行政院國家科學委員會補助專題研究計畫 □ 成果報告 ☑期中進度報告(精簡版)

六種新渾沌系統及三種新型的渾沌同步之研究(第二年)

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中文摘要

關鍵詞：雙 Duffing 系統，雙 van der Pol 系統，實用渾沌同步

渾沌系統之研究在物理、化學、生物學、生理學、各種工程等方面皆有日益重要之廣泛應用。Duffing 系統、van der Pol 系統都是典型的重要渾沌系統。本計畫(第二年)採取適當的耦合方式將它們推廣為雙 Duffing 系統、雙 van der Pol 系統。研究其渾沌性質，從而就典型重要渾沌系統而言，既擴大其研究範圍也深化其研究內容。渾沌同步之研究在秘密通訊、神經網路、自組織、物理系統、生態系統、工程系統等方面有長足之應用。本計畫(第二年)提出實用適應廣義同步法。用以糾正目前適應同步法中估值參數趨於未知參數未加證明之盲點。研究重點為：

1. 雙 Duffing 系統與雙 van der Pol 系統之渾沌研究。用相圖、分歧圖、功率譜圖、李雅普諾夫指數分析渾沌之行為，包括奇異吸引子之範圍及形狀、超渾沌之行為、碎形之維度等。
2. 應用實用穩定理論實現以實用適應渾沌同步，嚴格證明估值參數必然趨於未知參數，並由數值計算對雙 Duffing 系統與雙 van der Pol 系統加以驗證。

英文摘要

keywords: double Duffing system, double van der Pol system, pragmatical synchronization

The study of chaotic system has found wide applications in physics, chemistry, biology, physiology, and various engineerings. Duffing system and van der Pol system are paradigmatic important chaotic systems. In this project (second year), the study is extended to double Duffing system and double van der Pol system by suitable coupling. For these paradigmatic and important systems, the study will be extended and deepened.

Chaos synchronizations are applied in various regions, such as secure communication, neural networks, self-organization, physical systems, ecological systems and engineering systems, etc. In this project (second year), a new type of chaos synchronization with theoretical and practical importance are studied, i.e. pragmatical adaptive generalized synchronization, to correct the absence of proof of that estimated parameters approach the unknown parameters. The main parts of our study are:

1. The study of chaos of double Duffing system and double van der Pol system. By phase portraits, bifurcation diagrams, power spectra, Lyapunov exponents, the various chaotic behaviors of these systems will be studied. The regions and shapes of the strange attractors, hyperchaotic behaviors and fractal dimensions will also be studied.
2. By pragmatical stability theory, the pragmatical adaptive synchronization of the above systems will be obtained. That the estimated parameters approach the unknown parameters are rigorously proved and illustrated by simulation for double Duffing systems and double van der Pol systems.

報告內容

(一)前言及研究目的：

渾沌系統之研究除了在理論上的重要價值外，在物理、化學、生理學及各種工程等方面皆有廣泛之應用。Duffing 系統與 van der Pol 系統都是重要的典型渾沌系統。對於這兩個系統的極重要的渾沌現象及渾沌同步都已有豐富的研究成果，直到現在，這兩個重要典型系統仍為研究熱點[1-27]。本計畫(第二年)為了對這個著名系統，擴大其研究範圍並深化其研究內容，特提出兩種新系統，即雙 Duffing 系統與雙 van der Pol 系統。首先証明其為渾沌系統，其次研究其渾沌行為。渾沌同步之研究在秘密通訊、神經網路、自我組織等方面有長足之應用[28-81]。本計畫(第二年)研究一種新的渾沌同步方式及其對這兩個新系統的應用。

(二)研究方法及文獻探討：

(a)雙Duffing系統及雙van der Pol系統的渾沌行為

經典的Duffing系統是

$$\ddot{x} + a\dot{x} + bx + cx^3 = d\cos\omega t$$

或

$$\dot{x} = y$$
$$\dot{y} = -ay - bx - cx^3 + d\cos\omega t$$

其中 a,b,c,d 為常數，$d\cos\omega t$ 為外加激勵項。現將兩個Duffing系統的兩個激勵項中的 $\cos\omega t$ 交替換成

對方的狀態變量，即得到本計畫新創造的雙Duffing系統：

$$\dot{x} = y$$
$$\dot{y} = -ay - bx - cx^3 + du$$
$$\dot{u} = v$$
$$\dot{v} = -ev - gu - hu^3 + kx$$

這一交換使原來各不相同的兩個具有兩個狀態變量的非自治系統(nonautonomous system)變成一個具有四個狀態變量的自治系統(autonomous system)。自 x,y 構成之單Duffing方程而言，原來之激勵項為簡單的諧波，現在則變成渾沌變量 u，x,y 變成渾沌變量所激勵成的渾沌變量。這當然是單Duffing系統渾沌研究的延伸與深化。不僅如此，由於 du,kx 兩耦合渾沌激勵的同時存在，x,y 系統與 u,v 系統之間有相互影響，所以雙Duffing系統比單Duffing系統有更複雜的渾沌行為，當可預期。本計畫將研究其週期運動、準週期運動、渾沌運動及超渾沌運動。

經典的van der Pol系統是

$$\ddot{x} + \varphi x + a\dot{x}(x^2 - 1) - b\sin\omega t = 0$$

或

$$\dot{x} = y$$
$$\dot{y} = -\varphi x + a(1 - x^2)y + b\sin\omega t$$

其中 φ,a,b 是常數，$b\sin\omega t$ 為外加激勵項。現將兩個van der Pol系統的兩個激勵項中的 $\sin\omega t$ 交替換成對方的狀態變量，即得到本計畫新創造的雙van der Pol系統：

$$\dot{x} = y$$
$$\dot{y} = -x + b(1-cx^2)y + au$$
$$\dot{u} = v$$
$$\dot{v} = -u + e(1-fu^2)v + dx$$

同樣地，對此系統的研究，不僅是對單van der Pol系統渾沌行為研究之延伸與深化，而且此系統比單van der Pol系統有更複雜的渾沌行為，當可預期。本計畫將研究其週期運動、準週期運動、渾沌運動及超渾沌運動。

(b)實用適應廣義同步法及其對雙Duffing系統及雙van der Pol系統之應用

廣義渾沌同步為渾沌同步之進一步發展[82-85,87-92]。多數系統之參數值多為未知值，故採適應控制方法以達成同步之目的。但目前流行之適應同步，是應用Lyapunov漸近穩定定理及Babalat引理証明兩系統之狀態誤差趨於零。但對為何參數估計值會趨於其未知值這一問題並未証明[86, 93-97]。本計畫採用申請人提出之實用漸近穩定定理(pragmatical asymptotical stability theorem)[98, 99]，引用機率(probability)的概念嚴格證明了這一問題，稱之為實用適應廣義同步法。

所謂廣義同步乃指從系統變量 y 與主系統變量 x 之間有函數關係 $y = G(x)$，現在創造一個新式之函數關係

$$y = G(x) = x + F(t)$$

其中 $F(t)$ 為給定之渾沌函數。設主系統為

$$\dot{x} = Ax + f(x, B)$$

其中 $x = [x_1, \cdots, x_n]^T \in R^n$ 為狀態向量，A 為 $n \times n$ 未知參數矩陣，f 為非線性向量函數，B 為 f 中之未知參數向量。從系統為

$$\dot{y} = \hat{A}y + f(y, \hat{B}) + u(t)$$

其中 $\hat{A}, \hat{B}$ 分別為估計參數矩陣及估計參數向量，u 為控制輸入向量。誤差為

$$e = x - y + F(t)$$

當 $\lim_{t\to\infty} e = 0$，則廣義同步成立。將上式等號兩邊對時間求導，可得

$$\dot{e} = \dot{x} - \dot{y} + \dot{F}(t)$$

再在上式等號右邊加上控制項 $u(t)$，得

$$\dot{e} = Ax - \hat{A}y + f(x, B) - f(y, \hat{B}) + \dot{F}(t) - u(t)$$

今選定一定正之Lyapunov函數

$$V(e, \tilde{A}, \tilde{B}) = \frac{1}{2}e^T e + \frac{1}{2}\tilde{A}_c^T \tilde{A}_c + \frac{1}{2}\tilde{B}^T \tilde{B}$$

其中 $\tilde{A} = A - \hat{A}$, 將矩陣 $\tilde{A}$ 中之元素按列排成一個列陣，或向量，以 $\tilde{A}_c$ 表示，$\tilde{B} = B - \hat{B}$。V 沿誤差微分方程解之時間導數

$$\dot{V}(e) = e^T[Ax - \hat{A}y + f(x, B) - f(y, \hat{B}) + \dot{F}(t) - u(t)] + \tilde{A}_c \dot{\tilde{A}}_c + \tilde{B}\dot{\tilde{B}}$$

吾人如能應用技巧，選擇$u(t)$及$\dot{\tilde{A}}_c, \dot{\tilde{B}}$使$\dot{V}(e) = e^T Ce$，其中$C$為對角負定矩陣，則$\dot{V}$為$e$及$\tilde{A}_c, \tilde{B}$之負半定函數。只能證明$e, \tilde{A}_c, \tilde{B}$為穩定，而非漸近穩定。目前流行之做法[93-95]為：應用Babalat引理，可證明誤差趨於零，但對$\tilde{A}_c, \tilde{B}$趨於零未加以証明。

現用實用漸近穩定定理應用Lebesque測度及機率的觀點，在一定條件下証明$e, \tilde{A}_c, \tilde{B}$皆趨近於零：

定理：$V = [x_1, \cdots, x_n]^T : D \to R_+$在$D$上為定正而解析。$D$為$n-1$流形。$\dot{V}$為半負定而$\dot{V} = 0$之點集$x$為$m-1$流形。當$m+1 < n$時，所研究系統之零點為實際漸近穩定。
所謂“實際漸近穩定”是指：雖不符合傳統之漸近穩定，但非漸近穩定之機率為零，故實際上不會發生。吾人將以雙 Duffing 系統與雙 van der Pol 系統為例，實現實用適應廣義同步。

