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六種新渾沌系統及三種新型的渾沌同步之研究

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中英文摘要及關鍵詞

中文摘要

渾沌系統之研究在物理、化學、生物學、生理學、各種工程等方面皆有日益重要之廣泛應用。非線性 Mathieu 系統、Duffing 系統、van der Pol 系統都是典型的重要渾沌系統。Ikeda 系統及 Mackey-Glass 系統是典型的時滯光電及生理系統。本計畫採取適當的耦合方式將它們推廣為兩種雙 Mathieu 系統、雙 Duffing 系統、雙 van der Pol 系統、雙 Ikeda 系統及雙 Mackey-Glass 系統，研究其渾沌性質，從而就典型重要渾沌系統而言，既擴大其研究範圍也深化其研究內容。渾沌同步之研究在秘密通訊、神經網路、自組織、物理系統、生態系統、工程系統等方面有長足之應用。本計畫提出三種新型的渾沌同步，具有重要理論及實際意義：一、純誤差穩定的廣義同步法。用以改進目前需先由數值計算預先得出誤差方程中渾沌變量之最大值之有缺陷之方法。二、實用適應廣義同步法。用以糾正目前適應同步法中估值參數趨於未知參數未加證明之盲點。三、不同起始條件下的全同系統渾沌同步。根據渾沌理論之傳統說法，諸渾沌運動對初值極為敏感，隨時間按指數次數迅速分離。吾人發現對兩個全同而無任何連繫的雙 Ikeda 系統而言，不同的初值可導致二全同系統的延後同步等，對兩個全同而無任何連繫的雙 Mackey-Glass 系統而言，不同初值可導致各種暫時的延後同步。這些現象與傳統說法不同，極值得深入研究。研究重點為：

1. 兩種雙 Mathieu 系統之渾沌研究。用相圖、分歧圖、功率譜圖、李雅普諾夫指數、碎形維度等分析渾沌之行為。
2. 採用純誤差穩定理論及精緻之李雅普諾夫函數得出純誤差穩定的廣義同步法。並以對兩種雙 Mathieu 系統為例實現此種廣義同步。
3. 雙 Duffing 系統與雙 van der Pol 系統之渾沌研究。用相圖、分歧圖、功率譜圖、李雅普諾夫指數分析渾沌之行為，包括奇異吸引子之範圍及形狀、超渾沌之行為、碎形之維度等。
4. 應用實用穩定理論實現以實用適應渾沌同步，嚴格證明估值參數必然趨於未知參數，並由數值計算對雙 Duffing 系統與雙 van der Pol 系統加以驗證。
5. 雙 Ikeda 系統及雙 Mackey-Glass 系統的渾沌研究。
6. 找出各種不同初值之規律性，在此種初值下雙 Ikeda 系統呈現延後同步、超前同步、反延後同步及反超前同步。找出各種初值下之規律性，在此種初值下雙 Mackey-Glass 系統呈現暫時之延後同步、超前同步、反延後同步及反超前同步。並試圖解釋其原因。

關鍵詞：純誤差穩定的渾沌同步、實用渾沌同步、不同初值的渾沌同步、雙 Mathieu 系統、雙 Duffing 系統、雙 van der Pol 系統、雙 Ikeda 系統、雙 Mackey-Glass 系統

Abstract

The study of chaotic system has found wide applications in physics, chemistry, biology, physiology, and various engineerings. Nonlinear Mathieu system, Duffing system, and van der Pol system all are paradigmatic important chaotic systems. Ikeda system and Mackey-Glass system are paradigmatic important electro-optical and physiological time delay systems. In this project, the study is extended to two kinds of double Mathieu system, double Duffing system,

double van der Pol system, double Ikeda system, and double Mackey-Glass system by suitable coupling. For these paradigmatic and important systems, the study will be extended and deepened.

Chaos synchronizations are applied in various regions, such as secure communication, neural networks, self-organization, physical systems, ecological systems and engineering systems, etc. In this project, three new types of chaos synchronization with theoretical and practical importance are studied: 1. pure error stability synchronization, to improve the present defective method in which the maximum values of state variables appeared in error dynamics must be preliminarily calculated by simulations; 2. pragmatical adaptive generalized synchronization, to correct the absence of proof of that estimated parameters approach the unknown parameters; 3. different initial condition synchronization. By the traditional theory of chaos, the chaotic motions are very sensitive to initial conditions and separate each other exponentially. However, we discover that for two identical double Ikeda systems, lag synchronization, etc can be found for different initial condition, and for two identical double Mackey-Glass systems, various temporary lag synchronizations can be found for different initial conditions. These phenomina are contradictory to traditional theory. These should be studied seriously. The main parts of our study are:

1. The study of chaos of two kinds of double Mathieu system. By phase portraits, bifurcation diagrams, power spectra, Lyapunov exponents, fractal dimensions, the various chaotic behaviors of these systems will be studied.
2. By pure error stability theory and elaborate Lyapunov functions, the pure error generalized synchronization method is given, proved and illustrated by two kinds of double Mathieu systems.
3. The study of chaos of double Duffing system and double van der Pol system. By phase portraits, bifurcation diagrams, power spectra, Lyapunov exponents, the various chaotic behaviors of these systems will be studied. The regions and shapes of the strange attractors, hyperchaotic behaviors and fractal dimensions will also be studied.
4. By pragmatical stability theory, the pragmatical adaptive synchronization of the above systems will be obtained. That the estimated parameters approach the unknown parameters are rigorously proved and illustrated by simulation for double Duffing systems and double van der Pol systems.
5. The study of chaos of double Ikeda system and double Mackey-Glass system.
6. Discover the rule of the initial conditions, for which the double Ikeda systems appear to be in lag-synchronization, anticipated-synchronization, lag-anti-synchronization or anticipated-anti-synchronization, while the double Mackey-Glass systems appear to be in temporary ones. We will try to explain these phenomina.

key words: pure error stability generalized synchronization, pragmatical synchronization, different initial condition synchronization, double Mathieu system, double Duffing system, double van der Pol system, double Ikeda system, double Mackey-Glass system

前言與研究目的

渾沌系統之研究除了在理論上的重要價值外，在物理、化學、生理學及各種工程等方面皆有廣泛之應用。非線性Mathieu系統，Duffing系統與van der Pol系統都是重要的典型渾沌系統。Ikeda系統是重要的典型光電及生理時滯系統，而Mackey-Glass系統則是重要的典型生理時滯系統。對於這些極重要的渾沌現象及渾沌同步都已有豐富的研究成果，直到現在，這些重要典型系統仍為研究熱點。本計畫為了對這些著名系統，擴大其研究範圍並深化其研究內容，特提出六種新系統，即兩種雙Mathieu系統，雙Duffing系統，雙van der Pol系統，雙Ikeda系統及雙Mackey-Glass系統。首先証明其為渾沌系統，其次研究其渾沌行為。渾沌同步之研究在秘密通訊、神經網路、自我組織等方面有長足之應用。本計畫提出三種新的渾沌同步方式及其對這些新系統的應用。

(一)兩種雙Mathieu系統的渾沌行為與純誤差穩定的廣義同步及其對此二系統的應用

(a)兩種雙Mathieu系統的渾沌行為

經典的非線性Mathieu系統是

$$\ddot{x} + a(1+\sin\omega t)x + (1+\sin\omega t)x^3 + a\dot{x} = 0$$

或

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -a(1+\sin\omega t)x_1 - (1+\sin\omega t)x_1^3 - ax_2\end{aligned}$$

其中a,ω為常數。本計畫提出的創新系統，其一為自治的雙Mathieu系統

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -a(1+x_4)x_1 - (1+x_4)x_1^3 - ax_2 + bx_3\\ \dot{x}_3 &= x_4\\ \dot{x}_4 &= -(1+x_2)x_3 - a(1+x_2)x_3^3 - ax_4 + bx_1\end{aligned}$$

其中將兩個Mathieu系統的$\sin\omega t$交替換成對方的渾沌狀態x_4，x_2，並在第二、四式之末加上耦合項bx_3，bx_1。其二為非自治的雙Mathieu系統

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -a(1+\sin\omega t)x_1 - (1+\sin\omega t)x_1^3 - ax_2 + bx_3\\ \dot{x}_3 &= x_4\\ \dot{x}_4 &= -(1+\sin\omega t)x_3 - a(1+\sin\omega t)x_3^3 - ax_4 + bx_1\end{aligned}$$

其中保留$\sin\omega t$，僅在第二、四式之末加上耦合項bx_3，bx_1。同樣地，對它們的研究，一方面是對經典的單Mathieu系統研究之擴展及深化，且一方面它們也一定比單Mathieu系統更具複雜性。本計畫將研究其週期運動、準週期運動、渾沌運動及超渾沌運動。

(b)純誤差穩定的廣義同步及其對兩種雙Mathieu系統的應用

考慮最一般形式的主從系統

$$\begin{aligned}\dot{x} &= f(t,x)\\ \dot{y} &= f(t,y) + u(t,x,y)\end{aligned}$$

其中 $x, y \in R^n$ 為主從狀態向量，$f: R_+ \times R^n \to R^n$ 為非線性向量函數，$u: R_+ \times R \times R^n \to R^n$ 是控制向量。廣義同步指 $y = g(t,x)$，其中 g 為指定函數。$e = y - g(t,x)$ 是廣義同步誤差向量。誤差動力學為

$$\begin{aligned}\dot{e} &= \dot{y} - \dot{g}(t,x) \\ &= \dot{y} - \frac{\partial g(t,x)}{\partial x}\dot{x} - \frac{\partial g(t,x)}{\partial t}\end{aligned}$$

加上控制項後得：

$$\dot{e} = f(t,y) - \frac{\partial g(t,x)}{\partial x} f(t,x) - \frac{\partial g(t,x)}{\partial t} + u(t,x,y)$$

目前文獻中所用之Lyapunov函數千篇一律地採用 $V = \frac{1}{2}e^T e$ 平方和的形式，此種做法實為對Lyapunov直接法之極端自我窄化的做法。其實Lyapunov函數之形式千變萬化，運用得法，可得出人意外的佳績。今採用一個精緻的Lyapunov函數

$$\begin{aligned}V(t,e) &= \frac{1}{2}e^T \Lambda(t) e \\ &= \frac{1}{2}\lambda_{11}(t)e_1^2 + \cdots + \frac{1}{2}\lambda_{nn}(t)e_n^2\end{aligned}$$

其中 $\Lambda(t) = [\lambda_{ij}(t)] \in R^{n\times n}$ 為待求之可逆對角矩陣，$\lambda_{ij}(t)$ 皆為時間之函數。

上式可寫成

$$\begin{aligned}\dot{V}(t,e) = {} & G_1(\lambda_{11}, \dot{\lambda}_{11})e_1^2 + \cdots + G_n(\lambda_{nn}, \dot{\lambda}_{nn})e_n^2 \\ & + [H_1(\lambda_{11}, \cdots, \lambda_{nn}, x_1, \cdots, x_n, y_1, \cdots, y_n, t) + \lambda_{11}u_1]e_1 + \cdots \\ & + [H_n(\lambda_{11}, \cdots, \lambda_{nn}, x_1, \cdots, x_n, y_1, \cdots, y_n, t) + \lambda_{nn}u_n]e_n\end{aligned}$$

其中 G_i, H_i 為連續可微函數，u_i 為待求之控制器。此式可分為兩類：(1)所有 G_i 與 λ_{ii} 及 $\dot{\lambda}_{ii}$ 有關。(2)一些 G_j 與 $\lambda_{jj}, \dot{\lambda}_{jj}$ 有關，其他 G_k 僅與 $\dot{\lambda}_{kk}$ 有關。

對第(1)情況，設計 u_i 使

$$H_i + \lambda_{ii}u_i = 0 \qquad (i = 1, 2, \cdots, n)$$

則 $\dot{V}$ 中之狀態變量 x_i, y_i 皆不存在，乃得純誤差動力學。現在文獻中採用之 $\dot{V}$ 中皆含 x_i, y_i 狀態變量，為了保証 $\dot{V}$ 之負定性，或為了得出混沌同步條件，必須依賴數值計算，算出 x_i, y_i 之最大值。此方法有三缺點：1.同步理論其實並不限於兩渾沌系統間之同步，非渾沌系統之同步亦極有研究價值，其中包括 x_i 或 y_i 趨於無限大之非週期運動。即此時狀態變量不存在有限之最大值，故現在文獻所用方法成為無效。2.如果 $\dot{V}$ 中出現之狀態變量之最大值很大，則保證 $\dot{V}$ 為負定之條件將變得極為保守，而無足可取。3.需要以數值模擬計算之結果為條件之理論推導為有缺陷之理論(defective theory)，價值較低。

由上式，如令λ_{ii}滿足

$$\forall t \ge 0, \quad 0 < \lambda_{mii} \le \lambda_{ii}(t) \le \lambda_{Mii} \quad (i = 1, \cdots, n)$$

則可得

$$\forall t \ge 0, \quad G_i(\lambda_{ii}, \dot{\lambda}_{ii}) < 0 \quad (i = 1, \cdots, n)$$

即$\dot{V}$為負定。Lyapunov函數乃得到。

對第(2)情況，則設

$$\forall k, \quad \lambda_{kk} = 1$$
$$\forall k, \quad H_k + \lambda_{kk} u_k = -e_k$$
$$\forall j, \quad H_j + \lambda_{jj} u_j = 0$$

則可得純誤差動力學。再巧妙地適當設計u_i及λ_{jj}使

$$\forall t \ge 0, \quad 0 < \lambda_{mjj} \le \lambda_{jj}(t) \le \lambda_{Mjj}$$
$$\forall t \ge 0, \quad G_j(\lambda_{jj}, \dot{\lambda}_{jj}) < 0$$

即可得負定之$\dot{V}$，Lyapunov函數乃得到。

本計畫將對兩種雙Mathieu系統給出廣義同步$y_i = \alpha(t) x_i + \beta(t)$，其中$\alpha(t)$，$\beta(t)$為給定時間函數。由於每步設計都有賴於經驗及技巧的發揮，故難度較高。

（二）雙Duffing系統及雙van der Pol系統的渾沌行為與實用適應廣義同步法，及其對此二系統之應用

(a)雙Duffing系統及雙van der Pol系統的渾沌行為

經典的Duffing系統是

$$\ddot{x} + a\dot{x} + bx + cx^3 = d\cos\omega t$$

或

$$\dot{x} = y$$
$$\dot{y} = -ay - bx - cx^3 + d\cos\omega t$$

其中a, b, c, d為常數，$d\cos\omega t$為外加激勵項。現將兩個Duffing系統的兩個激勵項中的$\cos\omega t$交替換成

對方的狀態變量，即得到本計畫新創造的雙Duffing系統：

$$\dot{x} = y$$
$$\dot{y} = -ay - bx - cx^3 + du$$
$$\dot{u} = v$$
$$\dot{v} = -ev - gu - hu^3 + kx$$

這一交換使原來各不相同的兩個具有兩個狀態變量的非自治系統(nonautonomous system)變成一個具有四個狀態變量的自治系統(autonomous system)。自x, y構成之單Duffing方程而言，原來之激勵項為簡單的諧波，現在則變成渾沌變量u，x, y變成渾沌變量所激勵成的渾沌變量。這當然是單Duffing系統渾沌研究的延伸與深化。不僅如此，由於du, kx兩耦合渾沌激勵的同時存在，x, y系統與u, v系統之間有相互影響，所以雙Duffing系統比單

Duffing系統有更複雜的渾沌行為，當可預期。本計畫將研究其週期運動、準週期運動、渾沌運動及超渾沌運動。

經典的van der Pol系統是

$$\ddot{x}+\varphi x+a\dot{x}(x^2-1)-b\sin\omega t=0$$

或

$$\dot{x}=y$$
$$\dot{y}=-\varphi x+a(1-x^2)y+b\sin\omega t$$

其中 φ, a, b 是常數，$b\sin\omega t$ 為外加激勵項。現將兩個van der Pol系統的兩個激勵項中的 $\sin\omega t$ 交替換成對方的狀態變量，即得到本計畫新創造的雙van der Pol系統：

$$\dot{x}=y$$
$$\dot{y}=-x+b(1-cx^2)y+au$$
$$\dot{u}=v$$
$$\dot{v}=-u+e(1-fu^2)v+dx$$

同樣地，對此系統的研究，不僅是對單van der Pol系統渾沌行為研究之延伸與深化，而且此系統比單van der Pol系統有更複雜的渾沌行為，當可預期。本計畫將研究其週期運動、準週期運動、渾沌運動及超渾沌運動。

(b)實用適應廣義同步法及其對雙Duffing系統及雙van der Pol系統之應用

廣義渾沌同步為渾沌同步之進一步發展。多數系統之參數值多為未知值，故採適應控制方法以達成同步之目的。但目前流行之適應同步，是應用Lyapunov漸近穩定定理及Babalat引理証明兩系統之狀態誤差趨於零。但對為何參數估計值會趨於其未知值這一問題並未証明。本計畫採用申請人提出之實用漸近穩定定理(pragmatical asymptotical stability theorem)，引用機率(probability)的概念嚴格證明了這一問題，稱之為實用適應廣義同步法。

所謂廣義同步乃指從系統變量 y 與主系統變量 x 之間有函數關係 $y=G(x)$，現在創造一個新式之函數關係

$$y=G(x)=x+F(t)$$

其中 $F(t)$ 為給定之渾沌函數。設主系統為

$$\dot{x}=Ax+f(x,B)$$

其中 $x=[x_1,\cdots,x_n]^T\in R^n$ 為狀態向量，A 為 $n\times n$ 未知參數矩陣，f 為非線性向量函數，B 為 f 中之未知參數向量。從系統為

$$\dot{y}=\hat{A}y+f(y,\hat{B})+u(t)$$

其中 $\hat{A},\hat{B}$ 分別為估計參數矩陣及估計參數向量，u 為控制輸入向量。誤差為

$$e=x-y+F(t)$$

當 $\lim_{t\to\infty}e=0$，則廣義同步成立。將上式等號兩邊對時間求導，可得

$$\dot{e}=\dot{x}-\dot{y}+\dot{F}(t)$$

再在上式等號右邊加上控制項 $u(t)$，得

$$\dot{e} = Ax - \hat{A}y + f(x,B) - f(y,\hat{B}) + \dot{F}(t) - u(t)$$

今選定一定正之Lyapunov函數

$$V(e,\tilde{A},\tilde{B}) = \frac{1}{2}e^T e + \frac{1}{2}\tilde{A}_c^T \tilde{A}_c + \frac{1}{2}\tilde{B}^T \tilde{B}$$

其中 $\tilde{A} = A - \hat{A}$, 將矩陣 $\tilde{A}$ 中之元素按列排成一個列陣，或向量，以 $\tilde{A}_c$ 表示，$\tilde{B} = B - \hat{B}$。V 沿誤差微分方程解之時間導數

$$\dot{V}(e) = e^T[Ax - \hat{A}y + f(x,B) - f(y,\hat{B}) + \dot{F}(t) - u(t)] + \tilde{A}_c \dot{\tilde{A}}_c + \tilde{B}\dot{\tilde{B}}$$

吾人如能應用技巧，選擇 $u(t)$ 及 $\dot{\tilde{A}}_c, \dot{\tilde{B}}$ 使 $\dot{V}(e) = e^T Ce$，其中 C 為對角負定矩陣，則 $\dot{V}$ 為 e 及 $\tilde{A}_c, \tilde{B}$ 之負半定函數。只能證明 $e, \tilde{A}_c, \tilde{B}$ 為穩定，而非漸近穩定。目前流行之做法為：應用 Babalat引理，可證明誤差趨於零，但對 $\tilde{A}_c, \tilde{B}$ 趨於零未加以証明。

現用實用漸近穩定定理應用Lebesque測度及機率的觀點，在一定條件下証明 $e, \tilde{A}_c, \tilde{B}$ 皆趨近於零：

定理：$V = [x_1, \cdots, x_n]^T : D \to R_+$ 在 D 上為定正而解析。D 為 $n-1$ 流形。$\dot{V}$ 為半負定而 $\dot{V} = 0$ 之點集 x 為 $m-1$ 流形。當 $m+1 < n$ 時，所研究系統之零點為實際漸近穩定。

所謂“實際漸近穩定”是指：雖不符合傳統之漸近穩定，但非漸近穩定之機率為零，故實際上不會發生。吾人將以雙Duffing系統與雙van der Pol系統為例，實現實用適應廣義同步。

（三）雙Ikeda系統及雙Mackey-Glass系統的渾沌行為與不同起始條件下的全同系統渾沌同步及其在此兩系統的實現

(a)雙Ikeda系統及雙Mackey-Glass系統的渾沌行為

經典的Ikeda時滯系統為

$$\dot{x} + ax + b\sin x_\tau = 0$$

其中 a,b 為常數，$x_\tau = x(t-\tau)$，τ 為常數。此系統可用以表示有回授系統之B級雷射系統，其中B級之典型代表為固態、半導體及低壓 CO_2 雷射，也可表示生理(血液)系統。

本計畫新創的雙Ikeda系統為

$$\dot{x}_1 = -a_1 x_1 - b_1 \sin x_{1\tau_1} - c\sin y_1$$
$$\dot{y}_1 = -a_2 y_1 - b_2 \sin y_{1\tau_2}$$

其中 $c\sin y_1$ 為耦合項。對此系統之研究，不僅是對單Ikeda系統渾沌行為研究之延伸與深化，而且此系統比單Ikeda系統顯然有更複雜的渾沌行為，值得本計畫加以研究。

經典的Mackey-Glass系統是

$$\dot{x}=\frac{bx_\tau}{1+x_\tau^n}-rx$$

其中b,r為常數，n為正整數，τ為常數。此系統可以表示造血系統，其中x為t時刻之血液濃度，τ為造血所須之延遲時間，白血病病患之τ值增加即引起血液濃度產生渾沌變化。本計畫新創的雙Mackey-Glass系統是

$$\dot{x}_1=\frac{b_1x_{1\tau_1}}{1+x_{1\tau_1}^n}-rx_1$$

$$\dot{x}_2=\frac{b_2x_{2\tau_2}}{1+x_{2\tau_2}^n}-rx_2-x_1$$

其中第二式中之x_1為耦合項。對此系統之研究，不僅是對單Mackey-Glass系統渾沌行為研究之延伸與深化，而且此系統比單Mackey-Glass系統顯然有更複雜的渾沌行為，值得本計畫加以研究。

(b) 不同起始條件下的延遲同步(Lag synchronization)，預期同步(Anticipated synchronization)，延遲反同步(Lag anti-synchronization)或預期反同步(Anticipated anti-synchronization)及其在雙Ikeda系統及雙Mackey Glass系統中之實現

設第一個時滯系統是

$$\dot{x}=f(x,x_\tau,t)$$

第二個時滯系統是

$$\dot{y}=f(y,y_\tau,t)$$

其中$x,y\in R^n$是n維狀態向量，$x_\tau=x(t-\tau),y_\tau=y(t-\tau)$為時滯狀態向量。誤差定義為$e=x(t-\mathrm{T})-y(t)$，如果

$$e_i=x_{i\mathrm{T}_j}-y_i=0,\quad i=1,2,\cdots,p\le n,\ j=1,2,\cdots,m\quad \text{其中}\ t_{i\mathrm{T}_{j1}}\le t\le t_{i\mathrm{T}_{j2}}$$

T_j為在第j時間間隔內x_i延遲y_i之時間。T_j為負時則x_i超前(預期)y_i

在反同步的情況下誤差$e=x(t-\mathrm{T})+y(t)$將為零。故當

$$e_i=x_{i\mathrm{T}_j}+y_i=0,\quad i=1,2,\cdots,p\le n,\ j=1,2,\cdots,m\quad \text{其中}\ t_{i\mathrm{T}_{j1}}\le t\le t_{i\mathrm{T}_{j2}}$$

則x_i反延遲y_i。當T_j為負時則x_i反超前(反預期) y_i。

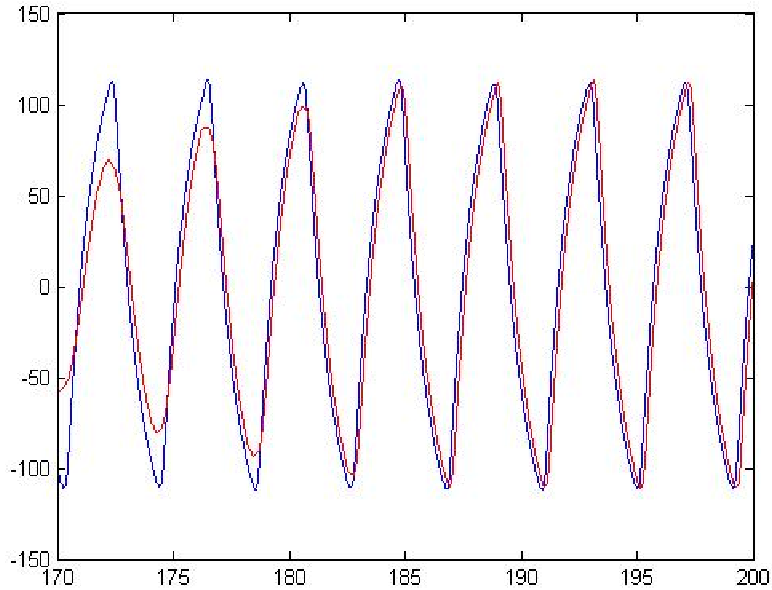
我們發現兩個雙Ikeda系統之初值取$x_{1_0}=1,y_{1_0}=1$，x_2,y_2取極小值時在180秒之後，x_1及y_1各延遲x_2,y_2 0.14868秒(Fig. 1)，此情況保持到20000秒。我們又發現兩個雙Mackey-Glass系統初值為$x_{1_0}=0.001,x_{2_0}=0.001,y_{1_0}=0.0015,y_2=0.0015$時，有間歇之延遲同步及預期同步。(Fig. 2)

本計畫將研究此二系統之其他不同初值下之長期及暫時延遲或預期，反延遲或反預期同步。試圖發現其規律性。特別是，長期之同步與傳統學說所稱渾沌系統對初值極敏感之

說不同，尤其值得研究。

Duffing系統，van der Pol系統與線性Mathieu系統原為振動學科之最重要最典型的系統。自渾沌動力學興起後，Duffing系統，van der Pol系統由於其為非線性系統故亦沿習成為渾沌動力學學科中最重要最典型的系統，四十年來對此二系統的渾沌研究之文獻可謂汗牛充棟，至今方興未艾。而線性Mathieu系統，則由於其為線性方程，不具渾沌性質，故在渾沌動力學學科中乃不再提及。人們忽視了非線性Mathieu系統實為Duffing系統中參數由常數轉為時間週期函數之推廣，實亦應成為渾沌動力學科之最重要最典型之系統。本計畫主持人率先研究非線性Mathieu系統之渾沌行為，可謂遲來之補求。眾所週知，此三種典型系統除理論意義外，廣泛應用於機械、電機、物理、化學、生科、奈米系統，本計畫今研究雙Duffing系統，雙van der Pol系統及雙種類型的非線性Mathieu系統，不僅對渾沌動力學學科中最重要最典型的三種渾沌系統的研究的拓廣與深化，更重要的是它們本身顯然具有更複雜的，未經發現的複雜渾沌行為，本研究對渾沌動力學學科具重大意義。其應用於機械、電機、物理、化學、生科、奈米之耦合系統，具有重要的實用價值。

渾沌同步除本身之重要理論價值外，其研究在秘密通訊、神經網路、自我組織等方面有日益廣泛之應用。廣義渾沌同步則為渾沌同步之進一步發展，其應用亦方興未艾。本計畫提出三種新的渾沌同步。實用適應廣義同步法糾正了目前國際文獻中未經証明即認為估值參數趨於為之參數之錯誤，首次在渾沌同步中引入概率概念，具重大理論及實用意義。由於$\dot{V}$之要求降低，實際應用亦較易實現。純誤差穩定的廣義同步，則彌補了國際文獻中需用數值計算結果為條件之理論，即有缺陷之理論。在理論與實用上有重要意義。不同起始條件的延遲同步等多種渾沌同步則為新發現的渾沌運動之現象，特別是Ikeda系統的永遠性延遲同步或反同步，不同於傳統理論，尤具重大意義。



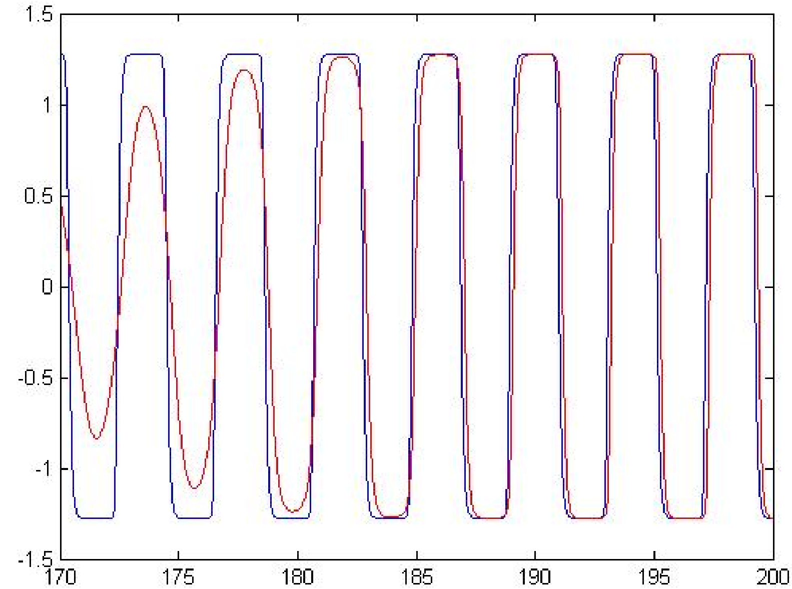


Fig.1 The time history of x_1 (blue) and x_2 (red), y_1 (blue) and y_2 (red) of double Ikeda systems.

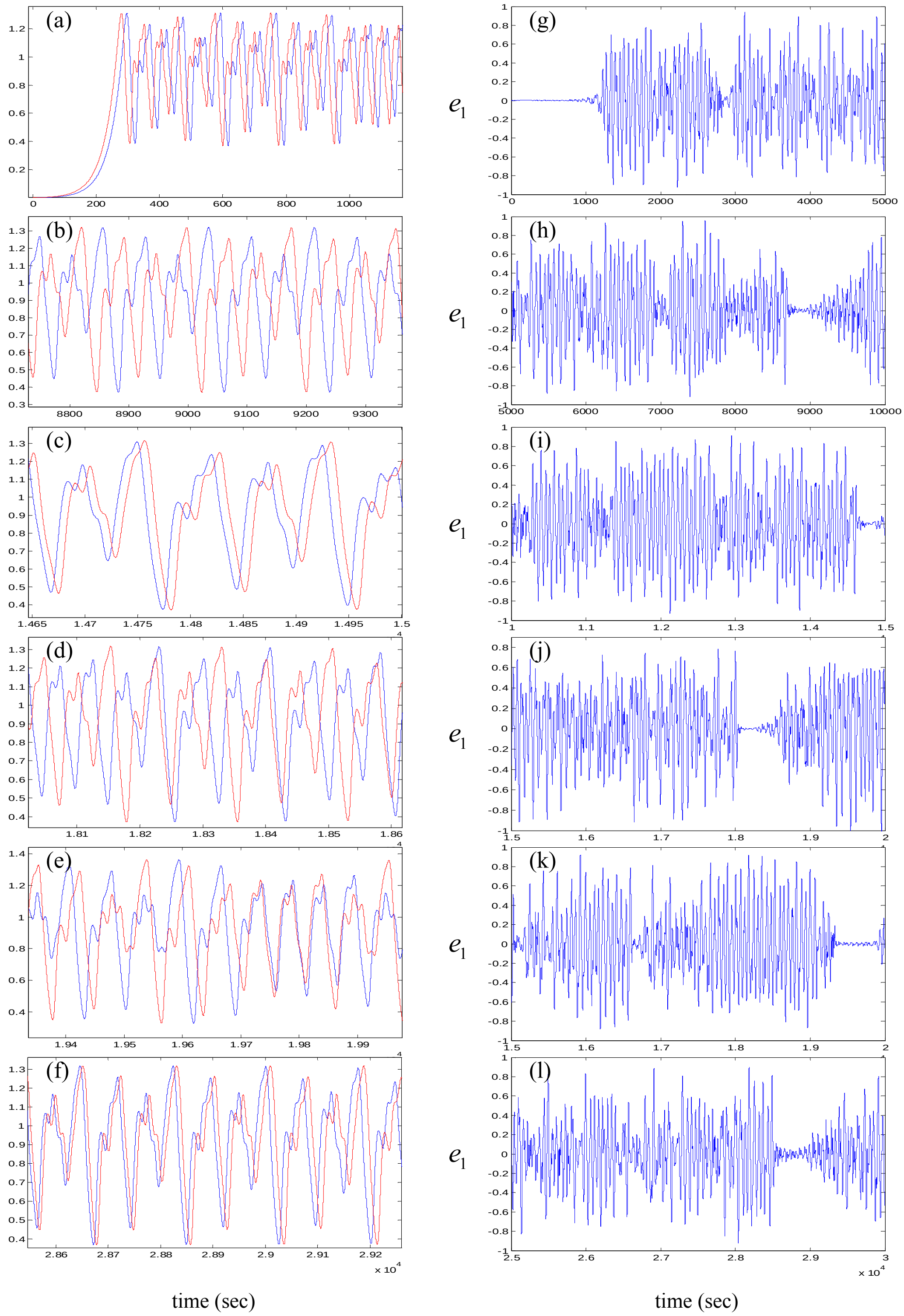


Fig.2 (a)~(f) The time histories of x_1 (blue) and y_1 (red) and (g)~(l) error $e_1 = x_{1T_j} - y_1$ of double Mackey-Glass systems with initial conditions (x_{10}, x_{20}) = (0.001, 0.001), (y_{10}, y_{20}) = (0.0015, 0.0015).

研究內容

Chapter 1
Introduction

Chaotic phenomena have been observed in physics, chemistry, physiology, and many disciplines [1-3]. In contrast with the famous chaotic systems, such as Lorenz system, Duffing system, and Rössler system, nonlinear Mathieu system is less mentioned [4-9]. However, nonlinear Mathieu system is important and can be applied in analysis of the resonant micro electro mechanical systems [10-12]. In this report, the new autonomous and new nonautonomous chaotic systems constructed by mutual linear coupling of two non-identical nonlinear damped Mathieu systems are studied.

Chaos synchronization has been widely applied in secure communication [13, 14], biological systems [15, 16], and many other fields [17, 18]. The generalized synchronization is a complex type of chaos synchronization and gives rise to extensive investigations recently [19-26]. The mixed error dynamics and the plain square sum Lyapunov function are currently applied in studying the generalized synchronization, but there are some shortcomings and restrictions in them. The auxiliary numerical simulation is unavoidable for current mixed error dynamics in which master state variables and slave state variables are presented while their maximum values must be determined by simulation [27-31]. However, the pure error dynamics can be analyzed theoretically without additional numerical simulation. Besides, monotonous and self-limited square sum Lyapunov function, $V(\mathbf{e}) = \frac{1}{2}\mathbf{e}^T\mathbf{e}$, is always used in most literatures [32-37], but the Lyapunov function can be chosen in a variety of elaborate and ingenious forms for different systems. Restricting Lyapunov function to square sum makes a long river brooklike, deeply weakens the powerfulness of Lyapunov direct method. Instead of current plain square sum Lyapunov function, the elaborate Lyapunov function is applied in this report. A systematic method of designing Lyapunov function is proposed based on the Lyapunov direct method [38].

The generalized synchronization is achieved for both new autonomous and nonautonomous chaotic systems by applying this technique.

Since Hertz [39] distinguished nonholonomic system from holonomic system in 1894, the study of nonholonomic system [40, 41] has been developed over one hundred years. A great number of studies in this field are connected with the extension of the developed analytical methods for holonomic system and for the systems with nonholonomic constraints. At present the dynamics of nonholonomic system has many applications in the problems of modern technology, such as the pursuit problems, the motion of automobiles, landing devices of airplanes, railway wheels, etc. However, the complete study of chaos in nonholonomic systems remains deficient. As far as we know, the only studies the chaos of nonholonomic system with an external constraint is Ref. [42], in which the chaotic phenomena of rattleback dynamics are studied. But in this , only Poincaré maps are given. As it is well-known, the only Poincaré map can not identify the existence of chaos reliably.

The moving target pursuit problem [43] is a typical example of nonholonomic system. In this report, chaos of nonholonomic systems with external nonholonomic constraint for two types of pursuit problems, a straightly oscillating target, and a circularly rotating target, is studied by applying the fundamental nonholonomic form of Lagrange's equations [44, 45]. Moreover, chaos of nonholonomic system with external nonlinear nonholonomic constraint, the magnitude of velocity keeping constant, is studied in this report by applying the nonlinear nonholonomic form of Lagrange's equations. All numerical criteria of chaos, i.e. the most reliable Lyapunov exponents [46], phase portraits, Poincaré maps and bifurcation diagrams are firstly wholly given to identify the existence of chaos of nonholonomic and nonlinear nonholonomic systems. Furthermore, it is found that the Feigenbaum number rule [47] still holds for nonlinear nonholonomic system.

There are various types of synchronization, such as complete synchronization [48], generalized synchronization [49], phase synchronization [50], lag synchronization [51], and so on. Among these types of synchronization, generalized synchronization is one of the most interesting topics. Generalized synchronization refers to a functional relation between the state vectors of master and slave, i.e. $\mathbf{y}=\mathbf{F}(\mathbf{x},t)$, where $\mathbf{x}$ and $\mathbf{y}$ are the state vectors of master and slave. In the work of Ref. [52], the generalized synchronization is extended to a more general form, $\mathbf{y}=\mathbf{F}(\mathbf{x},\mathbf{y},t)$, where the "slave" $\mathbf{y}$ is not a traditional pure slave obeying the "master" $\mathbf{x}$ completely but plays a role to determine the final desired state of the "slave". Since the "slave" $\mathbf{y}$ plays an "interwined" role, this type of synchronization is called "symplectic synchronization"[1], the master is called "partner A", and the slave is called "partner B". In this report, we propose two types of new chaos synchronization, "non-simultaneous symplectic synchronization" and "double symplectic synchronization".

We propose the "non-simultaneous symplectic synchronization", $\mathbf{y}(t)=\mathbf{F}(\mathbf{x}(\tau),\mathbf{y}(t),t)$, where τ is a given function of time t, so-called variable scale time. The synchronization is achieved at "different time" for "partner A" $\mathbf{x}(\tau)$ and "partner B" $\mathbf{y}(t)$, therefore we call this type of synchronization "non-simultaneous symplectic synchronization". When $\tau=t$, non-simultaneous symplectic synchronization reduces to symplectic synchronization. When applying the non-simultaneous symplectic synchronization in secret communication, since the functional relation of the non-simultaneous symplectic synchronization is more complex than that of the traditional generalized synchronization, and cracking the variable scale time τ is an extra task for the attackers in addition to cracking the system model and cracking the functional relation, the message is harder to be detected by applying the non-simultaneous symplectic synchronization than by applying traditional generalized synchronization. Therefore, the non-simultaneous symplectic synchronization may be applied to increase the security of secret communication. In order to achieve non-simultaneous symplectic synchronization, nonlinear control [53] and adaptive control are applied. In the work of Ref. [53], the induced matrix norm and the Lipschitz constant are obtained by auxiliary numerical simulation. However, they can be estimated theoretically by using the property of induced matrix norms [54a] and by applying

[1] The term "**symplectic**" comes from the Greek for "interwined". H. Weyl first introduced the term in 1939 in his book "The Classical Groups" (p. 165 in both the first edition, 1939, and second edition, 1946, Princeton University Press).

adaptive control. Furthermore, in our case, non-simultaneous symplectic synchronization, the complexity of the functional relation $\mathbf{F}(\mathbf{x}(\tau), \mathbf{y}(t), t)$ is greater than that studied in Ref. [53], thus the Lipschitz constant may be enormous. However, by applying adaptive control, the estimated Lipschitz constant is much less than the Lipschitz constant obtained by applying nonlinear control. This result in the reduction of the gain of the controller, i.e. the cost of controller is reduced. The proposed scheme is effective and feasible for both autonomous and nonautonomous chaotic systems, whether the dimensions of $\mathbf{x}(\tau)$ and $\mathbf{y}(t)$ systems are the same or not.

The "double symplectic synchronization", $\mathbf{G}(\mathbf{x}, \mathbf{y}) = \mathbf{F}(\mathbf{x}, \mathbf{y}, t)$, is proposed. Since the symplectic functions are presented at both the right hand side and the left hand side of the equality, it is called "double symplectic synchronization". It is an extension of symplectic synchronization, $\mathbf{y} = \mathbf{F}(\mathbf{x}, \mathbf{y}, t)$. When $\mathbf{G}(\mathbf{x}, \mathbf{y}) = \mathbf{y}$, the double symplectic synchronization is reduced to the symplectic synchronization. Due to the complexity of the form of the double symplectic synchronization, it may be applied to increase the security of secret communication. The double symplectic synchronization is obtained by applying active control. A scheme of synchronization is derived based on Barbalat's lemma [54b], and it is effective and feasible for both autonomous and nonautonomous chaotic systems.

The idea of fractional calculus has been known since the development of the regular calculus, with the first reference probably being associated with correspondence between Leibniz and L'Hospital in 1695, where the meaning of derivative of order one half was discussed [55-58]. Although fractional calculus has a 300-year-old history, its applications to physics and engineering are just a recent focus of interest [88-92]. It was found that many systems in interdisciplinary fields can be described by the fractional differential equations, such as viscoelastic systems, dielectric polarization [59], electrode electrolyte polarization [60], and electromagnetic waves [61]. More recently, many investigations are devoted to the control [62-66] and dynamics [67-79] of fractional order dynamical systems. In [67], it is shown that the fractional order Chua's circuit of order as low as 2.7 can produce a chaotic attractor. In [68], it is shown that nonautonomous Duffing systems of order less than 2 can still behave in a chaotic manner. In [69], chaotic behaviors of the fractional order "jerk" model is studied, in which chaotic attractor can be obtained with the system order as low as 2.1, and in [70] chaos control of this fractional order chaotic system is investigated. In [71], the fractional order Wien bridge oscillator is studied, where it is shown that limit cycle can be generated for any fractional order, with a proper value of the amplifier gain.

In 1990, the idea of synchronizing two identical chaotic systems with different initial conditions was introduced by Pecora and Carroll [92]. Since then, there has been particular interest in chaotic synchronization, due to many potential applications in secure communication, biological science, chemical reaction, social science, and many other fields. The concept of synchronization has been extended to the scope, such as complete synchronization (CS), phase synchronization (PS), lag synchronization (LS), anticipated synchronization (AS), and generalized synchronization (GS), etc [57-61, 91-109, 114-120]. However most of synchronizations can only be realized under the condition that there exists coupling between two chaotic systems. Sometimes, it is difficult even impossible to couple two chaotic systems such as in physical and electrical systems. In comparison with coupled chaotic systems, synchronization between the uncoupled chaotic systems has many advantages [99-100, 109-114]. In this report, synchronization of two double Duffing systems whose corresponding parameters are driven by a chaotic signal of a third system is analyzed. The chaos synchronizations of two uncoupled double Duffing systems are obtained by replacing their corresponding parameters by the same function of chaotic state variables of a third chaotic system. It is noted that whether CS or AS appears depends on the initial conditions. Besides, CS and AS are also characterized by great sensitivity to initial conditions and on the strengths of the substituting chaotic variable. It is found that CS or AS alternatively occurs under certain conditions [59-61, 68, 73]

Then we focus on the synchronization and antisynchronization of two identical double Duffing systems whose corresponding parameters are replaced by a white noise, a Rayleigh noise, a Rician noise or a uniform noise respectively. It is noted that whether CS or AS appears depends on the driving strength [57, 60, 73-74, 120-121].

In practice, some or all of the system parameters are uncertain. Moreover, these parameters change from time to time. Many researchers solve this problem by adaptive synchronization [122-127]. In current scheme of adaptive synchronization, traditional Lyapunov asymptotical stability theorem and Babalat lemma are used to prove the errors of synchronizing states approach zero. But the question that why the estimated parameters also approach the uncertain values, has still remained without answer. By the pragmatical asymptotical stability theorem [128-129] and an assumption of equal probability for ergodic initial conditions, the answer can be given.

Among many kinds of synchronizations, the generalized synchronization is investigated [130-142]. It means there exists a given functional relationship between the states of the master and that of the slave $y = G(x)$, where x, y are the states vector of master system and slave system respectively. In this report, a special kind of generalized synchronizations $y = G(x) = x + F(t)$ is studied, where $F(t)$ is a given vector function of time which may take various forms, either regular or chaotic function of time. When $F(t) = 0$, it reduces to a complete synchronization [143-144]. As a numerical example, two identical double Duffing chaotic systems [145] and a double van der Pol chaotic system [146-147] are used as master system, slave system, and goal system, respectively. The goal system gives chaotic $F(t)$. Next, the robustness of the generalized synchronization is also studied [148-154].

The contents of this report are as follows. Chapter 2 contains the dynamics of new autonomous and nonautonomous chaotic systems. The system models are described and the numerical results of regular and chaotic behaviors are presented. In Chapter 3, generalized synchronization of new chaotic systems is achieved by applying pure error dynamics and elaborate Lyapunov function. The methods of designing Lyapunov function are presented, and both new autonomous and new nonautonomous chaotic systems are illustrated in examples. By applying pure error dynamics and elaborate nondiagonal Lyapunov function, nonlinear generalized synchronization of new chaotic systems is obtained in Chapter 4. We propose the methods of designing Lyapunov function, and illustrate them by both new autonomous and new nonautonomous chaotic systems in examples. In Chapter 5, the dynamics of nonholonomic systems is studied by applying the fundamental nonholonomic form of Lagrange's equations. Two types of external nonholonomic constraints are studied for moving target pursuit problems: a straightly oscillating target and a circularly rotating target. Numerical results show that chaos exists in each case. By applying the nonlinear nonholonomic form of Lagrange's equations, the dynamics of nonlinear nonholonomic system is studied in Chapter 6. We investigate external nonlinear nonholonomic constraint: the magnitude of velocity keeping constant. Chaos is proved to exist in each case by numerical results. Furthermore, Feigenbaum number rule still holds for nonlinear nonholonomic system. In Chapter 7, the non-simultaneous symplectic synchronization is proposed, and it is achieved by applying adaptive control. The synchronization scheme is presented, and chaotic systems with the same or different dimensions are illustrated in examples. We investigate the double symplectic synchronization by applying active control in Chapter 8. The synchronization scheme is derived, and both autonomous and nonautonomous chaotic systems are illustrated in examples. In Chapter 9 the fractional derivative and its approximation are introduced. And then gives the dynamic equation of double Duffing system. The system under study is described both in its integer and fractional forms. Numerical simulation results are presented. In Chapter 10, a brief description of synchronization scheme based on the substitution of the strengths of the mutual coupling term of two identical chaotic double Duffing systems by the chaotic variable of a third double Duffing system are presented. And numerical simulations are given for illustration. It is found that one can obtain CS or AS by adjusting the driving strength and initial conditions. In Chapter 11, chaos synchronization and antisynchronization are obtained by replacing two corresponding parameters of two uncoupled identical double Duffing chaotic dynamical systems by a white noise, a Rayleigh noise, a Rician noise or a uniform noise respectively. It is found that one can obtain CS or AS by adjusting the driving strength. In Chapter 12, theoretical analyses of the pragmatical asymptotical stability are quoted. Adaptive controllers are designed for the pragmatical generalized synchronization of two double Duffing chaotic oscillators with a double van der Pol chaotic system as a goal system. High robustness of the generalized synchronization is also obtained in Chapter 12. In Chapter 13, chaotic behaviors of a fractional order double van der Pol system are studied by phase portraits and Poincaré maps. It is found that chaos exists in this system with order from 3.9 down to 0.4 much less than the number of states of the system. Linear transfer function approximations of the fractional integrator block are calculated for a set of fractional orders in [0.1, 0.9] based on frequency domain arguments. In Chapter 14, the variable of a third double van der Pol system substituted for the strength of two corresponding mutual coupling term of two identical chaotic double van der Pol system, give rise to their complete synchronization (CS) or anti-synchronization (AS). Numerical simulations show that either CS or AS depends on initial conditions and on the strengths of the substituted variable. In Chapter 15, we focus on the synchronization and antisynchronization of two identical double Duffing systems whose corresponding parameters are replaced by a white noise, a Rayleigh noise, a Rician noise or a uniform noise respectively. It is noted that whether CS or AS appear depends on the driving strength. In Chapter 16, based on a pragmatical theorem of asymptotical stability using the concept of probability, an adaptive control law is derived such that it can be proved strictly that the zero solution of error dynamics and of parameter dynamics is asymptotically stable. Numerical results are given for a chaotic double van der Pol system controlled to a double Duffing system. In Chapter 17, chaos in new integral and fractional order double Ikeda delay systems is studied. A double Ikeda delay system consists of two traditional Ikeda delay systems which are coupled together. Numerical simulations display the chaotic behaviors of the integral and fractional order delay systems by phase portraits, Poincaré maps and bifurcation diagrams. In Chapter 18, the chaotic behaviors of double Ikeda systems are obtained by replacing the original constant delay time by a function of chaotic state variable of a second chaotic double Ikeda system. The method is named delay time excited chaos synchronization which can be successfully obtained for some cases. Numerical simulations are illustrated by phase portraits. Phase portrait is expressed by numerical analysis. In Chapter 19, it is discovered that lag synchronization and lag anti-synchronization appear for two identical double Ikeda systems, without any control scheme or coupling terms, but with different initial conditions. In Chapter 20, the chaotic behaviors of double Ikeda systems are obtained by replacing the parameters by different chaotic state variables of a third chaotic double Ikeda system. The method is named parameter excited method for synchronization which will be successfully used for uncoupled synchronization. Numerical simulations are illustrated by phase portraits and time histories. In Chapter 21, a new double Mackey-Glass delay system, which consists of two coupled Mackey-Glass systems, is studied. Numerical simulations display the chaotic behaviors of the integral and fractional order delay systems by phase portraits and bifurcation diagrams. In Chapter 22, a control method called parameter excited method is applied to control a double Mackey-Glass chaotic system and to synchronize two uncoupled double Mackey-Glass systems. By replacing a parameter of the chaotic system by a noise signal, its chaotic motion can be eliminated. By replacing the corresponding parameters of two identical chaotic systems by a noise signal, these two chaotic systems with different initial conditions can be synchronized. For some chaotic systems, such as physical and electrical systems, which are difficult or even impossible to couple, this method is effective and potential in practice. In Chapter 23, it is discovered that TLS, TAS and TALS, TAAS appear for two identical double Mackey-Glass systems, without any control scheme or coupling terms, but with different initial conditions. In Chapter 24, the lag synchronization of two uncoupled double Mackey-Glass systems is achieved via the parameter excited method. This method is accomplished by replacing the corresponding parameters of the systems with two lag noise signals. By means of the difference of the timing between two replacements for the first system and the second system, the lag synchronization can be obtained. The parameter of the first system is substituted by a noise at $t = 0\,\text{sec}$, and the parameter of the second system is substituted by the noise at $t = d\,\text{sec}$. In other words, the control schemes do not work synchronously for these two systems. Parameter excited method is effective and potential in practice for some chaotic systems which are difficult or even impossible to be coupled. Temporary lag synchronization, partial lag synchronization, chaos control and robustness of lag synchronization are also obtained by this method. Finally, the conclusions of the whole report are drawn in Chapter 25.

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Chapter 2

Regular and Chaotic Dynamics of New Chaotic Systems

The nonlinear Mathieu system [1-6] is important and can be applied in analysis of the resonant micro electro mechanical systems [7-9]. In this Chapter, we propose new autonomous and new nonautonomous chaotic systems constructed by mutual linear coupling of two non-identical nonlinear damped Mathieu systems.

Consider two non-identical nonlinear damped Mathieu systems [5, 6] described by

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -a(1+\sin\omega t)x_1 - (1+\sin\omega t)x_1^3 - ax_2, \end{aligned} \tag{2.1}$$

$$\begin{aligned} \dot{x}_3 &= x_4, \\ \dot{x}_4 &= -(1+\sin\omega t)x_3 - a(1+\sin\omega t)x_3^3 - ax_4, \end{aligned} \tag{2.2}$$

where a and ω are constants.

A new autonomous chaotic system can be constructed by mutual linear coupling of two non-identical nonlinear damped Mathieu systems, Eq. (2.1) and Eq. (2.2). The term $\sin\omega t$ of one Mathieu system is replaced by one state of the other Mathieu system, and linear coupling terms are added to each other:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -a(1+x_4)x_1 - (1+x_4)x_1^3 - ax_2 + bx_3, \\ \dot{x}_3 &= x_4, \\ \dot{x}_4 &= -(1+x_2)x_3 - a(1+x_2)x_3^3 - ax_4 + bx_1. \end{aligned} \tag{2.3}$$

The parameters in simulation are $a = 0.5, b = 1 \sim 1.254$, and the initial condition is $x_1(0) = 0.1, x_2(0) = 0.1, x_3(0) = 0.2, x_4(0) = 0.2.$ The phase portraits, Poincaré maps, bifurcation diagram, and Lyapunov exponents of the new autonomous chaotic system are shown in Fig. 2.1-2.3. It can be observed that the motion is period 1 for $b = 1.1$, period 4 for $b = 1.243$, and period 8 for $b = 1.246$. For $b = 1.24$, the motion is chaotic.

A new nonautonomous chaotic system can also be constructed by mutual linear coupling of two non-identical nonlinear damped Mathieu systems, Eq. (2.1) and Eq. (2.2). The terms $\sin\omega t$ of each Mathieu system are preserved, and linear coupling terms are added to each other:

$$\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= -a(1+\sin\omega t)x_1 - (1+\sin\omega t)x_1^3 - ax_2 + bx_3, \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= -(1+\sin\omega t)x_3 - a(1+\sin\omega t)x_3^3 - ax_4 + bx_1.
\end{aligned} \tag{2.4}$$

The parameters in simulation are $a = 0.5, b = 0.9 \sim 1, \omega = 1,$ and the initial condition is $x_1(0) = 0.1, x_2(0) = 0.1, x_3(0) = 0.2, x_4(0) = 0.2.$ The phase portraits, Poincaré maps, bifurcation diagram, and Lyapunov exponents of the new nonautonomous chaotic system are shown in Fig. 2.4-2.6. It can be observed that the motion is period 1 for $b = 0.9$, period 2 for $b = 0.93$, and period 4 for $b = 0.934$. For $b = 1$, the motion is chaotic.

