Capacity Bounds of a Multiple-Access Rician Fading Channel

Final Report of NSC Project

"Capacity Analysis of Various Multiple-Antenna Multiple-Users Communication Channels with Joint Estimation and Detection"

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Author:	Stefan M. Moser
Organization:	Information Theory Laboratory Department of Electrical Engineering National Chiao Tung University
Address:	Engineering Building IV, Office 727 1001 Ta Hsueh Rd. Hsinchu 30010, Taiwan
E-mail:	stefan.moser@ieee.org

Abstract

The sum-rate capacity of a noncoherent multiple-access Rician fading channel is investigated under three different categories of power constraints: individual per user peak-power constraints, individual per user average-power constraints, or a global power-sharing average-power constraint. Upper and lower bounds on the sum-rate capacity are derived and it is shown that at high signal-to-noise ratio the sum-rate capacity only grows double-logarithmically in the available power. The asymptotic behavior of capacity is then analyzed in detail and the exact asymptotic expansion is derived including its second term, the so called *fading number*. It is shown that the fading number is identical to the fading number of the single-user Rician fading channel that is obtained when only the user seeing the best channel is transmitting and all other users are switched off at all times. This pessimistic result holds independently of the type of power constraint that is imposed.

Keywords: Channel capacity, fading number, flat fading, high signal-to-noise ratio (SNR), multiple-access channel (MAC), multiple-input single-output (MISO), multiple users, Rician fading.

Contents

1 Introduction

In a noncoherent fading channel where neither transmitter nor receiver know the fading realization, it has been shown in [1] that the capacity at high signal-to-noise ratio (SNR) behaves fundamentally different from the usual asymptotics seen in Gaussian channels or in coherent fading channels: instead of a logarithmic growth in the SNR, the capacity only grows double-logarithmically. To be precise, if the fading process is stationary, ergodic, and has a finite differential entropy rate and a finite expected second moment, then we have

$$\mathsf{C}(\textsc{snr}) = \log(1 + \log(1 + \textsc{snr})) + \chi + o(1) \tag{1}$$

where $o(1)$ denotes terms that tend to zero as the SNR tends to infinity; and where χ is a constant independent of the SNR that is called *fading number*. The value of χ depends on the exact specifications of the fading law. In the situation of a general memoryless fading process, i.e., a fading process that is independent and identically distributed (IID) over time and of a general law, the fading number has been derived for a single-input single-output (SISO) channel, a single-input multiple-output (SIMO) channel, and a multiple-input single-output (MISO) channel in [1], and the multiple-input multiple-output (MIMO) channel was solved in [2]. The more general setup of a stationary, ergodic and regular fading process has been analyzed in [1] for the SISO case, [3] solved the SIMO case, and the most general MIMO case was addressed in [4].

Note that even though the fading number is defined only in the limit when the available SNR tends to infinity, it has practical relevance also for finite SNR: it is a good estimator for the threshold where the capacity changes from the normal logarithmic growth to the highly inefficient double-logarithmic growth. For more details we refer to the discussion in Section 3.3 and to the introduction section in [4].

All the above mentioned results are restricted to the situation of a single transmitter (possibly with several antennas) and a single receiver. The present work is a first step towards generalizing the setup to a multiple-user situation. Concretely, we include m transmitters, each having a certain number n_i of antennas and trying to communicate to a common receiver with only one antenna. The fading law is assumed to be memoryless both over time and space and Gaussian distributed with line-of-sight (LOS) components. We will propose upper and lower bounds on the sum-rate capacity of this channel and derive the exact asymptotic expansion of the sum-rate capacity for the SNR tending to infinity. It will turn out that the asymptotic capacity corresponds to the single-user capacity for the case when all but one user are switched off at all times.

