# 行政院國家科學委員會補助專題研究計畫 □ 成果報告 ■期中進度報告

## 感知無線網路接取技術及資源管理之研究–子計畫五：

## 協力式感知無線網路之頻譜偵測及接收機設計

計畫類別：□ 個別型計畫 ■ 整合型計畫
計畫編號：NSC 97－2221－E－009－101－MY3
執行期間： 97 年 8 月 1 日至 100 年 7 月 31 日

計畫主持人：吳卓諭 助理教授
共同主持人：王蒞君 教授
計畫參與人員：

成果報告類型(依經費核定清單規定繳交)：■精簡報告 □完整報告

本成果報告包括以下應繳交之附件：

處理方式：除產學合作研究計畫、提升產業技術及人才培育研究計畫、列管計畫及下列情形者外，得立即公開查詢
□涉及專利或其他智慧財產權，□一年□二年後可公開查詢

執行單位：國立交通大學電信工程研究所

中 華 民 國 99 年 5 月 16 日

# 摘要

現有的具通道感知(Channel-aware)之決策融合(Decision fusion)架構，皆假設融合中心(Fusion center)已經知道局部準確偵測機率(Local detection probability)。然而，感測器警報可能會因為一些未知的原因而被觸發，如此假設則忽略了這個可能性。因此，本計劃即是在局部準確偵測機率是未知的假設下，探討二位元決策融合的問題。我們的研究方法首先是將感測器與決策中心之間的通訊鏈結(Communication links)視為二位元對稱通道(Binary symmetric channel)，並假設每個感測器會傳送一個簡單的位元訊號給決策中心；一開始的總體決策法則(Global fusion rule)是採用廣義概似比例檢定(generalized likelihood ratio test, GLRT)。執行廣義概似比例檢定時所需的未知參數是以最大概似估計(Maximum likelihood estimate)法求得，若假設此感測網路處於高訊雜比(Signal to noise ratio, SNR)的環境中，則可推導出一個可解析的近似解。然而，由於我們不易對廣義概似比例檢定進行效能分析，因此我們提出了另一個簡單的決策法則以進行效能分析。此外，若局部準確偵測機率在給定的合理範圍內，我們證明出降低平均的鏈結錯誤率可以提昇總體的準確偵測機率，因此，我們也提出了一個感測器的功率分配(Power allocation)機制，改善鏈結的品質以提昇偵測效能。模擬的結果顯示出(1) 替代的決策法則優於廣義概似比例檢定；(2) 當我們提出的功率分配機制被引入時，偵測效能可以進一步得到提昇。

**關鍵字**：分散式偵測、感測網路、通訊通道、功率分配

# Abstract

Existing channel-aware decision fusion schemes assume that the local detection probability is known at the fusion center (FC). However, this paradigm ignores the possibility of unknown sensor alarm responses to the occurrence of events. Accordingly, this paper examines the binary decision fusion problem under the assumption that the local detection probability is unknown. Treating the communication links between the nodes and the FC as binary symmetric channels and assuming that the sensor nodes transmit simple one-bit reports to the FC, the global fusion rule is formulated initially in terms of the generalized likelihood ratio test (GLRT). Adopting the assumption of a high SNR regime, an approximate maximum likelihood (ML) estimate is derived for the unknown parameter required to implement the GLRT that is affine in the received data. The GLRT-based formulation is intuitively straightforward, but does not permit a tractable performance analysis. Therefore, motivated by the affine nature of the approximate ML solution, a simple alternative fusion rule is proposed in which the test statistic remains affine in the received data. It is shown that the proposed fusion rule facilitates the analytic characterization of the channel effect on the global detection performance. In addition, given a reasonable range of the local detection probability, it is shown that the global detection probability can be improved by reducing the average link error. Thus, a sensor power allocation scheme is proposed for enhancing the detection performance by improving the link quality. Simulation results show that (i) the alternative fusion rule outperforms the GLRT, and (ii) the detection performance of the fusion rule is further improved when the proposed power loading method is applied.

**Keywords**: Distributed detection; Sensor networks; Communication channels; Power allocation.

# 前言

為滿足高品質、高速率的多媒體通訊需求以及增加通訊使用者用戶，無線通訊實體層設計著重於如何增加通訊系統的頻譜使用率及如何提高通訊的鏈結品質與通訊容量。因此，如何針對無線通訊的特性，設計合乎系統需求的通訊技術一直是新一代通訊標準所追求的目標。到目前為止，能達到高品質、高速率的關鍵無線傳輸技術即是在傳送與接收端配置多根天線，形成『多輸入多輸出系統』。然而，於實際通訊環境，因為使用者的手機體積限制，天線間的鏈結統計特性並未像學理所考慮的那樣完美，以致於多輸入多輸出系統效能表現出現瓶頸，因此，近年來協力式通訊(Cooperative communication)於感知無線網路系統中的應用已被廣泛地探討研究，其主要概念為利用中繼(Relay)節點幫忙傳送信號，使得系統能有較多且獨立的鏈結路徑，藉以實現多輸入多輸出的系統概念，因此，這種系統也稱為虛擬多輸入多輸出系統。由於協力式通訊系統的接收端會藉由這些獨立鏈結路徑所傳送來的訊號去判斷原始訊號為何，因此接收端的偵測問題其實可以看成是無線感測網路(Wireless sensor networks)中的分散式偵測(Distributed detection)問題，亦即將中繼節點當作感測器(sensor)、將接收端當作決策中心(Fusion center)，如此，我們便可以設計發展各種適用於分散式偵測中的演算法來處理協力式通訊系統的問題。本年度將針對無線感測網路中的分散式偵測進行深入的分析研究，有別於大部分文獻的假設，本計劃所考慮的感測網路與通訊鏈結環境不是理想的，除了提出一新穎的演算法以解決此問題，亦提出一功率分配(Power allocation)機制以提昇偵測效能[8]。

# 研究目的與文獻探討

在無線感測網路中，有關分散式偵測(Distributed detection)與決策融合(Decision Fusion)的問題已有相當多的研究[1], [5], [6]。然而，先前的研究皆假設決策中心(Fusion center)可以無誤地接收到感測器的偵測結果[6], [7]，亦即局部感測器與決策中心之間的通道鏈結是理想的，最近則有一些研究計劃開始探討分散式偵測中存在不理想的通道鏈結[2]。一般而言，具通道感知(Channel-aware)的通訊架構會假設決策中心已經知道局部感測器的偵測效能，但感測器警報可能會因為一些未知的原因而被觸發，如此假設則忽略了這個可能性。舉例來說，考慮一個監控室內溫度以偵測是否有火災發生的感測網路。實際上，因為火災的猛烈程度不同或是不同的起火原因，火災的平均溫度可能在 100 到 1000 度之間變化，所以火災發生的徵兆是不明確的，甚至會隨著時間而改變。因此，局部準確偵測機率(Local detection probability)是未知的。於是，有別於現有文獻所考慮的理想環境，本計劃將局部感測器與決策中心之間的通道視為二位元對稱通道(Binary symmetric channel)，局部準確偵測機率則視為一未知的參數，而為了處理這樣的不確定性，本計劃亦提出一個總體決策法則(Global fusion rule)以改良一般常使用的決策法則，並結合功率分配機制進一步提昇偵測效能。

# 研究方法

在此研究子題中，我們考慮一具有 $N$ 個感測器的感測網路，每一個感測器不是處於警報狀態就是處於靜止狀態；每一個感測器都會記錄自己目前的狀態並傳送一個位元訊號給決策中心以告知其狀態。假設每一個感測器的假警報機率(False alarm probability) $\pi_0$ 是已知的，準確偵測機率(Detection probability) $\pi_1$ 是未知的並且會落在此區間 $(\pi_0,1)$，接著假設第 $i$ 個感測器與決策中心之間的通道鏈結為二位元對稱通道，而交越機率(Cross-over probability)是$\varepsilon_i$，$1 \le i \le N$。於是，在決策中心所收到的位元訊息$r_i \in \{0,1\}$是一個白努力隨機變數(Bernoulli random variable)：

$$\Pr\left\{r_i = 1\right\} = \begin{cases} \pi_0\left(1-\varepsilon_i\right) + \left(1-\pi_0\right)\varepsilon_i, & (\text{事件不存在}), \\ \pi_1\left(1-\varepsilon_i\right) + \left(1-\pi_1\right)\varepsilon_i, & (\text{事件存在}). \end{cases}$$

進一步考慮決策中心所收到的$N$個位元訊號 $\mathbf{r} := [r_1 \cdots r_N]$，其結合機率質量函數(joint probability mass function)為

$$p(\mathbf{r};\pi_m) = \prod_{i=1}^{N}\left[(1-2\varepsilon_i)\pi_m + \varepsilon_i\right]^{r_i}\left[-(1-2\varepsilon_i)\pi_m + (1-\varepsilon_i)\right]^{1-r_i},\ m = 0,1.$$

因為$\pi_1$是未知的，此偵測問題可表示成二位元複合假設檢定(Binary composite hypothesis test)：

$$\begin{cases} \mathcal{H}_0 : p(\mathbf{r};\pi_0) & (\text{事件不存在}), \\ \mathcal{H}_1 : p(\mathbf{r};\pi_1), \pi_1 > \pi_0 & (\text{事件存在}). \end{cases}$$