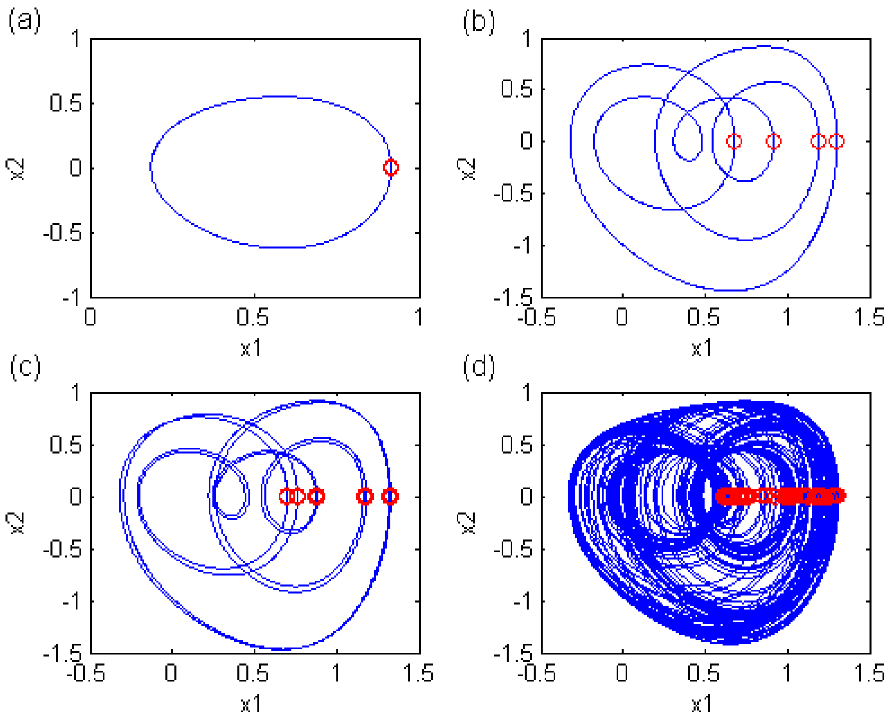

Fig. 2.1 Phase portraits and Poincaré maps of the new autonomous chaotic system: (a) period 1 for $b = 1.1$, (b) period 4 for $b = 1.243$, (c) period 8 for $b = 1.246$, (d) chaotic for $b = 1.24$.

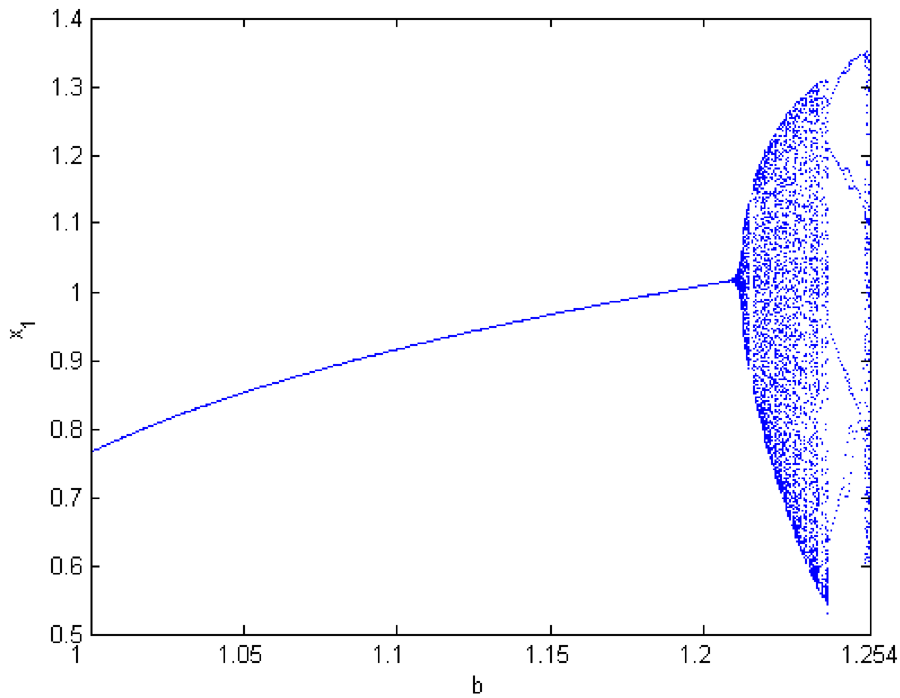

Fig. 2.2 Bifurcation diagram of the new autonomous chaotic system.

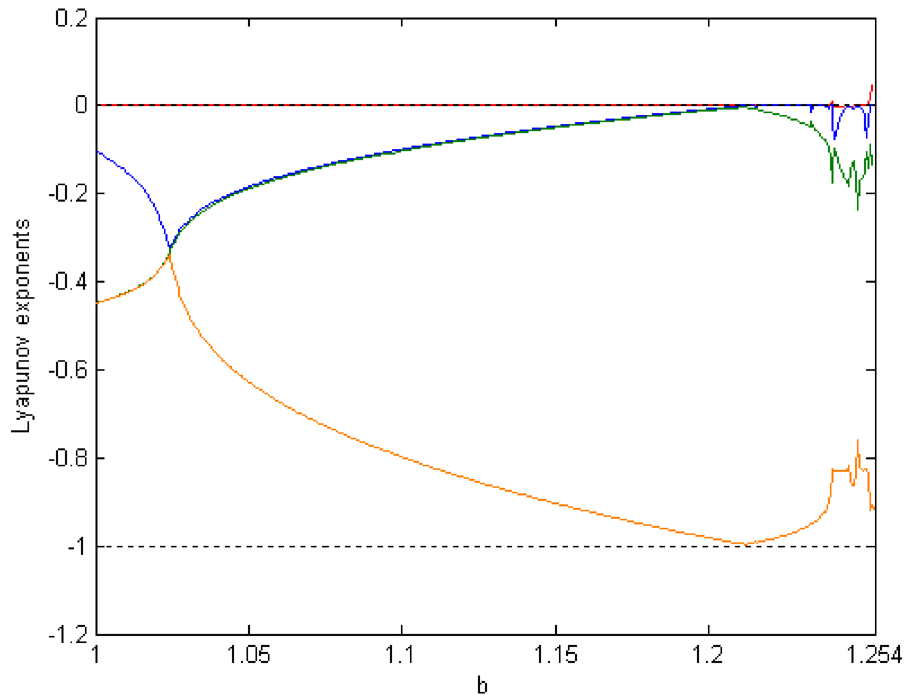

Fig. 2.3 Lyapunov exponents of the new autonomous chaotic system, where the sum of Lyapunov exponents is represented as a doted line at -1.

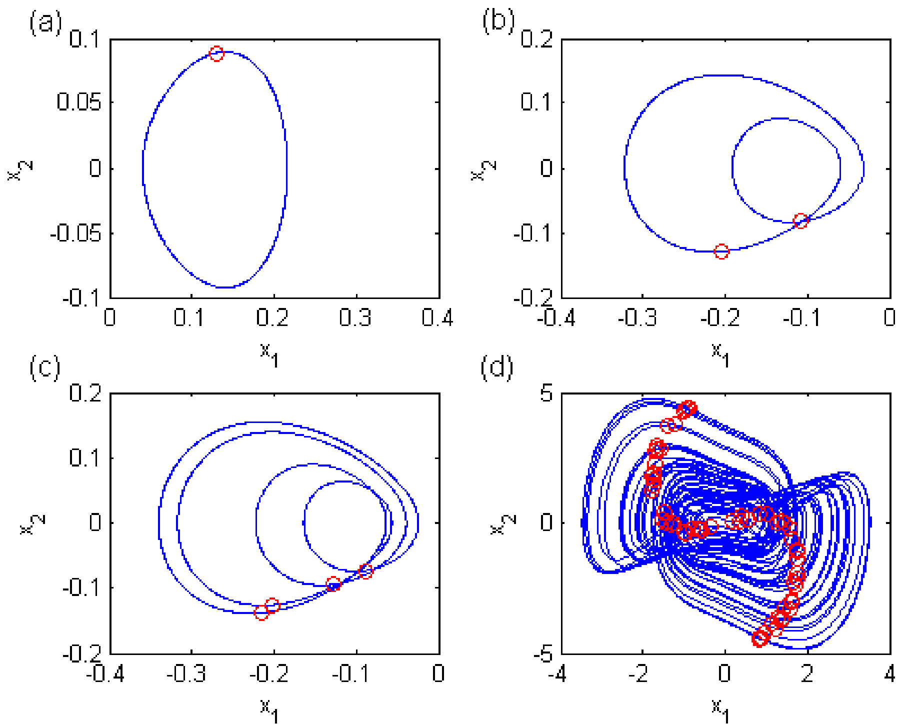

Fig. 2.4 Phase portraits and Poincaré maps of the new nonautonomous chaotic system: (a) period 1 for $b = 0.9$, (b) period 2 for $b = 0.93$, (c) period 4 for $b = 0.934$, (d) chaotic for $b = 1$.

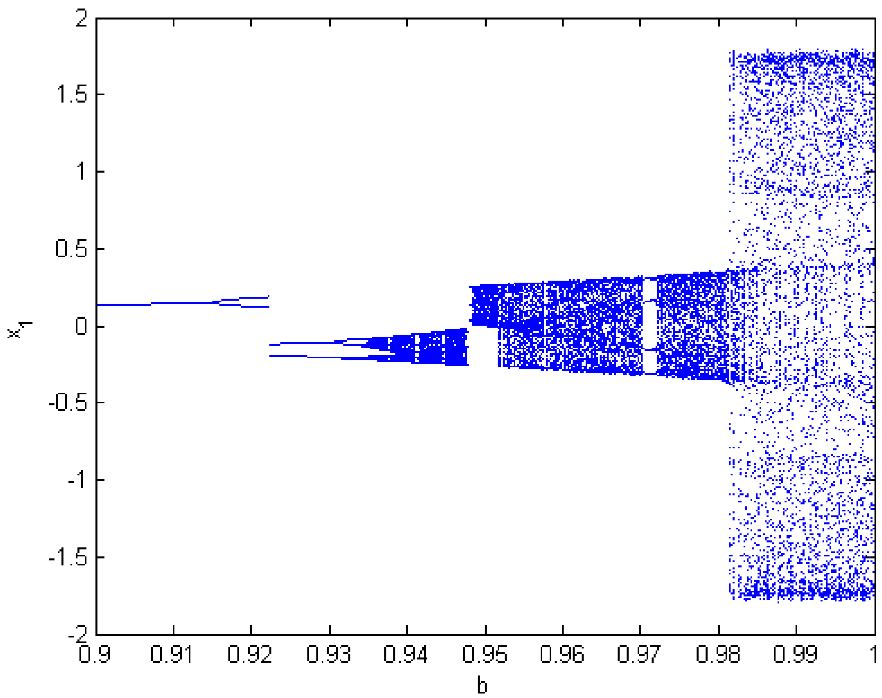

Fig 2.5 Bifurcation diagram of the new nonautonomous chaotic system.

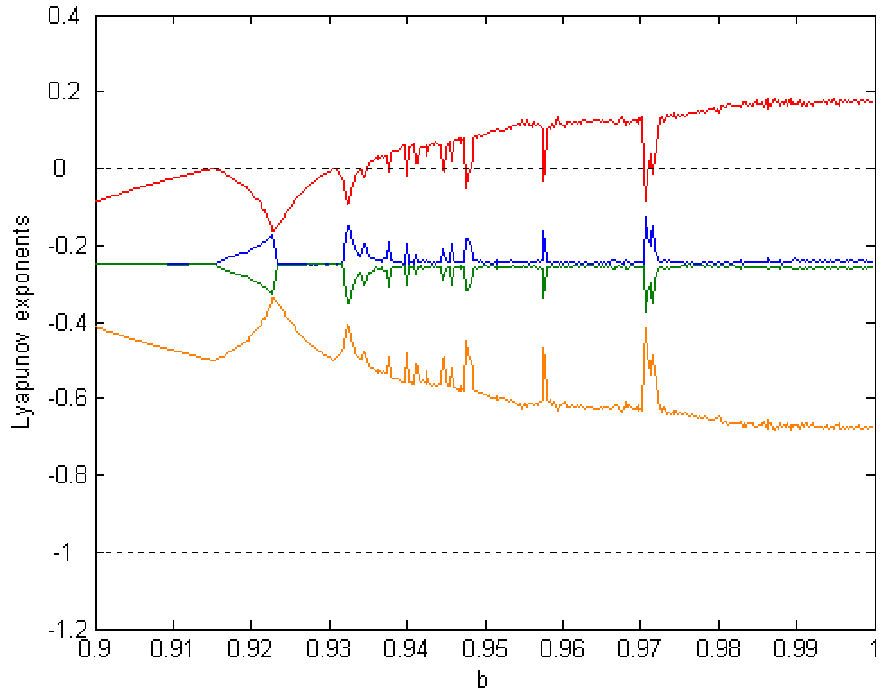

Fig. 2.6 Lyapunov exponents of the new nonautonomous chaotic system, where the sum of Lyapunov exponents is represented as a doted line at -1.

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Chapter 3

Generalized Synchronization of New Chaotic Systems by Pure Error Dynamics and Elaborate Lyapunov Function

3.1 Preliminaries

In this Chapter, the generalized synchronization is studied by applying pure error dynamics and elaborate Lyapunov function. The pure error dynamics can be analyzed theoretically without auxiliary numerical simulation, whereas the aid of additional numerical simulation is unavoidable for current mixed error dynamics in which master state variables and slave state variables are presented while their maximum values must be determined by simulation [1-5]. Besides, the elaborate Lyapunov function is applied rather than current plain square sum Lyapunov function, $V(\mathbf{e}) = \frac{1}{2}\mathbf{e}^T\mathbf{e}$, which is currently used [6-11] for convenience. However, Lyapunov function can be chosen in a variety of forms for different systems. Restricting Lyapunov function to square sum makes a long river brooklike, deeply weakens the powerfulness of Lyapunov direct method. Based on Lyapunov direct method [12], the generalized synchronization is achieved and a systematic method of designing Lyapunov function is proposed.

3.2 Design of Lyapunov Function

Consider the master and slave nonlinear dynamic systems described by

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}), \tag{3.1}$$

$$\dot{\mathbf{y}} = \mathbf{f}(t, \mathbf{y}) + \mathbf{u}(t, \mathbf{x}, \mathbf{y}), \tag{3.2}$$

where $\mathbf{x}, \mathbf{y} \in R^n$ are master and slave state vectors, $\mathbf{f} : R_+ \times R^n \to R^n$ is a nonlinear vector function, and $\mathbf{u}$: $R_+ \times R^n \times R^n \to R^n$ is controller vector.

Generalized synchronization means that there is a functional relation $\mathbf{y} = \mathbf{g}(t, \mathbf{x})$ between master and slave states as time goes to infinity, where $\mathbf{g}: R_+ \times R^n \to R^n$ is a continuously differentiable vector function. Define $\mathbf{e} = \mathbf{y} - \mathbf{g}(t, \mathbf{x})$ as generalized synchronization error vector, and the error dynamics can be obtained:

$$\begin{aligned}\dot{\mathbf{e}} &= \dot{\mathbf{y}} - \dot{\mathbf{g}}(t, \mathbf{x}) \\ &= \dot{\mathbf{y}} - \frac{\partial \mathbf{g}(t, \mathbf{x})}{\partial \mathbf{x}} \dot{\mathbf{x}} - \frac{\partial \mathbf{g}(t, \mathbf{x})}{\partial t} \\ &= \mathbf{f}(t, \mathbf{y}) - \frac{\partial \mathbf{g}(t, \mathbf{x})}{\partial \mathbf{x}} \mathbf{f}(t, \mathbf{x}) - \frac{\partial \mathbf{g}(t, \mathbf{x})}{\partial t} + \mathbf{u}(t, \mathbf{x}, \mathbf{y}).\end{aligned} \tag{3.3}$$

Based on Lyapunov direct method [12], the scheme of generalized synchronization and the procedure of designing Lyapunov function are described as follows:

Step 1. Construct a Lyapunov function

$$V(t, \mathbf{e}) = \frac{1}{2} \mathbf{e}^T \mathbf{\Lambda}(t) \mathbf{e} = \frac{1}{2} \lambda_{11}(t) e_1^2 + \frac{1}{2} \lambda_{22}(t) e_2^2 + \cdots + \frac{1}{2} \lambda_{nn}(t) e_n^2, \tag{3.4}$$

where $\mathbf{\Lambda}(t) = [\lambda_{ii}(t)] \in R^{n \times n}$ is an unknown continuously differentiable positive definite diagonal matrix to be designed. Its derivative is

$$\begin{aligned}\dot{V}(t, \mathbf{e}) &= \dot{\mathbf{e}}^T \mathbf{\Lambda}(t) \mathbf{e} + \frac{1}{2} \mathbf{e}^T \dot{\mathbf{\Lambda}}(t) \mathbf{e} \\ &= \lambda_{11}(t) e_1 \dot{e}_1 + \lambda_{22}(t) e_2 \dot{e}_2 + \cdots + \lambda_{nn}(t) e_n \dot{e}_n \\ &\quad + \frac{1}{2} \dot{\lambda}_{11}(t) e_1^2 + \frac{1}{2} \dot{\lambda}_{22}(t) e_2^2 + \cdots + \frac{1}{2} \dot{\lambda}_{nn}(t) e_n^2.\end{aligned} \tag{3.5}$$

Step 2. Eq. (3.5) can be rewritten in the following form:

$$\begin{aligned}\dot{V}(t, \mathbf{e}) &= G_1(\lambda_{11}, \dot{\lambda}_{11}) e_1^2 + G_2(\lambda_{22}, \dot{\lambda}_{22}) e_2^2 + \cdots + G_n(\lambda_{nn}, \dot{\lambda}_{nn}) e_n^2 \\ &\quad + [H_1(\lambda_{11}, \cdots, \lambda_{nn}, x_1, \cdots, x_n, y_1, \cdots, y_n, t) + \lambda_{11} u_1] e_1 \\ &\quad + [H_2(\lambda_{11}, \cdots, \lambda_{nn}, x_1, \cdots, x_n, y_1, \cdots, y_n, t) + \lambda_{22} u_2] e_2 \\ &\quad + \cdots + [H_n(\lambda_{11}, \cdots, \lambda_{nn}, x_1, \cdots, x_n, y_1, \cdots, y_n, t) + \lambda_{nn} u_n] e_n,\end{aligned} \tag{3.6}$$

where $G_i(\lambda_{ii}, \dot{\lambda}_{ii})$ and $H_i(\lambda_{11}, \cdots, \lambda_{nn}, x_1, \cdots, x_n, y_1, \cdots, y_n, t)$ $(i = 1, 2, \cdots, n)$ are continuous differentiable functions, u_i $(i = 1, 2, \cdots, n)$ are controllers to be determined.

Step 3. Eq. (3.6) may be classified as two general forms: (1) All $G_i(\lambda_{ii}, \dot{\lambda}_{ii})$ depend on $\lambda_{ii}(t)$

and $\dot{\lambda}_{ii}(t)$, (2) Some of $G_j(\lambda_{jj}, \dot{\lambda}_{jj})$ depend on $\lambda_{jj}(t)$ and $\dot{\lambda}_{jj}(t)$, the remaining $G_k(\lambda_{kk}, \dot{\lambda}_{kk})$ depend only on $\dot{\lambda}_{kk}(t)$.

Form (1): All $G_i(\lambda_{ii}, \dot{\lambda}_{ii})$ depend on $\lambda_{ii}(t)$ and $\dot{\lambda}_{ii}(t)$.

Step 4. Design the controllers u_i such that

$$H_i(\lambda_{11}, \cdots, \lambda_{nn}, x_1, \cdots, x_n, y_1, \cdots, y_n, t) + \lambda_{ii} u_i = 0 \quad (i = 1, 2, \cdots, n), \tag{3.7}$$

i.e. current mixed error dynamics has been replaced by pure error dynamics. By Eq. (3.7), we design the controllers u_i such that λ_{ii} $(i = 1,2,\cdots,n)$ are linear function of each other with positive coefficients.

Step 5. Design $\lambda_{11}(t), \lambda_{22}(t), \cdots, \lambda_{nn}(t)$ such that

$$\forall t \ge 0, \quad 0 < \lambda_{m\,ii} \le \lambda_{ii}(t) \le \lambda_{M\,ii} \quad (i = 1, 2, \cdots, n), \tag{3.8}$$

where $\lambda_{m\,ii}$, $\lambda_{M\,ii}$ are positive constants, and

$$\forall t \ge 0, \quad G_i(\lambda_{ii}, \dot{\lambda}_{ii}) < 0 \quad (i = 1, 2, \cdots, n), \tag{3.9}$$

then the Lyapunov function can be obtained and the generalized synchronization is achieved according to Lyapunov direct method.

Form (2): Some of $G_j(\lambda_{jj}, \dot{\lambda}_{jj})$ depend on $\lambda_{jj}(t)$ and $\dot{\lambda}_{jj}(t)$, and the remaining $G_k(\lambda_{kk}, \dot{\lambda}_{kk})$ depend only on $\dot{\lambda}_{kk}(t)$.

Step 4. Assume

$$\forall k, \quad \lambda_{kk}(t) = 1, \tag{3.10}$$

$$\forall k, \quad H_k(\lambda_{11}, \cdots, \lambda_{nn}, x_1, \cdots, x_n, y_1, \cdots, y_n, t) + \lambda_{kk}(t) u_k = -e_k, \tag{3.11}$$

$$\forall j, \quad H_j(\lambda_{11}, \cdots, \lambda_{nn}, x_1, \cdots, x_n, y_1, \cdots, y_n, t) + \lambda_{jj}(t) u_j = 0, \tag{3.12}$$

i.e. current mixed error dynamics has been replaced by pure error dynamics, and appropriately design the controllers $u_i \quad (i = 1,2,\cdots,n)$ and $\lambda_{jj}(t)$ such that

$$\forall t \geq 0, \quad 0 < \lambda_{m\,jj} \leq \lambda_{jj}(t) \leq \lambda_{M\,jj}, \tag{3.13}$$

where $\lambda_{m\,jj}$, $\lambda_{M\,jj}$ are positive constants, and

$$\forall t \geq 0, \quad G_j(\lambda_{jj}, \dot{\lambda}_{jj}) < 0, \tag{3.14}$$

then the Lyapunov function can be obtained and the generalized synchronization is achieved according to Lyapunov direct method.

3.3 Example for New Autonomous Chaotic Systems

In the following two Sections, the functional relation between master and slave states is $y_i = g_i(t, x_i) = \alpha(t)x_i + \beta(t) \quad (i = 1,2,\cdots,n)$.

The new autonomous chaotic system is constructed by mutual linear coupling of two non-identical nonlinear damped Mathieu systems, and the master and slave new autonomous chaotic systems can be described by

$$\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= -a(1+x_4)x_1 - (1+x_4)x_1^3 - ax_2 + bx_3, \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= -(1+x_2)x_3 - a(1+x_2)x_3^3 - ax_4 + bx_1,
\end{aligned} \tag{3.15}$$

$$\begin{aligned}
\dot{y}_1 &= y_2 + u_1, \\
\dot{y}_2 &= -a(1+y_4)y_1 - (1+y_4)y_1^3 - ay_2 + by_3 + u_2, \\
\dot{y}_3 &= y_4 + u_3, \\
\dot{y}_4 &= -(1+y_2)y_3 - a(1+y_2)y_3^3 - ay_4 + by_1 + u_4.
\end{aligned} \tag{3.16}$$

The parameters in simulation are $a = 0.5$, $b = 1.24$, and the initial condition is $x_1(0) = 0.1$, $x_2(0) = 0.1$, $x_3(0) = 0.2$, $x_4(0) = 0.2$, $y_1(0) = 0.3$, $y_2(0) = 0.3$, $y_3(0) = 0.4$, $y_4(0) = 0.4$. The phase portraits of the master new autonomous chaotic system are shown in Fig. 3.1.

Let $e_i = y_i - \alpha(t)x_i - \beta(t) \quad (i = 1,\cdots,4)$, then the error dynamics can be obtained:

$$
\begin{aligned}
\dot{e}_1 &= e_2 - \dot{\alpha}(t)x_1 + \beta(t) - \dot{\beta}(t) + u_1, \\
\dot{e}_2 &= -ae_1 - ae_2 + be_3 - a(y_1 y_4 - \alpha(t)x_1 x_4) - [(1+y_4)y_1^3 - \alpha(t)(1+x_4)x_1^3] \\
&\quad - \dot{\alpha}(t)x_2 + (b-2a)\beta(t) - \dot{\beta}(t) + u_2, \\
\dot{e}_3 &= e_4 - \dot{\alpha}(t)x_3 + \beta(t) - \dot{\beta}(t) + u_3, \\
\dot{e}_4 &= -e_3 - ae_4 + be_1 - (y_2 y_3 - \alpha(t)x_2 x_3) - a[(1+y_2)y_3^3 - \alpha(t)(1+x_2)x_3^3] \\
&\quad - \dot{\alpha}(t)x_4 + (b-a-1)\beta(t) - \dot{\beta}(t) + u_4.
\end{aligned}
\tag{3.17}
$$

Step 1. Construct a Lyapunov function

$$
V(t,\mathbf{e}) = \frac{1}{2}\mathbf{e}^T \mathbf{\Lambda}(t)\mathbf{e} = \frac{1}{2}\lambda_{11}(t)e_1^2 + \frac{1}{2}\lambda_{22}(t)e_2^2 + \frac{1}{2}\lambda_{33}(t)e_3^2 + \frac{1}{2}\lambda_{44}(t)e_4^2. \tag{3.18}
$$

Its derivative is

$$
\begin{aligned}
\dot{V}(t,\mathbf{e}) &= \frac{1}{2}\dot{\lambda}_{11}(t)e_1^2 + \lambda_{11}(t)e_1\dot{e}_1 + \frac{1}{2}\dot{\lambda}_{22}(t)e_2^2 + \lambda_{22}(t)e_2\dot{e}_2 \\
&\quad + \frac{1}{2}\dot{\lambda}_{33}(t)e_3^2 + \lambda_{33}(t)e_3\dot{e}_3 + \frac{1}{2}\dot{\lambda}_{44}(t)e_4^2 + \lambda_{44}(t)e_4\dot{e}_4.
\end{aligned}
\tag{3.19}
$$

Step 2. Eq. (3.19) can be rewritten in the following form

$$
\begin{aligned}
\dot{V}(t,\mathbf{e}) &= G_1(\lambda_{11},\dot{\lambda}_{11})e_1^2 + G_2(\lambda_{22},\dot{\lambda}_{22})e_2^2 + G_3(\lambda_{33},\dot{\lambda}_{33})e_3^2 + G_4(\lambda_{44},\dot{\lambda}_{44})e_4^2 \\
&\quad + [H_1(\lambda_{11},\cdots,\lambda_{44},x_1,\cdots,x_4,y_1,\cdots,y_4,t) + \lambda_{11}u_1]e_1 \\
&\quad + [H_2(\lambda_{11},\cdots,\lambda_{44},x_1,\cdots,x_4,y_1,\cdots,y_4,t) + \lambda_{22}u_2]e_2 \\
&\quad + [H_3(\lambda_{11},\cdots,\lambda_{44},x_1,\cdots,x_4,y_1,\cdots,y_4,t) + \lambda_{33}u_3]e_3 \\
&\quad + [H_4(\lambda_{11},\cdots,\lambda_{44},x_1,\cdots,x_4,y_1,\cdots,y_4,t) + \lambda_{44}u_4]e_4,
\end{aligned}
\tag{3.20}
$$

where

$$
\begin{aligned}
&G_1(\lambda_{11},\dot{\lambda}_{11}) = \frac{1}{2}\dot{\lambda}_{11}(t) - \lambda_{11}(t), \quad G_2(\lambda_{22},\dot{\lambda}_{22}) = \frac{1}{2}\dot{\lambda}_{22}(t) - a\lambda_{22}(t), \\
&G_3(\lambda_{33},\dot{\lambda}_{33}) = \frac{1}{2}\dot{\lambda}_{33}(t) - \lambda_{33}(t), \quad G_4(\lambda_{44},\dot{\lambda}_{44}) = \frac{1}{2}\dot{\lambda}_{44}(t) - a\lambda_{44}(t), \\
&H_1(\lambda_{11},\cdots,t) = \lambda_{11}(t)[-\dot{\alpha}(t)x_1 + \beta(t) - \dot{\beta}(t) + e_1] + b\lambda_{44}(t)e_4, \\
&H_2(\lambda_{11},\cdots,t) = \lambda_{11}(t)e_1 + \lambda_{22}(t)[-ae_1 - a(y_4 y_1 - \alpha(t)x_4 x_1) - ((1+y_4)y_1^3 \\
&\qquad\qquad - \alpha(t)(1+x_4)x_1^3) - \dot{\alpha}(t)x_2 + (b-2a)\beta(t) - \dot{\beta}(t)], \\
&H_3(\lambda_{11},\cdots,t) = b\lambda_{22}(t)e_2 + \lambda_{33}(t)[-\dot{\alpha}(t)x_3 + \beta(t) - \dot{\beta}(t) + e_3], \\
&H_4(\lambda_{11},\cdots,t) = \lambda_{33}(t)e_3 + \lambda_{44}(t)[-e_3 - (y_2 y_3 - \alpha(t)x_2 x_3) - a((1+y_2)y_3^3 \\
&\qquad\qquad - \alpha(t)(1+x_2)x_3^3) - \dot{\alpha}(t)x_4 + (b-a-1)\beta(t) - \dot{\beta}(t)].
\end{aligned}
\tag{3.21}
$$

Step 3. Since all $G_i(\lambda_{ii},\dot{\lambda}_{ii})$ depend on $\lambda_{ii}(t)$ and $\dot{\lambda}_{ii}(t)$ $(i=1,\cdots,4)$, Eq. (3.20) can be classified as form (1).

Step 4. Design the controllers

$$\begin{aligned}
u_1 &= -y_1 - by_4 + (\alpha(t)+\dot{\alpha}(t))x_1 + b\alpha(t)x_4 + b\beta(t) + \dot{\beta}(t),\\
u_2 &= a(y_1y_4 - \alpha(t)x_1x_4) + (1+y_4)y_1^3 - \alpha(t)(1+x_4)x_1^3\\
&\quad + \dot{\alpha}(t)x_2 - (b-2a)\beta(t) + \dot{\beta}(t),\\
u_3 &= -by_2 - y_3 + (\alpha(t)+\dot{\alpha}(t))x_3 + b\alpha(t)x_2 + b\beta(t) + \dot{\beta}(t),\\
u_4 &= y_2y_3 - \alpha(t)x_2x_3 + a(1+y_2)y_3^3 - \alpha(t)(1+x_2)x_3^3\\
&\quad + (1-\frac{1}{a})y_3 - (1-\frac{1}{a})\alpha(t)x_3 + \dot{\alpha}(t)x_4 - (b-a-\frac{1}{a})\beta(t) + \dot{\beta}(t),
\end{aligned} \tag{3.22}$$

such that

$$H_i(\lambda_{11},\cdots,\lambda_{44},x_1,\cdots,x_4,y_1,\cdots,y_4,t) + \lambda_{ii}(t)u_i = 0 \quad (i=1,\cdots,4), \tag{3.23}$$

and λ_{ii} $(i=1,\cdots,4)$ are linear function of each other with positive coefficients:

$$\lambda_{11}(t)=\lambda_{44}(t), \quad \lambda_{22}(t)=\frac{1}{a}\lambda_{11}(t), \quad \lambda_{33}(t)=\frac{1}{a}\lambda_{11}(t). \tag{3.24}$$

Now, the mixed error dynamics is replaced by pure error dynamics:

$$\dot{V}(t,\mathbf{e}) = G_1(\lambda_{11},\dot{\lambda}_{11})e_1^2 + G_2(\lambda_{22},\dot{\lambda}_{22})e_2^2 + G_3(\lambda_{33},\dot{\lambda}_{33})e_3^2 + G_4(\lambda_{44},\dot{\lambda}_{44})e_4^2. \tag{3.25}$$

Step 5. Design

$$\lambda_{11}(t)=\frac{1}{1+e^{-t}}, \lambda_{22}(t)=\frac{1}{a(1+e^{-t})}, \lambda_{33}(t)=\frac{1}{a(1+e^{-t})}, \lambda_{44}(t)=\frac{1}{1+e^{-t}}, \tag{3.26}$$

such that

$$\begin{aligned}
&\forall t\ge 0, \quad 0<\lambda_{m11}(t)=\frac{1}{2}\le\lambda_{11}(t)\le\lambda_{M11}(t)=1,\\
&\forall t\ge 0, \quad 0<\lambda_{m22}(t)=\frac{1}{2a}\le\lambda_{22}(t)\le\lambda_{M22}(t)=\frac{1}{a},\\
&\forall t\ge 0, \quad 0<\lambda_{m33}(t)=\frac{1}{2a}\le\lambda_{33}(t)\le\lambda_{M33}(t)=\frac{1}{a},\\
&\forall t\ge 0, \quad 0<\lambda_{m44}(t)=\frac{1}{2}\le\lambda_{44}(t)\le\lambda_{M44}(t)=1,
\end{aligned} \tag{3.27}$$

$$
\begin{aligned}
&\forall t \ge 0, \quad G_1(\lambda_{11}, \dot{\lambda}_{11}) = \frac{1}{2}\dot{\lambda}_{11}(t) - \lambda_{11}(t) = \frac{-2 - e^{-t}}{2(1+e^{-t})^2} < 0, \\
&\forall t \ge 0, \quad G_2(\lambda_{22}, \dot{\lambda}_{22}) = \frac{1}{2}\dot{\lambda}_{22}(t) - a\lambda_{22}(t) = \frac{-2a + (1-2a)e^{-t}}{2a(1+e^{-t})^2} = \frac{-1}{(1+e^{-t})^2} < 0, \\
&\forall t \ge 0, \quad G_3(\lambda_{33}, \dot{\lambda}_{33}) = \frac{1}{2}\dot{\lambda}_{33}(t) - \lambda_{33}(t) = \frac{-2 - e^{-t}}{2a(1+e^{-t})^2} = \frac{-2 - e^{-t}}{(1+e^{-t})^2} < 0, \\
&\forall t \ge 0, \quad G_4(\lambda_{44}, \dot{\lambda}_{44}) = \frac{1}{2}\dot{\lambda}_{44}(t) - \lambda_{44}(t) = \frac{-2a + (1-2a)e^{-t}}{2(1+e^{-t})^2} = \frac{-1}{2(1+e^{-t})^2} < 0,
\end{aligned}
\tag{3.28}
$$

then the Lyapunov function can be obtained

$$
V(t, \mathbf{e}) = \frac{1}{2(1+e^{-t})} e_1^2 + \frac{1}{2a(1+e^{-t})} e_2^2 + \frac{1}{2a(1+e^{-t})} e_3^2 + \frac{1}{2(1+e^{-t})} e_4^2, \tag{3.29}
$$

and

$$
\dot{V}(t, \mathbf{e}) = -\frac{2 + e^{-t}}{2(1+e^{-t})^2} e_1^2 - \frac{1}{(1+e^{-t})^2} e_2^2 - \frac{2 + e^{-t}}{(1+e^{-t})^2} e_3^2 - \frac{1}{2(1+e^{-t})^2} e_4^2 . \tag{3.30}
$$

Since Lyapunov global asymptotical stability theorem is satisfied, the global generalized synchronization is achieved. $\alpha(t) = \sin \omega t$, $\beta(t) = \cos \omega t$, $\omega = 1$ are chosen in simulation, and the results are shown in Fig. 3.2-3.3.

3.4 Example for New Nonautonomous Chaotic Systems

The new nonautonomous chaotic system is constructed by mutual linear coupling of two non-identical nonlinear damped Mathieu systems, and the master and slave new nonautonomous chaotic systems can be described by

$$
\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= -a(1 + \sin \omega t)x_1 - (1 + \sin \omega t)x_1^3 - ax_2 + bx_3, \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= -(1 + \sin \omega t)x_3 - a(1 + \sin \omega t)x_3^3 - ax_4 + bx_1,
\end{aligned}
\tag{3.31}
$$

$$
\begin{aligned}
\dot{y}_1 &= y_2 + u_1, \\
\dot{y}_2 &= -a(1 + \sin \omega t)y_1 - (1 + \sin \omega t)y_1^3 - ay_2 + by_3 + u_2, \\
\dot{y}_3 &= y_4 + u_3, \\
\dot{y}_4 &= -(1 + \sin \omega t)y_3 - a(1 + \sin \omega t)y_3^3 - ay_4 + by_1 + u_4,
\end{aligned}
\tag{3.32}
$$

The parameters in simulation are $a=0.5$, $b=1$, $\omega=1$, and the initial condition is $x_1(0)=0.1$, $x_2(0)=0.1$, $x_3(0)=0.2$, $x_4(0)=0.2$, $y_1(0)=0.3$, $y_2(0)=0.3$, $y_3(0)=0.4$, $y_4(0)=0.4$. The phase portraits of the master new nonautonomous chaotic system are shown in Fig. 3.4.

Let $e_i = y_i - \alpha(t)x_i - \beta(t) \quad (i=1,\cdots,4)$, then the error dynamics can be obtained:

$$\begin{aligned}
\dot{e}_1 &= e_2 - \dot{\alpha}(t)x_1 + \beta(t) - \dot{\beta}(t) + u_1, \\
\dot{e}_2 &= -a(1+\sin\omega t)e_1 - ae_2 + be_3 - (1+\sin\omega t)(y_1^3 - \alpha(t)x_1^3) - \dot{\alpha}(t)x_2 \\
&\quad + (-a(1+\sin\omega t) - a + b)\beta(t) - \dot{\beta}(t) + u_2, \\
\dot{e}_3 &= e_4 - \dot{\alpha}(t)x_3 + \beta(t) - \dot{\beta}(t) + u_3, \\
\dot{e}_4 &= -(1+\sin\omega t)e_3 - ae_4 + be_1 - a(1+\sin\omega t)(y_3^3 - \alpha(t)x_3^3) - \dot{\alpha}(t)x_4 \\
&\quad + (-(1+\sin\omega t) - a + b)\beta(t) - \dot{\beta}(t) + u_4,
\end{aligned} \tag{3.33}$$

Step 1. Construct a Lyapunov function

$$V(t,\mathbf{e}) = \frac{1}{2}\mathbf{e}^T\mathbf{\Lambda}(t)\mathbf{e} = \frac{1}{2}\lambda_{11}(t)e_1^2 + \frac{1}{2}\lambda_{22}(t)e_2^2 + \frac{1}{2}\lambda_{33}(t)e_3^2 + \frac{1}{2}\lambda_{44}(t)e_4^2. \tag{3.34}$$

Its derivative is

$$\begin{aligned}
\dot{V}(t,\mathbf{e}) &= \frac{1}{2}\dot{\lambda}_{11}(t)e_1^2 + \lambda_{11}(t)e_1\dot{e}_1 + \frac{1}{2}\dot{\lambda}_{22}(t)e_2^2 + \lambda_{22}(t)e_2\dot{e}_2 \\
&\quad + \frac{1}{2}\dot{\lambda}_{33}(t)e_3^2 + \lambda_{33}(t)e_3\dot{e}_3 + \frac{1}{2}\dot{\lambda}_{44}(t)e_4^2 + \lambda_{44}(t)e_4\dot{e}_4.
\end{aligned} \tag{3.35}$$

Step 2. Eq. (3.35) can be rewritten in the following form

$$\begin{aligned}
\dot{V}(t,\mathbf{e}) &= G_1(\lambda_{11},\dot{\lambda}_{11})e_1^2 + G_2(\lambda_{22},\dot{\lambda}_{22})e_2^2 + G_3(\lambda_{33},\dot{\lambda}_{33})e_3^2 + G_4(\lambda_{44},\dot{\lambda}_{44})e_4^2 \\
&\quad + [H_1(\lambda_{11},\cdots,\lambda_{44},x_1,\cdots,x_4,y_1,\cdots,y_4,t) + \lambda_{11}u_1]e_1 \\
&\quad + [H_2(\lambda_{11},\cdots,\lambda_{44},x_1,\cdots,x_4,y_1,\cdots,y_4,t) + \lambda_{22}u_2]e_2 \\
&\quad + [H_3(\lambda_{11},\cdots,\lambda_{44},x_1,\cdots,x_4,y_1,\cdots,y_4,t) + \lambda_{33}u_3]e_3 \\
&\quad + [H_4(\lambda_{11},\cdots,\lambda_{44},x_1,\cdots,x_4,y_1,\cdots,y_4,t) + \lambda_{44}u_4]e_4,
\end{aligned} \tag{3.36}$$

where

$$
\begin{aligned}
&G_1(\lambda_{11},\dot{\lambda}_{11}) = \frac{1}{2}\dot{\lambda}_{11}(t), \quad G_2(\lambda_{22},\dot{\lambda}_{22}) = \frac{1}{2}\dot{\lambda}_{22}(t) - a\lambda_{22}(t), \\
&G_3(\lambda_{33},\dot{\lambda}_{33}) = \frac{1}{2}\dot{\lambda}_{33}(t), \quad G_4(\lambda_{44},\dot{\lambda}_{44}) = \frac{1}{2}\dot{\lambda}_{44}(t) - a\lambda_{44}(t), \\
&H_1(\lambda_{11},\cdots,t) = \lambda_{11}(t)[-\dot{\alpha}(t)x_1 + \beta(t) - \dot{\beta}(t)] + b\lambda_{44}(t)e_4, \\
&H_2(\lambda_{11},\cdots,t) = \lambda_{11}(t)e_1 + \lambda_{22}(t)[-a(1+\sin\omega t)e_1 - (1+\sin\omega t)(y_1^3 - \alpha(t)x_1^3) \\
&\qquad - \dot{\alpha}(t)x_2 + (-a(1+\sin\omega t) - a + b)\beta(t) - \dot{\beta}(t)], \\
&H_3(\lambda_{11},\cdots,t) = b\lambda_{22}(t)e_2 + \lambda_{33}(t)[-\dot{\alpha}(t)x_3 + \beta(t) - \dot{\beta}(t)], \\
&H_4(\lambda_{11},\cdots,t) = \lambda_{33}(t)e_3 + \lambda_{44}(t)[-(1+\sin\omega t)e_3 - a(1+\sin\omega t)(y_3^3 - \alpha(t)x_3^3) \\
&\qquad - \dot{\alpha}(t)x_4 + (-(1+\sin\omega t) - a + b)\beta(t) - \dot{\beta}(t)].
\end{aligned} \tag{3.37}
$$

Step 3. Since some of $G_j(\lambda_{jj},\dot{\lambda}_{jj})$ depend on $\lambda_{jj}(t)$ and $\dot{\lambda}_{jj}(t)$ $(j=2,4)$, the remaining $G_k(\lambda_{kk},\dot{\lambda}_{kk})$ depend only on $\dot{\lambda}_{kk}(t)$ $(k=1,3)$, Eq. (3.37) can be classified as form (2).

Step 4. Assume

$$
\lambda_{11}(t) = 1, \quad \lambda_{33}(t) = 1, \tag{3.38}
$$

$$
\begin{aligned}
&H_1(\lambda_{11},\cdots,\lambda_{44},x_1,\cdots,x_4,y_1,\cdots,y_4,t) + \lambda_{11}(t)u_1 = -e_1, \\
&H_3(\lambda_{11},\cdots,\lambda_{44},x_1,\cdots,x_4,y_1,\cdots,y_4,t) + \lambda_{33}(t)u_3 = -e_3,
\end{aligned} \tag{3.39}
$$

$$
\begin{aligned}
&H_2(\lambda_{11},\cdots,\lambda_{44},x_1,\cdots,x_4,y_1,\cdots,y_4,t) + \lambda_{22}(t)u_2 = 0, \\
&H_4(\lambda_{11},\cdots,\lambda_{44},x_1,\cdots,x_4,y_1,\cdots,y_4,t) + \lambda_{44}(t)u_4 = 0,
\end{aligned} \tag{3.40}
$$

and appropriately design the controllers u_i $(i=1,\cdots,4)$ and $\lambda_{22}(t)$, $\lambda_{44}(t)$

$$
\begin{aligned}
&u_1 = -y_1 - \frac{b}{2+\sin\omega t}y_4 + (\alpha(t) + \dot{\alpha}(t))x_1 + \frac{b\alpha(t)}{2+\sin\omega t}x_4 + \frac{b\beta(t)}{2+\sin\omega t} + \dot{\beta}(t), \\
&u_2 = -ay_1 + a\alpha(t)x_1 + \dot{\alpha}(t)x_2 + (1+\sin\omega t)(y_1^3 - \alpha(t)x_1^3) \\
&\qquad + (a\sin\omega t + 3a - b)\beta(t) + \dot{\beta}(t), \\
&u_3 = -y_3 - \frac{b}{2+\sin\omega t}y_2 + (\alpha(t) + \dot{\alpha}(t))x_3 + \frac{b\alpha(t)}{2+\sin\omega t}x_2 + \frac{b\beta(t)}{2+\sin\omega t} + \dot{\beta}(t), \\
&u_4 = -y_3 + \alpha(t)x_3 + \dot{\alpha}(t)x_4 + a(1+\sin\omega t)(y_3^3 - \alpha(t)x_3^3) \\
&\qquad + (\sin\omega t + a - b + 2)\beta(t) + \dot{\beta}(t),
\end{aligned} \tag{3.41}
$$

$$
\lambda_{22}(t) = \frac{1}{a(2+\sin\omega t)}, \quad \lambda_{44}(t) = \frac{1}{2+\sin\omega t}, \tag{3.42}
$$

such that

$$\forall t \geq 0, \quad 0 < \lambda_{m22} = \frac{1}{3a} \leq \lambda_{22}(t) \leq \lambda_{M22} = \frac{1}{a},$$
$$\forall t \geq 0, \quad 0 < \lambda_{m44} = \frac{1}{3} \leq \lambda_{44}(t) \leq \lambda_{M44} = 1, \tag{3.43}$$

$$\begin{aligned} \forall t \geq 0, \quad G_2(\lambda_{22}, \dot{\lambda}_{22}) &= \frac{1}{2}\dot{\lambda}_{22}(t) - a\lambda_{22}(t) \\ &= \frac{-(4a + 2a\sin\omega t + \omega\cos\omega t)}{2a(2+\sin\omega t)^2} = \frac{-(2+\sin t + \cos t)}{(2+\sin t)^2} < 0, \\ \forall t \geq 0, \quad G_4(\lambda_{44}, \dot{\lambda}_{44}) &= \frac{1}{2}\dot{\lambda}_{44}(t) - a\lambda_{44}(t) \\ &= \frac{-(4a + 2a\sin\omega t + \omega\cos\omega t)}{2(2+\sin\omega t)^2} = \frac{-(2+\sin t + \cos t)}{2(2+\sin t)^2} < 0. \end{aligned} \tag{3.44}$$

Now, the mixed error dynamics is replaced by pure error dynamics:

$$\begin{aligned} \dot{V}(t, \mathbf{e}) &= [G_1(\lambda_{11}, \dot{\lambda}_{11}) - \lambda_{11}]e_1^2 + G_2(\lambda_{22}, \dot{\lambda}_{22})e_2^2 \\ &\quad + [G_3(\lambda_{33}, \dot{\lambda}_{33}) - \lambda_{33}]e_3^2 + G_4(\lambda_{44}, \dot{\lambda}_{44})e_4^2. \end{aligned} \tag{3.45}$$

Then the Lyapunov function can be obtained

$$V(t, \mathbf{e}) = \frac{1}{2}e_1^2 + \frac{1}{2a(2+\sin\omega t)}e_2^2 + \frac{1}{2}e_3^2 + \frac{1}{2(2+\sin\omega t)}e_4^2, \tag{3.46}$$

and

$$\dot{V}(t, \mathbf{e}) = -e_1^2 - \frac{2+\sin t + \cos t}{(2+\sin t)^2}e_2^2 - e_3^2 - \frac{2+\sin t + \cos t}{2(2+\sin t)^2}e_4^2. \tag{3.47}$$

Since Lyapunov global asymptotical stability theorem is satisfied, the global generalized synchronization is achieved. $\alpha(t) = \sin\omega t$, $\beta(t) = \cos\omega t$, $\omega = 1$ are chosen in simulation, and the results are shown in Fig. 3.5-3.6.

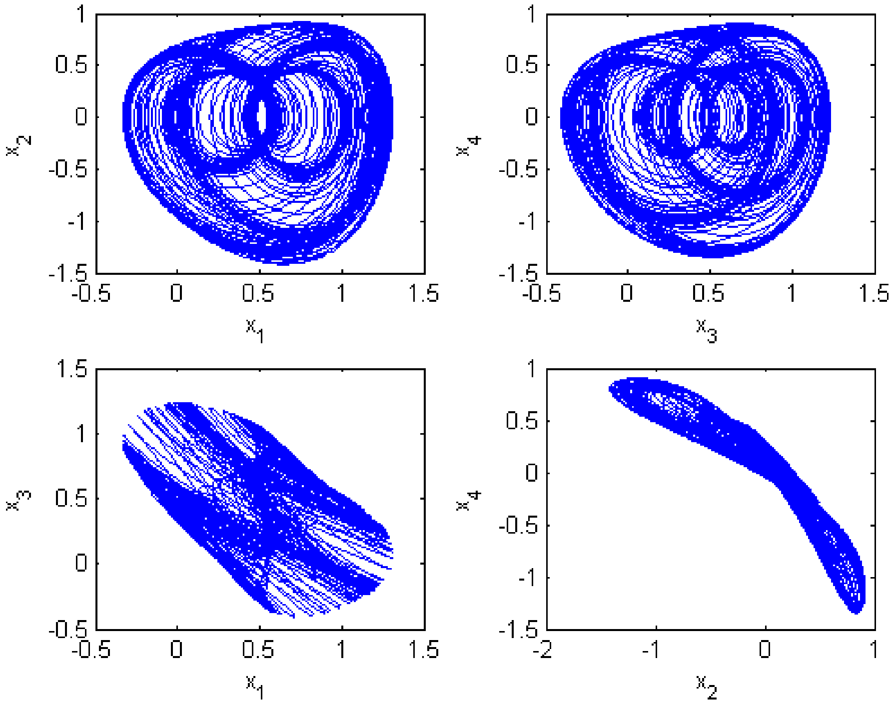

Fig. 3.1 Phase portraits of the master new autonomous chaotic system.

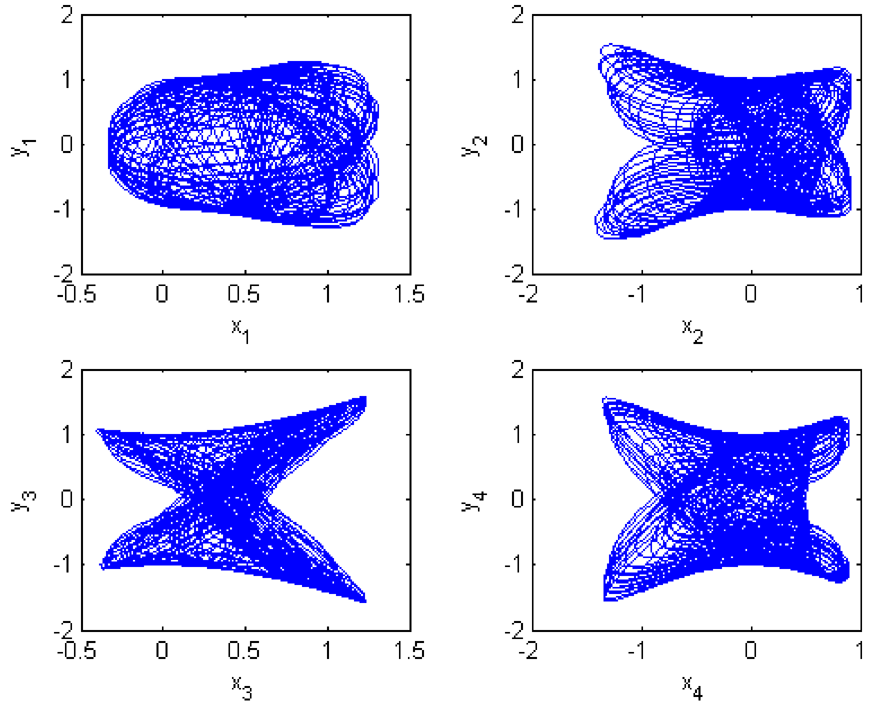

Fig. 3.2 Phase portraits of x_i to y_i ($i=1,\cdots,4$) for Section 3.3 when the generalized synchronization is obtained.

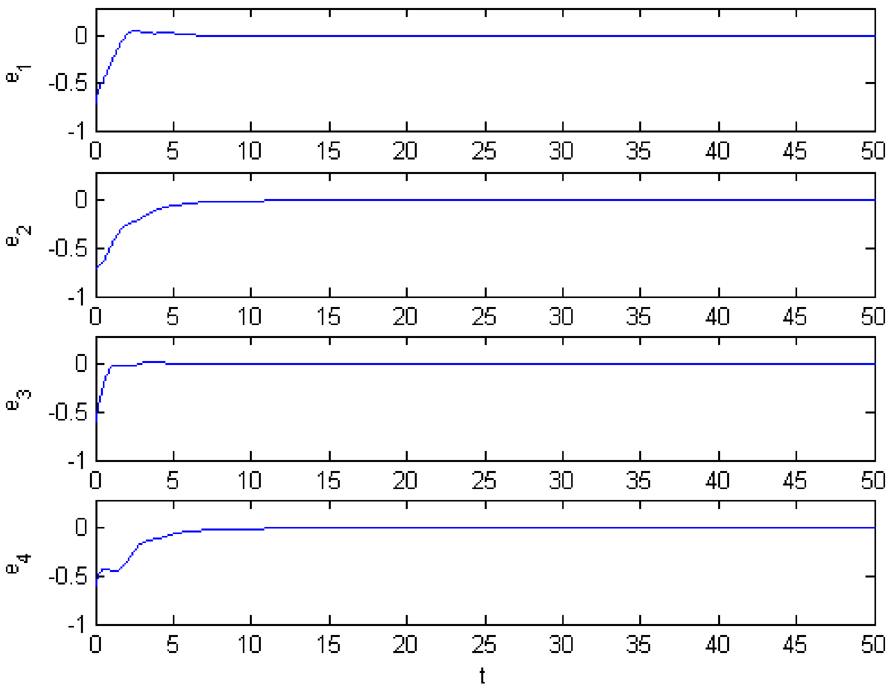

Fig. 3.3 Time histories of the state errors for Section 3.3.

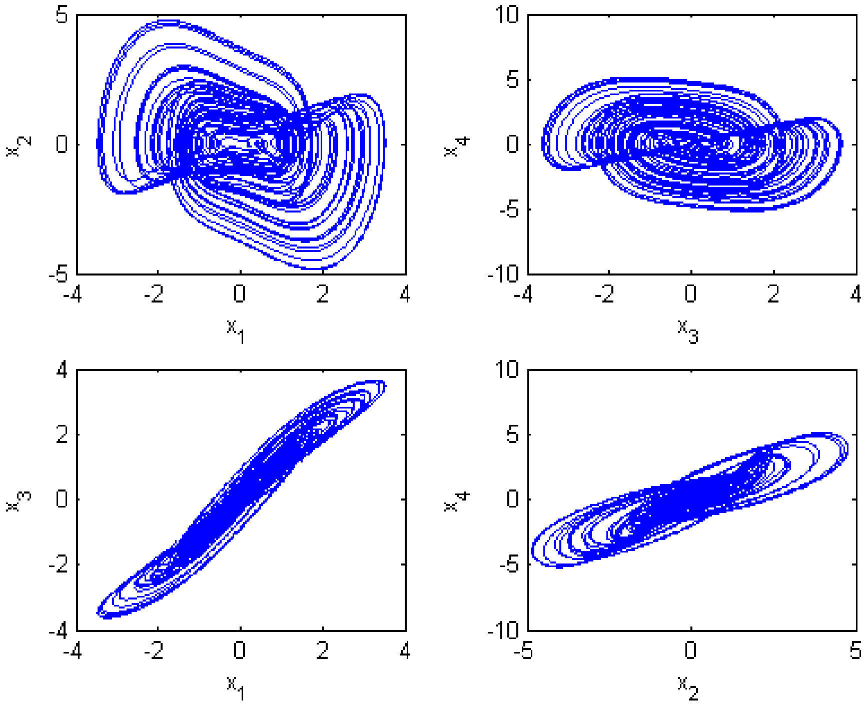

Fig. 3.4 Phase portraits of the master new nonautonomous chaotic system.