The remainder of this paper is structured as follows. After some short remarks about notation we will introduce the multiple-access (MAC) Rician fading channel and three different power constraints in Section 2. In Section 3 we will derive upper and lower bounds on the sum-rate capacity of this model that are valid for all SNR. These bounds are based on new bounds for the single-user MISO Rician fading channel. We will see there that in contrast to the low-SNR regime, at high SNR the capacity only grows double-logarithmically in the power.

To investigate the threshold between these two regimes, in Sections 4 and 5 the asymptotic behavior of the sum-rate capacity will be analyzed and stated exactly. The proof of the main result can be found in Section 6, while the derivations of some intermediate steps have been moved to the appendix. We conclude in Section 7.

We try to clearly distinguish random and constant quantities: while random quantities are denoted by capital Roman letters, constants are typeset in small Romans or the Greek alphabet. To distinguish numbers from vectors, vectors are in bold face. E.g., $\mathbf{X}$ denotes a random vector and $\mathbf{x}$ its realization, while Y is a random variable and y its realization. There are a few exceptions to this rule: matrices are denoted by capital letters, but of a different font, e.g., D; C stands for capacity; $\mathcal{E}$ for the available power; I denotes the mutual information functional; and Q is a cumulative distribution function (CDF) of the channel input.

The superscript $^{\mathsf{T}}$ refers to the transpose operation of vectors and matrices. We use $\|\cdot\|$ to denote the Euclidean norm of vectors. Sets are set in calligraphic font $\mathcal{D}$, and $\mathcal{D}^{\mathrm{c}}$ denotes the complement set.

All rates specified in this paper are in nats per channel use, i.e., $\log(\cdot)$ denotes the natural logarithmic function.

2 Channel Model and Power Constraints

We consider a multiple-access channel with m transmitters (users) and one receiver. The signals transmitted by the users are assumed to be independent. The receiver is assumed to have only one antenna, whereas each user i has some number n_i of transmit antennas, $i = 1, \ldots, m$, which yields a total number of antennas at the transmitter side of

$$n_{\mathrm{T}} = \sum_{i=1}^{m} n_i. \tag{2}$$

All channels between one of the n_{T} transmit antennas and the receiver antenna are assumed to be memoryless and independent Rician fading channels, i.e., the fading is complex Gaussian distributed with variance 1 and some mean (line-of-sight component) $d_i^{(\ell)} \in \mathbb{C}$. Note that in the following we will use i (and sometimes j) to denote the users, i.e., $i = 1, \ldots, m$, and ℓ to denote the antennas of user i, i.e., $\ell = 1, \ldots, n_i$.

To simplify our notation and because we assume all channels to be IID over time, we restrain ourselves from using time indices. We would like to point out that the assumption of memorylessness has been made for simplicity. We believe it is possible to extend the results to fading with memory (see also the discussion in Section 7).

So the channel output $Y \in \mathbb{C}$ can be written as

$$\begin{aligned} Y &= \sum_{i=1}^{m} (\mathbf{d}_i^{\mathsf{T}} + \mathbf{H}_i^{\mathsf{T}})\mathbf{x}_i + Z & (3)\\ &= \sum_{i=1}^{m} \sum_{\ell=1}^{n_i} \bigl(d_i^{(\ell)} + H_i^{(\ell)}\bigr) x_i^{(\ell)} + Z. & (4) \end{aligned}$$

Here $\mathbf{x}_i \in \mathbb{C}^{n_i}$ denotes the input vector for the n_i antennas of user i; the components of the random vector $\mathbf{d}_i + \mathbf{H}_i$ describe Rician fading

$$H_i^{(\ell)} + d_i^{(\ell)} \sim \mathcal{N}_{\mathbb{C}}\Bigl(d_i^{(\ell)}, 1\Bigr) \tag{5}$$

(hence, $H_i^{(\ell)}$ are zero-mean, unit-variance, circularly symmetric, complex Gaussian random variables) and are assumed to be independent

$$H_i^{(\ell)} \perp\!\!\!\perp H_{i'}^{(\ell')}, \qquad (i,\ell) \neq (i',\ell') \tag{6}$$

and $Z \sim \mathcal{N}_{\mathbb{C}}\left(0, \sigma^2\right)$ denotes additive, zero-mean, circularly symmetric Gaussian noise, independent from the fading $(\mathbf{H}_1, \ldots, \mathbf{H}_m)$.

