(\boldsymbol{\zeta}_F, \boldsymbol{\zeta}_\eta) = \mathbf{0}$. In addition, $Cov(\varepsilon_{jt}, \varepsilon_{j' t'}) =$

0, $j \neq j'$. Let Θ_{ε} , $\Psi_{\zeta_{\eta}}$, Ψ_{ζ_F} , and Φ_{ζ} denote, respectively, the variance-covariance matrices of ε , ζ_{η} , ζ_F , and ζ . The implied mean vector and the variance-covariance matrix of \mathbf{y} and \mathbf{x} are given in Appendix 2. Because Θ_{ε} and Ψ_{ζ_F} involve time occasions, their covariance structures need to be identified. Since the four level-1 error processes, $\varepsilon_1 = [\varepsilon_{11} \ \varepsilon_{12} \ \varepsilon_{13} \ \varepsilon_{14}]'$, $\varepsilon_2 = [\varepsilon_{21} \ \varepsilon_{22} \ \varepsilon_{23} \ \varepsilon_{24}]'$, $\varepsilon_3 = [\varepsilon_{31} \ \varepsilon_{32} \ \varepsilon_{33} \ \varepsilon_{34}]'$, and $\zeta_F = [\zeta_{F_1} \ \zeta_{F_2} \ \zeta_{F_3} \ \zeta_{F_4}]'$, associated with the indicators y_1 – y_3 and the construct F , respectively, are assumed to be uncorrelated, the approach proposed previously for identifying the first-level error covariance structure can be used individually for each of them. However, the assumption of the uncorrelatedness of ε_1 , ε_2 , and ε_3 may be violated due to systematic effects. The chi-square difference test can be used to test for the assumption.

 Insert Table 5 about here

Statements by SAS PROC CALIS/TCALIS

SAS Program 2 in Appendix 2 are used for fitting AR(1) for ε_j ($j = 1, 2, 3$) (the first-level errors associated with the same indicator across four time points) and ζ_F (the first-level errors associated with the construct across four time points), and fitting UN for ζ_{η} (the second-level errors) by using SAS PROC CALIS. There are two parameters in AR(1) for each error process. The parameters to be estimated simultaneously include $\sigma_{\varepsilon_j}^2$, $\phi_{1\varepsilon_j}$, $j = 1, 2, 3$, and $\sigma_{\zeta_F}^2$, $\phi_{1\zeta_F}$. The assumption of stationarity requires that $|\phi_{1\varepsilon_j}| < 1$ and $|\phi_{1\zeta_F}| < 1$.

In the STD statement, the first-level error variance estimates associated with the same indicator/the construct are set equal across time by using the same names. In the COV statement, we specify the same name for the lag-1 autocovariances to each indicator and the construct, and similarly for the lag-2 autocovariances. In the PARAMETERS statement, the additional covariance structure parameters for AR(1) need to be defined first, one for each indicator and the construct (i.e., $\phi_{1\varepsilon_j}$ and $\phi_{1\zeta_F}$), with initial values given in the parentheses, and then SAS programming statements are used to specify the relationships among the parameters delineating the particular error covariance structure for all indicators and the

construct. The principles are exactly the same as before. The requirements of ‘ $|\phi_{1\epsilon_j}| < 1$ and $|\phi_{1\zeta_F}| < 1$ ’ are specified by the BOUNDS statement.

The approach proposed for identifying the error covariance structure is also applicable for the second-order LGM. The first stage is to test, for stationarity,

$$H_{01_1} : \sigma_{\epsilon_{11}}^2 = \sigma_{\epsilon_{12}}^2 = \sigma_{\epsilon_{13}}^2 = \sigma_{\epsilon_{14}}^2, \sigma_{\epsilon_{12}\epsilon_{11}} = \sigma_{\epsilon_{13}\epsilon_{12}} = \sigma_{\epsilon_{14}\epsilon_{13}}, \sigma_{\epsilon_{13}\epsilon_{11}} = \sigma_{\epsilon_{14}\epsilon_{12}},$$

$$H_{01_2} : \sigma_{\epsilon_{21}}^2 = \sigma_{\epsilon_{22}}^2 = \sigma_{\epsilon_{23}}^2 = \sigma_{\epsilon_{24}}^2, \sigma_{\epsilon_{22}\epsilon_{21}} = \sigma_{\epsilon_{23}\epsilon_{22}} = \sigma_{\epsilon_{24}\epsilon_{23}}, \sigma_{\epsilon_{23}\epsilon_{21}} = \sigma_{\epsilon_{24}\epsilon_{22}},$$

$$H_{01_3} : \sigma_{\epsilon_{31}}^2 = \sigma_{\epsilon_{32}}^2 = \sigma_{\epsilon_{33}}^2 = \sigma_{\epsilon_{34}}^2, \sigma_{\epsilon_{32}\epsilon_{31}} = \sigma_{\epsilon_{33}\epsilon_{32}} = \sigma_{\epsilon_{34}\epsilon_{33}}, \sigma_{\epsilon_{33}\epsilon_{31}} = \sigma_{\epsilon_{34}\epsilon_{32}},$$

and

$$H_{01_4} : \sigma_{\zeta_{F1}}^2 = \sigma_{\zeta_{F2}}^2 = \sigma_{\zeta_{F3}}^2 = \sigma_{\zeta_{F4}}^2, \sigma_{\zeta_{F2}\zeta_{F1}} = \sigma_{\zeta_{F3}\zeta_{F2}} = \sigma_{\zeta_{F4}\zeta_{F3}}, \sigma_{\zeta_{F3}\zeta_{F1}} = \sigma_{\zeta_{F4}\zeta_{F2}},$$

individually for each of the first-level error processes, ϵ_1 , ϵ_2 , ϵ_3 , and ζ_F , by using the SIMTEST statement in PROC TCALIS mentioned in the previous section, followed by the steps in stage 2A for identifying the specific stationary error process or in stage 2B for identifying the specific nonstationary error process using SCDT.

Illustration

We demonstrate identifying the covariance structures of Θ_ϵ and Ψ_{ζ_F} with another dataset of size 300 generated from the second-order LGM in Figure 2. The population parameters with the AR(1) covariance structure for level-1 error processes ϵ_1 , ϵ_2 , ϵ_3 , and ζ_F and the sample covariance matrix of \mathbf{y} and \mathbf{x} resulting from the simulated dataset are presented in Table 5. The RANDNORMAL function in PROC IML (SAS 9.2) was used to generate multivariate normal data. Since the uncorrelatedness of ϵ_1 , ϵ_2 , and ϵ_3 was supported (chi-square difference = 30.866 with $df = 48$, $p = .97408$), stationarity could be assessed and specification search could be conducted individually. Since stationarity was further supported for all ϵ_1 , ϵ_2 , ϵ_3 , and ζ_F ($\chi_{df=6}^2 = 7.604, 7.049, 4.043, \text{ and } 2.691$ with $p = .269, .316, .671, \text{ and } .845$), we proceeded to search for stationary structures. When identifying the first-level ECM for a specific error process, TOEP was specified for the other three processes. The results of SCDT are reported in Table 6. AR(1) was identified for ϵ_1 , ϵ_2 , ϵ_3 , and TOEP(1) for ζ_F . The parameter estimates by fitting AR(1) for ϵ_1 , ϵ_2 , ϵ_3 and TOEP(1) for ζ_F were close to the

population parameters specified in Table 5. The model fit was satisfactory ($\chi^2_{df=112} = 107.53, p = .602, AGFI = .996, RMSEA = .000$).

Insert Table 6 about here

CONCLUSION

This article illustrates the use of the SAS PROC CALIS/TCALIS to fit error covariance structure of latent growth models. A tutorial on the syntax has been provided for both manifest variables and latent constructs. An effective approach for identifying the first-level error covariance structure based on the sequential chi-square difference test has been proposed and demonstrated. In particular, how to test for stationarity of an error process, not discussed in the SEM or HLM literature, was specifically addressed. The illustrations based on simulated data have well reflected the effectiveness of the approach for identifying the error covariance structures. It is our hope that the approach proposed will help applied researchers obtain a better understanding about the specification of error covariance structure in latent growth models.

The joint use of the STD, COV, PARAMETERS, LINCON, and BOUNDS statements in PROC CALIS can be extended for other types of ECM in a similar way to meet analysts' need. However, there exist some limitations. First, the design underlying LGM should be balanced. Second, time points should be equally spaced when testing for stationarity. Third, large samples are required to justify the use of SEM. In addition, if the errors associated with different indicators at the same occasion in the second-order LGM are correlated, a joint test for stationarity should be conducted, and the SCDT for identifying error covariance structures should also be carried out simultaneously. How to deal with these situations needs to be further studied.