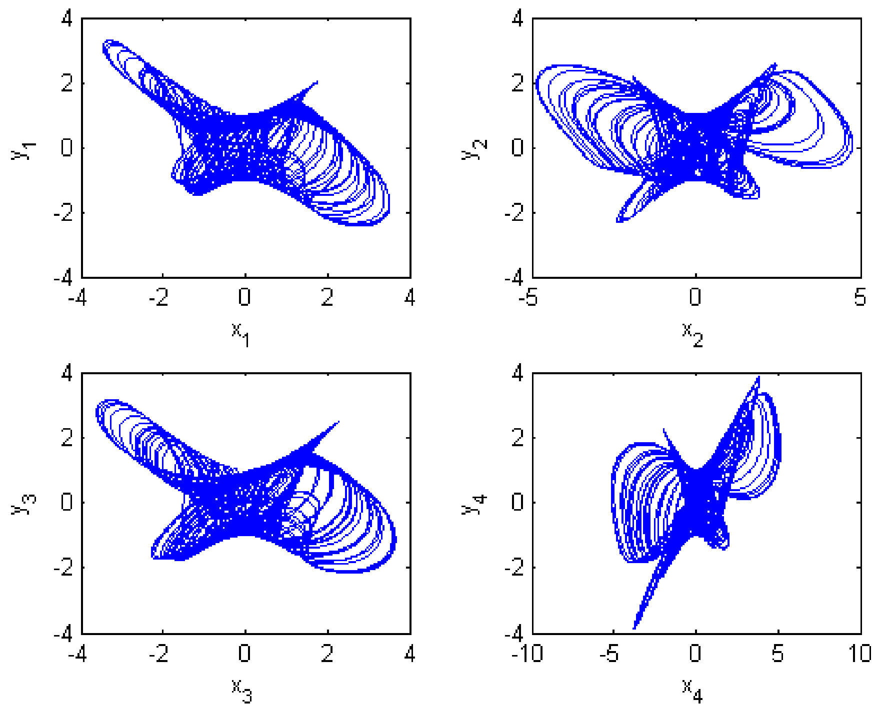

Fig. 3.5 Phase portraits of x_i to y_i ($i = 1,\cdots,4$) for Section 3.4 when the generalized synchronization is obtained.

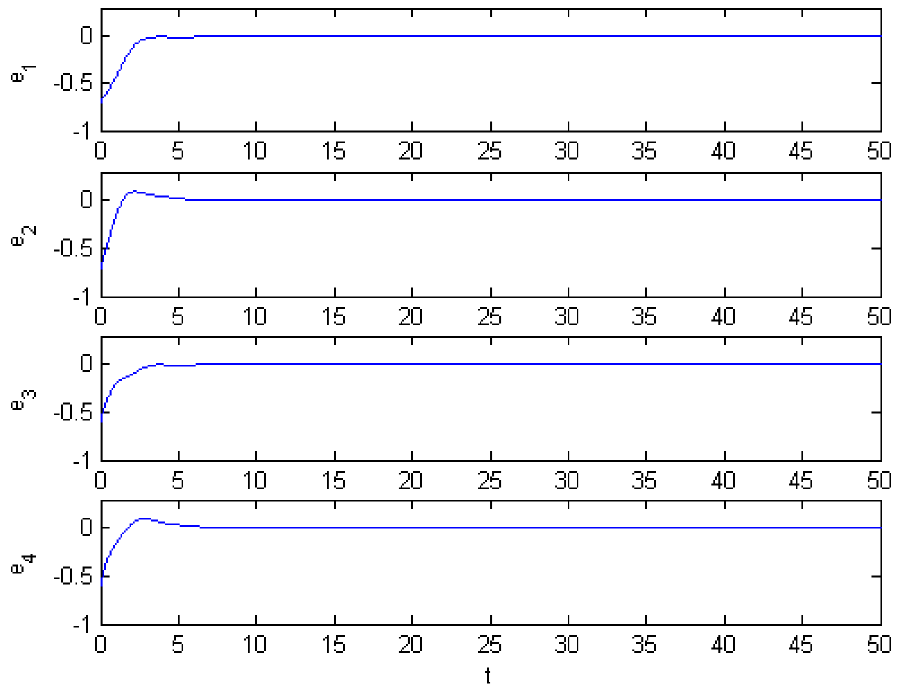

Fig. 3.6 Time histories of the state errors for Section 3.4.

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Chapter 4

Nonlinear Generalized Synchronization of New Chaotic Systems by Pure Error Dynamics and Elaborate Nondiagonal Lyapunov Function

4.1 Preliminaries

By applying pure error dynamics and elaborate nondiagonal Lyapunov function, the nonlinear generalized synchronization is studied in this Chapter. In stead of current plain square sum Lyapunov function [1-6], the elaborate nondiagonal Lyapunov function is applied in this study. A systematic method of designing Lyapunov function is proposed based on Lyapunov direct method [7], and the nonlinear generalized synchronization is achieved by applying this technique.

4.2 Design of Lyapunov Function

Consider the master and slave nonlinear dynamic systems described by

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}), \tag{4.1}$$

$$\dot{\mathbf{y}} = \mathbf{f}(t, \mathbf{y}) + \mathbf{u}(t, \mathbf{x}, \mathbf{y}), \tag{4.2}$$

where $\mathbf{x}, \mathbf{y} \in R^n$ are master and slave state vectors, $\mathbf{f} : R_+ \times R^n \to R^n$ is a nonlinear vector function, and $\mathbf{u}$: $R_+ \times R^n \times R^n \to R^n$ is controller vector.

Generalized synchronization means that there is a functional relation $\mathbf{y} = \mathbf{g}(\mathbf{x})$ between master and slave states as time goes to infinity, where $\mathbf{g} : R^n \to R^n$ is a continuously differentiable nonlinear vector function. Define $\mathbf{e} = \mathbf{y} - \mathbf{g}(\mathbf{x})$ as generalized synchronization error vector, and the error dynamics can be obtained:

$$\dot{\mathbf{e}} = \dot{\mathbf{y}} - \dot{\mathbf{g}}(\mathbf{x}) = \dot{\mathbf{y}} - \frac{d\mathbf{g}(\mathbf{x})}{d\mathbf{x}}\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{y}) - \frac{d\mathbf{g}(\mathbf{x})}{d\mathbf{x}}\mathbf{f}(t, \mathbf{x}) + \mathbf{u}(t, \mathbf{x}, \mathbf{y}). \tag{4.3}$$

Eq. (4.3) can be rewritten in the following form:

$$\dot{\mathbf{e}} = \mathbf{p}(t,\mathbf{e}) + \mathbf{q}(t,\mathbf{x},\mathbf{y}) + \mathbf{u}(t,\mathbf{x},\mathbf{y}), \tag{4.4}$$

where $\mathbf{p}: R_+ \times R^n \to R^n$ and $\mathbf{q}: R_+ \times R^n \times R^n \to R^n$ are continuous vector functions represent the error variable terms and the state variable terms in the error dynamics respectively.

In order to transform current mixed error dynamics into pure error dynamics, the controller vector is chosen as

$$\mathbf{u}(t,\mathbf{x},\mathbf{y}) = -\mathbf{q}(t,\mathbf{x},\mathbf{y}) + \mathbf{v}(t,\mathbf{e}), \tag{4.5}$$

where $\mathbf{v}: R_+ \times R^n \to R^n$ is a continuous vector function.

Now the pure error dynamics can be obtained:

$$\dot{\mathbf{e}} = \mathbf{p}(t,\mathbf{e}) + \mathbf{v}(t,\mathbf{e}), \tag{4.6}$$

Based on Lyapunov direct method [7], the scheme of nonlinear generalized synchronization and the procedure of designing elaborate nondiagonal Lyapunov function are described as follows:

Step 1. Construct a Lyapunov function

$$\begin{aligned} V(t,\mathbf{e}) &= \sum_{i=1}^{n} \frac{1}{2} \mathbf{e}_i^T \mathbf{\Lambda}_i(t) \mathbf{e}_i \\ &= [\frac{1}{2}\lambda_{11}(t)e_1^2 + \lambda_{12}e_1e_2 + \frac{1}{2}\lambda_{22}(t)e_2^2] + \cdots + [\frac{1}{2}\lambda_{nn}(t)e_n^2 + \lambda_{n1}e_ne_1 + \frac{1}{2}\lambda_{11}(t)e_1^2], \end{aligned} \tag{4.7}$$

where $\mathbf{e}_i = \begin{bmatrix} e_i \\ e_{i+1} \end{bmatrix}$ $(i = 1,2,\cdots,n-1)$, $\mathbf{e}_n = \begin{bmatrix} e_n \\ e_1 \end{bmatrix}$, $\mathbf{\Lambda}_i(t) = \begin{bmatrix} \lambda_{ii}(t) & \lambda_{ii+1} \\ \lambda_{ii+1} & \lambda_{i+1i+1}(t) \end{bmatrix}$ $(i = 1,2,\cdots,n-1)$,

$\mathbf{\Lambda}_n(t) = \begin{bmatrix} \lambda_{nn}(t) & \lambda_{n1} \\ \lambda_{n1} & \lambda_{11}(t) \end{bmatrix}$, and $\mathbf{\Lambda}_i(t) \in R^{2\times 2}$ $(i = 1,2,\cdots,n)$ are unknown continuously differentiable positive definite matrices to be designed and $\mathbf{\Lambda}_i(t)$, $\mathbf{\Lambda}_n(t)$ are nondiagonal. According to Sylvester's criterion, $\mathbf{\Lambda}_i(t)$ have to be chosen that

$$\begin{aligned} \forall t \ge 0, \quad & \lambda_{ii}(t) > 0, \lambda_{ii}(t)\lambda_{i+1i+1}(t) - \lambda_{ii+1}^2 > 0 \quad (i = 1,2,\cdots,n-1), \\ & \lambda_{nn}(t) > 0, \lambda_{nn}(t)\lambda_{11}(t) - \lambda_{n1}^2 > 0 \quad (i = n), \end{aligned} \tag{4.8}$$

and

$$\forall t \geq 0, \quad 0 < \lambda_{mii} \leq \lambda_{ii}(t) \leq \lambda_{Mii} \quad (i = 1, 2, \cdots, n), \tag{4.9}$$

where $\lambda_{mii}, \lambda_{Mii}$ are positive constants.

Step 2. The derivative of Lyapunov function is

$$\begin{aligned}
\dot{V}(t, \mathbf{e}) &= \sum_{i=1}^{n} [\dot{\mathbf{e}}_i^T \mathbf{\Lambda}_i(t) \mathbf{e}_i + \frac{1}{2} \mathbf{e}_i^T \dot{\mathbf{\Lambda}}_i(t) \mathbf{e}_i] \\
&= [\lambda_{11}(t) e_1 \dot{e}_1 + \lambda_{12} \dot{e}_1 e_2 + \lambda_{12} e_1 \dot{e}_2 + \lambda_{22}(t) e_2 \dot{e}_2 + \frac{1}{2} \dot{\lambda}_{11}(t) e_1^2 + \frac{1}{2} \dot{\lambda}_{22}(t) e_2^2] \\
&\quad + \cdots + [\lambda_{nn}(t) e_n \dot{e}_n + \lambda_{n1} \dot{e}_n e_1 + \lambda_{n1} e_n \dot{e}_1 + \lambda_{11}(t) e_1 \dot{e}_1 + \frac{1}{2} \dot{\lambda}_{nn}(t) e_n^2 + \frac{1}{2} \dot{\lambda}_{11}(t) e_1^2],
\end{aligned} \tag{4.10}$$

Eq. (4.10) can be rewritten in the following form:

$$\begin{aligned}
\dot{V}(t, \mathbf{e}) &= F_1(\dot{\lambda}_{11}, \lambda_{11}, \cdots, \lambda_{nn}, \lambda_{12}, \cdots, \lambda_{n1}, t) e_1^2 \\
&\quad + \cdots + F_n(\dot{\lambda}_{nn}, \lambda_{11}, \cdots, \lambda_{nn}, \lambda_{12}, \cdots, \lambda_{n1}, t) e_n^2 \\
&\quad + G_1(\lambda_{11}, \cdots, \lambda_{nn}, \lambda_{12}, \cdots, \lambda_{n1}, t) e_1 e_2 \\
&\quad + \cdots + G_m(\lambda_{11}, \cdots, \lambda_{nn}, \lambda_{12}, \cdots, \lambda_{n1}, t) e_{n-1} e_n \\
&\quad + (2\lambda_{11} v_1 + \lambda_{12} v_2 + \lambda_{n1} v_n) e_1 \\
&\quad + \cdots + (2\lambda_{nn} v_n + \lambda_{n1} v_1 + \lambda_{n-1n} v_{n-1}) e_n,
\end{aligned} \tag{4.11}$$

where $F_i(\dot{\lambda}_{ii}, \lambda_{11}, \cdots, \lambda_{nn}, \lambda_{12}, \cdots, \lambda_{n1}, t) \quad (i = 1, 2, \cdots, n)$, $G_j(\lambda_{11}, \cdots, \lambda_{nn}, \lambda_{12}, \cdots, \lambda_{n1}, t)$ $(j = 1, 2, \cdots, m, \quad m = \dfrac{n(n-1)}{2})$ are continuous differentiable functions, and $v_i \quad (i = 1, 2, \cdots, n)$ are controllers to be determined.

Step 3. Appropriately design the controllers v_i such that Eq. (4.11) can be reduced to

$$\begin{aligned}
\dot{V}(t, \mathbf{e}) &= \hat{F}_1(\dot{\lambda}_{11}, \lambda_{11}, \cdots, \lambda_{nn}, \lambda_{12}, \cdots, \lambda_{n1}, t) e_1^2 \\
&\quad + \cdots + \hat{F}_n(\dot{\lambda}_{nn}, \lambda_{11}, \cdots, \lambda_{nn}, \lambda_{12}, \cdots, \lambda_{n1}, t) e_n^2 \\
&\quad + \hat{G}_1(\lambda_{11}, \cdots, \lambda_{nn}, \lambda_{12}, \cdots, \lambda_{n1}, t) e_1 e_2 \\
&\quad + \cdots + \hat{G}_m(\lambda_{11}, \cdots, \lambda_{nn}, \lambda_{12}, \cdots, \lambda_{n1}, t) e_{n-1} e_n,
\end{aligned} \tag{4.12}$$

where $\hat{F}_i(\dot{\lambda}_{ii}, \lambda_{11}, \cdots, \lambda_{nn}, \lambda_{12}, \cdots, \lambda_{n1}, t) \quad (i = 1, 2, \cdots, n)$ and $\hat{G}_j(\lambda_{11}, \cdots, \lambda_{nn}, \lambda_{12}, \cdots, \lambda_{n1}, t)$ $(j = 1, 2, \cdots, m, \quad m = \dfrac{n(n-1)}{2})$ are continuous differentiable functions.

Step 4. Assume

$$\forall j,\quad \hat{G}_j(\lambda_{11},\cdots,\lambda_{nn},\lambda_{12},\cdots,\lambda_{n1},t)=0\,, \tag{4.13}$$

then the relationship between λ_{ij} can be obtained.

Step 5. Use the results of Step 4 to check if

$$\forall t\geq 0,\quad \hat{F}_i(\dot{\lambda}_{ii},\lambda_{11},\cdots,\lambda_{nn},\lambda_{12},\cdots,\lambda_{n1},t)<0\quad (i=1,2,\cdots,n)\,. \tag{4.14}$$

Step 6. If Eq. (4.14) can be satisfied, the conditions derived from Eq. (4.14) can be obtained. If Eq. (4.14) can not be satisfied, i.e.

$$\begin{aligned}\forall t\geq 0,\quad &\hat{F}_j(\dot{\lambda}_{jj},\lambda_{11},\cdots,\lambda_{nn},\lambda_{12},\cdots,\lambda_{n1},t)\geq 0,\\ &\hat{F}_k(\dot{\lambda}_{kk},\lambda_{11},\cdots,\lambda_{nn},\lambda_{12},\cdots,\lambda_{n1},t)<0,\end{aligned} \tag{4.15}$$

return to Step 3 and modify the controllers v_j by addition of $k_j e_j$, where k_j are constant gains to be determined. Repeat Step 4 and Step 5, then the conditions guarantee the validity of Eq. (4.14) can be assured.

Step 7. Appropriately design k_j and $\lambda_{ij}(t)$ such that each condition derived from the above procedure holds. Finally the elaborate nondiagonal Lyapunov function can be obtained and the generalized synchronization is achieved according to Lyapunov direct method.

4.3 Example for New Autonomous Chaotic Systems

In the following two Sections, the nonlinear functional relation between master and slave states is $y_i = g_i(x_i) = \alpha x_i^2 + \beta x_i + \gamma \quad (i=1,2,\cdots,n)$. The master and slave new autonomous chaotic systems can be described by Eq.(3.15) and Eq. (3.16), respectively. The parameters and the initial conditions of the master and slave systems are the same as shown in Section 3.3.

Let $e_i = y_i - \alpha x_i^2 - \beta x_i - \gamma \quad (i=1,\cdots,4)$, then the error dynamics can be obtained:

$$\dot{\mathbf{e}}=\mathbf{p}(\mathbf{e})+\mathbf{q}(\mathbf{x},\mathbf{y})+\mathbf{u}(\mathbf{x},\mathbf{y})\,, \tag{4.16}$$

where

$$
\begin{aligned}
&\mathbf{p}(\mathbf{e}) = \begin{bmatrix} p_1(\mathbf{e}) & p_2(\mathbf{e}) & p_3(\mathbf{e}) & p_4(\mathbf{e}) \end{bmatrix}^T, \\
&\mathbf{q}(\mathbf{x},\mathbf{y}) = \begin{bmatrix} q_1(\mathbf{x},\mathbf{y}) & q_2(\mathbf{x},\mathbf{y}) & q_3(\mathbf{x},\mathbf{y}) & q_4(\mathbf{x},\mathbf{y}) \end{bmatrix}^T, \\
&p_1(\mathbf{e}) = e_2, \quad p_2(\mathbf{e}) = -ae_1 - ae_2 + be_3, \quad p_3(\mathbf{e}) = e_4, \quad p_4(\mathbf{e}) = -e_3 - ae_4 + be_1, \\
&q_1(\mathbf{x},\mathbf{y}) = \alpha x_2^2 - 2\alpha x_1 x_2 + \gamma, \\
&q_2(\mathbf{x},\mathbf{y}) = -\alpha(ax_1^2 - ax_2^2 - bx_3^2) - a(y_4 y_1 - \beta x_4 x_1) + (b-2a)\gamma \\
&\qquad -[(1+y_4)y_1^3 - \beta(1+x_4)x_1^3] + 2\alpha x_2[a(1+x_4)x_1 + (1+x_4)x_1^3 - bx_3], \\
&q_3(\mathbf{x},\mathbf{y}) = \alpha x_4^2 - 2\alpha x_3 x_4 + \gamma, \\
&q_4(\mathbf{x},\mathbf{y}) = -\alpha(x_3^2 - ax_4^2 - bx_1^2) - (y_2 y_3 - \beta x_2 x_3) + (b-a-1)\gamma \\
&\qquad - a[(1+y_2)y_3^3 - \beta(1+x_2)x_3^3] + 2\alpha x_4[(1+x_2)x_3 + a(1+x_2)x_3^3 - bx_1].
\end{aligned}
\tag{4.17}
$$

In order to transform current mixed error dynamics into pure error dynamics, the controller vector is chosen as

$$\mathbf{u}(\mathbf{x},\mathbf{y}) = -\mathbf{q}(\mathbf{x},\mathbf{y}) + \mathbf{v}(\mathbf{e}). \tag{4.18}$$

Now the pure error dynamics can be obtained:

$$\dot{\mathbf{e}} = \mathbf{p}(\mathbf{e}) + \mathbf{v}(\mathbf{e}). \tag{4.19}$$

Step 1. Construct a Lyapunov function

$$
\begin{aligned}
V(\mathbf{e}) &= \sum_{i=1}^{4} \frac{1}{2} \mathbf{e}_i^T \mathbf{\Lambda}_i \mathbf{e}_i \\
&= [\frac{1}{2}\lambda_{11}e_1^2 + \lambda_{12}e_1e_2 + \frac{1}{2}\lambda_{22}e_2^2] + \cdots + [\frac{1}{2}\lambda_{44}e_4^2 + \lambda_{41}e_4e_1 + \frac{1}{2}\lambda_{11}e_1^2],
\end{aligned}
\tag{4.20}
$$

where $\mathbf{e}_i = \begin{bmatrix} e_i \\ e_{i+1} \end{bmatrix}$ $(i = 1,\cdots,3)$, $\mathbf{e}_4 = \begin{bmatrix} e_4 \\ e_1 \end{bmatrix}$, $\mathbf{\Lambda}_i = \begin{bmatrix} \lambda_{ii} & \lambda_{ii+1} \\ \lambda_{ii+1} & \lambda_{i+1i+1} \end{bmatrix}$ $(i = 1,\cdots,3)$, $\mathbf{\Lambda}_4 = \begin{bmatrix} \lambda_{44} & \lambda_{41} \\ \lambda_{41} & \lambda_{11} \end{bmatrix}$, and $\mathbf{\Lambda}_i$ $(i = 1,\cdots,4)$ are unknown continuously differentiable positive definite nondiagonal matrices to be designed. According to Sylvester's criterion, $\mathbf{\Lambda}_i$ have to be chosen that

$$
\begin{aligned}
&\lambda_{11} > 0, \lambda_{11}\lambda_{22} - \lambda_{12}^2 > 0, \\
&\lambda_{22} > 0, \lambda_{22}\lambda_{33} - \lambda_{23}^2 > 0, \\
&\lambda_{33} > 0, \lambda_{33}\lambda_{44} - \lambda_{34}^2 > 0, \\
&\lambda_{44} > 0, \lambda_{44}\lambda_{11} - \lambda_{41}^2 > 0.
\end{aligned}
\tag{4.21}
$$

Step 2. The derivative of Lyapunov function is

$$\begin{aligned}\dot{V}(\mathbf{e}) &= \sum_{i=1}^{4} \dot{\mathbf{e}}_i^T \boldsymbol{\Lambda}_i \mathbf{e}_i \\ &= [\lambda_{11} e_1 \dot{e}_1 + \lambda_{12} \dot{e}_1 e_2 + \lambda_{12} e_1 \dot{e}_2 + \lambda_{22} e_2 \dot{e}_2] \\ &\quad + \cdots + [\lambda_{44} e_4 \dot{e}_4 + \lambda_{41} \dot{e}_4 e_1 + \lambda_{41} e_4 \dot{e}_1 + \lambda_{11} e_1 \dot{e}_1].\end{aligned} \tag{4.22}$$

Eq. (4.22) can be rewritten in the following form:

$$\begin{aligned}\dot{V}(\mathbf{e}) &= F_1(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41})e_1^2 + F_2(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41})e_2^2 \\ &\quad + F_3(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41})e_3^2 + F_4(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41})e_4^2 \\ &\quad + G_1(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41})e_1e_2 + G_2(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41})e_1e_3 \\ &\quad + G_3(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41})e_1e_4 + G_4(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41})e_2e_3 \\ &\quad + G_5(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41})e_2e_4 + G_6(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41})e_3e_4 \\ &\quad + (2\lambda_{11}v_1 + \lambda_{12}v_2 + \lambda_{41}v_4)e_1 + (2\lambda_{22}v_2 + \lambda_{23}v_3 + \lambda_{12}v_1)e_2 \\ &\quad + (2\lambda_{33}v_3 + \lambda_{34}v_4 + \lambda_{23}v_2)e_3 + (2\lambda_{44}v_4 + \lambda_{41}v_1 + \lambda_{34}v_3)e_4,\end{aligned} \tag{4.23}$$

where

$$\begin{aligned}&F_1(\lambda_{11},\cdots,\lambda_{41}) = -a\lambda_{12} + b\lambda_{41}, \quad F_2(\lambda_{11},\cdots,\lambda_{41}) = \lambda_{12} - 2a\lambda_{22}, \\ &F_3(\lambda_{11},\cdots,\lambda_{41}) = b\lambda_{23} - \lambda_{34}, \quad F_4(\lambda_{11},\cdots,\lambda_{41}) = \lambda_{34} - 2a\lambda_{44}, \\ &G_1(\lambda_{11},\cdots,\lambda_{41}) = 2\lambda_{11} - a\lambda_{12} - 2a\lambda_{22}, \quad G_2(\lambda_{11},\cdots,\lambda_{41}) = b\lambda_{12} - a\lambda_{23} + b\lambda_{34} - \lambda_{41}, \\ &G_3(\lambda_{11},\cdots,\lambda_{41}) = 2b\lambda_{44} - a\lambda_{41}, \quad G_4(\lambda_{11},\cdots,\lambda_{41}) = 2b\lambda_{22} - a\lambda_{23}, \\ &G_5(\lambda_{11},\cdots,\lambda_{41}) = \lambda_{23} + \lambda_{41}, \quad G_6(\lambda_{11},\cdots,\lambda_{41}) = 2\lambda_{33} - a\lambda_{34} - 2\lambda_{44}.\end{aligned} \tag{4.24}$$

Step 3. Design the controllers

$$v_1 = -e_2, \quad v_2 = ae_1, \quad v_3 = -e_4, \quad v_4 = e_3, \tag{4.25}$$

such that Eq. (4.23) can be reduced to

$$\begin{aligned}\dot{V}(\mathbf{e}) &= \hat{F}_1(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41})e_1^2 + \hat{F}_2(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41})e_2^2 \\ &\quad + \hat{F}_3(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41})e_3^2 + \hat{F}_4(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41})e_4^2 \\ &\quad + \hat{G}_1(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41})e_1e_2 + \hat{G}_2(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41})e_1e_3 \\ &\quad + \hat{G}_3(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41})e_1e_4 + \hat{G}_4(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41})e_2e_3 \\ &\quad + \hat{G}_5(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41})e_2e_4 + \hat{G}_6(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41})e_3e_4,\end{aligned} \tag{4.26}$$

where

$$\begin{aligned}
&\hat{F}_1(\lambda_{11},\cdots,\lambda_{41}) = b\lambda_{41}, \quad \hat{F}_2(\lambda_{11},\cdots,\lambda_{41}) = -2a\lambda_{22}, \\
&\hat{F}_3(\lambda_{11},\cdots,\lambda_{41}) = b\lambda_{23}, \quad \hat{F}_4(\lambda_{11},\cdots,\lambda_{41}) = -2a\lambda_{44}, \\
&\hat{G}_1(\lambda_{11},\cdots,\lambda_{41}) = -a\lambda_{12}, \quad \hat{G}_2(\lambda_{11},\cdots,\lambda_{41}) = b\lambda_{12} + b\lambda_{34}, \\
&\hat{G}_3(\lambda_{11},\cdots,\lambda_{41}) = 2b\lambda_{44} - a\lambda_{41}, \quad \hat{G}_4(\lambda_{11},\cdots,\lambda_{41}) = 2b\lambda_{22} - a\lambda_{23}, \\
&\hat{G}_5(\lambda_{11},\cdots,\lambda_{41}) = 0, \quad \hat{G}_6(\lambda_{11},\cdots,\lambda_{41}) = -a\lambda_{34}.
\end{aligned} \tag{4.27}$$

Step 4. Assume

$$\forall j, \quad \hat{G}_j(\lambda_{11},\cdots,\lambda_{41}) = 0, \tag{4.28}$$

then the relationship between λ_{ij} can be obtained:

$$\lambda_{12} = 0, \quad \lambda_{23} = \frac{b}{2a}\lambda_{22}, \quad \lambda_{34} = 0, \quad \lambda_{41} = \frac{b}{2a}\lambda_{44}. \tag{4.29}$$

Step 5. Use the results of Step 4 to check if

$$\hat{F}_i(\lambda_{11},\cdots,\lambda_{41}) < 0 \quad (i = 1,\cdots,4). \tag{4.30}$$

It can be obtained that

$$\begin{aligned}
&\hat{F}_1(\lambda_{11},\cdots,\lambda_{41}) = b\lambda_{41} > 0, \quad \hat{F}_2(\lambda_{11},\cdots,\lambda_{41}) = -2a\lambda_{22} < 0, \\
&\hat{F}_3(\lambda_{11},\cdots,\lambda_{41}) = b\lambda_{23} > 0, \quad \hat{F}_4(\lambda_{11},\cdots,\lambda_{41}) = -2a\lambda_{44} < 0.
\end{aligned} \tag{4.31}$$

Step 6. Since Eq. (4.30) is not satisfied, i.e.

$$\begin{aligned}
&\hat{F}_j(\lambda_{11},\cdots,\lambda_{41}) \geq 0 \quad (j = 1,3), \\
&\hat{F}_k(\lambda_{11},\cdots,\lambda_{41}) < 0 \quad (k = 2,4),
\end{aligned} \tag{4.32}$$

return to Step 3 and modify the controllers v_1 and v_3 by addition of k_1e_1 and k_3e_3 respectively, where k_1 and k_3 are constant gains to be determined. Because $\dot{V}$ has been modified, Eq. (4.27) becomes

$$
\begin{aligned}
&\hat{F}_1(\lambda_{11},\cdots,\lambda_{41}) = b\lambda_{41} + 2k_1\lambda_{11}, \quad \hat{F}_2(\lambda_{11},\cdots,\lambda_{41}) = -2a\lambda_{22}, \\
&\hat{F}_3(\lambda_{11},\cdots,\lambda_{41}) = b\lambda_{23} + 2k_3\lambda_{33}, \quad \hat{F}_4(\lambda_{11},\cdots,\lambda_{41}) = -2a\lambda_{44}, \\
&\hat{G}_1(\lambda_{11},\cdots,\lambda_{41}) = (k_1 - a)\lambda_{12}, \quad \hat{G}_2(\lambda_{11},\cdots,\lambda_{41}) = b\lambda_{12} + b\lambda_{34}, \\
&\hat{G}_3(\lambda_{11},\cdots,\lambda_{41}) = 2b\lambda_{44} + (k_1 - a)\lambda_{41}, \quad \hat{G}_4(\lambda_{11},\cdots,\lambda_{41}) = 2b\lambda_{22} + (k_3 - a)\lambda_{23}, \\
&\hat{G}_5(\lambda_{11},\cdots,\lambda_{41}) = 0, \quad \hat{G}_6(\lambda_{11},\cdots,\lambda_{41}) = (k_3 - a)\lambda_{34}.
\end{aligned} \tag{4.33}
$$

Repeat Step 4 and Step 5, then the relationship between λ_{ij} becomes

$$
\lambda_{12} = 0, \quad \lambda_{22} = \frac{a - k_3}{2b}\lambda_{23}, \quad \lambda_{34} = 0, \quad \lambda_{44} = \frac{a - k_1}{2b}\lambda_{41}, \tag{4.34}
$$

and Eq. (4.30) can be satisfied if

$$
\lambda_{41} < \frac{-2k_1}{b}\lambda_{11}, \quad \lambda_{23} < \frac{-2k_3}{b}\lambda_{33}. \tag{4.35}
$$

Step 7. The conditions derived from the above procedure can be summed up as follows:

$$
\lambda_{12} = 0, \lambda_{34} = 0, \tag{4.36}
$$

$$
\lambda_{11} > 0, \lambda_{44} > 0, \lambda_{44}\lambda_{11} - \lambda_{41}^2 > 0, \lambda_{44} = \frac{a - k_1}{2b}\lambda_{41}, \lambda_{41} < \frac{-2k_1}{b}\lambda_{11}, \tag{4.37}
$$

$$
\lambda_{22} > 0, \lambda_{33} > 0, \lambda_{22}\lambda_{33} - \lambda_{23}^2 > 0, \lambda_{22} = \frac{a - k_3}{2b}\lambda_{23}, \lambda_{23} < \frac{-2k_3}{b}\lambda_{33}. \tag{4.38}
$$

Design

$$
\begin{aligned}
&k_1 = -a, \quad k_3 = -a, \\
&\lambda_{11} = b, \lambda_{22} = \frac{a^2}{2b}, \lambda_{33} = b, \lambda_{44} = \frac{a^2}{2b}, \\
&\lambda_{12} = 0, \lambda_{23} = \frac{a}{2}, \lambda_{34} = 0, \lambda_{41} = \frac{a}{2},
\end{aligned} \tag{4.39}
$$

such that each condition holds. Then the elaborate nondiagonal Lyapunov function can be obtained

$$
V(\mathbf{e}) = \frac{a^2}{2b}e_2^2 + \frac{a}{2}e_2e_3 + be_3^2 + \frac{a^2}{2b}e_4^2 + \frac{a}{2}e_4e_1 + be_1^2, \tag{4.40}
$$

and

$$
\dot{V}(\mathbf{e}) = -\frac{3ab}{2}e_1^2 - \frac{a^3}{b}e_2^2 - \frac{3ab}{2}e_3^2 - \frac{a^3}{b}e_4^2. \tag{4.41}
$$

Since Lyapunov global asymptotical stability theorem is satisfied, the global generalized synchronization is achieved. $\alpha = 1$, $\beta = 2$, $\gamma = 3$ are chosen in simulation, and the results are shown in Fig. 4.1-4.2.

4.4 Example for New Nonautonomous Chaotic Systems

The master and slave new nonautonomous chaotic systems can be described by Eq. (3.31) and Eq. (3.32), respectively. The parameters and the initial conditions of the master and slave systems are the same as shown in Section 3.4.

Let $e_i = y_i - \alpha x_i^2 - \beta x_i - \gamma \quad (i = 1, \cdots, 4)$, then the error dynamics can be obtained:

$$\dot{\mathbf{e}} = \mathbf{p}(t, \mathbf{e}) + \mathbf{q}(t, \mathbf{x}, \mathbf{y}) + \mathbf{u}(t, \mathbf{x}, \mathbf{y}), \tag{4.42}$$

where

$$\begin{aligned}
&\mathbf{p}(t,\mathbf{e}) = \begin{bmatrix} p_1(t,\mathbf{e}) & p_2(t,\mathbf{e}) & p_3(t,\mathbf{e}) & p_4(t,\mathbf{e}) \end{bmatrix}^T, \\
&\mathbf{q}(t,\mathbf{x},\mathbf{y}) = \begin{bmatrix} q_1(t,\mathbf{x},\mathbf{y}) & q_2(t,\mathbf{x},\mathbf{y}) & q_3(t,\mathbf{x},\mathbf{y}) & q_4(t,\mathbf{x},\mathbf{y}) \end{bmatrix}^T, \\
&p_1(t,\mathbf{e}) = e_2, \quad p_2(t,\mathbf{e}) = -a(1+\sin\omega t)e_1 - ae_2 + be_3, \\
&p_3(t,\mathbf{e}) = e_4, \quad p_4(t,\mathbf{e}) = -(1+\sin\omega t)e_3 - ae_4 + be_1, \\
&q_1(t,\mathbf{x},\mathbf{y}) = \alpha x_2^2 - 2\alpha x_1 x_2 + \gamma, \\
&q_2(t,\mathbf{x},\mathbf{y}) = -\alpha[a(1+\sin\omega t)x_1^2 - ax_2^2 - bx_3^2] + 2\alpha(1+\sin\omega t)x_1^3 x_2 \\
&\qquad\qquad + 2\alpha[a(1+\sin\omega t)x_1x_2 - bx_2x_3] - (1+\sin\omega t)(y_1^3 - \beta x_1^3) \\
&\qquad\qquad - \gamma[a(1+\sin\omega t) + a - b], \\
&q_3(t,\mathbf{x},\mathbf{y}) = \alpha x_4^2 - 2\alpha x_3 x_4 + \gamma, \\
&q_4(t,\mathbf{x},\mathbf{y}) = -\alpha[(1+\sin\omega t)x_3^2 - ax_4^2 - bx_1^2] + 2\alpha a(1+\sin\omega t)x_3^3 x_4 \\
&\qquad\qquad + 2\alpha[(1+\sin\omega t)x_3x_4 - bx_1x_4] - a(1+\sin\omega t)(y_3^3 - \beta x_3^3) \\
&\qquad\qquad - \gamma[1+\sin\omega t + a - b].
\end{aligned} \tag{4.43}$$

In order to transform current mixed error dynamics into pure error dynamics, the controller vector is chosen as

$$\mathbf{u}(t,\mathbf{x},\mathbf{y}) = -\mathbf{q}(t,\mathbf{x},\mathbf{y}) + \mathbf{v}(\mathbf{e}). \tag{4.44}$$

Now the pure error dynamics can be obtained:

$$\dot{\mathbf{e}} = \mathbf{p}(t,\mathbf{e}) + \mathbf{v}(\mathbf{e}) . \tag{4.45}$$

Step 1. Construct a Lyapunov function

$$\begin{aligned} V(t,\mathbf{e}) &= \sum_{i=1}^{4} \frac{1}{2} \mathbf{e}_i^T \mathbf{\Lambda}_i(t) \mathbf{e}_i \\ &= [\frac{1}{2}\lambda_{11}(t)e_1^2 + \lambda_{12}e_1e_2 + \frac{1}{2}\lambda_{22}(t)e_2^2] + \cdots + [\frac{1}{2}\lambda_{44}(t)e_4^2 + \lambda_{41}e_4e_1 + \frac{1}{2}\lambda_{11}(t)e_1^2], \end{aligned} \tag{4.46}$$

where $\mathbf{e}_i = \begin{bmatrix} e_i \\ e_{i+1} \end{bmatrix}$ $(i = 1,\cdots,3)$, $\mathbf{e}_4 = \begin{bmatrix} e_4 \\ e_1 \end{bmatrix}$, $\mathbf{\Lambda}_i(t) = \begin{bmatrix} \lambda_{ii}(t) & \lambda_{ii+1} \\ \lambda_{ii+1} & \lambda_{i+1i+1}(t) \end{bmatrix}$ $(i = 1,\cdots,3)$, $\mathbf{\Lambda}_4(t) = \begin{bmatrix} \lambda_{44}(t) & \lambda_{41} \\ \lambda_{41} & \lambda_{11}(t) \end{bmatrix}$, and $\mathbf{\Lambda}_i(t)$ $(i = 1,\cdots,4)$ are unknown continuously differentiable positive definite nondiagonal matrices to be designed. According to Sylvester's criterion, $\mathbf{\Lambda}_i(t)$ have to be chosen that

$$\begin{aligned} &\lambda_{11}(t) > 0, \lambda_{11}(t)\lambda_{22}(t) - \lambda_{12}^2 > 0, \\ &\lambda_{22}(t) > 0, \lambda_{22}(t)\lambda_{33}(t) - \lambda_{23}^2 > 0, \\ &\lambda_{33}(t) > 0, \lambda_{33}(t)\lambda_{44}(t) - \lambda_{34}^2 > 0, \\ &\lambda_{44}(t) > 0, \lambda_{44}(t)\lambda_{11}(t) - \lambda_{41}^2 > 0, \end{aligned} \tag{4.47}$$

and

$$\begin{aligned} &0 < \lambda_{m11} \le \lambda_{11}(t) \le \lambda_{M11}, \\ &0 < \lambda_{m22} \le \lambda_{22}(t) \le \lambda_{M22}, \\ &0 < \lambda_{m33} \le \lambda_{33}(t) \le \lambda_{M33}, \\ &0 < \lambda_{m44} \le \lambda_{44}(t) \le \lambda_{M44}, \end{aligned} \tag{4.48}$$

where $\lambda_{mii}, \lambda_{Mii}$ $(i = 1,\cdots,4)$ are positive constants.

Step 2. The derivative of Lyapunov function is

$$\begin{aligned} \dot{V}(t,\mathbf{e}) &= \sum_{i=1}^{4} \dot{\mathbf{e}}_i^T \mathbf{\Lambda}_i(t) \mathbf{e}_i \\ &= [\lambda_{11}(t)e_1\dot{e}_1 + \lambda_{12}\dot{e}_1e_2 + \lambda_{12}e_1\dot{e}_2 + \lambda_{22}(t)e_2\dot{e}_2 + \frac{1}{2}\dot{\lambda}_{11}(t)e_1^2 + \frac{1}{2}\dot{\lambda}_{22}(t)e_2^2] \\ &\quad + \cdots + [\lambda_{44}(t)e_4\dot{e}_4 + \lambda_{41}\dot{e}_4e_1 + \lambda_{41}e_4\dot{e}_1 + \lambda_{11}(t)e_1\dot{e}_1 + \frac{1}{2}\dot{\lambda}_{44}(t)e_4^2 + \frac{1}{2}\dot{\lambda}_{11}(t)e_1^2]. \end{aligned} \tag{4.49}$$

Eq. (4.49) can be rewritten in the following form:

$$
\begin{aligned}
\dot{V}(t,\mathbf{e}) = {} & F_1(\dot{\lambda}_{11},\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41},t)e_1^2 + F_2(\dot{\lambda}_{22},\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41},t)e_2^2 \\
& + F_3(\dot{\lambda}_{33},\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41},t)e_3^2 + F_4(\dot{\lambda}_{44},\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41},t)e_4^2 \\
& + G_1(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41},t)e_1e_2 + G_2(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41},t)e_1e_3 \\
& + G_3(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41},t)e_1e_4 + G_4(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41},t)e_2e_3 \\
& + G_5(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41},t)e_2e_4 + G_6(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41},t)e_3e_4 \\
& + (2\lambda_{11}v_1 + \lambda_{12}v_2 + \lambda_{41}v_4)e_1 + (2\lambda_{22}v_2 + \lambda_{23}v_3 + \lambda_{12}v_1)e_2 \\
& + (2\lambda_{33}v_3 + \lambda_{34}v_4 + \lambda_{23}v_2)e_3 + (2\lambda_{44}v_4 + \lambda_{41}v_1 + \lambda_{34}v_3)e_4,
\end{aligned} \tag{4.50}
$$

where

$$
\begin{aligned}
& F_1(\dot{\lambda}_{11},\cdots,t) = \dot{\lambda}_{11} - a(1+\sin\omega t)\lambda_{12} + b\lambda_{41}, \quad F_2(\dot{\lambda}_{22},\cdots,t) = \dot{\lambda}_{22} - 2a\lambda_{22} + \lambda_{12}, \\
& F_3(\dot{\lambda}_{33},\cdots,t) = \dot{\lambda}_{33} + b\lambda_{23} - (1+\sin\omega t)\lambda_{34}, \quad F_4(\dot{\lambda}_{44},\cdots,t) = \dot{\lambda}_{44} - 2a\lambda_{44} + \lambda_{34}, \\
& G_1(\lambda_{11},\cdots,t) = 2\lambda_{11} - a\lambda_{12} - 2a(1+\sin\omega t)\lambda_{22}, \\
& G_2(\lambda_{11},\cdots,t) = b\lambda_{12} - a(1+\sin\omega t)\lambda_{23} + b\lambda_{34} - (1+\sin\omega t)\lambda_{41}, \\
& G_3(\lambda_{11},\cdots,t) = 2b\lambda_{44} - a\lambda_{41}, \quad G_4(\lambda_{11},\cdots,t) = 2b\lambda_{22} - a\lambda_{23}, \\
& G_5(\lambda_{11},\cdots,t) = \lambda_{23} + \lambda_{41}, \quad G_6(\lambda_{11},\cdots,t) = 2\lambda_{33} - a\lambda_{34} - 2(1+\sin\omega t)\lambda_{44}.
\end{aligned} \tag{4.51}
$$

Step 3. Design the controllers

$$
v_1 = ae_1, \quad v_2 = -be_3 - ae_1, \quad v_3 = ae_3, \quad v_4 = -be_1 - e_3, \tag{4.52}
$$

such that Eq. (4.50) can be reduced to

$$
\begin{aligned}
\dot{V}(t,\mathbf{e}) = {} & \hat{F}_1(\dot{\lambda}_{11},\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41},t)e_1^2 + \hat{F}_2(\dot{\lambda}_{22},\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41},t)e_2^2 \\
& + \hat{F}_3(\dot{\lambda}_{33},\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41},t)e_3^2 + \hat{F}_4(\dot{\lambda}_{44},\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41},t)e_4^2 \\
& + \hat{G}_1(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41},t)e_1e_2 + \hat{G}_2(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41},t)e_1e_3 \\
& + \hat{G}_3(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41},t)e_1e_4 + \hat{G}_4(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41},t)e_2e_3 \\
& + \hat{G}_5(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41},t)e_2e_4 + \hat{G}_6(\lambda_{11},\cdots,\lambda_{44},\lambda_{12},\cdots,\lambda_{41},t)e_3e_4,
\end{aligned} \tag{4.53}
$$

where

$$
\begin{aligned}
& \hat{F}_1(\dot{\lambda}_{11},\cdots,t) = \dot{\lambda}_{11} + 2a\lambda_{11} - a(2+\sin\omega t)\lambda_{12}, \quad \hat{F}_2(\dot{\lambda}_{22},\cdots,t) = \dot{\lambda}_{22} - 2a\lambda_{22} + \lambda_{12}, \\
& \hat{F}_3(\dot{\lambda}_{33},\cdots,t) = \dot{\lambda}_{33} + 2a\lambda_{33} - (2+\sin\omega t)\lambda_{34}, \quad \hat{F}_4(\dot{\lambda}_{44},\cdots,t) = \dot{\lambda}_{44} - 2a\lambda_{44} + \lambda_{34}, \\
& \hat{G}_1(\lambda_{11},\cdots,t) = 2\lambda_{11} - 2a(1+\sin\omega t)\lambda_{22}, \\
& \hat{G}_2(\lambda_{11},\cdots,t) = -a(2+\sin\omega t)\lambda_{23} - (2+\sin\omega t)\lambda_{41}, \\
& \hat{G}_3(\lambda_{11},\cdots,t) = 0, \quad \hat{G}_4(\lambda_{11},\cdots,t) = 0, \\
& \hat{G}_5(\lambda_{11},\cdots,t) = \lambda_{23} + \lambda_{41}, \quad \hat{G}_6(\lambda_{11},\cdots,t) = 2\lambda_{33} - 2(2+\sin\omega t)\lambda_{44}.
\end{aligned} \tag{4.54}
$$

Step 4. Assume

$$\forall j, \quad \hat{G}_j(\lambda_{11},\cdots,t)=0, \tag{4.55}$$

then the relationship between λ_{ij} can be obtained:

$$\lambda_{11}=a(2+\sin\omega t)\lambda_{22}, \quad \lambda_{23}=0, \quad \lambda_{33}=(2+\sin\omega t)\lambda_{44}, \quad \lambda_{41}=0. \tag{4.56}$$

Step 5. Use the results of Step 4 to check if

$$\forall t\geq 0, \quad \hat{F}_i(\dot{\lambda}_{ii},\cdots,t)<0 \quad (i=1,\cdots,4). \tag{4.57}$$

Assume

$$\lambda_{11}=c_1, \quad \lambda_{22}=\frac{c_1}{a(2+\sin\omega t)}, \quad \lambda_{33}=c_2, \quad \lambda_{44}=\frac{c_2}{2+\sin\omega t}, \tag{4.58}$$

where c_1 and c_2 are positive constants to be designed. Eq. (4.57) can be satisfied if the following conditions hold:

$$\begin{aligned}
&\hat{F}_1(\dot{\lambda}_{11},\cdots,t)=2ac_1-a(2+\sin\omega t)\lambda_{12}<0,\\
&\hat{F}_2(\dot{\lambda}_{22},\cdots,t)=\frac{-2c_1}{2+\sin\omega t}-\frac{c_1\omega\cos\omega t}{a(2+\sin\omega t)^2}+\lambda_{12}<0,
\end{aligned} \tag{4.59}$$

$$\begin{aligned}
&\hat{F}_3(\dot{\lambda}_{33},\cdots,t)=2ac_2-(2+\sin\omega t)\lambda_{34}<0,\\
&\hat{F}_4(\dot{\lambda}_{44},\cdots,t)=\frac{-2ac_2}{2+\sin\omega t}-\frac{c_2\omega\cos\omega t}{(2+\sin\omega t)^2}+\lambda_{34}<0.
\end{aligned} \tag{4.60}$$

However, both results of Eq. (4.59) and Eq. (4.60) show the contradiction: $\hat{F}_1<0$ and $\hat{F}_2<0$ can not hold in the same time, neither can $\hat{F}_3<0$ and $\hat{F}_4<0$. To simplify the following work, assume only $\hat{F}_2<0$ and $\hat{F}_4<0$ can hold.

Step 6. Since Eq. (4.57) is not satisfied, i.e.

$$\begin{aligned}
\forall t\geq 0, \quad &\hat{F}_j(\lambda_{11},\cdots,\lambda_{41})\geq 0 \quad (j=1,3),\\
&\hat{F}_k(\lambda_{11},\cdots,\lambda_{41})<0 \quad (k=2,4),
\end{aligned} \tag{4.61}$$

return to Step 3 and modify the controllers v_1 and v_3 by addition of k_1e_1 and k_3e_3 respectively, where k_1 and k_3 are constant gains to be determined. Because $\dot{V}$ has been modified, Eq. (4.54) becomes

$$\begin{aligned}
&\hat{F}_1(\dot{\lambda}_{11},\cdots,t)=\dot{\lambda}_{11}+2(a+k_1)\lambda_{11}-a(2+\sin\omega t)\lambda_{12},\\
&\hat{F}_2(\dot{\lambda}_{22},\cdots,t)=\dot{\lambda}_{22}-2a\lambda_{22}+\lambda_{12},\\
&\hat{F}_3(\dot{\lambda}_{33},\cdots,t)=\dot{\lambda}_{33}+2(a+k_3)\lambda_{33}-(2+\sin\omega t)\lambda_{34},\\
&\hat{F}_4(\dot{\lambda}_{44},\cdots,t)=\dot{\lambda}_{44}-2a\lambda_{44}+\lambda_{34},\\
&\hat{G}_1(\lambda_{11},\cdots,t)=2\lambda_{11}+k_1\lambda_{12}-2a(1+\sin\omega t)\lambda_{22},\\
&\hat{G}_2(\lambda_{11},\cdots,t)=-a(2+\sin\omega t)\lambda_{23}-(2+\sin\omega t)\lambda_{41},\\
&\hat{G}_3(\lambda_{11},\cdots,t)=k_1\lambda_{41},\quad \hat{G}_4(\lambda_{11},\cdots,t)=k_3\lambda_{23},\\
&\hat{G}_5(\lambda_{11},\cdots,t)=\lambda_{23}+\lambda_{41},\quad \hat{G}_6(\lambda_{11},\cdots,t)=2\lambda_{33}+k_3\lambda_{34}-2(2+\sin\omega t)\lambda_{44}.
\end{aligned}\tag{4.62}$$

Repeat Step 4 and Step 5, then the relationship between λ_{ij} becomes

$$\lambda_{11}=a(2+\sin\omega t)\lambda_{22}-\frac{k_1}{2}\lambda_{12},\quad \lambda_{23}=0,\quad \lambda_{33}=(2+\sin\omega t)\lambda_{44}-\frac{k_3}{2}\lambda_{34},\quad \lambda_{41}=0\,. \tag{4.63}$$

Assume

$$\begin{aligned}
&\lambda_{11}=c_1-\frac{k_1}{2}c_3,\quad \lambda_{22}=\frac{c_1}{a(2+\sin\omega t)},\quad \lambda_{33}=c_2-\frac{k_3}{2}c_4,\quad \lambda_{44}=\frac{c_2}{2+\sin\omega t},\\
&\lambda_{12}=c_3,\quad \lambda_{34}=c_4,
\end{aligned}\tag{4.64}$$

where c_1, c_2, c_3, c_4 are constants to be designed, and c_1, c_2 are positive numbers. Eq. (4.57) can be satisfied if

$$\begin{aligned}
&2(a+k_1)c_1<(k_1^2+ak_1+2a+a\sin\omega t)c_3,\\
&(4a+2a\sin\omega t+\omega\cos\omega t)c_1>a(2+\sin\omega t)^2c_3,\\
&2(a+k_3)c_2<(k_3^2+ak_3+2+\sin\omega t)c_4,\\
&(4a+2a\sin\omega t+\omega\cos\omega t)c_2>(2+\sin\omega t)^2c_4.
\end{aligned}\tag{4.65}$$

Step 7. The conditions derived from the above procedure can be summed up as follows:

$$\lambda_{23}=0,\quad \lambda_{41}=0, \tag{4.66}$$

$$\begin{aligned}
&c_1>0,\quad c_1>\frac{k_1}{2}c_3,\\
&2(a+k_1)c_1<(k_1^2+ak_1+2a+a\sin\omega t)c_3,\\
&(4a+2a\sin\omega t+\omega\cos\omega t)c_1>a(2+\sin\omega t)^2c_3,\\
&(2c_1-k_1c_3)c_1>c_3^2(4a+2a\sin\omega t),
\end{aligned}\tag{4.67}$$

$$
\begin{aligned}
&c_2 > 0, \quad c_2 > \frac{k_3}{2} c_4, \\
&2(a + k_3)c_2 < (k_3^2 + ak_3 + 2 + \sin \omega t)c_4, \\
&(4a + 2a \sin \omega t + \omega \cos \omega t)c_2 > (2 + \sin \omega t)^2 c_4, \\
&(2c_2 - k_3 c_4)c_2 > c_4^2 (4 + 2\sin \omega t).
\end{aligned} \tag{4.68}
$$

Design

$$
\begin{aligned}
&k_1 = -0.4,\ k_3 = -0.4,\ \lambda_{23} = 0,\ \lambda_{41} = 0, \\
&c_1 = 20, c_3 = 9 \quad \Rightarrow \lambda_{12} = 9, \lambda_{11} = 21.8, \lambda_{22} = \frac{20}{a(2 + \sin \omega t)}, \\
&c_2 = 50, c_4 = 11 \ \Rightarrow \lambda_{34} = 11, \lambda_{33} = 52.2, \lambda_{44} = \frac{50}{2 + \sin \omega t},
\end{aligned} \tag{4.69}
$$

such that each condition can be satisfied. Then the elaborate nondiagonal Lyapunov function can be obtained

$$
V(t, \mathbf{e}) = 21.8 e_1^2 + 9 e_1 e_2 + \frac{20}{a(2 + \sin \omega t)} e_2^2 + 52.2 e_3^2 + 11 e_3 e_4 + \frac{50}{2 + \sin \omega t} e_4^2, \tag{4.70}
$$

and

$$
\begin{aligned}
\dot{V}(t, \mathbf{e}) = &-(4.64 + 4.5 \sin t) e_1^2 - \frac{44 + 4\sin t + 40 \cos t - 9 \sin^2 t}{(2 + \sin t)^2} e_2^2 \\
&-(11.56 + 11 \sin t) e_3^2 - \frac{56 + 6 \sin t + 50 \cos t - 11 \sin^2 t}{(2 + \sin t)^2} e_4^2.
\end{aligned} \tag{4.71}
$$

Since Lyapunov global asymptotical stability theorem is satisfied, the global generalized synchronization is achieved. $\alpha = 1$, $\beta = 2$, $\gamma = 3$ are chosen in simulation, and the results are shown in Fig. 4.3-4.4.

