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Optimal control of an M/G/1/K Queueing System with Combined *F* policy and Startup Time

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1. Introduction

A supplementary variable technique is used to study the optimal management problem of the *F* policy M/G/1/K queue where the server needs a startup time before start allowing customers in the system and $K < \infty$ denotes the maximum number of customers in the system. The method of controlling arrivals focuses on reducing the number of customers in the system. The model presented in this dissertation is very useful in real-life situations since the controlling of arriving customers is considered.

The primary objective of this chapter is threefold. Firstly, we develop a recursive method using the supplementary variable technique and treating the supplementary variable as the remaining service time, to develop the steady-state probability distributions of the number of customers for the *F* policy M/G/1/K queue. The method can be utilized for any service time distribution, such as deterministic (denoted D), exponential (denoted M) and k-stage Erlang (denoted E_k), etc. Secondly, to illustrate a recursive method we present three simple examples for three different service time distributions such as exponential, 3-stage Erlang, and deterministic. Thirdly, we study various system performance measures, such as the average number of customers in the system, the probability that the server is busy, the blocking probability, etc. The total expected cost function per unit time for the F policy M/G/1/K queue with startup times is developed. Numerical and comparative results are also provided.

2. Assumptions and Notations

We consider the controlling arrivals to a finite capacity M/G/1 queue with combined *F* policy and exponential startup time. It is assumed that customers arrive according to a Poisson process with parameter λ, and the service times of the successive customers are independently and identically distributed (i.i.d.) random variables having a distribution $S(u)$ $(u \geq 0)$, a probability density function $s(u)$ $(u \geq 0)$ and mean service time s_1. The arrival process is independent of the service process. We assume that arriving customers form a single waiting line based on the order of their arrivals; that is, the first-come, first-served discipline. Suppose that the server can serve only one customer at a time. Customers entering into the service facility and finding that the server is busy have to wait in the queue until the server is available. Gupta [6] first introduced the concept of a *F* policy. The definition of a *F* policy is described as follows: When the number of customers in the system reaches its capacity *K* (i.e. the system becomes full), no further arriving customers are allowed to enter the system until there are enough customers in the system have been served so that the number of customers in the system decreases to a threshold value F ($0 \le F < K-1$). At that time, the server requires to take an exponential startup time with parameter β to start allowing customers in the system. Thus, the system operates normally until the number of customers in the system reaches its capacity at which time the above process is repeated all over again.

The following notations and probabilities are used throughout this chapter.

F	threshold level
K	system capacity $(K > F+1)$
S	service time random variable
U	remaining service time random variable
$S(u)$	distribution function (d.f.) of S
$s(u)$	probability density function (p.d.f.) of S
$S^*(\theta)$	Laplace-Stieltjes transform (LST) of S
$S^{*(l)}(\theta)$	l th order derivative of $S^*(\theta)$ with respect to θ
$P_{0,0}(t)$	probability of no customers in the system at time t when the arrivals are not allowed to enter the system
$P_{0,n}(t)$	probability of n customers in the system at time t when the arrivals are not allowed to enter the system, where $n = 1, 2, \ldots, K$.
$P_{1,0}(t)$	probability of no customers in the system at time t when the arrivals are allowed to enter the system
$P_{1,n}(t)$	probability of n customers in the system at time t when the arrivals are allowed to enter the system, where $n = 1, 2, \ldots, K-1$.
$P_{0,0}$	steady state probability of no customers in the system when the arrivals are not allowed to enter the system
$P_{0,n}$	steady state probability of n customers in the system when the arrivals are not allowed to enter the system, where $n = 1, 2, \ldots, K$.
$P_{1,0}$	steady state probability of no customers in the system when the arrivals are allowed to enter the system
$P_{1,n}$	steady state probability of n customers in the system when the arrivals are allowed to enter the system, where $n = 1, 2, \ldots, K-1$.
s_1	mean service time

The special case with system capacity K=F+1 is presented in the appendix.

3. Development of the Equations and Solutions

We use the following supplementary variable: $U \equiv$ remaining service time for the customer in service. The state of the system at time t is given by

$N(t) \equiv$ number of customers in the system, and

$U(t) \equiv$ remaining service time for the customer being served.

Let us define

$$P_{0,n}(u,t)du = \Pr\{N(t)=n, u<U(t)\le u+\mathrm{d}u\},\ u\ge 0,\quad n=0,1,\ldots,K.$$

$$P_{1,n}(u,t)du = \Pr\{N(t)=n, u<U(t)\le u+\mathrm{d}u\},\ u\ge 0,\quad n=0,1,\ldots,K-1.$$

$$P_{0,n}(t)=\int_0^\infty P_{0,n}(u,t)\,\mathrm{d}u,\quad n=0,1,\ldots,K.$$

$$P_{1,n}(t)=\int_0^\infty P_{1,n}(u,t)\,\mathrm{d}u,\quad n=0,1,\ldots,K-1.$$

Relating the state of the system at time t and $t+\mathrm{d}t$, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}P_{0,0}(t)=-\beta P_{0,0}(t)+P_{0,1}(0,t), \tag{1}$$

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_{0,n}(u,t)=-\beta P_{0,n}(u,t)+P_{0,n+1}(0,t)s(u),\quad 1\le n\le F, \tag{2}$$

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_{0,n}(u,t)=P_{0,n+1}(0,t)s(u),\quad F+1\le n\le K-1, \tag{3}$$

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_{0,K}(u,t)=\lambda P_{1,K-1}(u,t), \tag{4}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}P_{1,0}(t)=-\lambda P_{1,0}(t)+\beta P_{0,0}(t)+P_{1,1}(0,t), \tag{5}$$

$$\begin{aligned}\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_{1,1}(u,t)&=-\lambda P_{1,1}(u,t)+\beta P_{0,1}(u,t)+\lambda P_{1,0}(t)s(u)+\\ &P_{1,2}(0,t)s(u),\end{aligned} \tag{6}$$

$$\begin{aligned}\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_{1,n}(u,t)&=-\lambda P_{1,n}(u,t)+\beta P_{0,n}(u,t)+\lambda P_{1,n-1}(u,t)s(u)\\ &+P_{1,n+1}(0,t)s(u),\quad 2\le n\le F,\end{aligned} \tag{7}$$

$$\begin{aligned}\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_{1,n}(u,t)&=-\lambda P_{1,n}(u,t)+\lambda P_{1,n-1}(u,t)s(u)+P_{1,n+1}(0,t)s(u),\\ &F+1\le n\le K-2,\end{aligned} \tag{8}$$

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_{1,K-1}(u,t)=-\lambda P_{1,K-1}(u,t)+\lambda P_{1,K-2}(u,t). \tag{9}$$