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TABLE 1
SAS Statements in PROC CALIS for Specifying Different Types of the First-Level Error
Covariance Structure with Four Occasions

Structure (Θ_ε)	Statements in PROC CALIS
<p>AR(1) (first-order autoregressive):</p> $\varepsilon_t = \phi_1 \varepsilon_{t-1} + v_t,$ $\sigma_\varepsilon^2 \begin{bmatrix} 1 & & & \\ \phi_1 & 1 & & \\ \phi_1^2 & \phi_1 & 1 & \\ \phi_1^3 & \phi_1^2 & \phi_1 & 1 \end{bmatrix},$ <p>$\phi_1 < 1.$</p>	<p>STD E1=VARE, E2=VARE, E3=VARE, E4=VARE, D0 =VARD0, D1 =VARD1;</p> <p>COV E1 E2=COV_lag1, E2 E3=COV_lag1, E3 E4=COV_lag1, E1 E3=COV_lag2, E2 E4=COV_lag2, E1 E4=COV_lag3, D0 D1=CD0D1;</p> <p>PARAMETERS PHI1 (0.6); COV_lag1= PHI1*VARE; COV_lag2=(PHI1**2)*VARE; COV_lag3= (PHI1**3) *VARE;</p> <p>BOUNDS -1. < PHI1 < 1. ;</p>
<p>ARMA(1,1) (first-order autoregressive moving average):</p> $\varepsilon_t = \phi_1 \varepsilon_{t-1} + v_t - \theta_1 v_{t-1},$ $\sigma_\varepsilon^2 \begin{bmatrix} 1 & & & \\ \rho_1 & 1 & & \\ \rho_2 & \rho_1 & 1 & \\ \rho_3 & \rho_2 & \rho_1 & 1 \end{bmatrix},$ $\rho_1 = \frac{(\phi_1 - \theta_1)(1 - \phi_1 \theta_1)}{(1 - 2\phi_1 \theta_1 + \theta_1^2)},$ <p>$\rho_k = \phi_1 \rho_{k-1}, k = 2, 3, \phi_1 < 1, \rho_1 < 1.$</p>	<p>STD E1=VARE, E2=VARE, E3=VARE, E4=VARE, D0=VARD0, D1=VARD1;</p> <p>COV E1 E2=COV_lag1, E2 E3=COV_lag1, E3 E4=COV_lag1, E1 E3=COV_lag2, E2 E4=COV_lag2, E1 E4=COV_lag3, D0 D1=CD0D1;</p> <p>PARAMETERS PHI1 RHO1 (0.6 0.7) ; COV_lag1=RHO1*VARE; COV_lag2=PHI1* COV_lag1; COV_lag3=PHI1* COV_lag2;</p> <p>BOUNDS -1. < PHI1 < 1., -1. < RHO1 < 1. ;</p>
<p>AR(2) (second-order AR):</p> $\varepsilon_t = \phi_1 \varepsilon_{t-1} + \phi_2 \varepsilon_{t-2} + v_t,$ $\sigma_\varepsilon^2 \begin{bmatrix} 1 & & & \\ \rho_1 & 1 & & \\ \rho_2 & \rho_1 & 1 & \\ \rho_3 & \rho_2 & \rho_1 & 1 \end{bmatrix},$ <p>$\rho_0 = 1, \rho_1 = \phi_1 / (1 - \phi_2),$ $\rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2}, k = 2, 3,$ $\phi_2 < 1, \phi_2 + \phi_1 < 1, \phi_2 - \phi_1 < 1.$</p>	<p>STD E1=VARE, E2=VARE, E3=VARE, E4=VARE, D0=VARD0, D1=VARD1;</p> <p>COV E1 E2=COV_lag1, E2 E3=COV_lag1, E3 E4=COV_lag1, E1 E3=COV_lag2, E2 E4=COV_lag2, E1 E4=COV_lag3, D0 D1=CD0D1;</p> <p>PARAMETERS PHI1 PHI2 (0.5 0.4); RHO1= PHI1/(1-PHI2); COV_lag1=RHO1*VARE; COV_lag2=PHI1*COV_lag1+ PHI2 *VARE; COV_lag3=PHI1*COV_lag2+PHI2*COV_lag1;</p> <p>LINCON PHI2 + PHI1 < 1., PHI2 -PHI1 < 1.;</p> <p>BOUNDS -1. < PHI2 < 1.;</p>

TABLE 1 (Continued)

Structure	Statements in PROC CALIS
UN (unstructured): $\begin{bmatrix} \sigma_{\varepsilon_1}^2 & & & \\ \sigma_{\varepsilon_2\varepsilon_1} & \sigma_{\varepsilon_2}^2 & & \\ \sigma_{\varepsilon_3\varepsilon_1} & \sigma_{\varepsilon_3\varepsilon_2} & \sigma_{\varepsilon_3}^2 & \\ \sigma_{\varepsilon_4\varepsilon_1} & \sigma_{\varepsilon_4\varepsilon_2} & \sigma_{\varepsilon_4\varepsilon_3} & \sigma_{\varepsilon_4}^2 \end{bmatrix}$	STD E1=VARE1, E2=VARE2, E3=VARE3, E4=VARE4, D0=VARD0, D1=VARD1; COV E1 E2=COVE1E2, E1 E3=COVE1E3, E1 E4=COVE1E4, E2 E3=COVE2E3, E2 E4=COVE2E4, E3 E4=COVE3E4, D0 D1=CD0D1;
TOEPH (heterogeneous Toeplitz): $\begin{bmatrix} \sigma_{\varepsilon_1}^2 & & & \\ \sigma_{\varepsilon_2} \sigma_{\varepsilon_1} \rho_1 & \sigma_{\varepsilon_2}^2 & & \\ \sigma_{\varepsilon_3} \sigma_{\varepsilon_1} \rho_2 & \sigma_{\varepsilon_3} \sigma_{\varepsilon_2} \rho_1 & \sigma_{\varepsilon_3}^2 & \\ \sigma_{\varepsilon_4} \sigma_{\varepsilon_1} \rho_3 & \sigma_{\varepsilon_4} \sigma_{\varepsilon_2} \rho_2 & \sigma_{\varepsilon_4} \sigma_{\varepsilon_3} \rho_1 & \sigma_{\varepsilon_4}^2 \end{bmatrix},$ $ \rho_1 < 1, \rho_2 < 1, \rho_3 < 1$	STD E1=VARE1, E2=VARE2, E3=VARE3, E4=VARE4, D0=VARD0, D1=VARD1; COV E1 E2=COVE1E2, E1 E3=COVE1E3, E1 E4=COVE1E4, E2 E3=COVE2E3, E2 E4=COVE2E4, E3 E4=COVE3E4, D0 D1=CD0D1; PARAMETERS RHO1 RHO2 RHO3 (0.7 0.6 0.7); COVE1E2=SQRT(VARE1)*SQRT(VARE2)*RHO1; COVE1E3=SQRT(VARE1)*SQRT(VARE3)*RHO2; COVE1E4=SQRT(VARE1)*SQRT(VARE4)*RHO3; COVE2E3=SQRT(VARE2)*SQRT(VARE3)*RHO1; COVE2E4=SQRT(VARE2)*SQRT(VARE4)*RHO2; COVE3E4=SQRT(VARE3)*SQRT(VARE4)*RHO1; BOUNDS -1. < RHO1 < 1. , -1.< RHO2 <1., -1.< RHO3 < 1.;
ARH(1) (heterogeneous AR(1)): $\begin{bmatrix} \sigma_{\varepsilon_1}^2 & & & \\ \sigma_{\varepsilon_2} \sigma_{\varepsilon_1} \rho & \sigma_{\varepsilon_2}^2 & & \\ \sigma_{\varepsilon_3} \sigma_{\varepsilon_1} \rho^2 & \sigma_{\varepsilon_3} \sigma_{\varepsilon_2} \rho & \sigma_{\varepsilon_3}^2 & \\ \sigma_{\varepsilon_4} \sigma_{\varepsilon_1} \rho^3 & \sigma_{\varepsilon_4} \sigma_{\varepsilon_2} \rho^2 & \sigma_{\varepsilon_4} \sigma_{\varepsilon_3} \rho & \sigma_{\varepsilon_4}^2 \end{bmatrix},$ $ \rho < 1$	STD E1=VARE1, E2=VARE2, E3=VARE3, E4=VARE4, D0=VARD0, D1=VARD1; COV E1 E2=COVE1E2, E1 E3=COVE1E3, E1 E4=COVE1E4, E2 E3=COVE2E3, E2 E4=COVE2E4, E3 E4=COVE3E4, D0 D1=CD0D1; PARAMETERS RHO (0.7); COVE1E2=SQRT(VARE1)*SQRT(VARE2)*RHO; COVE1E3=SQRT(VARE1)*SQRT(VARE3)*RHO**2; COVE1E4=SQRT(VARE1)*SQRT(VARE4)*RHO**3; COVE2E3=SQRT(VARE2)*SQRT(VARE3)*RHO; COVE2E4=SQRT(VARE2)*SQRT(VARE4)*RHO**2; COVE3E4=SQRT(VARE3)*SQRT(VARE4)*RHO; BOUNDS -1. < RHO < 1.;

