行政院國家科學委員會專題研究計畫 期中進度報告

在排隊系統裡控制到達與控制服務之理論與實務應用研究--子計畫一:探討有限容量F方策M/G/1排隊系統之操作特性與敏感度分析(第1年)

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（期中進度報告）

子計畫一： 探討有限容量 F 方策 M/G/1 排隊系統之操作特性與敏感度分析（1/3）

計畫編號：NSC 96-2628-E-009-025-MY3

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一、摘要

在計畫的第一年，我們探討有限容量 M/G/1 排隊系統結合 F 方策及啟動時間之最佳控制。當服務者開始允許顧客進入系統時，服務者需要啟動時間，而啟動時間具指數分配。F 方策排隊問題是研究控制到達的顧客進入排隊系統之最普遍議題。我們提出遞迴方法及使用輔助變數技巧，並將輔助變數視為剩餘服務時間，來推導在系統中顧客數之機率分配。此遞迴方法可用三種不同服務時間分配：指數分配，三階段 Erlang 分配，及常數分配來分析解釋。我們並提出一些數值例子。

關鍵詞：F 方策，M/G/1/K 排隊，遞迴方法，啟動時間，輔助變數

二、計劃緣由與目的

我們研究有限容量 F 方策 M/G/1 排隊系統之最佳控制。假設顧客到達時間呈指數分配，而服務時間則為一般分配。當服務者開始允許顧客進入系統時，服務者需要具有指數分配之啟動時間。所謂的 F 方策其特性為當在系統裡的顧客數達到系統容量時（也就是系統達飽和狀態時），便不允許任何一位顧客進到系統來接受服務，一直到在系統裡有足夠的顧客已經完成服務，此時在系統裡的顧客數會遞減到一個門檻值 F $(0 \le F < K-1)$。而在那時，服務者會要求一段具指數分配之啟動時間來開始允許對顧客服務。此系統維持正常操作直到系統裡的顧客數再次達到系統之容量。在此計畫，我們首先參考 Wang 和 Ke [16] 所提出之輔助變數的技巧，將輔助變數視為剩餘顧客服務時間並用來建立系統之**穩態方程式**（steady-state equations）。接者利用遞迴方法，經過複雜之數學運算，推導出在系統中顧客數之機率分配，並以 closed-form 表示。我們針對三種不同的顧客服務時間分配：指數分配，三階段 Erlang 分配及常數分配，並提供 ”The Solution Algorithm”來分析解釋如何利用遞迴方法。最後，選擇不同系統參數的數值，使用 Maple 及 Matlab 等科學計算軟體撰寫電腦程式，以計算不同系統之執行測度，如系統平均長度（mean system length），並提供一些數值例子。我們也推導出成本模式，並針對此成本模式來求得其在最低成本時之最佳 F 方策。

顧客到達時間呈指數分配，而服務時間為一般分配之 F 方策 M/G/1/K 排隊系統，至今尚無學者研究過。因此本計劃將探討在 F 方策條件下，顧客到達時間呈指數分配，而服務時間具一般分配之 F 方策 M/G/1/K 排隊系統。本子計畫一的主要目的有下列四項：（1）針對 F 方策 M/G/1/K 排隊系統含有啟動時間是指數分配進行探討（2）針對 F 方策 M/G/1/K 排隊系統之操作特性公式進行推導（3）利用建立單位時間總期望成本函數，來決定在最低成本時之最佳解 F^*（4）進行敏感度分相關研

究。即研究系統不同參數及成本元素分別對最佳解 F^* 之影響程度。

三、 研究方法

本子計畫一將推廣 Gupta [4] 的文章(即 N 方策 M/M/1/K 排隊系統含有指數分配之啟動時間），來研究 F 方策 M/G/1/K 排隊系統含有指數分配之啟動時間。我們首先利用遞迴方法來推導在系統中顧客數之機率分配，並以 closed-form 之形式來表示。其次，我們提供 ”The Solution Algorithm” 來解釋如何利用遞迴方法，並以三種不同的顧客服務時間分配：指數分配，三階段 Erlang 分配及常數分配來舉例說明。最後，我們建立成本函數，求出在最低成本時之最佳解 F^* ，並提供在最低成本之一些不同系統執行測度下之數值例子。