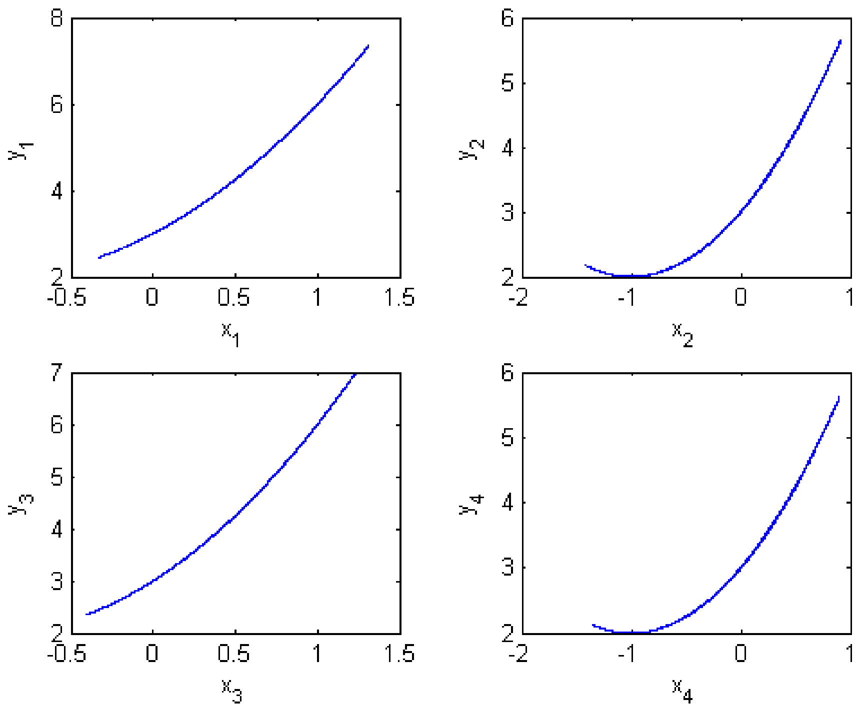

Fig. 4.1 Phase portraits of x_i to y_i $(i = 1, \cdots, 4)$ for Section 4.3 when the nonlinear generalized synchronization is obtained.

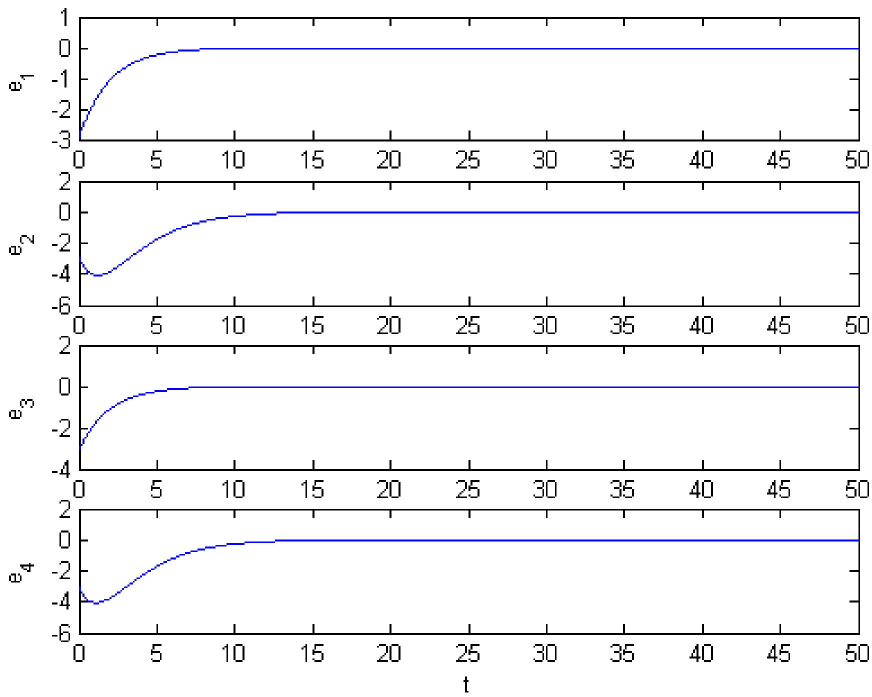

Fig. 4.2 Time histories of the state errors for Section 4.3.

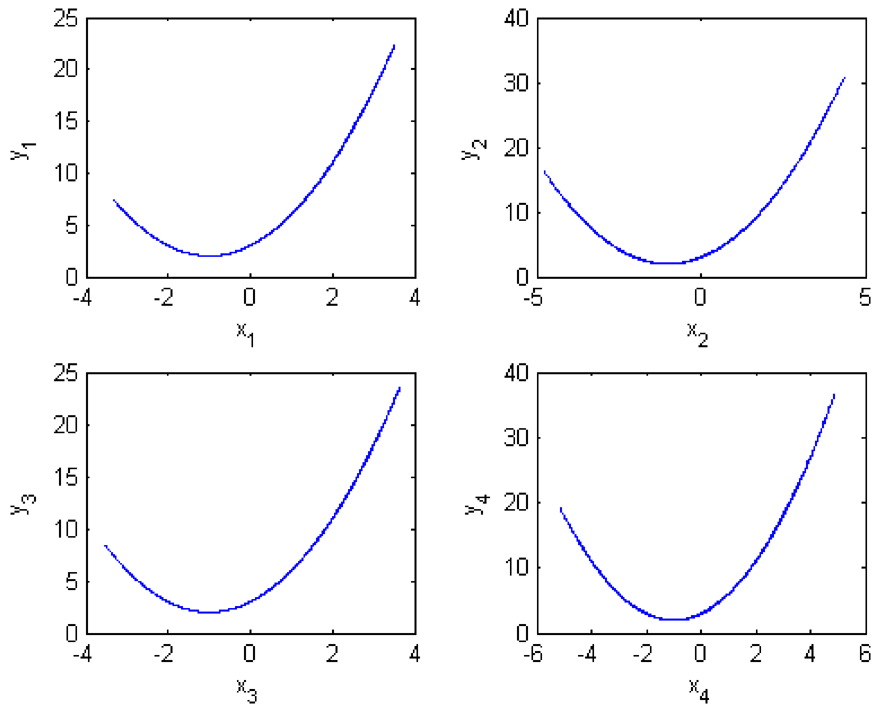

Fig. 4.3 Phase portraits of x_i to y_i ($i = 1,\cdots,4$) for Section 4.4 when the nonlinear generalized synchronization is obtained.

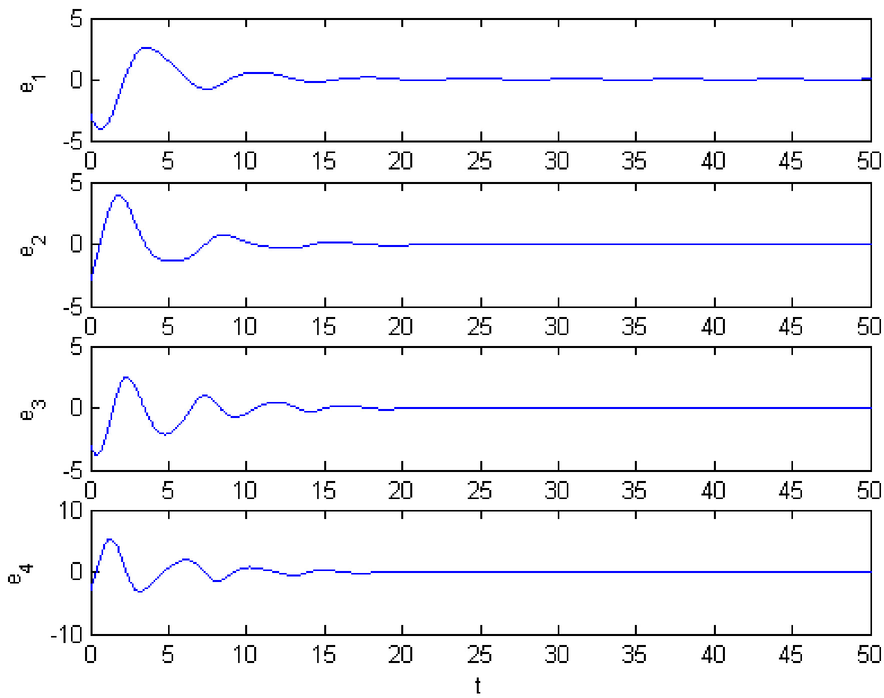

Fig. 4.4 Time histories of the state errors for Section 4.4.

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Chapter 12

Highly Robust Pragmatical Generalized Synchronization of Double Duffing Systems with Uncertain Parameters via Adaptive Control

12.1 Preliminaries

A scheme is proposed to achieve generalized synchronization for two chaotic systems with uncertain parameters. By the pragmatical asymptotical stability theorem [1-2] using the concept of probability, we can prove strictly that the common null solution of error dynamics and of parameter dynamics is actually asymptotically stable.

12.2 Pragmatical Generalized Synchronization Scheme by Adaptive Control

There are two identical nonlinear dynamical systems, and the master system controls the slave system. The master system is given by

$$\dot{x} = Ax + f(x, B) \tag{12.1}$$

where $x = [x_1, x_2, \cdots x_n]^T \in R^n$ denotes a state vector, A is an $n \times n$ uncertain constant coefficient matrix, f is a nonlinear vector function, and B is a vector of uncertain constant coefficients in f.

The slave system is given by

$$\dot{y} = \hat{A}y + f(y, \hat{B}) + u(t) \tag{12.2}$$

where $y = [y_1, y_2, \cdots y_n]^T \in R^n$ denotes a state vector, $\hat{A}$ is an $n \times n$ estimated coefficient matrix, $\hat{B}$ is a vector of estimated coefficients in f, and $u(t) = [u_1(t), u_2(t), \cdots u_n(t)]^T \in R^n$ is a control input vector.

Our goal is to design a controller $u(t)$ so that the state vector of the slave system (12.2) asymptotically approaches the state vector of the master system (12.1) plus a given chaotic vector function $F(t)=[F_1(t),F_2(t),\cdots F_n(t)]^T$. This is a special kind of generalized synchronization:

$$y = G(x) = x + F(t)\,. \tag{12.3}$$

The synchronization is accomplished when $t \to \infty$, the limit of the error vector $e(t)=[e_1,e_2,\cdots e_n]^T$ approaches zero:

$$\lim_{t\to\infty} e = 0 \tag{12.4}$$

where

$$e = x - y + F(t)\,. \tag{12.5}$$

From Eq. (12.5) we have

$$\dot{e} = \dot{x} - \dot{y} + \dot{F}(t) \tag{12.6}$$

$$\dot{e} = Ax - \hat{A}y + f(x,B) - f(y,\hat{B}) + \dot{F}(t) - u(t)\,. \tag{12.7}$$

A Lyapunov function $V(e,\widetilde{A}_c,\widetilde{B}_c)$ is chosen as a positive definite function

$$V(e,\widetilde{A}_c,\widetilde{B}_c) = \frac{1}{2}e^T e + \frac{1}{2}\widetilde{A}_c^{\,T}\widetilde{A}_c + \frac{1}{2}\widetilde{B}_c^{\,T}\widetilde{B}_c \tag{12.8}$$

where $\widetilde{A} = A - \hat{A}$, $\widetilde{B} = B - \hat{B}$, $\widetilde{A}_c$ and $\widetilde{B}_c$ are two column matrices whose elements are all the elements of matrix $\widetilde{A}$ and of matrix $\widetilde{B}$, respectively.

Its derivative along any solution of the differential equation system consisting of Eq. (12.7) and update parameter differential equations for $\widetilde{A}_c$ and $\widetilde{B}_c$ is

$$\therefore \dot{V}(e,\widetilde{A}_c,\widetilde{B}_c) = e^T[Ax - \hat{A}y + Bf(x) - \hat{B}f(y) + \dot{F}(t) - u(t)] + \widetilde{A}_c\dot{\widetilde{A}}_c + \widetilde{B}_c\dot{\widetilde{B}}_c \tag{12.9}$$

where $u(t)$, $\dot{\widetilde{A}}_c$ and $\dot{\widetilde{B}}_c$ are chosen so that $\dot{V} = e^T Ce$, C is a diagonal negative definite matrix, and $\dot{V}$ is a negative semi-definite function of e and parameter differences $\widetilde{A}_c$ and $\widetilde{B}_c$. In current scheme of adaptive synchronization [3-5], traditional Lyapunov asymptotical stability theorem and Babalat lemma are used to prove the error vector approaches zero, as time approaches infinity. But the question, why the estimated parameters also approach to the uncertain parameters, remains no answer. By pragmatical asymptotical stability theorem, the question can be answered strictly.

The stability for many problems in real dynamical systems is actual asymptotical stability, although may not be mathematical asymptotical stability. The mathematical asymptotical stability demands that trajectories from all initial states in the neighborhood of zero solution must approach the origin as $t \to \infty$. If there are only a small part or even a few of the initial states from which the trajectories do not approach the origin as $t \to \infty$, the zero solution is not mathematically asymptotically stable. However, when the probability of occurrence of an event is zero, it means the event does not occur actually. If the probability of occurrence of the event that the trajectries from the initial states are that they do not approach zero when $t \to \infty$, is zero, the stability of zero solution is actual asymptotical stability though it is not mathematical asymptotical stability. In order to analyze the asymptotical stability of the equilibrium point of such systems, the pragmatical asymptotical stability theorem is used.

Let X and Y be two manifolds of dimensions m and n $(m < n)$, respectively, and φ be a differentiable map from X to Y, then $\varphi(X)$ is a subset of Lebesque measure 0 of Y. For an autonomous system

$$\frac{dx}{dt} = f(x_1, x_2, \cdots x_n) \tag{12.10}$$

where $x = [x_1, x_2, \cdots x_n]^T$ is a state vector, the function $f = [f_1, f_2, \cdots f_n]^T$ is defined on $D \subset R^n$ and $\|x\| \leq H > 0$. Let $x = 0$ be an equilibrium point for the system (12.10). Then

$$f(0) = 0 \tag{12.11}$$

Definition : The equilibrium point for the system (12.11) is pragmatically asymptotically stable provided that with initial points on C which is a subset of Lebesque measure 0 of D, the behaviors of the corresponding trajectories cannot be determined, while with initial points on $D - C$, the corresponding trajectories behave as that agree with traditional asymptotical stability.

Theorem : Let $V = [x_1, x_2, \cdots x_n]^T : D \to R_+$ be positive definite and analytic on *D*, such that the derivative of V through Eq. (12.10), $\dot{V}$ is negative semi-definite.

Let *X* be the m-manifold consisted of point set for which $\forall x \neq 0$, $\dot{V}(x) = 0$ and *D* is a n-manifold. If $m+1 < n$, then the equilibrium point of the system is pragmatically asymptotically stable.

Proof : Since every point of *X* can be passed by a trajectory of Eq. (12.10), which is one dimensional, the collection of these trajectories, *C*, is a $(m+1)$-manifold [6-7]. If $(m+1) < n$, then the collection *C* is a subset of Lebesque measure 0 of *D*. By the above definition, the equilibrium point of the system is pragmatically asymptotically stable.

If an initial point is ergodicly chosen in *D*, *the probability of that the initial point falls on the collection C is zero*. *Here, equal probability is assumed for every point chosen as an initial point in the neighborhood of the equilibrium point.* Hence, the event that the initial point is chosen from collection *C does not occur actually*. Therefore, under the equal probability assumption, pragmatical asymptotical stability becomes actual asymptotical stability. When the initial point falls on $D - C$, $\dot{V}(x) < 0$, the corresponding trajectories behave as that agree with traditional asymptotical stability because by the existence and uniqueness of the solution of initial-value problem, these trajectories never meet *C*.

In Eq. (12.8) *V* is a positive definite function of *n* variables, i.e. *p* error state variables and $n - p = m$ differences between unknown and estimated parameters, while $\dot{V} = e^T C e$ is a negative semi-definite function of *n* variables. Since the number of error state variables is always more than one, $p > 1$, $(m+1) < n$ is always satisfied, by pragmatical asymptotical stability theorem we have

$$\lim_{t \to \infty} e = 0 \tag{12.12}$$

and the estimated parameters approach the uncertain parameters. The pargmatical generalized synchronizations is obtained. Therefore, *the equilibrium point of the system is pragmatically asymptotically stable. Under the equal probability assumption, it is actually asymptotically* *stable for both error state variables and parameter variables.*

12.3 Numerical Results of Pragmatical Generalized Chaos Synchronization of Two Double Duffing systems by Adaptive Control

12.3.1 Two double Duffing systems with double van der Pol system as goal system

The chaotic states of a goal system, a double van der Pol chaotic system, used as $F(t)$. For a double Duffing system [8], the following differential equations are used as master system:

$$\begin{cases} \dfrac{dx_1}{dt} = x_2 \\ \dfrac{dx_2}{dt} = -ax_2 - bx_1 - cx_1^{\ 3} + dx_3 \\ \dfrac{dx_3}{dt} = x_4 \\ \dfrac{dx_4}{dt} = -fx_4 - gx_3 - hx_3^{\ 3} + kx_1 \end{cases} \tag{12.13}$$

It consists of two Duffing systems in which two external excitations are replaced by two coupling terms. It is an autonomous system with four states where *a*, *b*, *c*, *d*, *e*, *g*, *h*, and *k* are constant unknown parameters of the systems. The chaotic phase portraits for double Duffing system and double van der Pol system are shown in Fig.12.1

In numerical simulation, we take $a = 0.05, b = 1, c = 3, d = 7, f = 0.0005, g = 1$ $, h = 3$ and $k = -7$. The slave system is described by

$$\begin{cases} \dfrac{dy_1}{dt} = y_2 \\ \dfrac{dy_2}{dt} = -\hat{a}y_2 - \hat{b}y_1 - \hat{c}y_1^{\ 3} + \hat{d}y_3 \\ \dfrac{dy_3}{dt} = y_4 \\ \dfrac{dy_4}{dt} = -\hat{f}y_4 - \hat{g}y_3 - \hat{h}y_3^{\ 3} + \hat{k}y_1 \end{cases} \tag{12.14}$$

where $\hat{a}$, $\hat{b}$, $\hat{c}$, $\hat{d}$, $\hat{f}$, $\hat{g}$, $\hat{h}$ and $\hat{k}$ are estimated parameters.

In order to lead (y_1, y_2, y_3, y_4) to $(x_1 + F_1(t), x_2 + F_2(t), x_3 + F_3(t), x_4 + F_4(t))$, we add u_1, u_2, u_3, and u_4 to each equation of Eq. (12.14), respectively:

$$\begin{cases} \dfrac{dy_1}{dt} = y_2 + u_1 \\ \dfrac{dy_2}{dt} = -\hat{a}y_2 - \hat{b}y_1 - \hat{c}{y_1}^3 + \hat{d}y_3 + u_2 \\ \dfrac{dy_3}{dt} = y_4 + u_3 \\ \dfrac{dy_4}{dt} = -\hat{f}y_4 - \hat{g}y_3 - \hat{h}{y_3}^3 + \hat{k}y_1 + u_4 \end{cases} \tag{12.15}$$

Subtracting Eq. (12.15) from Eq. (12.13), we obtain an error dynamics. The initial values of the master system and the slave system are taken as $x_1(0) = 2,\quad x_2(0) = 5,$ $x_3(0) = 1,\ x_4(0) = 3, y_1(0) = 2.1, y_2(0) = 4.9, y_3(0) = 0.9$ and $y_4(0) = 3.1,$ respectively.

The goal system for generalized synchronization is a double van der Pol chaotic system [9-10].

$$\begin{cases} \dfrac{dz_1}{dt} = z_2 \\ \dfrac{dz_2}{dt} = -z_1 + a_3(1 - b_3 z_1^2)z_2 + c_3 z_3 \\ \dfrac{dz_3}{dt} = z_4 \\ \dfrac{dz_4}{dt} = -z_3 + a_4(1 - b_4 z_3^2)z_4 + c_4 z_1 \end{cases} \tag{12.16}$$

where a_3=0.2, b_3=1, c_3= -0.01, a_4=-2,b_4=1, c_4=0.3, $z_1(0)$=3, $z_2(0)$=4, $z_3(0)$=3, and $z_4(0)$=4. We have

$$\lim_{t\to\infty} e_i = \lim_{t\to\infty}(x_i - y_i + z_i) = 0 \qquad i = 1, 2, 3, 4 \tag{12.17}$$

where $\dot{e} = \dot{x} - \dot{y} + \dot{z}$, and

$$\begin{cases} \dot{e}_1 = x_2 - y_2 - u_1 + \dot{z}_1 \\ \dot{e}_2 = -ax_2 - bx_1 - c{x_1}^3 + dx_3 + \hat{a}y_2 + \hat{b}y_1 + \hat{c}{y_1}^3 - \hat{d}y_3 - u_2 + \dot{z}_2 \\ \dot{e}_3 = x_4 - y_4 - u_3 + \dot{z}_3 \\ \dot{e}_4 = -fx_4 - gx_3 - h{x_3}^3 + kx_1 + \hat{f}y_4 + \hat{g}y_3 + \hat{h}{y_3}^3 - \hat{k}y_1 - u_4 + \dot{z}_4 \end{cases} \tag{12.18}$$

where $e_1 = x_1 - y_1 + z_1$, $e_2 = x_2 - y_2 + z_2$, $e_3 = x_3 - y_3 + z_3$, and $e_4 = x_4 - y_4 + z_4$.

Choose a Lyapunov function in the form of a positive definite function:

$$V = \frac{1}{2}({e_1}^2 + {e_2}^2 + {e_3}^2 + {e_4}^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2 + \tilde{f}^2 + \tilde{g}^2 + \tilde{h}^2 + \tilde{k}^2) \tag{12.19}$$

where $\tilde{a}=(a-\hat{a})$, $\tilde{b}=(b-\hat{b})$, $\tilde{c}=(c-\hat{c})$, $\tilde{d}=(d-\hat{d})$, $\tilde{f}=(f-\hat{f})$, $\tilde{g}=(g-\hat{g})$, $\tilde{h}=(h-\hat{h})$, $\tilde{k}=(k-\hat{k})$ and $\hat{a}$, $\hat{b}$, $\hat{c}$, $\hat{d}$, $\hat{f}$, $\hat{g}$, $\hat{h}$ and $\hat{k}$ are estimates of uncertain parameters a , b , c , d , f , g , h and k respectively. Its time derivative is

$$
\begin{aligned}
\dot{V} &= e_1(x_2 - y_2 - u_1 + z_2) \\
&+ e_2[-ax_2 - bx_1 - cx_1^3 + dx_3 + \hat{a}y_2 + \hat{b}y_1 + \hat{c}y_1^3 - \hat{d}y_3 - u_2 \\
&- z_1 + a_3(1 - b_3 z_1^2)z_2 + c_3 z_3] \\
&+ e_3(x_4 - y_4 - u_3 + z_4) \\
&+ e_4[-fx_4 - gx_3 - hx_3^3 + kx_1 + \hat{f}y_4 + \hat{g}y_3 + \hat{h}y_3^3 - \hat{k}y_1 - u_4 \\
&- z_3 + a_4(1 - b_4 z_3^2)z_4 + c_4 z_1] \\
&+ \tilde{a}(-\dot{\hat{a}}) + \tilde{b}(-\dot{\hat{b}}) + \tilde{c}(-\dot{\hat{c}}) + \tilde{d}(-\dot{\hat{d}}) + \tilde{f}(-\dot{\hat{f}}) + \tilde{g}(-\dot{\hat{g}}) + \tilde{h}(-\dot{\hat{h}}) + \tilde{k}(-\dot{\hat{k}})
\end{aligned}
\tag{12.20}
$$

Choose

$$
\begin{cases}
u_1 = x_2 - y_2 + z_2 + e_1 \\
u_2 = \hat{a}(y_2 - x_2) + \hat{b}(y_1 - x_1) + \hat{c}(y_1^3 - x_1^3) + \hat{d}(x_3 - y_3) \\
+[-z_3 + a_4(1 - b_4 z_3^2)z_4 + c_4 z_1] + e_2(1 - \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2) \\
u_3 = x_4 - y_4 + z_4 + e_3 \\
u_4 = \hat{f}(y_4 - x_4) + \hat{g}(y_3 - x_3) + \hat{h}(y_3^3 - x_3^3) + \hat{k}(x_1 - y_1) \\
+[-z_3 + a_4(1 - b_4 z_3^2)z_4 + c_4 z_1] + e_4(1 - \tilde{f}^2 + \tilde{g}^2 + \tilde{h}^2 + \tilde{k}^2)
\end{cases}
\tag{12.21}
$$

$$
\begin{cases}
\dot{\tilde{a}} = -\dot{\hat{a}} = x_2 e_2 - \tilde{a} e_2 \\
\dot{\tilde{b}} = -\dot{\hat{b}} = x_1 e_2 - \tilde{b} e_2 \\
\dot{\tilde{c}} = -\dot{\hat{c}} = x_1^3 e_2 - \tilde{c} e_2 \\
\dot{\tilde{d}} = -\dot{\hat{d}} = -x_3 e_2 - \tilde{d} e_2 \\
\dot{\tilde{f}} = -\dot{\hat{f}} = x_4 e_2 - \tilde{f} e_2 \\
\dot{\tilde{g}} = -\dot{\hat{g}} = x_3 e_2 - \tilde{g} e_2 \\
\dot{\tilde{h}} = -\dot{\hat{h}} = x_3^3 e_2 - \tilde{h} e_4 \\
\dot{\tilde{k}} = -\dot{\hat{k}} = -x_1 e_2 - \tilde{k} e_4
\end{cases}
\tag{12.22}
$$

The initial values of estimate for uncertain parameters are $\hat{a}(0)=\hat{b}(0)=\hat{c}(0)=\hat{d}(0)=\hat{f}(0)=\hat{g}(0)=\hat{h}(0)=\hat{k}(0)=0$. Substituting Eq. (12.21) and Eq. (12.22) into Eq. (12.20), we obtain

$$
\dot{V} = -e_1^2 - e_2^2 - e_3^2 - e_4^2 \le 0 \tag{12.23}
$$

which is negative semi-definite function of $e_1, e_2, e_3, e_4, \tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}, \tilde{f}, \tilde{g}, \tilde{h}, \tilde{k}$. The Lyapunov asymptotical stability theorem is not satisfied. We cannot obtain that the common origin of error dynamics (12.18) and parameter dynamics (12.22) is asymptotically stable. Now, D is an 8-manifold, n=8 and the number of error state variables p=4. When e_1=e_2=e_3=e_4=0 and $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}, \tilde{f}, \tilde{g}, \tilde{h}, \tilde{k}$ take arbitrary values, $\dot{V} = 0$, so X is 4-manifold, $m = n - p = 8 - 4 = 4$. $m + 1 < n$ is satisfied. By pragmatical asymptotical stability theorem, error vector e approaches zero and the estimated parameters also approach the uncertain parameters. The pragmatical generalized synchronization is obtained.

The equilibrium point $e_1 = e_2 = e_3 = e_4 = \tilde{a} = \tilde{b} = \tilde{c} = \tilde{d} = \tilde{f} = \tilde{g} = \tilde{h} = \tilde{k} = 0$ is pragmatically asymptotically stable. *Under the assumption of equal probability, it is actually asymptotically stable*. State errors versus time and the estimates of uncertain parameters are shown in Fig. 12.2.

12.3.2 Robustness of the above generalized synchronization

For a double Duffing system, the following differential equations are used as master system:

$$\begin{cases} \dfrac{dx_1}{dt} = x_2 \\ \dfrac{dx_2}{dt} = -ax_2 - bx_1 - cx_1^{\ 3} + dx_3 + \Delta f_1 \\ \dfrac{dx_3}{dt} = x_4 \\ \dfrac{dx_4}{dt} = -fx_4 - gx_3 - hx_3^{\ 3} + kx_1 + \Delta f_1 \end{cases} \tag{12.24}$$

A slave system is described by

$$\begin{cases} \dfrac{dy_1}{dt} = y_2 + u_1 \\ \dfrac{dy_2}{dt} = -\hat{a}y_2 - \hat{b}y_1 - \hat{c}y_1^{\ 3} + \hat{d}y_3 + u_2 + \Delta f_2 \\ \dfrac{dy_3}{dt} = y_4 + u_3 \\ \dfrac{dy_4}{dt} = -\hat{f}y_4 - \hat{g}y_3 - \hat{h}y_3^{\ 3} + \hat{k}y_1 + u_4 + \Delta f_2 \end{cases} \tag{12.25}$$

They are two double Duffing systems with disturbance $\Delta f_1(t,x,y,z)$ and $\Delta f_2(t,x,y,z)$ respectively.

In simulation, the parameters of the master system in chosen as $a = 0.05, b = 1, c = 3, d = 7, f = 0.0005, g = 1, h = 3, k = -7$. The initial values of the master system and the slave system are taken as $x_1(0)$=2, $x_2(0)$=5, $x_3(0)$=1, $x_4(0)$=3, $y_1(0)$=2.1, $y_2(0)$=4.9, $y_3(0)$=0.9, and $y_4(0)$=3.1, respectively.

Let

$$\begin{aligned} \Delta f_1(t,x,y) &= \alpha(x_i - y_i + z_i)\Gamma_1(t,x) \\ \Delta f_2(t,x,y) &= \alpha(x_i - y_i + z_i)\Gamma_2(t,y) \end{aligned} \qquad i = 2, 4 \tag{12.26}$$

where $\Gamma_1(t,x)$ is the Gaussian noise and $\Gamma_2(t,y)$ is the Rayleigh noise, α is the strength constant. Since both disturbances are the products of chaos and noise, they are highly perturbative.

The goal system for generalized synchronization is a double Van der Pol chaotic

$$\begin{cases} \dfrac{dz_1}{dt} = z_2 \\ \dfrac{dz_2}{dt} = -z_1 + a_3(1 - b_3 z_1^2)z_2 + c_3 z_3 \\ \dfrac{dz_3}{dt} = z_4 \\ \dfrac{dz_4}{dt} = -z_3 + a_4(1 - b_4 z_3^2)z_4 + c_4 z_1 \end{cases} \tag{12.27}$$

We demand

$$\lim_{t\to\infty} e_i = \lim_{t\to\infty}(x_i - y_i + z_i) = 0, \qquad i = 1, 2, 3, 4$$

then

$\dot{e} = \dot{x} - \dot{y} + \dot{z}$,

and

$$\begin{cases} \dot{e}_1 = x_2 - y_2 - u_1 + \dot{z}_1 \\ \dot{e}_2 = -ax_2 - bx_1 - cx_1{}^3 + dx_3 + \hat{a}y_2 + \hat{b}y_1 + \hat{c}y_1{}^3 - \hat{d}y_3 - u_2 + (\Delta f_1 - \Delta f_2) + \dot{z}_2 \\ \dot{e}_3 = x_4 - y_4 - u_3 + \dot{z}_3 \\ \dot{e}_4 = -fx_4 - gx_3 - hx_3{}^3 + kx_1 + \hat{f}y_4 + \hat{g}y_3 + \hat{h}y_3{}^3 - \hat{k}y_1 - u_4 + (\Delta f_1 - \Delta f_2) + \dot{z}_4 \end{cases} \quad (12.28)$$

where $e_1 = x_1 - y_1 + z_1$, $e_2 = x_2 - y_2 + z_2$, $e_3 = x_3 - y_3 + z_3$, and $e_4 = x_4 - y_4 + z_4$. Choose a Lyapunov function in the form of a positive definite function:

$$V = \frac{1}{2}(e_1{}^2 + e_2{}^2 + e_3{}^2 + e_4{}^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2 + \tilde{f}^2 + \tilde{g}^2 + \tilde{h}^2 + \tilde{k}^2) \quad (12.29)$$

where $\tilde{a} = (a - \hat{a})$, $\tilde{b} = (b - \hat{b})$, $\tilde{c} = (c - \hat{c})$, $\tilde{d} = (d - \hat{d})$, $\tilde{f} = (f - \hat{f})$, $\tilde{g} = (g - \hat{g})$, $\tilde{h} = (h - \hat{h})$, $\tilde{k} = (k - \hat{k})$ and $\hat{a}$, $\hat{b}$, $\hat{c}$, $\hat{d}$, $\hat{f}$, $\hat{g}$, $\hat{h}$ and $\hat{k}$ are estimates of uncertain parameters a, b, c, d, f, g, h and k respectively.

Its time derivative is

$$\begin{aligned} \dot{V} &= e_1(x_2 - y_2 - u_1 + z_2) \\ &+ e_2[-ax_2 - bx_1 - cx_1{}^3 + dx_3 + \hat{a}y_2 + \hat{b}y_1 + \hat{c}y_1{}^3 - \hat{d}y_3 - u_2 \\ &- z_1 + a_3(1 - b_3 z_1^2)z_2 + c_3 z_3] \\ &+ e_3(x_4 - y_4 - u_3 + z_4) \\ &+ e_4[-fx_4 - gx_3 - hx_3{}^3 + kx_1 + \hat{f}y_4 + \hat{g}y_3 + \hat{h}y_3{}^3 - \hat{k}y_1 - u_4 \\ &- z_3 + a_4(1 - b_4 z_3^2)z_4 + c_4 z_1] \\ &+ \tilde{a}(-\dot{\hat{a}}) + \tilde{b}(-\dot{\hat{b}}) + \tilde{c}(-\dot{\hat{c}}) + \tilde{d}(-\dot{\hat{d}}) + \tilde{f}(-\dot{\hat{f}}) + \tilde{g}(-\dot{\hat{g}}) + \tilde{h}(-\dot{\hat{h}}) + \tilde{k}(-\dot{\hat{k}}) \end{aligned} \quad (12.30)$$

Choose

$$\begin{cases} u_1 = x_2 - y_2 + z_2 + e_1 \\ u_2 = \hat{a}(y_2 - x_2) + \hat{b}(y_1 - x_1) + \hat{c}(y_1{}^3 - x_1{}^3) + \hat{d}(x_3 - y_3) \\ + [-z_3 + a_4(1 - b_4 z_3^2)z_4 + c_4 z_1] + e_2(1 - \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2) \\ u_3 = x_4 - y_4 + z_4 + e_3 \\ u_4 = \hat{f}(y_4 - x_4) + \hat{g}(y_3 - x_3) + \hat{h}(y_3{}^3 - x_3{}^3) + \hat{k}(x_1 - y_1) \\ + [-z_3 + a_4(1 - b_4 z_3^2)z_4 + c_4 z_1] + e_4(1 - \tilde{f}^2 + \tilde{g}^2 + \tilde{h}^2 + \tilde{k}^2) \end{cases} \quad (12.31)$$

$$\begin{cases} \dot{\tilde{a}} = -\dot{\hat{a}} = x_2 e_2 - \tilde{a} e_2 \\ \dot{\tilde{b}} = -\dot{\hat{b}} = x_1 e_2 - \tilde{b} e_2 \\ \dot{\tilde{c}} = -\dot{\hat{c}} = x_1^{\ 3} e_2 - \tilde{c} e_2 \\ \dot{\tilde{d}} = -\dot{\hat{d}} = -x_3 e_2 - \tilde{d} e_2 \\ \dot{\tilde{f}} = -\dot{\hat{f}} = x_4 e_2 - \tilde{f} e_2 \\ \dot{\tilde{g}} = -\dot{\hat{g}} = x_3 e_2 - \tilde{g} e_2 \\ \dot{\tilde{h}} = -\dot{\hat{h}} = x_3^{\ 3} e_2 - \tilde{h} e_4 \\ \dot{\tilde{k}} = -\dot{\hat{k}} = -x_1 e_2 - \tilde{k} e_4 \end{cases} \tag{12.32}$$

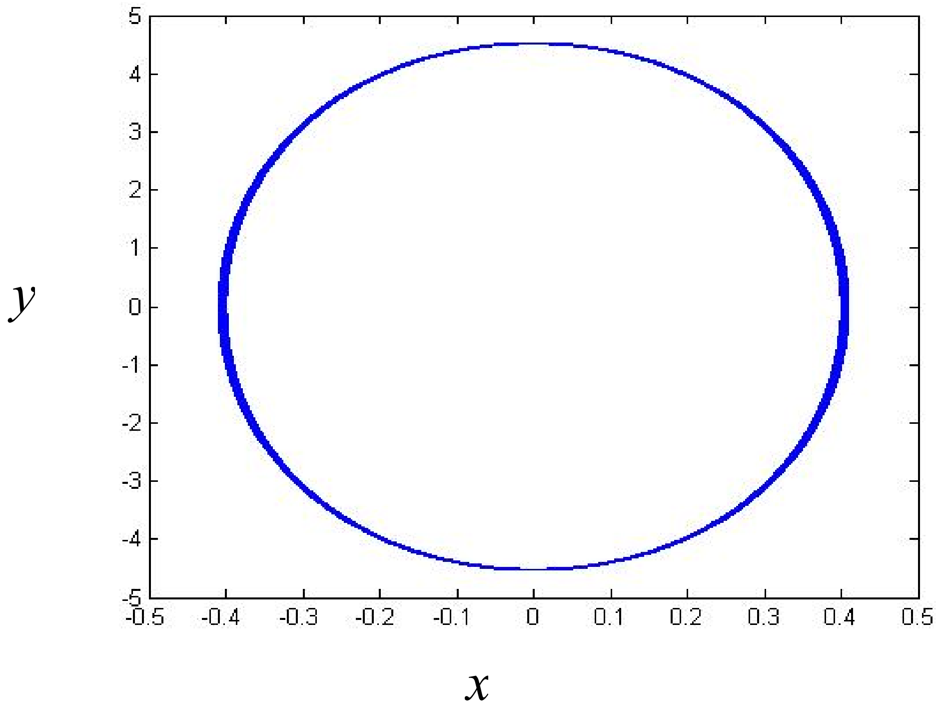
The initial values of estimate for uncertain parameters are $\hat{a}(0) = \hat{b}(0) = \hat{c}(0) = \hat{d}(0)$ $= \hat{f}(0) = \hat{g}(0) = \hat{h}(0) = \hat{k}(0) = 0$.

Substituting Eq. (12.31) and Eq. (12.32) into Eq. (12.30), we obtain

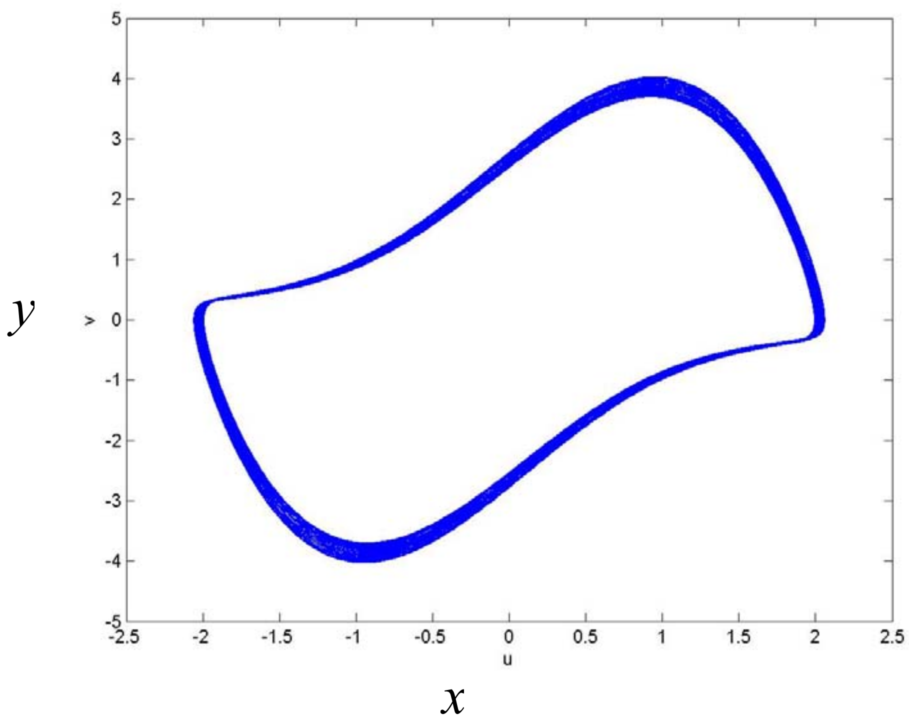
$$\dot{V} = -e_1^{\ 2} - e_2^{\ 2} - e_3^{\ 2} - e_4^{\ 2} \le 0 \tag{12.33}$$

which is negative semi-definite function of $e_1, e_2, e_3, e_4, \tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}, \tilde{f}, \tilde{g}, \tilde{h}, \tilde{k}$. The Lyapunov asymptotical stability theorem is not satisfied. We cannot obtain that the common origin of error dynamics (12.28) and parameter dynamics (12.32) is asymptotically stable. Now, D is an 8-manifold, n=8 and the number of error state variables p=4. When e_1=e_2=e_3= e_4=0 and $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}, \tilde{f}, \tilde{g}, \tilde{h}, \tilde{k}$ take arbitrary values, $\dot{V} = 0$, so X is 4-manifold, $m = n - p = 8 - 4 = 4$. $m + 1 < n$ is satisfied. By pragmatical asymptotical stability theorem, when $\alpha = 0 \sim 12$, the error vector e approaches zero and the estimated parameters also approach the uncertain parameters. The pragmatical generalized synchronization is obtained.

The equilibrium point $e_1 = e_2 = e_3 = e_4 = \tilde{a} = \tilde{b} = \tilde{c} = \tilde{d} = \tilde{f} = \tilde{g} = \tilde{h} = \tilde{k} = 0$ is pragmatically asymptotically stable. Under the assumption of equal probability, it is actually asymptotically stable. State errors versus time and the estimates of uncertain parameters with $\alpha = 11$ are shown in Fig.12.3. From Fig.12.3, the robustness of the generalized synchronization is very satisfactory when $\alpha \leq 11$. i.e. when there are strong highly perturbative disturbances. The robustness obtained is very high.



(a)



(b)

Fig 12.1 (a) The chaotic phase portrait for the double Duffing system,
(b) The chaotic phase portrait for the double van der Pol system.

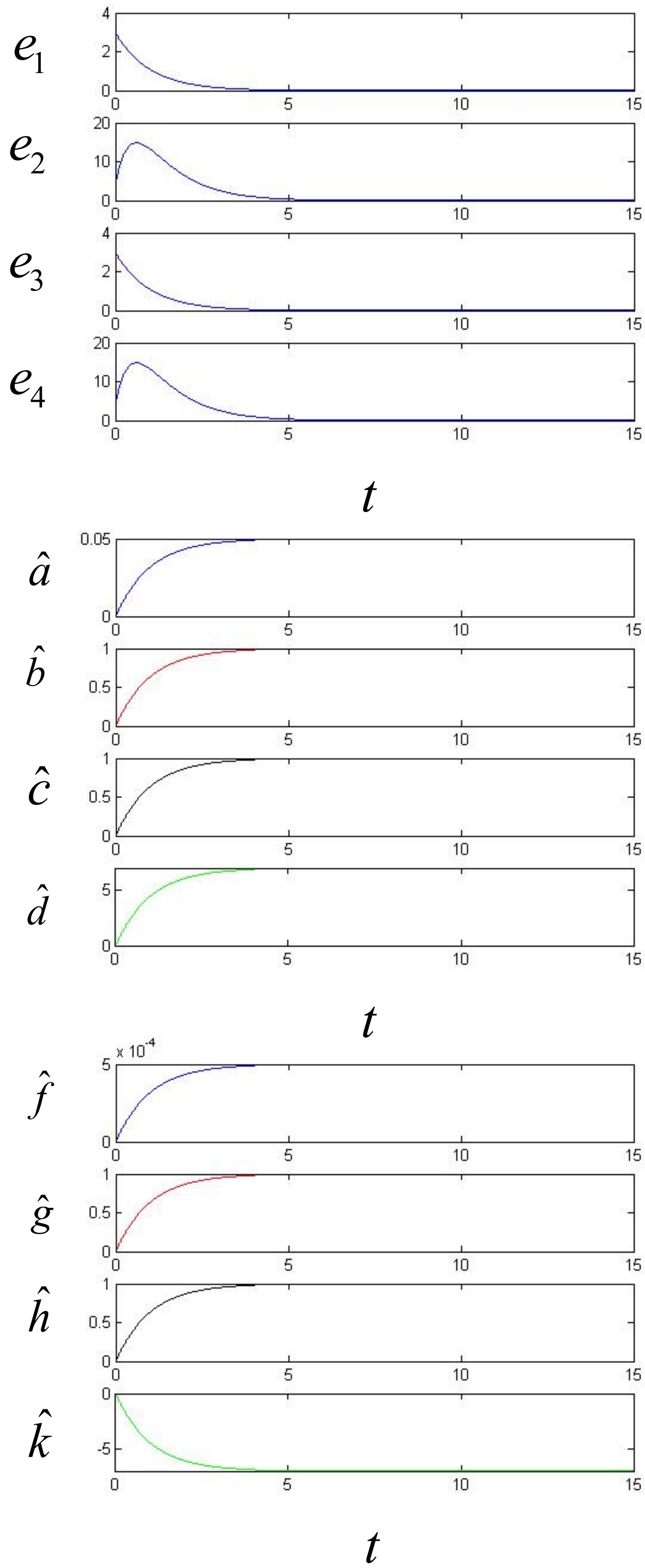


Fig 12.2 Time histories of state errors, $\hat{a}$, $\hat{b}$, $\hat{c}$, $\hat{d}$, $\hat{f}$, $\hat{g}$, $\hat{h}$ and $\hat{k}$ for Case 1 with $a = 0.05, b = 1, c = 3, d = 7, f = 0.0005, g = 1, h = 3, k = -7$.

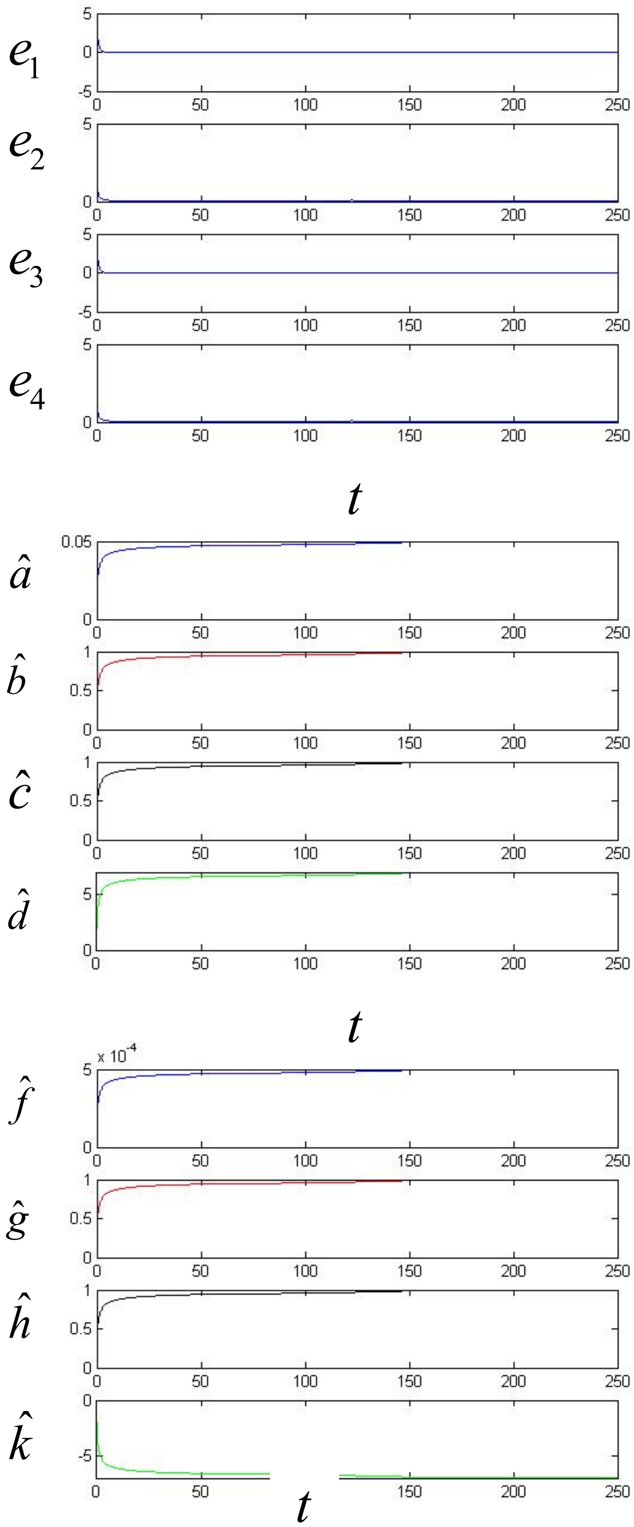


Fig 12.3 Time histories of state errors, $\hat{a}$, $\hat{b}$, $\hat{c}$, $\hat{d}$, $\hat{f}$, $\hat{g}$, $\hat{h}$ and $\hat{k}$ for Case 2 with $\alpha = 11$ and $a = 0.05, b = 1, c = 3, d = 7, f = 0.0005, g = 1, h = 3, k = -7$.

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Chapter 14

Chaos-excited Synchronization of Uncopuled Double Van der Pol systems

14.1 Preliminaries

Chaos synchronization is an important problem in nonlinear science. Since the discovery of chaos synchronization by Pecora and Carroll [1], there have been tremendous interests in studying the synchronization of various chaotic systems [2–16]. Most of synchronizations can only realize when there exist various couplings between two chaotic systems. A major drawback of these approaches is that they, to some extent, require mutually coupled structures. In practice, such as in physical and electrical systems, sometimes it is difficult even impossible to couple two chaotic systems. In comparison with coupled chaotic systems, synchronization between the uncoupled chaotic systems has many advantages [17,18]. In this Chapter, the variable of a third double van der Pol system substituted for the strength of two corresponding mutual coupling term of two identical chaotic double van der Pol system, give rise to their complete synchronization (CS) or anti-synchronization (AS). Numerical simulations show that either CS or AS depends on initial conditions and on the strengths of the substituted variable.

14.2 Synchronization between uncoupled double van der Pol system

A van der Pol [19-21] oscillator driven by a periodic excitation is considered. The equation of motion can be written as:

$$\ddot{x} + \varphi x + a\dot{x}(x^2 - 1) - b\sin \omega t = 0 \tag{14.1}$$

where φ, a, b are constant parameters and $b\sin\omega t$ is an external excitation . In Eq. (14.1), the linear term stands for a conservative harmonic force which determines the intrinsic oscillation frequency. The self-sustaining mechanism which is responsible for the perpetual oscillation rests on the nonlinear term. Energy exchange with the external agent depends on the magnitude of displacement |x| and on the sign of velocity $\dot{x}$. During a complete cycle of oscillation, the energy is dissipated if displacement x(t) is large than one, and that energy is fed-in if |x| < 1. The time-dependent term stands for the external driving force with amplitude b and frequency ω.Eq. (14.1) can be rewritten as two first order equations:

$$\begin{cases} \dot{x} = y \\ \dot{y} = -\varphi x + a(1 - x^2)y + b\sin \omega t \end{cases} \tag{14.2}$$

The double van der Pol system studied in this Chapter consists of two van der Pol systems with mutual coupling terms instead of two external excitations:

$$\begin{cases} \dfrac{dx_1}{dt} = y_1 \\ \dfrac{dy_1}{dt} = -x_1 + b(1 - cx_1^2)y_1 + au_1 \\ \dfrac{du_1}{dt} = v_1 \\ \dfrac{dv_1}{dt} = -u_1 + e(1 - fu_1^2)v_1 + dx_1 \end{cases} \tag{14.3}$$

where au_1, dx_1 are mutual coupling terms. When a = 0.04 , b = 0.2, c = 12 , d = -0.3, e = 2, f = 1, chaos of the system are illustrated by Lyapunov exponent diagram (Fig.14.1) and phase portrait (Fig.14.2).

Take system (14.3) as master system, the slave system is

$$\begin{cases} \dfrac{dx_2}{dt} = y_2 \\ \dfrac{dy_2}{dt} = -x_2 + b(1 - cx_2^2)y_2 + au_2 \\ \dfrac{du_2}{dt} = v_2 \\ \dfrac{dv_2}{dt} = -u_2 + e(1 - fu_2^2)v_2 + dx_2 \end{cases} \tag{14.4}$$

A third double van der Pol system is given:

$$\begin{cases} \dfrac{dx_3}{dt} = y_3 \\ \dfrac{dy_3}{dt} = -x_3 + b(1 - cx_3^2)y_3 + au_3 \\ \dfrac{du_3}{dt} = v_3 \\ \dfrac{dv_3}{dt} = -u_3 + e(1 - fu_3^2)v_3 + dx_3 \end{cases} \tag{14.5}$$

Substituting kx_3 or ky_3 for both *a* in system (14.3) and system (14.4), respectively. and giving suitable values for k and initial conditions, we obtain that two system (14.3) and system (14.4) are either synchronized or anti-synchronized.

14.3 Numerical simulations

Matlab method is used to all of the simulations with time step 0.01. The parameters of two systems (14.3) and system (14.4) are given as a = 0.04, b = 0.2, c = 12, d = -0.3, e = 2, f = 1 to ensure the chaotic behavior. To verify CS and AS, it is convenient to introduce the following coordinate transformation: $E_1 = (x_1 + x_2)$ and $e_1 = (x_1 - x_2)$ and the same transformation for *y, u* and v variables. Therefore, the new coordinate systems (E_1, E_2, E_3, E_4) and (e_1, e_2, e_3, e_4) represent the sum and difference motions of the original coordinate system, respectively. We can easily see that the (e_1, e_2, e_3, e_4) subspace represents the CS case, and the (E_1, E_2, E_3, E_4) subspace for the AS one.

Choice A

Take kx_3 instead of both a in system (14.3) and system (14.4), and take (x_1, y_1, u_1, v_1) = (3, 4, 3, 4), (x_2, y_2, u_2, v_2) = (-3, 4, -3, 4) as the initial conditions of system (14.3) and system (14.4). For Fig. 14.3, k = 1 and for Fig. 14.4, k = 0.9. Fig. 14.3 and Fig. 14.4 show the time-series of AS (case (a)) and CS (case (b)) phenomena for different k, respectively. These simulation results indicate that the final state develops to CS or AS, depending sensitively on k in spite of the identical initial conditions in both cases. For Fig. 14.3, e_4 (CS), E_2 (AS), E_3 (AS), converge to zero, while the other coordinates remain chaotic. For Fig. 14.4, on the other hand, only E_2 (AS) converges to zero.

Depending on the initial conditions both AS and CS can also be observed. To study how these phenomena depend upon the initial conditions, we change the initial conditions for fixed k. The results are shown in Figs. 14.5 and 14.6. Fig. 14.5 (a) shows that the differences $e_2 = y_1 - y_2$ and $e_3=u_1 - u_2$ tend to zero. In Fig. 14.5 (b), the sum $E_4=v_1 + v_2$ tends to zero. Comparing Fig. 14.3 with Fig. 14.5, one can find that they have different behaviors. The only reason lies in the different initial conditions. Similar result also exists by comparing Fig. 14.4 with Fig. 14.6. But we have not observed the intermittent synchronization and AS states as declared in Ref. [22].

The simulation results are shown in Fig. 14.7 for different value of k. The solid circle "•" and triangle "▲" correspond to CS where parameter values k leads to synchronized behavior. While white circle "○" and triangle "△" indicate AS. The blank space means no exist AS or CS. We can see that the system (14.3) and system (14.4) tend to either AS or CS by using combination of different value of k and initial values. However, as we can see from Fig. 14.7, both cases agree well with the fact that the system goes to either synchronized state or anti-synchronized state depending on initial values and on k. When k = 0.8 ~ 0.82, neither synchronization nor anti-synchronization is found.

Choice B

Take kx_3 instead of both a in system (14.3) and system (14.4), and take (x_1, y_1, u_1, v_1) = (3, 4, 3, 4), (x_2, y_2, u_2, v_2) = (-3, 4, 3, 4) as the initial conditions of system (14.3) and system (14.4). For Fig. 14.8, k = 0.97 and for Fig. 14.9, k = 1.02. Fig. 14.8 and Fig. 14.9 show the time-series of AS (case (c)) and CS (case (d)) phenomena for different k, respectively. These simulation results indicate that the final state develops to CS or AS, depending sensitively on k in spite of the identical initial conditions in both cases. For Fig. 14.8, e_2 (CS), e_3 (CS), E_4 (AS), converge to zero, while the other coordinates remain chaotic. For the Fig. 14.9, on the other hand, e_2 (CS) , e_3 (CS), E_4 (AS) also converge to zero.