In steady state, let us define

$$P_{0,n}=\lim_{t\to\infty}P_{0,n}(t),\quad n=0,1,\ldots,K.$$

$$P_{1,n}=\lim_{t\to\infty}P_{0,n}(t),\quad n=0,1,\ldots,K-1.$$

$$P_{0,n}(u)=\lim_{t\to\infty}P_{0,n}(u,t),\quad n=1,2,\ldots,F.$$

$$P_{1,n}(u)=\lim_{t\to\infty}P_{1,n}(u,t),\quad n=0,1,\ldots,K-1.$$

and further define

$$P_{0,n}(u)=P_{0,n}s(u),\quad n=1,2,\ldots,F\ . \tag{10}$$

From (1)-(10), we can easily obtain the following steady state equations:

$$0=-\beta P_{0,0}+P_{0,1}(0), \tag{11}$$

$$-\frac{\mathrm{d}}{\mathrm{d}u}P_{0,n}(u)=-\beta P_{0,n}s(u)+P_{0,n+1}(0)s(u),\quad 1\le n\le F, \tag{12}$$

$$-\frac{\mathrm{d}}{\mathrm{d}u}P_{0,n}(u)=P_{0,n+1}(0)s(u),\quad F+1\le n\le K-1, \tag{13}$$

$$-\frac{\mathrm{d}}{\mathrm{d}u}P_{0,K}(u)=\lambda P_{1,K-1}(u), \tag{14}$$

$$0=-\lambda P_{1,0}+\beta P_{0,0}+P_{1,1}(0), \tag{15}$$

$$-\frac{\mathrm{d}}{\mathrm{d}u}P_{1,1}(u)=-\lambda P_{1,1}(u)+\beta P_{0,1}s(u)+\lambda P_{1,0}s(u)+P_{1,2}(0)s(u), \tag{16}$$

$$-\frac{\mathrm{d}}{\mathrm{d}u}P_{1,n}(u)=-\lambda P_{1,n}(u)+\beta P_{0,n}s(u)+\lambda P_{1,n-1}(u)+P_{1,n+1}(0)s(u),\quad 2\le n\le F, \tag{17}$$

$$-\frac{\mathrm{d}}{\mathrm{d}u}P_{1,n}(u)=-\lambda P_{1,n}(u)+\lambda P_{1,n-1}(u)+P_{1,n+1}(0)s(u),\quad F+1\le n\le K-2, \tag{18}$$

$$-\frac{\mathrm{d}}{\mathrm{d}u}P_{1,K-1}(u)=-\lambda P_{1,K-1}(u)+\lambda P_{1,K-2}(u). \tag{19}$$

Further define

$$S^*(\theta) = \int_0^\infty e^{-\theta u} \mathrm{d}S(u) = \int_0^\infty e^{-\theta u} s(u)\,\mathrm{d}u,$$

$$P_{0,n}^*(\theta) = \int_0^\infty e^{-\theta u} P_{0,n}(u)\,\mathrm{d}u,$$

$$P_{1,n}^*(\theta) = \int_0^\infty e^{-\theta u} P_{1,n}(u)\,\mathrm{d}u,$$

$$P_{0,n} = P_{0,n}^*(0) = \int_0^\infty P_{0,n}(u)\,\mathrm{d}u,$$

$$P_{1,n} = P_{1,n}^*(0) = \int_0^\infty P_{1,n}(u)\,\mathrm{d}u,$$

$$\int_0^\infty e^{-\theta u} \frac{\partial}{\partial u} P_{0,n}(u)\,\mathrm{d}u = \theta P_{0,n}^*(\theta) - P_{0,n}(0),$$

and

$$\int_0^\infty e^{-\theta u} \frac{\partial}{\partial u} P_{1,n}(u)\,\mathrm{d}u = \theta P_{1,n}^*(\theta) - P_{1,n}(0).$$

Therefore, we take the LST on both sides of (12)-(14) and (16)-(19). It yields

$$-\theta P_{0,n}^*(\theta) = -\beta P_{0,n} S^*(\theta) + P_{0,n+1}(0) S^*(\theta) - P_{0,n}(0), \quad 1 \le n \le F, \tag{20}$$

$$-\theta P_{0,n}^*(\theta) = P_{0,n+1}(0) S^*(\theta) - P_{0,n}(0), \quad F+1 \le n \le K-1, \tag{21}$$

$$-\theta P_{0,K}^*(\theta) = \lambda P_{1,K}^*(\theta) - P_{0,K}(0), \tag{22}$$

$$(\lambda - \theta) P_{1,1}^*(\theta) = \beta P_{0,1} S^*(\theta) + \lambda P_{1,0} S^*(\theta) + P_{1,2}(0) S^*(\theta) - P_{1,1}(0), \tag{23}$$

$$(\lambda - \theta) P_{1,n}^*(\theta) = \beta P_{0,n} S^*(\theta) + \lambda P_{1,n-1}^*(\theta) + P_{1,n+1}(0) S^*(\theta) - P_{1,n}(0), \quad 2 \le n \le F, \tag{24}$$

$$(\lambda - \theta) P_{1,n}^*(\theta) = \lambda P_{1,n-1}^*(\theta) + P_{1,n+1}(0) S^*(\theta) - P_{1,n}(0), \quad F+1 \le n \le K-2, \tag{25}$$

$$(\lambda - \theta) P_{1,K-1}^*(\theta) = \lambda P_{1,K-2}^*(\theta) - P_{1,K-1}(0). \tag{26}$$

The recursive method is developed to obtain $P_{0,n}^*(0)$ and $P_{1,n}^*(0)$. Our solution algorithm will first obtain $P_{0,n}(0)$ $(1 \le n \le K)$ which are then used for finding $P_{0,n}^*(0)$.