Note. The second-level ECM, $\Psi_{\zeta\eta} = \begin{bmatrix} \sigma_{\zeta\eta\alpha}^2 & \\ \sigma_{\zeta\eta\alpha\zeta\eta\beta} & \sigma_{\zeta\eta\beta}^2 \end{bmatrix}$, is estimated with type = UN.

TABLE 2
Population Parameters of the Model in Figure 1 Based on the First-Level Error Covariance Structure of TOEPH and the Sample Covariance Matrix of y_1 - y_4 and X Resulting from a Dataset of Size 300 Generated from the Model

$$\Lambda_y^* = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}, \Gamma_x = \begin{bmatrix} \gamma_{01} \\ \gamma_{11} \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}, \Sigma_{xx} = \sigma_X^2 = 1, \mu_x = \mu_X = 0, \Psi_{\zeta_\eta} = \begin{bmatrix} \sigma_{\zeta_{\eta\alpha}}^2 & \\ \sigma_{\zeta_{\eta\alpha}\zeta_{\eta\beta}} & \sigma_{\zeta_{\eta\beta}}^2 \end{bmatrix} = \begin{bmatrix} 15 \\ 7 & 10 \end{bmatrix},$$

$$\Gamma_0 = \begin{bmatrix} \gamma_{00} \\ \gamma_{10} \end{bmatrix} = \begin{bmatrix} 10 \\ 4 \end{bmatrix}, \Theta_\varepsilon = \begin{bmatrix} \sigma_{\varepsilon_1}^2 & & & & \\ \sigma_{\varepsilon_2}\sigma_{\varepsilon_1}\rho_1 & \sigma_{\varepsilon_2}^2 & & & \\ \sigma_{\varepsilon_3}\sigma_{\varepsilon_1}\rho_2 & \sigma_{\varepsilon_3}\sigma_{\varepsilon_2}\rho_1 & \sigma_{\varepsilon_3}^2 & & \\ \sigma_{\varepsilon_4}\sigma_{\varepsilon_1}\rho_3 & \sigma_{\varepsilon_4}\sigma_{\varepsilon_2}\rho_2 & \sigma_{\varepsilon_4}\sigma_{\varepsilon_3}\rho_1 & \sigma_{\varepsilon_4}^2 & \end{bmatrix}$$

$$\sigma_{\varepsilon_1}^2 = 36, \sigma_{\varepsilon_2}^2 = 25, \sigma_{\varepsilon_3}^2 = 49, \sigma_{\varepsilon_4}^2 = 40$$

$$\rho_1 = .7, \rho_2 = .6, \rho_3 = .7$$

Sample Covariance Matrix

	y_1	y_2	y_3	y_4	X
y_1	72.272	99.943	140.297	175.400	5.497
y_2	99.943	200.547	292.249	374.457	12.923
y_3	140.297	292.249	462.949	587.170	20.360
y_4	175.400	374.457	587.170	786.616	27.348
X	5.497	12.923	20.360	27.348	1.205

TABLE 3
 Summary of SCDT for Identifying the First-Level ECM and Parameter Estimates by Fitting
 ARH(1) Based on the Sample Covariance Matrix Shown in Table 2

Step	Structure	$H_{02} : M_T = M_S$ (SCDT)					Assessment of model fit		
		χ^2	df	$\Delta\chi^2$	Δdf	$P_r > \Delta\chi^2_{\Delta df}$	$P_r > \chi^2$	AGFI	RMSEA
0	UN	1.042	1				.307	.984	.012
1	UN(1)	16.945	7	15.903	6	.014	.018	.966	.069
2	ARH(1)	1.483	6	.441	5	.994	.961	.996	.000
	TOEPH	1.042	4	.000	3	1.000	.903	.996	.000

Parameter estimates by fitting ARH(1)

Parameters	Estimates by using PROC CALIS	Estimates by using PROC MIXED
ARH(1) for $\Theta_\varepsilon :$ $\begin{bmatrix} \sigma_{\varepsilon_1}^2 & & & & & & & \\ \sigma_{\varepsilon_2} \sigma_{\varepsilon_1} \rho & \sigma_{\varepsilon_2}^2 & & & & & & \\ \sigma_{\varepsilon_3} \sigma_{\varepsilon_1} \rho^2 & \sigma_{\varepsilon_3} \sigma_{\varepsilon_2} \rho & \sigma_{\varepsilon_3}^2 & & & & & \\ \sigma_{\varepsilon_4} \sigma_{\varepsilon_1} \rho^3 & \sigma_{\varepsilon_4} \sigma_{\varepsilon_2} \rho^2 & \sigma_{\varepsilon_4} \sigma_{\varepsilon_3} \rho & \sigma_{\varepsilon_4}^2 & & & & \end{bmatrix}$	$\begin{bmatrix} 36.48^{**} & & & & & & & \\ 21.32^* & 27.57^{**} & & & & & & \\ 18.54^* & 23.97^* & 46.07^{**} & & & & & \\ 12.06 & 15.60 & 29.98^* & 43.18^{**} & & & & \\ \hat{\rho} = .67^{***} & & & & & & & \end{bmatrix}$	$\begin{bmatrix} 36.34^{**} & & & & & & & \\ 21.24^a & 27.46^{**} & & & & & & \\ 18.46^a & 23.87^a & 45.89^{**} & & & & & \\ 12.01^a & 15.53^a & 29.85^a & 42.96^{**} & & & & \\ \hat{\rho} = .67^{***} & & & & & & & \end{bmatrix}$
UN for $\Psi_{\zeta_\eta} :$ $\begin{bmatrix} \sigma_{\zeta_{\eta\alpha}}^2 & & & \\ \sigma_{\zeta_{\eta\alpha} \zeta_{\eta\beta}} & \sigma_{\zeta_{\eta\beta}}^2 & & \end{bmatrix}$	$\begin{bmatrix} 10.46 & & & \\ 9.10^{***} & 6.26^{***} & & \end{bmatrix}$	$\begin{bmatrix} 10.45 & & & \\ 9.07^{***} & 6.24^{***} & & \end{bmatrix}$
$\begin{pmatrix} \gamma_{00} & \gamma_{01} \\ \gamma_{10} & \gamma_{11} \end{pmatrix}$	$\begin{pmatrix} 10.58^{***} & 4.03^{***} \\ 4.60^{***} & 6.05^{***} \end{pmatrix}$	$\begin{pmatrix} 10.58^{***} & 4.03^{***} \\ 4.60^{***} & 6.05^{***} \end{pmatrix}$

Note. $M_T = \{\text{UN}(1), \text{ARH}(1), \text{TOEPH}(1)\}$, $M_S = \text{UN}$, $\Delta\chi^2$ = the chi-square difference between M_T and M_S , Δdf = the difference of df associated with M_T and M_S , $P_r > \Delta\chi^2_{\Delta df}$ denotes the p -value of the chi-square difference test.

^a Test for significance cannot be achieved.

* $p < .05$, ** $p < .01$, *** $p < .001$.