對於此類問題，廣義概似比例檢定(Generalized likelihood ratio test, GLRT) [3, chap. 6]是一種常用的決策法則，若以下方程式成立則判斷事件存在：

$$\ln\frac{p(\mathbf{r};\hat{\pi}_{1.ML})}{p(\mathbf{r};\pi_0)} = \sum_{i=1}^{N} r_i \log\left[\frac{(1-2\varepsilon_i)\hat{\pi}_{1,ML} + \varepsilon_i}{(1-2\varepsilon_i)\pi_0 + \varepsilon_i}\right] + \sum_{i=1}^{N}(1-r_i)\log\left[\frac{(1-2\varepsilon_i)\hat{\pi}_{1,ML} - (1-\varepsilon_i)}{(1-2\varepsilon_i)\pi_0 - (1-\varepsilon_i)}\right] \ge \gamma,$$

其中$\hat{\pi}_{1,ML}$是$\pi_1$的最大概似估計，而門檻值$\gamma$可以由假警報機率所決定。

為了求得 $\hat{\pi}_{1,ML}$，我們必須求解以下方程式：

$$\frac{\partial \ln p(\mathbf{r};\pi_1)}{\partial \pi_1} = \sum_{i=1}^{N}\frac{r_i}{\pi_1 + \left[\varepsilon_i / (1-2\varepsilon_i)\right]} + \sum_{i=1}^{N}\frac{1-r_i}{\pi_1 - \left[(1-\varepsilon_i)/(1-2\varepsilon_i)\right]} = 0.$$

但由於此方程式無法得到一個可解析的解，因此我們假設此無線感測網路處於高訊雜比(Signal to noise ratio)的環境中，則可得到一個近似的最大概似估計

$$\hat{\pi}_1 = \frac{1}{N}\sum_{i=1}^{N}\left[(1+2\varepsilon_i)r_i - \varepsilon_i\right].$$

然而，即使引入了近似解，我們仍然不易分析廣義概似比例檢定的偵測效能，因此，我們根據[4, p-204]提出另一種偵測法則，不僅複雜度較廣義概似比例檢定低，也較易於分析其偵測效能。替代的檢定準則如下：

$$\begin{cases} \mathcal{H}_0 : \hat{\pi}_1 - \pi_0 \le \gamma, \\ \mathcal{H}_1 : \hat{\pi}_1 - \pi_0 > \gamma. \end{cases}$$

將$\hat{\pi}_1$改寫成下式：

$$\hat{\pi}_1 = \frac{1}{N}\sum_{i=1}^{N}\left[(1+2\varepsilon_i)r_i - \varepsilon_i\right] = \underbrace{\frac{1}{N}\sum_{i=1}^{N}(1+2\varepsilon_i)r_i}_{:=T} - \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i,$$

其中 $T$ 是等效的統計檢定。對於每一個 $0 \le k \le N$，令 $I^{(k)} := \left\{I_1^{(k)}, I_2^{(k)} \cdots, I_{C_k^N}^{(k)}\right\}$ 為 $\{1,\cdots,N\}$ 所有相異子集合之集合，這些子集合皆由 $k$ 個元素所構成，其中 $C_k^N := N!/[k!(n-k)!]$ 且 $I^{(0)} = \{\phi\}$。

經過一些推導[8]，我們可以得到準確偵測機率與假警報機率的下限，如下所示：

$$
\begin{aligned}
P_d &= \Pr\left\{T \ge \pi_0 + \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i + \gamma \,\middle|\, \mathcal{H}_1\right\} \\
&\ge P_d^{(L)} \\
&= \sum_{k=k_l+1}^{N}\sum_{l=1}^{C_k^N}\left\{\prod_{i\in I_l^{(k)}}\left[\left(1-2\varepsilon_i\right)\pi_1 + \varepsilon_i\right]\prod_{i\notin I_l^{(k)}}\left[-\left(1-2\varepsilon_i\right)\pi_1 + \left(1-\varepsilon_i\right)\right]\right\}
\end{aligned},
$$

$$
\begin{aligned}
P_f &= \Pr\left\{T \ge \pi_0 + \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i + \gamma \,\middle|\, \mathcal{H}_0\right\} \\
&\ge P_f^{(L)} \\
&= \sum_{k=k_l+1}^{N}\sum_{l=1}^{C_k^N}\left\{\prod_{i\in I_l^{(k)}}\left[\left(1-2\varepsilon_i\right)\pi_0 + \varepsilon_i\right]\prod_{i\notin I_l^{(k)}}\left[-\left(1-2\varepsilon_i\right)\pi_0 + \left(1-\varepsilon_i\right)\right]\right\}
\end{aligned}.
$$

此外，藉由一些推導與討論後(細節請參閱[8])，我們歸納出以下定理：

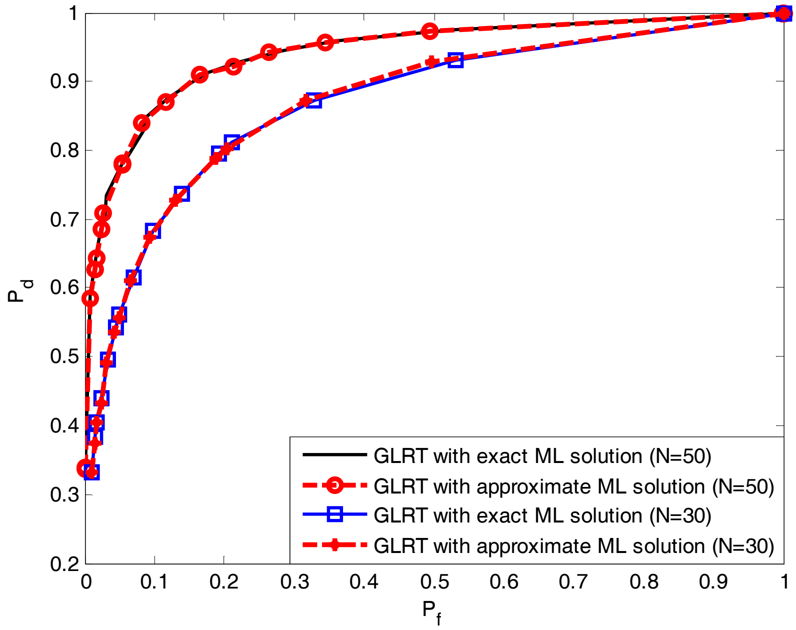
定理

假設 $\pi_0 < 0.5 < \pi_1$ 並固定假警報機率 $P_f$。對於兩個不同的總和鏈結錯誤 $E = \sum_{i=1}^{N}\varepsilon_i$ 和 $E' = \sum_{i=1}^{N}\varepsilon_i'$，令其相對的準確偵測機率下限分別為 $P_d^{(L)}$ 和 $P_d'^{(L)}$。若 $E' < E$，則 $P_d'^{(L)} > P_d^{(L)}$。

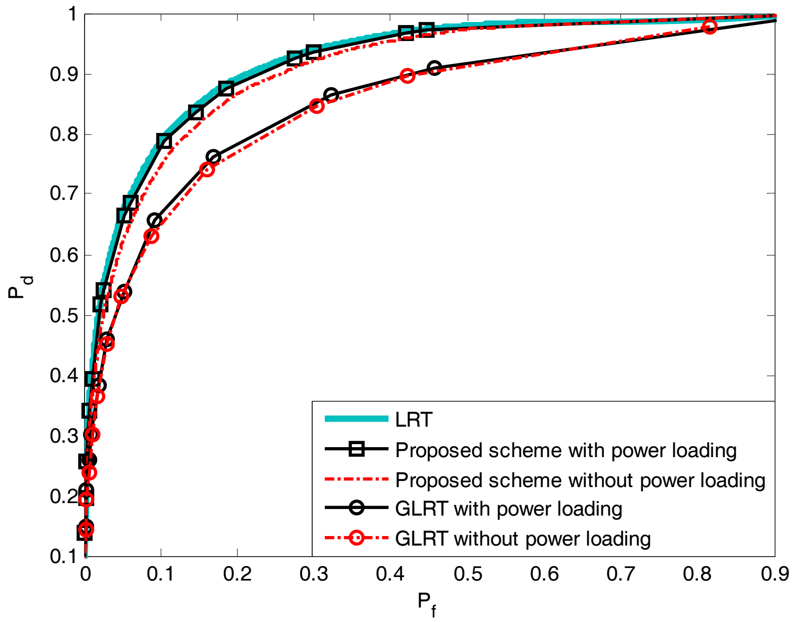
由此定理可知，若 $\pi_0 < 0.5 < \pi_1$，則降低總和鏈結錯誤率可以提昇總體準確偵測機率(Global detection probability)。於是，我們可以設計出一個最佳的功率分配架構以提昇總體準確偵測機率。而此最佳化問題已被記載於一些多重輸入多重輸出(Multiple input multiple output)的無線通訊系統相關之文獻中[9], [10]。

## 結果與討論

圖一為局部準確偵測機率的最大概似估計之真實解與近似解比較。由圖可知，採用近似解的偵測效能與採用真實解的偵測效能幾乎一樣。圖四為所提出方法與廣義概似比例檢定的效能比較以及引入功率分配機制的效能比較。由圖可知，所提出方法的偵測效能比起廣義概似比例檢定有顯著的提昇，此外，若皆引入功率分配機制，所提出方法獲得的效能提昇比起廣義概似比例檢定要顯著許多，這是因為此功率分配機制是針對替代的檢定所設計而不是針對廣義概似比例檢定。




圖一：廣義概似比例檢定中，局部準確偵測機率的最大概似估計之真實解與近似解比較。




圖二：本計劃所提出的偵測法則與廣義概似比例檢定之比較。

# 成果自評

計劃執行將滿兩年，研究成果相當豐碩，目前已有五篇論文被刊載在 IEEE Transactions on Wireless Communications 與 IEEE Transactions on Signal Processing 期刊中（詳見 publication list），並有三篇期刊論文正在接受審查，可見本研究的成果有相當優異且具體的學術價值。

# Publication List

[1] Jwo-Yuh Wu, "How much coherent interval should be dedicated to non-redundant diagonal precoding for blind channel estimation in single carrier block transmission?" ***IEEE Trans. Wireless Communications*** [SCI, EI], accepted and to appear.