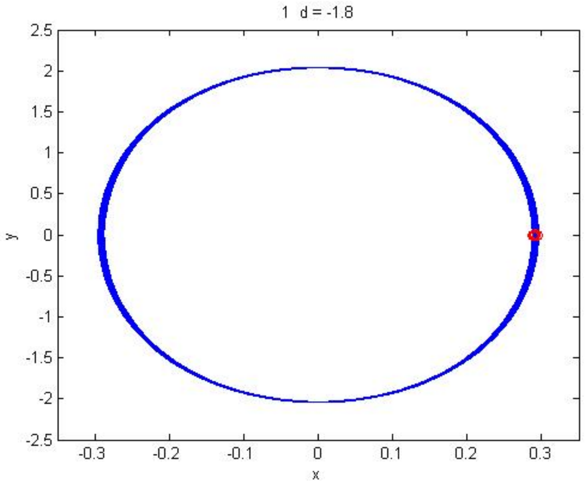
(三)討論與結果：

Duffing系統，van der Pol系統原為振動學科之最重要最典型的系統。自渾沌動力學興起後，Duffing系統，van der Pol系統由於其為非線性系統故亦沿習成為渾沌動力學學科中最重要最典型的系統，四十年來對此二系統的渾沌研究之文獻可謂汗牛充棟，至今方興未艾。眾所週知，此兩種典型系統除理論意義外，廣泛應用於機械、電機、物理、化學、生科、奈米系統，本計畫(第二年)研究雙Duffing系統與雙van der Pol系統，不僅對渾沌動力學學科中最重要最典型的渾沌系統的研究的拓廣與深化，更重要的是它們本身顯然具有更複雜的，未經發現的複雜渾沌行為，本研究對渾沌動力學學科具重大意義。其應用於機械、電機、物理、化學、生科、奈米之耦合系統，具有重要的實用價值。

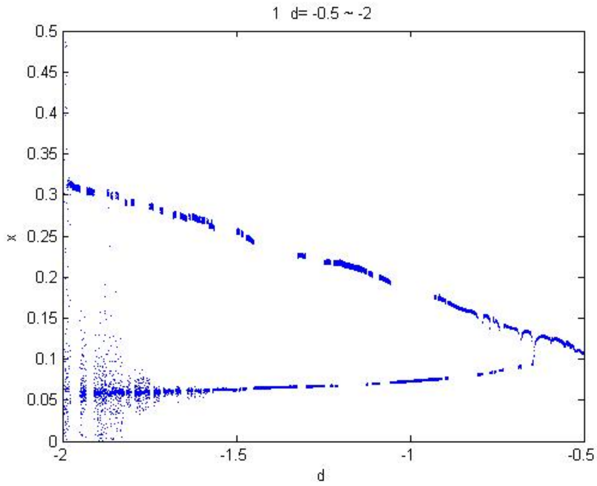
渾沌同步除本身之重要理論價值外，其研究在秘密通訊、神經網路、自我組織等方面有日益廣泛之應用。廣義渾沌同步則為渾沌同步之進一步發展，其應用亦方興未艾。本計畫(第二年)提出實用適應廣義同步法，糾正了目前國際文獻中未經証明即認為估值參數趨於為之參數之錯誤，首次在渾沌同步中引入概率概念，具重大理論及實用意義。由於$\dot{V}$之要求降低，實際應用亦較易實現。

雙 Duffing 系統及雙 van der Pol 系統的渾沌行為與實用適應廣義同步法，及對此二系統的應用：

1. 研究獲得諸多相圖、分歧圖、功率譜圖、參數圖及李亞普諾夫指數及碎形維度等研究雙 Duffing 系統週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。



(a) d = -1.8



(b) d = -2 ~ -0.5

Fig. 1 The phase portraits, Poincaré maps and the bifurcation diagram for the double Duffing system: (a) Phase portraits and Poincaré maps for $d=-1.8$, (b) Bifurcation diagram for $d=-2\sim-0.5$.

2. 研究獲得諸多相圖、分歧圖、功率譜圖、參數圖及李亞普諾夫指數及碎形維度等研究雙 van der Pol 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。

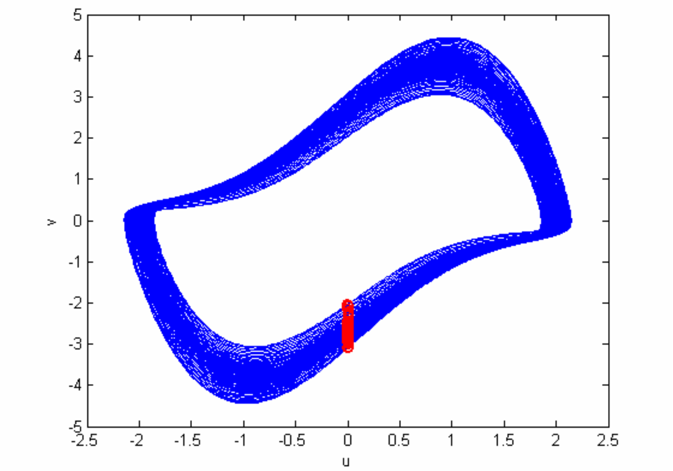


Fig2: The phase portrait and Poincaré map for the double van der Pol system.

3. 研究獲得實用適應廣義同步法對雙 Duffing 系統之應用。

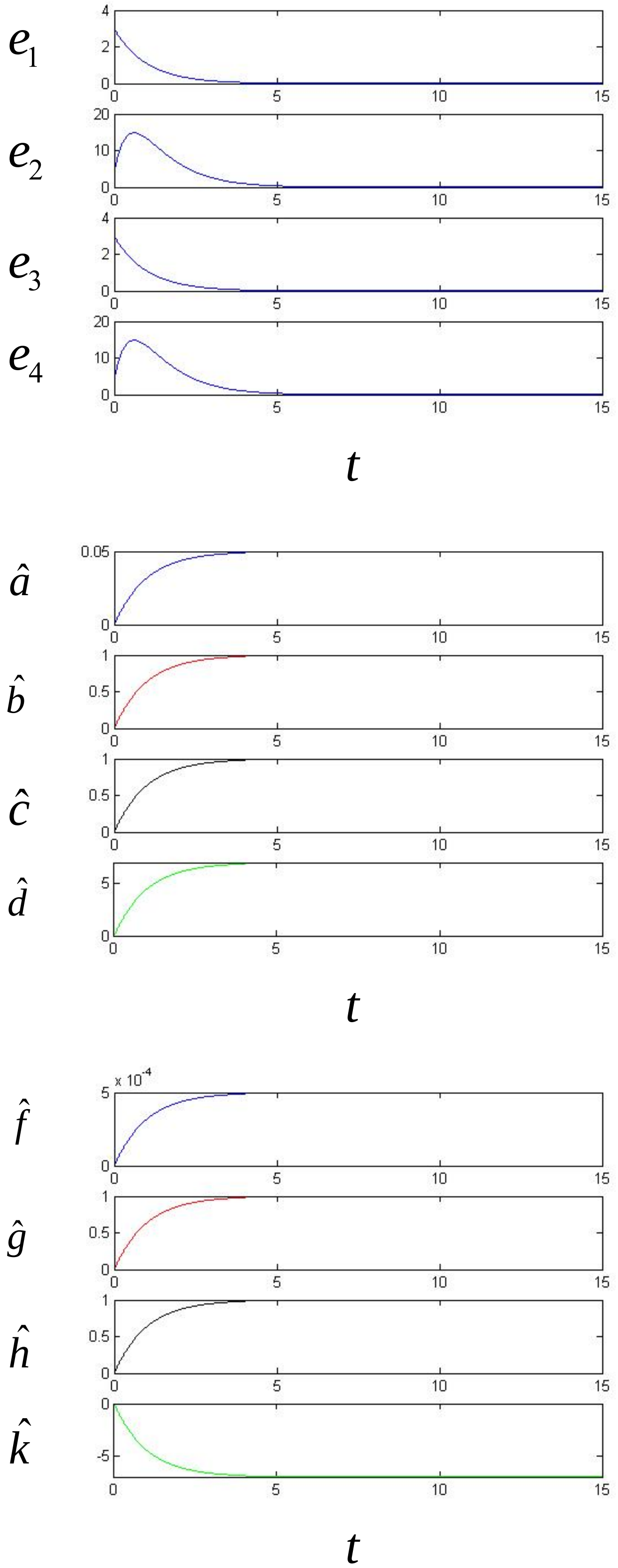


Fig 3 Time histories of state errors, a, $\hat{b}$, $\hat{c}$, $\hat{d}$, $\hat{f}$, $\hat{g}$, $\hat{h}$ and $\hat{k}$ with