TABLE 4
Summary of SCDT for Identifying the First-Level ECM with the Dataset in Singer and Willetts (2003, Chap. 7) and Parameter Estimates by Fitting TOEP(1)

Step	Structure	$H_{02} : M_T = M_S$ (SCDT)					Assessment of model fit		
		χ^2	df	$\Delta\chi^2$	Δdf	$P_r > \Delta\chi^2_{\Delta df}$	$P_r > \chi^2$	AGFI	RMSEA
0	TOEP	3.879	6				.693	.988	.000
1	TOEP(1)	6.680	10	2.801	4	.592	.755	.987	.000
Parameter estimates by fitting TOEP(1)									
Parameters		Estimates by using PROC CALIS				Estimates by using PROC MIXED			
TOEP(1) for $\Theta_\varepsilon :$	$\begin{bmatrix} \sigma_\varepsilon^2 & & & \\ 0 & \sigma_\varepsilon^2 & & \\ 0 & 0 & \sigma_\varepsilon^2 & \\ 0 & 0 & 0 & \sigma_\varepsilon^2 \end{bmatrix}$	$\begin{bmatrix} 164.1^{***} & & & \\ 0 & 164.1^{***} & & \\ 0 & 0 & 164.1^{***} & \\ 0 & 0 & 0 & 164.1^{***} \end{bmatrix}$				$\begin{bmatrix} 159.5^{***} & & & \\ 0 & 159.5^{***} & & \\ 0 & 0 & 159.5^{***} & \\ 0 & 0 & 0 & 159.5^{***} \end{bmatrix}$			
UN for $\Psi_{\zeta_\eta} :$	$\begin{bmatrix} \sigma_{\zeta_{\eta\alpha}}^2 & & \\ \sigma_{\zeta_{\eta\alpha}\zeta_{\eta\beta}} & & \sigma_{\zeta_{\eta\beta}}^2 \end{bmatrix}$	$\begin{bmatrix} 1194.00^{***} & & \\ -170.19^{**} & 102.23^{**} & \end{bmatrix}$				$\begin{bmatrix} 1159.38^{***} & & \\ -165.31^{**} & 99.29^{**} & \end{bmatrix}$			
	$\begin{pmatrix} \gamma_{00} & \gamma_{01} \\ \gamma_{10} & \gamma_{11} \end{pmatrix}$	$\begin{pmatrix} 164.40^{***} & 26.96^{***} \\ -.11 & .43^{**} \end{pmatrix}$				$\begin{pmatrix} 164.37^{***} & 26.96^{***} \\ -.12 & .43^{**} \end{pmatrix}$			

Note. $M_T = \text{TOEP}(1)$, $M_S = \text{TOEP}$, $\Delta\chi^2$ = the chi-square difference between M_T and M_S , Δdf = the difference of df associated with M_T and M_S , $P_r > \Delta\chi^2_{\Delta df}$ denotes the p -value of the chi-square difference test.

* $p < .05$, ** $p < .01$, *** $p < .001$.

TABLE 5 (Continued)

Sample Covariance Matrix															
	y_{11}	y_{21}	y_{31}	y_{12}	y_{22}	y_{32}	y_{13}	y_{23}	y_{33}	y_{14}	y_{24}	y_{34}	x_1	x_2	x_3
y_{11}	2.913	1.971	2.264	4.399	3.157	3.604	5.876	4.324	5.006	7.515	5.578	6.348	2.356	2.120	1.767
y_{21}	1.971	1.817	1.648	3.018	2.474	2.529	4.131	3.254	3.542	5.286	4.053	4.464	1.679	1.459	1.279
y_{31}	2.264	1.648	2.038	3.542	2.616	3.059	4.855	3.613	4.224	6.215	4.625	5.325	1.941	1.732	1.440
y_{12}	4.399	3.018	3.542	8.107	5.715	6.551	11.064	8.111	9.336	14.236	10.600	12.042	4.339	3.795	3.212
y_{22}	3.157	2.474	2.616	5.715	4.504	4.782	7.973	6.159	6.825	10.380	7.915	8.810	3.227	2.770	2.373
y_{32}	3.604	2.529	3.059	6.551	4.782	5.616	9.111	6.765	7.869	11.794	8.831	10.096	3.643	3.167	2.674
y_{13}	5.876	4.131	4.855	11.064	7.973	9.111	16.115	11.807	13.508	20.694	15.446	17.487	6.146	5.398	4.509
y_{23}	4.324	3.254	3.613	8.111	6.159	6.765	11.807	9.164	10.076	15.353	11.756	13.052	4.655	4.031	3.359
y_{33}	5.006	3.542	4.224	9.336	6.825	7.869	13.508	10.076	11.685	17.517	13.160	15.015	5.285	4.590	3.851
y_{14}	7.515	5.286	6.215	14.236	10.380	11.794	20.694	15.353	17.517	27.455	20.437	23.108	8.119	7.070	5.884
y_{24}	5.578	4.053	4.625	10.600	7.915	8.831	15.446	11.756	13.160	20.437	15.666	17.349	6.140	5.315	4.400
y_{34}	6.348	4.464	5.325	12.042	8.810	10.096	17.487	13.052	15.015	23.108	17.349	19.815	6.902	5.994	4.986
x_1	2.356	1.679	1.941	4.339	3.227	3.643	6.146	4.655	5.285	8.119	6.140	6.902	4.330	3.038	2.648
x_2	2.120	1.459	1.732	3.795	2.770	3.167	5.398	4.031	4.590	7.070	5.315	5.994	3.038	2.893	2.111
x_3	1.767	1.279	1.440	3.212	2.373	2.674	4.509	3.359	3.851	5.884	4.400	4.986	2.648	2.111	2.755

TABLE 6

Summary of SCDT for Identifying the First-Level ECM for $\varepsilon_1, \varepsilon_2, \varepsilon_3$, and ζ_F and Parameter Estimates by Fitting the Final Model Based on the Sample Covariance Matrix Shown in Table 5

Process	Step	Structure	$H_{02} : M_T = M_S$ (SCDT)					Assessment of model fit		
			χ^2	df	$\Delta\chi^2$	Δdf	$P_r > \Delta\chi^2_{\Delta df}$	$P_r > \chi^2$	AGFI	RMSEA
ε_1	0	TOEP	95.01	103				.700	.966	.000
	1	TOEP(1)	157.70	106	62.69	3	<.0001	.001	.994	.040
	2	AR(1)	96.59	105	1.58	2	.454	.709	.996	.000
ε_2	0	TOEP	95.01	103				.700	.966	.000
	1	TOEP(1)	408.04	106	313.04	3	<.0001	<.0001	.978	.098
	2	AR(1)	95.97	105	.96	2	.619	.724	.996	.000
ε_3	0	TOEP	95.01	103				.700	.966	.000
	1	TOEP(1)	154.27	106	59.26	3	<.0001	.002	.993	.039
	2	AR(1)	99.11	105	4.10	2	.129	.644	.996	.000
ζ_F	0	TOEP	95.01	103				.700	.966	.000
	1	TOEP(1)	100.99	106	5.98	3	.112	.619	.996	.000

Parameter estimates by fitting the final model (AR(1) for $\varepsilon_1, \varepsilon_2, \varepsilon_3$, TOEP(1) for ζ_F , and UN for ζ_η)

$$\begin{pmatrix} \hat{\lambda}_{y_{2t}} & \hat{\lambda}_{y_{3t}} \\ \hat{\lambda}_{x_{2t}} & \hat{\lambda}_{x_{3t}} \end{pmatrix} = \begin{pmatrix} .75^{***} & .85^{***} \\ .76^{***} & .71^{***} \end{pmatrix}, \begin{pmatrix} \hat{\gamma}_{00} & \hat{\gamma}_{01} \\ \hat{\gamma}_{10} & \hat{\gamma}_{11} \end{pmatrix} = \begin{pmatrix} 11.56^{***} & .64^{***} \\ 1.79^{***} & .52^{***} \end{pmatrix}, \begin{pmatrix} \hat{\mu}_\zeta \\ \hat{\sigma}_\zeta^2 \end{pmatrix} = \begin{pmatrix} 12.95^{***} \\ 3.92^{***} \end{pmatrix}, \begin{bmatrix} \hat{\sigma}_{\zeta_{\eta\alpha}}^2 & \\ & \hat{\sigma}_{\zeta_{\eta\beta}}^2 \end{bmatrix} = \begin{bmatrix} .945^{***} \\ .313^{***} \ .599^{***} \end{bmatrix},$$