四、 研究成果

本子計畫一之研究成果敘述如下：

1. 利用遞迴方法推導在此穩態方程式之穩態解析解，並以 closed-form 表示；
2. 以三種不同的顧客服務時間分配:指數分配，三階段 Erlang 分配及常數分配來舉例說明，並提供”The Solution Algorithm”來解釋如何使用遞迴方法；
3. 當顧客服務時間是指數分配時,其所獲得的結果跟 Gupta [4] 的結果做比較，來驗證此穩態分析解是否正確,同時也可以證明本計劃結果比 Gupta [4]的結果更具一般性；
4. 給予不同的系統參數之數值,提供一些不同系統執行測度之重要數值結果；
5. 建立此 F 方策之成本函數，並獲得最佳解 F^*;
6. 在最低成本時,提供一些不同系統執行測度之數值結果。

本子計畫一的主要目的是：

1. 我們參考 Wang 和 Ke [16]所提出的輔助變數技巧 (其探討 N 方策 M/G/1/K 排隊系統)來建立 F 方策 M/G/1/K 排隊系統之穩態方程式；
2. 使用遞迴方法來推導此穩態方程式之穩態解析解；
3. 以三種不同顧客服務時間分配來舉例說明如何利用遞迴方法；
4. 在顧客服務時間呈指數分配時,將其所獲得的結果，來和 Gupta [4] 的結果做比較，可以驗證此穩態解析解是否正確；
5. 求出在最低成本時，此控制參數 F 之最佳值；
6. 在最低成本時,提供一些不同系統執行測度之重要數值結果。

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備註：本計畫之研究成果會投著名之 SCI 期刊。

Appendix 1

Let

$$P_{0,n}(u) = P_{0,n} s(u),\ n = 1,2,\ldots,F\ .$$

Steady state equations:

$$0 = -\beta P_{0,0} + P_{0,1}(0), \tag{1}$$

$$-(d/du)P_{0,n}(u) = -\beta P_{0,n} s(u) + P_{0,n+1}(0)s(u),\ 1 \le n \le F, \tag{2}$$

$$-(d/du)P_{0,n}(u) = P_{0,n+1}(0)s(u),\ F+1 \le n \le K-1, \tag{3}$$

$$-(d/du)P_{0,K}(u) = \lambda P_{1,K-1}(u), \tag{4}$$

$$0 = -\lambda P_{1,0} + \beta P_{0,0} + P_{1,1}(0), \tag{5}$$

$$-(d/du)P_{1,1}(u) = -\lambda P_{1,1}(u) + \beta P_{0,1} s(u) + \lambda P_{1,0} s(u) + P_{1,2}(0)s(u), \tag{6}$$

$$-(d/du)P_{1,n}(u) = -\lambda P_{1,n}(u) + \beta P_{0,n} s(u) + \lambda P_{1,n-1}(u) + P_{1,n+1}(0)s(u),\ 2 \le n \le F, \tag{7}$$

$$-(d/du)P_{1,n}(u) = -\lambda P_{1,n}(u) + \lambda P_{1,n-1}(u) + P_{1,n+1}(0)s(u),\ F+1 \le n \le K-2, \tag{8}$$

$$-(d/du)P_{1,K-1}(u) = -\lambda P_{1,K-1}(u) + \lambda P_{1,K-2}(u). \tag{9}$$

Further define

$$S^*(\theta) = \int_0^\infty e^{-\theta u} \mathrm{d}S(u) = \int_0^\infty e^{-\theta u} s(u)\,\mathrm{d}u,$$