$$a = 0.05, b = 1, c = 3, d = 7, f = 0.0005, g = 1, h = 3, k = -7.$$

4. 研究獲得實用適應廣義同步法對雙 Van der Pol 系統之應用。

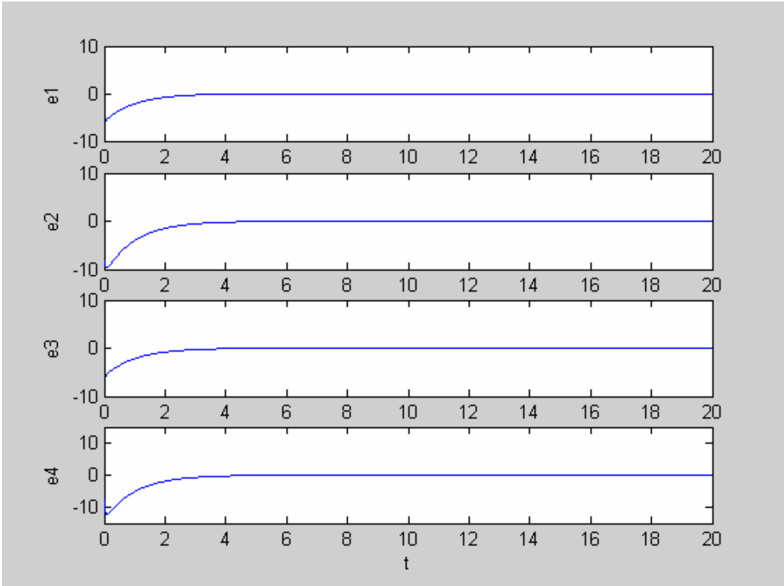
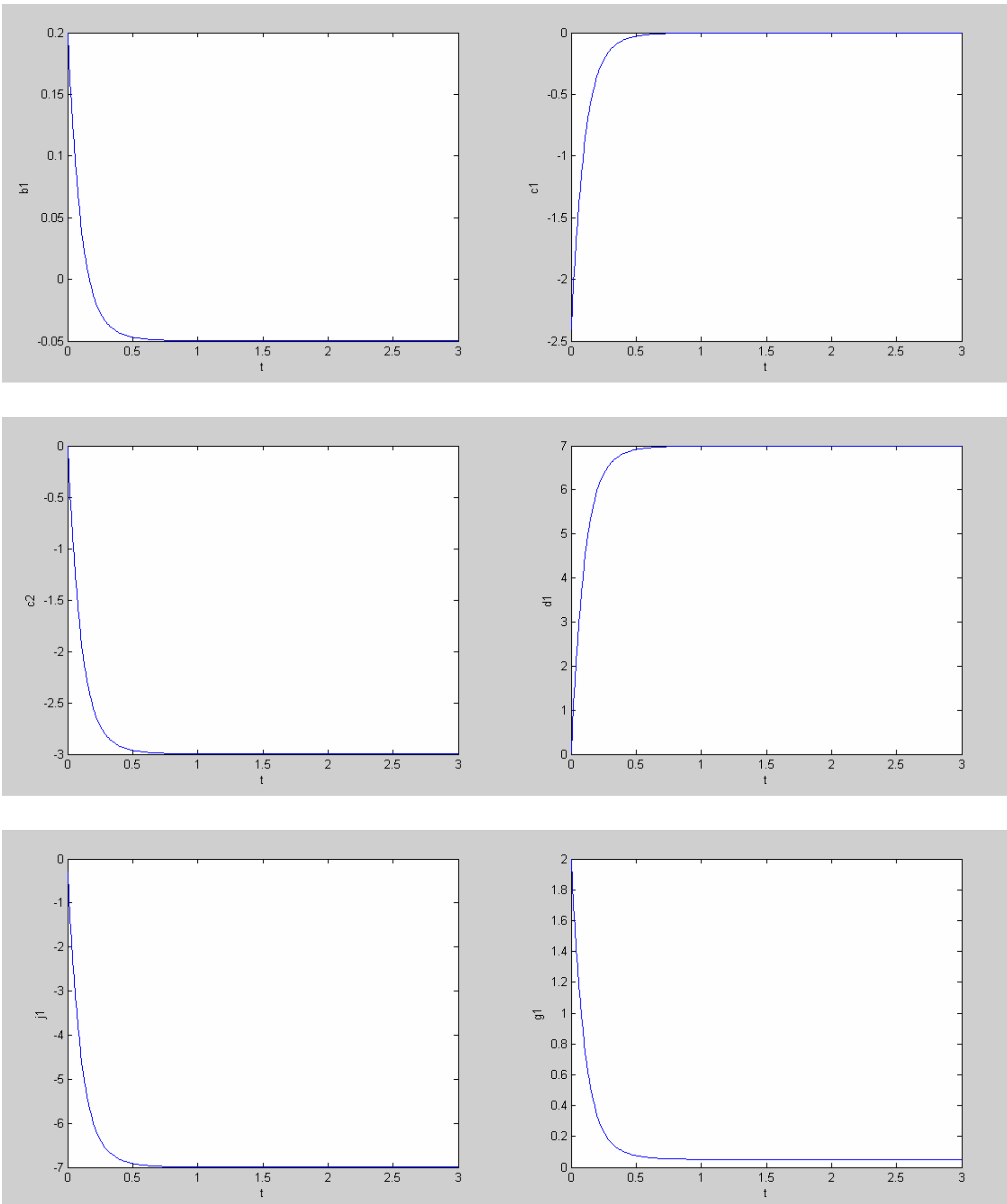


Fig. 4 Time histories of state errors for e_1, e_2, e_3, e_4.



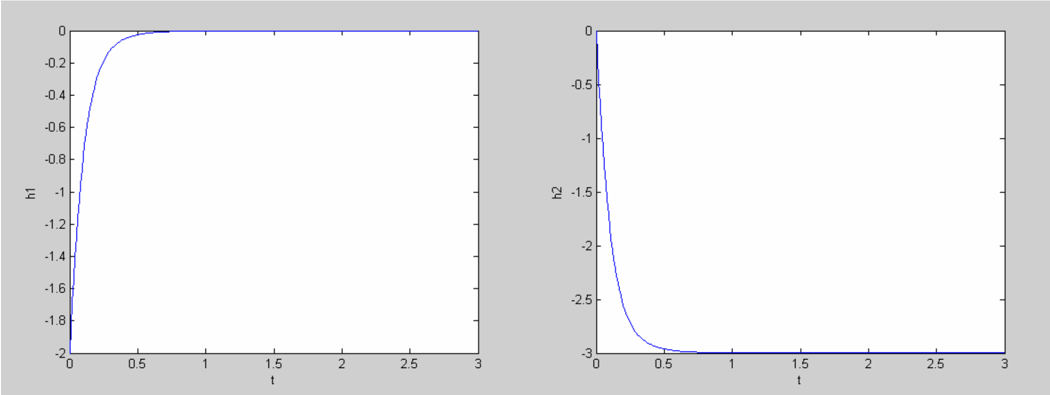


Fig. 5 Time histories of coefficients $a_1, b_1, c_1, c_2, d_1, j_1, f_1, g_1, h_1, h_2$.

參考文獻

1. F. Battelli and K. J. Palmer, 1993, "Chaos in the Duffing Equation", Journal of Differential Equations, Vol. 101, pp. 276-301.
2. G. Osipov, L. Glatz and H. Troger, 1998, "Suppressing Chaos in the Duffing Oscillator by Impulsive Actions", Chaos, Solitons and Fractals, Vol. 9, pp. 307-321.
3. S. Paul Raj, S. Rajasekar, and K. Murali, 1999, "Coexisting Chaotic Attractors, Their Basin of Attractions and Synchronization of Chaos in Two Coupled Duffing Oscillators", Physics Letters A, Vol. 264, pp. 283-288.
4. B.R. Nana Nbendjo, R. Tchoukuegno, and P. Woafo, 2003, "Active Control with Delay of Vibration and Chaos in a Double-well Duffing Oscillator", Chaos, Solitons and Fractals, Vol. 18, pp. 345-353.
5. B.R. Nana Nbendjo, Y. Salissou, and P. Woafo, 2005, "Active Control with Delay of Catastrophic Motion and Horseshoes Chaos in a Single Well Duffing Oscillator", Chaos, Solitons and Fractals, Vol. 23, pp. 809-816.
6. Chunbiao Gan, "Noise-Induced Chaos and Basin Erosion in Softening Duffing Oscillator", 2005, Chaos, Solitons and Fractals, Vol. 25, pp. 1069-1081.
7. Zheng-Ming Ge, Chun-Lai Hsiao and Yen-Sheng Chen, 2005, "Nonlinear Dynamics and Chaos Control for a Time Delay Duffing System", International Journal of Nonlinear Sciences and Numerical Simulation, Vol. 6, No. 2, pp. 187-199.
8. Cunli Wu, Youming Lei, and Tong Fang, 2006, "Stochastic Chaos in a Duffing Oscillator and Its Control", Chaos, Solitons and Fractals, Vol. 27, pp. 459-469.
9. Ruiqi Wang, Jin Deng, and Zhujun Jing, 2006, "Chaos Control in Duffing System", Chaos, Solitons and Fractals, Vol. 27, pp. 249-257.
10. Zhongkui Sun, Wei Xu, Xiaoli Yang, and Tong Fang, 2006, "Inducing or Suppressing Chaos in a Double-Well Duffing Oscillator by Time Delay Feedback", Chaos, Solitons and Fractals, Vol. 27, pp. 705-714.
11. Samuel Bowong, F. M. Moukam Kakmeni, and Jean Luc Dimi, 2006, "Chaos Control in the Uncertain Duffing Oscillator" Journal of Sound and Vibration, Vol. 292, pp. 869-880.
12. Cunli Wu, Tong Fang, Haiwu Rong, 2007, "Chaos Synchronization of Two Stochastic

Duffing Oscillators by Feedback Control", Chaos, Solitons and Fractals, Vol. 32, pp. 1201-1207.