We assume a noncoherent situation, i.e., neither transmitters nor receiver have knowledge of the current fading realization, they only know the fading distributions.[1] Note that we do not restrict the receiver and/or transmitters to try to gain such knowledge. Any power or bandwidth used for such estimation schemes, however, are taken into account for the capacity analysis and are not given for free as in a coherent setup. Neither will it be possible for the receiver to gain *perfect* channel knowledge.

We do not allow cooperation between the users, i.e., we assume that the input vectors of the different users are statistically independent:

$$\mathbf{X}_i \perp\!\!\!\perp \mathbf{X}_j, \quad i \neq j. \tag{7}$$

For completeness we also mention that the users' input vectors are assumed to be independent from fading and noise.

For simplicity of notation we will sometimes collect all LOS vectors $\mathbf{d}_i$ into one n_{T}-vector $\mathbf{d}$:

$$\mathbf{d} \triangleq (\mathbf{d}_1^{\mathsf{T}}, \ldots, \mathbf{d}_m^{\mathsf{T}})^{\mathsf{T}} \tag{8}$$

the fading vectors $\mathbf{H}_i$ into one fading vector $\mathbf{H}$ of length n_{T}:

$$\mathbf{H} \triangleq (\mathbf{H}_1^{\mathsf{T}}, \ldots, \mathbf{H}_m^{\mathsf{T}})^{\mathsf{T}} \tag{9}$$

and the input vectors $\mathbf{X}_i$ of all users into one n_{T}-vector $\mathbf{X}$:

$$\mathbf{X} \triangleq (\mathbf{X}_1^{\mathsf{T}}, \ldots, \mathbf{X}_m^{\mathsf{T}})^{\mathsf{T}}. \tag{10}$$

In the given setup we can consider several possible constraints on the power. We will analyze three different scenarios:

- **Peak-Power Constraint:** At every time-step every user i is allowed to use a power of at most $\frac{\kappa_i}{m}\mathcal{E}$:

  $$\Pr\left[\|\mathbf{X}_i\|^2 > \frac{\kappa_i}{m}\mathcal{E}\right] = 0 \tag{11}$$

  for some fixed number $\kappa_i > 0$.

- **Average-Power Constraint:** Averaged over the length of a codeword, every user i is allowed to use a power of at most $\frac{\kappa_i}{m}\mathcal{E}$:

  $$\mathsf{E}\left[\|\mathbf{X}_i\|^2\right] \leq \frac{\kappa_i}{m}\mathcal{E} \tag{12}$$

  for some fixed number $\kappa_i > 0$.

- **Power-Sharing Average-Power Constraint:** Averaged over the length of a codeword all users together are allowed to use a power of at most $\bar{\kappa}\mathcal{E}$:

  $$\mathsf{E}\left[\sum_{i=1}^{m} \|\mathbf{X}_i\|^2\right] \leq \bar{\kappa}\mathcal{E} \tag{13}$$

  for some fixed number $\bar{\kappa} > 0$.

[1] Note that the constant LOS vectors $\mathbf{d}_i$ are part of the distributions and are therefore known everywhere!

Note that if $\kappa_i = 1$ for all i, we have the special case where all users have an equal power available. Also note that in (11) and (12) we have normalized the power to the number of users m. This might be strange from an engineering point of view, however, in regard of our freedom to choose κ_i it is irrelevant, and it simplifies our analysis because we can easily connect the power-sharing average-power constraint with the other two constraints. Indeed, if we define $\bar{\kappa}$ to be the average of the constants $\{\kappa_i\}_{i=1}^m$, i.e.,

$$\bar{\kappa} = \frac{1}{m}\sum_{i=1}^{m} \kappa_i \tag{14}$$

then the three constraints are in order of strictness: the peak-power constraint is the most stringent of the three constraints in the sense that if (11) is satisfied for all $i = 1, \ldots, m$, then the other two constraints are also satisfied; and the average-power constraint is the second most stringent in the sense that if (12) is satisfied for all i, then also the power-sharing average-power constraint (13) is satisfied. In the remainder of this paper we will always assume that (14) holds.