$$P_{0,n}^*(\theta) = \int_0^\infty e^{-\theta u} P_{0,n}(u)\,\mathrm{d}u,$$

$$P_{1,n}^{*}(\theta)=\int_{0}^{\infty}e^{-\theta u}P_{1,n}(u)\mathrm{d}u,$$

$$P_{0,n}=P_{0,n}^{*}(0)=\int_{0}^{\infty}P_{0,n}(u)\mathrm{d}u,$$

$$P_{1,n}=P_{1,n}^{*}(0)=\int_{0}^{\infty}P_{1,n}(u)\mathrm{d}u,$$

$$\int_{0}^{\infty}e^{-\theta u}\frac{\partial}{\partial u}P_{0,n}(u)\mathrm{d}u=\theta P_{0,n}^{*}(\theta)-P_{0,n}(0),$$

and

$$\int_{0}^{\infty}e^{-\theta u}\frac{\partial}{\partial u}P_{1,n}(u)\mathrm{d}u=\theta P_{1,n}^{*}(\theta)-P_{1,n}(0).$$

Therefore if the LST is taken of both sides of (1)-(9) , it is found that

$$-\theta P_{0,n}^{*}(\theta)=-\beta P_{0,n}S^{*}(\theta)+P_{0,n+1}(0)S^{*}(\theta)-P_{0,n}(0),\ 1\le n\le F, \tag{10}$$

$$-\theta P_{0,n}^{*}(\theta)=P_{0,n+1}(0)S^{*}(\theta)-P_{0,n}(0),\ F+1\le n\le K-1, \tag{11}$$

$$-\theta P_{0,K}^{*}(\theta)=\lambda P_{1,K}^{*}(\theta)-P_{0,K}(0), \tag{12}$$

$$(\lambda-\theta)P_{1,1}^{*}(\theta)=\beta P_{0,1}S^{*}(\theta)+\lambda P_{1,0}S^{*}(\theta)+P_{1,2}(0)S^{*}(\theta)-P_{1,1}(0), \tag{13}$$

$$(\lambda-\theta)P_{1,n}^{*}(\theta)=\beta P_{0,n}S^{*}(\theta)+\lambda P_{1,n-1}^{*}(\theta)+P_{1,n+1}(0)S^{*}(\theta)-P_{1,n}(0),\ 2\le n\le F, \tag{14}$$

$$(\lambda-\theta)P_{1,n}^{*}(\theta)=\lambda P_{1,n-1}^{*}(\theta)+P_{1,n+1}(0)S^{*}(\theta)-P_{1,n}(0),\ F+1\le n\le K-2, \tag{15}$$

$$(\lambda-\theta)P_{1,K-1}^{*}(\theta)=\lambda P_{1,K-2}^{*}(\theta)-P_{1,K-1}(0). \tag{16}$$

Appendix 2

The recursive method is developed to obtain $P^*_{0,n}(0)$ and $P^*_{1,n}(0)$. Our solution algorithm will first obtain $P_{0,n}(0)$ $(1 \le n \le K)$ which are then used for finding $P^*_{0,n}(0)$. We get

$$P_{0,n}(0) = \beta \sum_{i=0}^{\zeta_{n-1}} P_{0,i}, \ 1 \le n \le K, \quad \text{where} \quad \zeta_n = \begin{cases} n, \ 0 \le n \le F-1, \\ F, \ F \le n \le K, \end{cases} \tag{17}$$

and

$$P_{0,n+1}(0) = -\beta \varphi_{n,F} P_{0,n} + P_{0,n}(0), \ 1 \le n \le K-1, \quad \text{where} \quad \varphi_{n,F} = \begin{cases} 1, \ 1 \le n \le F, \\ 0, \ \text{otherwise}. \end{cases} \tag{18}$$

Using (28) in (20) and (21), we get

$$P^*_{0,n}(\theta) = \{[1 - S^*(\theta)] / \theta\} P_{0,n}(0), \ 1 \le n \le K-1. \tag{19}$$

Taking $\lim_{\theta \to 0}$ in (29) and using L'Hôspital's rule once gives

$$P^*_{0,n}(0) = s_1 P_{0,n}(0), \ 1 \le n \le K-1, \tag{20}$$

where $s_1 = -S^{*(1)}(0)$ is the mean service time.