[2] Jwo-Yuh Wu, Chan-Wei Wu, Tsang-Yi Wang, and Ta-Sung Lee, "Channel-aware decision fusion with unknown local sensor detection probability," ***IEEE Trans. Signal Processing*** [SCI, EI], vol. 58, no 3, pp. 1457-1463, March 2010.

[3] Wen-Fang Yang, Jwo-Yuh Wu, Li-Chun Wang, and Ta-Sung Lee, "A cooperative multi-group priority MAC protocol for multi-packet reception channels," ***IEEE Trans. Wireless Communications*** [SCI, EI], vol. 8, no. 11, pp 5416-5421, Nov. 2009.

[4] Fang-Shuo Tseng, Wen-Rong Wu, and Jwo-Yuh Wu, "Joint source/relay precoder design in nonregenerative cooperative systems using an MMSE criterion," ***IEEE Trans. Wireless Communications*** [SCI, EI], vol. 8, no. 10, pp. 4928-4933, Oct. 2009.

[5] Chih-Yuan Lin, Jwo-Yuh Wu, and Ta-Sung Lee, "Robust receiver design for MIMO single-carrier block transmission over time-varying dispersive channels against imperfect channel knowledge," ***IEEE Trans. Wireless Communications*** [SCI, EI], vol. 7, no. 10, pp. 3799-3812, Oct. 2008.

# 參考文獻


[1] J. F. Chamberland and V. V. Veeravalli. "Wireless sensors in distributed detection applications," *IEEE Signal Processing Magazine,* vol. 24, no. 3, pp. 16-25, May 2007.

[2] B. Chen, T. Lang, and P. K. Varshney, "Channel aware distributed detection in wireless sensor networks," *IEEE Signal Processing Magazines*, vol. 23, no. 4, pp. 16-26, July 2006.

[3] S. M. Kay, *Fundamentals of Statistical Signal Processing: Detection Theory*, Prentice-Hall PTR, 1998.

[4] B. C. Levy, *Principles of Signal Detection and Parameter Estimation*, Springer, 2008.

[5] A. Swami, Q. Zhao, Y. W. Hong, and L. Tong, *Wireless Sensor Networks: Signal Processing and Communications Perspectives*, John Wiley & Sons Ltd., 2007.

[6] P. K. Varshney, *Distributed Detection and Data Fusion*, New York, Springer 1997.

[7] Q. Zhao, A. Swami, and L. Tong, "The interplay between signal processing and networking in sensor networks," *IEEE Signal Processing Magazine*, vol. 23, no. 4, pp. 84-93, July 2006.

[8] J. Y. Wu, C. W. Wu, T. Y. Wang, and T. S. Lee, "Channel-aware decision fusion with unknown local sensor detection probability," *IEEE Trans. Signal Processing*, vol. 58, no 3, pp. 1457-1463, March 2010.

[9] N. Wang and S. D. Blostein, "Minimum BER power allocation for MIMO spatial multiplexing systems," *Proc. ICC'05*, vol. 4, pp. 2282-2286, 2005.

[10] Z. Yan, K. M. Wong, and Z. Q. Luo, "Optimal diagonal precoder for multiantenna communication systems," *IEEE Trans. Signal Processing*, vol. 53, no. 6, pp. 2089-2100, June 2005.

# Channel-Aware Decision Fusion With Unknown Local Sensor Detection Probability

Jwo-Yuh Wu, Chan-Wei Wu, Tsang-Yi Wang, and Ta-Sung Lee

*Abstract*—**Existing channel-aware decision fusion schemes assume that the local detection probability is known at the fusion center (FC). However, this paradigm ignores the possibility of unknown sensor alarm responses to the occurrence of events. Accordingly, this correspondence examines the binary decision fusion problem under the assumption that the local detection probability is unknown. Treating the communication links between the nodes and the FC as binary symmetric channels and assuming that the sensor nodes transmit simple one-bit reports to the FC, the global fusion rule is formulated initially in terms of the generalized likelihood ratio test (GLRT). Adopting the assumption of a high SNR regime, an approximate maximum likelihood (ML) estimate is derived for the unknown parameter required to implement the GLRT that is affine in the received data. The GLRT-based formulation is intuitively straightforward, but does not permit a tractable performance analysis. Therefore, motivated by the affine nature of the approximate ML solution, a simple alternative fusion rule is proposed in which the test statistic remains affine in the received data. It is shown that the proposed fusion rule facilitates the analytic characterization of the channel effect on the global detection performance. In addition, given a reasonable range of the local detection probability, it is shown that the global detection probability can be improved by reducing the total link error. Thus, a sensor power allocation scheme is proposed for enhancing the detection performance by improving the link quality. Simulation results show that: 1) the alternative fusion rule outperforms the GLRT; and 2) the detection performance of the fusion rule is further improved when the proposed power loading method is applied.**

*Index Terms*—**Communication channels, distributed detection, power allocation, sensor networks.**

## I. Introduction

The problem of distributed signal/event detection and decision fusion in wireless sensor networks has attracted significant attention in the literature [1], [12], [13]. However, most previous studies are based on the idealized assumption that the sensor reports are received at the fusion center (FC) without error [13], [16]. Recently, there have been several proposals that further take into account the communication channel impairments [2], [3], [6], [7], [11]; see [4] for a tutorial introduction to distributed detection in the presence of nonideal channel links. In general, these channel-aware schemes assume that the local sensor detection performance, characterized by the detection probability and the false-alarm probability, is known to the FC. However, this paradigm ignores the possibility of unknown sensor alarm responses to the occurrence of events within the sensing field. For example, consider a sensor network designed to monitor the rise in temperature within a room in order to detect the potential outbreak of a fire. In practice, the characteristics of a fire are uncertain, e.g., the mean temperature may vary from $100^{\circ}$ to $1000^{\circ}$ depending on the severity of the fire or the type of fire. Moreover, the characteristics of the fire may vary over time. As a result, the local detection probability (under a fixed threshold) could be unknown due to the response to the uncertain temperature of fire events. To accommodate such variations in the sensing field conditions, one conceivable approach is to simply model the local detection probability as an unknown parameter and to design suitable global decision rules for tackling such uncertainty.

This correspondence proposes a channel-aware decision fusion scheme tailored for the above-mentioned scenario. The communication links between the sensor nodes and the FC are modeled as binary symmetric channels. Each sensor, when triggered, sends a single bit to the FC to inform it of its local decision. Note that since the FC treats the local detection probability as an unknown parameter, the nodes do not need to send an additional message regarding the current local detection performance, and thus the communication overhead is reduced. Based solely on the received sensor reports, the global decision rule is formulated intuitively as a generalized likelihood ratio test (GLRT) [9]. The implementation of this test calls for the maximum likelihood (ML) estimate of the unknown parameter, which, in the current case, does not allow for a closed-form solution. Thus, under a high signal-to-noise ratio (SNR) assumption, an approximate ML estimate is derived that is *affine* in the received data. However, even when adopting this approximation, the detection performance of the GLRT decision rule remains difficult to characterize. Therefore, based on the approximated ML scheme, a simple alternative fusion rule is proposed in which the test statistic remains affine in the received data. The proposed fusion rule enables the derivation of a closed-form expression for the detection performance and, therefore, facilitates the analytic characterization of the channel effect. In addition, it is shown that, for reasonable ranges of the local detection and false alarm probabilities, the global detection performance can be improved by enhancing the communication-link quality, specifically, reducing the total link bit-error rate (BER). Hence, an optimal power allocation scheme is proposed to minimize the total BER subject to a total power budget. Simulations show that the proposed fusion rule outperforms the GLRT; in addition, the detection performances of both the proposed fusion rule and the GLRT decision rule are seen to be further improved via the application of the optimal sensor power loading scheme. The remainder of this correspondence is organized as follows. Section II formulates the problem, while Section III presents the GLRT based detection scheme and derives the approximate ML solution. Section IV introduces the proposed fusion rule and derives the associated analytic performance results. Section V formulates an algorithm for improving the channel link quality in order to improve the detection performance. Section VI presents the simulation results. Finally, Section VII provides some brief concluding remarks.

## II. Problem Statement

Consider a sensor network with $N$ identical binary nodes designed to monitor the occurrence of a certain event. Each sensor exists in one of two different states, namely active (e.g., when the measurement is above a certain threshold) or silent (e.g., the measurement is below the threshold, and the sensor simply remains quiet to conserve energy). Assume that each node is subject to a *known* false-alarm probability

Manuscript received June 05, 2009; accepted October 08, 2009. First published November 06, 2009; current version published February 10, 2010. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Pramod K. Varshney. This work was supported by the National Science Council of Taiwan under grants NSC 97-2221-E-009-101-MY3 and NSC 98-2221-E-110-043-MY3, by the Ministry of Education of Taiwan under the MoE ATU Program, by the Academia-Research Cooperation Program of MoEA under Grant 98-EC-17-A-02-S2-0050, and by MediaTek Research Center at National Chiao Tung University, Taiwan.