Using (11) and setting $\theta = 0$ in (20) and (21), we get

$$P_{0,n}(0)=\beta\sum_{i=0}^{\zeta_{n-1}}P_{0,i},\quad 1\le n\le K,\ \text{ where }\ \zeta_n=\begin{cases}n, & 0\le n\le F-1,\\ F, & F\le n\le K,\end{cases}\tag{27}$$

and

$$P_{0,n+1}(0)=-\beta\varphi_{n,F}P_{0,n}+P_{0,n}(0),\quad 1\le n\le K-1,$$
$$\text{where }\ \varphi_{n,F}=\begin{cases}1, & 1\le n\le F,\\ 0, & \text{otherwise.}\end{cases}\tag{28}$$

Using (28) in (20) and (21), we get

$$P_{0,n}^{*}(\theta)=\frac{1-S^{*}(\theta)}{\theta}P_{0,n}(0),\quad 1\le n\le K-1.\tag{29}$$

Taking $\lim_{\theta\to 0}$ in (29) and using L'Hôspital's rule once gives

$$P_{0,n}^{*}(0)=s_1P_{0,n}(0),\quad 1\le n\le K-1,\tag{30}$$

where $s_1=-S^{*(1)}(0)$ is the mean service time.
Using (27) in (30), we have

$$P_{0,n}^{*}(0)=\phi_nP_{0,0},\quad 1\le n\le K-1,\tag{31}$$

$$\text{where }\ \phi_n=\begin{cases}1, & n=0,\\ s_1\beta(1+s_1\beta)^{\zeta_{n-1}}, & 1\le n\le K.\end{cases}\tag{32}$$

Thus, $P_{0,1}^{*}(0)$, $P_{0,2}^{*}(0),\ldots,$ $P_{0,K-1}^{*}(0)$ can be obtained by using (31).

Next, we derive the expressions of $P_{1,n}(0)$ $(1\le n\le K)$ in terms of $P_{1,0}$ and $P_{0,0}$. Using (31) in (23)-(24) and then setting $\theta=\lambda$ in (23)-(26), we finally obtain

$$P_{1,2}(0)=\frac{P_{1,1}(0)-\beta\phi_1P_{0,0}S^{*}(\lambda)-\lambda P_{1,0}S^{*}(\lambda)}{S^{*}(\lambda)},\tag{33}$$

$$P_{1,n+1}(0)=\frac{P_{1,n}(0)-\beta\varphi_{n,F}\phi_nP_{0,0}S^{*}(\lambda)-\lambda P_{1,n-1}^{*}(\lambda)}{S^{*}(\lambda)},\quad 2\le n\le K-2,\tag{34}$$

$$P_{1,K-1}(0)=\lambda P_{1,K-2}^{*}(\lambda).\tag{35}$$

To obtain $P_{1,n-1}^{*}(\lambda)$ $(1\le n\le K-1)$ in (34)-(35), using (31) in (23)-(24) again, differentiating (23)-(26) $(l-1)$ times with respect to θ and setting $\theta=\lambda$, we finally get

$$P_{1,1}^{*(l-1)}(\lambda)=-\frac{S^{*(l)}(\lambda)}{l}\left[\lambda P_{1,0}+\beta\phi_1 P_{0,0}+\lambda P_{1,2}(0)\right],\quad l=1,\ldots,K-2\,, \tag{36}$$

$$\begin{aligned}P_{1,n}^{*(l-1)}(\lambda)=&-\frac{1}{l}\left[P_{1,n+1}(0)S^{*(l)}(\lambda)+\beta\varphi_{n,F}\phi_n P_{0,0}S^{*(l)}(\lambda)+\lambda P_{1,n-1}^{*(l)}(\lambda)\right],\\&2\le n\le K-2,\quad l=1,\ldots,K-n-1,\end{aligned} \tag{37}$$

$$P_{1,K-1}^{*}(\lambda)=-\lambda P_{1,K-2}^{*(1)}(\lambda), \tag{38}$$

where $P_{1,n}^{*(0)}(\lambda)=P_{1,n}^{*}(\lambda)$ and $S^{*(l)}(\theta)=\left[\left(\mathrm{d}^l/\mathrm{d}\theta^l\right)S^{*}(\theta)\right]$ denotes the lth derivative of $S^{*}(\theta)$.

Solving (36)-(38) recursively, we obtain

$$\begin{aligned}P_{1,n}^{*}(\lambda)=&-\ell_n S^{*}(\lambda)P_{1,0}-\sum_{i=1}^{\zeta_n}\frac{\beta\ell_{n-i+1}\phi_i S^{*}(\lambda)}{\lambda}P_{0,0}-\sum_{i=1}^{n}\frac{\ell_{n-i+1}S^{*}(\lambda)}{\lambda}P_{1,i+1}(0),\\&1\le n\le K-1,\end{aligned} \tag{39}$$

where

$$\ell_n=\begin{cases}-\dfrac{(-\lambda)^n S^{*(n)}(\lambda)}{n!\,S^{*}(\lambda)}, & 1\le n\le K-1,\\ 0, & \text{otherwise.}\end{cases} \tag{40}$$

Using (39) in (34), we can obtain

$$\begin{aligned}P_{1,n}(0)=&\frac{1}{S^{*}(\lambda)}P_{1,n-1}(0)+\sum_{i=1}^{n-2}\ell_{n-i-1}P_{1,i+1}(0)+\\&\beta\left[\sum_{i=1}^{\zeta_{n-2}}\ell_{n-i-1}\phi_i-\varphi_{n-1,F}\phi_{n-1}\right]P_{0,0}+\lambda\ell_{n-2}P_{1,0},\quad 3\le n\le K-1.\end{aligned} \tag{41}$$

We further define

$$\Psi_n=\begin{cases}1, & n=0,\\ \displaystyle\sum_{1\le k\le n}\ \sum_{\substack{\tau_1+\tau_2+\cdots+\tau_k=n\\ \tau_1,\tau_2,\cdots,\tau_k\in\{1,2,\cdots,n\}}}\kappa_{\tau_1}\kappa_{\tau_2}\cdots\kappa_{\tau_k}, & n=1,\ 2,\ldots,\ K-3,\\ 0, & \text{otherwise,}\end{cases} \tag{42}$$

where

$$\kappa_n = \begin{cases} \frac{1}{S^*(\lambda)} + \ell_1, & n = 1, \\ \ell_n, & n = 2, 3, \cdots, K-3, \\ 0, & \text{otherwise.} \end{cases} \tag{43}$$

Remark: The representative meaning of the above formulation (42) is to sum up all possible products of k κs in which the total of subscript values of κ equals n. We give an easily understood example for n = 4:

$$\begin{aligned} \Psi_4 &= \kappa_4 + \kappa_3\kappa_1 + \kappa_2\kappa_2 + \kappa_1\kappa_3 + \kappa_1\kappa_1\kappa_2 + \kappa_1\kappa_2\kappa_1 + \kappa_2\kappa_1\kappa_1 + \kappa_1\kappa_1\kappa_1\kappa_1 \\ &= \kappa_4 + 2\kappa_3\kappa_1 + \kappa_2^2 + 3\kappa_1^2\kappa_2 + \kappa_1^4. \end{aligned}$$