$$\begin{bmatrix} \hat{\sigma}_{\varepsilon_1}^2 & & & \\ \hat{\phi}_{1\varepsilon_1} \hat{\sigma}_{\varepsilon_1}^2 & \hat{\sigma}_{\varepsilon_1}^2 & & \\ \hat{\phi}_{1\varepsilon_1}^2 \hat{\sigma}_{\varepsilon_1}^2 & \hat{\phi}_{1\varepsilon_1} \hat{\sigma}_{\varepsilon_1}^2 & \hat{\sigma}_{\varepsilon_1}^2 & \\ \hat{\phi}_{1\varepsilon_1}^3 \hat{\sigma}_{\varepsilon_1}^2 & \hat{\phi}_{1\varepsilon_1}^2 \hat{\sigma}_{\varepsilon_1}^2 & \hat{\phi}_{1\varepsilon_1} \hat{\sigma}_{\varepsilon_1}^2 & \hat{\sigma}_{\varepsilon_1}^2 \end{bmatrix} = \begin{bmatrix} .251^{***} & & & \\ .121^{***} & .251^{***} & & \\ .058^{***} & .121^{***} & .251^{***} & \\ .028^{**} & .058^{***} & .121^{***} & .251^{***} \end{bmatrix}, \begin{bmatrix} \hat{\sigma}_{\varepsilon_2}^2 & & & \\ \hat{\phi}_{1\varepsilon_2} \hat{\sigma}_{\varepsilon_2}^2 & \hat{\sigma}_{\varepsilon_2}^2 & & \\ \hat{\phi}_{1\varepsilon_2}^2 \hat{\sigma}_{\varepsilon_2}^2 & \hat{\phi}_{1\varepsilon_2} \hat{\sigma}_{\varepsilon_2}^2 & \hat{\sigma}_{\varepsilon_2}^2 & \\ \hat{\phi}_{1\varepsilon_2}^3 \hat{\sigma}_{\varepsilon_2}^2 & \hat{\phi}_{1\varepsilon_2}^2 \hat{\sigma}_{\varepsilon_2}^2 & \hat{\phi}_{1\varepsilon_2} \hat{\sigma}_{\varepsilon_2}^2 & \hat{\sigma}_{\varepsilon_2}^2 \end{bmatrix} = \begin{bmatrix} .353^{***} & & & \\ .241^{***} & .353^{***} & & \\ .165^{***} & .241^{***} & .353^{***} & \\ .113^{**} & .165^{***} & .241^{***} & .353^{***} \end{bmatrix}$$

$$\hat{\phi}_{1\varepsilon_1} = .482^{***}, \hat{\sigma}_{\varepsilon_1}^2 = .251^{***} \quad \hat{\phi}_{1\varepsilon_2} = .684^{***}, \hat{\sigma}_{\varepsilon_2}^2 = .353^{***}$$

$$\begin{bmatrix} \hat{\sigma}_{\varepsilon_3}^2 & & & \\ \hat{\phi}_{1\varepsilon_3} \hat{\sigma}_{\varepsilon_3}^2 & \hat{\sigma}_{\varepsilon_3}^2 & & \\ \hat{\phi}_{1\varepsilon_3}^2 \hat{\sigma}_{\varepsilon_3}^2 & \hat{\phi}_{1\varepsilon_3} \hat{\sigma}_{\varepsilon_3}^2 & \hat{\sigma}_{\varepsilon_3}^2 & \\ \hat{\phi}_{1\varepsilon_3}^3 \hat{\sigma}_{\varepsilon_3}^2 & \hat{\phi}_{1\varepsilon_3}^2 \hat{\sigma}_{\varepsilon_3}^2 & \hat{\phi}_{1\varepsilon_3} \hat{\sigma}_{\varepsilon_3}^2 & \hat{\sigma}_{\varepsilon_3}^2 \end{bmatrix} = \begin{bmatrix} .160^{***} & & & \\ .097^{***} & .160^{***} & & \\ .059^{***} & .097^{***} & .160^{***} & \\ .036^{**} & .059^{***} & .097^{***} & .160^{***} \end{bmatrix}, \text{Diag}[\hat{\sigma}_{\delta_1}^2 \ \hat{\sigma}_{\delta_2}^2 \ \hat{\sigma}_{\delta_3}^2] = \text{Diag} [.646^{***} \ .413^{***} \ .902^{***}],$$

$$\text{Diag}[\hat{\sigma}_{\zeta_F}^2 \ \hat{\sigma}_{\zeta_F}^2 \ \hat{\sigma}_{\zeta_F}^2 \ \hat{\sigma}_{\zeta_F}^2] = \text{Diag} [.142^{***} \ .142^{***} \ .142^{***} \ .142^{***}]$$

$$\hat{\phi}_{1\varepsilon_3} = .608^{***}, \hat{\sigma}_{\varepsilon_3}^2 = .160^{***}$$

$$\chi^2_{df=112} = 107.53, p = .602, \text{AGFI} = .996, \text{RMSEA} = .000.$$

Note. $M_T = \{\text{AR}(1), \text{TOEP}(1)\}$, $M_S = \text{TOEP}$, $\Delta\chi^2$ = the chi-square difference between M_T and M_S , Δdf = the difference of df associated with M_T and M_S , $P_r > \Delta\chi^2_{\Delta df}$ denotes the p -value of the chi-square difference test.

* $p < .05$, ** $p < .01$, *** $p < .001$.

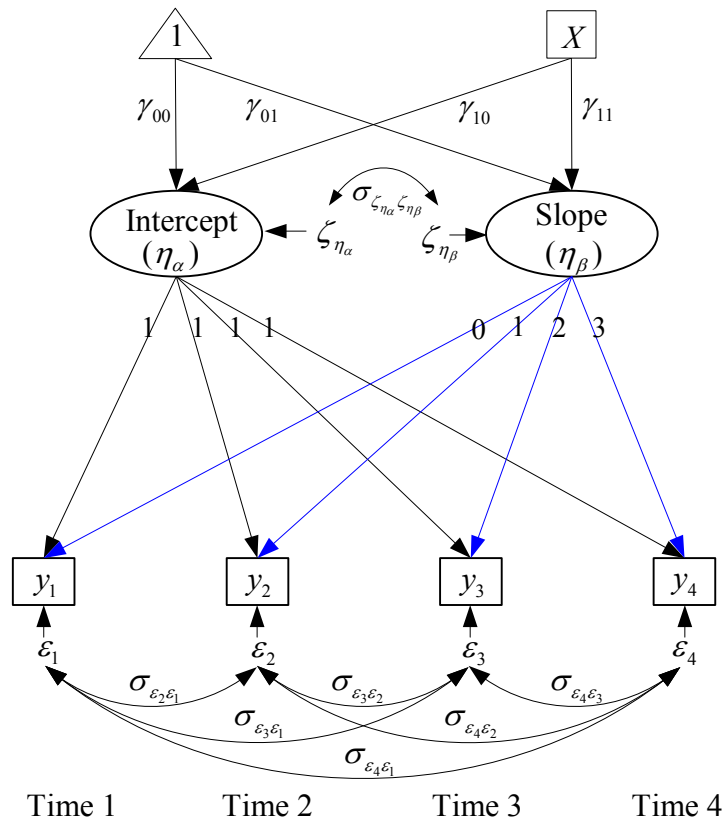


FIGURE 1. Linear latent curve model with four repeated measures and a covariate X .