Using (17) in (20), we have

$$P^*_{0,n}(0) = \phi_n P_{0,0}, \ 1 \le n \le K-1, \tag{21}$$

$$\text{where} \quad \phi_n = \begin{cases} 1, & n = 0, \\ s_1 \beta (1 + s_1 \beta)^{\zeta_{n-1}}, & 1 \le n \le K. \end{cases} \tag{22}$$

Thus, $P_{0,1}^{*}(0),\ P_{0,2}^{*}(0),\ldots,\ P_{0,K-1}^{*}(0)$ can be obtained by using (21).

Next, we derive the expressions of $P_{1,n}(0)\ (1\le n\le K)$ in terms of $P_{1,0}$ and $P_{0,0}$. Using (11) in (13)-(14) and then setting $\theta=\lambda$ in (13)-(16), we finally obtain

$$P_{1,2}(0)=\left[P_{1,1}(0)-\beta\phi_1 P_{0,0}S^{*}(\lambda)-\lambda P_{1,0}S^{*}(\lambda)\right]/S^{*}(\lambda), \tag{23}$$

$$P_{1,n+1}(0)=\left[P_{1,n}(0)-\beta\varphi_{n,F}\phi_n P_{0,0}S^{*}(\lambda)-\lambda P_{1,n-1}^{*}(\lambda)\right]/S^{*}(\lambda),\ 2\le n\le K-2, \tag{24}$$

$$P_{1,K-1}(0)=\lambda P_{1,K-2}^{*}(\lambda). \tag{25}$$

To obtain $P_{1,n-1}^{*}(\lambda)\ (1\le n\le K-1)$ in (24)-(25), using (21) in (13)-(14) again, differentiating (13)-(16) $(l-1)$ times with respect to θ and setting $\theta=\lambda$, we finally get

$$P_{1,1}^{*(l-1)}(\lambda)=-\left(S^{*(l)}(\lambda)/l\right)\left[\lambda P_{1,0}+\beta\phi_1 P_{0,0}+\lambda P_{1,2}(0)\right],\ l=1,\ldots,K-2, \tag{26}$$

$$P_{1,n}^{*(l-1)}(\lambda)=-(1/l)\left[P_{1,n+1}(0)S^{*(l)}(\lambda)+\beta\varphi_{n,F}\phi_n P_{0,0}S^{*(l)}(\lambda)+\lambda P_{1,n-1}^{*(l)}(\lambda)\right],$$
$$\text{where } 2\le n\le K-2,\ l=1,\ldots,K-n-1, \tag{27}$$

$$P_{1,K-1}^{*}(\lambda)=-\lambda P_{1,K-2}^{*(1)}(\lambda), \tag{28}$$

where $P_{1,n}^{*(0)}(\lambda)=P_{1,n}^{*}(\lambda)$ and $S^{*(l)}(\theta)=(\mathrm{d}^l/\mathrm{d}\theta^l)S^{*}(\theta)$ denote the lth derivative of $S^{*}(\theta)$.

Solving (26)-(28) recursively, we obtain

$$P_{1,n}^{*}(\lambda)=-\ell_n S^{*}(\lambda)P_{1,0}-\sum_{i=1}^{\zeta_n}[\beta\ell_{n-i+1}\phi_i S^{*}(\lambda)/\lambda]P_{0,0}-\sum_{i=1}^{n}[\ell_{n-i+1}S^{*}(\lambda)/\lambda]P_{1,i+1}(0),$$
$$1\le n\le K-1, \tag{29}$$

where

$$\ell_n = \begin{cases} -[(-\lambda)^n S^{*(n)}(\lambda)/n!S^*(\lambda)], & 1 \le n \le K-1, \\ 0, & \text{otherwise.} \end{cases} \tag{30}$$