Using (42) and (43) to solve (2.41) recursively, and including (15) and (33), we finally get

$$P_{1,1}(0) = A(1)P_{1,0} + B(1)P_{0,0}, \tag{44}$$

$$P_{1,n}(0) = \sum_{i=2}^{n} \Psi_{n-i}\left[A(i)P_{1,0} + B(i)P_{0,0}\right], \quad 2 \le n \le K-1, \tag{45}$$

where

$$A(n) = \begin{cases} \lambda, & n = 1, \\ \lambda\left[\frac{1 - S^*(\lambda)}{S^*(\lambda)}\right], & n = 2, \\ \lambda\ell_{n-2}, & 3 \le n \le K-1, \end{cases} \tag{46}$$

$$B(n) = \begin{cases} -\beta, & n = 1, \\ -\beta\left[\frac{1 + \varphi_{1,F}\phi_1 S^*(\lambda)}{S^*(\lambda)}\right], & n = 2, \\ \beta\sum_{i=1}^{\zeta_{n-2}} \ell_{n-i-1}\phi_i - \beta\varphi_{n-1,F}\phi_{n-1}, & 3 \le n \le K-1. \end{cases} \tag{47}$$

Substituting (45), (44), and (35) into (39) finally yields

$$P_{1,0}=\frac{-\left[\sum_{i=1}^{K-2}\ell_{K-i-1}\sum_{j=2}^{i+1}\Psi(i-j+1)B(j)+\sum_{i=2}^{K-1}\frac{\Psi(K-i-1)B(i)}{S^{*}(\lambda)}+\sum_{i=1}^{\zeta_{K-2}}\beta\ell_{K-i-1}\phi_{i}\right]}{\left[\sum_{i=1}^{K-2}\ell_{K-i-1}\sum_{j=2}^{i+1}\Psi(i-j+1)A(j)+\sum_{i=2}^{K-1}\frac{\Psi(K-i-1)A(i)}{S^{*}(\lambda)}+\lambda\ell_{K-2}\right]}P_{0,0}. \tag{48}$$

Finally, we develop the steady-state probabilities $P_{1,n}^{*}(0)$ in terms of $P_{0,0}$. Setting $\theta=0$ in (23)-(26) we have

$$P_{1,n}^{*}(0)=\frac{1}{\lambda}\left[\beta\sum_{i=0}^{\zeta_n}\phi_i P_{0,0}+P_{1,n+1}(0)\right],\quad 0\le n\le K-2, \tag{49}$$

$$P_{1,K-1}^{*}(0)=\frac{\beta}{\lambda}\sum_{i=0}^{F}\phi_i P_{0,0}. \tag{50}$$

As $P_{1,1}(0)$, $P_{1,2}(0)$,..., $P_{1,K-1}(0)$ and $P_{1,0}$ are known, $P_{1,1}^{*}(0)$, $P_{1,2}^{*}(0)$, ..., $P_{1,K-1}^{*}(0)$ can be determined recursively using (49) and (50) in terms of $P_{0,0}$.

Now the only unknown quantity is $P_{0,K}^{*}(0)$ which can be obtained from (22). To find it, differentiating (22) with respect to θ and setting $\theta=0$, we have

$$P_{0,K}^{*}(0)=-\lambda P_{1,K-1}^{*(1)}(0). \tag{51}$$

To find $\lambda P_{1,K-1}^{*(1)}(0)$, differentiating (23)-(26) with respect to θ and setting $\theta=0$, we finally obtain

$$P_{1,1}^{*(1)}(0)=\frac{P_{1,1}+\beta\phi_1 P_{0,0}S^{*(1)}(0)+\lambda P_{1,0}S^{*(1)}(0)+P_{1,2}(0)S^{*(1)}(0)}{\lambda}, \tag{52}$$

$$P_{1,n}^{*(1)}(0)=\frac{P_{1,n}+\beta\varphi_{n,F}\phi_n P_{0,0}S^{*(1)}(0)+\lambda P_{1,n-1}^{*(1)}(0)+P_{1,n+1}(0)S^{*(1)}(0)}{\lambda}, \tag{53}$$
$$2\le n\le K-2,$$

$$P_{1,K-1}^{*(1)}(0)=\frac{P_{1,K-1}+\lambda P_{1,K-2}^{*(1)}(0)}{\lambda}. \tag{54}$$

As $P_{1,1}^{*(1)}(0)$ is known completely from (52), the values $P_{1,n}^{*(1)}(0)$ for $n=2,3,\dots,K-1$ can be found recursively from (53) and (54). Therefore we obtain

$$P_{1,K-1}^{*(1)}(0)=\frac{1}{\lambda}\left[\sum_{i=1}^{K-1}P_{1,i}+\beta S^{*(1)}(0)\sum_{i=1}^{F}\phi_n P_{0,0}+S^{*(1)}(0)\sum_{i=2}^{K-1}P_{1,i}(0)+\lambda P_{1,0}S^{*(1)}(0)\right]. \tag{55}$$

Substituting (55) into (51), we have

$$P_{0,K}^{*}(0) = -\left[\sum_{i=1}^{K-1} P_{1,i} + \beta S^{*(1)}(0)\sum_{i=1}^{F}\phi_n P_{0,0} + S^{*(1)}(0)\sum_{i=2}^{K-1} P_{1,i}(0) + \lambda P_{1,0} S^{*(1)}(0)\right]. \quad (56)$$

So $P_{0,1}^{*}(0), P_{0,2}^{*}(0), \ldots, P_{0,K}^{*}(0)$ is known in terms of $P_{0,0}$, which can be determined using the normalizing condition

$$\sum_{i=0}^{K} P_{0,i} + \sum_{i=0}^{K-1} P_{1,i} = 1. \quad (57)$$

To demonstrate the working of the recursive method, we first describe the solution algorithm for calculating the steady state probabilities, $P_{0,n}^{*}(0)$ $(0 \le n \le K)$ and $P_{1,n}^{*}(0)$ $(0 \le n \le K-1)$. Next, to illustrate the solution algorithm, we provide three simple examples where the service time distributions are exponential, k-stage Erlang, and deterministic, respectively.

Let F be the threshold, K be the maximum capacity of the system, and let $S^{*(l)}(\theta)$ be the l-th derivative of $S^{*}(\theta)$, where $l = 1, 2, \ldots, K$. We set the values of F, K, and the LST expression of the service time distribution, namely $S^{*}(\theta)$. The steps of the solution algorithm are stated as follows:

Step 1. For each $n = 0, 1, \ldots, K$, compute ϕ_n using (32).

Step 2. For each $n = 1, 2, \ldots, K-1$, compute $P_{0,n}^{*}(0)$ using (31) in terms of $P_{0,0}$.

Step 3. Compute ℓ_n $(1 \le n \le K-2)$ and κ_n $(1 \le n \le K-3)$ using (40) and (43), respectively.