13. Zheng-Ming Ge and Chan-Yi Ou, 2005, "Chaos in a Fractional Order Modified Duffing System", accepted by Chaos, Solitons and Fractals.
14. Zheng-Ming Ge and Chan-Yi Ou, 2006, "Chaos Synchronization of Fractional Order Modified Duffing Systems with Parameters Excited by a Chaotic Signal", accepted by Chaos, Solitons and Fractals.
15. B. van der Pol, 1920, "A Theory of the Amplitude of Free and Forced Triode Vibrations", Radio Review, Vol. 1, 701-710.
16. B. van der Pol and J. van der Mark, 1927, "Frequency Demultiplication", Nature, Vol. 120, pp. 363-364.
17. B. van der Pol, 1927, "Forced Oscillations in a Circuit with Non-Linear Resistance (Reception with Reactive Triode)", The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science Ser. 7, Vol. 3, pp. 65-80.
18. Y. Ashkenazy, C. Goren, and L. P. Horwitz, 1998, "Chaos of the Relativistic Parametrically Forced van der Pol Oscillator", Physics Letters A, Vol. 243, pp. 195-204.
19. Gamal M. Mahmoud, Ahmed A. M. Farghaly, 2004, "Chaos Control of Chaotic Limit Cycles of Real and Complex van der Pol Oscillators", Chaos, Solitons and Fractals, Vol. 21, pp. 915-924.
20. Munehisa Sekikawa, Naohiko Inaba, and Takashi Tsubouchi, 2004, "Chaos via Duck Solution Breakdown in a Piecewise Linear van der Pol Oscillator Driven by an Extremely Small Periodic Perturbation", Physica D, Vol. 194, pp. 227-249.
21. F. M. Moukam Kakmeni, Samuel Bowong, Clement Tchawoua, and Ernest Kaptouom, 2004, "Chaos Control and Synchronization of a Φ6-van der Pol Oscillator", Physics Letters A, Vol. 322, pp. 305-323.
22. M. Siewe Siewe, F. M. Moukam Kakmeni, C. Tchawoua, and P. Woafo, 2005, "Bifurcations and Chaos in the Triple-Well Formula Not Shown -van der Pol Oscillator Driven by External and Parametric Excitations", Physica A, Vol. 357, pp. 383-396.
23. Cristina Stan, C. P. Cristescu, and M. Agop, 2007, "Golden Mean Relevance for Chaos Inhibition in a System of Two Coupled Modified van der Pol Oscillators", Chaos, Solitons and Fractals, Vol. 31, pp. 1035-1040.
24. Zheng-Ming-Ge and An-Ray Zhang, 2005, "Chaos in a Modified Van der Pol System and in Its Fractional Order Systems", accepted by Chaos, Solitons amd Fractals.
25. Zheng-Ming Ge and An-Ray Zhang, 2006, "Anticontrol of Chaos of Fractional Order Modified Van der Pol Systems", accepted by Applied Mathematics and Computation.
26. Zheng-Ming Ge and Mao-Yuan Hsu, 2006, "Chaos Excited Chaos Synchronizations of Integral and Fractional Oredr Generalized Van der Pol Systems", accepted by Chaos, Solitons and Fractals.
27. Zheng-Ming Ge and Mao-Yuan Hsu, 2006, "Chaos in a Generalized Van der Pol System and in Its Fractional Order System", accepted by Chaos, Solitons and Fractals.
28. G. Álvarez, Shujun Li, F. Montoya, G. Pastor, and M. Romera, 2005, "Breaking Projective

Chaos Synchronization Secure Communication Using Filtering and Generalized Synchronization", Chaos, Solitons and Fractals, Vol. 24, pp. 775-783.
29. M. Feki, 2003, "An Adaptive Chaos Synchronization Scheme Applied to Secure Communication", Chaos, Solitons and Fractals, Vol. 18, pp. 141-148.
30. Zhigang Li and Daolin Xu, 2004, "A Secure Communication Scheme Using Projective Chaos Synchronization", Chaos, Solitons and Fractals, Vol. 22, pp. 477-481.
31. Chil-Min Kim, Won-Ho Kye, Sunghwan Rim, and Soo-Young Lee, 2004, "Communication Key Using Delay Times in Time-Delayed Chaos Synchronization", Physics Letters A Vol. 333, pp. 235-240.
32. J. Zhou, H. B. Huang, G. X. Qi, P. Yang, and X. Xie, 2005, "Communication with Spatial Periodic Chaos Synchronization", Physics Letters A, Vol. 335, pp. 191-196.
33. Atsushi Uchida and Shigeru Yoshimori, 2004, "Synchronization of Chaos in Microchip Lasers and Its Communication Applications", Comptes Rendus Physique, Vol. 5, pp. 643-656.
34. Qing Yun Wang, Qi Shao Lu, Guan Rong Chen, and Ding Hui Guo, 2006, "Chaos Synchronization of Coupled Neurons with Gap Junctions", Physics Letters A, Vol. 356, pp. 17-25.
35. E. N. Sanchez, L. J. Ricalde, 2003, "Chaos Control and Synchronization, with Input Saturation, via Recurrent Neural Networks", Neural Networks, Vol. 16, pp. 711-717.
36. Jinhu Lü, Xinghuo Yu, and Guanrong Chen, 2004, "Chaos Synchronization of General Complex Dynamical Networks", Physica A, Vol. 334, pp. 281-302.
37. Yao-Chen Hung, Ming-Chung Ho, Jiann-Shing Lih, and I-Min Jiang, 2006, "Chaos Synchronization of Two Stochastically Coupled Random Boolean Networks", Physics Letters A, Vol. 356, pp. 35-43.
38. Xiaoming Zhang, Maolin Fu, Jinghua Xiao, Gang Hu, 2006, "Self-Organization of Chaos Synchronization and Pattern Formation in Coupled Chaotic Oscillators", Physical Review E, Vol. 74, pp. 015202-1-4.
39. A. Raffone and C. van Leeuwen, 2003 "Dynamic Synchronization and Chaos in an Associative Neural Network with Multiple Active Memories", Chaos, Vol. 13, pp. 1090-104.
40. J. M. Liu, H. F. Chen, S. Tang, 2001, "Synchronization of Chaos in Semiconductor Lasers", Nonlinear Analysis, Vol. 47, pp. 5741-5751.
41. Kanako Suzuki, Yoh Imai, 2004, "Periodic Chaos Synchronization in Slave Subsystems Using Optical Fiber Ring Resonators", Optics Communications, Vol. 241, pp. 507-512.
42. Jianping Yan and Changpin Li, 2007, "On Chaos Synchronization of Fractional Differential Equations", Chaos, Solitons and Fractals, Vol. 32, pp. 725-735.
43. Er-Wei Bai, Karl E. Lonngren, and J. C. Sprott, 2002, "On the Synchronization of a Class of Electronic Circuits that Exhibit Chaos", Chaos, Solitons and Fractals, Vol. 13, pp. 1515-1521.
44. A. Barsella and C. Lepers, 2002, "Chaotic Lag Synchronization and Pulse-Induced Transient Chaos in Lasers Coupled by Saturable Absorber", Optics Communications, Vol. 205, pp. 397-403.
45. Y. Zhang, S. Q. Hu, and G. H. Du, 1999, "Chaos Synchronization of Two Parametrically Excited Pendulums", Journal of Sound and Vibration, Vol. 223, pp. 247-254.

46. Liguo Luo, P. L. Chu, T. Whitbread, and R. F. Peng, 2000, “Experimental Observation of Synchronization of Chaos in Erbium-Doped Fiber Lasers”, Optics Communications, Vol. 176, pp. 213-217.
47. A. Uchida, S. Kinugawa, and S. Yoshimori, 2003, “Synchronization of Chaos in Two Microchip Lasers by Using Incoherent Feedback Method”, Chaos, Solitons and Fractals, Vol. 17, pp. 363-368.
48. P. Saha, S. Banerjee, and A. R. Chowdhury, 2002, “Some Aspects of Synchronization and Chaos in a Coupled Laser System”, Chaos, Solitons and Fractals, Vol. 14, pp. 1083-1093.
49. Fan Zhang and Pak L. Chu, 2004, “Effect of Coupling Strength on Chaos Synchronization Generated by Erbium-Doped Fiber Ring Laser”, Optics Communications, Vol. 237, pp. 213-219.
50. Yan-Ni Li, Lan Chen, Zun-Sheng Cai, and Xue-Zhuang Zhao, 2003, “Study on Chaos Synchronization in the Belousov–Zhabotinsky Chemical System”, Chaos, Solitons and Fractals, Vol. 17, pp. 699-707.
51. Yan-Ni Li, Lan Chen, Zun-Sheng Cai, and Xue-zhuang Zhao, 2004, “Experimental Study of Chaos Synchronization in the Belousov–Zhabotinsky chemical system”, Chaos, Solitons and Fractals, Vol. 22, pp. 767-771.
52. A. Kittel, J. Parisi, and K. Pyragas, 1998, “Generalized Synchronization of Chaos in Electronic Circuit Experiments”, Physica D, Vol. 112, pp. 459-471.
53. A. Ucar, K. E. Lonngren, and Er-Wei Bai, 2007, “Chaos Synchronization in RCL-Shunted Josephson Junction via Active Control”, Chaos, Solitons and Fractals, Vol. 31, pp. 105-111.
54. F. T. Arecchi, R. Meucci, A. Di Garbo, and E. Allaria, 2003, “Homoclinic Chaos in a Laser: Synchronization and Its Implications in Biological Systems”, Optics and Lasers in Engineering, Vol. 39, pp. 293-304.
55. Y. Imai, H. Murakawa, and T. Imoto, 2003, “Chaos Synchronization Characteristics in Erbium-Doped Fiber Laser Systems”, Optics Communications, Vol. 217, pp. 415-420.
56. Qingxian Xie, Guanrong Chen, E. M. Bollt, 2002, “Hybrid Chaos Synchronization and Its Application in Information Processing”, Mathematical and Computer Modelling, Vol. 35, pp. 145-163.
57. R. McAllister, A. Uchida, R. Meucci, R. Roy, 2004, “Generalized Synchronization of Chaos: Experiments on a Two-Mode Microchip Laser with Optoelectronic Feedback”, Physica D, Vol. 195, pp. 244-262.
58. A. Shabunin, V. Astakhov, V. Demidov, A. Provata, F. Baras, G. Nicolis, and V. Anishchenko, 2003, “Modeling Chemical Reactions by Forced Limit-Cycle Oscillator: Synchronization Phenomena and Transition to Chaos”, Chaos, Solitons and Fractals, Vol. 15, pp. 395-405.
59. Z.-M. Ge and T.-N. Lin, 2001, “Chaos, Chaos Control and Synchronization of Gyrostat System”, Journal of Sound and Vibration, Vol. 251, pp.519-542.
60. Zheng-Ming Ge, Tsung-Chih Yu and Yen-Sheng Chen, 2003, “Chaos Synchronization of a Horizontal Platform System”, Journal of Sound and Vibration, Vol. 268, pp. 731-749.
61. Z.-M. Ge and T.-N. Lin, 2003, “Chaos, Chaos Control and Synchronization of Electro-Mechanical Gyrostat System”, Journal of Sound and Vibration, Vol.259, pp. 585-603.