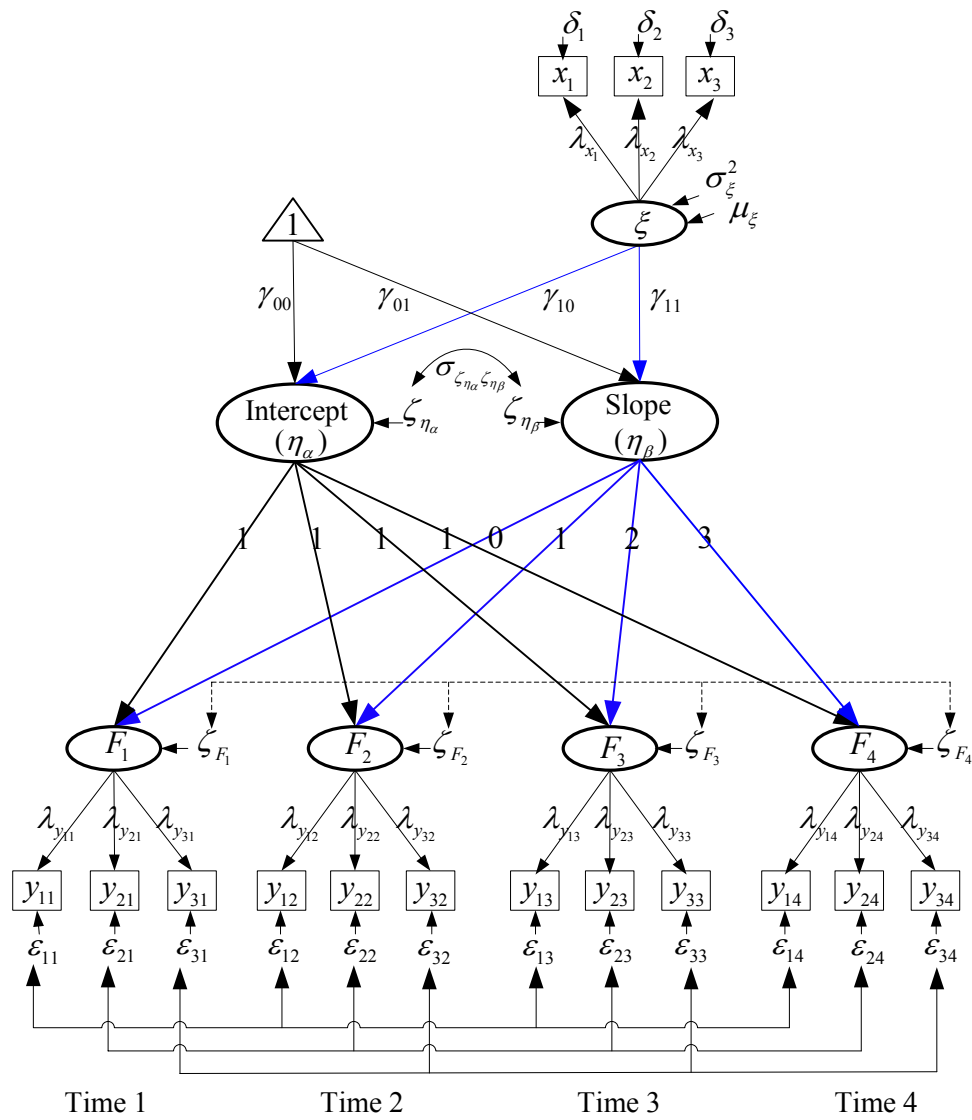


FIGURE 2. A second-order growth model, representing linear change in four repeated measures of a latent construct with three indicators (adapted from Chan (1998) and Preachers et al. (2008)).

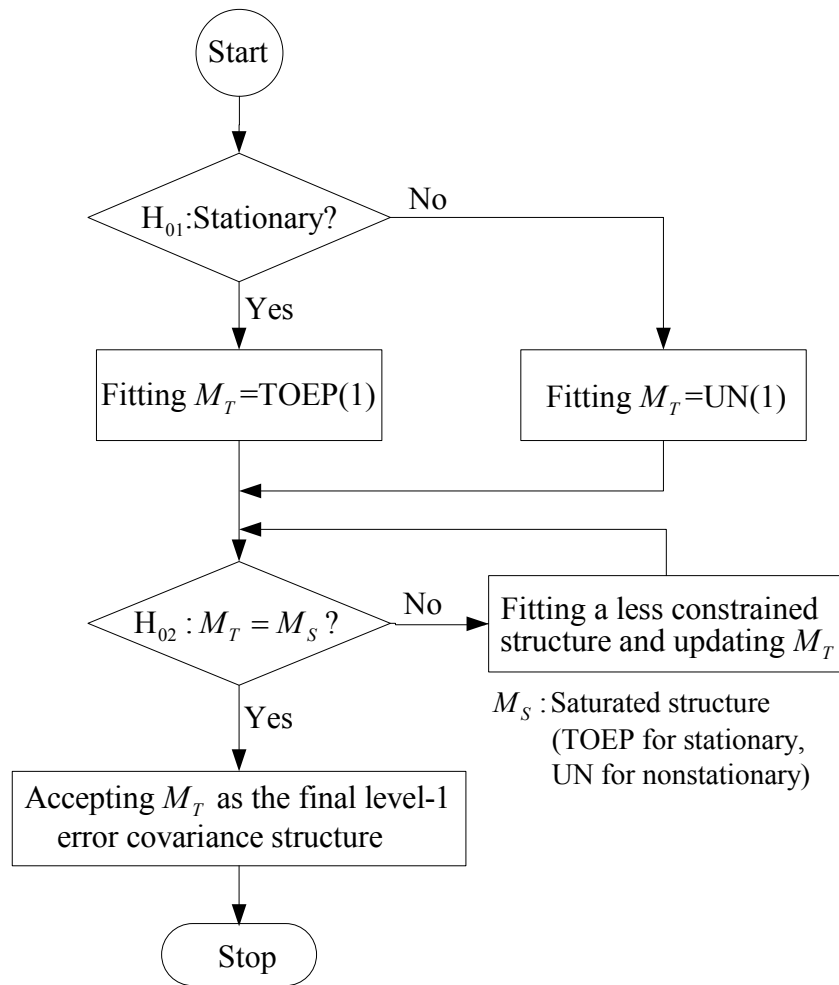


FIGURE 3. A flow chart for identifying the first-level error covariance structure.

APPENDIX 1

Sample SAS programs for LGM

Program 1

A SAS Program for LGM by Specifying the First-Level ECM to be ARMA(1,1) with the SEM Approach (PROC CALIS) and the HLM Approach (PROC MIXED)

/* The dataset used for PROC CALIS should be a multi variable dataset.
 The dataset used for PROC MIXED should be a multi record dataset.
 SAS codes for manipulating datasets were given in Singer (1998) */

```
PROC CALIS METHOD=ML UCOV AUG;
  LINEQS
    Y1 = 1 F_Alpha + 0 F_Beta + E1,
    Y2 = 1 F_Alpha + 1 F_Beta + E2,
    Y3 = 1 F_Alpha + 2 F_Beta + E3,
    Y4 = 1 F_Alpha + 3 F_Beta + E4,
    F_Alpha = GA00 INTERCEPT + GA01 X + D0,
    F_Beta = GA10 INTERCEPT + GA11 X + D1;
  STD
    E1=VARE, E2=VARE, E3=VARE, E4=VARE, X=VARX,
    D0=VARD0, D1=VARD1;
  COV
    E1 E2=COV_lag1, E2 E3=COV_lag1, E3 E4=COV_lag1,
    E1 E3=COV_lag2, E2 E4=COV_lag2, E1 E4=COV_lag3,
    D0 D1=COVD0D1;
  PARAMETERS PHI1 RHO1 (0.6 0.7);
    COV_lag1=RHO1*VARE;
    COV_lag2=PHI1* COV_lag1;
    COV_lag3=PHI1* COV_lag2;
  BOUNDS
    -1. < PHI1 < 1., -1. < RHO1 < 1. ;
  VAR Y1 Y2 Y3 Y4 X;
  TITLE 'LGM_SEM with ECM=ARMA(1,1)';
RUN;

PROC MIXED METHOD=ML COVTEST;
  CLASS ID;
  MODEL Y=TIME X TIME*X / SOLUTION DDFM=BW;
  REPEATED / SUBJECT=ID TYPE=ARMA(1,1) R RCORR;
  RANDOM INTERCEPT TIME / SUBJECT=ID TYPE=UN G GCORR;
  TITLE 'LGM_HLM with ECM=ARMA(1,1)';
RUN;
```