Step 4. For each $n = 0, 1, \ldots, K-3$, compute Ψ_n using (42).

Step 5. For each $n = 1, 2, \ldots, K-1$, compute $A(n)$ and $B(n)$ using (46) and (47).

Step 6. For each $n = 1, 2, \ldots, K-1$, compute $P_{1,n}(0)$ using (44) and (45) in terms of $P_{1,0}$ and $P_{0,0}$.

Step 7. Compute $P_{1,0}$ using (48) in terms of $P_{0,0}$. Thus $P_{1,n}(0)$ $(1 \le n \le K-1)$ are achieved from *Step* 6.

Step 8. For each $n = 1, 2, \ldots, K-1$, compute $P_{1,n}^{*}(0)$ using (49) and (50) in terms of $P_{0,0}$.

Step 9. For $n = K$, compute $P_{0,n}^{*}(0)$ using (56) in terms of $P_{0,0}$.

Step 10. Determine $P_{0,0}$ using (57). Thus $P_{0,n}^{*}(0)$ $(n = 1, 2, \ldots, K)$ are achieved from *Steps* 2 and 9, and $P_{1,n}^{*}(0)$ $(n = 0, 1, \ldots, K-1)$ are achieved from *Steps* 7 to 8.

4. Simple Examples

We use the solution algorithm to illustrate a recursive method. We provide three simple examples for three different service time distributions such as exponential, 3-stage Erlang, and deterministic, respectively.

Example 1 (For M/M/1 queue). We set the mean service time $s_1 = 1/\mu$, where μ is the service rate. Assume that $F = 1$ and $K = 4$. In this case, we have

$$S^*(\theta) = \frac{\mu}{\mu+\theta}.$$

Step 1. For each $n = 0, 1, \ldots, 4$, compute ϕ_n.

Using (32), we obtain

$$\phi_0 = 1,\ \phi_1 = (1-\alpha)/\alpha, \text{ and } \phi_2 = \phi_3 = \phi_4 = (1-\alpha)/\alpha^2, \text{ where } \alpha = \mu/(\mu+\beta).$$

Step 2. For each $n = 1, 2, 3$, compute $P^*_{0,n}(0)$ using (31) in terms of $P_{0,0}$.

From (31), we finally get

$$P^*_{0,1}(0) = \phi_1 P_{0,0} = \frac{1-\alpha}{\alpha} P_{0,0},$$

$$P^*_{0,2}(0) = P^*_{0,3}(0) = \phi_2 P_{0,0} = \frac{1-\alpha}{\alpha^2} P_{0,0}.$$

Step 3. For each $n = 1, 2$, compute ℓ_n and κ_n using (40) and (43), respectively.

For each $n = 1, 2$, using (40) yields $\ell_1 = -1/(1+\sigma)$ and $\ell_2 = -1/(1+\sigma)^2$, where $\sigma = \mu/\lambda$.

For each $n = 1$, we find from (43) that $\kappa_1 = (1+\sigma+\sigma^2)/\sigma(1+\sigma)$.

Step 4. For each $n = 0, 1$, compute Ψ_n.

It implies from (42) that $\Psi_0 = 1$ and $\Psi_1 = (1+\sigma+\sigma^2)/\sigma(1+\sigma)$.

Step 5. For each $n = 1, 2, 3$, compute $A(n)$ and $B(n)$.

Using (46) and (47), it follows that

$$A(1) = \mu/\sigma,\ A(2) = \mu/\sigma^2, \text{ and } A(3) = -\mu/\sigma(1+\sigma).$$

$$B(1) = -\frac{(1-\alpha)\mu}{\alpha},\ B(2) = -\frac{(\alpha+\sigma)(1-\alpha)\mu}{\sigma\alpha^2}, \text{ and } B(3) = -\frac{(1-\alpha)^2\mu}{(1+\sigma)\alpha^2}.$$

Step 6. For each $n = 1, 2, 3$, compute $P_{1,n}(0)$ using (44) and (45) in terms of $P_{1,0}$ and $P_{0,0}$.

It yields from (44) and (45) that

$$P_{1,1}(0) = A(1)P_{1,0} + B(1)P_{0,0},$$

$$P_{1,2}(0) = \Psi_0\left[A(2)P_{1,0} + B(2)P_{0,0}\right],$$

$$P_{1,3}(0) = \Psi_1\left[A(2)P_{1,0} + B(2)P_{0,0}\right] + \Psi_0\left[A(3)P_{1,0} + B(3)P_{0,0}\right].$$

Step 7. Compute $P_{1,0}$ using (48) in terms of $P_{0,0}$. Thus $P_{1,n}(0)$ $(1 \le n \le 3)$ are achieved from *Step* 6.

From (48), we finally have

$$P_{1,0} = \frac{\sigma(1-\alpha)\left(\alpha + \sigma + \sigma^2 + \sigma^3\right)}{\alpha^2} P_{0,0}, \quad \left(P_{1,0}^*(0) = P_{1,0}\right),$$

$$P_{1,1}(0) = \frac{\sigma\mu(1-\alpha)\left(1 + \sigma + \sigma^2\right)}{\alpha^2} P_{0,0},$$

$$P_{1,2}(0) = \frac{\sigma\mu(1-\alpha)(1+\sigma)}{\alpha^2} P_{0,0},$$

$$P_{1,3}(0) = \frac{\sigma\mu(1-\alpha)}{\alpha^2} P_{0,0}.$$

Step 8. For each $n = 1, 2, 3$, compute $P_{1,n}^*(0)$ using (49) and (50) in terms of $P_{0,0}$. Using (49) and (50) yields

$$P_{1,1}^*(0) = \frac{\sigma(1-\alpha)\left(1 + \sigma + \sigma^2\right)}{\alpha^2} P_{0,0}, \quad P_{1,2}^*(0) = \frac{\sigma(1-\alpha)(1+\sigma)}{\alpha^2} P_{0,0},$$

and $P_{1,3}^*(0) = \dfrac{\sigma(1-\alpha)}{\alpha^2} P_{0,0}$.

Step 9. For $n = 4$, compute $P_{0,n}^*(0)$ using (56) in terms of $P_{0,0}$.

Using (56), it follows that

$$P_{0,4}^*(0) = \frac{(1-\alpha)}{\alpha^2} P_{0,0}.$$

Step 10. Determine $P_{0,0}$ using (57). Thus $P_{0,n}^*(0)$ $(n = 0, 1, \ldots, 4)$ are achieved from *Steps* 2 and 9, and $P_{1,n}^*(0)$ $(n = 0, 1, 2, 3)$ are achieved from *Steps* 7 to 8.

$$P_{0,0} = \frac{\alpha^2}{\alpha^2 + \alpha(1-\alpha) + 3(1-\alpha) + \sigma(1-\alpha)\left(3 + \alpha + 3\sigma + 2\sigma^2 + \sigma^3\right)}.$$

It is to be noted that these results are the same as those given in Gupta [錯誤! 找不到參照來源。, p1006].