62. Z.-M. Ge and Hong-Wen Wu, 2004, "Chaos Synchronization and Chaos Anticontrol of a Suspended Track with Moving Loads", Journal of Sound and Vibration, Vol. 270, pp. 685-712.
63. Zheng-Ming Ge and Yen-Sheng Chen, 2004, "Synchronization of Unidirectional Coupled Chaotic Systems via Partial Stability", Chaos, Solitons and Fractals, Vol. 21, pp. 101-111.
64. Zheng-Ming Ge, Chia-Yang Yu and Yen-Sheng Chen, 2004, "Chaos Synchronization and Anticontrol of a Rotationally Supported Simple Pendulum", JSME International Journal, Series C, Vol. 47, No. 1, pp. 233-241.
65. Zheng-Ming Ge and Wei-Ying Leu, 2004, "Anti-Control of Chaos of Two-degrees-of-Freedom Louderspeaker System and Chaos Synchronization of Different Order Systems", Chaos, Solitons & Fractals, Vol. 20, pp.503-521.
66. Zheng-Ming Ge and Chien-Cheng Chen, 2004, "Phase Synchronization of Coupled Chaotic Multiple Time Scales Systems", Chaos, Solitons & Fractals, Vol. 20, pp. 639-647.
67. Z.-M. Ge and C.-M. Chang, 2004, "Chaos Synchronization and Parameters Identification of Single Time Scale Brushless DC Motors", Chaos, Solitons and Fractals, Vol. 20, pp. 883-903.
68. Zheng-Ming Ge and Wei-Ying Leu, 2004, "Chaos Synchronization and Parameter Identification for Loudspeaker System", Chaos, Solitons & Fractals, Vol. 21, pp. 1231-1247.
69. Zheng-Ming Ge, Chui-Chi Lin and Yen-Sheng Chen, 2004, "Chaos, Chaos Control and Synchronization of Vibrometer System", Journal of Mechanical Engineering Science, Vol.218, pp.1001-1020.
70. Zheng-Ming Ge, Jui-Wen Cheng and Yen-Sheng Chen, 2004, "Chaos Anticontrol and Synchronization of Three Time Scales Brushless DC Motor System", Chaos, Solitons & Fractals Vol. 22, pp.1165-1182.
71. Zheng-Ming Ge and Jui-Kai Lee, 2005, "Chaos Synchronization and Parameter Identification for Gyroscope System", Applied Mathematics and Computation, Vol. 163, pp. 667-682.
72. Z.-M. Ge and C.-I Lee, 2005, "Anticontrol and Synchronization of Chaos for an Autonomous Rotational Machine System with a Hexagonal Centrifugal Governor", Journal of Sound and Vibration Vol. 282, pp. 635-648.
73. Zheng-Ming Ge and Ching-I Lee, 2005, "Control, Anticontrol and Synchronization of Chaos for an Autonomous Rotational Machine System with Time-Delay", Chaos, Solitons and Fractals Vol.23, pp.1855-1864.
74. Zheng-Ming Ge and Jui-Wen Cheng, 2005, "Chaos Synchronization and Parameter Identification of Three Time Scales Brushless DC Motor System", Chaos, Solitons and Fractals Vol. 24, pp.597-616.
75. Zheng-Ming Ge, Cheng-Hsiung Yang, 2005, "The Generalized Synchronization of Quantum-CNN Chaotic Oscillator with Different Order Systems", accepted by Chaos, Solitons and Fractals.
76. Zheng-Ming Ge, Yen-Sheng Chen, 2005, "Adaptive Synchronization of Unidirectional and Mutual Coupled Chaotic Systems", Chaos, Solitons and Fractals. Vol. 26, pp. 881-888.
77. Z.-M. Ge, C.-M. Chang, Y.-S. Chen, 2006, "Anti-Control of Chaos of Single Time Scale Brushless DC Motor and Chaos Synchronization of Different Order Systems", Chaos,

Solitons and Fractals, Vol. 27, pp.1298-1315
78. Zheng-Ming Ge and Guo-Hua Lin, 2005, “Complete, Lag and Anticipated Synchronization of a BLDCM Chaotic System”, accepted by Chaos, Solitons and Fractals.
79. Zheng-Ming Ge and Yen-Sheng Chen, 2005, “Synchronization of Mutual Coupled Chaotic Systems via Partial Stability Theory”, accepted by Chaos, Solitons and Fractals.
80. Zheng-Ming-Ge and Wei-Ren Jhuang, 2005, “Chaos, Its Control and Synchronization of a Fractional Order Rotational Mechanical System with a Centrifugal Governor”, accepted by Chaos, Solitons and Fractals.
81. Zheng-Ming Ge and Cheng-Hsiung Yang, 2006, “Synchronization of Complex Chaotic Systems in Series Expansion Form”, accepted by Chaos, Solitons and Fractals.
82. Changpin Li and Jianping Yan, 2006, “Generalized Projective Synchronization of Chaos: The Cascade Synchronization Approach”, Chaos, Solitons and Fractals, Vol. 30, pp. 140-146.
83. Jianping Yan and Changpin Li, 2005, “Generalized Projective Synchronization of a Unified Chaotic System”, Chaos, Solitons and Fractals, Vol. 26, pp. 1119-1124.
84. Guo Hui Li, Shi Ping Zhou, and Kui Yang, 2006, “Generalized Projective Synchronization Between Two Different Chaotic Systems Using Active Backstepping Control”, Physics Letters A, Vol. 355, pp. 326-330.
85. Gang Zhang, Zengrong Liu, and Zhongjun Ma, 2007, “Generalized Synchronization of Different Dimensional Chaotic Dynamical Systems”, Chaos, Solitons and Fractals, Vol. 32, pp. 773-779.
86. E. M. Elabbasy, H. N. Agiza, and M. M. El-Dessoky, 2005, “Global Synchronization Criterion and Adaptive Synchronization for New Chaotic System”, Chaos, Solitons and Fractals, Vol. 23, pp. 1299-1309.
87. Tao Yang and Leon O. Chua, 1999, “Generalized Synchronization of Chaos via Linear Transformation”, International Journal of Bifurcation and Chaos, Vol. 9, pp. 215-219.
88. M. Brucoli, D. Cafagna, L. Carnimeo, and G. Grassi, 1999, “Design of a Hyperchaotic Cryptosystem Based on Identical and Generalized Synchronization”, International Journal of Bifurcation and Chaos, Vol. 9, pp. 2027-2037.
89. L. M. Pecora and T. L. Carroll, 2000, “Detecting Chaotic Drive-Response Geometry in Generalized Synchronization”, International Journal of Bifurcation and Chaos, Vol. 10, pp.875-889.
90. Wen-Xin Qin, 2001, “Generalized Synchronization, Frequency-Locking and Phase-Locking of Coupled Sine Circle Maps”, International Journal of Bifurcation and Chaos, Vol. 11, pp. 2245-2253.
91. S. Cincotti and A. Teglio, 2004, “A Receiver Design Approach to Generalized Synchronization on a Linear Manifold”, International Journal of Bifurcation and Chaos, Vol. 14, pp. 2493-2502.
92. Lequan Min, Xiaodan Zhang, and Guanrong Chen, 2005, “A Generalized Synchronization Theorem for an Array of Differential Equations with Application to Secure Communication”, International Journal of Bifurcation and Chaos, Vol. 15, pp. 119-135.
93. Ju H. Park, 2005, “Adaptive Synchronization of Rössler System with Uncertain Parameters”,

Chaos, Solitons and Fractals, Vol. 25, pp. 333-338.
94. Ju H. Park, 2005, “Adaptive Synchronization of Hyperchaotic Chen System with Uncertain Parameters”, Chaos, Solitons and Fractals, Vol. 26, pp. 959-964.
95. E. M. Elabbasy, H. N. Agiza, and M. M. El-Desoky, 2006, “Adaptive Synchronization of a Hyperchaotic System with Uncertain Parameter”, Chaos, Solitons and Fractals, Vol. 30, pp. 1133-1142.
96. Jianwen Feng, Shihua Chen, and Changping Wang, 2005, “Adaptive Synchronization of Uncertain Hyperchaotic Systems Based on Parameter Identification”, Chaos, Solitons and Fractals, Vol. 26, pp. 1163-1169.
97. A. El-Gohary and R. Yassen, 2006, “Adaptive Control and Synchronization of a Coupled Dynamo System with Uncertain Parameters”, Chaos, Solitons and Fractals, Vol. 29, pp. 1085-1094.
98. Zheng-Ming Ge, Jung-Kui Yu and Yu-Jung Chen, 1999, "Pragmatical Asymptotical stability Theorem with Application to Satellite System", Japanese Journal of Applied Physics, Vol.38, Part 1, No. 10.
99. Zheng-Ming Ge and Jung-Kui Yu, 2000, "Pragmatical Asymptopical Stability Theorem on Pratial Region and for Partial Variables with Applications to Gyroscopic Systems", The Chinese Journal of Mechanics, Vol. 16, No. 4.