J.-Y. Wu, C.-W. Wu, and T.-S. Lee are with the Department of Electrical Engineering, National Chiao Tung University, Hsinchu, Taiwan (e-mail: jywu@cc.nctu.edu.tw; xl3505@gmail.com; tslee@mail.nctu.edu.tw).

T.-Y. Wang is with Institute of Communications Engineering, National Sun Yat-sen University, Kaohsiung, Taiwan (e-mail: tcwang@faculty.nsysu.edu.tw).

Color versions of one or more of the figures in this correspondence are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TSP.2009.2036065

$\pi_0$ due to small ambient perturbations.[1] When triggered by the occurrence of the event of interest, the local detection probability across the sensors, $\pi_1$, falls within the interval $(\pi_0, 1)$. However, the exact value of $\pi_1$ is assumed to be *unknown*. The status of the $i$th sensor can thus be represented using a binary random variable $s_i \in \{0,1\}$, with $\Pr\{s_i = 1\} = \pi_0$ in the absence of the event of interest, and $\Pr\{s_i = 1\} = \pi_1$ otherwise. Each sensor records its status using a single bit and then transmits this bit to the FC. Assume that the communication link between the $i$th sensor node and the FC is nonideal and is modeled by a binary-symmetric channel with a crossover probability $\varepsilon_i$, $1 \le i \le N$. At the FC, the bit received from the $i$th sensor is decoded, and the resultant message $r_i \in \{0,1\}$ is a Bernoulli random variable

$$\Pr\{r_i = 1\} = \begin{cases} \pi_0(1-\varepsilon_i)+(1-\pi_0)\varepsilon_i, & \text{(event is absent)} \\ \pi_1(1-\varepsilon_i)+(1-\pi_1)\varepsilon_i, & \text{(event is present).} \end{cases} \quad (2.1)$$

Based on a single snapshot of the observed sensor reports $r_i$, $1 \le i \le N$, the FC utilizes a predefined decision rule in order to make a final decision regarding the occurrence (or absence) of the event of interest within the sensing field. The main objectives of this study include: 1) to formulate a suitable fusion rule for the case in which the local detection probability is unknown; and 2) to devise a technique for mitigating the channel effect on the global detection performance.

## III. GLRT-Based Detection Scheme

### A. GLRT Scheme

Assuming that the set of Bernoulli random variables $\{r_i\}$ are conditionally independent given the event under test, the joint probability mass functions of $\mathbf{r} := [r_1 \cdots r_N]$ under either $\pi_0$ and $\pi_1$ are given by

$$p(\mathbf{r};\pi_m) = \prod_{i=1}^{N} [(1-2\varepsilon_i)\pi_m + \varepsilon_i]^{r_i} \times [-(1-2\varepsilon_i)\pi_m + (1-\varepsilon_i)]^{1-r_i}, \quad m = 0,1. \quad (3.1)$$

Since $\pi_1$ is unknown, the detection problem can be formulated intuitively as the following binary composite hypothesis test:

$$\begin{cases} \mathcal{H}_0 : p(\mathbf{r};\pi_0), & \text{(event is absent)} \\ \mathcal{H}_1 : p(\mathbf{r};\pi_1), \pi_1 > \pi_0, & \text{(event is present).} \end{cases} \quad (3.2)$$

The GLRT [9, Ch. 6] is a typical decision rule for problems of this type and decides $\mathcal{H}_1$ if

$$\ln \frac{p(\mathbf{r};\widehat{\pi}_{1,\mathrm{ML}})}{p(\mathbf{r};\pi_0)} = \sum_{i=1}^{N} r_i \log\left[\frac{(1-2\varepsilon_i)\widehat{\pi}_{1,\mathrm{ML}} + \varepsilon_i}{(1-2\varepsilon_i)\pi_0 + \varepsilon_i}\right] + \sum_{i=1}^{N}(1-r_i)\log\left[\frac{(1-2\varepsilon_i)\widehat{\pi}_{1,\mathrm{ML}} - (1-\varepsilon_i)}{(1-2\varepsilon_i)\pi_0 - (1-\varepsilon_i)}\right] \ge \gamma \quad (3.3)$$

where $\hat{\pi}_{1,\mathrm{ML}}$ is the ML estimate of $\pi_1$ and the threshold $\gamma$ is determined from the prescribed false-alarm probability. However, to implement (3.3), it is first necessary to find $\hat{\pi}_{1,\mathrm{ML}}$. This can be achieved by solving the equation $\partial \ln p(\mathbf{r};\pi_1)/\partial\pi_1 = 0$, which based on (3.1) can determined directly as

$$\sum_{i=1}^{N} \frac{r_i}{\pi_1 + [\varepsilon_i/(1-2\varepsilon_i)]} + \sum_{i=1}^{N} \frac{1-r_i}{\pi_1 - [(1-\varepsilon_i)/(1-2\varepsilon_i)]} = 0. \quad (3.4)$$

[1]Note that knowledge about $\pi_0$ can be acquired in a training process conducted in the absence of the event of interest.

In other words, finding the unknown parameter $\hat{\pi}_{1,\mathrm{ML}}$ involves solving certain roots of the polynomial given in (3.4), which is of the order $N-1$. While this can be achieved using numerical techniques, an analytic solution does not exist. Accordingly, given the assumption of a high SNR regime, the following section derives a simple closed-form approximate ML solution.

### B. Approximate ML Estimate

Crucially, if the link error probability $\varepsilon_i$ is small, it follows that

$$\frac{\varepsilon_i}{1-2\varepsilon_i} = \left(1 + 2\varepsilon_i + 4\varepsilon_i^2 + \cdots\right)\varepsilon_i \approx \varepsilon_i$$

and

$$\frac{1-\varepsilon_i}{1-2\varepsilon_i} = \left(1 + 2\varepsilon_i + 4\varepsilon_i^2 + \cdots\right)(1-\varepsilon_i) \approx 1 + \varepsilon_i \quad (3.5)$$

by neglecting the higher order terms. In accordance with (3.5), (3.4) can be well approximated by

$$\sum_{i=1}^{N} \frac{r_i}{\pi_1 + \varepsilon_i} + \sum_{i=1}^{N} \frac{1-r_i}{\pi_1 - (1+\varepsilon_i)} = \sum_{i=1}^{N} \frac{r_i(\pi_1 - 1 - \varepsilon_i) + (1-r_i)(\pi_1 + \varepsilon_i)}{\pi_1^2 - \pi_1 - \varepsilon_i(1-\varepsilon_i)} = 0. \quad (3.6)$$

Retaining only the first-order term in the denominator in each summand and rearranging, (3.6) becomes

$$\sum_{i=1}^{N} \frac{\pi_1 + \varepsilon_i - (2\varepsilon_i + 1)r_i}{\pi_1^2 - \pi_1 - \varepsilon_i} = 0. \quad (3.7)$$

Given the assumption that $\varepsilon_i$ is small, $\pi_1^2 - \pi_1 - \varepsilon_i \approx \pi_1^2 - \pi_1$. Therefore, (3.7) can be further reduced to

$$\frac{1}{\pi_1^2 - \pi_1} \sum_{i=1}^{N} [\pi_1 + \varepsilon_i - (2\varepsilon_i + 1)r_i] = 0. \quad (3.8)$$

Hence, provided that $\pi_1 \neq \{0,1\}$, $\hat{\pi}_1$ (hereafter denoting the approximate ML estimate of $\pi_1$) can be found by solving

$$\sum_{i=1}^{N} [\pi_1 + \varepsilon_i - (2\varepsilon_i + 1)r_i] = 0. \quad (3.9)$$

Therefore, the following approximate ML scheme is obtained

$$\hat{\pi}_1 = \frac{1}{N} \sum_{i=1}^{N} [(1 + 2\varepsilon_i)r_i - \varepsilon_i]. \quad (3.10)$$

### C. Discussions

1) Although the estimate given in (3.10) is only an approximation to the true ML solution, it is nevertheless attractive since it is *affine* in the received data samples, $r_i$, and is therefore potentially amenable to analysis. Furthermore, extensive simulations reveal that the detection performances of the GLRT decision rule based on $\hat{\pi}_1$ and the true ML solution, respectively, are very similar (see Section VI).
2) Even with the approximate ML estimate $\hat{\pi}_1$ given in (3.10), the achievable detection performance of the GLRT (3.3), in particular, the impact due to channel uncertainty, remains quite difficult to characterize especially when the number of sensors is finite. Thus, the following section proposes an alternative fusion test that exploits the affine nature of $\hat{\pi}_1$ to facilitate the analytic characterization of the link error effect.

□

## IV. Proposed Detection Scheme

### A. Proposed Approach

It is widely known that the GLRT is merely a heuristic approach and does not take account of any specific optimality criteria regarding the detection performance [9], [10]. As suggested by [10, p. 204], an alternative (yet simple and intuitive) strategy is to simply compare the ML estimate against the known parameter $\pi_0$ and decide in favor of the null hypothesis whenever the resultant difference (measured using some appropriate metric) is less than a certain threshold. Utilizing this approach, and in order to further exploit the affine nature of the approximate ML estimate in (3.10), the following alternative test criterion is proposed:

$$\begin{cases} \mathcal{H}_0 : \hat{\pi}_1 - \pi_0 \leq \gamma \\ \mathcal{H}_1 : \hat{\pi}_1 - \pi_0 > \gamma. \end{cases} \tag{4.1}$$

The main advantage of this decision rule is that, unlike the GLRT in (3.3), the test statistic in (4.1) is affine in the estimate $\hat{\pi}_1$ and is therefore also affine in the received data samples $r_i$. This attractive feature enables the resultant decision performance to be analytically characterized, as shown below.