Example 2 (For M/E_3/1 queue). The 3-stage Erlang distribution is made up of three independent and identical exponential stages, each with mean $1/3\mu$. We set the mean service time $s_1 = 1/\mu$, $F = 1$, and $K = 3$. In this case, we have

$$S^*(\theta) = \left(\frac{3\mu}{3\mu + \theta} \right)^3 .$$

Step 1. For each $n = 0, 1, \ldots, 3$, compute ϕ_n.

From (32), we finally obtain

$$\phi_0 = 1, \ \phi_1 = 3(1-\gamma)/\gamma, \text{ and } \phi_2 = \phi_3 = 3(1-\gamma)(3-2\gamma)/\gamma^2,$$

where $\gamma = 3\mu/(3\mu + \beta)$.

Step 2. For each $n = 1, 2$, compute $P^*_{0,n}(0)$ using (31) in terms of $P_{0,0}$.

From (31), it follows that

$$P^*_{0,1}(0) = \phi_1 P_{0,0} = 3\frac{1-\gamma}{\gamma} P_{0,0},$$

$$P^*_{0,2}(0) = \phi_2 P_{0,0} = 3\frac{(1-\gamma)(3-2\gamma)}{\gamma^2} P_{0,0}.$$

Step 3. For each $n = 1$, compute ℓ_n.

Using (40) yields $\ell_1 = -3/(1+\tau)$, where $\tau = 3\mu/\lambda$.

Step 4. For each $n = 0$, compute Ψ_n.

It implies from (42) that $\Psi_0 = 1$.

Step 5. For each $n = 1, 2$, compute $A(n)$ and $B(n)$.

It yields from (46) and (47) that

$$A(1) = 3\mu/\tau \ \text{ and } \ A(2) = 3\mu\left(1 + 3\tau + 3\tau^2\right)/\tau^4 .$$

$$B(1) = -3\frac{(1-\gamma)\mu}{\gamma} \ \text{ and } \ B(2) = -3\frac{\left[\alpha\left(1+3\tau+3\tau^2\right) + \tau^3(3-2\gamma)\right](1-\gamma)\mu}{\tau^3\gamma^2}.$$

Step 6. For each $n = 1, 2$, compute $P_{1,n}(0)$ using (44) and (45) in terms of $P_{1,0}$ and $P_{0,0}$.

From (44) and (45), we find that

$$P_{1,1}(0) = A(1)P_{1,0} + B(1)P_{0,0},$$

$$P_{1,2}(0) = \Psi_0\left[A(2)P_{1,0} + B(2)P_{0,0}\right],$$

Step 7. Compute $P_{1,0}$ using (48) in terms of $P_{0,0}$. Thus $P_{1,n}(0)$ $(1 \le n \le 2)$ are achieved from *Step* 6.

It implies from (48) that

$$P_{1,0} = \frac{\tau(1-\gamma)\left[\tau^3(1+\tau)(3-2\gamma)+\gamma\left(1+4\tau+6\tau^2\right)\right]}{\left(1+4\tau+6\tau^2\right)\gamma^2}P_{0,0}, \ \left(P_{1,0}^*(0) = P_{1,0}\right),$$

$$P_{1,1}(0) = 3\frac{\tau^3\mu(1+\tau)(1-\gamma)(3-2\gamma)}{\left(1+4\tau+6\tau^2\right)\gamma^2}P_{0,0},$$

$$P_{1,2}(0) = 9\frac{\tau^3\mu(1-\gamma)(3-2\gamma)}{\left(1+4\tau+6\tau^2\right)\gamma^2}P_{0,0}.$$

Step 8. For each $n = 1,\ 2$, compute $P_{1,n}^*(0)$ using (49) and (50) in terms of $P_{0,0}$.

Using (49) and (50) yields

$$P_{1,1}^*(0) = \frac{\tau(1+\tau)(1-\gamma)(3-2\gamma)\left(1+3\tau+3\tau^2\right)}{\left(1+4\tau+6\tau^2\right)\gamma^2}P_{0,0},$$

and $P_{1,2}^*(0) = \dfrac{\tau(1-\gamma)(3-2\gamma)\left(1+3\tau+3\tau^2\right)}{\gamma^2}P_{0,0}.$

Step 9. For $n = 3$, compute $P_{0,n}^*(0)$ using (56) in terms of $P_{0,0}$.

It follows from (56) that

$$P_{0,3}^*(0) = \frac{(1-\gamma)(3-2\gamma)\left(3+10\tau+10\tau^2\right)}{\left(1+4\tau+6\tau^2\right)\gamma^2}P_{0,0}.$$

Step 10. Determine $P_{0,0}$ using (57). Thus $P_{0,n}^*(0)$ $(n = 0,\ 1, \ldots, 3)$ are achieved from *Steps* 2 and 9, and $P_{1,n}^*(0)$ $(n = 0,\ 1,\ 2)$ are achieved from *Steps* 7 to 8.

$$P_{0,0}=\left(1+4\tau+6\tau^2\right)\gamma^2\times$$
$$\left\{\left(1+4\tau+6\tau^2\right)\left[\gamma^2+(1-\gamma)\left(9-\gamma^2\right)\right]\right.$$
$$\left.+(1-\gamma)(3-2\gamma)\left(3+11\tau+14\tau^2+6\tau^3+4\tau^4+\tau^5\right)\right\}^{-1}.$$

Example 3 (For M/D/1 queue). We set the mean service time $s_1=1/\mu$, $F=1$, and $K=3$. In this case,

$$S^*(\theta)=e^{-\theta/\mu}.$$

Step 1. For each $n=0, 1, \ldots, 3$, compute ϕ_n.

Using (32) yields

$$\phi_0=1,\ \phi_1=(1-\alpha)/\alpha, \text{ and } \phi_2=\phi_3=(1-\alpha)/\alpha^2, \text{ where } \alpha=\mu/(\mu+\beta).$$

Step 2. For each $n=1, 2$, compute $P^*_{0,n}(0)$ using (31) in terms of $P_{0,0}$.

Using (31), we finally get

$$P^*_{0,1}(0)=\phi_1 P_{0,0}=\frac{1-\alpha}{\alpha}P_{0,0},$$
$$P^*_{0,2}(0)=\phi_2 P_{0,0}=\frac{1-\alpha}{\alpha^2}P_{0,0}.$$

Step 3. For each $n=1$, compute ℓ_n.