Program 2

A SAS Program for a Second-Order Growth Model by Specifying the First-Level ECM to be AR(1) with the SEM Approach (PROC CALIS)

```
PROC CALIS METHOD=ML UCOV AUG;
  LINEQS
    Y11 = 1 F1 + EY11,   Y21 = LY21F1 F1 + EY21,   Y31 = LY31F1 F1 + EY31,
    Y12 = 1 F2 + EY12,   Y22 = LY22F2 F2 + EY22,   Y32 = LY32F2 F2 + EY32,
    Y13 = 1 F3 + EY13,   Y23 = LY23F3 F3 + EY23,   Y33 = LY33F3 F3 + EY33,
    Y14 = 1 F4 + EY14,   Y24 = LY24F4 F4 + EY24,   Y34 = LY34F4 F4 + EY34,
    X1 = 1 F7 + EX1,     X2 = LX2F7 F7 + EX2,     X3 = LX3F7 F7 + EX3,
    F1 = 1 F_Alpha + 0 F_Beta + EZF1,
    F2 = 1 F_Alpha + 1 F_Beta + EZF2,
    F3 = 1 F_Alpha + 2 F_Beta + EZF3,
    F4 = 1 F_Alpha + 3 F_Beta + EZF4,
    F_Alpha = GA00 INTERCEPT + GA10 F7 + EZF5,
    F_Beta = GA01 INTERCEPT + GA11 F7 + EZF6,
    F7 = F7_int INTERCEPT + EZF7;
  STD
    EY11-EY14=4*VARE1, EY21-EY24=4*VARE2, EY31-EY34=4*VARE3,
    EX1=VAREX1, EX2=VAREX2, EX3=VAREX3,
    EZF1-EZF4=4*VARZF, EZF5=VAR_Intercept, EZF6=VAR_Slope, EZF7=VARZF7;
  COV
    /* for the first-level errors associated with indicator 1 */
    EY11 EY12=COV1_lag1, EY12 EY13=COV1_lag1, EY13 EY14=COV1_lag1,
    EY11 EY13=COV1_lag2, EY12 EY14=COV1_lag2, EY11 EY14=COV1_lag3,
    /* for the first-level errors associated with indicator 2 */
    EY21 EY22=COV2_lag1, EY22 EY23=COV2_lag1, EY23 EY24=COV2_lag1,
    EY21 EY23=COV2_lag2, EY22 EY24=COV2_lag2, EY21 EY24=COV2_lag3,
    /* for the first-level errors associated with indicator 3 */
    EY31 EY32=COV3_lag1, EY32 EY33=COV3_lag1, EY33 EY34=COV3_lag1,
    EY31 EY33=COV3_lag2, EY32 EY34=COV3_lag2, EY31 EY34=COV3_lag3,
    /* for the first-level errors associated with the construct */
    EZF1 EZF2=COVZ_lag1, EZF2 EZF3=COVZ_lag1, EZF3 EZF4=COVZ_lag1,
    EZF1 EZF3=COVZ_lag2, EZF2 EZF4=COVZ_lag2, EZF1 EZF4=COVZ_lag3,
    /* for the second-level errors associated with growth factors */
    EZF5 EZF6=CZF5ZF6;
  PARAMETERS PHI1 PHI2 PHI3 PHIZ (0.5 0.7 0.6 0.4);
    /* for the first-level errors associated with indicator 1 */
    COV1_lag1=PHI1*VARE1; COV1_lag2=(PHI1**2)*VARE1;
    COV1_lag3=(PHI1**3)*VARE1;
    /* for the first-level errors associated with indicator 2 */
    COV2_lag1=PHI2*VARE2; COV2_lag2=(PHI2**2)*VARE2;
    COV2_lag3=(PHI2**3)*VARE2;
    /* for the first-level errors associated with indicator 3 */
    COV3_lag1=PHI3*VARE3; COV3_lag2=(PHI3**2)*VARE3;
    COV3_lag3=(PHI3**3)*VARE3;
    /* for the first-level errors associated with the construct */
    COVZ_lag1=PHIZ*VARZF; COVZ_lag2=(PHIZ**2)*VARZF;
    COVZ_lag3=(PHIZ**3)*VARZF;
```

```
BOUNDS
  -1.<PHI1<1., -1.<PHI2<1., -1.<PHI3<1., -1.<PHIZ<1. ;
LINCON /* invariance across time is assumed */
  LY21F1=LY22F2, LY21F1=LY23F3, LY21F3=LY24F4,
  LY31F1=LY32F2, LY31F1=LY33F3, LY31F3=LY34F4;
VAR Y11 Y21 Y31 Y12 Y22 Y32 Y13 Y23 Y33 Y14 Y24 Y34 X1 X2 X3;
TITLE 'A Second-Order Growth Model with Four Occasions by Specifying AR(1) for';
TITLE2 'The First-Level Errors Associated with the Construct Measured with Three';
TITLE3 'Indicators and Those Associated with the Indicators';
RUN;
```

APPENDIX 2

Derivation of the implied mean vector $\boldsymbol{\mu}$ and variance-covariance matrix $\boldsymbol{\Sigma}$ of \mathbf{y} and \mathbf{x} in Figure 2

Based on Equation 11, we have

$$\mathbf{F} = \Lambda_y^* (\Gamma_0 + \Gamma_{\xi} \zeta + \zeta_{\eta}) + \zeta_F = \Lambda_y^* \Gamma_0 + \Lambda_y^* \Gamma_{\xi} \zeta + \Lambda_y^* \zeta_{\eta} + \zeta_F,$$

$$\mathbf{y} = \Lambda_y \mathbf{F} + \boldsymbol{\varepsilon}$$

$$= \Lambda_y (\Lambda_y^* \Gamma_0 + \Lambda_y^* \Gamma_{\xi} \zeta + \Lambda_y^* \zeta_{\eta} + \zeta_F) + \boldsymbol{\varepsilon}$$

$$= \Lambda_y \Lambda_y^* \Gamma_0 + \Lambda_y \Lambda_y^* \Gamma_{\xi} \zeta + \Lambda_y \Lambda_y^* \zeta_{\eta} + \Lambda_y \zeta_F + \boldsymbol{\varepsilon},$$

$$\mathbf{x} = \Lambda_x \boldsymbol{\xi} + \boldsymbol{\delta}.$$

Therefore

$$\boldsymbol{\mu}_y = E(\mathbf{y}) = \Lambda_y \Lambda_y^* \Gamma_0 + \Lambda_y \Lambda_y^* \Gamma_{\xi} \boldsymbol{\mu}_{\xi},$$

$$\boldsymbol{\mu}_x = E(\mathbf{x}) = \Lambda_x \boldsymbol{\mu}_{\xi},$$

that is,

$$\boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}_y \\ \boldsymbol{\mu}_x \end{bmatrix} = \begin{bmatrix} \Lambda_y \Lambda_y^* \Gamma_0 + \Lambda_y \Lambda_y^* \Gamma_{\xi} \boldsymbol{\mu}_{\xi} \\ \Lambda_x \boldsymbol{\mu}_{\xi} \end{bmatrix},$$

and

$$\boldsymbol{\Sigma}_{yy} = Cov(\mathbf{y}) = \Lambda_y \Lambda_y^* \Gamma_{\xi} \boldsymbol{\Phi}_{\zeta} \Gamma'_{\xi} \Lambda_y^{*'} \Lambda'_y + \Lambda_y \Lambda_y^* \boldsymbol{\Psi}_{\zeta_{\eta}} \Lambda_y^{*'} \Lambda'_y + \Lambda_y \boldsymbol{\Psi}_{\zeta_F} \Lambda'_y + \boldsymbol{\Theta}_{\boldsymbol{\varepsilon}}$$

$$= \Lambda_y (\Lambda_y^* \Gamma_{\xi} \boldsymbol{\Phi}_{\zeta} \Gamma'_{\xi} \Lambda_y^{*'} + \Lambda_y^* \boldsymbol{\Psi}_{\zeta_{\eta}} \Lambda_y^{*'} + \boldsymbol{\Psi}_{\zeta_F}) \Lambda'_y + \boldsymbol{\Theta}_{\boldsymbol{\varepsilon}},$$

$$\boldsymbol{\Sigma}_{xx} = Cov(\mathbf{x}) = \Lambda_x \boldsymbol{\Phi}_{\zeta} \Lambda'_x + \boldsymbol{\Theta}_{\boldsymbol{\delta}},$$

$$\boldsymbol{\Sigma}_{yx} = Cov(\mathbf{y}, \mathbf{x}) = \Lambda_y \Lambda_y^* \Gamma_{\xi} \boldsymbol{\Phi}_{\zeta} \Lambda'_x,$$

$$\boldsymbol{\Sigma}_{xy} = \Lambda_x \boldsymbol{\Phi}'_{\zeta} \Gamma'_{\xi} \Lambda_y^{*'} \Lambda'_y.$$

that is,

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}_{yy} & \boldsymbol{\Sigma}_{yx} \\ \boldsymbol{\Sigma}_{xy} & \boldsymbol{\Sigma}_{xx} \end{bmatrix} = \begin{bmatrix} \Lambda_y (\Lambda_y^* \Gamma_{\xi} \boldsymbol{\Phi}_{\zeta} \Gamma'_{\xi} \Lambda_y^{*'} + \Lambda_y^* \boldsymbol{\Psi}_{\zeta_{\eta}} \Lambda_y^{*'} + \boldsymbol{\Psi}_{\zeta_F}) \Lambda'_y + \boldsymbol{\Theta}_{\boldsymbol{\varepsilon}} & \Lambda_y \Lambda_y^* \Gamma_{\xi} \boldsymbol{\Phi}_{\zeta} \Lambda'_x \\ \Lambda_x \boldsymbol{\Phi}'_{\zeta} \Gamma'_{\xi} \Lambda_y^{*'} \Lambda'_y & \Lambda_x \boldsymbol{\Phi}_{\zeta} \Lambda'_x + \boldsymbol{\Theta}_{\boldsymbol{\delta}} \end{bmatrix}.$$