For some comments about even more general types of power constraints, we refer to the discussion in Section 7.

It is worth mentioning that the slackest constraint, i.e., the power-sharing average-power constraint, implicitly allows a form of cooperation: while the users are still assumed to be statistically independent, we do allow cooperation concerning power distribution. This is not very realistic, however, we include it anyway because it will help in deriving bounds on the sum-rate capacity. As a matter of fact, it will turn out that the asymptotic sum-rate capacity is unchanged irrespective of which constraint is assumed.

The sum-rate capacity $\mathsf{C}_{\text{MAC}}(\mathcal{E})$ of the channel (3) is given by

$$\mathsf{C}_{\text{MAC}}(\mathcal{E}) = \sup_{\substack{Q_{\mathbf{X}_1}\cdots Q_{\mathbf{X}_m} \\ \text{power constraint}}} I(\mathbf{X}_1, \ldots, \mathbf{X}_m; Y) \tag{15}$$

where the supremum is over the set of all probability distributions of the m input vectors such that the users are statistically independent of each other (7), and such that one particular power constraint (11), (12), or (13) is satisfied.

3 Nonasymptotic Bounds on the Sum-Rate Capacity

3.1 Relationship between MAC and MISO

We derive an upper and a lower bound on the sum-rate capacity (15) by properly changing the setup to a single-user situation.

Firstly, we upper-bound $\mathsf{C}_{\text{MAC}}(\mathcal{E})$ by dropping the independence-constraint (7), i.e., allowing full cooperation among all users. Moreover, we choose the most relaxed power constraint (13):

$$\begin{aligned}
\mathsf{C}_{\text{MAC}}(\mathcal{E}) &= \sup_{\substack{Q_{\mathbf{X}_1}\cdots Q_{\mathbf{X}_m} \\ \text{power constraint}}} I(\mathbf{X}_1, \ldots, \mathbf{X}_m; Y) && (16) \\
&\le \sup_{\substack{Q_{\mathbf{X}_1,\ldots,\mathbf{X}_m} \\ \mathsf{E}\left[\sum_{i=1}^m \|\mathbf{X}_i\|^2\right] \le \bar{\kappa}\mathcal{E}}} I(\mathbf{X}_1, \ldots, \mathbf{X}_m; Y) && (17) \\
&= \sup_{\substack{Q_{\mathbf{X}} \\ \mathsf{E}\left[\|\mathbf{X}\|^2\right] \le \bar{\kappa}\mathcal{E}}} I(\mathbf{X}; Y) && (18)
\end{aligned}$$

$$= \mathsf{C}_{\text{MISO,av},n_\text{T}}(\bar{\kappa}\mathcal{E}). \tag{19}$$

Here $\mathsf{C}_{\text{MISO,av},n_\text{T}}(\Upsilon)$ denotes the (single-user) capacity of the MISO Rician fading channel with n_T transmitter antennas (and one receiver antenna)

$$Y = \mathbf{d}^\mathsf{T}\mathbf{x} + \mathbf{H}^\mathsf{T}\mathbf{x} + Z \tag{20}$$