From (40), we find that $\ell_1=-\rho$, where $\rho=\lambda/\mu$.

Step 4. For each $n=0$, compute Ψ_n.

It implies from (42) that $\Psi_0=1$.

Step 5. For each $n=1, 2$, compute $A(n)$ and $B(n)$.

From (46) and (47), it follows that

$$A(1)=\rho\mu \text{ and } A(2)=\rho\mu\left(1-e^{\rho}\right).$$

$$B(1)=-\frac{(1-\alpha)\mu}{\alpha} \text{ and } B(2)=-\frac{\mu(1-\alpha)\left(1-\alpha+e^{\rho}\right)}{\alpha^2}.$$

Step 6. For each $n=1, 2$, compute $P_{1,n}(0)$ using (44) and (45) in terms of $P_{1,0}$ and $P_{0,0}$.

Using (44) and (45) yields

$$P_{1,1}(0) = A(1)P_{1,0} + B(1)P_{0,0},$$

$$P_{1,2}(0) = \Psi_0\left[A(2)P_{1,0} + B(2)P_{0,0}\right],$$

Step 7. Compute $P_{1,0}$ using (48) in terms of $P_{0,0}$. Thus $P_{1,n}(0)$ $(1 \le n \le 2)$ are achieved from *Step* 6.

We find from (48) that

$$P_{1,0} = \frac{(1-\alpha)\left[\alpha(1+\rho)-1-\alpha e^{\rho}\right]}{\alpha^2\rho\left(1+\rho-e^{\rho}\right)}P_{0,0}, \ \left(P^*_{1,0}(0) = P_{1,0}\right),$$

$$P_{1,1}(0) = -\frac{(1-\alpha)\mu}{\alpha^2\left(1+\rho-e^{\rho}\right)}P_{0,0},$$

$$P_{1,2}(0) = -\frac{(1-\alpha)\rho\mu}{\alpha^2\left(1+\rho-e^{\rho}\right)}P_{0,0}.$$

Step 8. For each $n = 1, 2$, compute $P^*_{1,n}(0)$ using (49) and (50) in terms of $P_{0,0}$.

Using (49) and (50) yields

$$P^*_{1,1}(0) = -\frac{(1-\alpha)\left(1-e^{\rho}\right)}{\rho\alpha^2\left(1+\rho-e^{\rho}\right)}P_{0,0} \ \text{and} \ P^*_{1,2}(0) = -\frac{(1-\alpha)}{\rho\alpha^2}P_{0,0}.$$

Step 9. For $n = 3$, compute $P^*_{0,n}(0)$ using (56) in terms of $P_{0,0}$.

It follows from (56) that

$$P^*_{0,3}(0) = \frac{(1-\alpha)\left[2\left(e^{\rho}-1\right)-\rho\left(1+e^{\rho}\right)\right]}{\rho\alpha^2\left(1+\rho-e^{\rho}\right)}P_{0,0}.$$

Step 10. Determine $P_{0,0}$ using (57). Thus $P^*_{0,n}(0)$ $(n = 0, 1, \ldots, 3)$ are achieved from *Steps* 2 and 9, and $P^*_{1,n}(0)$ $(n = 0, 1, 2)$ are achieved from *Steps* 7 to 8.

$$P_{0,0} = \rho\alpha^2\left(1+\rho-e^{\rho}\right)\times$$
$$\left[\rho\left(1+\rho+\alpha-\alpha^2\right)+e^{\rho}\left(\alpha\rho+\alpha^2-2\rho^2-\alpha\right)-(1-\alpha)^2\right]^{-1}.$$

5. Optimal *F* policy

Our analysis is based on the following system performance measures of the F

policy M/G/1/K queue with exponential startup time. Let

$L_s \equiv$ the average number of customers in the system;

$P_b \equiv$ the probability that the server is busy;

$P_s \equiv$ the probability that the server requires a startup time before starting the service;

$P_{bl} \equiv$ the probability that the server is blocked.

The expressions for L_s, P_b, P_s, and P_{bl} are give by

$$L_s = \sum_{n=1}^{K} nP_{0,n} + \sum_{n=1}^{K-1} nP_{1,n} ,$$

$$P_b = \sum_{n=0}^{K} P_{0,n} + \sum_{n=0}^{K-1} P_{1,n} ,$$

$$P_s = \sum_{n=0}^{F} P_{0,n} ,$$

$$P_{bl} = \sum_{n=0}^{K} P_{0,n} .$$

We develop the total expected cost function per unit time for the F policy M/G/1/K queue with startup times, in which F is a management decision variable. The main purpose of this subsection is to determine the optimum management F policy so as to minimize this total expected cost function. Let

$C_h \equiv$ holding cost per unit time for each customer present in the system;

$C_b \equiv$ busy cost per unit time for a busy server;

$C_s \equiv$ startup cost per unit time for the preparatory work of the server before starting the service;

$C_{bl} \equiv$ fixed cost for every lost customer when the system is blocked.

Utilizing the definitions of each cost element listed above, the total expected cost function per unit time is given by

$$TC(F) = C_h L_s + C_b P_b + C_s P_s + C_{bl} \lambda P_{bl} . \tag{58}$$

The optimal value of F, F^* is determined by satisfying the following inequality

$$TC(F^* - 1) \geq TC(F^*) \quad \text{and} \quad TC(F^* + 1) \geq TC(F^*) . \tag{59}$$

We now perform a sensitivity analysis on the optimum value F^* based on changes in specific values of the system parameters and fix the system capacity K=15. We consider the three simple examples for three different service time distributions such as exponential, 3-stage Erlang, and deterministic and employ the following cost elements:

Case 1: $C_h = 5,\ C_b = 200,\ C_s = 250,\ C_{bl} = 300.$

Case 2: $C_h = 5,\ C_b = 200,\ C_s = 250,\ C_{bl} = 350.$

Case 3: $C_h = 5,\ C_b = 200,\ C_s = 300,\ C_{bl} = 350.$

Case 4: $C_h = 5,\ C_b = 225,\ C_s = 300,\ C_{bl} = 350.$
Case 5: $C_h = 10,\ C_b = 225,\ C_s = 300,\ C_{bl} = 350.$

In this section we provide the numerical results of the optimal value F^* and the minimum expected cost for three interarrival time distributions and specific values of λ, μ, β. We first fix (μ, β) = (1.0, 3.0) and choose different values of λ = 0.5, 0.6, 0.7. Next, we fix (λ, β) = (0.8, 3.0) and consider various values of μ = 1.0, 1.1, 1.2. Finally, we fix (λ, μ) = (0.8, 1.0) and select different values of β = 2.0, 4.0, 5.0.