### B. Analytic Performance

To proceed, let us write

$$\hat{\pi}_1 = \frac{1}{N}\sum_{i=1}^{N}[(1+2\varepsilon_i)r_i - \varepsilon_i] = \underbrace{\frac{1}{N}\sum_{i=1}^{N}(1+2\varepsilon_i)r_i}_{:=T} - \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i, \tag{4.2}$$

where $T$ denotes the equivalent test statistic. Since $r_i \in \{0,1\}$, $T$ assumes a finite number of alphabets, which are to be specified first. Thus, for each $0 \leq k \leq N$, let $I^{(k)} := \{I_1^{(k)}, I_2^{(k)} \cdots, I_{C_k^N}^{(k)}\}$ be the collection of all the distinct $k$-element subsets of $\{1,\cdots,N\}$, where $C_k^N := N!/[k!(n-k)!]$ and $I^{(0)} = \{\phi\}$. Also, for each $0 \leq k \leq N$, let $S^{(k)}$ be a set consisting of all possible values of $T$ when $k$ sensors are active, i.e.,

$$\begin{aligned} S^{(k)} &:= \{T | k \text{ sensors are active}\} \\ &= \left\{S_1^{(k)}, S_2^{(k)}, \cdots, S_{C_k^N}^{(k)}\right\} \\ \text{where} \quad S_l^{(k)} &:= N^{-1}\left(k + 2\sum_{i\in I_l^{(k)}}\varepsilon_i\right). \end{aligned} \tag{4.3}$$

As a result, it follows that

$$T \in \bigcup_{k=0}^{N} S^{(k)}, \quad \text{where} \quad S^{(0)} = \{0\}. \tag{4.4}$$

It is noted from (4.4) that there are a total of $C_0^N + C_1^N + \cdots + C_N^N = (1+1)^N = 2^N$ possible levels of $T$. To assess the performance of the proposed decision rule given in (4.1), assume without loss of generality that for each $1 \leq k \leq N$, the elements in $S^{(k)}$ are arranged as $S_1^{(k)} \leq S_2^{(k)} \leq \cdots \leq S_{C_k^N}^{(k)}$, i.e.,

$$N^{-1}\left(k + 2\sum_{i\in I_1^{(k)}}\varepsilon_i\right) \leq N^{-1}\left(k + 2\sum_{i\in I_2^{(k)}}\varepsilon_i\right) \leq \cdots \leq N^{-1}\left(k + 2\sum_{i\in I_{C_k^N}^{(k)}}\varepsilon_i\right). \tag{4.5}$$

Also, let $1 \leq k_l \leq N$ be such that

$$N^{-1}\left(k_l + 2\sum_{i\in I_1^{(k_l)}}\varepsilon_i\right) \leq \pi_0 + \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i + \gamma < N^{-1}\left(k_l + 1 + 2\sum_{i\in I_1^{(k_l+1)}}\varepsilon_i\right). \tag{4.6}$$

In accordance with the definition of the detection probability, $P_d = \Pr\{T \geq \pi_0 + (1/N)\sum_{i=1}^{N}\varepsilon_i + \gamma | \mathcal{H}_1\}$, the following lower bound for $P_d$ can be derived:

$$P_d \geq P_d^{(L)} \tag{4.7}$$

in which based on (4.6) and (4.3)

$$\begin{aligned} P_d^{(L)} &= \Pr\left\{S^{(k_l+1)} \cup S^{(k_l+2)} \cup \ldots \cup S^{(N)} | \pi_1 \text{ is true}\right\} \\ &= \sum_{k=k_l+1}^{N}\sum_{l=1}^{C_k^N}\left\{\prod_{i\in I_l^{(k)}}[(1-2\varepsilon_i)\pi_1 + \varepsilon_i] \times \prod_{i\notin I_l^{(k)}}[-(1-2\varepsilon_i)\pi_1 + (1-\varepsilon_i)]\right\}. \end{aligned} \tag{4.8}$$

Similarly, for the false-alarm probability $P_f = \Pr\{T \geq \pi_0 + (1/N)\sum_{i=1}^{N}\varepsilon_i + \gamma | \mathcal{H}_0\}$, the associated lower bound is obtained as[2]

$$P_f \geq P_f^{(L)} \tag{4.9}$$

where

$$\begin{aligned} P_f^{(L)} &= \Pr\left\{S^{(k_l+1)} \cup S^{(k_l+2)} \cup \ldots \cup S^{(N)} | \pi_0 \text{ is true}\right\} \\ &= \sum_{k=k_l+1}^{N}\sum_{l=1}^{C_k^N}\left\{\prod_{i\in I_l^{(k)}}[(1-2\varepsilon_i)\pi_0 + \varepsilon_i] \times \prod_{i\notin I_l^{(k)}}[-(1-2\varepsilon_i)\pi_0 + (1-\varepsilon_i)]\right\}. \end{aligned} \tag{4.10}$$

It is observed that the performance bounds in (4.8) and (4.10) depend on the link error probability $\varepsilon_i$. Thus, the performance bounds provide a convenient means of evaluating the effect of channel link uncertainties on the detection performance, as discussed in the following section.

## V. Impact of Channel Link Uncertainty on Detection Performance

### A. Tractable Approximation of Performance Bounds

The performance bounds in (4.8) and (4.10) vary as a nonlinear function of the link error probability $\varepsilon_i$. Therefore, the original forms of (4.8) and (4.10) do not permit a straightforward analysis of the impact of nonideal communication channels on the detection performance. It will be recalled that the approximate ML estimate given in (3.10) is

[2]Note that the upper bounds for both $P_d$ and $P_f$ can be directly obtained by replacing the lower summation indexes in (4.8) and (4.10) by $k_l$. Simulations indicate that in both cases the gap between the lower and upper bounds is small.

based on the assumption of a high SNR regime (i.e., the link error probability has a low value). Adopting the same assumption here, the lower bounds given in (4.8) and (4.10) can be simplified considerably, as established in the following lemma.

*Lemma 5.1:* For a small value of $\varepsilon_i$, it can be shown that

$$P_d^{(L)} = \sum_{k=k_l+1}^{N} A_k + \left(\sum_{i=1}^{N} \varepsilon_i\right) \sum_{k=k_l+1}^{N} B_k \tag{5.1}$$

where

$$\begin{aligned} A_k &:= C_k^N (1-\pi_1)^{N-k} \pi_1^k \\ B_k &:= \pi_1^{k-1}(1-2\pi_1)(1-\pi_1)^{N-k-1} \\ &\quad \times \left(C_{k-1}^{N-1}(1-\pi_1) - C_k^{N-1}\pi_1\right). \end{aligned} \tag{5.2}$$

Similarly

$$P_f^{(L)} = \sum_{k=k_l+1}^{N} C_k + \left(\sum_{i=1}^{N} \varepsilon_i\right) \sum_{k=k_l+1}^{N} D_k \tag{5.3}$$

where

$$\begin{aligned} C_k &:= C_k^N (1-\pi_0)^{N-k} \pi_0^k \\ D_k &:= \pi_0^{k-1}(1-2\pi_0)(1-\pi_0)^{N-k-1} \\ &\quad \times \left(C_{k-1}^{N-1}(1-\pi_0) - C_k^{N-1}\pi_0\right). \end{aligned} \tag{5.4}$$

*Proof:* Based on (4.8) and (4.10), the results can be obtained by neglecting the high-order terms of $\varepsilon_i$ and performing some direct manipulations. □

## B. Characterization of Channel Effects

Lemma 5.1 shows that while (4.8) and (4.10) are complicated functions of $\varepsilon_i$, in the high SNR regime the detection performance is closely related to the total link error rate, namely $\sum_{i=1}^{N} \varepsilon_i$. Although $\sum_{i=1}^{N} \varepsilon_i$ provides a measure of the aggregate end-to-end communication link quality, it does not necessarily directly reflect the overall detection performance. However, given certain assumptions regarding the sensor alarm rates, the lower bounds of the global detection probability can be enlarged provided that $\sum_{i=1}^{N} \varepsilon_i$ is kept small. More precisely, the following theorem applies.

*Theorem 5.2:* Assume that $\pi_0 < 0.5 < \pi_1$. With a fixed false-alarm probability $P_f$, let $P_d^{(L)}$ and $P_d^{\prime(L)}$ be the detection probability lower bounds associated with two different summed link errors, i.e., $E = \sum_{i=1}^{N} \varepsilon_i$ and $E' = \sum_{i=1}^{N} \varepsilon_i'$, respectively. If $E' < E$, then it follows that $P_d^{\prime(L)} > P_d^{(L)}$.

*Proof:* See Appendix. □

Theorem 5.2 suggests that, when the condition $\pi_0 < 0.5 < \pi_1$ is satisfied, the global detection probability tends to improve as the value of the total link error rate is reduced. Note that the assumption $\pi_0 < 0.5 < \pi_1$ is not too restricted for any reasonable detectors. Inspired by Theorem 5.2, the following section develops a sensor power allocation scheme designed to enhance the global detection performance by reducing the total link error.