Depending on the initial conditions, both AS and CS can also be observed. To study how these phenomena depend upon the initial conditions, we change the initial conditions for fixed k.

The results are shown in Figs. 14.10 and 14.11. Fig. 14.10 (a) shows that the difference $e_4=v_1 - v_2$ tends to zero. In Fig. 14.10 (b), the sums $E_2=y_1 + y_2$, $E_3=u_1 + u_2$ tend to zero. Comparing Fig. 14.8 with Fig. 14.10 one can find that they have different behaviors. The only reason lies in the different initial conditions. Similar result also exists by comparing Fig. 14.9 with Fig. 14.11. But we have not observed the intermittent synchronization and AS states as declared in Ref. [22].

The simulation results are shown in Fig. 14.12 for different value of k. The solid circle "•" and triangle "▲" correspond to CS where parameter values k leads to synchronized behavior. While white circle "○" and triangle "△" indicate AS. The blank space means no exist AS or CS. We can see that the system (14.3) and system (14.4) tend to either AS or CS by using combination of different value of k and initial values. However, as we can see from Fig. 14.12, both cases agree well with the fact that the system goes to either synchronized state or anti-synchronized state depending on initial values and on k. When k = 0.9 ~ 0.96 and 1.04 ~ 1.10, neither synchronization nor anti-synchronization is found.

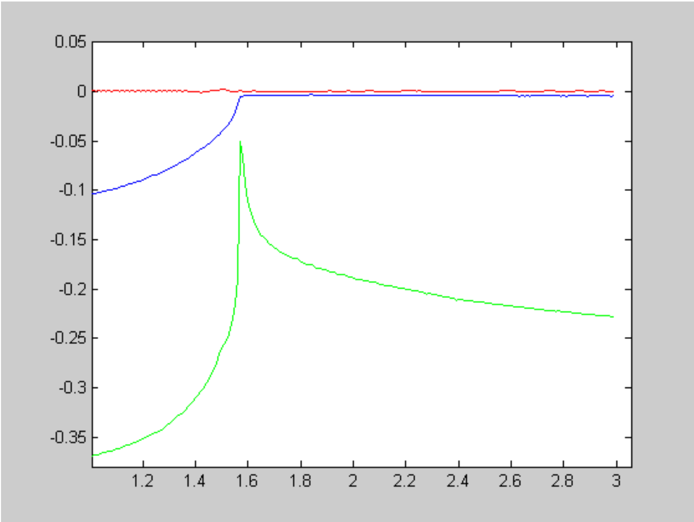


Fig. 14.1. Lyapunov exponent diagram of the double van der Pol system for c between 1.0 and 3.0

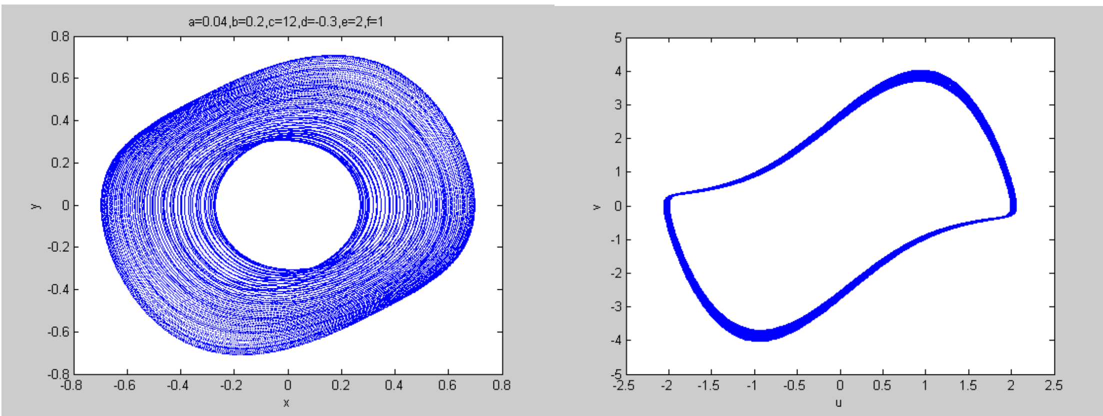


Fig. 14.2. Phase portraits of the double van der Pol system

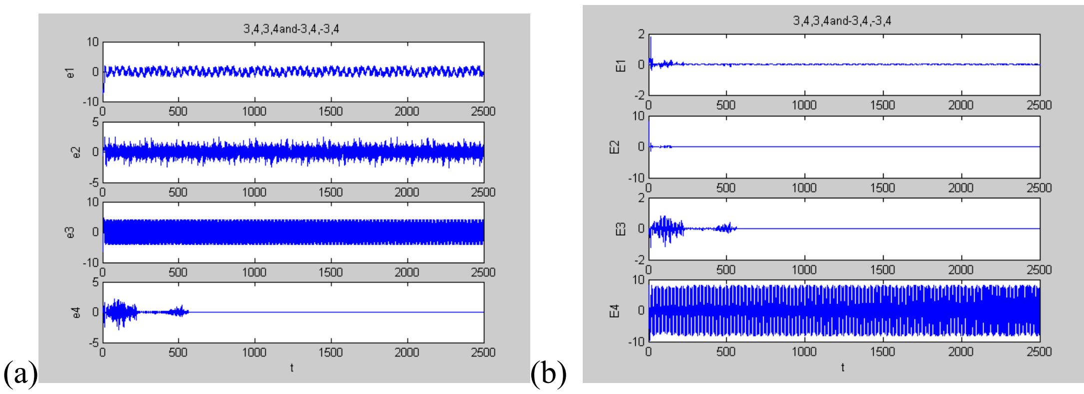
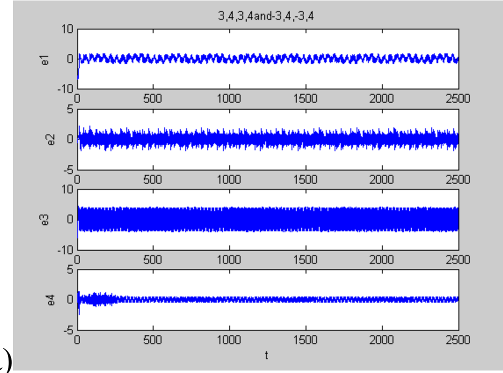


Fig. 14.3. CS and AS for initial condition(x_2, y_2, u_2, v_2)= (-3, 4, -3, 4) and k=1, (a) e_1, e_2, e_3, e_4 (b) E_1, E_2, E_3, E_4.

(a) 

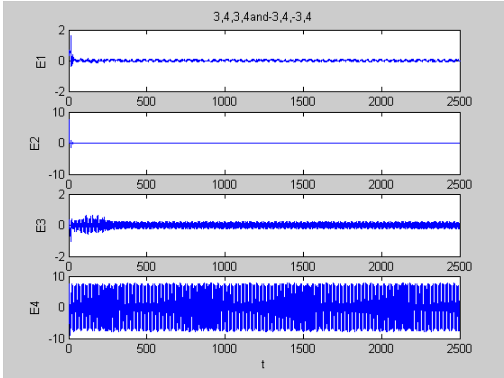
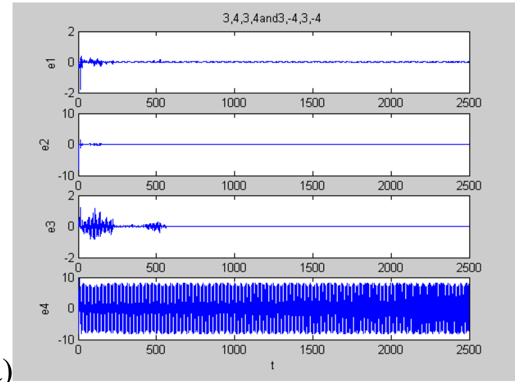
(b) 

Fig. 14.4. AS for initial condition $(x_2, y_2, u_2, v_2) = (-3, 4, -3, 4)$ and k=0.9, (a) e_1, e_2, e_3, e_4 (b) E_1, E_2, E_3, E_4.

(a) 

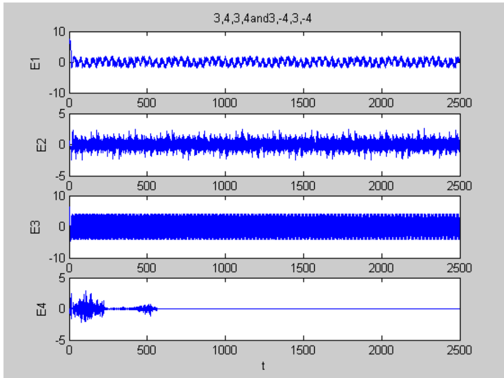
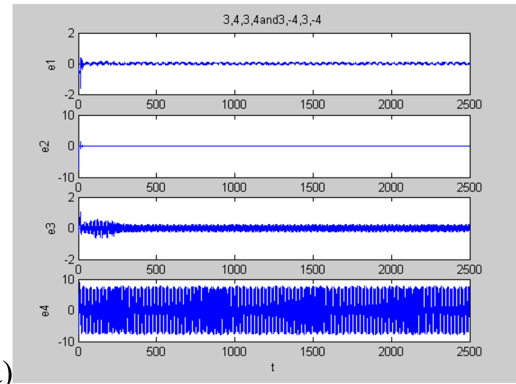
(b) 

Fig. 14.5. CS and AS for initial condition $(x_2, y_2, u_2, v_2) = (3, -4, 3, -4)$ and k=1, (a) e_1, e_2, e_3, e_4 (b) E_1, E_2, E_3, E_4.

(a) 

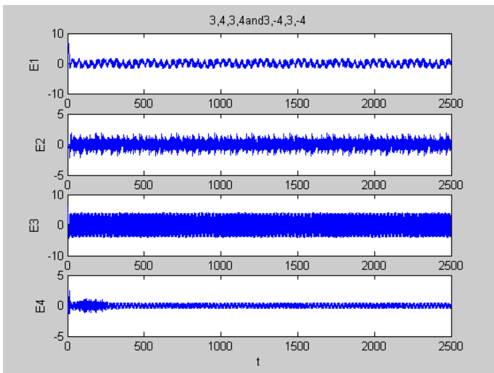
(b) 

Fig. 14.6. CS for initial condition$(x_2, y_2, u_2, v_2) = (3, -4, 3, -4)$ and k=0.9, (a) e_1, e_2, e_3, e_4 (b) E_1, E_2, E_3, E_4.

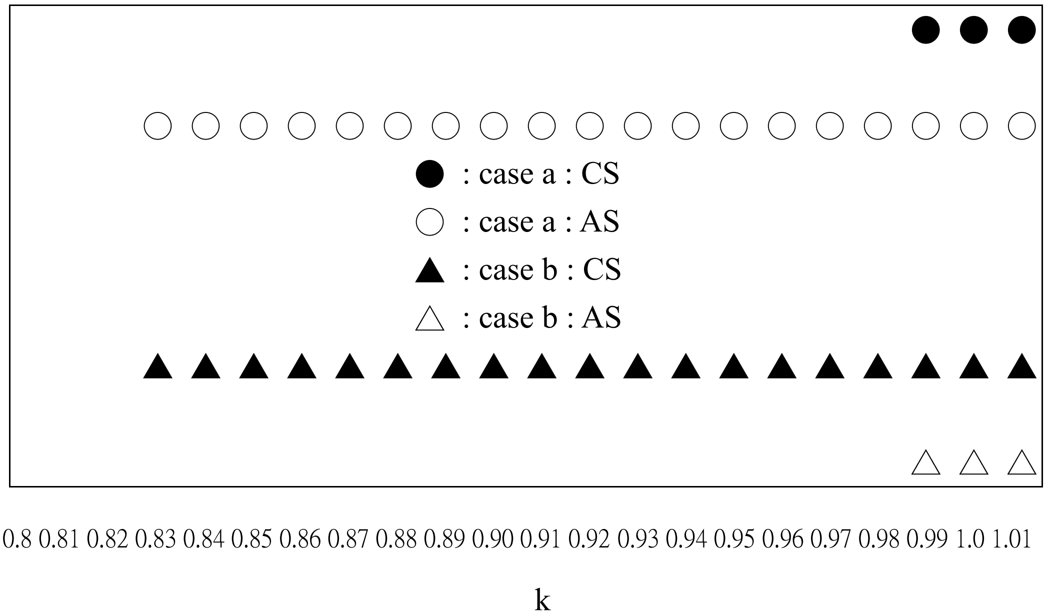


Fig. 14.7. CS or AS vs. the k for different initial conditions,
case a: $(x_1, y_1, u_1, v_1) = (3, 4, 3, 4)$ and $(x_2, y_2, u_2, v_2) = (-3, 4, -3, 4)$
case b: $(x_1, y_1, u_1, v_1) = (3, 4, 3, 4)$ and $(x_2, y_2, u_2, v_2) = (3, -4, 3, -4)$

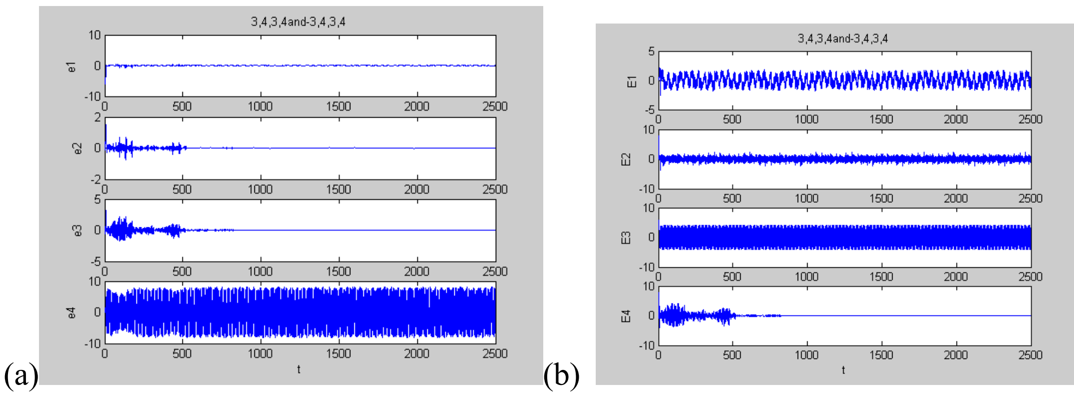


Fig. 14.8. CS and AS for initial condition$(x_2, y_2, u_2, v_2) = (-3, 4, 3, 4)$ and k=0.97,
(a) e_1, e_2, e_3, e_4 (b) E_1, E_2, E_3, E_4.

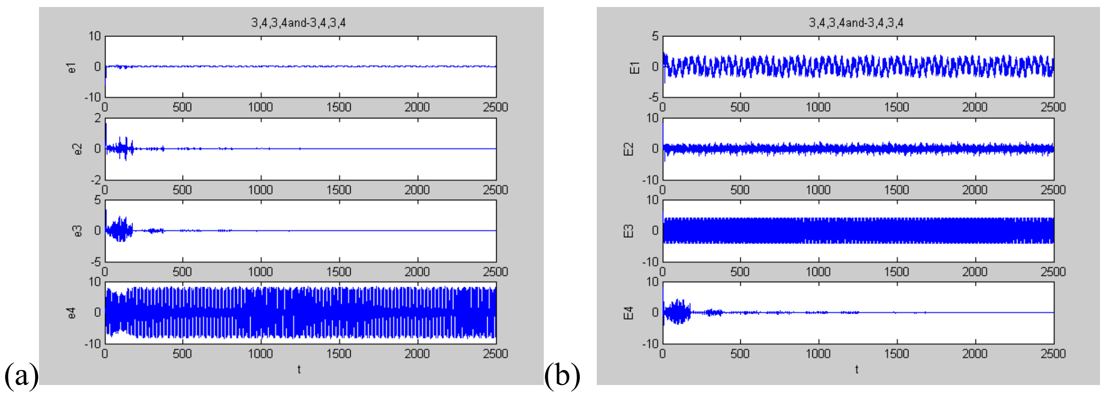


Fig. 14.9. CS and AS for initial condition $(x_2, y_2, u_2, v_2) = (-3, 4, 3, 4)$ and k=1.02,
(a) e_1, e_2, e_3, e_4 (b) E_1, E_2, E_3, E_4.

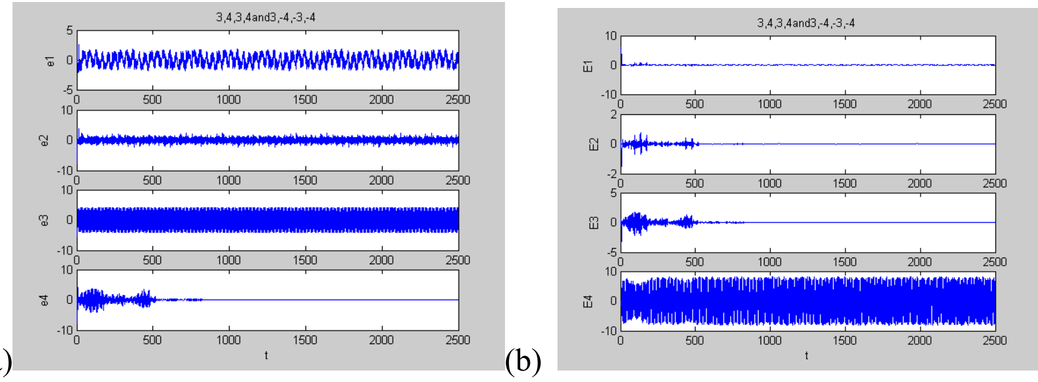


Fig. 14.10. CS and AS for initial condition $(x_2, y_2, u_2, v_2) = (3, -4, -3, -4)$ and k=0.97, (a) e_1, e_2, e_3, e_4 (b) E_1, E_2, E_3, E_4.

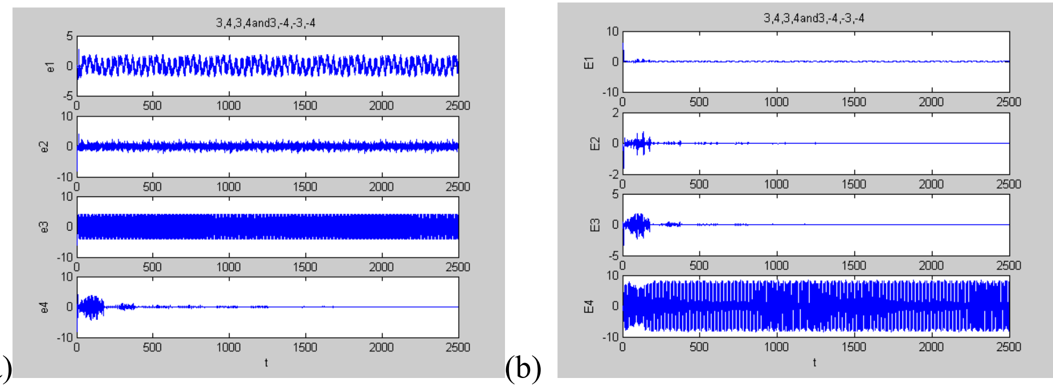


Fig. 14.11. CS and AS for initial condition $(x_2, y_2, u_2, v_2) = (3, -4, -3, -4)$ and k=1.02, (a) e_1, e_2, e_3, e_4 (b) E_1, E_2, E_3, E_4.

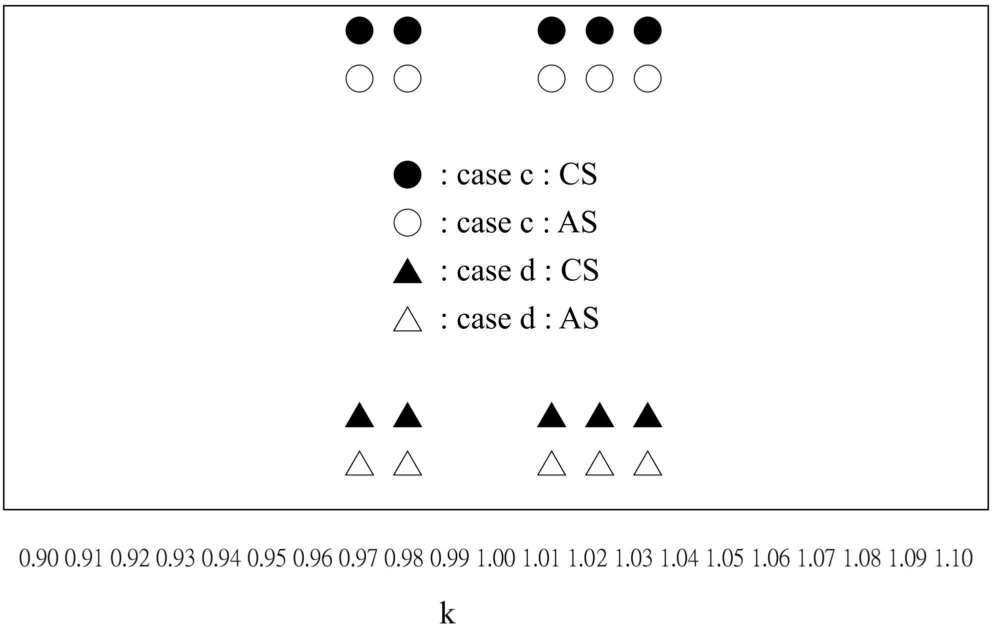


Fig. 14.12. CS or AS vs. the k for different initial conditions,
case c: $(x_1, y_1, u_1, v_1) = (3, 4, 3, 4)$ and $(x_2, y_2, u_2, v_2) = (-3, 4, 3, 4)$
case d: $(x_1, y_1, u_1, v_1) = (3, 4, 3, 4)$ and $(x_2, y_2, u_2, v_2) = (3, -4, -3, -4)$

Reference

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Chapter 22
Chaos Control and Synchronization of Double Mackey-Glass System by Noise Excitation of Parameters

22.1 Preliminaries

In recent years, chaos control and chaos synchronization have been received a great deal of interests among scientists from various fields. The chaotic system performances are often desired to be avoided and to be controlled to achieve some specific behavior. In 1990, Ott et al. [1] utilizeed small time-dependent perturbations of an available system parameter to convert a chaotic attractor to any one of a large number of possible attracting time-periodic motions. In [2] a linear feedback controller is designed to control the Chen system. An algorithm for suppressing the chaotic oscillation in non-linear systems with singular Jacobian matrices has been developed based on the Lyapunov–Krasovskii method by Kuang et al. [3]. Linear feedback control and adaptive control algorithm are used to control chaos effectively in [4]. Many different techniques and methods have been proposed to achieve chaos control, which include sliding method control, impulsive control method, linear feedback control, nonlinear feedback control and H_∞ control method etc.

Since the pioneering work was given by Pecorra and Carroll [5], chaos synchronization [6-12] has become an important topic in physical engineering sciences. Many effective control schemes have been developed in a variety of fields, such as parameters adaptive control [13-21], observer based control [22, 23], variable structure control [24, 25], active control [26-35], nonlinear control [36-38] and so on. The applications of chaos synchronization are implemented extensively in secure communications, chemical, physical, and biological systems and neural networks.

In this , a control method called parameter excited method [39] is applied to control a double Mackey-Glass chaotic system and to synchronize two uncoupled double Mackey-Glass systems. By replacing a parameter of the chaotic system by a noise signal, its chaotic motion can be eliminated. By replacing the corresponding parameters of two identical chaotic systems by a noise signal, these two chaotic systems with different initial conditions can be synchronized. For some chaotic systems, such as physical and electrical systems, which are difficult or even impossible to couple, this method is effective and potential in practice [40].

22.2 Chaos control and synchronization for uncoupled double Mackey-Glass system by parameter excited method

We consider a double Mackey-Glass system as follow:

$$\begin{cases} \dot{x}_1 = \dfrac{bx_{1\tau}}{1+x_{1\tau}^n} - rx_1 \\ \dot{x}_2 = \dfrac{bx_{2\tau}}{1+x_{2\tau}^n} - rx_2 - x_1 \end{cases} \tag{22.1}$$

where x_1, x_2 are state variables and $x_{i\tau} = x_i(t-\tau)$, $(i=1,2)$, τ is a time delay, and b, r, n are constant parameter. Eq. (22.1) is a generalized system of a classical system established by Mackey and Glass [41]. It is a model of blood production of patient with leukemia.

We keep the delay time fixed at 20 second ($\tau = 20$) and the parameters are taken as $b = 0.2$, $r = 0.1$, $n = 10$. The initial values are given as $(x_{10}, x_{20}) = (0.1, 0.1)$. With these data, the equilibrium point (0,0,0) of Eq. (22.1) is unstable and leads to chaotic motion. The bifurcation diagram is shown in Fig. 22.1 [42]. By replacing a parameter by a noise signal, the chaotic motion can be eliminated and the equilibrium point becomes asymptotically stable.

Next, a second identical double Mackey-Glass system is given by

$$\begin{cases} \dot{y}_1 = \dfrac{by_{1\tau}}{1+y_{1\tau}^n} - ry_1 \\ \dot{y}_2 = \dfrac{by_{2\tau}}{1+y_{2\tau}^n} - ry_2 - y_1 \end{cases} \tag{22.2}$$

where the parameters and time delay τ are the same as Eq. (22.1) but with different initial values $(y_{10}, y_{20}) = (0.2, 0.2)$. We use the parameter excited method to synchronize these two identical double Mackey-Glass chaotic systems with different initial conditions. By replacing the corresponding parameters of these two chaotic systems by a noise signal, the synchronizations are achieved successfully in major cases.

22.3 Numerical simulations of chaos control

In Sections 22.3 and 22.4, the numerical simulations which carried out by Simulink environment of MATLAB are presented. The corresponding parameter is replaced by Gaussian noise, Rayleigh noise, Rician noise and uniform noise respectively and the noise strength is adjustable. With suitable noise strengths, the chaotic motions of double Mackey-Glass system can be eliminated, and the motions converge to zero.

22.3.1 Gaussian noise

The noise that has a probability density function (PDF) of the normal distribution

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} \tag{22.3}$$

is called Gaussian distributed noise, where μ is the mean and σ^2 is the variance of the random variable. The Simulink Communication toolbox provides the Gaussian noise generator block. In our case, we take the mean as 0 and the variance as 1. Therefore, μ is a constant vector and K is a constant matrix.

Parameter b and parameter r of Eq. (22.1) are substituted respectively by p_1F_1 where F_1 is Gaussian noise and p_1 is the noise strength. When b is replaced, the chaotic behavior is suppressed and the system is asymptotically stable at the origin as $p_1 < 0.6$. Fig 22.2 shows the time histories of the variables x_1 and x_2 with noise strength $p_1 = 0.5$. When r is replaced, the trajectories gradually increase unbounded when $p_1 > 0.5$ and chaotic behavior cannot eliminated with any noise strength.

22.3.2 Rayleigh noise

The probability density function of Rayleigh distributed noise is

$$f(x)=\begin{cases}\frac{x}{\sigma^2}\exp\left(\frac{-x^2}{2\sigma^2}\right) & x\geq 0\\ 0 & x<0\end{cases} \tag{22.4}$$

where σ^2 is known as the fading envelope of the Rayleigh distribution. The Simulink Communication toolbox provides the Rayleigh noise generator block. We specify $\sigma=1$ in the case.

Parameter r of Eq. (22.1) is substituted by p_2F_2 where F_2 is Rayleigh noise and p_2 is the noise strength. The chaotic motion of the system can be eliminated when $p_2\geq 0.165$. In other words, noise excitation of parameters makes the double Mackey-Glass system asymptotically stable at the origin. The time histories of the variables x_1 and x_2 with noise strength 0.2 are shown in Fig. 22.3.

22.3.3 Rician noise

The probability density function of Rician distributed noise is

$$f(x)=\begin{cases}\frac{x}{\sigma^2}I_0\left(\frac{mx}{\sigma^2}\right)\exp\left(-\frac{x^2+m^2}{2\sigma^2}\right) & x\geq 0\\ 0 & x<0\end{cases} \tag{22.5}$$

where σ is the standard deviation of the Gaussian distribution that underlies the Rician distribution noise, I_0 is the modified 0th-order Bessel function of the first kind given by

$$I_0(y)=\frac{1}{2\pi}\int_{-\pi}^{\pi}e^{y\cos t}dt \tag{22.6}$$

And m is defined as $m^2=m_I^2+m_Q^2$ where m_I and m_Q are the mean values of two independent Gaussian components. The Simulink Communication toolbox provides the Rician noise generator block. We assign that $\sigma=1$ and K-factor 2 in the case, which the K-factor has a definition as a form of $K=m^2/2\sigma^2$.

Parameter r of Eq. (22.1) is substituted by p_3F_3 where F_3 is Rician noise and p_3 is the noise strength. The chaotic motion of the system can be eliminated as $p_3 > 0.08$. Numerical simulation, illustrated in Fig. 22.4, shows that the motion is asymptotically stabilized to the equilibrium point (0,0,0) by the noise excitation method with noise strength $p_3 = 0.1$.

22.3.4 Uniform noise

The probability density function of Uniform distributed noise is

$$f(x) = \begin{cases} \dfrac{1}{d-c} & \text{if } c \le x \le d \\ 0 & \text{otherwise} \end{cases} \tag{22.7}$$

The mean value of this density function μ and its variance σ are given as follow:

$$\mu = \frac{c+d}{2} \tag{22.8}$$

$$\sigma^2 = \frac{(d-c)^2}{12} \tag{22.9}$$

The Simulink Communication toolbox provides the Rician noise generator block. We specify lower bound $c = 0$ and upper bound $d = 1$ in the case.

Parameter r of Eq. (22.1) is substituted by p_4F_4 where F_4 is uniform noise and p_4 is the noise strength. When $p_4 > 0.4$, the chaotic motion can be eliminated and the system is asymptotically stable at the origin. Fig. 22.5 illustrates the time histories of the states of the system with noise strength $p_4 = 0.5$.

22.4 Numerical simulations of chaos synchronizations

In this section, we use the parameter excited method by replacing the corresponding parameters by Gaussian noise, Rayleigh noise, Rician noise and uniform noise respectively, to synchronize two uncoupled double Mackey-Glass systems. The system parameter b and r are substituted by noise respectively and the noise strength is variable. The error states which are defined as $e_i = x_i - y_i$, $(i = 1,2)$ will converge to zero as $t \to \infty$ when the strength is chosen properly. The results of simulations show that the synchronizations are successfully achieved via parameter excited method in major cases.

22.4.1 Gaussian noise

We replace two corresponding parameters b and two corresponding parameters r of the system (22.1) and (22.2) by p_1F_1 respectively where F_1 is Gaussian noise and p_1 is the noise strength. When b is replaced, the trajectory of the states converge to zero with $p_1 < 0.6$. When the strength is increased, the error states oscillate. However, in a small range of 0.61~0.625, the systems show temporary chaos synchronization [43]. Fig.22.6 shows error e_1, e_2 and the time histories of the state variables with noise strength $p_1 = 0.625$. As r is replaced, the trajectory of the states gradually increase unbounded when the strength is larger than 0.5. In other words, Gaussian noise excitation can be used only when the noise strength $p_1 < 0.5$ to synchronize two identical double Mackey-Glass systems with different initial conditions. Fig.22.7 shows error e_1, e_2 and the time histories of the state variables with noise strength $p_1 = 0.05$.

22.4.2 Rayleigh noise

We replace two corresponding parameters b and two corresponding parameters r of the system (22.1) and (22.2) by p_2F_2 respectively where F_2 is Rayleigh noise and p_2 is the noise strength. The synchronizations are successfully achieved in both cases. When b is replaced, we assume that the noise strength

$$p_2 = 0.25i, \quad i = 1,2,...,50 \tag{22.10}$$

Fig.22.8 shows the result of the simulation. We find that the synchronizations of two double Mackey-Glass systems are achieved with major noise strength, but failed with minor cases. Fig.22.9 and Fig.22.10 show the error states and the phase portraits of the systems with $p_2 = 0.5$ and $p_2 = 9.25$. The error states approach to zero in the former case, but not in the latter case. However, if we choose the strength appropriately, the chaos synchronizations are accomplished.

When two corresponding r are substituted, only a small interval of the noise strength $0.105 \le p_2 \le 0.16$ leads to synchronization. Error e_1, e_2 and the phase portraits of the systems with $p_2 = 0.16$ are shown in Fig. 22.11. Besides, in a small range of 0.08~0.1, the systems show temporary chaos synchronization. Fig.22.12 shows error e_1, e_2 and the time histories of the state variables with noise strength $p_2 = 0.08$. For the values of p_2 other than these ranges, chaos synchronization cannot be obtained.

22.4.3 Rician noise

We replace two corresponding parameter b and two corresponding parameters r of the system (22.1) and (22.2) with p_3F_3 respectively where F_3 is Rician noise and p_3 is the noise strength. In the case of b, we assume that the noise strength

$$p_3 = 0.25i, \quad i = 1,2,...,50 \tag{22.11}$$

As shown in Fig. 22.12, the Rician noise is more effective than Rayleigh noise. In the range of $0.25 \le p_3 \le 12.5$, the synchronization is achieved except p_3 takes 1.25, 4.75, 11.25, 12 and 12.25. Fig. 22.14 shows the error states and the phase portraits of the systems with noise strength $p_3 = 5$.

In the case of r, synchronization is obtained only when $0.06 \le p_3 \le 0.08$. Error e_1, e_2 and the phase portraits of the systems with noise strength $p_3 = 0.07$ are given in Fig. 22.15. The error states oscillate when $p_3 < 0.06$ and the state variables of the system converge to zero as $p_3 > 0.08$. Chaos synchronization cannot obtain for the values of p_2 other than 0.105~0.16.

22.4.4 Uniform noise

We replace two corresponding parameters b and two corresponding parameters r of the system (22.1) and (22.2) by p_4F_4 respectively where F_4 is uniform noise and p_4 is the noise strength. In the case of b, we assume that the noise strength

$$p_3 = 0.25i, \quad i = 1,2,...,50 \tag{22.13}$$

As shown in Fig. 22.16, the synchronization is achieved except for a few p_4 in the range of $0.25 \le p_4 \le 12.5$. Fig. 22.17 shows the error states and the phase portraits of the systems with noise strength $p_4 = 10.25$.

In the case of r, synchronization is obtained only when $0.26 \le p_4 \le 0.4$. Error e_1, e_2 and the phase portraits of the systems with noise strength $p_4 = 0.27$ are given in Fig. 22.18. The error states oscillate while $p_4 < 0.26$ and the states of the system converge to zero as $p_4 > 0.4$. Two systems cannot be synchronized with the values of the noise strength other than 0.26~0.4.

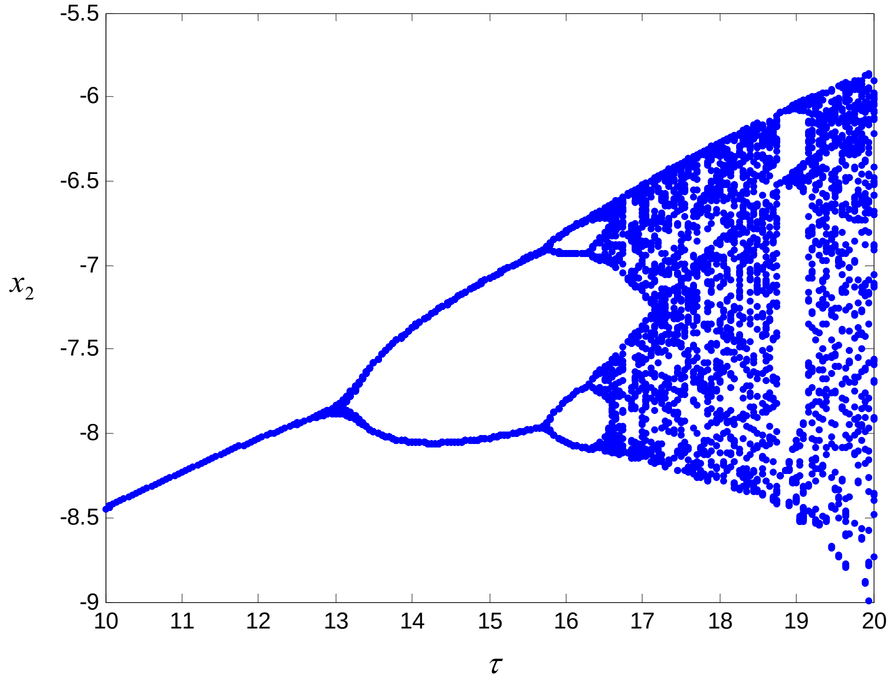

Fig.22.1 The bifurcation diagram for Double Mackey-Glass system.

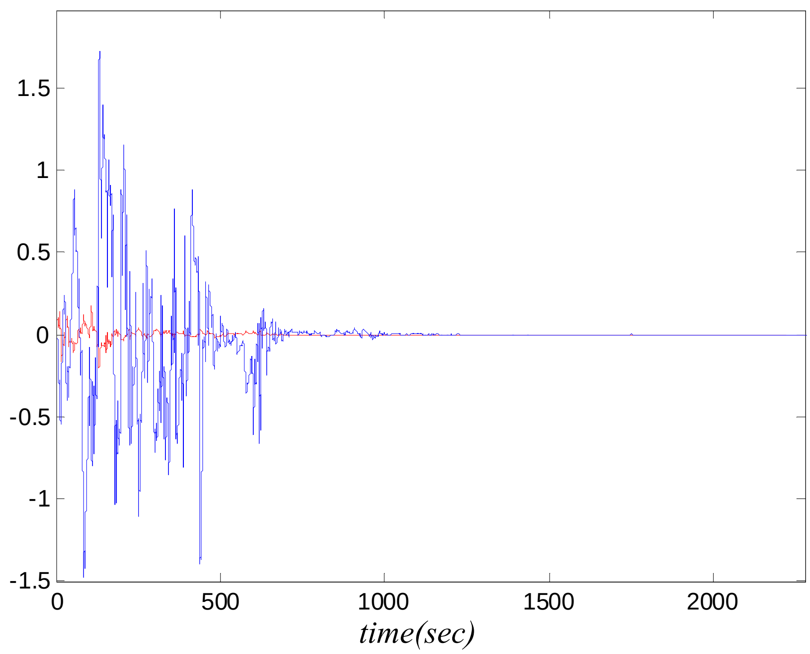

Fig.22.2 The time histories of x_1 (red) and x_2 (blue) of the double Mackey-Glass system when parameter b is substituted by a Gaussian noise with noise strength $p_1 = 0.5$.

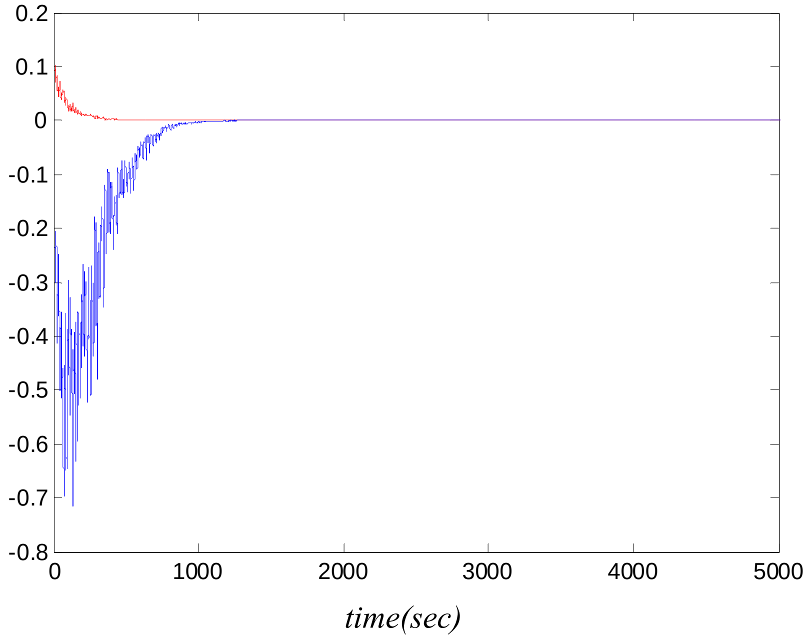

Fig.22.3 The time histories of x_1 (red) and x_2 (blue) of the double Mackey-Glass system when parameter b is substituted by a Rayleigh noise with noise strength $p_2 = 0.2$.

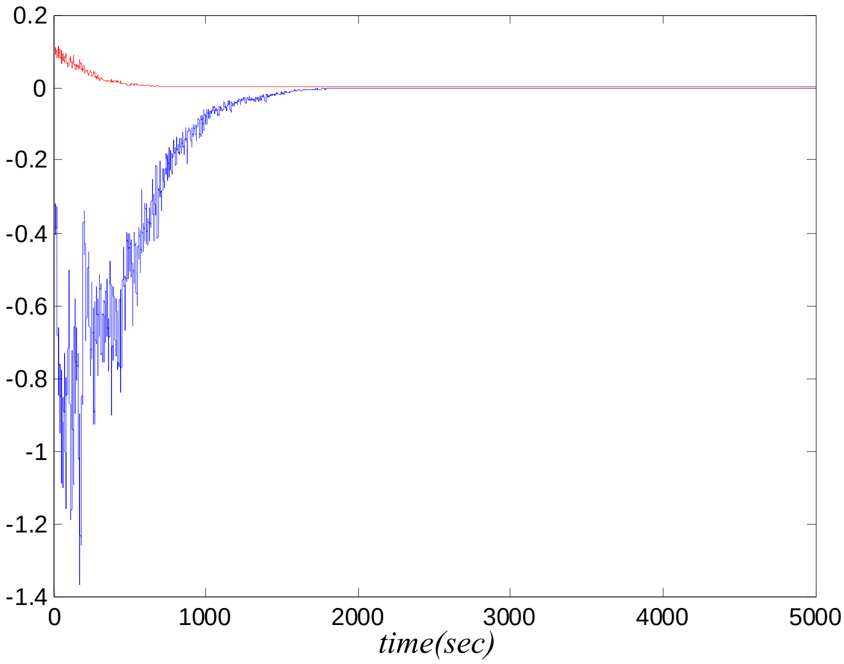

Fig.22.4 The time histories of x_1 (red) and x_2 (blue) of the double Mackey-Glass system when parameter b is substituted by a Rician noise with noise strength $p_3 = 0.1$.

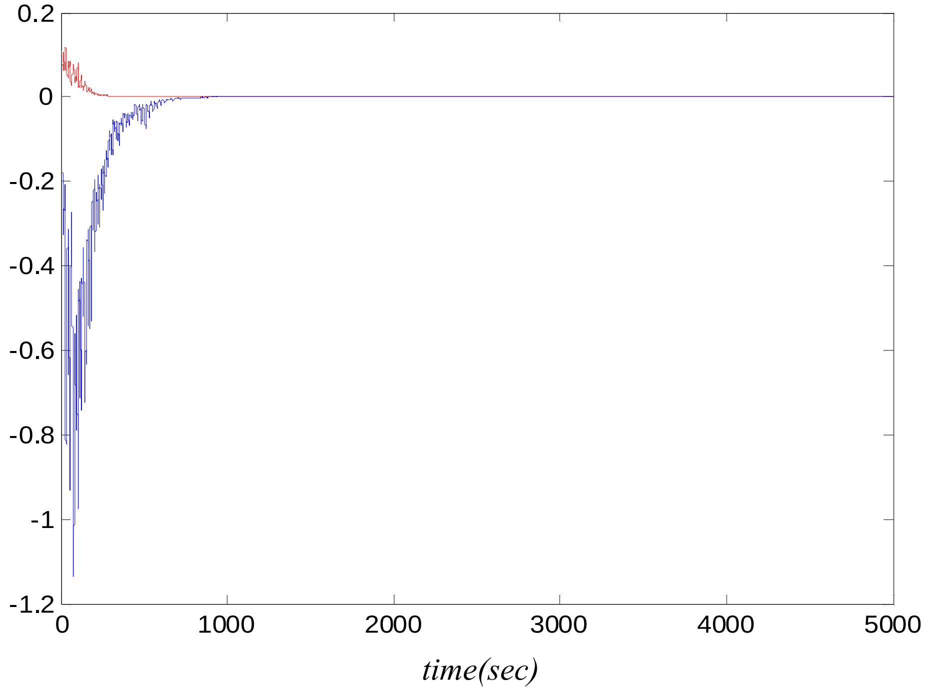

Fig.22.5 The time histories of x_1 (red) and x_2 (blue) of the double Mackey-Glass system when parameter b is substituted by a uniform noise with noise strength $p_4 = 0.5$.

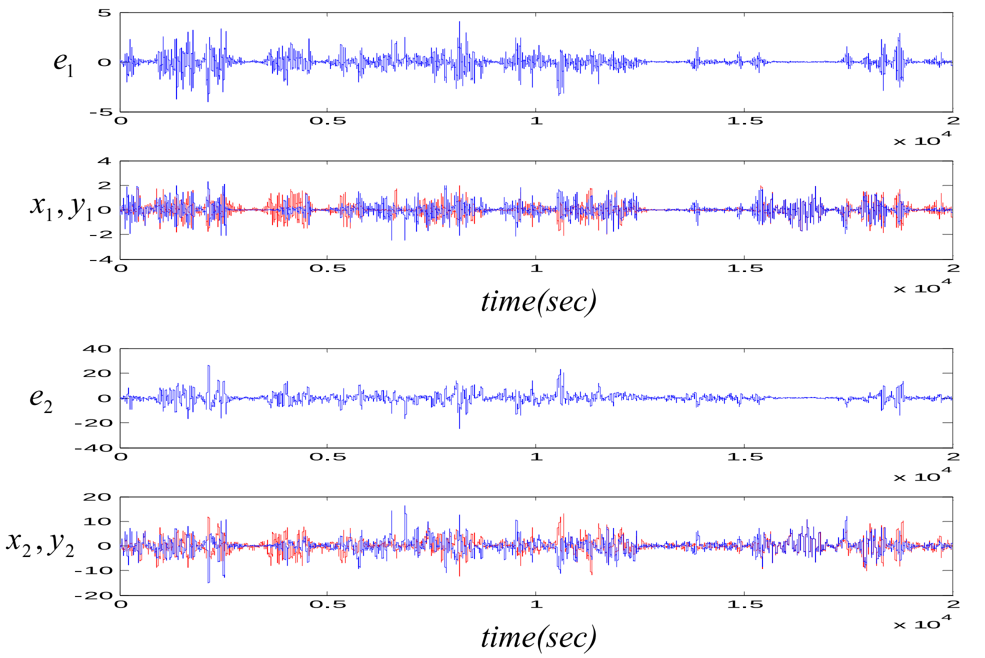

Fig.22.6 The error states and the time histories of x_1, y_1 (red) and x_2, y_2 (blue) of the double Mackey-Glass systems when two corresponding parameters r are substituted by a Gaussian noise with noise strength $p_1 = 0.625$.

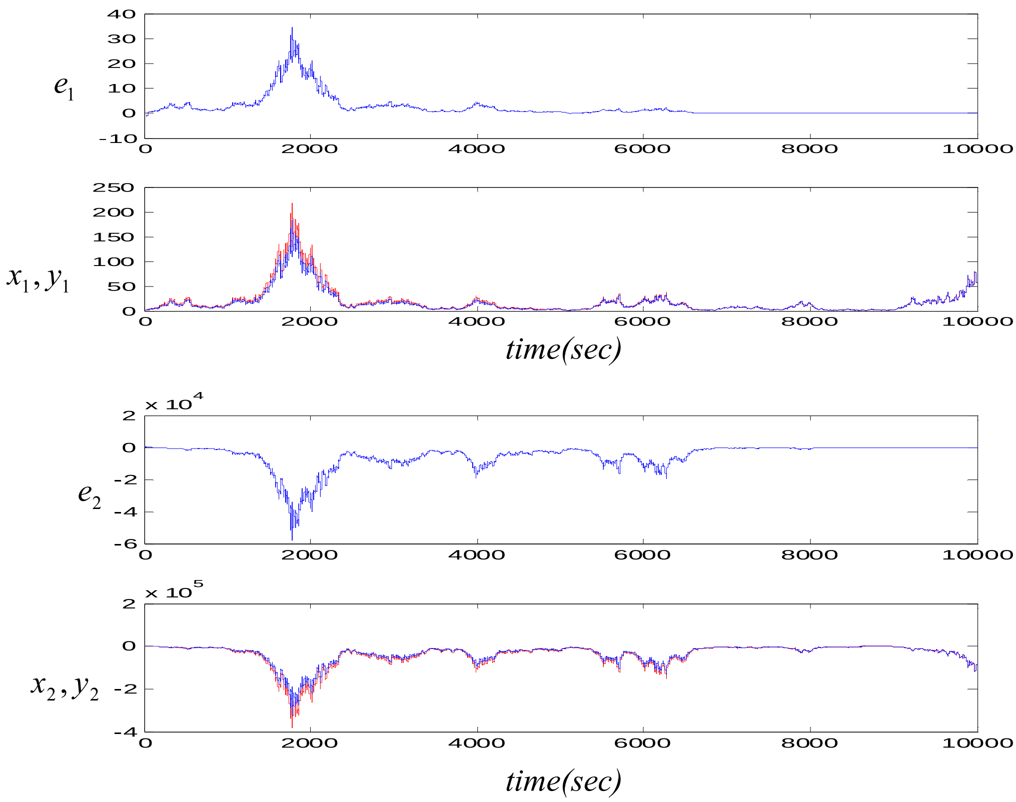

Fig.22.7 The error states and the time histories of x_1, y_1 (red) and x_2, y_2 (blue) of the double Mackey-Glass systems when two corresponding parameters r are substituted by a Gaussian noise with noise strength $p_1 = 0.05$.

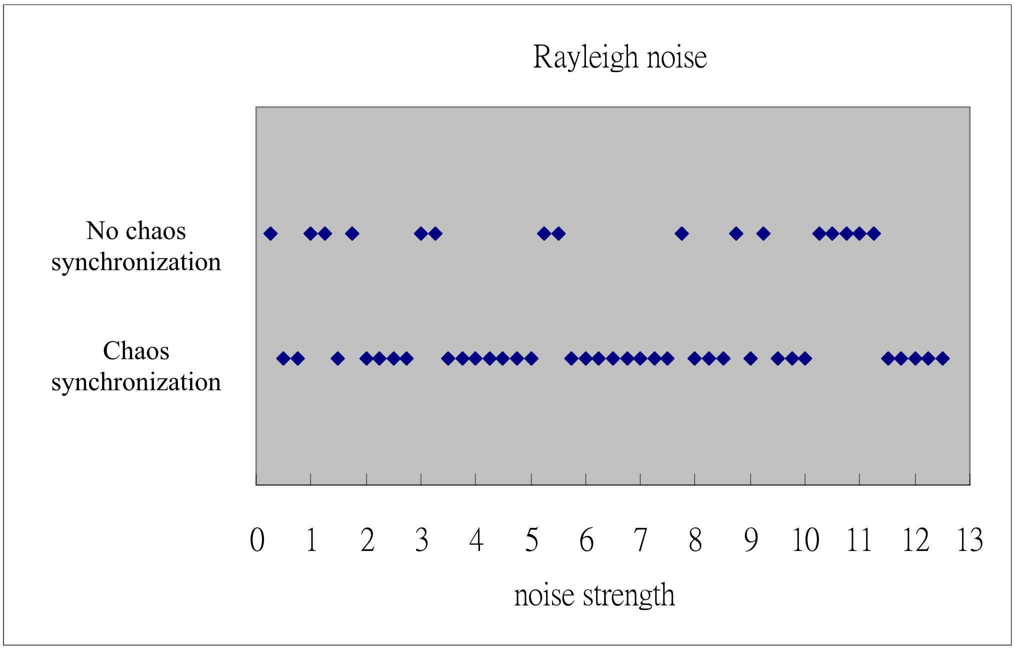

Fig.22.8 Two corresponding parameters b are substituted by a Rayleigh noise with different noise strengths.

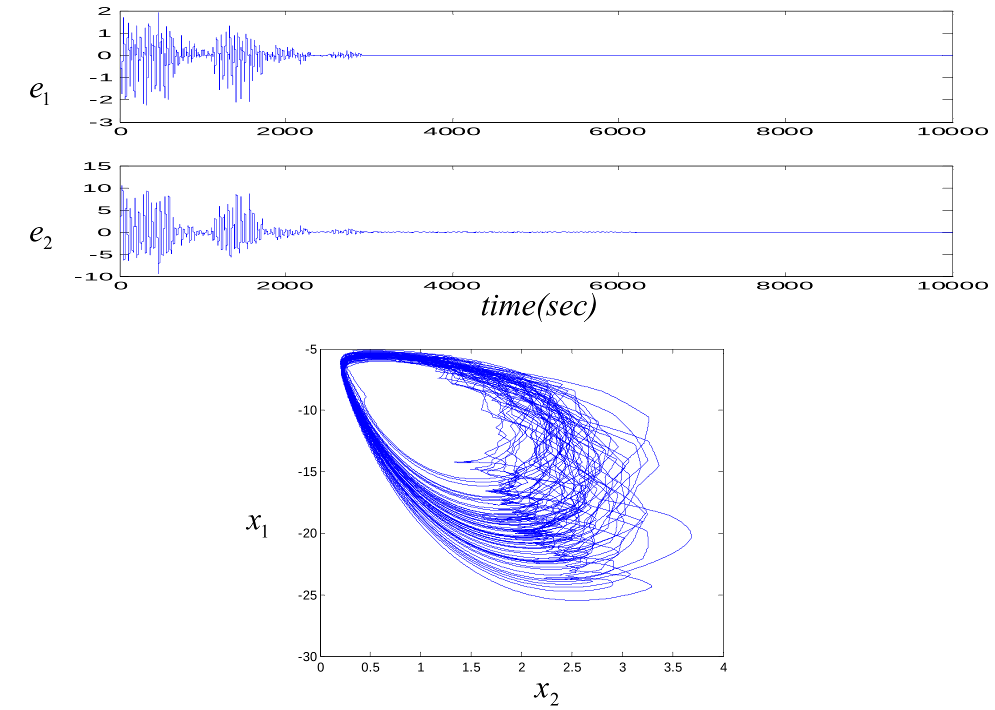

Fig.22.9 The error states and the phase portraits of the double Mackey-Glass systems when two corresponding parameters b are substituted by a Rayleigh noise with noise strength $p_2 = 0.5$.

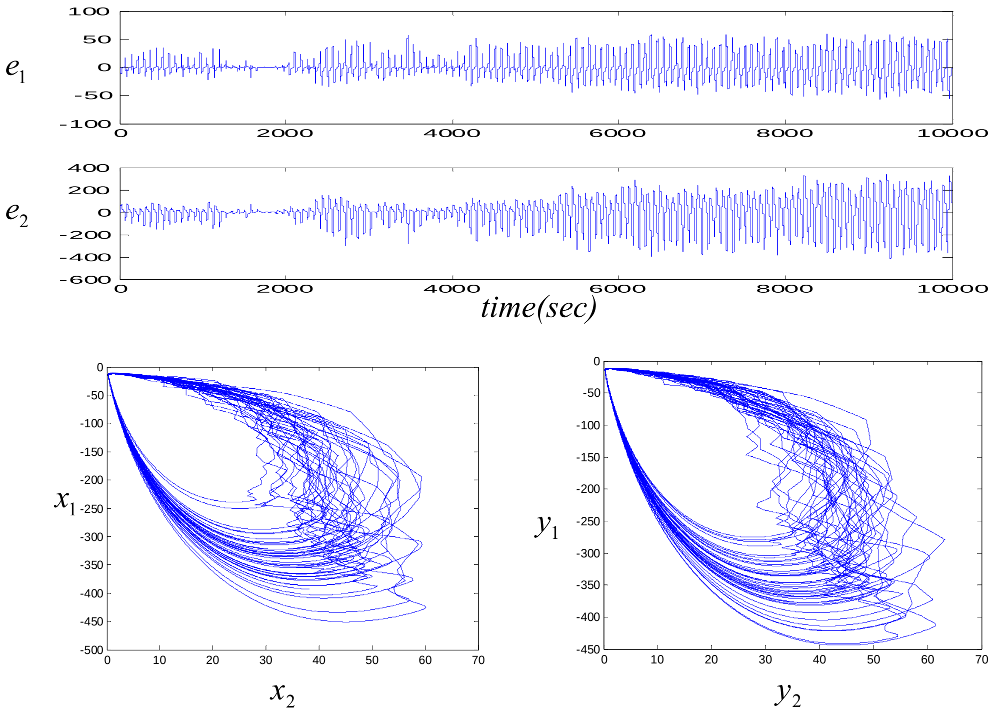

Fig.22.10 The error states and the phase portraits of the double Mackey-Glass systems when two corresponding parameters b are substituted by a Rayleigh noise with noise strength $p_2 = 9.25$.