(where $\mathbf{d}$, $\mathbf{H}$, and $\mathbf{x}$ are defined in (8), (9), and (10), respectively) under the average-power constraint

$$\mathsf{E}\bigl[\|\mathbf{X}\|^2\bigr] \leq \Upsilon. \tag{21}$$

On the other hand, obviously the sum rate cannot be smaller than the single-user rate that can be achieved if all but one user are switched off, assuming the most stringent type of power constraint (11), and assuming the minimal amount of power among all users. I.e., for an arbitrary $i \in \{1, \dots, m\}$,

$$\begin{aligned}
\mathsf{C}_{\text{MAC}}(\mathcal{E}) &= \sup_{\substack{Q_{\mathbf{X}_1}\cdots Q_{\mathbf{X}_m}\\ \text{power constraint}}} I(\mathbf{X}_1, \dots, \mathbf{X}_m; Y) &(22)\\
&\geq \sup_{\substack{Q_{\mathbf{X}_1}\cdots Q_{\mathbf{X}_m}\\ \Pr[\|\mathbf{X}_j\|^2 > \frac{\kappa_{\min}}{m}\mathcal{E}]=0,\ \forall j}} I(\mathbf{X}_1, \dots, \mathbf{X}_m; Y)\Big|_{\substack{\mathbf{X}_j\equiv\mathbf{0},\\ \forall j\neq i}} &(23)\\
&= \sup_{\substack{Q_{\mathbf{X}_i}\\ \Pr[\|\mathbf{X}_i\|^2 > \frac{\kappa_{\min}}{m}\mathcal{E}]=0}} I(\mathbf{X}_i; Y) &(24)\\
&= \mathsf{C}_{\text{MISO,pp},n_i}\left(\frac{\kappa_{\min}}{m}\mathcal{E}\right). &(25)
\end{aligned}$$

Here, $\mathsf{C}_{\text{MISO,pp},n_i}(\Upsilon)$ denotes the (single-user) capacity of the MISO Rician fading channel with n_i transmitter antennas (and one receiver antenna)

$$Y = \mathbf{d}_i^\mathsf{T}\mathbf{x}_i + \mathbf{H}_i^\mathsf{T}\mathbf{x}_i + Z \tag{26}$$

under the peak-power constraint

$$\Pr\bigl[\|\mathbf{X}_i\|^2 > \Upsilon\bigr] = 0 \tag{27}$$

and we define

$$\kappa_{\min} \triangleq \min_{i\in\{1,\dots,m\}} \kappa_i. \tag{28}$$

Hence, we have the following first important result.

Theorem 1. *The sum-rate capacity* (15) *of the multiple-access Rician fading channel* (3) *under one of the three power constraints* (11)*,* (12)*, or* (13) *is bounded as follows:*

$$\max_i \mathsf{C}_{\text{MISO,pp},n_i}\left(\frac{\kappa_{\min}}{m}\mathcal{E}\right) \leq \mathsf{C}_{\text{MAC}}(\mathcal{E}) \leq \mathsf{C}_{\text{MISO,av},n_\text{T}}(\bar{\kappa}\mathcal{E}). \tag{29}$$

3.2 Bounds on MISO Rician Fading Channel

In order to be able derive more explicit bounds on the MAC sum-rate capacity, we make a small detour and develop some bounds on the MISO Rician fading channel. We start with an upper bound, which is a generalization of a bound from [1], based on a dual expression of mutual information.

Proposition 2. *The capacity of the MISO Rician fading channel* (20) *under an average-power constraint* (21) *is upper-bounded as follows:*

$$\begin{aligned} C_{\text{MISO,av},n_\text{T}}(\Upsilon) \leq \inf_{\substack{0<\alpha\leq 1\\ \beta>0,\nu\geq 0}} \Bigg\{ & \alpha \log\left(\frac{\beta}{\sigma^2}\right) - 1 + \log\Gamma\left(\alpha, \frac{\nu}{\beta}\right) + \frac{(\|\mathbf{d}\|^2+1)\Upsilon+\sigma^2}{\beta} \\ & + \frac{\nu}{\beta} + (1-\alpha)\left(\log\left(\frac{\|\mathbf{d}\|^2\Upsilon}{\Upsilon+\sigma^2}\right) - \text{Ei}\left(-\frac{\|\mathbf{d}\|^2\Upsilon}{\Upsilon+\sigma^2}\right)\right) \\ & + (1-\alpha)\left(\log\left(\frac{\nu}{\sigma^2}\right) - e^{\frac{\nu}{\sigma^2}}\,\text{Ei}\left(-\frac{\nu}{\sigma^2}\right) + \gamma\right)\Bigg\} \end{aligned} \tag{30}$$