計畫(第二年)成果自評

對雙Duffing系統與雙van der Pol系統的渾沌研究，不但拓廣與深化了典型Duffing系統與van der Pol系統的研究，並且發現了它們本身具有更複雜的複雜渾沌行為，本計畫(第二年)對雙Duffing系統與雙van der Pol系統之渾沌性質作全面而詳盡之研究，對渾沌研究而言，其重要性不言而喻。本計畫(第二年)提出實用適應廣義同步法，有三大貢獻：1. 糾正了目前國際文獻中未經証明即認為估值參數趨於為之參數之錯誤。2. 首次在渾沌同步中引入概率概念，具重大理論及實用意義。3. 由於$\dot{V}$之要求降低，實際應用亦較易實現。故此理論有重大意義。已投出之國際著名論文已達8篇，其中已被接受者2篇。

1. Zheng-Ming Ge, Shih-Chung Li, Shih-Yu Li and Ching-Ming Chang, 2008, “Pragmatical Adaptive Chaos Control from a New Double Van der Pol System to a New Double Duffing System”, accepted by Applied Mathematics and Computation. (SCI, Impact factor: 0.821)
2. Zheng-Ming Ge, Chien-Hao Li, Shih-Yu Li and Ching-Ming Chang, 2008, “Chaos Synchronization of Double Duffing Systems with Parameters Excited by a Chaotic Signal”, Journal of Sound and Vibration, Vol. 317, pp. 449-455. (SCI, Impact factor: 1.024)
3. Zheng-Ming Ge and Chien-Hao Li, 2008, “Chaos in a Double Duffing System and in Its Fractional Order System”, submitted to Journal of Computational and Applied Mathematics. (SCI, Impact factor: 0.943)
4. Zheng-Ming Ge and Chien-Hao Li, 2008, “Highly Robust Pragmatical Generalized Synchronization of Double Duffing Systems with Uncertain Parameters via Adaptive Control”, submitted to Mathematics and Computers in Simulation. (SCI, Impact factor: 0.738)
5. Zheng-Ming Ge and Chien-Hao Li, 2008, “Uncoupled Chaos Synchronization and Antisynchronization of Double Duffing Systems by Noise Excited Parameters”, submitted to International Journal of Engineering Science. (SCI, Impact factor: 0.966)
6. Zheng-Ming Ge and Shih-Chung Li, 2008, “Chaos in a Double Van der Pol System and in Its Fractional Order System”, submitted to Mathematics and Computers in Simulation. (SCI,

Impact factor: 0.738)

7. Zheng-Ming Ge and Shih-Chung Li, 2008, “Chaos-excited Synchronization of Uncopuled Double Van der Pol systems”, submitted to Journal of Computational and Applied Mathematics. (SCI, Impact factor: 0.943)
8. Zheng-Ming Ge and Shih-Chung Li, 2008, “Uncoupled Chaos Synchronization and Antisynchronization of Double Van der Pol Systems by Noise Excited Parameters”, submitted to International Journal of Engineering Science. (SCI, Impact factor: 0.966)

另外由本計畫經費贊助已接受或出版之國際期刊論文 3 篇；見附錄。

附錄

Paper List:

1. Zheng-Ming Ge and Pu-Chien Tzen, 2008, “Chaos Synchronization by Variable Strength Linear Coupling and Lyapunov Function Derivative in Series Form”, Nonlinear Analysis: Theory, Mehtods, and Applications, Vol. 69, pp.4604-4613. (SCI, Impact factor: 1.097).
2. Zheng-Ming Ge and Cheng-Hsiung Yang, 2008, “Synchronization of Chaotic Systems with Uncertain Chaotic Parameters by Linear Coupling and Pragmatical Adaptive Tracking”, Chaos, Vol. 18, pp. 043129-043129-11. (SCI, Impact factor: 2.188)
3. Cheng-Hsiung Yang, Zheng-Ming Ge, Ching-Ming Chang and Shih-Yu Li, 2009, “Chaos Synchronization and Chaos Control of Quantum-CNN Chaotic System by Variable Structure Control and Impulse Control”, accepted by Nonlinear Analysis: Real World Applications. (SCI, Impact factor: 1.232)
4. Zheng-Ming Ge and Cheng-Hsiung Yang, 2009, “Symplectic Synchronization of Different Chaotic Systems”, Chaos, Solitons and Fractals, Vol. 40, pp. 2536-2543 (SCI, Impact factor: 3.025).

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Chaos synchronization by variable strength linear coupling and Lyapunov function derivative in series form

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Abstract

A new general strategy to achieve chaos synchronization by variable strength linear coupling without another active control is proposed. They give the criteria of chaos synchronization for two identical chaotic systems and two different chaotic dynamic systems with variable strength linear coupling. In this method, the time derivative of Lyapunov function in series form is firstly used. Lorenz system, Duffing system, Rössler system and Hyper-Rössler system are presented as simulated examples.

Keywords: Chaos; Synchronization; Linear coupling; Coupled chaotic systems

1. Introduction

In recent years, synchronization in chaotic dynamic system has been a very interesting problem and has been widely studied [1–3]. Synchronization means that the state variables of a response system approach eventually to that of a drive system. There are many control techniques to synchronize chaotic systems, such as linear error feedback control, adaptive control, active control [2–17].

In this paper, a new general strategy to achieve chaos synchronization by variable strength linear coupling is proposed. This method, in which the time derivative of Lyapunov function in series form is firstly used, can give either local synchronization which is usually good enough or global synchronization which is usually an unnecessary high demand [18–21].

This paper is organized as follows. In Section 2, synchronization strategy by variable strength linear coupling without another active control is proposed, in which the Lyapunov function derivative in series form is first used. In Section 3, Lorenz system, Duffing system, Rössler system and Hyper-Rössler system are presented as simulated examples. In Section 4, conclusions are given.

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References

[1] L.M. Pecora, T.L. Carroll, Synchronization in chaotic system, Physical Review Letters 64 (1990) 821–824.
[2] R. Femat, G.S. Perales, On the chaos synchronization phenomenon, Physics Letters A262 (1999) 50–60.
[3] A. Krawiecki, A. Sukiennicki, Generalizations of the concept of marginal synchronization of chaos, Chaos, Solitons and Fractals 11 (9) (2000) 1445–1458.
[4] C. Wang, S.S. Ge, Adaptive synchronization of uncertain chaotic systems via backstepping design, Chaos, Solitons and Fractals 12 (2001) 1199–1206.
[5] R. Femat, J.A. Ramirez, G.F. Anaya, Adaptive synchronization of high-order chaotic systems: A feedback with low-order parameterization, Physica D 139 (2000) 231–246.
[6] O. Morgul, M. Feki, A chaotic masking scheme by using synchronized chaotic systems, Physics Letters A251 (1999) 169–176.
[7] S. Chen, J. Lu, Synchronization of uncertain unified chaotic system via adaptive control, Chaos, Solitons and Fractals 14 (4) (2002) 643–647.
[8] Ju H. Park, Adaptive synchronization of hyperchaotic Chen system with uncertain parameters, Chaos, Solitons and Fractals 26 (2005) 959–964.
[9] Ju H. Park, Adaptive synchronization of rossler system with uncertain parameters, Chaos, Solitons and Fractals 25 (2005) 333–338.
[10] E.M. Elabbasy, H.N. Agiza, M.M. El-Desoky, Adaptive synchronization of a hyperchaotic system with uncertain parameter, Chaos, Solitons and Fractals 30 (2006) 1133–1142.
[11] Z.-M. Ge, C.-C. Chen, Phase synchronization of coupled chaotic multiple time scales systems, Chaos, Solitons and Fractals 20 (2004) 639–647.
[12] Z.-M. Ge, W.-Y. Leu, Chaos synchronization and parameter identification for identical system, Chaos, Solitons and Fractals 21 (2004) 1231–1247.
[13] Z.-M. Ge, W.-Y. Leu, Anti-control of chaos of two-degrees-of- freedom louderspeaker system and chaos synchronization of different order systems, Chaos, Solitons and Fractals 20 (2004) 503–521.
[14] Z.-M. Ge, Y.-S. Chen, Synchronization of unidirectional coupled chaotic systems via partial stability, Chaos, Solitons and Fractals 21 (2004) 101–111.
[15] Z.-M. Ge, J.-K. Yu, Pragmatical asymptotical stability theorem on partial region and for partial variable with applications to gyroscopic systems, The Chinese Journal of Mechanics 16 (4) (2000) 179–187.
[16] Z.-M. Ge, C.-M. Chang, Chaos synchronization and parameters identification of single time scale brushless dc motors, Chaos, Solitons and Fractals 20 (2004) 883–903.
[17] Zheng-Ming Ge, Yen-Sheng Chen, Synchronization of unidirectional coupled chaotic systems via partial stability, Chaos, Solitons and Fractals 21 (2004) 101.
[18] F. Liu, Y. Ren, X. Shan, Z. Qiu, A linear feedback synchronization theorem for a class of chaotic systems, Chaos, Solitons and Fractals 13 (4) (2002) 723–730.
[19] J. Lü, T. Zhou, S. Zhang, Chaos synchronization between linearly coupled chaotic systems, Chaos, Solitons and Fractals 14 (4) (2002) 529–541.
[20] J. Lu, Y. Xi, Linear generalized synchronization of continuous-time chaotic systems, Chaos, Solitons and Fractals 17 (2003) 825–831.
[21] Z.-M. Ge, C.-H. Yang, Synchronization of complex chaotic systems in series expansion form, Chaos, Solitons, and Fractals 34 (2007) 1649–1658.
[22] I.G. Malkin, Theory of Stability of Motion, The State Publishing House of Technical–Theoretical Literature, Moscow-Leningrad, 1952. Translated by the Language Service Bureau, Washington, D.C., published by Office of Technical Services, Dept. of Commerce, Washington 25, DC, p. 21.
[23] Zheng-Ming Ge, Developing Theory of Motion Stability, Gaulih Book Company, Taipei, 2001, pp. 10–11.