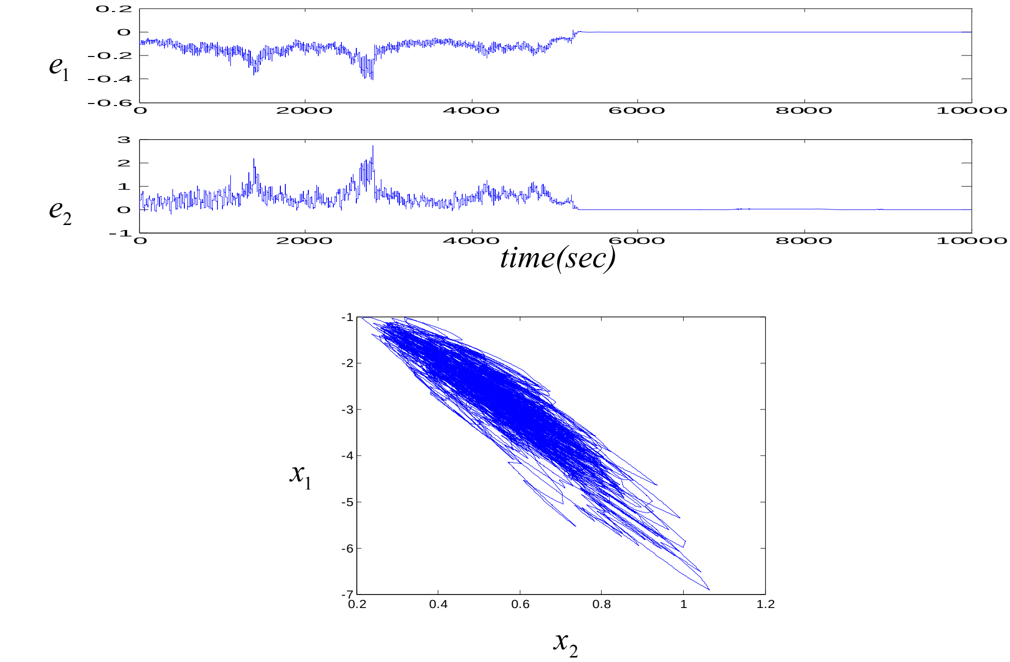

Fig.22.11 The error states and the phase portraits of the double Mackey-Glass systems which two corresponding parameters r are substituted by a Rayleigh noise with noise strength $p_2 = 0.16$.

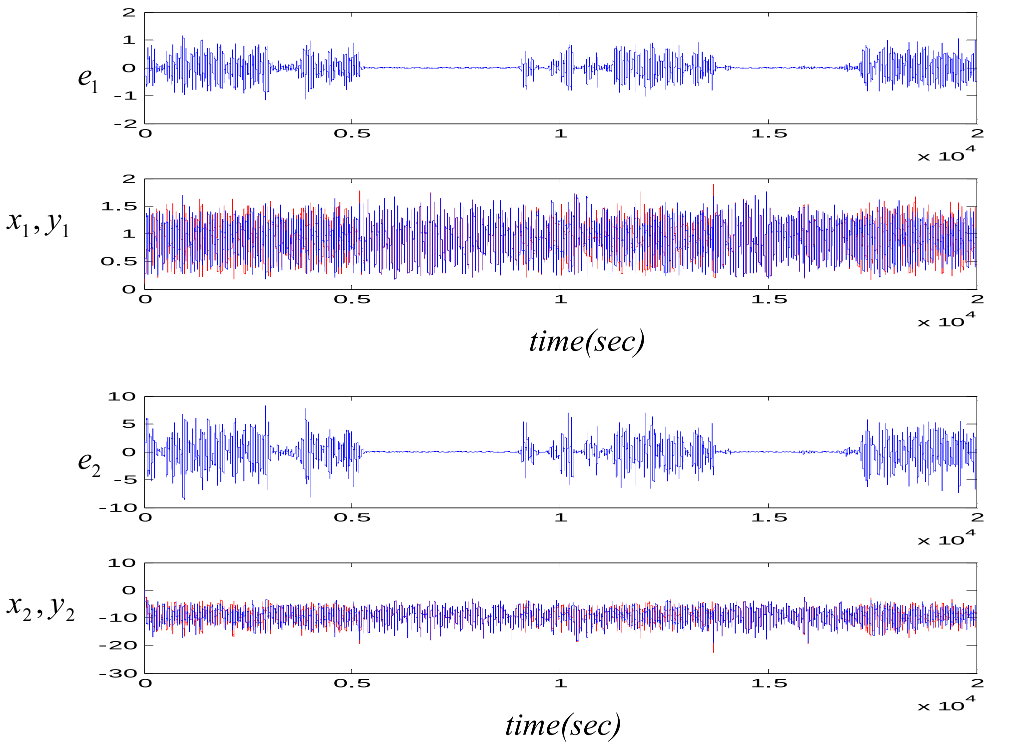

Fig.22.12 The error states and the time histories of x_1, y_1 (red) and x_2, y_2 (blue) of the double Mackey-Glass systems when two corresponding parameters r are substituted by a Rayleigh noise with noise strength $p_2 = 0.08$.

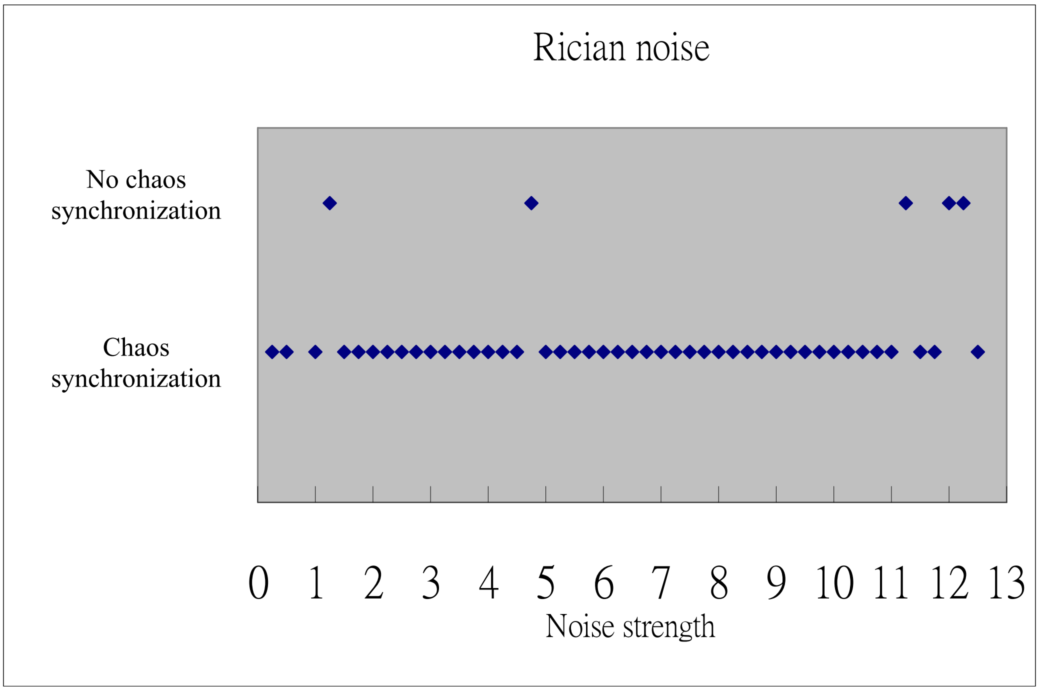

Fig.22.13 Two corresponding parameters b are substituted by a Rician noise with different noise strengths.

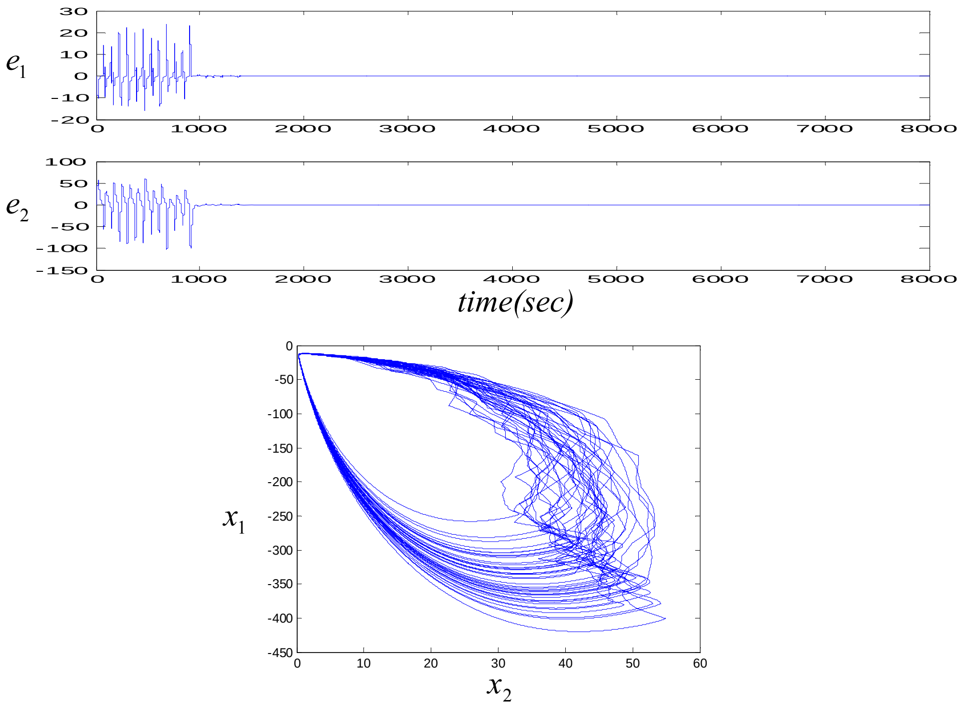

Fig.22.14 The error states and the phase portraits of the double Mackey-Glass systems when two corresponding parameters b are substituted by a Rician noise with noise strength $p_3 = 5$.

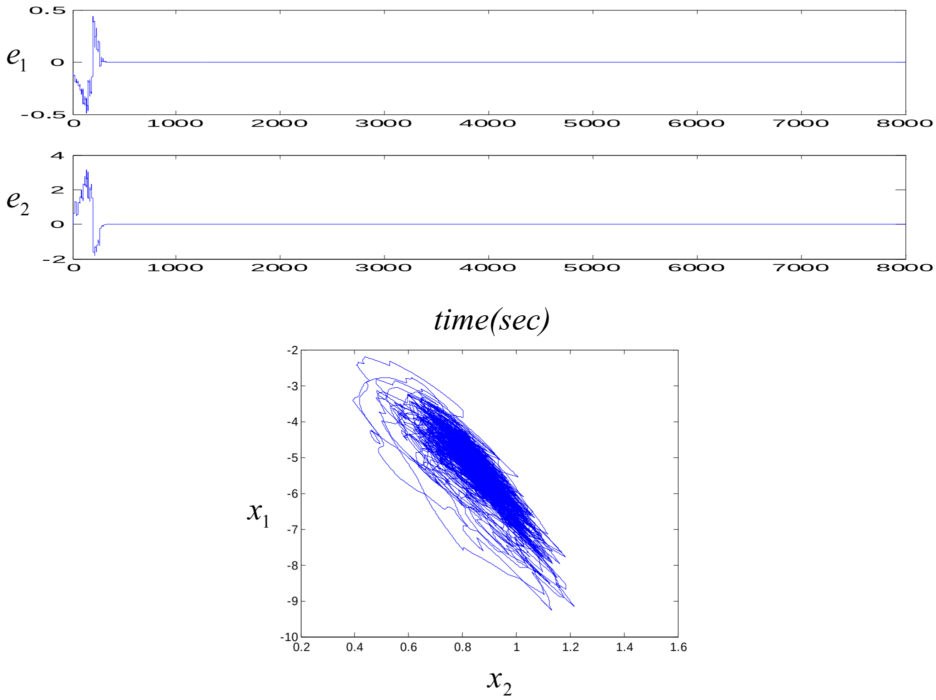

Fig.22.15 The error states and the phase portraits of the double Mackey-Glass systems when two corresponding parameters r are substituted by a Rician noise with noise strength $p_3 = 0.07$.

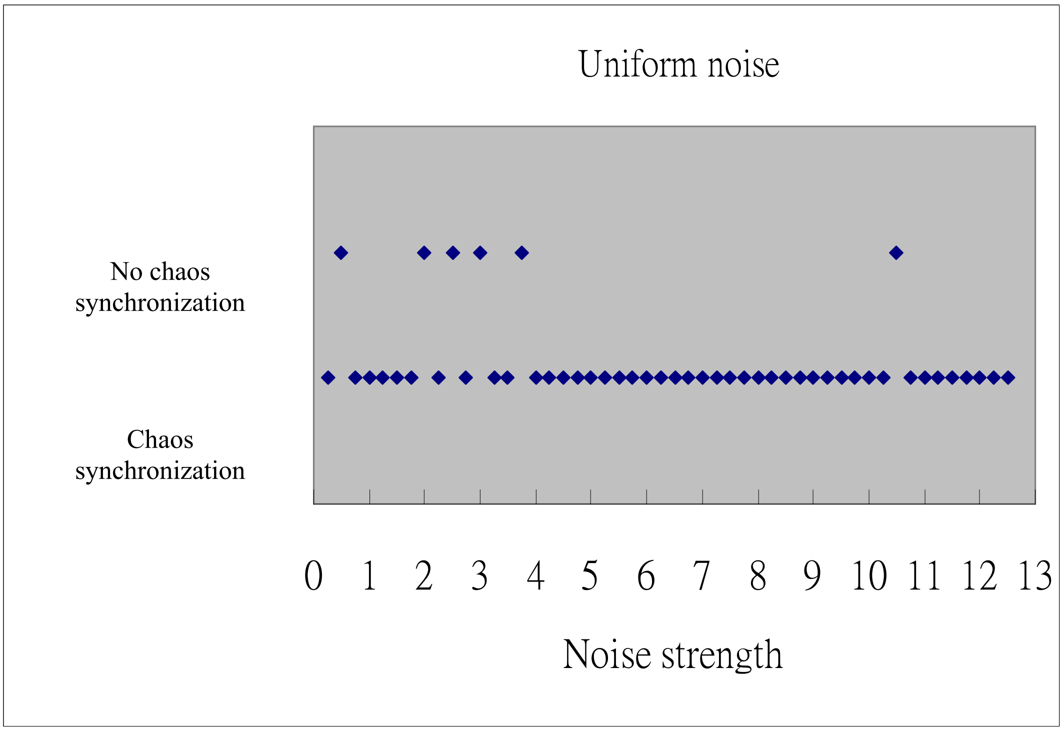

Fig.22.16 Two corresponding parameters b are substituted by a Rician noise with different noise strengths.

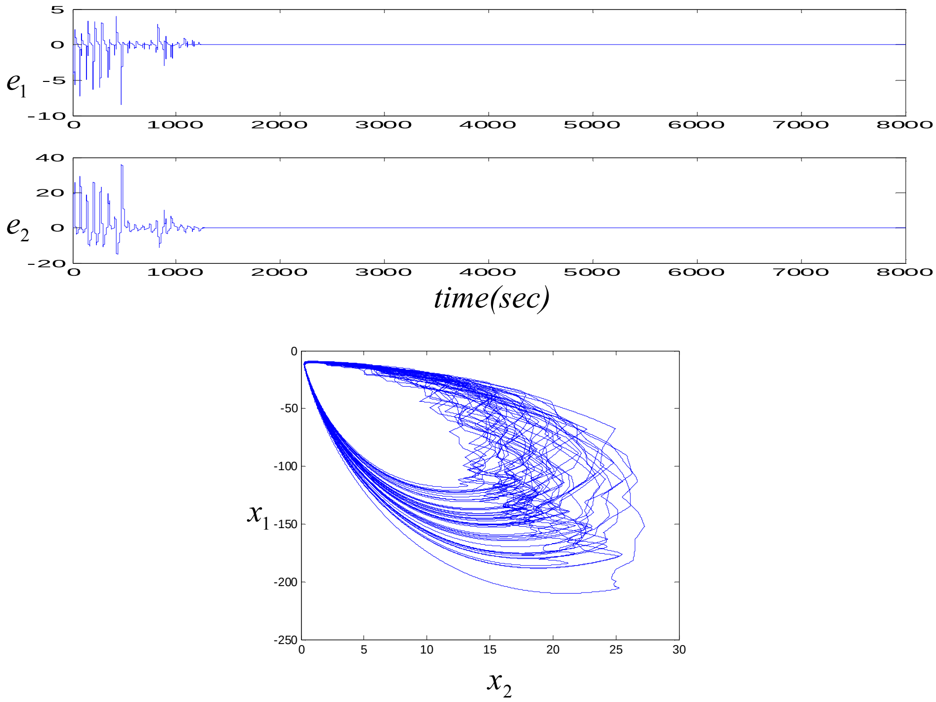

Fig.22.17 The error states and the phase portraits of the double Mackey-Glass systems when two corresponding parameters b are substituted by a uniform noise with noise strength $p_4 = 10.25$.

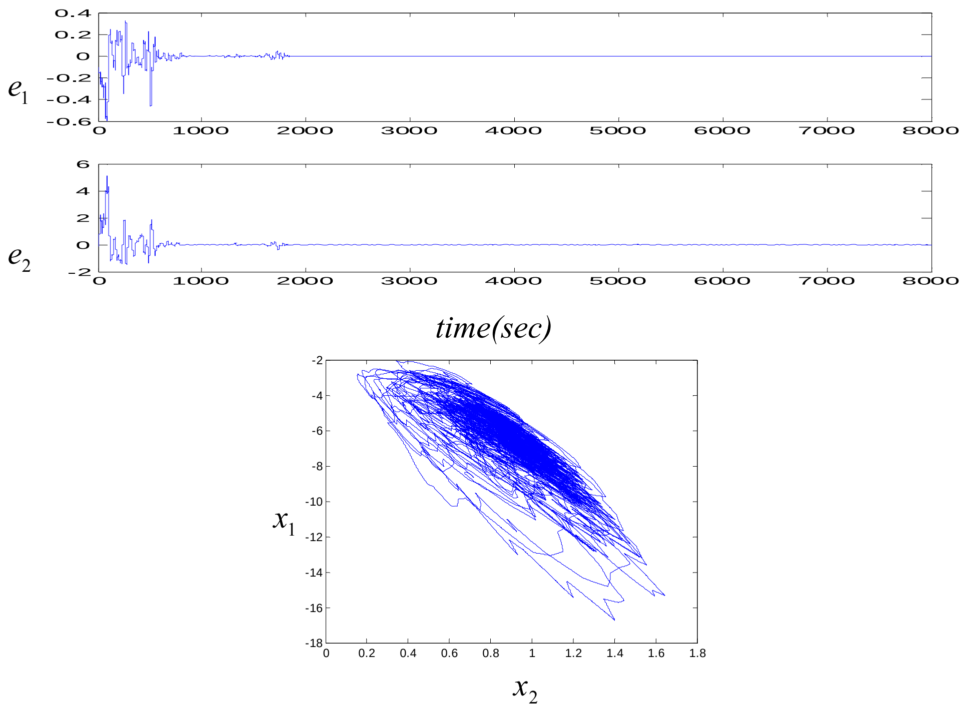

Fig.22.18 The error states and the phase portraits of the double Mackey-Glass systems when two corresponding parameters r are substituted by a uniform noise with noise strength $p_4 = 0.27$.

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Chapter 23

Temporary Lag and Anticipated Synchronization and Anti-synchronization of Uncoupled Time-delayed Chaotic Systems

23.1 Preliminaries

Since the pioneering work was investigated by Pecorra and Carroll [1], Chaos synchronization [2-8] has become an important topic in engineering science. Many effective control schemes have been developed in a variety of fields, such as parameters adaptive control [9-17], observer based control [18, 19], variable structure control [20, 21], active control [22-26], anti-control [27-33], nonlinear control [34-36] and so on. The applications of chaos synchronization are implemented extensively including secure communications, chemical, physical, and biological systems and neural networks.

Recently, the concept of synchronization has been extended to the scope, such as generalized, lag, anticipating, phase and anti-synchronization. The basic synchronization called complete synchronization is that the state vectors of the first system $x(t)$ is equal to the state vectors of the second system $y(t)$: $y(t)=x(t)$. The lag synchronization [37] is that the state vector of the second system y delay that of driver system x: $y(t)=x(t-T)$ with positive T. If T is negative, we have anticipated synchronization. If the synchronizations are temporary and intermittent, they are called temporary lag synchronization (TLS) and temporary anticipated synchronization (TAS). Lag anti-synchronization [38] means $y(t)=-x(t-T)$. When T is negative, we have anticipated anti-synchronization. If they are temporary and intermittent, they are called temporary lag anti-synchronization (TLAS) and temporary anticipated anti-synchronization (TAAS) [39].

It is discovered that TLS, TAS and TALS, TAAS appear for two identical double

Mackey-Glass systems, without any control scheme or coupling terms, but with different initial conditions.

23.2 Temporary lag and anticipated synchronization and temporary lag and anticipated anti-synchronization

Consider the first time-delay chaotic system

$$\dot{x} = f(x, x_\tau, t) \tag{23.1}$$

and second time-delay chaotic system

$$\dot{y} = f(y, y_\tau, t) \tag{23.2}$$

where $x, y \in R^n$ are n-dimensional state vectors, $x_\tau = x(t-\tau)$ are corresponding time-delay state vectors, and $f : R^n \to R^n$ defines a vector function in n-dimensional space. The error are defined as $e = x(t-T) - y(t)$. If the following conditions hold, the systems are in temporary lag synchronization.

$$e_i = x_{iT_j} - y_i = 0, \quad i = 1,2,....,p \le n, \quad j = 1,2,....,m \qquad \text{for } t_{iT_{j1}} \le t \le t_{iT_{j2}} \tag{23.3}$$

where x_i, y_i are the state vectors of the system, T_j is the time which x_i lag behind y_i in the j-th intervals. When T_j is negative, we have temporary anticipated synchronization.

In the case of anti-synchronization, the states of the systems which have opposite signs, the error $e = x(t-T) + y(t)$ will converge to zero. Therefore, we can say the temporary lag anti-synchronization is achieved when the following conditions are satisfied:

$$e_i = x_{iT_j} + y_i = 0, \quad i = 1,2,....,p \le n, \quad j = 1,2,....,m \qquad \text{for } t_{iT_{j1}} \le t \le t_{iT_{j2}} \tag{23.4}$$

where x_i, y_i are the state vectors of the system, T_j is the time which x_i lag behind y_i in the j-th intervals. When T_j is negative, we have temporary anticipated anti-synchronization.

23.3 The lag and anticipated synchronization of two identical double

Mackey-Glass systems

We consider two double Mackey-Glass systems which consist of two coupled Mackey-Glass equations [40]:

$$\begin{cases} \dot{x}_1 = \dfrac{bx_{1\tau}}{1+x_{1\tau}^n} - rx_1 \\ \dot{x}_2 = \dfrac{bx_{2\tau}}{1+x_{2\tau}^n} - rx_2 - x_1 \end{cases} \tag{23.5}$$

and

$$\begin{cases} \dot{y}_1 = \dfrac{by_{1\tau}}{1+y_{1\tau}^n} - ry_1 \\ \dot{y}_2 = \dfrac{by_{2\tau}}{1+y_{2\tau}^n} - ry_2 - y_1 \end{cases} \tag{23.6}$$

The system is a model of blood production of patients with leukemia. The variables x_1, x_2 are the concentration of the mature blood cells in the blood, and $x_{1\tau}$, $x_{2\tau}$ are presented the request of the cells which is made after τ seconds, i.e. $x_{i\tau} = x_i(t-\tau), (i = 1,2)$. The time delay τ indicates the difference between the time of cellular production in the bone marrow and of the release of mature cells into the blood. According to the observations, the time τ is large in the patients with leukemia and the concentration of the blood cells becomes oscillatory.

In our study, we keep the delay time fixed in 20 second ($\tau = 20$) and the parameters are shown as follow: $b = 0.2$, $r = 0.1$, and $n = 10$. The system is chaotic in foregoing conditions as shown in Fig. 23.1 [41]. All the numerical simulations are implemented by Matlab. The initial conditions we choose are constant, i.e. the variable $x(t+\theta)$ maintains a constant for all $\theta \in (-\tau, 0)$.

Fig.23.2 shows the time histories of double Mackey-Glass system with initial conditions (x_{10}, x_{20}) = (0.001, 0.001), (y_{10}, y_{20}) = (0.0015, 0.0015) respectively. Because the similar characteristics exist for x_1, y_1 and for x_2, y_2, we only draw the time histories of x_1, y_1 (Fig. 23.2 (a)~(f)) and the time histories of error, $e_1 = x_{1T_j} - y_1$ (Fig. 23.2 (g)~(l)). From Fig. 23.2, the temporary lag and anticipated synchronizations appear intermittently. Lag synchronizations are more than anticipated synchronization. In Table I, we marshal the length of the temporary lag (anticipated) synchronization and the lag (anticipated) of x_1 to y_1, which are varied in each intervals. There are four lag synchronous intervals and two anticipated synchronous intervals between 30000 seconds. Notice that the longest interval occur at the first interval, about 1200 seconds. Others are hundreds seconds long.

We also find the trend of decreasing the length of the temporary synchronization with increasing initial conditions. As the initial values increase, the time intervals for temporary lag or anticipated synchronization decrease. Table II show the lengths of the first time interval where the initial values are varied from 0.00001 to 0.1, L_1 and L_2 indicate the length of first temporary synchronization of x_1, y_1 and of x_2, y_2, respectively. From the curve fitting presented in Fig. 23.2 and Fig. 23.3, the relations between L_1, L_2 and x_{10}, x_{20} are obtained as follow:

$$L_1 = -229.93\ln(x_{10}) - 262.06 \tag{23.7}$$

and

$$L_2 = -229.88\ln(x_{20}) - 261.58 \tag{23.8}$$

They are essentially identical.

Table I. The length of temporary lag (anticipated) synchronization and the lag (anticipated) of x_1, x_2 to y_1, y_2.

	x_1, y_1			x_2, y_2		
	time intervals (sec)	length of temporary synchronization (sec)	lag of x_1 to y_1 (sec)	time intervals (sec)	length of temporary synchronization (sec)	lag of x_2 to y_2 (sec)
1	0—1187	1187	17	0—1194	1194	17
2	8730—9215	485	37	8740—9360	620	38
3	14630—15000	370	−8	14640—15010	370	−8
4	18103—18611	508	77	18111—18658	547	77
5	19387—19983	596	55	19390—19990	600	55

6	28580－29010	430	－7	28530－28980	450	－6

Table II. The lengths of the first time intervals of TLS and TAS where the initial values are varied from 0.00001 to 0.1.

Initial conditions $(x_{10}=x_{20}, y_{10}=y_{20})$	L_1	L_2
$(10^{-5}, 1.5\times10^{-5})$	2593	2593
$(5\times10^{-5}, 7.5\times10^{-5})$	1759	1759
$(10^{-4}, 1.5\times10^{-4})$	1683	1683
$(5\times10^{-4}, 7.5\times10^{-4})$	1806	1806
$(10^{-3}, 1.5\times10^{-3})$	1187	1186
$(5\times10^{-3}, 7.5\times10^{-3})$	843	843
(0.01,0.015)	1031	1033
(0.05,0.075)	382	382
(0.1,0.15)	231	231

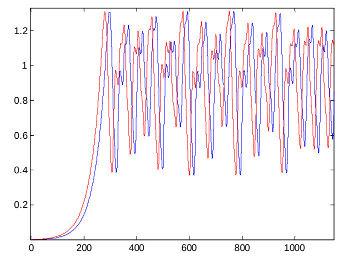
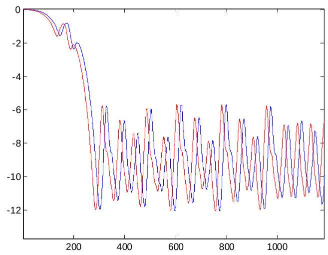
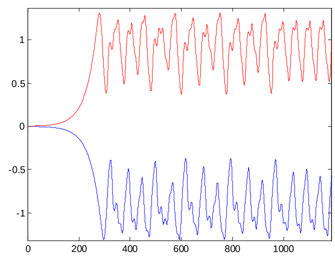
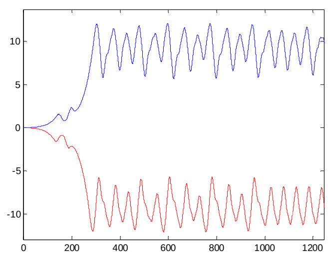
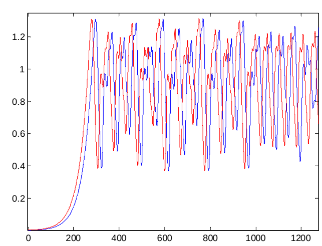
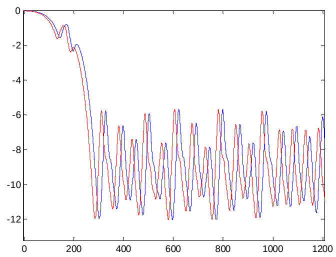
23.4 The lag and anticipated anti-synchronization of two identical double Mackey-Glass systems

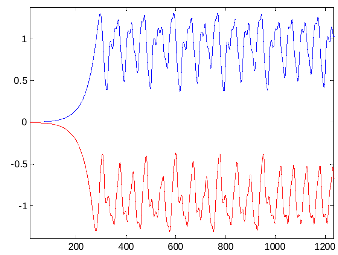
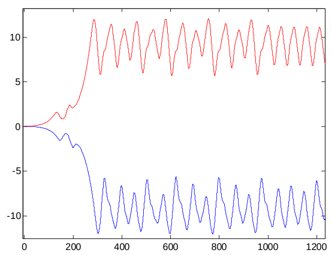
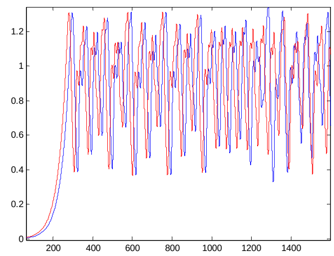
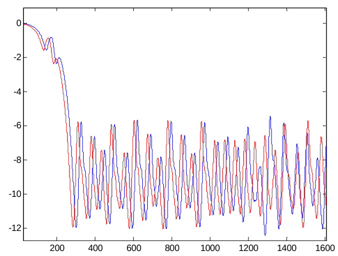
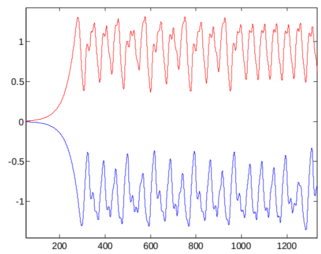
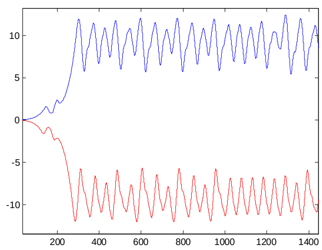
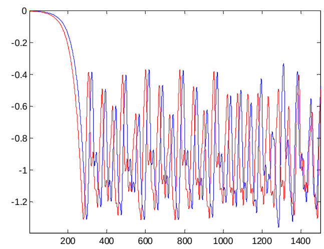
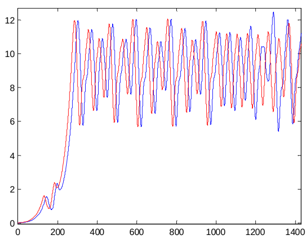
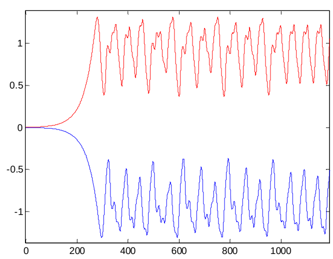
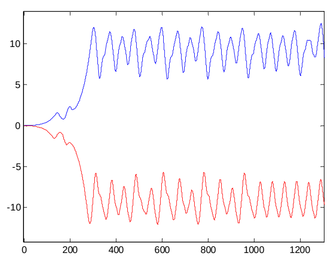
In this section, we add one, two, three or four minus sign to the initial conditions, TLS and TLAS occur alternatively.

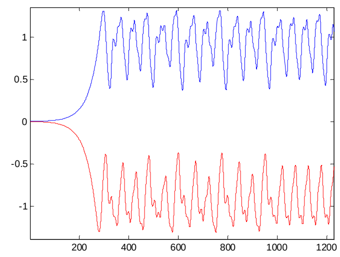
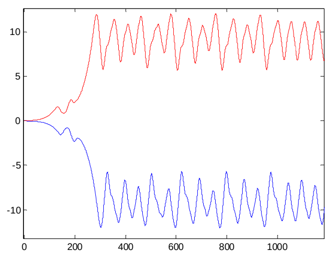
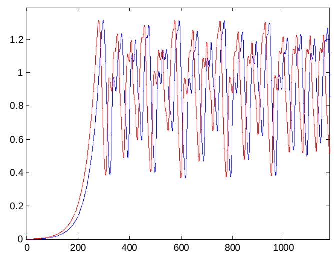
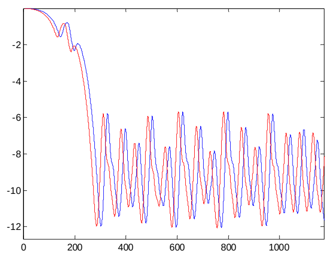
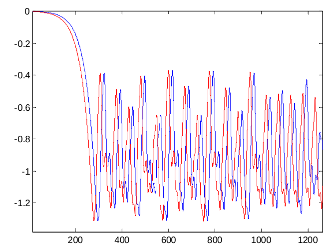
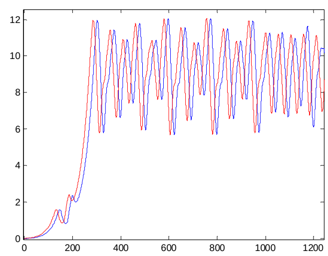
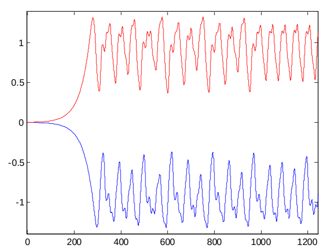
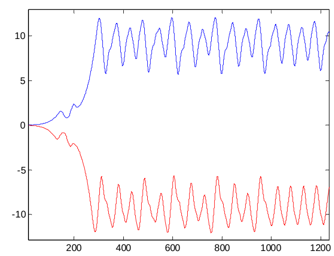
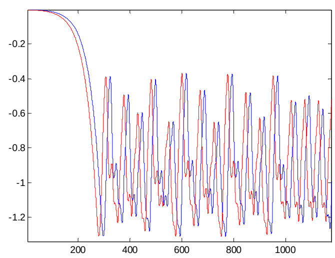
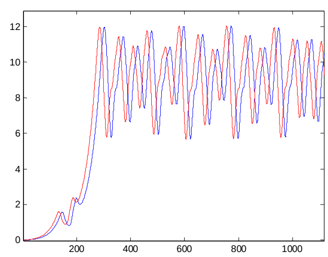
Table III shows the results of the simulations. There are interesting phenomena. The minus sign makes the original time history inverse but with same magnitude, i.e. two time histories are symmetric to the abscissa. From Case 1~4, it is found that the inverse effect only appears when the initial condition x_{10} or y_{10} is negative. On the contrary, it does not work for x_{20} and y_{20}. The trajectories of x_1 and x_2 are upside down as x_{10} is negative, and the trajectories of y_1 and y_2 show the similar characteristics with negative y_{10}. In these two cases, the lag anti-synchronizations exist. Because the negative initial conditions x_{20}, y_{20} have no influence on the systems, there are still lag synchronizations in Case 2 and 4. Case 5~9 show the results where there are two negative initial conditions at the same time. In Case 5 and 7, only the inverses of x_1 and x_2 occur, so two systems are in lag anti-synchronization. Case 6 and 9 maintain lag synchronization because both trajectories are opposite in the former case and no inversion exists in the latter case. Case 8 shows the lag anti-synchronization where the trajectory of y_1 and y_2 is reversed. Finally, the simulations where there are three and four negative initial values, are presented respectively. It is easy to know that Case 10 is the same as Case 6 and Case 11 and Case 1 are quite alike.

According to the symmetric relations between cases with negative initial conditions and the original cases, the lengths of the lag anti-synchronizations and the lags of x_1 to y_1 are all invariant, just as that in Table I which is listed in Section 23.2.

Table III. The time histories of double Mackey-Glass system with negative initial values.

Case	Initial conditions $(x_{10},x_{20}),(y_{10},y_{20})$	$x_1 : blue, y_1 : red$	$x_2 : blue, y_2 : red$
0	(0.001,0.001), (0.0015,0.0015)	 Lag synchronization	 Lag synchronization
1	(−0.001,0.001), (0.0015,0.0015)	 Lag anti-synchronization	 Lag anti-synchronization
2	(0.001,−0.001), (0.0015,0.0015)	 Lag synchronization	 Lag synchronization

3	$(0.001,0.001)$, $(-0.0015,0.0015)$	 Lag anti-synchronization	 Lag anti-synchronization
4	$(0.001,0.001)$, $(0.0015,-0.0015)$	 Lag synchronization	 Lag synchronization
5	$(-0.001,-0.001)$, $(0.0015,0.0015)$	 Lag anti-synchronization	 Lag anti-synchronization
6	$(-0.001,0.001)$, $(-0.0015,0.0015)$	 Lag synchronization	 Lag synchronization
7	$(-0.001,0.001)$, $(0.0015,-0.0015)$	 Lag anti-synchronization	 Lag anti-synchronization

8	$(0.001,-0.001)$, $(-0.0015,0.0015)$		
		Lag anti-synchronization	Lag anti-synchronization
9	$(0.001,-0.001)$, $(0.0015,-0.0015)$		
		Lag synchronization	Lag synchronization
10	$(-0.001,-0.001)$, $(-0.0015,0.0015)$		
		Lag synchronization	Lag synchronization
11	$(-0.001,-0.001)$, $(0.0015,-0.0015)$		
		Lag anti-synchronization	Lag anti-synchronization
12	$(-0.001,-0.001)$, $(-0.0015,-0.0015)$		
		Lag synchronization	Lag synchronization

The time histories and the error dynamics e with initial conditions $(x_{10}, y_{10}) = (-0.001, 0.001)$, $(x_{20}, y_{20}) = (0.0015, 0.0015)$ are shown in Fig. 23.4. Comparing with Fig. 23.1, nothing is changed except the inverse of x_1 and y_1.

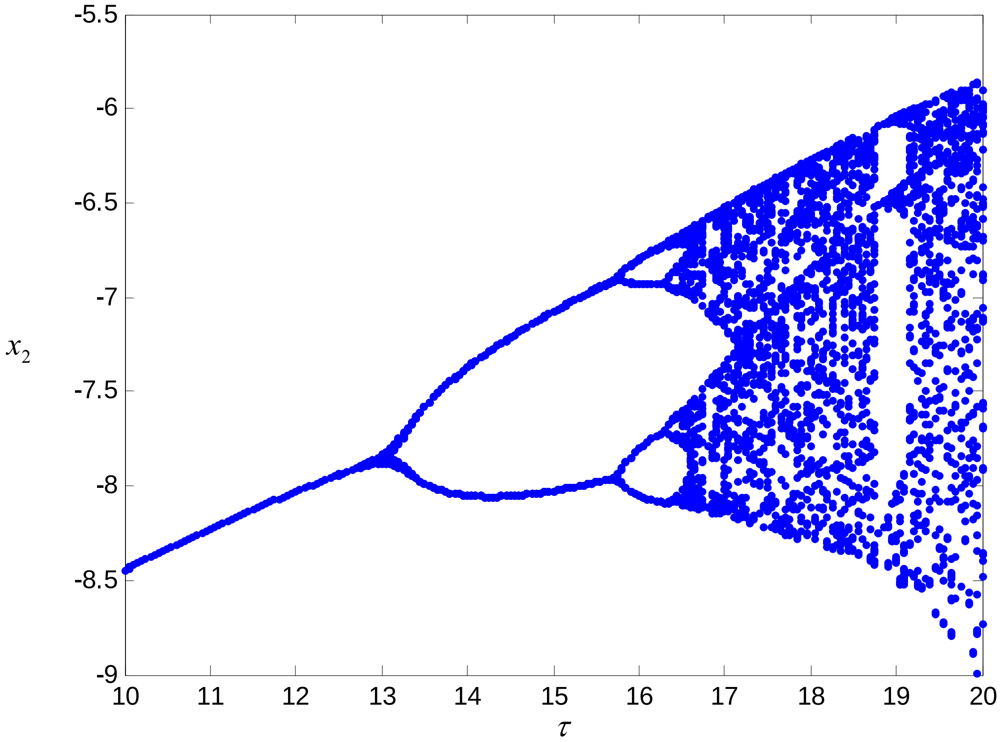


Fig.23.1 The phase portraits and the bifurcation diagram for Double Macky-Glass system.

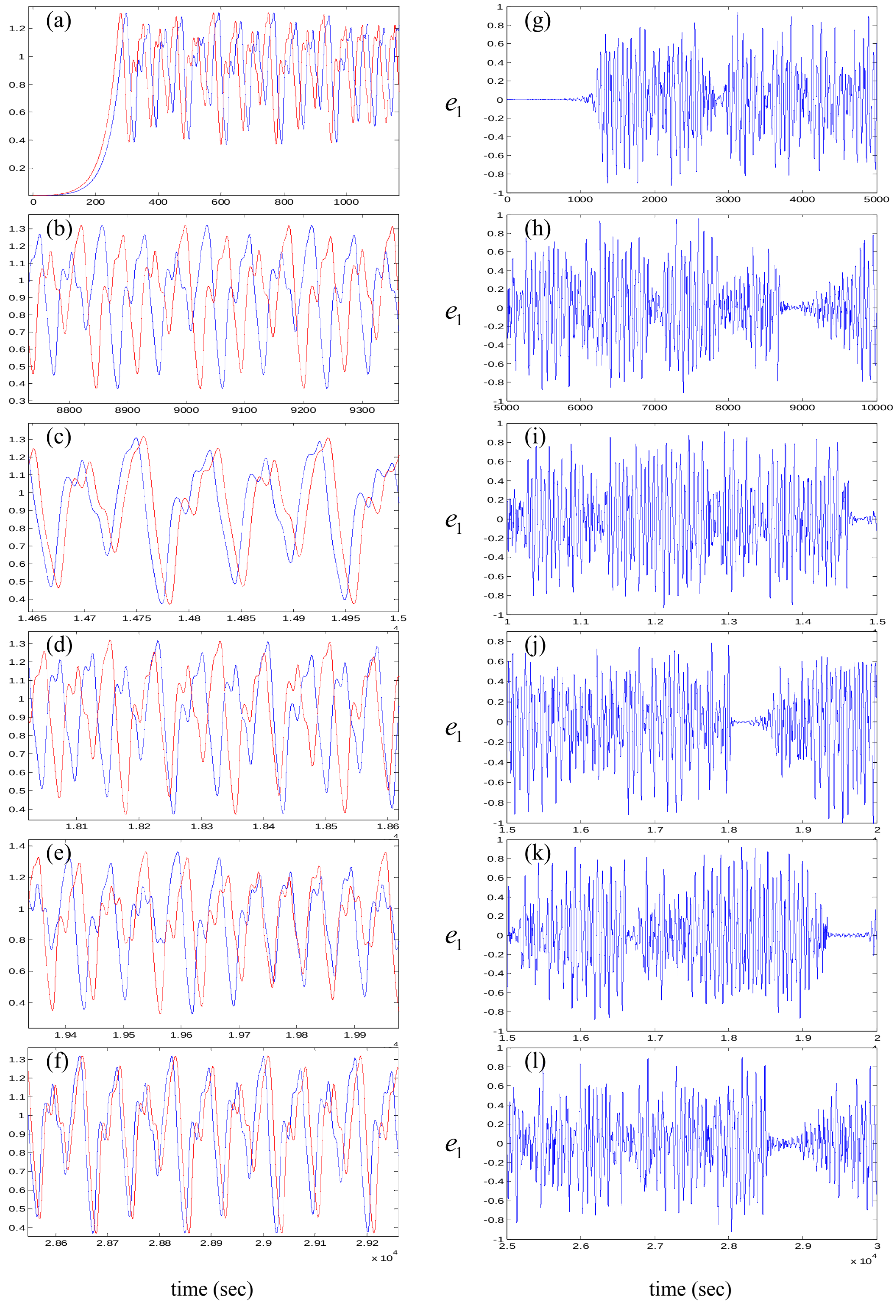


Fig.23.2 (a)~(f) The time histories of x_1 (blue) and y_1 (red) and (g)~(l) error $e_1 = x_{1T_j} - y_1$ of double Mackey-Glass systems with initial conditions $(x_{10}, x_{20}) =$ (0.001, 0.001), $(y_{10}, y_{20}) =$ (0.0015, 0.0015).

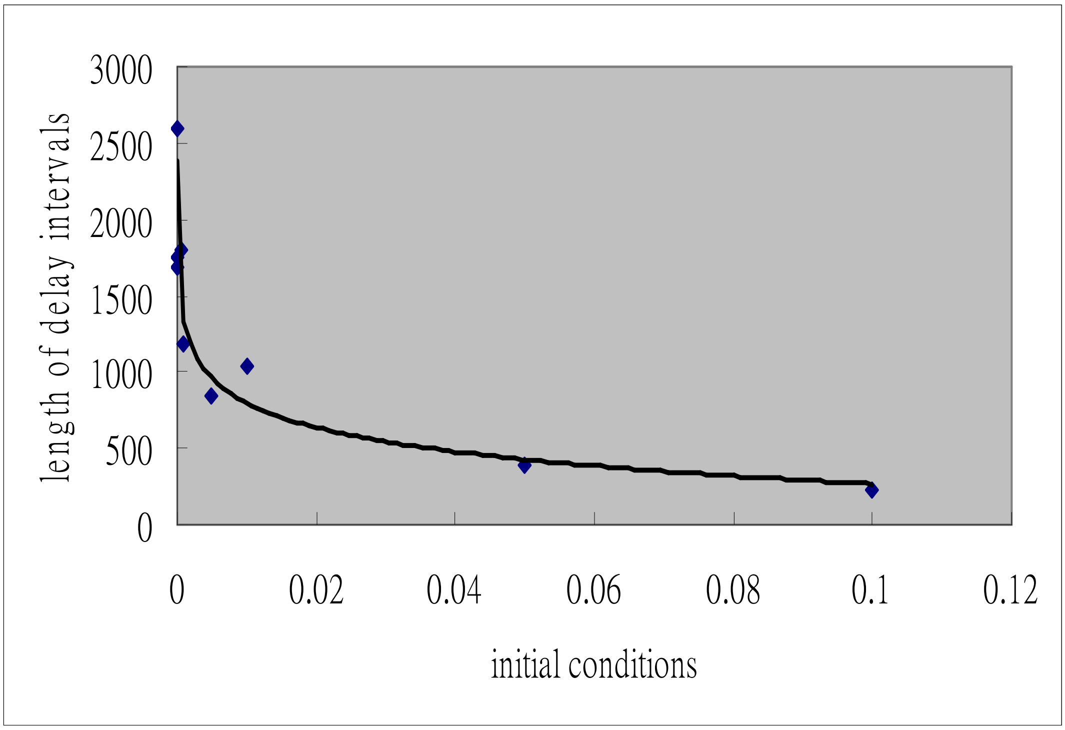


Fig.23.3 The curve fitting of initial condition x_0 to the length of temporary lag or anticipated synchronization L_1.

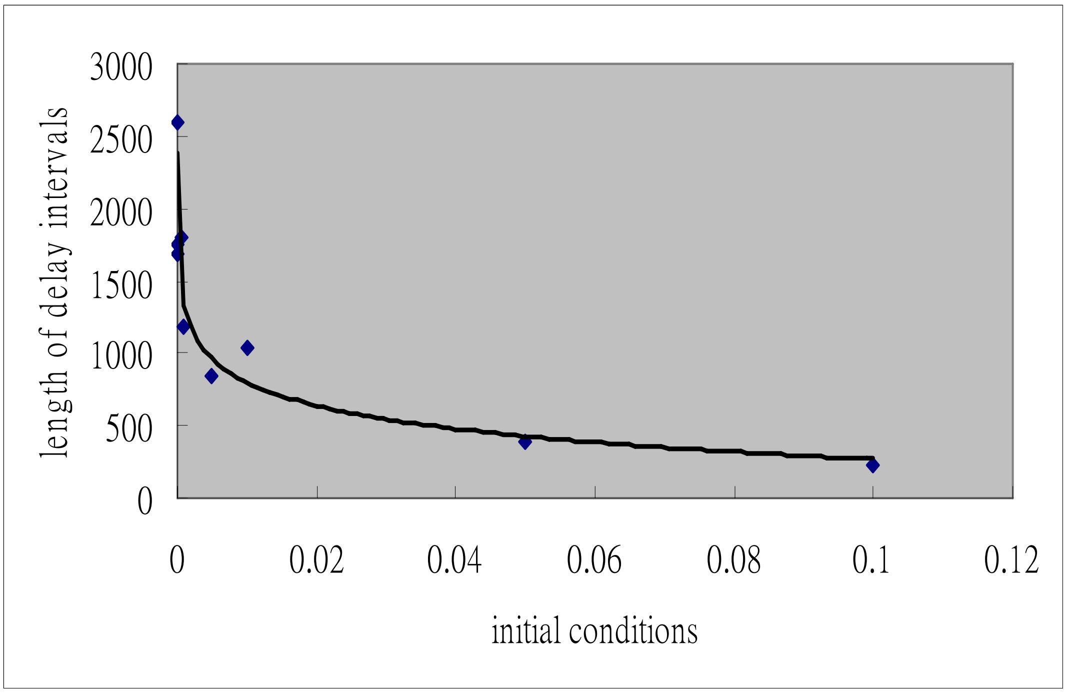


Fig.23.4 The curve fitting of initial condition x_0 to the length of temporary lag or anticipated synchronization L_2.

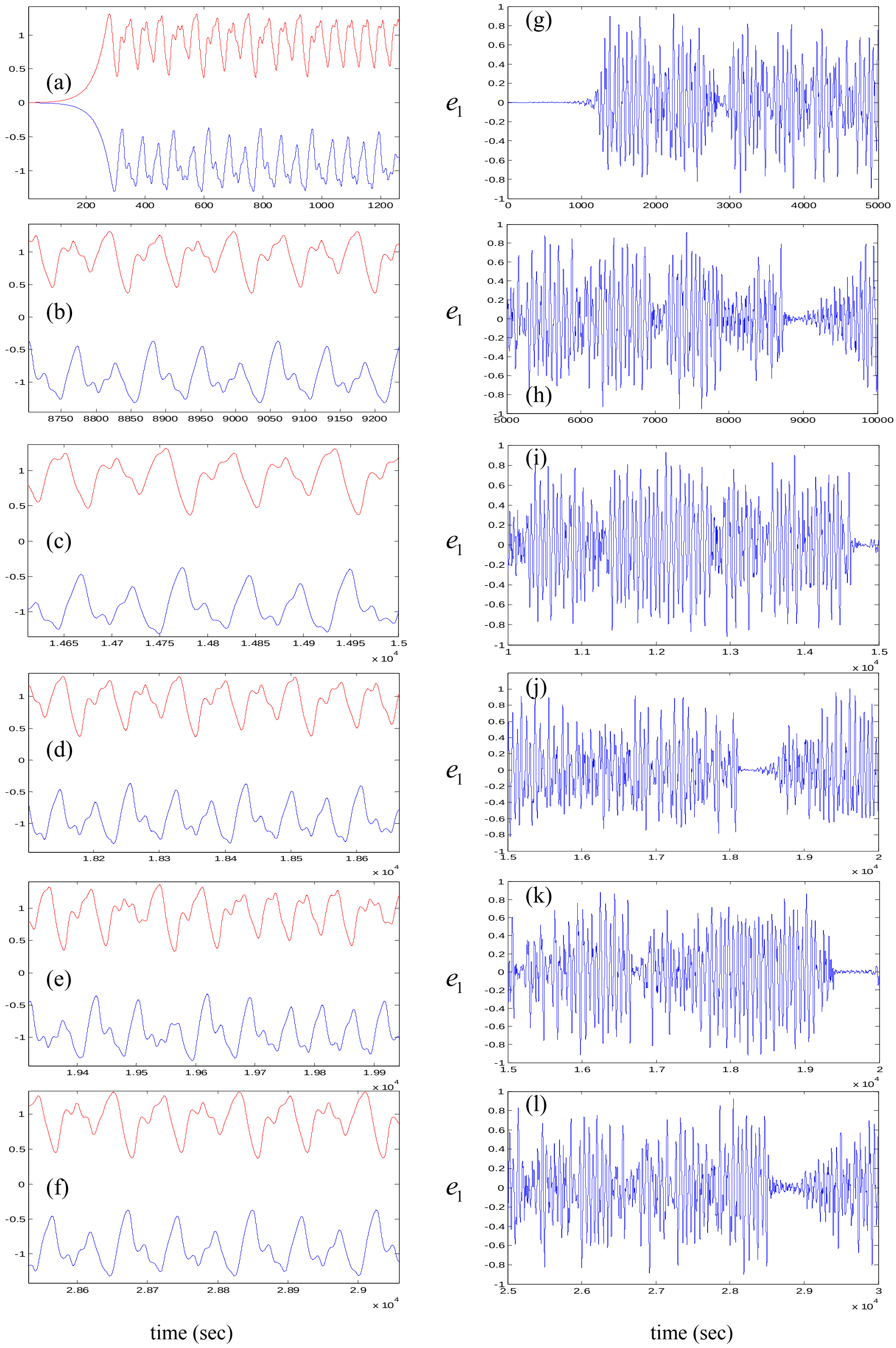


Fig.23.5 (a)~(f) The time histories of x_1 (blue) and y_1 (red) and (g)~(l) error $e_1 = x_{1T_j} + y_1$ of double Mackey-Glass systems with initial conditions $(x_{10}, x_{20}) = (-0.001, 0.001)$, $(y_{10}, y_{20}) = (0.0015, 0.0015)$.

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Chapter 24

Robust Chaos Lag Synchronization and Chaos Control of Double Mackey-Glass System by Noise Excitation of Parameters

24.1 Preliminaries

In the past few decades, chaos synchronization has attracted a great deal of attention in various fields [1-7] since the pioneering work was given by Pecorra and Carroll [8]. Due to its potential applications in secure communication [9,10], chemical and biological systems [11] and so on, many control schemes or strategies are proposed, which include adaptive control [12-19], impulsive control [20,21], nonlinear control [22-24], active control [25-30], backstepping design approach [31,32], etc.