Using (29) in (24), we can obtain

$$P_{1,n}(0) = [1/S^*(\lambda)]P_{1,n-1}(0) + \sum_{i=1}^{n-2} \ell_{n-i-1} P_{1,i+1}(0) + \beta\left[\sum_{i=1}^{\zeta_{n-2}} \ell_{n-i-1}\phi_i - \varphi_{n-1,F}\phi_{n-1}\right]P_{0,0} + \lambda\ell_{n-2}P_{1,0},\ 3 \le n \le K-1. \tag{31}$$

We further define

$$\Psi_n = \begin{cases} 1, & n = 0, \\ \sum_{1\le k\le n} \sum_{\tau_1+\tau_2+\cdots+\tau_k=n} \kappa_{\tau_1}\kappa_{\tau_2}\cdots\kappa_{\tau_k}, & n = 1,\ 2,\ldots,\ K-3, \quad \tau_1,\tau_2,\cdots,\tau_k \in \{1,2,\cdots,n\} \\ 0, & \text{otherwise,} \end{cases} \tag{32}$$

where

$$\kappa_n = \begin{cases} [1/S^*(\lambda)] + \ell_1, & n = 1, \\ \ell_n, & n = 2,3,\cdots,K-3, \\ 0, & \text{otherwise.} \end{cases} \tag{33}$$

Remark: The representative meaning of the above formulation (32) is to sum up all possible products of k κs in which the total of subscript values of κ equals n. We give an easily understood example for $n = 4$:

$$\Psi_4 = \kappa_4 + \kappa_3\kappa_1 + \kappa_2\kappa_2 + \kappa_1\kappa_3 + \kappa_1\kappa_1\kappa_2 + \kappa_1\kappa_2\kappa_1 + \kappa_2\kappa_1\kappa_1 + \kappa_1\kappa_1\kappa_1\kappa_1$$
$$= \kappa_4 + 2\kappa_3\kappa_1 + \kappa_2^2 + 3\kappa_1^2\kappa_2 + \kappa_1^4.$$

Using (32) and (33) to solve (31) recursively, and including (5) and (23), we finally get

$$P_{1,1}(0) = A(1)P_{1,0} + B(1)P_{0,0}, \tag{34}$$

$$P_{1,n}(0) = \sum_{i=2}^{n} \Psi_{n-i}\left[A(i)P_{1,0} + B(i)P_{0,0}\right],\ 2 \le n \le K-1, \tag{35}$$

where

$$A(n)=\begin{cases}\lambda, & n=1,\\ \lambda[1-S^*(\lambda)]/S^*(\lambda), & n=2,\\ \lambda\ell_{n-2}, & 3\le n\le K-1,\end{cases} \tag{36}$$

$$B(n)=\begin{cases}-\beta, & n=1,\\ -\beta[1+\varphi_{1,F}\phi_1 S^*(\lambda)]/S^*(\lambda), & n=2,\\ \beta\sum_{i=1}^{\zeta_{n-2}}\ell_{n-i-1}\phi_i-\beta\varphi_{n-1,F}\phi_{n-1}, & 3\le n\le K-1.\end{cases} \tag{37}$$

Substituting (35), (34), and (25) into (29) finally yields

$$P_{1,0}=-\left\{\left[\sum_{i=1}^{K-2}\ell_{K-i-1}\sum_{j=2}^{i+1}\Psi(i-j+1)B(j)+\sum_{i=2}^{K-1}[\Psi(K-i-1)B(i)/S^*(\lambda)]+\sum_{i=1}^{\zeta_{K-2}}\beta\ell_{K-i-1}\phi_i\right]\Big/\left[\sum_{i=1}^{K-2}\ell_{K-i-1}\sum_{j=2}^{i+1}\Psi(i-j+1)A(j)+\sum_{i=2}^{K-1}[\Psi(K-i-1)A(i)/S^*(\lambda)]+\lambda\ell_{K-2}\right]\right\}P_{0,0}. \tag{38}$$