APPENDIX 3

More types of stationary and nonstationary level-1 error covariance structures with four occasions

Structure	ECM								
Stationary:									
CS (Compound symmetry)	$\begin{bmatrix} \sigma_1 + \sigma^2 & & & \\ \sigma_1 & \sigma_1 + \sigma^2 & & \\ \sigma_1 & \sigma_1 & \sigma_1 + \sigma^2 & \\ \sigma_1 & \sigma_1 & \sigma_1 & \sigma_1 + \sigma^2 \end{bmatrix}$								
TOEP (Toeplitz)	$\begin{bmatrix} \sigma^2 & & & \\ \sigma_1 & \sigma^2 & & \\ \sigma_2 & \sigma_1 & \sigma^2 & \\ \sigma_3 & \sigma_2 & \sigma_1 & \sigma^2 \end{bmatrix}$								
TOEP(q) (Toeplitz with q bands, $q = 1, 2, 3, 4$. Only the first q bands of the matrix are estimated, setting all higher bands equal to zero.)	<table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;">TOEP(1)</td> <td style="text-align: center;">TOEP(2)</td> </tr> <tr> <td style="text-align: center;"> $\begin{bmatrix} \sigma^2 & & & \\ 0 & \sigma^2 & & \\ 0 & 0 & \sigma^2 & \\ 0 & 0 & 0 & \sigma^2 \end{bmatrix}$ </td> <td style="text-align: center;"> $\begin{bmatrix} \sigma^2 & & & \\ \sigma_1 & \sigma^2 & & \\ 0 & \sigma_1 & \sigma^2 & \\ 0 & 0 & \sigma_1 & \sigma^2 \end{bmatrix}$ </td> </tr> <tr> <td style="text-align: center;">TOEP(3)</td> <td style="text-align: center;">TOEP(4)</td> </tr> <tr> <td style="text-align: center;"> $\begin{bmatrix} \sigma^2 & & & \\ \sigma_1 & \sigma^2 & & \\ \sigma_2 & \sigma_1 & \sigma^2 & \\ 0 & \sigma_2 & \sigma_1 & \sigma^2 \end{bmatrix}$ </td> <td style="text-align: center;"> $\begin{bmatrix} \sigma^2 & & & \\ \sigma_1 & \sigma^2 & & \\ \sigma_2 & \sigma_1 & \sigma^2 & \\ \sigma_3 & \sigma_2 & \sigma_1 & \sigma^2 \end{bmatrix}$ </td> </tr> </table>	TOEP(1)	TOEP(2)	$\begin{bmatrix} \sigma^2 & & & \\ 0 & \sigma^2 & & \\ 0 & 0 & \sigma^2 & \\ 0 & 0 & 0 & \sigma^2 \end{bmatrix}$	$\begin{bmatrix} \sigma^2 & & & \\ \sigma_1 & \sigma^2 & & \\ 0 & \sigma_1 & \sigma^2 & \\ 0 & 0 & \sigma_1 & \sigma^2 \end{bmatrix}$	TOEP(3)	TOEP(4)	$\begin{bmatrix} \sigma^2 & & & \\ \sigma_1 & \sigma^2 & & \\ \sigma_2 & \sigma_1 & \sigma^2 & \\ 0 & \sigma_2 & \sigma_1 & \sigma^2 \end{bmatrix}$	$\begin{bmatrix} \sigma^2 & & & \\ \sigma_1 & \sigma^2 & & \\ \sigma_2 & \sigma_1 & \sigma^2 & \\ \sigma_3 & \sigma_2 & \sigma_1 & \sigma^2 \end{bmatrix}$
TOEP(1)	TOEP(2)								
$\begin{bmatrix} \sigma^2 & & & \\ 0 & \sigma^2 & & \\ 0 & 0 & \sigma^2 & \\ 0 & 0 & 0 & \sigma^2 \end{bmatrix}$	$\begin{bmatrix} \sigma^2 & & & \\ \sigma_1 & \sigma^2 & & \\ 0 & \sigma_1 & \sigma^2 & \\ 0 & 0 & \sigma_1 & \sigma^2 \end{bmatrix}$								
TOEP(3)	TOEP(4)								
$\begin{bmatrix} \sigma^2 & & & \\ \sigma_1 & \sigma^2 & & \\ \sigma_2 & \sigma_1 & \sigma^2 & \\ 0 & \sigma_2 & \sigma_1 & \sigma^2 \end{bmatrix}$	$\begin{bmatrix} \sigma^2 & & & \\ \sigma_1 & \sigma^2 & & \\ \sigma_2 & \sigma_1 & \sigma^2 & \\ \sigma_3 & \sigma_2 & \sigma_1 & \sigma^2 \end{bmatrix}$								
Nonstationary:									
CSH (Heterogeneous CS), $ \rho < 1$.	$\begin{bmatrix} \sigma_1^2 & & & \\ \sigma_2 \sigma_1 \rho & \sigma_2^2 & & \\ \sigma_3 \sigma_1 \rho & \sigma_3 \sigma_2 \rho & \sigma_3^2 & \\ \sigma_4 \sigma_1 \rho & \sigma_4 \sigma_2 \rho & \sigma_4 \sigma_3 \rho & \sigma_4^2 \end{bmatrix}$								

APPENDIX 3 (Continued)

Structure	ECM
Nonstationary:	
<p>TOEPH(q) (Heterogeneous Toeplitz with q bands, $q = 1, 2, 3, 4$), $\rho_1 < 1, \rho_2 < 1, \rho_3 < 1$.</p>	<p style="text-align: center;">TOEPH(1) TOEPH(2)</p> $\begin{bmatrix} \sigma_1^2 & & & \\ 0 & \sigma_2^2 & & \\ 0 & 0 & \sigma_3^2 & \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & & & \\ \sigma_2 \sigma_1 \rho_1 & \sigma_2^2 & & \\ 0 & \sigma_3 \sigma_2 \rho_1 & \sigma_3^2 & \\ 0 & 0 & \sigma_4 \sigma_3 \rho_1 & \sigma_4^2 \end{bmatrix},$ <p style="text-align: center;">TOEPH(3) TOEPH(4)</p> $\begin{bmatrix} \sigma_1^2 & & & \\ \sigma_2 \sigma_1 \rho_1 & \sigma_2^2 & & \\ \sigma_3 \sigma_1 \rho_2 & \sigma_3 \sigma_2 \rho_1 & \sigma_3^2 & \\ 0 & \sigma_4 \sigma_2 \rho_2 & \sigma_4 \sigma_3 \rho_1 & \sigma_4^2 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & & & \\ \sigma_2 \sigma_1 \rho_1 & \sigma_2^2 & & \\ \sigma_3 \sigma_1 \rho_2 & \sigma_3 \sigma_2 \rho_1 & \sigma_3^2 & \\ \sigma_4 \sigma_1 \rho_3 & \sigma_4 \sigma_2 \rho_2 & \sigma_4 \sigma_3 \rho_1 & \sigma_4^2 \end{bmatrix}$
<p>UN(q) (UN with q bands, $q = 1, 2, 3, 4$.)</p>	<p style="text-align: center;">UN(1) UN(2)</p> $\begin{bmatrix} \sigma_1^2 & & & \\ 0 & \sigma_2^2 & & \\ 0 & 0 & \sigma_3^2 & \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & & & \\ \sigma_{21} & \sigma_2^2 & & \\ 0 & \sigma_{32} & \sigma_3^2 & \\ 0 & 0 & \sigma_{43} & \sigma_4^2 \end{bmatrix}$ <p style="text-align: center;">UN(3) UN(4)</p> $\begin{bmatrix} \sigma_1^2 & & & \\ \sigma_{21} & \sigma_2^2 & & \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 & \\ 0 & \sigma_{42} & \sigma_{43} & \sigma_4^2 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & & & \\ \sigma_{21} & \sigma_2^2 & & \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 & \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_4^2 \end{bmatrix}$

Note. TOEP(4)=TOEP, TOEPH(1)=UN(1), TOEPH(4)=TOEPH, UN(4)=UN.

三、計畫成果自評

A paper, “Using SAS PROC CALIS/TCALIS to Fit Error Covariance Structures of Latent Growth Models,” derived from the first-year part of the project, has been submitted to *Structural Equation Modeling* on March 29, 2010, to be considered for publication, and is currently under review. The contribution of the paper is to facilitate fitting error covariance structures of latent growth models.

四、誌謝

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