The optimal value of F, F^*, and its minimum expected cost $TC(F^*)$ for the above five cases are shown in Tables 1-3. For fixed values of (μ, β) and various values of λ in Tables 1-3, we observe that (i) $TC(F^*)$ increases as λ increases for any case; and (ii) F^* decreases as λ increases for any case. For fixed values of (λ, β) and various values of μ in Tables 1-3, we find that (i) $TC(F^*)$ decreases as μ increases for any case; and (ii) F^* increases as μ increases for any case. Again, for fixed (λ, μ) and various values of β in Tables 1-3, we observe that (i) $TC(F^*)$ slightly decreases as β increases for any case; and (ii) F^* does not change at all when β changes from 2.0 to 5.0 for any case. Intuitively, F^* is insensitive to changes in β.

It can be easily see from Tables 1 through 3 that (i) F^* increases as C_h decreases or C_{bl} increases (see cases 4-5 and cases 1-2); and (ii) C_h and C_{bl} have a larger effect on F^* than C_b and C_s (see cases 3-4 and cases 2-3).

Table 1. The optimal value of F and its minimum expected cost for the service time distribution such as exponential.

		$\lambda\ (\mu,\beta)=(1.0,3.0)$			$\mu\ (\lambda,\beta)=(0.8,3.0)$			$\beta\ (\lambda,\mu)=(0.8,1.0)$		
		0.5	0.6	0.7	1.0	1.1	1.2	2.0	4.0	5.0
Case1	F^*	9	7	5	4	7	10	5	4	4
	$TC(F^*)$	105.000	127.486	151.420	177.597	158.454	143.314	177.680	177.561	177.540
Case2	F^*	12	11	9	6	10	12	6	6	6
	$TC(F^*)$	105.001	127.501	151.554	178.285	158.655	143.367	178.361	178.247	178.225
Case3	F^*	12	11	8	6	10	12	6	6	6
	$TC(F^*)$	105.001	127.502	151.562	178.314	158.669	143.374	178.404	178.269	178.242
Case4	F^*	11	9	7	4	8	11	5	5	5
	$TC(F^*)$	117.500	142.496	169.000	197.985	176.767	160.020	198.072	197.941	197.915
Case5	F^*	5	4	3	2	4	6	2	2	2
	$TC(F^*)$	122.470	149.933	180.049	213.873	189.095	169.773	213.960	213.830	213.804

Table 2. The optimal value of F and its minimum expected cost for the service time distribution such as 3-stage Erlang.

		$\lambda\ (\mu,\beta)=(1.0,3.0)$			$\mu\ (\lambda,\beta)=(0.8,3.0)$			$\beta\ (\lambda,\mu)=(0.8,1.0)$		
		0.5	0.6	0.7	1.0	1.1	1.2	2.0	4.0	5.0
Case1	F^*	9	7	6	4	7	10	4	4	4
	$TC(F^*)$	104.167	125.999	148.912	173.998	155.504	141.111	174.022	173.986	173.979
Case2	F^*	12	11	9	6	10	12	6	6	6
	$TC(F^*)$	104.167	126.000	148.932	174.216	155.541	141.116	174.241	174.204	174.197
Case3	F^*	12	11	9	6	10	12	6	6	6
	$TC(F^*)$	104.167	126.000	148.933	174.226	155.544	141.117	174.255	174.211	174.202
Case4	F^*	11	9	7	5	9	12	5	5	5
	$TC(F^*)$	116.667	141.000	166.424	194.121	173.710	157.781	194.150	194.107	194.099
Case5	F^*	6	4	3	2	4	6	2	2	2
	$TC(F^*)$	120.833	146.996	175.278	207.572	183.632	165.532	207.601	207.557	207.548

Table 3. The optimal value of F and its minimum expected cost for the service time distribution such as deterministic.

		$\lambda\ (\mu,\beta)=(1.0,3.0)$			$\mu\ (\lambda,\beta)=(0.8,3.0)$			$\beta\ (\lambda,\mu)=(0.8,1.0)$		
		0.5	0.6	0.7	1.0	1.1	1.2	2.0	4.0	5.0
Case1	F^*	10	8	6	4	7	10	4	4	4
	$TC(F^*)$	103.750	125.250	147.578	171.798	153.930	140.000	171.806	171.794	171.792
Case2	F^*	12	11	9	6	10	12	6	6	6
	$TC(F^*)$	103.750	125.250	147.582	171.869	153.938	140.001	171.877	171.865	171.863
Case3	F^*	12	11	9	6	10	12	6	6	6
	$TC(F^*)$	103.750	125.250	147.582	171.872	153.938	140.001	171.882	171.867	171.864
Case4	F^*	12	10	7	5	9	12	5	5	5
	$TC(F^*)$	116.250	140.250	165.080	191.839	172.117	156.667	191.848	191.834	191.831
Case5	F^*	7	5	3	2	4	6	2	2	2
	$TC(F^*)$	120.000	145.500	172.649	203.459	180.568	163.331	203.469	203.454	203.451

無研發成果推廣資料

96 年度專題研究計畫研究成果彙整表

計畫主持人：彭文理	計畫編號：96-2628-E-009-025-MY3

計畫名稱：在排隊系統裡控制到達與控制服務之理論與實務應用研究--子計畫一:探討有限容量 F 方策 M/G/1 排隊系統之操作特性與敏感度分析

成果項目			量化			單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）
			實際已達成數（被接受或已發表）	預期總達成數(含實際已達成數)	本計畫實際貢獻百分比		
國內	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%		
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（本國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		
國外	論文著作	期刊論文	1	1	100%	篇	Optimal Control of an M/G/1/K Queueing System with Combined F policy and Startup Time
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%	章/本	
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（外國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		

其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)	無

	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

■達成目標

□未達成目標（請說明，以 100 字為限）

　　□實驗失敗

　　□因故實驗中斷

　　□其他原因

說明：

2. 研究成果在學術期刊發表或申請專利等情形：

論文：■已發表 □未發表之文稿 □撰寫中 □無

專利：□已獲得 □申請中 ■無

技轉：□已技轉 □洽談中 ■無

其他：（以 100 字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限）

針對於不同分佈下的各別排隊系統，我們提供了有效率的求解方法。並探討 M/G/1/K 與 G/M/1/K 排隊含啟動時間的 F 方策 與 N 方策 之相互關係，建立出兩模式之間穩態解之間的相互轉換方法，可經由其中一個排隊模式的穩態解，應用轉換方法而得到另一模式之穩態解。此結果可以推廣至兩模式可靠度分析與最佳解之相互關係的研究。