## C. Optimal Sensor Power Allocation Scheme

Assume that each sensor utilizes an ON–OFF signaling technique to transmit its one-bit reports to the FC. Assume also that the node-to-FC communications take place over flat fading channels. Thus, the following discrete-time baseband channel model can be applied:

$$y_i = h_i p_i s_i + v_i, \quad 1 \le i \le N \tag{5.5}$$

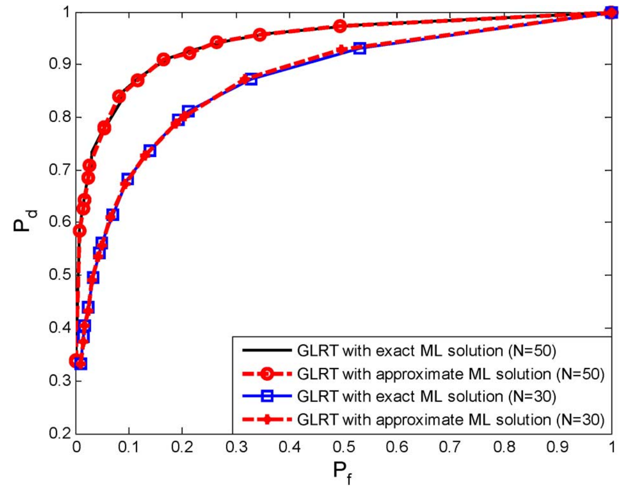



Fig. 1. ROC curves of GLRT (3.3) with exact and approximate ML solutions.

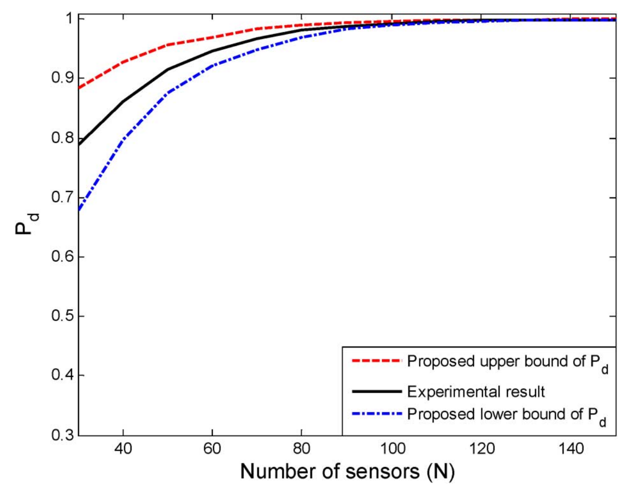



Fig. 2. Detection probability and theoretical performance bounds $(P_f = 0.1)$.

where $y_i$ is the data received at the FC from the $i$th sensor, $h_i$ is the instantaneous channel gain of the $i$th link (assumed to be perfectly known to the FC), $p_i^2$ is the power allocation factor for the $i$th node, and $v_i$ is zero-mean Gaussian measurement noise with variance $\sigma_v^2$. From (5.5), the crossover probability is given by $\varepsilon_i = Q(|h_i p_i|/\sigma_v)$, where $Q(t) := (\sqrt{2\pi})^{-1} \int_t^{\infty} \exp[-u^2/2] du$ is the $Q$-function. Given a fixed total power budget $\mathcal{P}$, the optimal sensor power allocation problem can be formulated as

$$\text{Minimize} \sum_{i=1}^{N} Q\left(|h_i p_i|/\sigma_v\right), \quad \text{subject to} \sum_{i=1}^{N} p_i^2 = \mathcal{P}. \tag{5.6}$$

Note that the optimization problem given in (5.6) has been addressed in the context of MIMO wireless communications in [14] and [15]. Thus, the algorithm proposed therein is used directly here to establish the optimal sensor power allocation factor for each of the nodes within the network.

## VI. Simulation Results

In this section, the performance of the proposed scheme is investigated by means of numerical simulation. The channel gains of the communication paths are assumed to be complex Gaussian with zero mean and unit variance and are i.i.d. across sensors. Throughout the simulation, the noise variance is set to be $\sigma_v^2 = 0.05$; in Figs. 1–6, the local sensor alarm probabilities are $(\pi_0, \pi_1) = (0.4, 0.6)$. Fig. 1 presents the ROC curves of the GLRT test (3.3) implemented using the exact

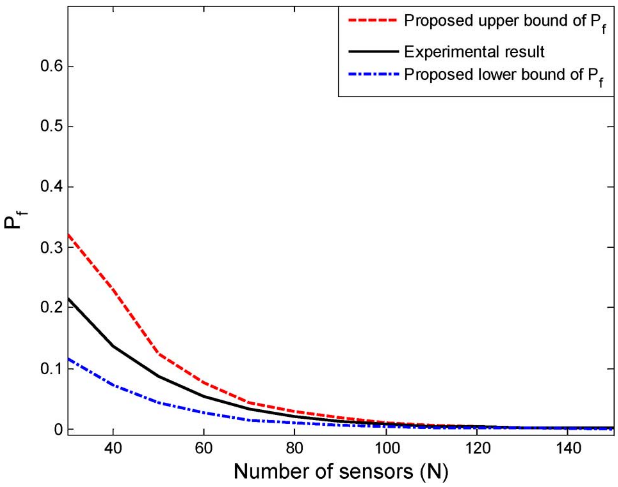



Fig. 3. False-alarm probability and theoretical performance bounds ($P_d = 0.9$).

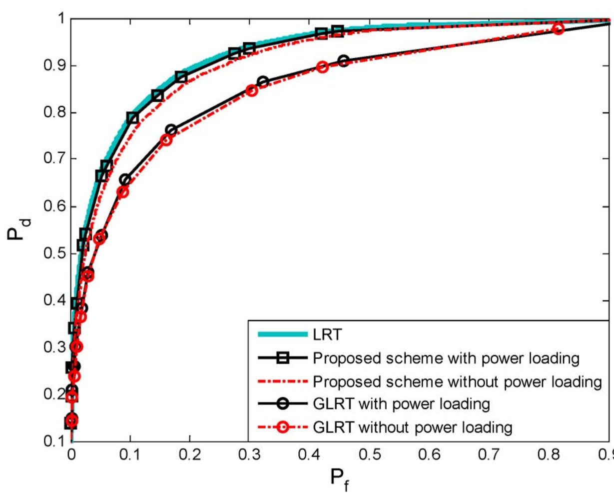



Fig. 4. ROC curves of GLRT (3.3) and proposed scheme (4.1) ($N = 30$).

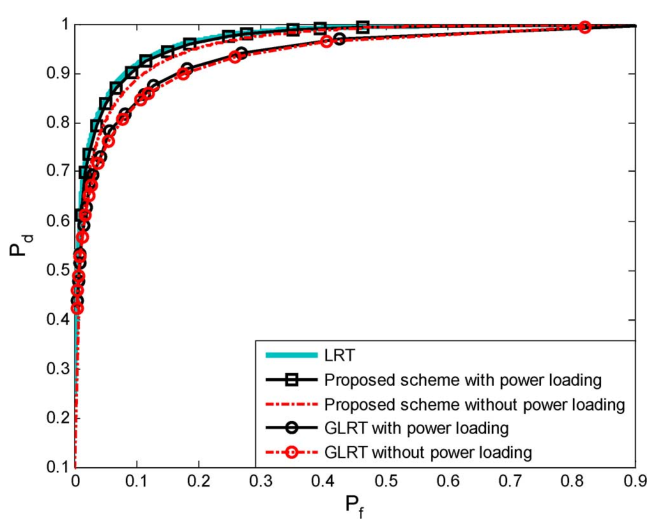



Fig. 5. ROC curves of GLRT (3.3) and proposed scheme (4.1) ($N = 50$).

ML solution [via solving the polynomial (3.4)] and the proposed approximate ML solution ($\hat{\pi}_1$ in (3.10)), respectively. It is seen that the detection performances achieved using $\hat{\pi}_1$ and the true ML solution are almost identical. Figs. 2 and 3 examine the tightness of the performance

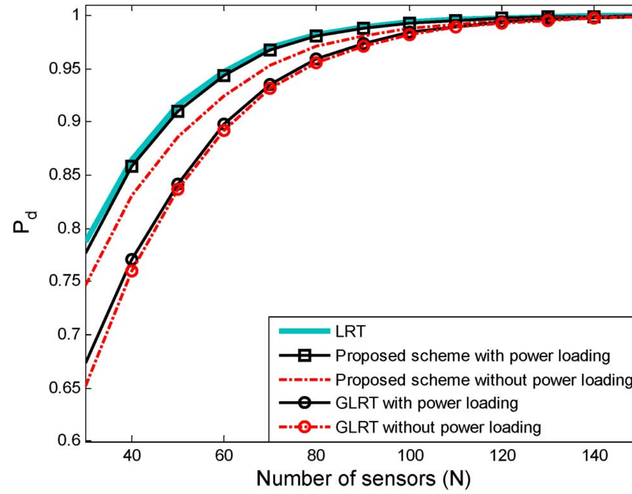



Fig. 6. Detection probabilities for different numbers of sensors ($P_f = 0.1$).

