行政院國家科學委員會補助專題研究計畫成果報告

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計畫主持人：徐力行

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本成果報告包括以下應繳交之附件：

□赴國外出差或研習心得報告一份

□赴大陸地區出差或研習心得報告一份

□出席國際學術會議心得報告及發表之論文各一份

□國際合作研究計畫國外研究報告書一份

執行單位：國立交通大學資訊科學系

中　華　民　國　　90 年　　7 月　　31 日

有錯遞迴式循環圖中無錯漢米爾頓迴圈之嵌入

The embedding of fault-free hamiltonian cycles in faulty recursive circulant graphs

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一、中文摘要

近來先進的積體電路技術使得建構超大型連接網路變得可行，於是很多的連接網路拓樸一一的被提出來，並且在文獻中也有研究，其中遞迴式循環圖(recursive ciculant graph) RC(c,d,k)，在 1997 年被 Park 和 Chwa 提出。RC(c,d,k)有一漢米爾頓迴圈除了節點個數小於或等於 2 以外，RC(c,d,k)是點對稱和正規化圖形，遞迴式循環圖(recursive ciculant graph)有很多好的性質被提出並且用在連接網路設計上。

在本計劃中，我們將研究 RC(c,d,k)-F 仍然是漢米爾頓圖形，假如 $|F| \leq \deg(RC(c,d,k))-2$，更進一步的，我們將證明假如 $|F| \leq \deg(RC(c,d,k))-3$，對任何在 RC(c,d,k)-F 上之兩點 u 和 v 一定存在一條漢米爾頓路徑連接它們。

一個 k-正規化圖形 G，假如它的邊集合可以被分割成 k/2 條漢米爾頓迴圈，其中 k 是偶數或可以被分割成(k-1)/2 條漢米爾頓迴圈和一對完美配對，其中 k 是奇數則稱 G 是可漢米爾頓分解的。本計劃中，我們研究了遞迴式循環圖(recursive ciculant graph)是可漢米爾頓分解的。 其結果已發表在 C.H.Tsai, Y.C.Chen, Jimmy J.M.Tan and L.H.Hsu (1999), ``Hamiltonian Decompositions of Recursive Circulant Graphs," *Proceedings of 3rd World Multiconference on Systemics, Cybernetics, and Informatics and 5th International Conference on Information Systems Analysis and Synthesis*, Vol.5, pp.446-449.

在分散式作業系統，環狀區域拓譜網路在資料傳遞方面是備受重視的。容錯對於分散式作業系統是相當重要的。在本計劃中，我們將研究一些建構的機制，來建構一些具有好的容錯性質的連結網路，我們叫它容錯漢米爾頓和漢米爾頓連接。 其結果以發表在 *Y-Chuang Chen, Chang-Hsiung Tsai, Lih-Hsing Hsu, and Jimmy J. M. Tan (2001), "Construction Schemes of Fault-Tolerant Hamiltonian Graphs", Proceedings of the World Multiconference, Cybernetics and Informatics, Communicaion Systems and Internet: Part 1, Vol.5, pp.183-187.*

關鍵詞：連結網路、遞迴式循環圖、漢米爾頓圖形、容錯

英文摘要

Recent advance in integrated circuit technology make it possible to construct very large interconnection networks. Therefore, many interconnection network topologies have been proposed and investigated in the literature. A recursive circulant network RC(c,d,k) have been proposed by Park and Chwa in 1997. RC(c,d,k) has a hamiltonian cycle unless $cd^k \leq 2$. RC(c,d,k) is node symmetric and thus regular. The family of recursive circulant network has a lot of nice properties so that it is used for interconnection networks.

In this proposal, we concern that RC(c,d,k)-F remains hamiltonian if $|F| \leq \deg(RC(c,d,k)) - 2$. Moreover, we will prove that for any two vertices u and v in RC(c,d,k)-F, there exists a hamiltonian path in RC(c,d,k)-F joining u and v if $|F| \leq \deg(RC(c,d,k)) - 3$.
A k-regular graph G is hamiltonian decomposable if its edge-set can be partitioned into k/2 hamiltonian cycles when k is even or (k-1)/2 hamiltonian cycles and a perfect matching when k is odd. In this proposal, we concern that every recursive circulant graph is hamiltonian decomposable. The result was publication in C.H.Tsai, Y.C.Chen, Jimmy J.M.Tan and L.H.Hsu (1999), ``Hamiltonian Decompositions of Recursive Circulant Graphs," *Proceedings of 3rd World Multiconference on Systemics, Cybernetics, and Informatics and 5th International Conference on Information Systems Analysis and Synthesis*, Vol.5, pp.446-449.

The token ring topology is required in token passing approach in distributed operating systems. Fault tolerance is also required in the design of distributed systems. In this proposal, we concern some construction schemes to construct interconnection networks with some good fault-tolerant properties, namely fault-tolerant Hamiltonicity and fault-tolerant Hamiltonian connectivity. The result was publication in *Y-Chuang Chen, Chang-Hsiung Tsai, Lih-Hsing Hsu, and Jimmy J. M. Tan (2001), "Construction Schemes of Fault-Tolerant Hamiltonian Graphs", Proceedings of the World Multiconference, Cybernetics and Informatics, Communicaion Systems and Internet: Part 1, Vol.5, pp.183-187.*

Keywords: Interconnection Network, Recursive circulant network, Hamiltonian graph, Fault-tolerant

二、計劃緣由及目的

The ring structure is important for distributed computing. It allows communication with loss cost because the number of edges of the ring is low, is free of branching, and is often used in local area networks, for example, Token Ring[ ]. Hence it is useful to construct a hamiltonian cycle or ring structure in the network.
The faulty network is practically meaningful because node faults and link faults may happen when a network is used. A graph is called k-node hamiltonian if G remains hamiltonian after removing at most k nodes. Similarly, a graph G is called k-edges hamiltonian if G remains hamiltonian after removing at most k edges. The k-nodes and k-edges hamiltonicity is proposed by Hsieh, Chen, and Ho [3], measures the performance of the hamiltonian property in the faulty networks. Obviously, $k \leq \deg(G) - 2$ where

$\deg(G)=\min\{\deg(v)\mid v\in V(G)\}$. Many related works have appeared in literature, for example [3,4,5].