There are different regimes of synchronization in interacting chaotic systems: complete synchronization, lag synchronization [33], anticipated synchronization [34], generalized synchronization [35], and phase synchronization [36], etc. In this , the lag synchronization of two uncoupled double Mackey-Glass systems is achieved via the parameter excited method [37]. This method is accomplished by replacing the corresponding parameters of the systems with two lag noise signals. By means of the difference of the timing between two replacements for the first system and the second system, the lag synchronization can be obtained. The parameter of the first system is substituted by a noise at $t = 0\sec$, and the parameter of the second system is substituted by the noise at $t = d\sec$. In other words, the control schemes do not work synchronously for these two systems. Parameter excited method is effective and potential in practice for some chaotic systems which are difficult or even impossible to be coupled [38]. Temporary lag synchronization, partial lag synchronization, chaos control and robustness of lag synchronization are also obtained by this method.

24.2 Lag synchronization of double Mackey-Glass system by parameter excited method

We consider a double Mackey-Glass system described by

$$\begin{cases} \dot{x}_1 = \dfrac{bx_{1\tau}}{1+x_{1\tau}^n} - r_1 x_1 \\ \dot{x}_2 = \dfrac{bx_{2\tau}}{1+x_{2\tau}^n} - r_2 x_2 - kx_1 \end{cases} \tag{24.1}$$

and a second identical double Mackey-Glass system described by

$$\begin{cases} \dot{y}_1 = \dfrac{by_{1\tau}}{1+y_{1\tau}^n} - r_1 y_1 \\ \dot{y}_2 = \dfrac{by_{2\tau}}{1+y_{2\tau}^n} - r_2 y_2 - ky_1 \end{cases} \tag{24.2}$$

where x_1, x_2, y_1, y_2 are state variables and $x_{i\tau} = x_i(t-\tau)$, $y_{i\tau} = y_i(t-\tau)$ $(i=1,2)$, τ is a time delay, and b, r_1, r_2, n, k are constant parameters. Double Mackey-Glass system is a generalized system of a well-known blood production model established by Mackey and Glass [39]. For the patient with leukemia, the concentration of the blood can vary chaotically because of the excessively large time delay τ.

The parameters and the time delay of Eq. (24.1) and (24.2) are chosen as follows: $b=0.2$, $r_1 = r_2 = 0.1$, $n=10$, $k=1$ and $\tau = 20$. The initial values are given as $(x_{10}, x_{20}) = (0.1, 0.1)$ and $(y_{10}, y_{20}) = (0.2, 0.2)$. Both systems are chaotic in foregoing conditions [40] as shown in Fig. 24.1. The lag synchronization is obtained by using the control scheme called parameter excited method. The designated parameter is replaced by a noise signal, but there exist a time difference between two replacements for the first system and for the second system. The illustrations will show that the system (24.1) and system (24.2) are in lag synchronization.

24.3 Numerical simulation results of lag synchronizations

All simulations are carried out by Simulink environment of MATLAB. By replacing the corresponding parameter b, r_1, r_2 or k by a Rayleigh noise signal respectively, lag synchronizations of two uncoupled double Mackey-Glass systems can be achieved with appropriate noise strengths. The probability density function of Rayleigh distributed noise is

$$f(x) = \begin{cases} \dfrac{x}{\sigma^2} \exp\left(\dfrac{-x^2}{2\sigma^2} \right) & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{24.3}$$

where σ^2 is known as the fading envelope of the Rayleigh distribution. The Simulink Communication toolbox provides the Rayleigh noise generator block. We specify $\sigma = 1$ in the case. Errors are defined as $e_1(t) = x_1(t) - y_1(t+d)$, $e_2(t) = x_2(t) - y_2(t+d)$, where d is the lag of the states of the second system lag behind the states of the first system and also the time difference of the control schemes acting on these two systems. In our study, d is kept a constant, d=30. e_1 and e_2 will converge to zero as $t \to \infty$ and the lag synchronization is obtained.

Firstly, two corresponding parameters b of systems (24.1) and (24.2) are replaced by pN where N is a Rayleigh noise and p is the noise strength. In this case, we take the noise strength

$$p = 0.25i, \quad i = 1,2,...,50 \tag{24.4}$$

The simulation results are shown in Fig. 24.2. It is found that the lag synchronization is successfully achieved with most noise strengths. Fig. 24.3 shows the error states e_1, e_2 and the time histories of $x_i, y_i \, (i = 1,2)$ with noise strength $p = 11$. Lag synchronization is accomplished when $t > 4000\,\text{sec}$. It is noted that some lag synchronizations need more time (>30000sec). For instance, in Fig. 24.4, when the noise strength is taken as $p = 8.5$, the error states converge to zero at $t > 37500\,\text{sec}$.

Then the corresponding parameters r_1 and r_2 are replaced by a Rayleigh noise signal pN where p is the noise strength. When the noise strength is in the range of $0.105 \leq p \leq 0.16$, the lag synchronization is obtained. Error e_1, e_2 and the time histories of the state variables with noise strength $p = 0.15$ are given in Fig. 24.5. As $p \geq 0.165$, the state variables of the system (24.1) and (24.2) approach zero and Fig. 24.6 shows the time histories of $x_i, y_i \, (i = 1,2)$ with noise strength $p = 0.165$. *It's important to point out that parameter excited* *method is an effective chaos control method which controls the chaotic states of systems (24.1) and (24.2) to zero.* No lag synchronization is found in the rest range of the noise strength.

Next, we replace two corresponding parameters r_1 and k of the systems (24.1) and (24.2) by pN where N is a Rayleigh noise and p is the noise strength. As the noise strength in the range of $0.02 \le p \le 0.05$ and $0.105 \le p \le 0.16$, the lag synchronization can be accomplished. Fig. 24.7 and Fig. 24.8 show the error states and the time histories of the states variables of two systems with noise strengths $p = 0.03$ and $p = 0.12$ respectively. When the noise strength p is taken between two foregoing ranges, $0.05 < p < 0.105$, a phenomenon called temporary lag synchronization (TLS) is found [41]. Fig. 24.9 shows the error states and the time histories of the state variables with $p = 0.103$. When the noise strength decreases as $p \le 0.01$, the error state e_1 converge to zero and the lag synchronization for x_1 and y_1 is achieved. However, the error state e_2 is chaotic and the lag synchronization for x_2 and y_2 can not be obtained. This phenomenon is called *partial lag synchronization*. The error states e_1, e_2 and the time histories of x_1, y_1 are shown in Fig. 24.10. When the noise strength increases to $p > 0.16$, the trajectories of x_1, y_1 approach to zero and the difference between x_2 and y_2 is chaotic. The error states e_1, e_2 and the time histories of x_1, y_1 are shown in Fig. 24.11.

In order to verify the robustness of lag synchronization, a small disturbance $\varepsilon(x_1 - y_1)\cos t$ is added in two double Mackey-Glass systems:

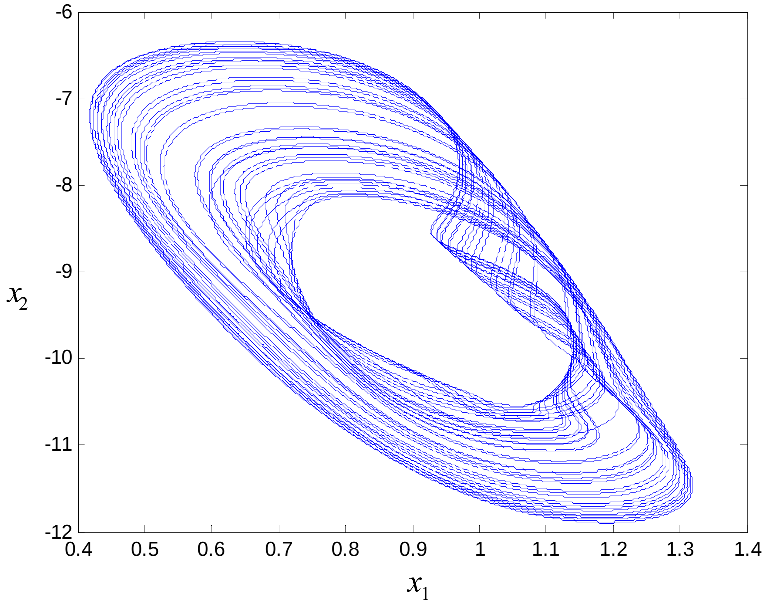
$$\begin{cases} \dot{x}_1 = \dfrac{bx_{1\tau}}{1 + x_{1\tau}^n} - r_1 x_1 + \varepsilon(x_1 - y_1)\cos t \\ \dot{x}_2 = \dfrac{bx_{2\tau}}{1 + x_{2\tau}^n} - r_2 x_2 - kx_1 \end{cases} \tag{24.5}$$

and

$$\begin{cases} \dot{y}_1 = \dfrac{by_{1\tau}}{1 + y_{1\tau}^n} - r_1 y_1 + \varepsilon(x_1 - y_1)\cos t \\ \dot{y}_2 = \dfrac{by_{2\tau}}{1 + y_{2\tau}^n} - r_2 y_2 - ky_1 \end{cases} \tag{24.6}$$

where ε is a small number which is taken as 10^{-5}. The lag synchronization is accomplished as well via the parameter excited method. In the case of replacing b, Fig. 24.12 and Fig. 24.13

show the error states and the time histories of the state variables of systems (24.5) and (24.6) with different noise strengths. One can find that the error states approach to zero in the case with $p = 8.5$ and the lag synchronization is obtain temporarily in the case with $p = 5.25$ which is defined as temporary lag synchronization (TLS). In the cases of replacing r_1, r_2 and r_1, k, Fig. 24.14 and 24.15 indicate that the error states practically approach zero which imply that lag synchronization by parameter excited method is robust in the presence of small disturbances.



Chaos, $\tau = 17$

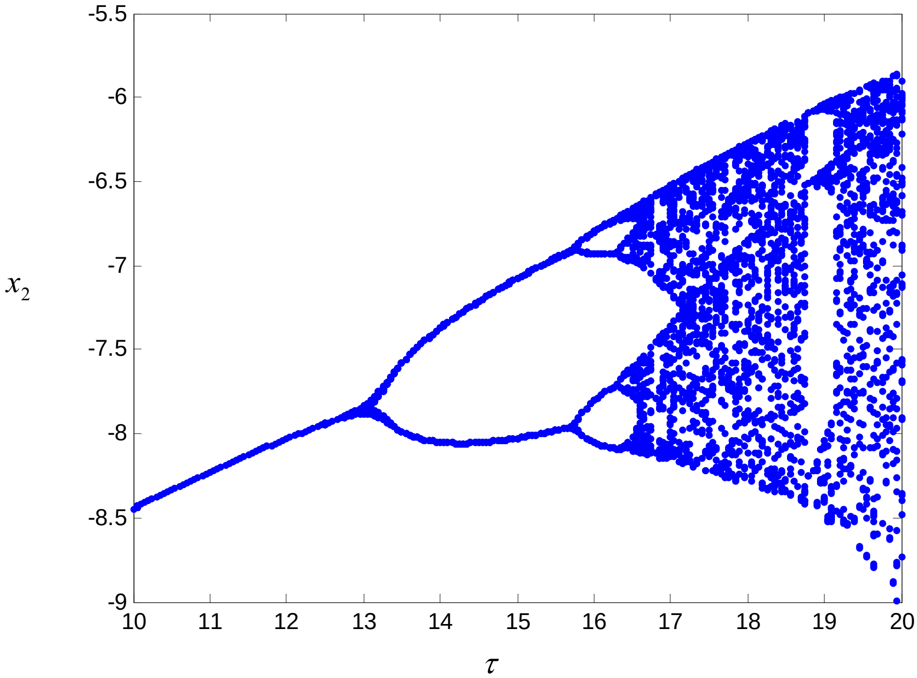


Fig. 24.1 The phase portrait and the bifurcation diagram for Double Mackey-Glass system.

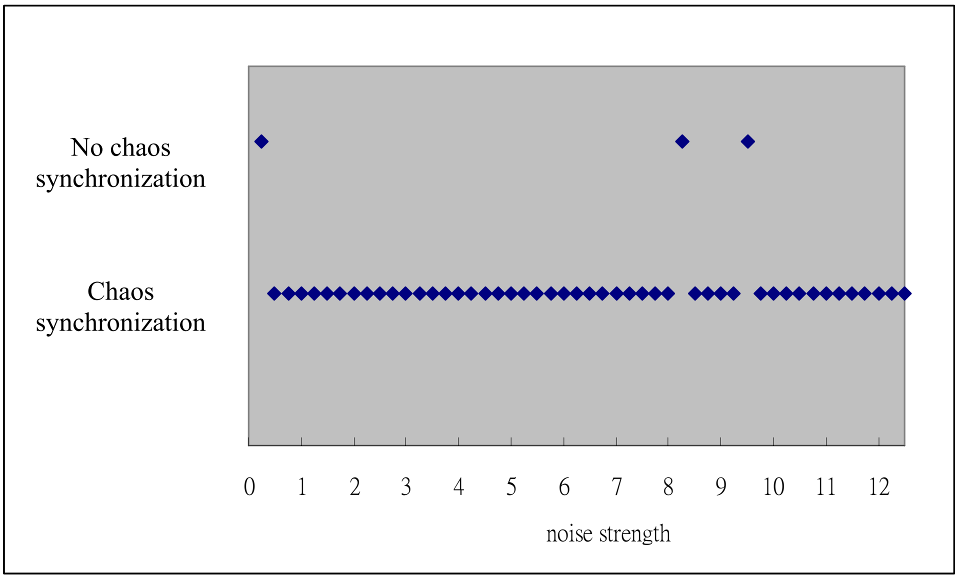


Fig. 24.2 Two corresponding parameters b are substituted by a Rayleigh noise with different noise strengths.

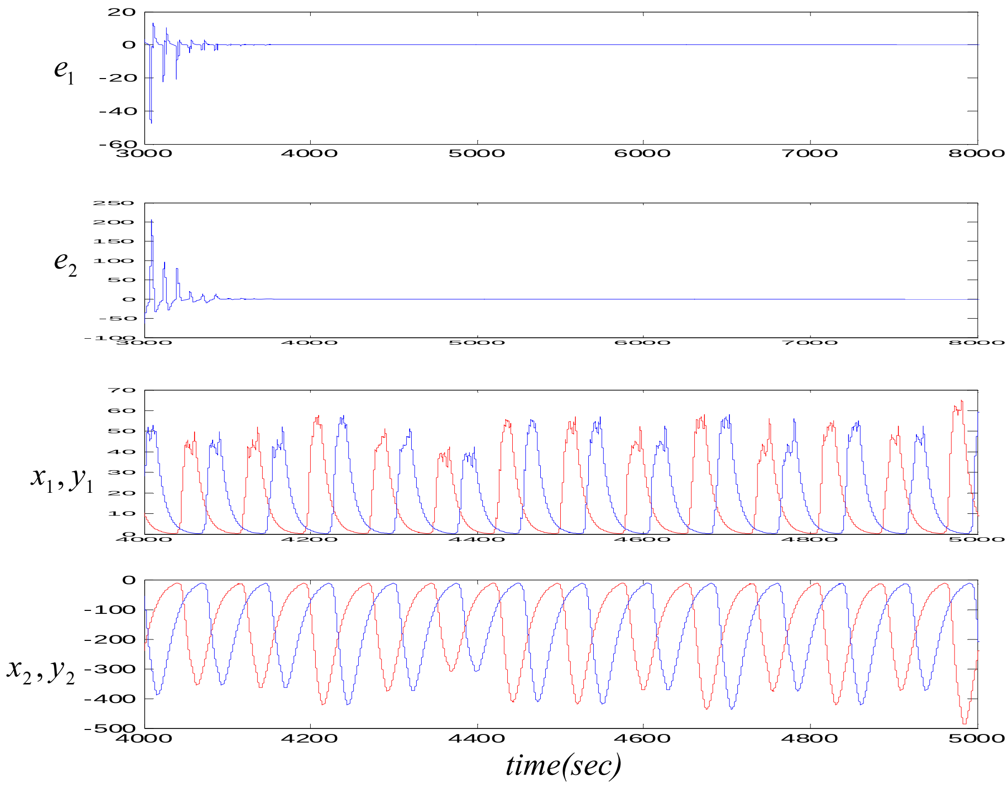


Fig. 24.3 The error states and the time histories of x_1, x_2 (red) and y_1, y_2 (blue) of the double Mackey-Glass systems when two corresponding parameters b are substituted by a Rayleigh noise with noise strength $p = 11$.

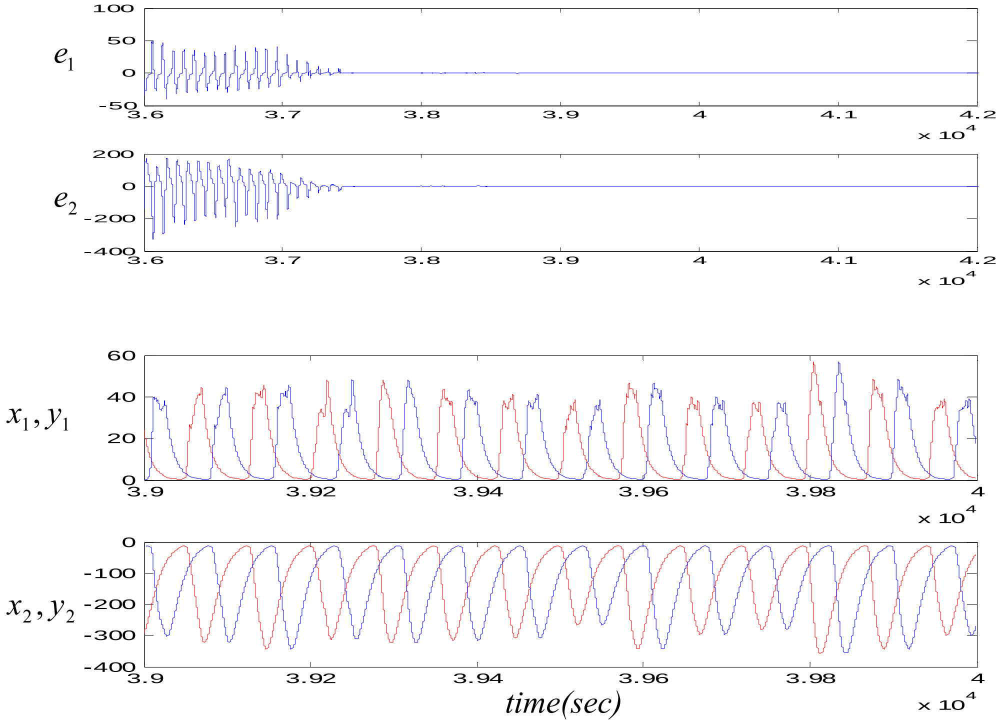


Fig. 24.4 The error states and the time histories of x_1, x_2 (red) and y_1, y_2 (blue) of the double Mackey-Glass systems when two corresponding parameters b are substituted by a Rayleigh noise with noise strength $p = 8.5$.

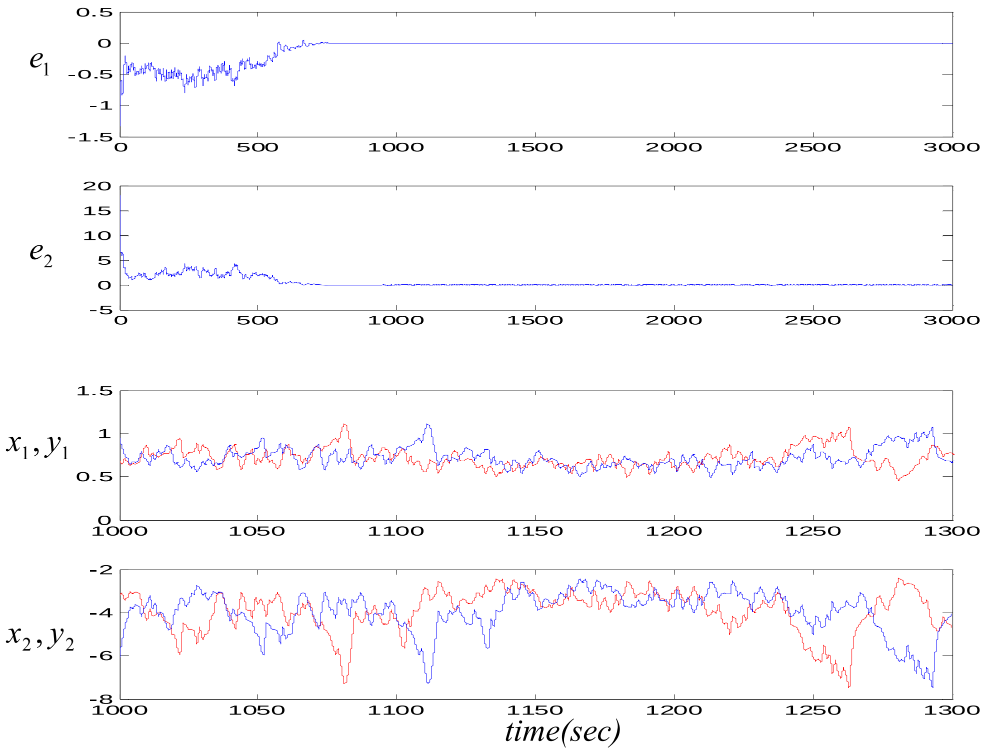


Fig. 24.5 The error states and the time histories of x_1, x_2 (red) and y_1, y_2 (blue) of the double Mackey-Glass systems when two corresponding parameters r are substituted by a Rayleigh noise with noise strength $p = 0.15$.

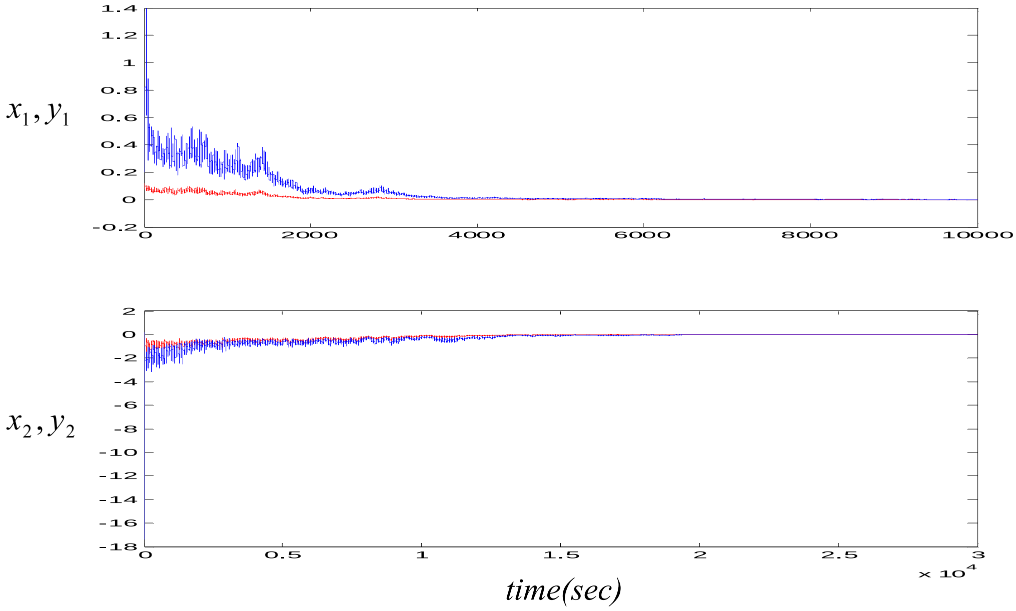


Fig. 24.6 Chaos control by parameter excited method. The time histories of x_1, x_2 (red) and y_1, y_2 (blue) of the double Mackey-Glass system when parameter r is substituted by a uniform noise with noise strength $p = 0.165$.

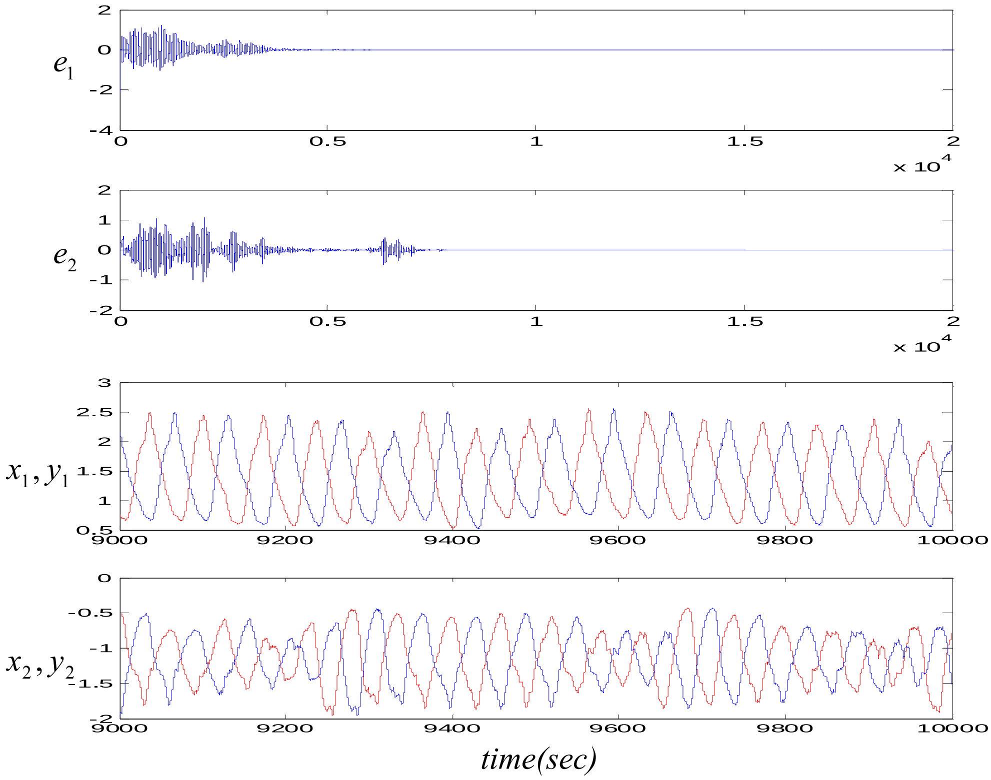


Fig. 24.7 The error states and the time histories of x_1, x_2 (red) and y_1, y_2 (blue) of the double Mackey-Glass systems when two corresponding parameters r_1 and k are substituted by a Rayleigh noise with noise strength $p = 0.03$.

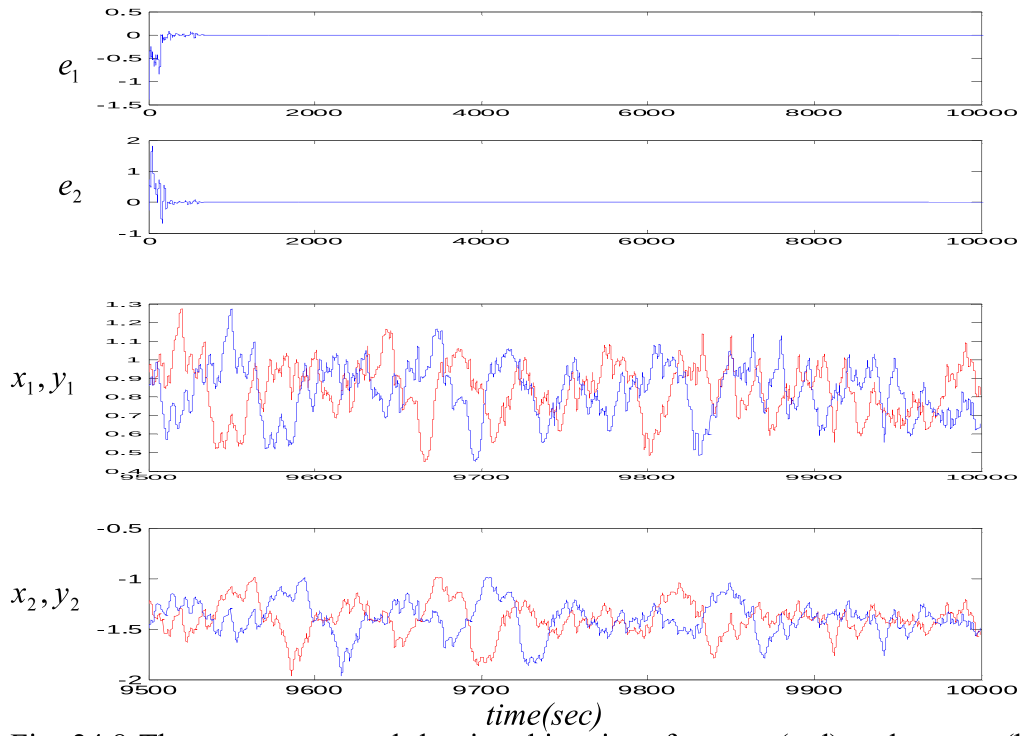


Fig. 24.8 The error states and the time histories of x_1, x_2 (red) and y_1, y_2 (blue) of the double Mackey-Glass systems when two corresponding parameters r_1 and k are substituted by a Rayleigh noise with noise strength $p = 0.12$.

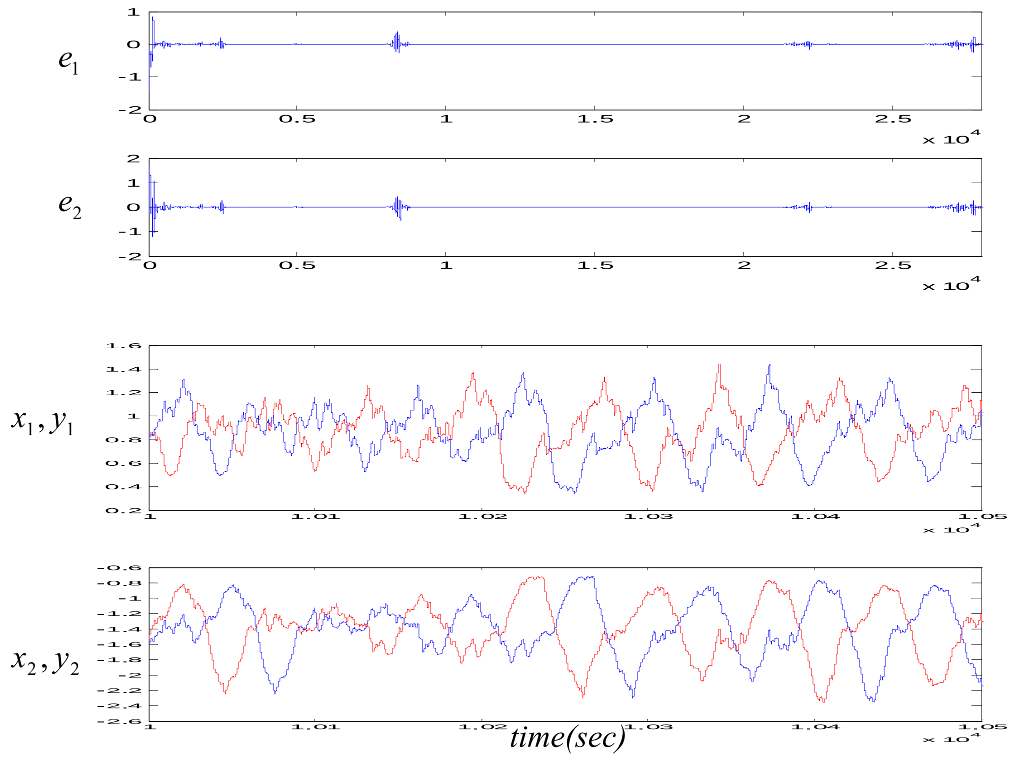


Fig.24.9 Temporary lag synchronization. The error states and the time histories of x_1, x_2 (red) and y_1, y_2 (blue) of the double Mackey-Glass systems when the two corresponding parameters r_1 and k are substituted by a Rayleigh noise with noise strength $p = 0.103$.

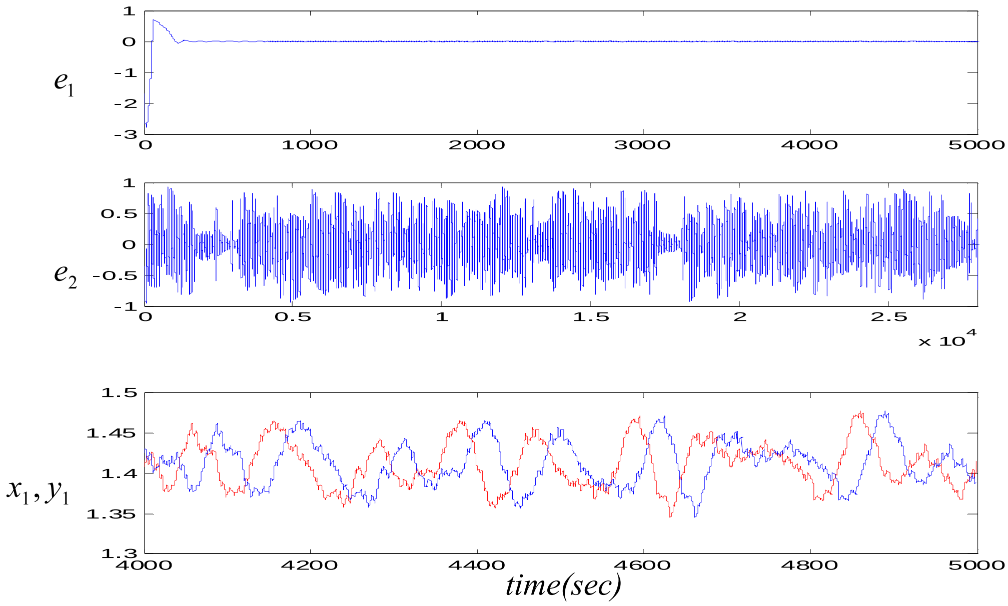


Fig. 24.10 Partial lag synchronization. The error states and the time histories of x_1 (red) and y_1 (blue) of the double Mackey-Glass systems when the two corresponding parameters r_1 and k are substituted by a Rayleigh noise with noise strength $p = 0.005$.

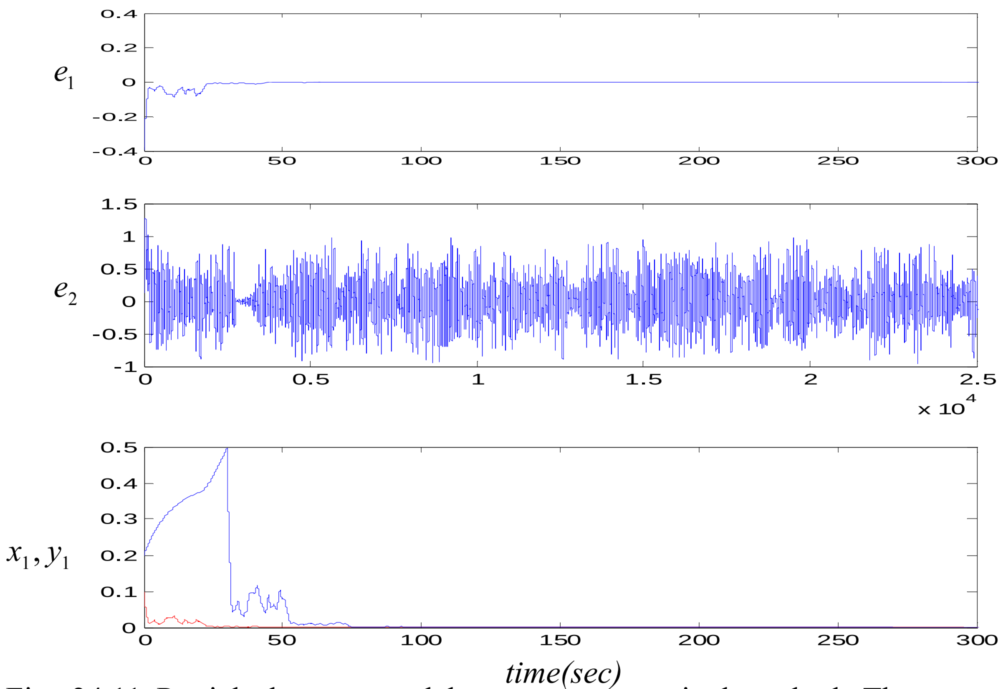


Fig. 24.11 Partial chaos control by parameter excited method. The error states and the time histories of x_1 (red) and y_1 (blue) of the double Mackey-Glass systems when the two corresponding parameters r_1 and k are substituted by a Rayleigh noise with noise strength $p = 1$.

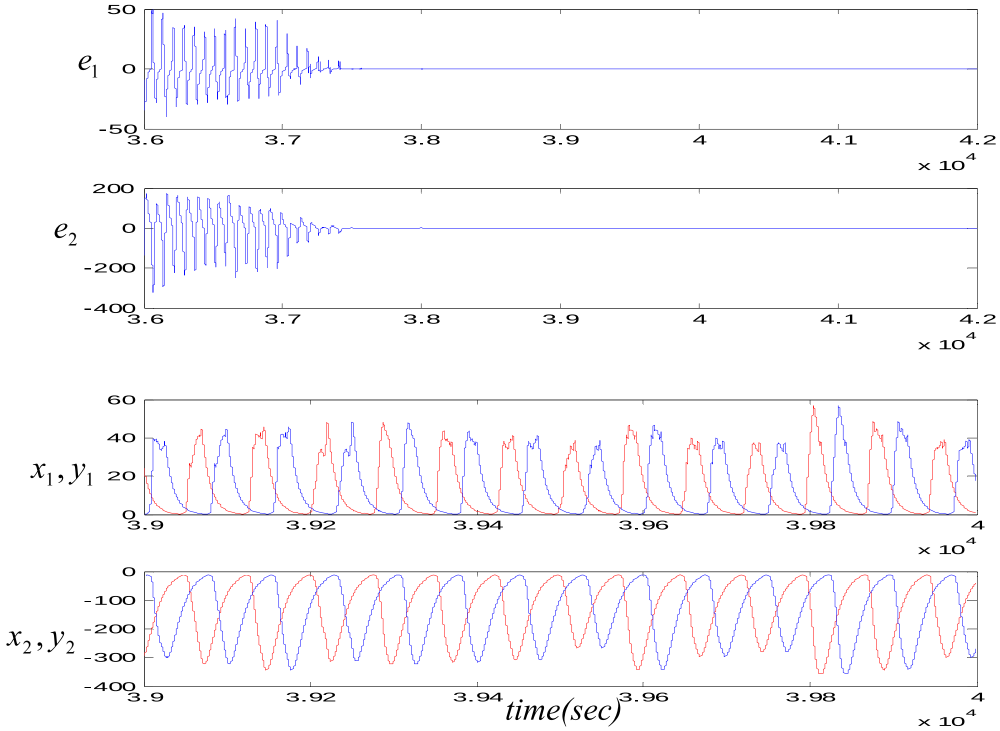


Fig 24.12 The error states and the time histories of x_1, x_2 (red) and y_1, y_2 (blue) when the two corresponding parameters b are substituted by a Rayleigh noise with noise strength $p = 8.5$ in presence of a small disturbance.

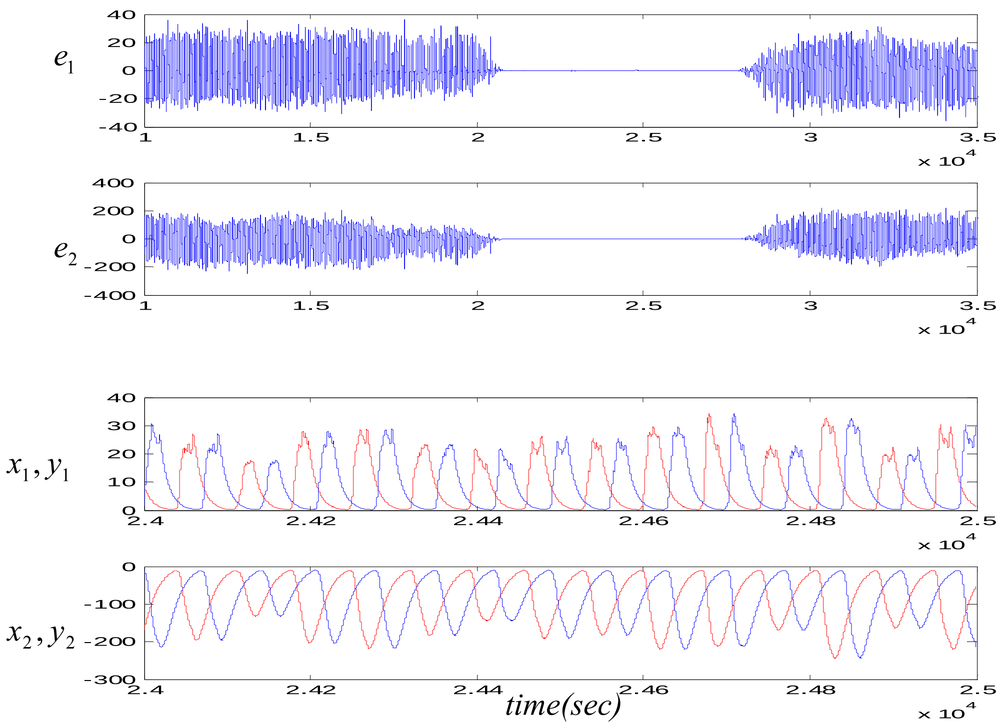


Fig. 24.13 The error states and the time histories of x_1, x_2 (red) and y_1, y_2 (blue) when the two corresponding parameters b are substituted by a Rayleigh noise with noise strength $p = 5.25$ in presence of a small disturbance.

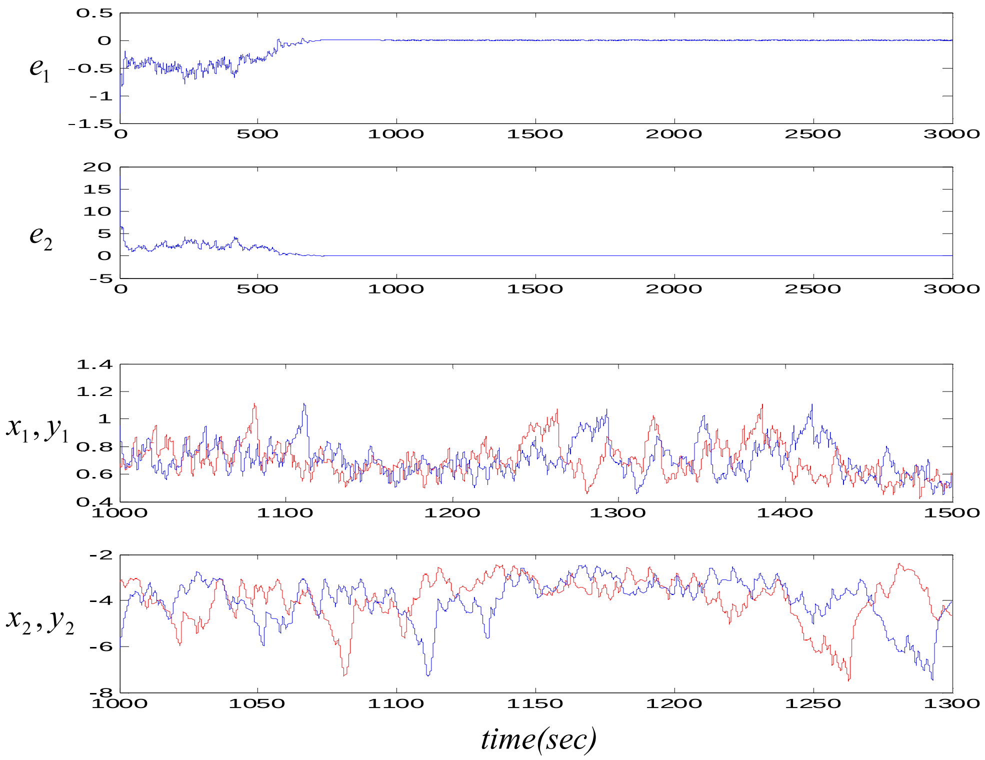


Fig. 24.14 The error states and the time histories of x_1, x_2 (red) and y_1, y_2 (blue) when the two corresponding parameters r are substituted by a Rayleigh noise with noise strength $p = 0.03$ in presence of a small disturbance.

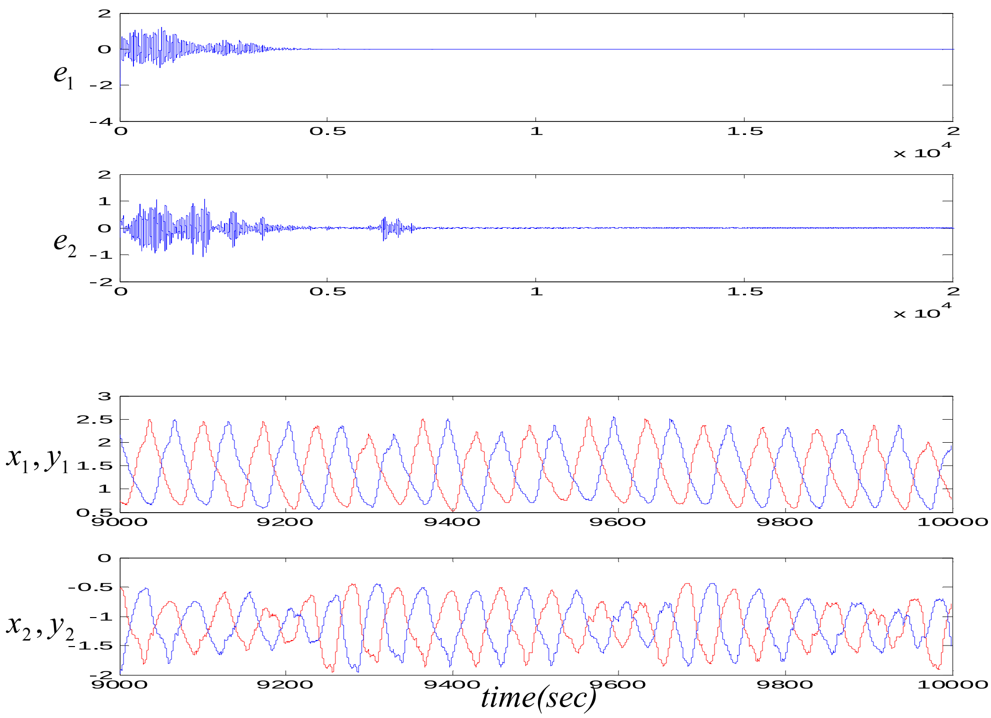


Fig. 24.15 The error states and the time histories of x_1, x_2 (red) and y_1, y_2 (blue) when the two corresponding parameters r_1 and k are substituted by a Rayleigh noise with noise strength $p = 0.03$ in presence of a small disturbance.

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Chapter 25
Conclusions

In this report, the generalized synchronization of new chaotic systems by pure error dynamics and elaborate Lyapunov function, chaos of nonholonomic systems, non-simultaneous symplectic synchronization of different chaotic systems with variable scale time, double symplectic synchronization of different chaotic systems, chaos and chaos synchronization of double Duffing system, chaos and chaos synchronization of double van der Pol system, chaos and chaos synchronization of double Ikeda system, chaos and chaos synchronization of double Macky Glass system, are studied.

Chapter 2 contains the dynamics of new autonomous and new nonautonomous chaotic systems. The system model and the numerical results of regular and chaotic phenomena are presented.

In Chapter 3, the generalized synchronization is studied by applying pure error dynamics and elaborate Lyapunov function. In Chapter 4, by applying pure error dynamics and elaborate nondiagonal Lyapunov function, the nonlinear generalized synchronization is achieved. The methods give rigorous theories for generalized synchronization and nonlinear generalized synchronization and greatly extend the use of various forms of Lyapunov function while current method only gives semi-simulation theory for generalized synchronization, in which the maximum values of state variables must be given by simulation, and monotonous square sum Lyapunov function is used. By the systematic procedures, the complexity of designing suitable elaborate Lyapunov function and elaborate nondiagonal Lyapunov function is reduced greatly. The proposed methods are effectively applied to both new autonomous and new nonautonomous chaotic systems.

Complete identification of chaos in nonholonomic systems and nonlinear nonholonomic systems is firstly presented in Chapter 5 and Chapter 6. The scope of chaos study has been extended to nonholonomic systems and nonlinear nonholonomic system. By applying the fundamental nonholonomic form of Lagrange's equations, the chaos of two nonholonomic moving target pursuit systems is studied in Chapter 5, in which nonholonomic pursuit system with a straightly oscillating target and nonholonomic pursuit system with a circularly rotating target are investigated. In Chapter 6, chaos of nonlinear nonholonomic problem, the magnitude of velocity keeping constant, is studied by applying the nonlinear nonholonomic form of Lagrange's equations. Complete identification of chaotic phenomena is obtained in nonlinear nonholonomic system by Lyapunov exponents, phase portraits, Poincaré maps, and bifurcation diagrams. Futhermore, the Feigenbaum number rule still holds for nonlinear nonholonomic system.

In Chapter 7, the non-simultaneous symplectic synchronization with variable scale time, $\mathbf{y}(t) = \mathbf{F}(\mathbf{x}(\tau), \mathbf{y}(t), t)$, is studied. By applying adaptive control, the non-simultaneous symplectic synchronization is achieved and the estimated Lipschitz constant is much less than the Lipschitz constant obtained by applying nonlinear control. This result in the reduction of the gain of the controller, i.e. the cost of controller is reduced. The simulation results show that the proposed scheme is feasible for both autonomous and nonautonomous chaotic systems, whether the dimensions of $\mathbf{x}(\tau)$ and $\mathbf{y}(t)$ are the same or not. Furthermore, when applying the non-simultaneous symplectic synchronization in secret communication, since the functional relation of the non-simultaneous symplectic synchronization is more complex than that of traditional generalized synchronization, and cracking the variable scale time τ is an extra task for the attackers in addition to cracking the system model and cracking the functional relation, the non-simultaneous symplectic synchronization may be applied to increase the security of secret communication.

In Chapter 8, the double symplectic synchronization, $\mathbf{G}(\mathbf{x}, \mathbf{y}) = \mathbf{F}(\mathbf{x}, \mathbf{y}, t)$, is studied. It is an extension of symplectic synchronization, $\mathbf{y} = \mathbf{F}(\mathbf{x}, \mathbf{y}, t)$. By applying active control, the double symplectic synchronization is achieved. By simulation results, it is shown that the proposed scheme is effective and feasible for both autonomous and nonautonomous chaotic systems. Furthermore, the double symplectic synchronization may be applied to increase the security of secret communication due to the complexity of its synchronization form.

In Chapter 9, we have studied the chaos in the integral order and fractional order double Duffing system by phase portraits, Poincaré maps and bifurcation diagrams. The total orders of the system for the existence of chaos are 0.1 to 0.7 and 1.

In Chapter 10, parameter excited chaos synchronizations of two identical double Duffing systems are studied by adjusting the strengths of the substituting state variables. Numerical simulations are illustrated for CS or AS of which the occurrence depends on initial conditions and driving strength. Besides, alternative CS and AS is also discovered with same initial conditions and same driving strengths.

In Chapter 11, synchronization and antisynchronization scheme based on the substitution of the corresponding parameters in two identical chaotic double Duffing systems by a white noise, a Rayleigh noise, a Rician noise or a uniform noise respectively. For the white noise case, neither CS and AS are found. For the Rayleigh noise case, CS and AS are obtained for different noise strengths. For the Rician noise case and the uniform noise case, only AS is obtained. Numerical simulations show that whether CS or AS occurs is sensitive to the noise strength.

In Chapter 12, a new scheme to achieve the pragmatical generalized synchronization of adaptive control via the pragmatical asymptotical stability theorem is gavien. By the procedure of the proposed scheme, two double Duffing systems and a double van der Pol system are used as master system, slave system, and goal system, respectively. The validity of this approach is verified theoretically and numerically. Based on pragmatical asymptotical stability theorem, using this theorem, we can obtain the generalized synchronization of chaotic systems and prove that the estimated parameters approach the uncertain values.

In Chapter 13, chaos in double van der Pol system and in its fractional order systems is studied. It is found that with reducing the total derivative order $\alpha_1+\beta_1+\alpha_2+\beta_2$ the ranges of the chaotic phase portraits of the system decrease and its shape changes differently for different choices of parameters.Twenty-one chaotic cases for $0.4 \le (\alpha_1+\beta_1+\alpha_2+\beta_2) \le 4.0$ are studied, and the lowest total order for chaos existence in the system is found to be 0.4. Thirty nonchaotic cases are found.

In Chapter 14, the variable with adjustable strength of a third double van der Pol system substituted for the strength of two corresponding mutual coupling terms of two uncoupled identical chaotic double van der Pol system, gives rise to their synchronization or anti-synchronization. Both CS and AS can be achived by adjusting the strength of the substituted variable and the initial conditions.

In Chapter 15, complete synchronization and antisynchronization scheme based on the substitution of two same parameters in two identical chaotic double van der Pol systems by a white noise, a Rayleigh noise respectively. For the white noise case and Rayleigh noise case, CS and AS are obtained for different noise strengths and initial conditions. Numerical simulations show that whether CS or AS occurs is sensitive to the noise strength.

In Chapter 16, controling chaotic systems to different systems is studied by new pragmatical adaptive control method. The pragmatical asymptotical stability theorem fills the vacancy between the actual asymptotical stability and mathematical asymptotical stability, the conditions of the Lyapunov function for pragmatical asymptotical stability are lower than that for traditional asymptotical stability. By using this theorem, with the same conditions for Lyapunov function, $V > 0$, $\dot{V} \leq 0$, as that in current scheme of adaptive chaos control, we not only obtain the adaptive control of chaotic systems but so prove that the estimated parameters approach the uncertain values. Traditional chaos control is limited for the same system. This method enlarges the function of chaos control. We can control a chaotic system to a given chaotic system. The method also downhill simplex the controllers and reduce their cost.

In Chapter 17, the chaos in integral and fractional order double Ikeda systems with total order of derivatives from 2 to 0.2 are studied by phase portraits, Poincaré maps and bifurcation diagrams. It is found that chaos exists in all cases.

In Chapter 18, the chaotic behaviors of double Ikeda systems are obtained by replacing their delay time by a function of chaotic state variables of a second chaotic system. It is found that chaos exists for Case 1, 3, 5, 6. The chaotization of a double Ikeda system is studied by using a function of state variable of a second identical system to replace a parameter of the first system. It is found that in Case 9, 10, 11, chaotization exists.

In Chapter 19, lag or anticipated synchronization and the lag or anticipated anti-synchronization of two double Ikeda systems with different initial conditions are discovered. There are two situations in all possible initial conditions. Cases 1~8 are the lag or anticipated synchronizations. Cases 9~16 are the lag or anticipated anti-synchronizations.

In Chapter 20, robust lag chaos synchronization, lag quasi-synchronization and chaos control of two uncoupled double Ikeda system, are achieved by replacing the corresponding parameters of two systems by different chaotic state variables of a third chaotic system. Robustness of synchronization is studied by addition of various noises. The results are satisfactory.

In Chapter 21, first, we introduce the definition and approximation of fractional order operator briefly. Then the double Mackey-Glass delay systems in integral and fractional forms are described. We find the chaos which exists in the integral system and in fractional systems with orders 0.9, 0.8, 0.1 by phase portraits and the bifurcation diagrams.

In Chapter 22, we apply the parameter excited method to control the double Mackey-Glass system and to synchronize two uncoupled double Mackey-Glass systems. By replacing the corresponding parameters of chaotic system with noise, chaos control and chaos synchronization can be accomplished. This method is effective to synchronize two systems, for which coupling method of synchronization is difficult or even impossible. Finally, numerical simulations show the proposed method is effective to suppress the chaotic behavior and drag the trajectories to the origin. Also, chaos synchronizations are successfully achieved in many cases with Rayleigh noise Rician noise, and uniform noise respectively.