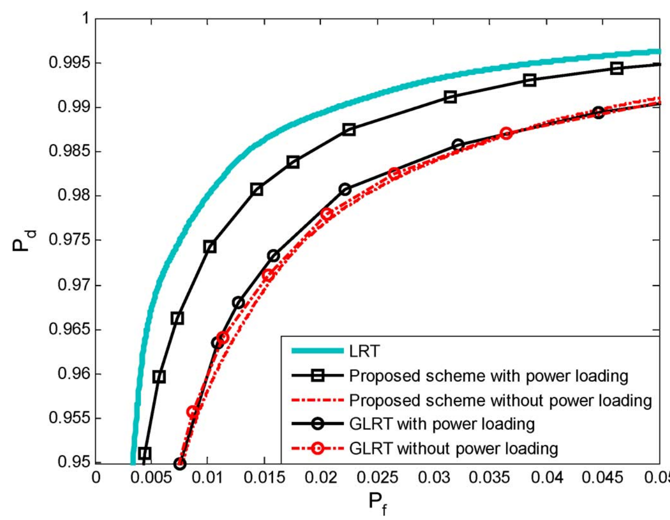



Fig. 7. ROC curves of GLRT (3.3) and proposed scheme (4.1) ($\pi_0 = 0.2$).

bounds for $P_d$ and $P_f$ derived in Section IV-B for different number of sensors $N$. As we can see, the theoretical bounds well predict the experimental results, especially when $N$ is large. Figs. 4 and 5 compare the GLRT (3.3) (with the exact ML solution) and the alternative test (4.1) for, respectively, $N = 30$ and $N = 50$. Both methods with and without power allocation are considered. Note that by "without power allocation" we mean $p_i = 1$ for all $i$ in (5.5), and the channel gains $h_i$'s are drawn from the standard Gaussian distribution in each Monte Carlo run. Also, the ROC curves obtained by the likelihood ratio test (LRT) assuming that $\pi_1$ is exactly known are also included as the benchmark. It is seen that the detection performance of the proposed alternative rule (4.1) is discriminably improved via the application of the sensor power allocation scheme (especially when the network size is small) and is almost identical to the ideal LRT. For the GLRT, the performance improvement attained via power loading is only slight. This is reasonable since the proposed power allocation scheme is specifically aimed at enhancing the detection probability of the alternative test (4.1) but not for the GLRT. With fixed $P_f = 0.1$, Fig. 6 further depicts the detection probabilities of all methods for $30 \leq N \leq 150$. The results show that as the number of sensors increases, the performances of all methods improve and converge to the ideal LRT solution. The proposed test (4.1), however, performs quite close to the ideal LRT, irrespective of the network size. Finally, Fig. 7 shows the ROC curve when $\pi_0$ is set instead as $\pi_0 = 0.2$ ($N = 30$). Compared to Fig. 4, the detection

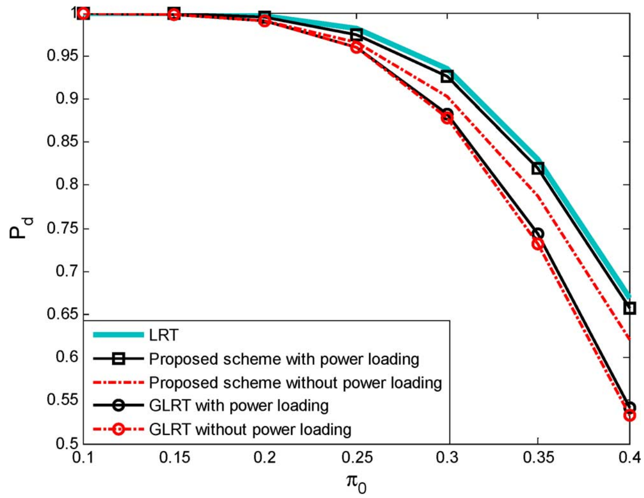



Fig. 8. Detection probabilities for different $\pi_0$ $(P_f = 0.05)$.

performances of all methods improve. This is reasonable since when $\pi_0$ is small, the occurrence of change is potentially more discernible. With $P_f = 0.05$ and $N = 30$, Fig. 8 shows the detection probability for $0.1 \le \pi_0 \le 0.4$. As we can see, the proposed test (4.1) with power loading still outperforms the GLRT-based solution.

## VII. Conclusions

This correspondence has presented an original contribution to binary decision fusion with identical sensors when the local detection probability is unknown. It has been shown that while the global fusion rule can be formulated in terms of the GLRT, the need for an ML estimate of the unknown parameter when implementing the GLRT decision rule prevents an analytic evaluation of the detection performance, even when a closed-form approximation of the ML solution is used. Thus, exploiting the affine nature of the approximate ML solution, a simple alternative fusion statistic has been proposed, which remains affine in the received sensor reports. Such an alternative scheme not only facilitates a tractable performance analysis, but also enables the analytic characterization of the effect of channel impairments on the global decision performance. Given a reasonable range of the local detection and false alarm probabilities, it has been shown that a higher aggregate link quality leads to an improved global detection probability. Therefore, a sensor power allocation scheme has been proposed to minimize the summed link errors. The simulation study has shown that the proposed alternative fusion rule outperforms the GLRT and yields a further improved performance when combined with the proposed power loading method. In the future, we will extend the current study to the scenario with nonidentical sensors and investigate the problem with the Bayesian approach.

## Appendix
## Proof of Theorem 5.2

To prove the theorem, let us define

$$S_k^A := \sum_{i=k}^{N} A_i,\ S_k^B := \sum_{i=k}^{N} B_i,\ S_k^C := \sum_{i=k}^{N} C_i,\ S_k^D := \sum_{i=k}^{N} D_i \tag{A.1}$$

where $A_i$ and $B_i$ are defined in (5.2) and $C_i$ and $D_i$ are defined in (5.4). The theorem is proven using the following two technical lemmas.

*Lemma A.1:* Assume that $\pi_0 < 0.5 < \pi_1$. The following results hold:

1) Both $S_k^A > 0$ and $S_k^C > 0$ are monotonically decreasing in $k$.
2) $S_k^B \le 0$ and $S_k^D \ge 0$ for all $k$.

□

*Proof of Lemma A.1:* We note that 1) follows immediately by definition, and thus it remains to prove 2). Let us write

$$B_k = \underbrace{\pi_1^{k-1}(1-2\pi_1)(1-\pi_1)^{N-k}C_{k-1}^{N-1}}_{:=Q_k} - \underbrace{\pi_1^{k}(1-2\pi_1)(1-\pi_1)^{N-k-1}C_{k}^{N-1}}_{:=R_k}. \tag{A.2}$$

Note that $Q_0 = R_N = 0$ since $C_{-1}^{N-1} = 0$ and $C_N^{N-1} = 0$. Since

$$\sum_{k=1}^{N} Q_k = (1-2\pi_1)\sum_{k=1}^{N}\pi_1^{k-1}(1-\pi_1)^{N-k}C_{k-1}^{N-1} = (1-2\pi_1)(\pi_1+1-\pi_1)^{N-1} = (1-2\pi_1) \tag{A.3}$$

and

$$\sum_{k=0}^{N-1} R_k = (1-2\pi_1)\sum_{k=0}^{N-1}\pi_1^{k}(1-\pi_1)^{N-k-1}C_{k}^{N-1} = (1-2\pi_1)(\pi_1+1-\pi_1)^{N-1} = (1-2\pi_1) \tag{A.4}$$

we have

$$S_0^B = \sum_{i=0}^{N} B_i = \sum_{i=0}^{N} Q_i - \sum_{i=0}^{N} R_i = \sum_{i=1}^{N} Q_i - \sum_{i=0}^{N-1} R_i = 0. \tag{A.5}$$

Furthermore, since $\pi_1 > 0.5$, it follows that

$$S_N^B = \pi_1^{N-1}(1-2\pi_1) \to 0^- \text{ as } N \text{ gets large.} \tag{A.6}$$

We further observe that

$$\begin{aligned} &C_{k-1}^{N-1}(1-\pi_1) - C_k^{N-1}\pi_1 \\ &= \frac{(1-\pi_1)(N-1)!}{(k-1)!(N-k)!} - \frac{\pi_1(N-1)!}{(k)!(N-k-1)!} \\ &= (N-1)!\left[\frac{k-N\pi_1}{(k)!(N-k)!}\right]. \end{aligned} \tag{A.7}$$

From (A.7), and by the definition of $B_k$ in (5.2), it follows immediately that

$$B_k > 0 \text{ for } k < N\pi_1 \quad \text{and} \quad B_k < 0 \text{ for } k > N\pi_1. \tag{A.8}$$

From (A.8), $S_k^B$ decreases when $0 \le k < N\pi_1$ and increases for $N\pi_1 < k \le N$. This result, together with (A.5) and (A.6), implies that $S_k^B \le 0$. Following a similar approach, it can be shown that $S_k^D \ge 0$. □

*Lemma A.2:* The following results hold:

1) If $\sum_{i=1}^{N}\varepsilon_i \le (\pi_1/(2\pi_1-1))$, then $S_k^A + S_k^B(\sum_{i=1}^{N}\varepsilon_i)$ is monotonically decreasing.
2) If $\sum_{i=1}^{N}\varepsilon_i \le (\pi_0/(1-2\pi_0))$, then $S_k^C + S_k^D(\sum_{i=1}^{N}\varepsilon_i)$ is monotonically decreasing.