In this proposal, a more general parameter will be considered. A graph G is k-hamiltonian if G-F remains hamiltonian for every $F\subset V(G)\bigcup E(G)$ with $|F|\le k$. Obviously, $k\le \deg(G)-2$. We will research that recursive circulant graphs are k-hamiltonian graphs with $k\le \deg(RC(c,d,k))-2$ and that for any two vertices u and v in RC(c,d,k)-F, there exists a hamiltonian path joining u and v if $F\subset V(RC(c,d,k))\bigcup E(RC(c,d,k))$ with $|F|\le \deg(RC(c,d,k))-2$.

A k-regular graph G is hamiltonian decomposable if its edge-set can be partitioned into k/2 hamiltonian cycles when k is even or (k-1)/2 hamiltonian cycles and a perfect matching when k is odd. In this proposal, we will prove that every recursive circulant graphs is hamiltonian decomposable.

The fault-tolerant Hamiltonicity and the fault-tolerant Hamiltonian connectivity are essential parameters of an interconnection network. In this propose, we propose a family of k-regular, (k-2)-Hamiltonian, and (k-3)-Hamiltonian connected graphs. These graphs are maximally fault-tolerant, and we call them super fault-tolerant Hamiltonian graphs.

One of the contributions of this proposal is the following. We propose a construction scheme to construct, with flexibility, many k-regular super fault-tolerant Hamiltonian graphs for k ≥ 6. As for small values of k, k ≤ 5, there are some examples in literature, such as Twisted-cubes, Crossed-cubes, Möbius cubes, and recursive circulant graphs, etc.

There are many popular interconnection networks which are k-regular graphs. Some of them, e.g., Twisted-cubes, Crossed-cubes, Möbius cubes, and recursive circulant graphs, can be recursively constructed using our construction schemes. And therefore, they are in fact a subclass of our proposed family of graphs. Then, we know that they are super fault-tolerant Hamiltonian as long as the case is true for initial cases k ≤ 5. For small values of k, k ≤ 5, we may use a computer program to check if it is (k-2)-Hamiltonian and (k-3)-Hamiltonian connected.

三、研究方法與成果：

步驟一: 文獻收集

透過各學校圖書館、研究機構、研討會及 Internet 上的各資料庫，收集一切關於遞迴式循環圖(recursive circulant graphs)問題的資料，以其能全面了解遞迴式循環圖(recursive circulant graphs)的意義及特性並希望能做一全面之研究。

步驟二 研讀及討論

定期研讀、報告所收集之資料及論文，比較分析探討文獻，經由全方位的討論找出遞迴式循環圖(recursive circulant graphs)容錯的性質。

步驟三 模擬系統研究及理論推導

針對目前尚未解決之問題設計軟體模擬，並推導出其理論架構並加以學術證明。

步驟四成果發表

將研究心得及成果撰寫成論文，並發表於國內外知名學術期刊及研討會，並將此計劃執行過程心得，在教學上與學生分享，讓教學與研究能相輔相成。 我們已經成果發表在 *Proceedings of 3rd World Multiconference on Systemics, Cybernetics, and Informatics and 5th International Conference on Information Systems Analysis and Synthesis*

本計劃最後完成的成果如下

1. 遞迴式循環圖(recursive circulant graphs)是可以被漢米爾頓分解的(hamiltonian decomposition)。
2. 得到一結果—遞迴式循環圖(recursive

circulant graphs)之最佳容錯性質一:即可以嵌入一無錯之漢米爾頓迴圈到一有錯的遞迴式循環圖(recursive circulant graphs)，且其中有錯的個數小於 deg(RC(c,d,k))-2。

3. 遞迴式循環圖(recursive circulant graphs)之最佳容錯性質二:即在一有小於 deg(RC(c,d,k))-3 錯的遞迴式循環圖(recursive circulant graphs)中，任何兩點都被一條漢米爾頓路徑所連接。
4. 我們提出一些富有彈性的建構機制，以建構出許多 k-正規的超級容錯漢米爾頓圖漢超級容錯漢米爾頓連接圖。
5. 有很多受歡迎的連結網路都是 k-正規。其中包括：扭曲圖(Twisted-cubes)，相交圖(Crossed-cubes)，梅氏圖(Möbius cubes)，和遞迴式循環圖 (recursive circulant graphs)，而這些圖都能夠用我們的建構機制遞迴的建構出來。所以我們得知他們大部分都是超級容錯漢米爾頓圖漢超級容錯漢米爾頓連接圖。

四、結論與討論：

在計畫主持人的帶領之下，定期研讀、報告所收集到的資料，比較、分析各種方法的優缺點，透過全體多方面的反覆討論，使我們對漢米爾頓分解問題和解決方式有清楚的了解。

在本計劃中，我們對於漢米爾頓相關問題，先前已有相當的經驗及基礎，爲本計劃的執行奠定良好的根基，而能順利完成預定之研究進度。我們希望這些相關問題的研究能提升有關領域之應用拓展；而研究所得之經驗、知識以及部份未解或仍可發揮之處，當可做爲日後更深入的研究討論。

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