Synchronization of chaotic systems with uncertain chaotic parameters by linear coupling and pragmatical adaptive tracking

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We study the synchronization of general chaotic systems which satisfy the Lipschitz condition only, with uncertain **chaotic** parameters by linear coupling and pragmatical adaptive tracking. The uncertain parameters of a system vary with time due to aging, environment, and disturbances. A sufficient condition is given for the asymptotical stability of common zero solution of error dynamics and parameter update dynamics by the Ge–Yu–Chen pragmatical asymptotical stability theorem based on equal probability assumption. Numerical results are studied for a Lorenz system and a quantum cellular neural network oscillator to show the effectiveness of the proposed synchronization strategy. © *2008 American Institute of Physics*. [DOI: 10.1063/1.3049320]

Theoretical and experimental investigations have shown that synchronization, in particular chaos synchronization, has great potential in a large amount of application areas ranging from secure communications to modeling brain activity. In this paper, we introduce a synchronization of chaotic systems with uncertain chaotic parameters by linear coupling and pragmatical adaptive tracking. Based on pragmatical stability theorem and Lipschitz condition, some less conservative conditions for determining linear coupling synchronization of general chaotic systems are obtained. Two examples are simulated to illustrate the validity of the theoretical analysis.

I. INTRODUCTION

The idea of synchronizing two identical chaotic systems with different initial conditions was introduced by Pecora and Carroll.[1] Since then there has been particular interest in chaotic synchronization, due to many potential applications in secure communication,[2] chemical and biological systems.[3,4] There are many control methods to synchronize chaotic systems, such as, linear coupling, for which the implementation is rather easy, adaptive control, impulsive control, sliding mode control, and other methods.[5] Most of them are based on the exact knowledge of the system structure and parameters. But in practice, some or all of the system parameters are uncertain. Moreover these parameters may change from time to time and become chaotic because of chaotic disturbances. For uncertain parameters, a lot of works have proceeded to solve this problem by adaptive synchronization.[6–12] In the current scheme of adaptive synchronization,[13–15] the traditional Lyapunov stability theorem and Barbalat lemma are used to prove that the error vector approaches zero as time approaches infinity. But the question, why the estimated parameters also approach the uncertain parameters, has remained without answer. From the Ge–Yu–Chen (GYC) pragmatical asymptotical stability theorem,[16–18] the question is strictly answered. In this paper, the synchronization of general chaotic systems which satisfy the Lipschitz condition only, with unknown parameters which are altered under some **chaotic** disturbances, by linear coupling and pragmatical adaptive tracking, is studied first.

As numerical examples, the Lorenz system and recently developed quantum cellular neural network (Quantum-CNN) chaotic oscillator are used. Pragmatical adaptive tracking is used to track **chaotic** parameters in unidirectional coupled systems. Two Lorenz systems and two Quantum-CNN systems by pragmatical adaptive tracking are given as simulation examples. Quantum-CNN oscillator equations are derived from a Schrödinger equation taking into account quantum dots cellular automata structures to which in the last decade a wide interest has been devoted with particular attention towards quantum computing.[19–21]

This paper is organized as follows: In Sec. II, by pragmatical asymptotical stability theorem and by using Lipschitz conditions, theoretical analysis of synchronization is given. In Sec. III linear feedback controllers are used. By pragmatical adaptive tracking, chaos synchronization of two Lorenz systems and of two Quantum-CNN oscillator systems are achieved by numerical simulations. Conclusions are given in Sec. IV. GYC pragmatical asymptotical stability theorem is presented in the Appendix. Intuitively this theorem is different from traditional Lyapunov stability theorem at that when the points in the neighborhood of zero solution initiating trajectories not approaching zero with time are "not too many," i.e., in a subset of Lebesque measure 0 in mathematical language,[22] we can neglect their existence, i.e., the zero solution is actually asymptotically stable.

II. STRATEGY OF THE CHAOTIC SYNCHRONIZATION

Consider a nonautonomous system in the form as follows:

$$\dot{x} = F[t, x, B(t)]. \tag{1}$$

The slave system is given by

X is a 5-manifold, $m=n-p=9-4=5$. $m+1<n$ is satisfied. From the GYC pragmatical asymptotical stability theorem, error vector e approaches zero and the estimated parameters also approach the uncertain parameters. The equilibrium point $e_i=\tilde{a}_j=\tilde{\omega}_j=\tilde{G}=0$ $(i=1,2,3,4;\ j=1,2)$ is asymptotically stable. Moreover, the result is global asymptotically stable (see Appendix). The numerical results of the time series of states, state errors, parameters and estimated Lipschitz constant $\hat{G}$ are shown in Fig. 6. The chaos synchronization is accomplished near 3 s. $\hat{G}$ approaches constant also near 3 s. The coupling strength required is $K=2G=5.62$.

IV. CONCLUSIONS

Using the Lipschitz condition, the synchronization of Lorenz chaotic systems and of Quantum-CNN chaotic oscillator systems with uncertain **chaotic** parameters by linear coupling and pragmatical adaptive tracking are accomplished by the GYC pragmatical asymptotical stability theorem. Tracking uncertain **chaotic** parameters is first studied in this paper. This is of practical interest, because system parameters may be varied chaotically due to aging, environment, and disturbances. Two Lorenz systems are synchronized for chaotic parameters $M<n$. Two Quantum-CNN systems are synchronized for chaotic parameters $M=n$. The simulation results imply that this scheme is very effective. By GYC pragmatical asymptotical stability theorem, the question, why the estimated parameters approach the uncertain parameters, has been strictly answered and verified by numerical simulations.

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APPENDIX: GYC PRAGMATICAL ASYMPTOTICAL STABILITY THEOREM

The stability for many problems in real dynamical systems is actual asymptotical stability, although it may not be mathematical asymptotical stability. The mathematical asymptotical stability demands that trajectories from all initial states in the neighborhood of zero solution must approach the origin as $t\to\infty$. If there is only a small part or even a few of the initial states from which the trajectories do not approach the origin as $t\to\infty$, the zero solution is not mathematically asymptotically stable. If the probability of occurrence of the event that the trajectories from the initial states are that they do not approach zero when $t\to\infty$, i.e., these trajectories are not asymptotical stable for zero solution, is zero, the stability of zero solution is actual asymptotical stability though it is not mathematical asymptotical stability. In order to analyze the asymptotical stability of the equilibrium point of such systems, the pragmatical asymptotical stability theorem is used. The conditions for pragmatical asymptotical stability are more slack than that for traditional Lyapunov asymptotical stability.

Let X and Y be two manifolds of dimensions m and n $(m<n)$, respectively, and φ be a differentiable map from X to Y; then $\varphi(X)$ is a subset of the Lebesque measure 0 of Y.[22] For an autonomous system

$$\dot{x}=f(x_1,\ldots,x_n), \tag{A1}$$

where $x=[x_1,\ldots,x_n]^T$ is a state vector, the function $f=[f_1,\ldots,f_n]^T$ is defined on $D\subset R^n$, an n-manifold.

Let $x=0$ be an equilibrium point for the system (A1). Then

$$f(0)=0. \tag{A2}$$

For a nonautonomous system,

$$\dot{x}=f(x_1,\ldots,x_{n+1}), \tag{A3}$$

where $x=[x_1,\ldots,x_{n+1}]^T$, the function $f=[f_1,\ldots,f_n]^T$ is defined on $D\subset R^n\times R_+$, here $t=x_{n+1}\subset R_+$. The equilibrium point is

$$f(0,x_{n+1})=0. \tag{A4}$$

Definition. The equilibrium point for the system is pragmatically asymptotically stable provided that with initial points on C which is a subset of the Lebesque measure 0 of D, the behaviors of the corresponding trajectories cannot be determined, while with initial points on $D-C$, the corresponding trajectories behave as those that agree with traditional asymptotical stability.

Theorem: Let $V=[x_1,x_2,\ldots,x_n]^T$: $D\to R_+$ be positive definite and analytic on D, where $x_1,x_2,\ldots,x_n$ are all space coordinates such that the derivative of V through Eqs. (A1) or (A3), $\dot{V}$, is negative semidefinite of $[x_1,x_2,\ldots,x_n]^T$.

For an autonomous system, let X be the m-manifold consisting of a point set for which $\forall x\neq 0$, $\dot{V}(x)=0$ and D is an m-manifold. If $m+1<n$, then the equilibrium point of the system is pragmatically asymptotically stable.

For a nonautonomous system, let X be the $m+1$-manifold consisting of the point set for which $\forall x\neq 0$, $\dot{V}(x_1,x_2,\ldots,x_n)=0$, and D is an $n+1$-manifold. If $m+1+1<n+1$, i.e., $m+1<n$, then the equilibrium point of the system is pragmatically asymptotically stable. Therefore, for both autonomous and nonautonomous systems, the formula $m+1<n$ is universal. So the following proof is only for an autonomous system. The proof for the nonautonomous system is similar.

Proof: Since every point of X can be passed by a trajectory of Eq. (A1), which is one-dimensional, the collection of these trajectories, C, is an $(m+1)$-manifold.[16,17]

If $m+1<n$, then the collection C is a subset of Lebesque measure 0 of D. By the above definition, the equilibrium point of the system is pragmatically asymptotically stable.