In Chapter 23, temporary lag or anticipated synchronization and the lag or anticipated anti-synchronization of double Mackey-Glass systems with small and similar initial conditions are discovered. For the first interval of TLS, when all initial values are positive, temporary lag synchronizations are found. The trajectory will be reversed if the initial condition of x_1 or y_1 is negative. In these cases, the lag or anticipated anti-synchronization exists. From the results of simulation, we find six temporary lag (anticipated) synchronization intervals in 30000seconds. Although the numerical simulations of temporary lag and anticipated synchronization and anti-synchronization are showed in this . However, the theoretical analysis and its applications should be open for further work in the future.

In Chapter 24, the parameter excited method is applied to synchronize two uncoupled double Mackey-Glass systems. By replacing the corresponding parameters with a Rayleigh noise and choose the appropriate noise strength, the lag synchronization can be successfully obtained. Temporary lag synchronization, partial lag synchronization, chaos control and robustness of lag synchronization are also obtained. The abundance of various phenomena fully exhibits the potential application of this method.

計畫成果自評

研究成果之學術或應用價值：

Duffing系統，van der Pol系統與線性Mathieu系統原為振動學科之最重要最典型的系統。自渾沌動力學興起後，Duffing系統，van der Pol系統由於其為非線性系統故亦沿習成為渾沌動力學學科中最重要最典型的系統，四十年來對此二系統的渾沌研究之文獻可謂汗牛充棟，至今方興未艾。而線性Mathieu系統，則由於其為線性方程，不具渾沌性質，故在渾沌動力學學科中乃不再提及。人們忽視了非線性Mathieu系統實為Duffing系統中參數由常數轉為時間週期函數之推廣，實亦應成為渾沌動力學科之最重要最典型之系統。本計畫主持人率先研究非線性Mathieu系統之渾沌行為[6]，可謂遲來之補求。眾所週知，此三種典型系統除理論意義外，廣泛應用於機械、電機、物理、化學、生科、奈米系統，本計畫今研究雙Duffing系統，雙van der Pol系統及雙種類型的非線性Mathieu系統，不僅對渾沌動力學學科中最重要最典型的三種渾沌系統的研究的拓廣與深化，更重要的是它們本身顯然具有更複雜的，未經發現的複雜渾沌行為，本研究對渾沌動力學學科具重大意義。其應用於機械、電機、物理、化學、生科、奈米之耦合系統，具有重要的實用價值。

渾沌同步除本身之重要理論價值外，其研究在秘密通訊、神經網路、自我組織等方面有日益廣泛之應用。廣義渾沌同步則為渾沌同步之進一步發展，其應用亦方興未艾。本計畫提出三種新的渾沌同步。實用適應廣義同步法糾正了目前國際文獻中未經証明即認為估值參數趨於為之參數之錯誤，首次在渾沌同步中引入概率概念，具重大理論及實用意義。由於$\dot{V}$之要求降低，實際應用亦較易實現。純誤差穩定的廣義同步，則彌補了國際文獻中需用數值計算結果為條件之理論，即有缺陷之理論。在理論與實用上有重要意義。不同起始條件的延遲同步等多種渾沌同步則為新發現的渾沌運動之現象，特別是 Ikeda 系統的永遠性延遲同步或反同步，不同於傳統理論，尤具重大意義。

達成預期目標情況：

第一年：完成兩種雙 Mathieu 系統的渾沌行為與純誤差穩定的廣義同步及其對此二系統的應用

1. 完成採用諸多相圖、分歧圖、功率譜圖、參數圖及李亞普諾夫指數及碎形維度等研究自治的雙 Mathieu 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。(二個月)
2. 完成採用諸多相圖、分歧圖、功率譜圖、參數圖及李亞普諾夫指數及碎形維度等研究非自治的雙 Mathieu 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。(二個月)
3. 完成研究純誤差穩定的廣義同步法之理論。(三個月)
4. 完成研究純誤差穩定的廣義同步法對自治的雙 Mathieu 系統之應用。(二個月)
5. 完成研究純誤差穩定的廣義同步法對非自治的雙 Mathieu 系統之應用。(二個月)
6. 完成撰寫年度報告書。(一個月)

第二年：完成雙 Duffing 系統及雙 van der Pol 系統的渾沌行為與實用適應廣義同步法，及對此二系統的應用

1. 完成採用諸多相圖、分歧圖、功率譜圖、參數圖、李亞普諾夫指數及碎形維度等研究雙 Duffing 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。(二個月)
2. 完成採用諸多相圖、分歧圖、功率譜圖、參數圖、李亞普諾夫指數及碎形維度等研究雙 van der Pol 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。(二個月)
3. 完成研究實用適應廣義同步法之理論。(三個月)
4. 完成研究實用適應廣義同步法對雙 Duffing 系統之應用。(二個月)
5. 完成研究實用適應廣義同步法對雙 Van der Pol 系統之應用。(二個月)
6. 完成撰寫年度報告書。(一個月)

第三年：完成雙 Ikeda 系統及雙 Mackey-Glass 系統與不同起始條件下的延遲或預期同步及其在此兩系統的實現

1. 完成採用諸多相圖、分歧圖、功率譜圖、參數圖研究雙 Ikeda 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。(二個月)
2. 完成採用諸多相圖、分歧圖、功率譜圖、參數圖研究雙 Mackey-Glass 系統之週期運動、

準週期運動、渾沌運動及超渾沌運動各種行為。(二個月)

3. 完成研究在各種不同起始條件下，雙 Ikeda 系統之延遲或預期同步。(二個月)
4. 完成研究在各種不同起始條件下，雙 Mackey-Glass 系統之延遲或預期同步。(二個月)
5. 完成研究在各種不同起始條件下，雙 Ikeda 系統之延遲或預期反同步。(一個月)
6. 完成研究在各種不同起始條件下，雙 Mackey-Glass 系統之延遲或預期反同步。(一個月)
7. 完成歸納並分析在各種不同起始條件下，雙 Ikeda 系統及雙 Mackey-Glass 系統發生延遲或預期同步之規律與原因。(一個月)
8. 完成撰寫年度報告書。(一個月)

具體成果：

1. 獲得兩種雙 Mathieu 系統、雙 Duffing 系統、雙 Van der Pol 系統、雙 Ikeda 系統及雙 Mackey-Glass 系統共六種新型系統的渾沌行為之完整而詳盡之資料。同時獲得共三種新型的渾沌同步成果：(1)發展出三種新渾沌同步方法之一般理論。(2)將純誤差穩定廣義同步法及精緻之李雅普諾夫函數設計法應用在兩種雙 Mathieu 系統。(3)將實用適應廣義同步法應用在雙 Duffing 系統及雙 Van der Pol 系統。(4)獲得在各種不同起始條件下，雙 Ikeda 系統及雙 Mackey-Glass 系統呈現永久或暫時之延後同步、超前同步、反延後同步及反超前同步之完備資料，並歸結其規律與成因。
2. 所得結果在渾沌系統研究與應用方面具重要學術及實用價值。
3. 培養研究生科學研究能力。

研究內容寫成之期刊論文已有 22 篇，其中已有 5 篇被接受：

1. Z. M. Ge and C. M. Chang, “Nonlinear Generalized Synchronization of Chaotic Systems by Pure Error Dynamics and Elaborate Nondiagonal Lyapunov Function”, 2009, Chaos, Solitons and Fractals, Vol. 39, pp. 1959-1974. (SCI, Impact Factor: 3.025)
2. Z. M. Ge and C. M. Chang, “Generalized Synchronization of Chaotic Systems by Pure Error Dynamics and Elaborate Lyapunov Function”, 2009, accepted by Nonlinear Analysis: Theory, Methods, and Applications. (SCI, Impact factor: 1.097)
3. Zheng-Ming Ge, Shih-Chung Li, Shih-Yu Li and Ching-Ming Chang, 2008, “Pragmatical Adaptive Chaos Control from a New Double Van der Pol System to a New Double Duffing System”, Applied Mathematics and Computation, Vol. 203, pp. 513-522. (SCI, Impact factor: 0.821)
4. Zheng-Ming Ge, Chien-Hao Li, Shih-Yu Li and Ching-Ming Chang, 2008, “Chaos

Synchronization of Double Duffing Systems with Parameters Excited by a Chaotic Signal", Journal of Sound and Vibration, Vol. 317, pp. 449-455. (SCI, Impact factor: 1.024)
5. Zheng-Ming Ge, Yu-Ting Wong, and Shih-Yu Li, 2008, "Temporary Lag and Anticipated Synchronization and Anti-synchronization of Uncoupled Time-delayed Chaotic Systems", Journal of Sound and Vibration, Vol. 318, pp. 267-278. (SCI, Impact factor: 1.024)
6. Ching-Ming Chang and Zheng-Ming Ge, "Complete Identification of Chaos of Nonholonomic Systems", 2009, submitted to International Journal of Bifurcation and Chaos. (SCI, Impact factor: 0.910).
7. Ching-Ming Chang and Zheng-Ming Ge, "Complete Identification of Chaos of Nonlinear Nonholonomic Systems", 2009, submitted to Nonlinear Dynamics. (SCI, Impact factor: 1.045).
8. Ching-Ming Chang and Zheng-Ming Ge, "Non-simultaneous Symplectic Synchronization of Different Chaotic Systems with Variable Scale Time by Adaptive Control", 2009, submitted to Chaos. (SCI, Impact factor: 2.188).
9. Ching-Ming Chang and Zheng-Ming Ge, "Double Symplectic Synchronization of Different Chaotic Systems by Active Control", 2009, submitted to Journal of Sound and Vibration. (SCI, Impact factor: 1.024).
10. Zheng-Ming Ge and Chien-Hao Li, 2008, "Chaos in a Double Duffing System and in Its Fractional Order System", submitted to Journal of Computational and Applied Mathematics. (SCI, Impact factor: 0.943)
11. Zheng-Ming Ge and Chien-Hao Li, 2008, "Highly Robust Pragmatical Generalized Synchronization of Double Duffing Systems with Uncertain Parameters via Adaptive Control", submitted to Mathematics and Computers in Simulation. (SCI, Impact factor: 0.738)
12. Zheng-Ming Ge and Chien-Hao Li, 2008, "Uncoupled Chaos Synchronization and Antisynchronization of Double Duffing Systems by Noise Excited Parameters", submitted to International Journal of Engineering Science. (SCI, Impact factor: 0.966)
13. Zheng-Ming Ge and Shih-Chung Li, 2008, "Chaos in a Double Van der Pol System and in Its Fractional Order System", submitted to Mathematics and Computers in Simulation. (SCI, Impact factor: 0.738)
14. Zheng-Ming Ge and Shih-Chung Li, 2008, "Chaos-excited Synchronization of Uncopuled Double Van der Pol systems", submitted to Journal of Computational and Applied Mathematics. (SCI, Impact factor: 0.943)
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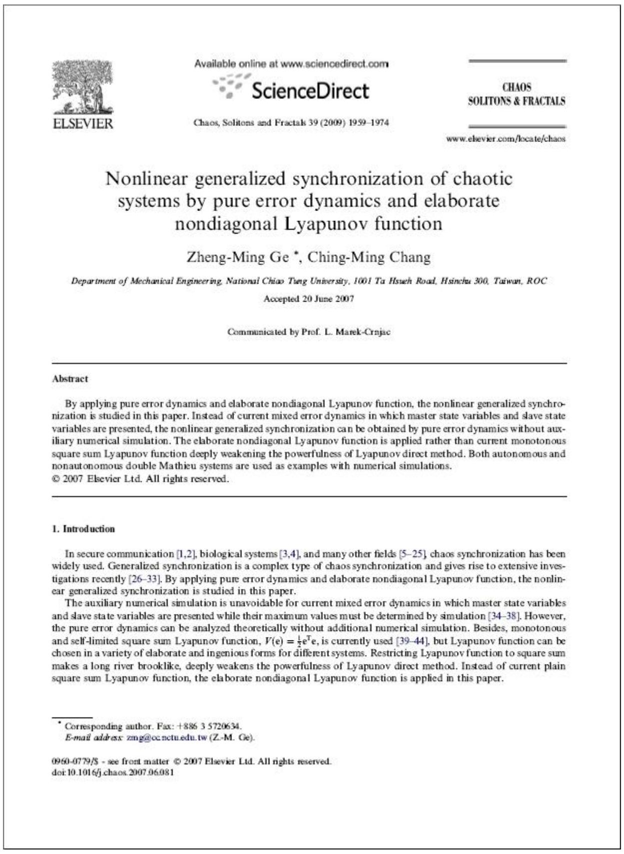

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Nonlinear generalized synchronization of chaotic systems by pure error dynamics and elaborate nondiagonal Lyapunov function

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Abstract

By applying pure error dynamics and elaborate nondiagonal Lyapunov function, the nonlinear generalized synchronization is studied in this paper. Instead of current mixed error dynamics in which master state variables and slave state variables are presented, the nonlinear generalized synchronization can be obtained by pure error dynamics without auxiliary numerical simulation. The elaborate nondiagonal Lyapunov function is applied rather than current monotonous square sum Lyapunov function deeply weakening the powerfulness of Lyapunov direct method. Both autonomous and nonautonomous double Mathieu systems are used as examples with numerical simulations.

1. Introduction

In secure communication [1,2], biological systems [3,4], and many other fields [5–25], chaos synchronization has been widely used. Generalized synchronization is a complex type of chaos synchronization and gives rise to extensive investigations recently [26–33]. By applying pure error dynamics and elaborate nondiagonal Lyapunov function, the nonlinear generalized synchronization is studied in this paper.

The auxiliary numerical simulation is unavoidable for current mixed error dynamics in which master state variables and slave state variables are presented while their maximum values must be determined by simulation [34–38]. However, the pure error dynamics can be analyzed theoretically without additional numerical simulation. Besides, monotonous and self-limited square sum Lyapunov function, $V(e) = \frac{1}{2}e^T e$, is currently used [39–44], but Lyapunov function can be chosen in a variety of elaborate and ingenious forms for different systems. Restricting Lyapunov function to square sum makes a long river brooklike, deeply weakens the powerfulness of Lyapunov direct method. Instead of current plain square sum Lyapunov function, the elaborate nondiagonal Lyapunov function is applied in this paper.

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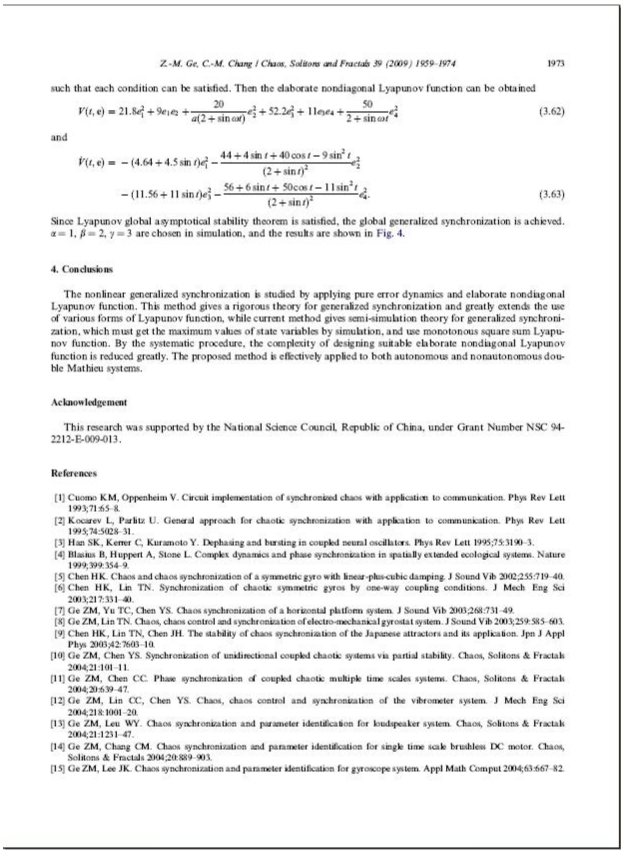

such that each condition can be satisfied. Then the elaborate nondiagonal Lyapunov function can be obtained

$$V(t, e) = 21.8e_1^2 + 9e_1e_2 + \frac{20}{a(2+\sin\omega t)}e_2^2 + 52.2e_3^2 + 11e_3e_4 + \frac{50}{2+\sin\omega t}e_4^2 \quad (3.62)$$

and

$$\dot{V}(t, e) = -(4.64 + 4.5\sin t)e_1^2 - \frac{44 + 4\sin t + 40\cos t - 9\sin^2 t}{(2+\sin t)^2}e_2^2 - (11.56 + 11\sin t)e_3^2 - \frac{56 + 6\sin t + 50\cos t - 11\sin^2 t}{(2+\sin t)^2}e_4^2. \quad (3.63)$$

Since Lyapunov global asymptotical stability theorem is satisfied, the global generalized synchronization is achieved. $\alpha = 1$, $\beta = 2$, $\gamma = 3$ are chosen in simulation, and the results are shown in Fig. 4.

4. Conclusions

The nonlinear generalized synchronization is studied by applying pure error dynamics and elaborate nondiagonal Lyapunov function. This method gives a rigorous theory for generalized synchronization and greatly extends the use of various forms of Lyapunov function, while current method gives semi-simulation theory for generalized synchronization, which must get the maximum values of state variables by simulation, and use monotonous square sum Lyapunov function. By the systematic procedure, the complexity of designing suitable elaborate nondiagonal Lyapunov function is reduced greatly. The proposed method is effectively applied to both autonomous and nonautonomous double Mathieu systems.

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Generalized synchronization of chaotic systems by pure error dynamics and elaborate Lyapunov function

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ABSTRACT

The generalized synchronization is studied by applying pure error dynamics and elaborate Lyapunov function in this paper. Generalized synchronization can be obtained by pure error dynamics without auxiliary numerical simulation, instead of current mixed error dynamics in which master state variables and slave state variables are presented. The elaborate Lyapunov function is applied rather than the current plain square sum Lyapunov function, deeply weakening the power of Lyapunov direct method. The scheme is successfully applied to both autonomous and nonautonomous double Mathieu systems with numerical simulations.

1. Introduction

Chaos synchronization has been applied in secure communication [1,2], biological systems [3,4], and many other fields [5–25]. One of the intricate types of chaos synchronization is generalized synchronization, which has been extensively investigated recently [26–33]. The generalized synchronization is studied by applying pure error dynamics and elaborate Lyapunov function in this paper.

The pure error dynamics can be analyzed theoretically without auxiliary numerical simulation, whereas the aid of additional numerical simulation is unavoidable for current mixed error dynamics in which master state variables and slave state variables are presented, while their maximum values must be determined by simulation [34–38]. Besides, the elaborate Lyapunov function is applied rather than current plain square sum Lyapunov function, $V(\mathbf{e}) = \frac{1}{2}\mathbf{e}^T\mathbf{e}$, which is currently used for convenience. However, the Lyapunov function can be chosen in a variety of forms for different systems. Restricting Lyapunov function to square sum makes a long river brook-like, and greatly weakens the power of Lyapunov direct method.

Based on the Lyapunov direct method [39], generalized synchronization is achieved and a systematic method of designing Lyapunov function is proposed. The technique is successfully applied to both autonomous and nonautonomous double Mathieu systems. This paper is organized as follows. In Section 2, the method of designing Lyapunov function is presented.

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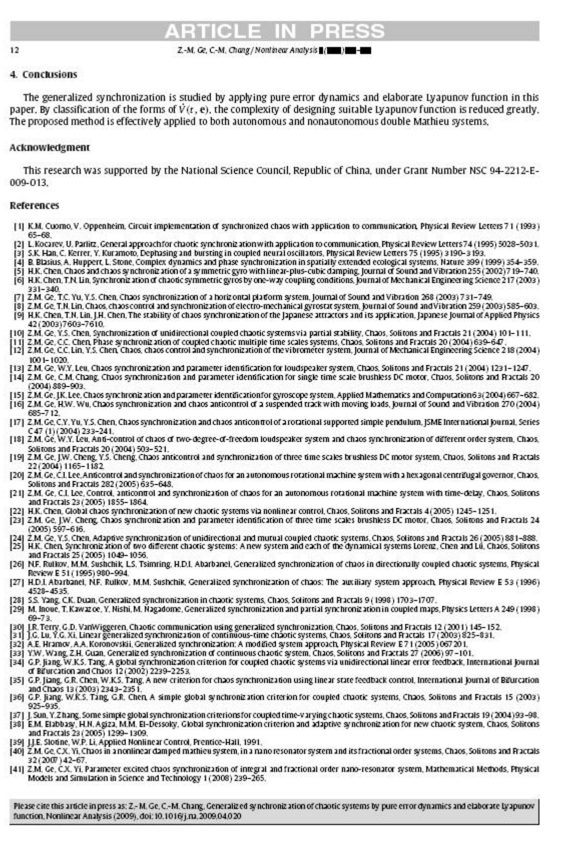

4. Conclusions

The generalized synchronization is studied by applying pure error dynamics and elaborate Lyapunov function in this paper. By classification of the forms of $\dot{V}(t, \mathbf{e})$, the complexity of designing suitable Lyapunov function is reduced greatly. The proposed method is effectively applied to both autonomous and nonautonomous double Mathieu systems.

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Pragmatical adaptive chaos control from a new double van der Pol system to a new double Duffing system

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ABSTRACT

A new pragmatical adaptive control method for different chaotic systems is proposed. Traditional chaos control is limited to decrease chaos of one chaotic system. This method enlarges the effective scope of chaos control. We can control a chaotic system, e.g. a new chaotic double van der Pol system, to a given chaotic or regular system, e.g. a new chaotic double Duffing system or to a damped simple harmonic system. By a pragmatical theorem of asymptotical stability based on an assumption of equal probability of initial point, an adaptive control law is derived such that it can be proved strictly that the common zero solution of error dynamics and of parameter dynamics is asymptotically stable. Numerical simulations are given to show the effectiveness of the proposed scheme.

1. Introduction

Since chaos control was firstly used by Ott et al. [1], it has been studied extensively. Many control methods have been employed to control chaos [2–6]. Simple linear feedback control was proposed [7–9]. Time delay feedback control [10–13], sliding mode control [14–17], backstepping method [18] and adaptive control [19–22] were widely used. However, traditional adaptive chaos control is limited to control the chaotic motion of one chaotic system to regular motion or to fixed point. Proposed pragmatical adaptive control method enlarges the scope of chaos control. We can control a chaotic system to a given simple unchaotic system or to a more complex chaotic system. In current scheme of adaptive control of chaotic motion [23–25], traditional Lyapunov stability theorem and Babalat lemma are used to prove the error vector approaches zero, as time approaches infinity. But the question, why the estimated or given parameters also approach to the uncertain or goal parameters, remains no answer. By a pragmatical theorem of asymptotical stability [29–31] based on an assumption of equal probability of initial points, an adaptive control law is derived such that it can be proved strictly that the common zero solution of error dynamics and of parameter dynamics is asymptotically stable. Numerical results are given for a chaotic double van der Pol system to be controlled to a chaotic double Duffing system and to a regular damped simple harmonic system.

This paper is organized as follows: In Section 2, a pragmatical adaptive control scheme is given. In Section 3 numerical results of chaos control are given. A chaotic double van der Pol system is controlled to a chaotic double Duffing system and to a regular damped simple harmonic system. Finally, conclusions are given in Section 4.

2. Pragmatical adaptive control scheme

Consider the following chaotic system

$$\dot{x} = f(x, A) + u(t), \tag{1}$$

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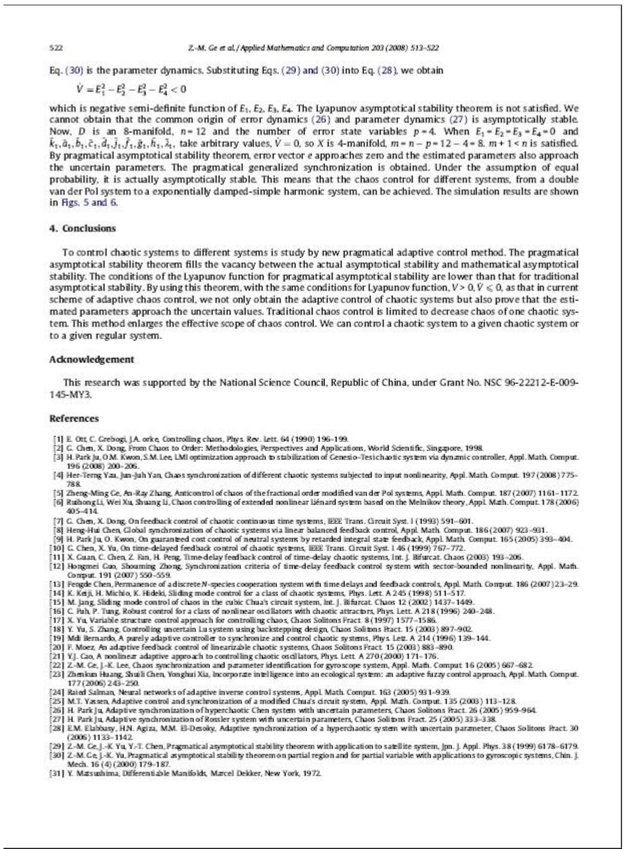

Eq. (30) is the parameter dynamics. Substituting Eqs. (29) and (30) into Eq. (28), we obtain

$$\dot{V} = E_1^2 - E_2^2 - E_3^2 - E_4^2 < 0$$

which is negative semi-definite function of E_1, E_2, E_3, E_4. The Lyapunov asymptotical stability theorem is not satisfied. We cannot obtain that the common origin of error dynamics (26) and parameter dynamics (27) is asymptotically stable. Now, D is an 8-manifold, $n = 12$ and the number of error state variables $p = 4$. When $E_1 = E_2 = E_3 = E_4 = 0$ and $\tilde{k}_1, \tilde{a}_1, \tilde{b}_1, \tilde{c}_1, \tilde{d}_1, \tilde{j}_1, \tilde{f}_1, \tilde{g}_1, \tilde{h}_1, \tilde{\lambda}_1$, take arbitrary values, $\dot{V} = 0$, so X is 4-manifold, $m = n - p = 12 - 4 = 8$. $m + 1 < n$ is satisfied. By pragmatical asymptotical stability theorem, error vector e approaches zero and the estimated parameters also approach the uncertain parameters. The pragmatical generalized synchronization is obtained. Under the assumption of equal probability, it is actually asymptotically stable. This means that the chaos control for different systems, from a double van der Pol system to a exponentially damped-simple harmonic system, can be achieved. The simulation results are shown in Figs. 5 and 6.

4. Conclusions

To control chaotic systems to different systems is study by new pragmatical adaptive control method. The pragmatical asymptotical stability theorem fills the vacancy between the actual asymptotical stability and mathematical asymptotical stability. The conditions of the Lyapunov function for pragmatical asymptotical stability are lower than that for traditional asymptotical stability. By using this theorem, with the same conditions for Lyapunov function, $V > 0, \dot{V} \leqslant 0$, as that in current scheme of adaptive chaos control, we not only obtain the adaptive control of chaotic systems but also prove that the estimated parameters approach the uncertain values. Traditional chaos control is limited to decrease chaos of one chaotic system. This method enlarges the effective scope of chaos control. We can control a chaotic system to a given chaotic system or to a given regular system.

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Rapid Communication

Chaos synchronization of double Duffing systems with parameters excited by a chaotic signal

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Abstract

Chaos synchronization by driving parameter for two uncoupled identical chaotic double Duffing systems is presented. Replacing two corresponding parameters of the identical systems by the same function of chaotic state variables of a third chaotic system, the synchronization or anti-synchronization of two uncoupled systems can be obtained. Numerical simulations are illustrated for either synchronization or anti-synchronization of which the occurrence depends significantly on initial conditions and on driving strength. Alternative complete synchronization and anti-synchronization is also discovered.

1. Introduction

Various synchronization phenomena are being reported for chaotic systems, such as complete synchronization (CS), anti-synchronization (AS), phase synchronization (PS), lag synchronization, and generalized synchronization [1–20,29–38]. However, most of synchronizations can only realize under the condition that there exists coupling between two chaotic systems. In practice, such as in physical and electrical systems, sometimes, it is difficult even impossible to couple two chaotic systems. In comparison with coupled chaotic systems, synchronization between the uncoupled chaotic systems has many advantages [20–29].

In this paper, synchronization of two double Duffing systems whose corresponding parameter is driven by a chaotic signal of a third system is analyzed. The chaos synchronizations of two uncoupled double Duffing systems are obtained by replacing their corresponding parameters by the same function of chaotic state variables of a third chaotic system. It is noted that whether CS or AS appear depends on the initial conditions. Besides, CS and AS are also characterized by great sensitivity to initial conditions and on the strengths of the substituted variable. It is found that CS or AS alternatively occurs under certain conditions [38–42].

This paper is organized as follows. In Section 2, a brief description of synchronization scheme based on the substitution of the strengths of the mutual coupling term of two identical chaotic double Duffing systems by

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4. Conclusions

In this paper, parameter excited chaos synchronizations of two identical double Duffing systems are studied by adjusting the strength of the substituting variable. Numerical simulations are illustrated for CS or AS of which the occurrence depends on initial conditions and driving strength. Besides, alternative CS and AS is also discovered with the same initial conditions and the same driving strength.

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Temporary lag and anticipated synchronization and anti-synchronization of uncoupled time-delayed chaotic systems

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Abstract

Without any control scheme and coupling terms, temporary lag and anticipated synchronization and temporary lag and anticipated anti-synchronization are newly discovered in two identical double Mackey–Glass systems with different initial conditions. When all initial conditions are positive, the lag synchronization is obtained. The negative initial values make the time history inverse and temporary lag anti-synchronization occur. The phenomena both appear intermittently.

1. Introduction

Since the first idea of synchronizing two identical chaotic systems with different initial conditions was investigated by Pecorra and Carroll [1], chaos synchronization [2–8] has become an important topic in engineering science. In Ref. [2], this study demonstrates that chaos synchronization between two different chaotic systems using active control has been achieved. The Lorenz, Chen and Lü systems have been controlled to be the new system. In Ref. [3], chaos synchronization of two identical chaotic motions of symmetric gyros is presented. It has been demonstrated that applying four different kinds of one-way coupling conditions can synchronize two identical chaotic systems. In Ref. [4], the dynamic behavior of a symmetric gyro with linear-plus-cubic damping, which is subjected to a harmonic excitation, is studied in this paper. In Ref. [5], synchronization of feedback method in two identical non-autonomous coupled systems has been studied. Then the phase effect of two coupled systems and the transient time in unidirectional synchronization also have been researched. In Ref. [6], the dynamic behavior of electro-mechanical gyrostat system subjected to external disturbance is studied. In Ref. [7], a general scheme is proposed to achieve chaos synchronization via stability with respect to partial variables. Three theorems for synchronization of unidirectional coupled non-autonomous (also autonomous) systems by linear feedback are developed for systems with and without system structure perturbations. In Ref. [8], the dynamic system of the vibrometer is shown to produce regular and chaotic behavior as the parameters are varied. When the system is non-autonomous, the periodic and chaotic motions are obtained by numerical methods. Many effective control schemes have been developed in a

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5. Conclusions

In this paper, temporary lag or anticipated synchronization and the lag or anticipated anti-synchronization of double Mackey–Glass systems with small and similar initial conditions are discovered. For the first interval of TLS, when all initial values are positive, temporary lag synchronizations are found. The trajectory will be reversed if the initial condition of x_1 or y_1 is negative. In these cases, the lag or anticipated anti-synchronization exists. From the results of simulation, we find six temporary lag (anticipated) synchronization intervals in 30,000 s. The numerical simulations of temporary lag and anticipated synchronization and anti-synchronization are showed in this paper. In fact, our new double Mackey–Glass systems with different delay time τ can be used in transfusion of blood between two persons. Our future work will study model for different persons with different initial conditions in transfusion of blood. The theoretical analysis and its applications should be open for further work in the future.

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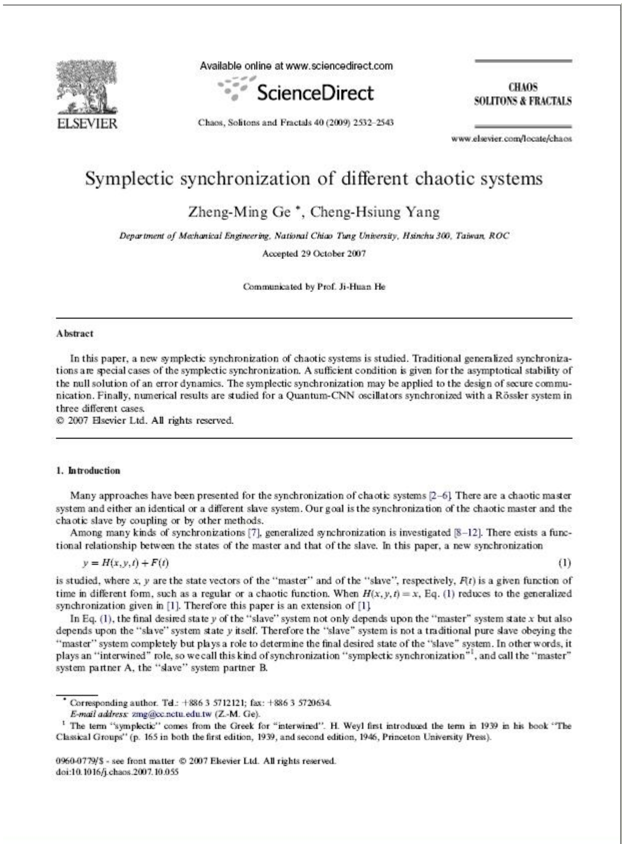

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Symplectic synchronization of different chaotic systems

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Abstract

In this paper, a new symplectic synchronization of chaotic systems is studied. Traditional generalized synchronizations are special cases of the symplectic synchronization. A sufficient condition is given for the asymptotical stability of the null solution of an error dynamics. The symplectic synchronization may be applied to the design of secure communication. Finally, numerical results are studied for a Quantum-CNN oscillators synchronized with a Rössler system in three different cases.

1. Introduction

Many approaches have been presented for the synchronization of chaotic systems [2–6]. There are a chaotic master system and either an identical or a different slave system. Our goal is the synchronization of the chaotic master and the chaotic slave by coupling or by other methods.

Among many kinds of synchronizations [7], generalized synchronization is investigated [8–12]. There exists a functional relationship between the states of the master and that of the slave. In this paper, a new synchronization

$$y = H(x, y, t) + F(t) \quad (1)$$

is studied, where x, y are the state vectors of the “master” and of the “slave”, respectively, $F(t)$ is a given function of time in different form, such as a regular or a chaotic function. When $H(x, y, t) = x$, Eq. (1) reduces to the generalized synchronization given in [1]. Therefore this paper is an extension of [1].

In Eq. (1), the final desired state y of the “slave” system not only depends upon the “master” system state x but also depends upon the “slave” system state y itself. Therefore the “slave” system is not a traditional pure slave obeying the “master” system completely but plays a role to determine the final desired state of the “slave” system. In other words, it plays an “interwined” role, so we call this kind of synchronization “symplectic synchronization”[1], and call the “master” system partner A, the “slave” system partner B.

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[1] The term “symplectic” comes from the Greek for “interwined”. H. Weyl first introduced the term in 1939 in his book “The Classical Groups” (p. 165 in both the first edition, 1939, and second edition, 1946, Princeton University Press).

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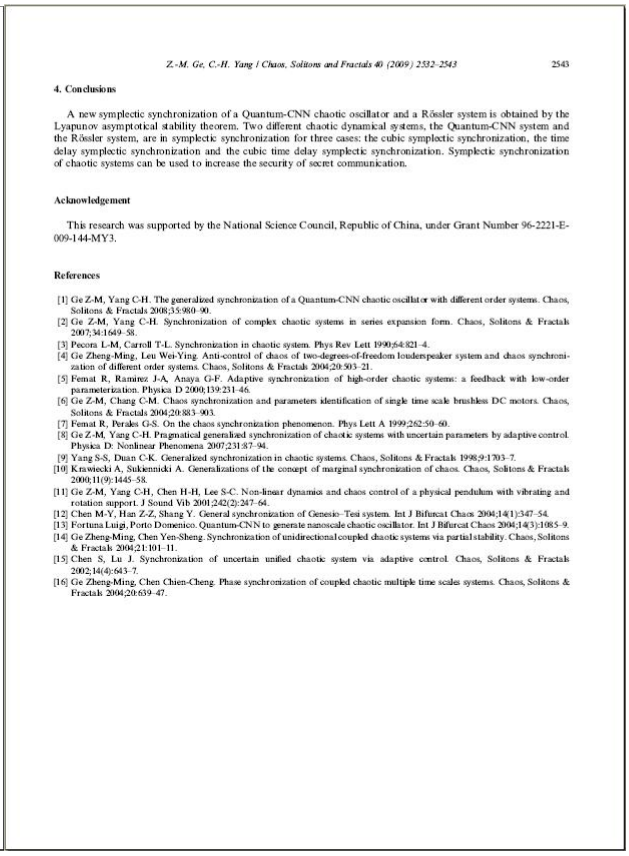

4. Conclusions

A new symplectic synchronization of a Quantum-CNN chaotic oscillator and a Rössler system is obtained by the Lyapunov asymptotical stability theorem. Two different chaotic dynamical systems, the Quantum-CNN system and the Rössler system, are in symplectic synchronization for three cases: the cubic symplectic synchronization, the time delay symplectic synchronization and the cubic time delay symplectic synchronization. Symplectic synchronization of chaotic systems can be used to increase the security of secret communication.

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Chaos Synchronization and Chaos Control of Quantum-CNN Chaotic System by Variable Structure Control and Impulse Control

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Keywords : Quantum Cellular Neural Network (Quantum -CNN), Chaos Synchronization, Chaos Control

ABSTRCACT

In this paper, we derive some less stringent conditions for the exponential and asymptotic stability of impulsive control systems with impulses at fixed times. These conditions are then used to design an impulsive control law for Quantum Cellular Neural Network chaotic system, which drives the chaotic state to zero equilibrium and synchronizes two chaotic systems. An active sliding mode control method is synchronizing two chaotic systems and controlling chaotic state to periodic motion state. And a sufficient condition is drawn for the robust stability of the error dynamics, and is applied to guiding the design of the controllers. Finally, numerical results are used to show the robustness and effectiveness of the proposed control strategy.

1. Introduction.

Chaotic system exhibits unpredictable and irregular dynamics and it has been found in many engineering systems. Interestingly, chaotic models can describe complex dynamics with only few nonlinear equations without any random external inputs and small differences in the initial state can lead to extraordinary differences in the system state. Since Ott, Grebogi, and Yorke proposed the OGY method [1], a method of controlling chaos, 'controlling of chaos' is receiving increasing attention within the area of non-linear dynamics [2,3]. It has many applications in various systems while it is unfavorable in many other cases due to its irregular behavior. Therefore, both chaos utilization and elimination are important depending on the specific applications. Chaos control is an effective method for both chaos utilization and elimination and has been thoroughly studied in various fields of science.

Since the seminal work of Pecora and Carroll [4], there has been an interesting and potential topic in recent years in the study of chaos synchronization in physics, mathematics and engineering

$\rho_{11}=32, \rho_{12}=27, \rho_{13}=49, \rho_{14}=31, \rho_{21}=32, \rho_{22}=27, \rho_{23}=49, \rho_{24}=31$.

The result is shown in Figs. 4~5 for unidirectional linear coupling and bi-directional linear coupling, respectively.

4. Conclusions

Two chaotic Quantum-CNN systems are synchronized by three methods: unidirectional linear coupling by impulse control, bi-directional linear coupling by impulse control and variable structure control. The chaos controls of a Quantum-CNN system are also studied. The impulse control, and variable structure control are used to suppress chaos to fixed point or regulation motion. Numerical simulations are used to verify the effectiveness of the proposed controllers. These chaos synchronization and control methods can be also used for other chaotic systems.

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Synchronization of chaotic systems with uncertain chaotic parameters by linear coupling and pragmatical adaptive tracking

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We study the synchronization of general chaotic systems which satisfy the Lipschitz condition only, with uncertain **chaotic** parameters by linear coupling and pragmatical adaptive tracking. The uncertain parameters of a system vary with time due to aging, environment, and disturbances. A sufficient condition is given for the asymptotical stability of common zero solution of error dynamics and parameter update dynamics by the Ge–Yu–Chen pragmatical asymptotical stability theorem based on equal probability assumption. Numerical results are studied for a Lorenz system and a quantum cellular neural network oscillator to show the effectiveness of the proposed synchronization strategy. © *2008 American Institute of Physics*. [DOI: 10.1063/1.3049320]

Theoretical and experimental investigations have shown that synchronization, in particular chaos synchronization, has great potential in a large amount of application areas ranging from secure communications to modeling brain activity. In this paper, we introduce a synchronization of chaotic systems with uncertain chaotic parameters by linear coupling and pragmatical adaptive tracking. Based on pragmatical stability theorem and Lipschitz condition, some less conservative conditions for determining linear coupling synchronization of general chaotic systems are obtained. Two examples are simulated to illustrate the validity of the theoretical analysis.

I. INTRODUCTION

The idea of synchronizing two identical chaotic systems with different initial conditions was introduced by Pecora and Carroll.[1] Since then there has been particular interest in chaotic synchronization, due to many potential applications in secure communication,[2] chemical and biological systems.[3,4] There are many control methods to synchronize chaotic systems, such as, linear coupling, for which the implementation is rather easy, adaptive control, impulsive control, sliding mode control, and other methods.[5] Most of them are based on the exact knowledge of the system structure and parameters. But in practice, some or all of the system parameters are uncertain. Moreover these parameters may change from time to time and become chaotic because of chaotic disturbances. For uncertain parameters, a lot of works have proceeded to solve this problem by adaptive synchronization.[6–12] In the current scheme of adaptive synchronization,[13–15] the traditional Lyapunov stability theorem and Barbalat lemma are used to prove that the error vector approaches zero as time approaches infinity. But the question, why the estimated parameters also approach the uncertain parameters, has remained without answer. From the Ge–Yu–Chen (GYC) pragmatical asymptotical stability theorem,[16–18] the question is strictly answered. In this paper, the synchronization of general chaotic systems which satisfy the Lipschitz condition only, with unknown parameters which are altered under some **chaotic** disturbances, by linear coupling and pragmatical adaptive tracking, is studied first.

As numerical examples, the Lorenz system and recently developed quantum cellular neural network (Quantum-CNN) chaotic oscillator are used. Pragmatical adaptive tracking is used to track **chaotic** parameters in unidirectional coupled systems. Two Lorenz systems and two Quantum-CNN systems by pragmatical adaptive tracking are given as simulation examples. Quantum-CNN oscillator equations are derived from a Schrödinger equation taking into account quantum dots cellular automata structures to which in the last decade a wide interest has been devoted with particular attention towards quantum computing.[19–21]

This paper is organized as follows: In Sec. II, by pragmatical asymptotical stability theorem and by using Lipschitz conditions, theoretical analysis of synchronization is given. In Sec. III linear feedback controllers are used. By pragmatical adaptive tracking, chaos synchronization of two Lorenz systems and of two Quantum-CNN oscillator systems are achieved by numerical simulations. Conclusions are given in Sec. IV. GYC pragmatical asymptotical stability theorem is presented in the Appendix. Intuitively this theorem is different from traditional Lyapunov stability theorem at that when the points in the neighborhood of zero solution initiating trajectories not approaching zero with time are "not too many," i.e., in a subset of Lebesque measure 0 in mathematical language,[22] we can neglect their existence, i.e., the zero solution is actually asymptotically stable.

II. STRATEGY OF THE CHAOTIC SYNCHRONIZATION

Consider a nonautonomous system in the form as follows:

$$\dot{x} = F[t,x,B(t)]. \quad (1)$$

The slave system is given by

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X is a 5-manifold, $m=n-p=9-4=5$. $m+1<n$ is satisfied. From the GYC pragmatical asymptotical stability theorem, error vector e approaches zero and the estimated parameters also approach the uncertain parameters. The equilibrium point $e_i=\tilde{a}_j=\tilde{\omega}_j=\tilde{G}=0$ ($i=1,2,3,4$; $j=1,2$) is asymptotically stable. Moreover, the result is global asymptotically stable (see Appendix). The numerical results of the time series of states, state errors, parameters and estimated Lipschitz constant $\hat{G}$ are shown in Fig. 6. The chaos synchronization is accomplished near 3 s. $\hat{G}$ approaches constant also near 3 s. The coupling strength required is $K=2G=5.62$.

IV. CONCLUSIONS

Using the Lipschitz condition, the synchronization of Lorenz chaotic systems and of Quantum-CNN chaotic oscillator systems with uncertain **chaotic** parameters by linear coupling and pragmatical adaptive tracking are accomplished by the GYC pragmatical asymptotical stability theorem. Tracking uncertain **chaotic** parameters is first studied in this paper. This is of practical interest, because system parameters may be varied chaotically due to aging, environment, and disturbances. Two Lorenz systems are synchronized for chaotic parameters $M<n$. Two Quantum-CNN systems are synchronized for chaotic parameters $M=n$. The simulation results imply that this scheme is very effective. By GYC pragmatical asymptotical stability theorem, the question, why the estimated parameters approach the uncertain parameters, has been strictly answered and verified by numerical simulations.

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APPENDIX: GYC PRAGMATICAL ASYMPTOTICAL STABILITY THEOREM

The stability for many problems in real dynamical systems is actual asymptotical stability, although it may not be mathematical asymptotical stability. The mathematical asymptotical stability demands that trajectories from all initial states in the neighborhood of zero solution must approach the origin as $t\to\infty$. If there is only a small part or even a few of the initial states from which the trajectories do not approach the origin as $t\to\infty$, the zero solution is not mathematically asymptotically stable. If the probability of occurrence of the event that the trajectories from the initial states are that they do not approach zero when $t\to\infty$, i.e., these trajectories are not asymptotical stable for zero solution, is zero, the stability of zero solution is actual asymptotical stability though it is not mathematical asymptotical stability. In order to analyze the asymptotical stability of the equilibrium point of such systems, the pragmatical asymptotical stability theorem is used. The conditions for pragmatical asymptotical stability are more slack than that for traditional Lyapunov asymptotical stability.

Let X and Y be two manifolds of dimensions m and n ($m<n$), respectively, and φ be a differentiable map from X to Y; then $\varphi(X)$ is a subset of the Lebesque measure 0 of Y.[22] For an autonomous system

$$\dot{x} = f(x_1, \ldots, x_n), \quad (A1)$$

where $x=[x_1,\ldots,x_n]^T$ is a state vector, the function $f=[f_1,\ldots,f_n]^T$ is defined on $D\subset R^n$, an n-manifold.

Let $x=0$ be an equilibrium point for the system (A1). Then

$$f(0) = 0. \quad (A2)$$

For a nonautonomous system,

$$\dot{x} = f(x_1, \ldots, x_{n+1}), \quad (A3)$$

where $x=[x_1,\ldots,x_{n+1}]^T$, the function $f=[f_1,\ldots,f_n]^T$ is defined on $D\subset R^n\times R_+$, here $t=x_{n+1}\subset R_+$. The equilibrium point is

$$f(0,x_{n+1}) = 0. \quad (A4)$$

Definition. The equilibrium point for the system is pragmatically asymptotically stable provided that with initial points on C which is a subset of the Lebesque measure 0 of D, the behaviors of the corresponding trajectories cannot be determined, while with initial points on $D-C$, the corresponding trajectories behave as those that agree with traditional asymptotical stability.

Theorem: Let $V=[x_1,x_2,\ldots,x_n]^T$: $D\to R_+$ be positive definite and analytic on D, where $x_1,x_2,\ldots,x_n$ are all space coordinates such that the derivative of V through Eqs. (A1) or (A3), $\dot{V}$, is negative semidefinite of $[x_1,x_2,\ldots,x_n]^T$.

For an autonomous system, let X be the m-manifold consisting of a point set for which $\forall x\neq 0$, $\dot{V}(x)=0$ and D is an m-manifold. If $m+1<n$, then the equilibrium point of the system is pragmatically asymptotically stable.

For a nonautonomous system, let X be the $m+1$-manifold consisting of the point set for which $\forall x\neq 0$, $\dot{V}(x_1,x_2,\ldots,x_n)=0$, and D is an $n+1$-manifold. If $m+1+1<n+1$, i.e., $m+1<n$, then the equilibrium point of the system is pragmatically asymptotically stable. Therefore, for both autonomous and nonautonomous systems, the formula $m+1<n$ is universal. So the following proof is only for an autonomous system. The proof for the nonautonomous system is similar.

Proof: Since every point of X can be passed by a trajectory of Eq. (A1), which is one-dimensional, the collection of these trajectories, C, is an $(m+1)$-manifold.[16,17]

If $m+1<n$, then the collection C is a subset of Lebesque measure 0 of D. By the above definition, the equilibrium point of the system is pragmatically asymptotically stable.

If an initial point is ergodicly chosen in D, *the probability of that the initial point falls on the collection C is zero. Here, equal probability is assumed for every point chosen as an initial point in the neighborhood of the equilibrium point.*

Chaos Synchronization and Chaos Control of Quantum-CNN Chaotic System by Variable Structure Control and Impulse Control

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Keywords : Quantum Cellular Neural Network (Quantum -CNN), Chaos Synchronization, Chaos Control

ABSTRCACT

In this paper, we derive some less stringent conditions for the exponential and asymptotic stability of impulsive control systems with impulses at fixed times. These conditions are then used to design an impulsive control law for Quantum Cellular Neural Network chaotic system, which drives the chaotic state to zero equilibrium and synchronizes two chaotic systems. An active sliding mode control method is synchronizing two chaotic systems and controlling chaotic state to periodic motion state. And a sufficient condition is drawn for the robust stability of the error dynamics, and is applied to guiding the design of the controllers. Finally, numerical results are used to show the robustness and effectiveness of the proposed control strategy.

1. Introduction.

Chaotic system exhibits unpredictable and irregular dynamics and it has been found in many engineering systems. Interestingly, chaotic models can describe complex dynamics with only few nonlinear equations without any random external inputs and small differences in the initial state can lead to extraordinary differences in the system state. Since Ott, Grebogi, and Yorke proposed the OGY method [1], a method of controlling chaos, 'controlling of chaos' is receiving increasing attention within the area of non-linear dynamics [2,3]. It has many applications in various systems while it is unfavorable in many other cases due to its irregular behavior. Therefore, both chaos utilization and elimination are important depending on the specific applications. Chaos control is an effective method for both chaos utilization and elimination and has been thoroughly studied in various fields of science.

Since the seminal work of Pecora and Carroll [4], there has been an interesting and potential topic in recent years in the study of chaos synchronization in physics, mathematics and engineering

ρ_{11}=32, ρ_{12}=27, ρ_{13}=49, ρ_{14}=31, ρ_{21}=32, ρ_{22}=27, ρ_{23}=49, ρ_{24}=31.

The result is shown in Figs. 4~5 for unidirectional linear coupling and bi-directional linear coupling, respectively.

4. Conclusions

Two chaotic Quantum-CNN systems are synchronized by three methods: unidirectional linear coupling by impulse control, bi-directional linear coupling by impulse control and variable structure control. The chaos controls of a Quantum-CNN system are also studied. The impulse control, and variable structure control are used to suppress chaos to fixed point or regulation motion. Numerical simulations are used to verify the effectiveness of the proposed controllers. These chaos synchronization and control methods can be also used for other chaotic systems.

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國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

■ 達成目標

□ 未達成目標（請說明，以 100 字為限）

□ 實驗失敗

□ 因故實驗中斷

□ 其他原因

第一年：完成兩種雙 Mathieu 系統的渾沌行為與純誤差穩定的廣義同步及其對此二系統的應用

1.完成採用諸多相圖、分歧圖、功率譜圖、參數圖及李亞普諾夫指數及碎形維度等研究自治的雙 Mathieu 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。(二個月)
2.完成採用諸多相圖、分歧圖、功率譜圖、參數圖及李亞普諾夫指數及碎形維度等研究非自治的雙 Mathieu 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。(二個月)
3.完成研究純誤差穩定的廣義同步法之理論。(三個月)
4.完成研究純誤差穩定的廣義同步法對自治的雙 Mathieu 系統之應用。(二個月)
5.完成研究純誤差穩定的廣義同步法對非自治的雙Mathieu系統之應用。(二個月)
6.完成撰寫年度報告書。(一個月)

第二年：完成雙 Duffing 系統及雙 van der Pol 系統的渾沌行為與實用適應廣義同步法，及對此二系統的應用 1.完成採用諸多相圖、分歧圖、功率譜圖、參數圖、李亞普諾夫指數及碎形維度等研究雙 Duffing 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。(二個月) 2.完成採用諸多相圖、分歧圖、功率譜圖、參數圖、李亞普諾夫指數及碎形維度等研究雙 van der Pol 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。(二個月) 3.完成研究實用適應廣義同步法之理論。(三個月) 4.完成研究實用適應廣義同步法對雙 Duffing 系統之應用。(二個月) 5.完成研究實用適應廣義同步法對雙 Van der Pol 系統之應用。(二個月) 6.完成撰寫年度報告書。(一個月)
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論文：■已發表 □未發表之文稿 □撰寫中 □無

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研究內容寫成之期刊論文已有 22 篇，其中已有 5 篇被接受：

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4. Cheng-Hsiung Yang, Zheng-Ming Ge, Ching-Ming Chang and Shih-Yu Li, 2009, “Chaos Synchronization and Chaos Control of Quantum-CNN Chaotic System by Variable Structure Control and Impulse Control”, accepted by Nonlinear Analysis: Real World Applications. (SCI, Impact factor: 1.232)

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以500字為限）

Duffing系統，van der Pol系統與線性Mathieu系統原為振動學科之最重要最典型的系統。自渾沌動力學興起後，Duffing系統，van der Pol系統由於其為非線性系統故亦沿習成為渾沌動力學學科中最重要最典型的系統，四十年來對此二系統的渾沌研究之文獻可謂汗牛充棟，至今方興未艾。而線性Mathieu系統，則由於其為線性方程，不具渾沌性質，故在渾沌動力學學科中乃不再提及。人們忽視了非線性Mathieu系統實為Duffing系統中參數由常數轉為時間週期函數之推廣，實亦應成為渾沌動力學科之最重要最典型之系統。本計畫主持人率先研究非線性Mathieu系統之渾沌行為[6]，可謂遲來之補求。眾所週知，此三種典型系統除理論意義外，廣泛應用於機械、電機、物理、化學、生科、奈米系統，本計畫今研究雙Duffing系統，雙van der Pol系統及雙種類型的非線性Mathieu系統，不僅對渾沌動力學學科中最重要最典型的三種渾沌系統的研究的拓廣與深化，更重要的是它們本身顯然具有更複雜的，未經發現的複雜渾沌行為，本研究對渾沌動力學學科具重大意義。其應用於機械、電機、物理、化學、生科、奈米之耦合系統，具有重要的實用價值。

渾沌同步除本身之重要理論價值外，其研究在秘密通訊、神經網路、自我組織等方面有日益廣泛之應用。廣義渾沌同步則為渾沌同步之進一步發展，其應用亦方興未艾。本計畫提出三種新的渾沌同步。實用適應廣義同步法糾正了目前國際文獻中未經証明即認為估值參數趨於為之參數之錯誤，首次在渾沌同步中引入概率概念，具重大理論及實用意義。由於$\dot{V}$之要求降低，實際應用亦較易實現。純誤差穩定的廣義同步，則彌補了國際文獻中需用數值計算結果為條件之理論，即有缺陷之理論。在理論與實用上有重要意義。不同起始條件的延遲同步等多種渾沌同步則為新發現的渾沌運動之現象，特別是Ikeda系統的永遠性延遲同步或反同步，不同於傳統理論，尤具重大意義。