□

*Proof:* We shall only prove 1) since 2) can be verified using a similar approach. To proceed, let us first focus on the case in which $k > N\pi_1$. By assumption we have

$$\begin{aligned} \sum_{i=1}^{N}\varepsilon_i &\le \frac{\pi_1}{2\pi_1-1} = \frac{\pi_1}{2\pi_1-1} \times \frac{C_k^N(1-\pi_1)}{C_k^N(1-\pi_1)} \\ &= \frac{C_k^N}{2\pi_1-1} \times \frac{\pi_1(1-\pi_1)}{C_k^N - \pi_1 C_k^N} \\ &\le \frac{C_k^N}{2\pi_1-1} \times \frac{\pi_1(1-\pi_1)}{C_{k-1}^{N-1} - \pi_1 C_k^N} \end{aligned} \tag{A.9}$$

where the last equality follows since $C_{k-1}^{N-1} < C_k^N$ and $k > N\pi_1$. From (A.9), we immediately have

$$\frac{C_{k-1}^{N-1} - \pi_1 C_k^N}{\pi_1(1-\pi_1)} \cdot \left(\sum_{i=1}^{N} \varepsilon_i\right) \leq \frac{C_k^N}{2\pi_1 - 1}. \tag{A.10}$$

Since $C_k^N = C_{k-1}^{N-1} + C_k^{N-1}$, we can rewrite (A.10) as

$$\begin{aligned} &\frac{C_{k-1}^{N-1} - \pi_1\left(C_{k-1}^{N-1} + C_k^{N-1}\right)}{\pi_1(1-\pi_1)} \cdot \left(\sum_{i=1}^{N} \varepsilon_i\right) \\ &= \frac{(1-\pi_1)C_{k-1}^{N-1} - \pi_1 C_k^{N-1}}{\pi_1(1-\pi_1)} \cdot \left(\sum_{i=1}^{N} \varepsilon_i\right) \\ &= \left[\frac{C_{k-1}^{N-1}}{\pi_1} - \frac{C_k^{N-1}}{(1-\pi_1)}\right] \cdot \left(\sum_{i=1}^{N} \varepsilon_i\right) \\ &\leq \frac{C_k^N}{2\pi_1 - 1}. \end{aligned} \tag{A.11}$$

The last inequality in (A.11) is equivalent to

$$C_k^N + \left(C_{k-1}^{N-1}\frac{1-2\pi_1}{\pi_1} - C_k^{N-1}\frac{1-2\pi_1}{1-\pi_1}\right)\left(\sum_{i=1}^{N} \varepsilon_i\right) \geq 0. \tag{A.12}$$

Multiplying both sides of (A.12) by $(1-\pi_1)^{N-k}\pi_1^k$ and rearranging, we obtain

$$\begin{aligned} C_k^N(1-\pi_1)^{N-k}\pi_1^k + \pi_1^{k-1}(1-2\pi_1)(1-\pi_1)^{N-k-1} \times \\ \left(C_{k-1}^{N-1}(1-\pi_1) - C_k^{N-1}\pi_1\right)\left(\sum_{i=1}^{N} \varepsilon_i\right) \geq 0. \end{aligned} \tag{A.13}$$

From the definition of the sequences $A_k$ and $B_k$ in (5.2), inequality (A.13) essentially asserts that

$$A_k + B_k\left(\sum_{i=1}^{N} \varepsilon_i\right) \geq 0. \tag{A.14}$$

Since $S_k^A - S_{k+1}^A = A_k$ and $S_k^B - S_{k+1}^B = B_k$, (A.14) implies that

$$S_k^A + S_k^B\left(\sum_{i=1}^{N} \varepsilon_i\right) \geq S_{k+1}^A + S_{k+1}^B\left(\sum_{i=1}^{N} \varepsilon_i\right) \tag{A.15}$$

which proves 1) for $k > N\pi_1$. If $k < N\pi_1$, we have $[(C_{k-1}^{N-1}/\pi_1) - (C_k^{N-1}/(1-\pi_1))] < 0$, and hence the last inequality in (A.11) still holds. Repeating the procedures shown in (A.12)–(A.14), the relation given in (A.15) is obtained. The proof is thus completed. □

*Proof of Theorem 5.2:* Associated with the total error rate $E$, let $(S_k^A, S_k^B, S_k^C, S_k^D)$ be accordingly defined as in (A.1). For a given threshold $\gamma$, and with the given $E$, we can then express the performance bounds in (5.1) and (5.3) as

$$P_d^{(L)} = S_{k_l+1}^A + S_{k_l+1}^B E \quad \text{and} \quad P_f^{(L)} = S_{k_l}^C + S_{k_l}^D E \tag{A.16}$$

where $k_l$ is some positive integer. If $E$ is reduced to $E' < E$, it follows from part 2) of Lemma A.1 that

$$S_{k_l}^A + S_{k_l}^B E < S_{k_l}^A + S_{k_l}^B E' \text{ and } S_{k_l}^C + S_{k_l}^D E > S_{k_l}^C + S_{k_l}^D E'. \tag{A.17}$$

Since $\pi_0 < 0.5 < \pi_1$, we have $(\pi_1/(2\pi_1 - 1)) > 0$ and $(\pi_0/(1 - 2\pi_0)) > 0$. Under the assumptions of Lemma A.2, $S_k^C + S_k^D E'$ is monotonically decreasing. Let $k_l' < k_l$ be such that

$$k_l' = \min\left\{k | S_k^C + S_k^D E' \leq S_{k_l}^C + S_{k_l}^D E\right\}. \tag{A.18}$$

For such $k_l'$, it follows that $P_f^{(L)}(k_l') := S_{k_l'}^C + S_{k_l'}^D E' \leq S_{k_l}^C + S_{k_l}^D E = P_f^{(L)} \leq P_f$. The corresponding detection probability lower bound satisfies

$$P_d^{(L)}(k_l') := S_{k_l'}^A + S_{k_l'}^B E' \overset{(a)}{>} S_{k_l}^A + S_{k_l}^B E' \overset{(b)}{>} S_{k_l}^A + S_{k_l}^B E = P_d^{(L)} \tag{A.19}$$

where (a) holds since $S_k^A + S_k^B E$ is also monotonically decreasing (see Lemma A.2) and (b) follows from the first inequality in (A.17). Hence, as $E$ is reduced to $E'$, we have $P_d^{(L)}(k_l') > P_d^{(L)}$ whenever $P_f^{(L)}(k_l') \leq P_f$. This implies that the detection probability lower bound $P_d^{(L)\prime}$ corresponding to $P_f$ must exceed $P_d^{(L)}$. □

## REFERENCES


[1] J. F. Chamberland and V. V. Veeravalli, "Wireless sensors in distributed detection applications," *IEEE Signal Process. Mag.*, vol. 24, no. 3, pp. 16–25, May 2007.

[2] B. Chen, R. Jiang, T. Kasetkasem, and P. K. Varshney, "Channel aware decision fusion in wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 52, no. 12, pp. 3454–3458, Dec. 2004.

[3] Q. Cheng, B. Chen, and P. K. Varshney, "Detection performance limits for distributed sensor networks in the presence of non-ideal channels," *IEEE Trans. Wireless Commun.*, vol. 5, no. 11, pp. 3034–3038, Nov. 2006.

[4] B. Chen, T. Lang, and P. K. Varshney, "Channel aware distributed detection in wireless sensor networks," *IEEE Signal Process. Mag.*, vol. 23, no. 4, pp. 16–26, Jul. 2006.

[5] R. V. Hogg, J. W. McKean, and A. T. Craig, *Introduction to Mathematical Statistics*, 6th ed. Englewood Cliffs, NJ: Prentice-Hall, 2005.

[6] S. K. Jayaweera, "Large system decentralized detection performance under communication constraints," *IEEE Commun. Lett.*, vol. 9, no. 9, pp. 769–771, Sep. 2005.

[7] V. R. Kanchumarthy, R. Viswanathan, and M. Madishetty, "Impact of channel errors on decentralized detection performance of wireless sensor networks: A study of binary modulations, Rayleigh-fading and nonfading channels, and fusion-combiners," *IEEE Trans. Signal Process.*, vol. 56, no. 5, pp. 1761–1769, May 2008.

[8] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Englewood Cliffs, NJ: Prentice-Hall PTR, 1993.

[9] S. M. Kay, *Fundamentals of Statistical Signal Processing: Detection Theory*. Englewood Cliffs, NJ: Prentice-Hall PTR, 1998.

[10] B. C. Levy, *Principles of Signal Detection and Parameter Estimation*. New York: Springer, 2008.

[11] R. Niu, B. Chen, and P. K. Vahshney, "Fusion of decisions over Rayleigh fading channels in wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 54, no. 3, pp. 1018–1027, Mar. 2006.

[12] A. Swami, Q. Zhao, Y. W. Hong, and L. Tong, *Wireless Sensor Networks: Signal Processing and Communications Perspectives*. Hoboken, NJ: Wiley, 2007.

[13] P. K. Varshney, *Distributed Detection and Data Fusion*. New York: Springer, 1997.

[14] N. Wang and S. D. Blostein, "Minimum BER power allocation for MIMO spatial multiplexing systems," in *Proc. ICC*, 2005, vol. 4, pp. 2282–2286.

[15] Z. Yan, K. M. Wong, and Z. Q. Luo, "Optimal diagonal precoder for multiantenna communication systems," *IEEE Trans. Signal Process.*, vol. 53, no. 6, pp. 2089–2100, Jun. 2005.

[16] Q. Zhao, A. Swami, and L. Tong, "The interplay between signal processing and networking in sensor networks," *IEEE Signal Process. Mag.*, vol. 23, no. 4, pp. 84–93, Jul. 2006.