If an initial point is ergodicly chosen in D, *the probability of that the initial point falls on the collection C is zero. Here, equal probability is assumed for every point chosen as an initial point in the neighborhood of the equilibrium point.*

Chaos Synchronization and Chaos Control of Quantum-CNN Chaotic System by Variable Structure Control and Impulse Control

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Keywords : Quantum Cellular Neural Network (Quantum -CNN), Chaos Synchronization, Chaos Control

ABSTRCACT

In this paper, we derive some less stringent conditions for the exponential and asymptotic stability of impulsive control systems with impulses at fixed times. These conditions are then used to design an impulsive control law for Quantum Cellular Neural Network chaotic system, which drives the chaotic state to zero equilibrium and synchronizes two chaotic systems. An active sliding mode control method is synchronizing two chaotic systems and controlling chaotic state to periodic motion state. And a sufficient condition is drawn for the robust stability of the error dynamics, and is applied to guiding the design of the controllers. Finally, numerical results are used to show the robustness and effectiveness of the proposed control strategy.

1. Introduction.

Chaotic system exhibits unpredictable and irregular dynamics and it has been found in many engineering systems. Interestingly, chaotic models can describe complex dynamics with only few nonlinear equations without any random external inputs and small differences in the initial state can lead to extraordinary differences in the system state. Since Ott, Grebogi, and Yorke proposed the OGY method [1], a method of controlling chaos, 'controlling of chaos' is receiving increasing attention within the area of non-linear dynamics [2,3]. It has many applications in various systems while it is unfavorable in many other cases due to its irregular behavior. Therefore, both chaos utilization and elimination are important depending on the specific applications. Chaos control is an effective method for both chaos utilization and elimination and has been thoroughly studied in various fields of science.

Since the seminal work of Pecora and Carroll [4], there has been an interesting and potential topic in recent years in the study of chaos synchronization in physics, mathematics and engineering

ρ_{11}=32, ρ_{12}=27, ρ_{13}=49, ρ_{14}=31, ρ_{21}=32, ρ_{22}=27, ρ_{23}=49, ρ_{24}=31.

The result is shown in Figs. 4~5 for unidirectional linear coupling and bi-directional linear coupling, respectively.

4. Conclusions

Two chaotic Quantum-CNN systems are synchronized by three methods: unidirectional linear coupling by impulse control, bi-directional linear coupling by impulse control and variable structure control. The chaos controls of a Quantum-CNN system are also studied. The impulse control, and variable structure control are used to suppress chaos to fixed point or regulation motion. Numerical simulations are used to verify the effectiveness of the proposed controllers. These chaos synchronization and control methods can be also used for other chaotic systems.

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References

[1] E. Ott, C. Grebogi, J.A. Yorke, Phys. Rev. Lett. 64 (1990) 1196.

[2] G. Chen, X. Dong, Int. J. Bifur. Chaos 3 (1993) 1363.

[3] G. Chen, X. Dong, From Chaos to Order, Methodologies, Perspectives and Applications, World Scientific, Singapore, 1998.

[4] Pecora, L. M. and Carroll, T. L., ‘‘Synchronization in chaotic system”, Physical Review Letters’’, 64, pp. 821-824, 1990.

[5] Ott E, Grebogi C, Yorke JA., ‘‘Controlling chaos’’, Phys Rev Lett, 64, pp.1196-9, 1990.

[6] WeiGuo Xu, HuiZhang Shen, DaiPing Hu, and AiZhong Lei, “Impulse tuning of Chua chaos”, International Journal of Engineering Science, 43, 275–280, 2005.

[7] Keiji K, Michio H, Hideki K., ‘‘Sliding mode control for a class of chaotic systems’’, Phys Lett A, 245, pp.511-7, 1998.

[8] Jang M., ‘‘Sliding mode control of chaos in the cubic Chua’s circuit system’’, Int J Bifurcat Chaos, 12, pp.1437-49, 2002.

[9] Fuh C, Tung P., ‘‘Robust control for a class of nonlinear oscillators with chaotic attractors’’, Phys Lett A, 218, pp.240-8, 1996.

[10] Yu X., ‘‘Variable structure control approach for controlling chaos’’, Chaos, Solitons and Fractals, 8, pp.1577-86, 1997.

[11] Yu Y, Zhang S., ‘‘Controlling uncertain Lü system using backstepping design’’, Chaos, Solitons and Fractals, 15, pp.897-902, 2003.

[12] Bernardo Mdi., ‘‘A purely adaptive controller to synchronize and control chaotic systems’’, Phys Lett A, 214, pp.139-44, 1996.

[13] Moez F. An adaptive feedback control of linearizable chaotic systems’’, Chaos, Solitons & Fractals, 15, pp.883-90, 2003.

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Symplectic synchronization of different chaotic systems

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Abstract

In this paper, a new symplectic synchronization of chaotic systems is studied. Traditional generalized synchronizations are special cases of the symplectic synchronization. A sufficient condition is given for the asymptotical stability of the null solution of an error dynamics. The symplectic synchronization may be applied to the design of secure communication. Finally, numerical results are studied for a Quantum-CNN oscillators synchronized with a Rössler system in three different cases.

1. Introduction

Many approaches have been presented for the synchronization of chaotic systems [2–6]. There are a chaotic master system and either an identical or a different slave system. Our goal is the synchronization of the chaotic master and the chaotic slave by coupling or by other methods.

Among many kinds of synchronizations [7], generalized synchronization is investigated [8–12]. There exists a functional relationship between the states of the master and that of the slave. In this paper, a new synchronization

$$y = H(x, y, t) + F(t) \quad (1)$$

is studied, where x, y are the state vectors of the "master" and of the "slave", respectively, $F(t)$ is a given function of time in different form, such as a regular or a chaotic function. When $H(x, y, t) = x$, Eq. (1) reduces to the generalized synchronization given in [1]. Therefore this paper is an extension of [1].

In Eq. (1), the final desired state y of the "slave" system not only depends upon the "master" system state x but also depends upon the "slave" system state y itself. Therefore the "slave" system is not a traditional pure slave obeying the "master" system completely but plays a role to determine the final desired state of the "slave" system. In other words, it plays an "interwined" role, so we call this kind of synchronization "symplectic synchronization"[1], and call the "master" system partner A, the "slave" system partner B.

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[1] The term "symplectic" comes from the Greek for "interwined". H. Weyl first introduced the term in 1939 in his book "The Classical Groups" (p. 165 in both the first edition, 1939, and second edition, 1946, Princeton University Press).

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4. Conclusions

A new symplectic synchronization of a Quantum-CNN chaotic oscillator and a Rössler system is obtained by the Lyapunov asymptotical stability theorem. Two different chaotic dynamical systems, the Quantum-CNN system and the Rössler system, are in symplectic synchronization for three cases: the cubic symplectic synchronization, the time delay symplectic synchronization and the cubic time delay symplectic synchronization. Symplectic synchronization of chaotic systems can be used to increase the security of secret communication.

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References

[1] Ge Z-M, Yang C-H. The generalized synchronization of a Quantum-CNN chaotic oscillator with different order systems. Chaos, Solitons & Fractals 2008;35:980–90.
[2] Ge Z-M, Yang C-H. Synchronization of complex chaotic systems in series expansion form. Chaos, Solitons & Fractals 2007;34:1649–58.
[3] Pecora L-M, Carroll T-L. Synchronization in chaotic system. Phys Rev Lett 1990;64:821–4.
[4] Ge Zheng-Ming, Leu Wei-Ying. Anti-control of chaos of two-degrees-of-freedom louderspeaker system and chaos synchronization of different order systems. Chaos, Solitons & Fractals 2004;20:503–21.
[5] Femat R, Ramirez J-A, Anaya G-F. Adaptive synchronization of high-order chaotic systems: a feedback with low-order parameterization. Physica D 2000;139:231–46.
[6] Ge Z-M, Chang C-M. Chaos synchronization and parameters identification of single time scale brushless DC motors. Chaos, Solitons & Fractals 2004;20:883–903.
[7] Femat R, Perales G-S. On the chaos synchronization phenomenon. Phys Lett A 1999;262:50–60.
[8] Ge Z-M, Yang C-H. Pragmatical generalized synchronization of chaotic systems with uncertain parameters by adaptive control. Physica D: Nonlinear Phenomena 2007;231:87–94.
[9] Yang S-S, Duan C-K. Generalized synchronization in chaotic systems. Chaos, Solitons & Fractals 1998;9:1703–7.
[10] Krawiecki A, Sukiennicki A. Generalizations of the concept of marginal synchronization of chaos. Chaos, Solitons & Fractals 2000;11(9):1445–58.
[11] Ge Z-M, Yang C-H, Chen H-H, Lee S-C. Non-linear dynamics and chaos control of a physical pendulum with vibrating and rotation support. J Sound Vib 2001;242(2):247–64.
[12] Chen M-Y, Han Z-Z, Shang Y. General synchronization of Genesio–Tesi system. Int J Bifurcat Chaos 2004;14(1):347–54.
[13] Fortuna Luigi, Porto Domenico. Quantum-CNN to generate nanoscale chaotic oscillator. Int J Bifurcat Chaos 2004;14(3):1085–9.
[14] Ge Zheng-Ming, Chen Yen-Sheng. Synchronization of unidirectional coupled chaotic systems via partial stability. Chaos, Solitons & Fractals 2004;21:101–11.
[15] Chen S, Lu J. Synchronization of uncertain unified chaotic system via adaptive control. Chaos, Solitons & Fractals 2002;14(4):643–7.
[16] Ge Zheng-Ming, Chen Chien-Cheng. Phase synchronization of coupled chaotic multiple time scales systems. Chaos, Solitons & Fractals 2004;20:639–47.