Finally, we develop the steady-state probabilities $P^*_{1,n}(0)$ in terms of $P_{0,0}$. Setting $\theta=0$ in (13)-(16) we have

$$P^*_{1,n}(0)=(1/\lambda)\left[\beta\sum_{i=0}^{\zeta_n}\phi_i P_{0,0}+P_{1,n+1}(0)\right],\ 0\le n\le K-2, \tag{39}$$

$$P^*_{1,K-1}(0)=(\beta/\lambda)\sum_{i=0}^{F}\phi_i P_{0,0}. \tag{40}$$

As $P_{1,1}(0), P_{1,2}(0),..., P_{1,K-1}(0)$ and $P_{1,0}$ are known, $P^*_{1,1}(0), P^*_{1,2}(0),..., P^*_{1,K-1}(0)$ can be determined recursively using (39) and (40) in terms of $P_{0,0}$.

Now the only unknown quantity is $P^*_{0,K}(0)$ which can be obtained from (12). To find it, differentiating (12) with respect to θ and setting $\theta=0$, we have

$$P_{0,K}^{*}\left(0\right)=-\lambda P_{1,K-1}^{*(1)}\left(0\right). \tag{41}$$

To find $\lambda P_{1,K-1}^{*(1)}\left(0\right)$, differentiating (13)-(16) with respect to θ and setting $\theta=0$, we finally obtain

$$P_{1,1}^{*(1)}\left(0\right)=\left[P_{1,1}+\beta\phi_1 P_{0,0}S^{*(1)}(0)+\lambda P_{1,0}S^{*(1)}(0)+P_{1,2}\left(0\right)S^{*(1)}(0)\right]/\lambda, \tag{42}$$

$$P_{1,n}^{*(1)}\left(0\right)=\left[P_{1,n}+\beta\varphi_{n,F}\phi_n P_{0,0}S^{*(1)}(0)+\lambda P_{1,n-1}^{*(1)}\left(0\right)+P_{1,n+1}\left(0\right)S^{*(1)}(0)\right]/\lambda, \quad 2\le n\le K-2, \tag{43}$$

$$P_{1,K-1}^{*(1)}\left(0\right)=\left[P_{1,K-1}+\lambda P_{1,K-2}^{*(1)}\left(0\right)\right]/\lambda. \tag{44}$$

As $P_{1,1}^{*(1)}\left(0\right)$ is known completely from (52), the values $P_{1,n}^{*(1)}\left(0\right)$ for $n=2,3,\ldots,K-1$ can be found recursively from (43) and (44). Therefore we obtain

$$P_{1,K-1}^{*(1)}\left(0\right)=(1/\lambda)\left[\sum_{i=1}^{K-1}P_{1,i}+\beta S^{*(1)}(0)\sum_{i=1}^{F}\phi_n P_{0,0}+S^{*(1)}(0)\sum_{i=2}^{K-1}P_{1,i}\left(0\right)+\lambda P_{1,0}S^{*(1)}(0)\right]. \tag{45}$$

Substituting (45) into (41), we have

$$P_{0,K}^{*}\left(0\right)=-\left[\sum_{i=1}^{K-1}P_{1,i}+\beta S^{*(1)}(0)\sum_{i=1}^{F}\phi_n P_{0,0}+S^{*(1)}\left(0\right)\sum_{i=2}^{K-1}P_{1,i}\left(0\right)+\lambda P_{1,0}S^{*(1)}\left(0\right)\right]. \tag{46}$$

So $P_{0,1}^{*}\left(0\right)$, $P_{0,2}^{*}\left(0\right),\ldots,$ $P_{0,K}^{*}\left(0\right)$ is known in terms of $P_{0,0}$, which can be determined using the normalizing condition

$$\sum_{i=0}^{K}P_{0,i}+\sum_{i=0}^{K-1}P_{1,i}=1. \tag{47}$$