where $\text{Ei}(\cdot)$ *denotes the exponential integral function*

$$\text{Ei}(-\xi) \triangleq -\int_\xi^\infty \frac{e^{-t}}{t}\,\text{d}t, \qquad \xi > 0 \tag{31}$$

and where $\gamma \approx 0.57$ *denotes Euler's constant.*

Proof. See Appendix A. □

In order to be able to apply any lower bound on the MISO Rician fading channel to Theorem 1, we need to consider a peak-power constraint instead of an average-power constraint. We will derive two different lower bounds. The first bound relies on an input chosen such that the logarithm of its magnitude is uniformly distributed in the interval $\left[\frac{1}{2}\log\Upsilon_0, \frac{1}{2}\log\Upsilon\right]$ for some constant $0 < \Upsilon_0 < \Upsilon$.

The second lower bound is based on a binary input

$$\mathbf{X}_i \triangleq \sqrt{\Upsilon}\cdot\Xi\cdot\frac{\mathbf{d}_i}{\|\mathbf{d}_i\|}e^{\text{i}\Phi} \tag{32}$$

with $\Pr[\Xi = 1] = 1 - \Pr[\Xi = 0] = p$ and Φ (independent of Ξ) being uniform between 0 and 2π, $\Phi \sim \mathcal{U}([0, 2\pi))$. The induced mutual information is then computed numerically.

Proposition 3. *The capacity of the MISO Rician fading channel* (26) *under a peak-power constraint* (27) *is lower-bounded as follows:*

$$C_{\text{MISO,pp},n_i}(\Upsilon) \geq \text{conv.-hull}\big\{\max\{C_{\text{L1},n_i}(\Upsilon), C_{\text{L2},n_i}(\Upsilon)\}\big\} \tag{33}$$

where

$$\begin{aligned} C_{\text{L1},n_i}(\Upsilon) \triangleq \max_{0<\Upsilon_0<\Upsilon} \Bigg\{ & \log\log\left(\frac{\Upsilon}{\Upsilon_0}\right) + \log\left(\|\mathbf{d}_i\|^2\right) - \text{Ei}\left(-\|\mathbf{d}_i\|^2\right) \\ & - 1 - \log\left(1 + \frac{\sigma^2}{\Upsilon_0}\right)\Bigg\} \end{aligned} \tag{34}$$

and

$$\begin{aligned} C_{\text{L2},n_i}(\Upsilon) \triangleq \max_{0\leq p\leq 1} \Bigg\{ & -\int_0^\infty f_{R_i^2}(t)\log f_{R_i^2}(t)\,\text{d}t - 1 \\ & - p\log\left(\Upsilon+\sigma^2\right) - (1-p)\log\left(\sigma^2\right)\Bigg\} \end{aligned} \tag{35}$$

with

$$f_{R_i^2}(t) \triangleq \frac{1-p}{\sigma^2}e^{-\frac{t}{\sigma^2}} + \frac{p}{\Upsilon+\sigma^2}e^{-\frac{t+\|\mathbf{d}_i\|^2\Upsilon}{\Upsilon+\sigma^2}} I_0\left(\frac{2\|\mathbf{d}_i\|\sqrt{\Upsilon t}}{\Upsilon+\sigma^2}\right). \tag{36}$$

Here $I_0(\cdot)$ *denotes the modified Bessel function of order zero, and* $\text{Ei}(\cdot)$ *is defined in* (31).

Proof. See Appendix B. □

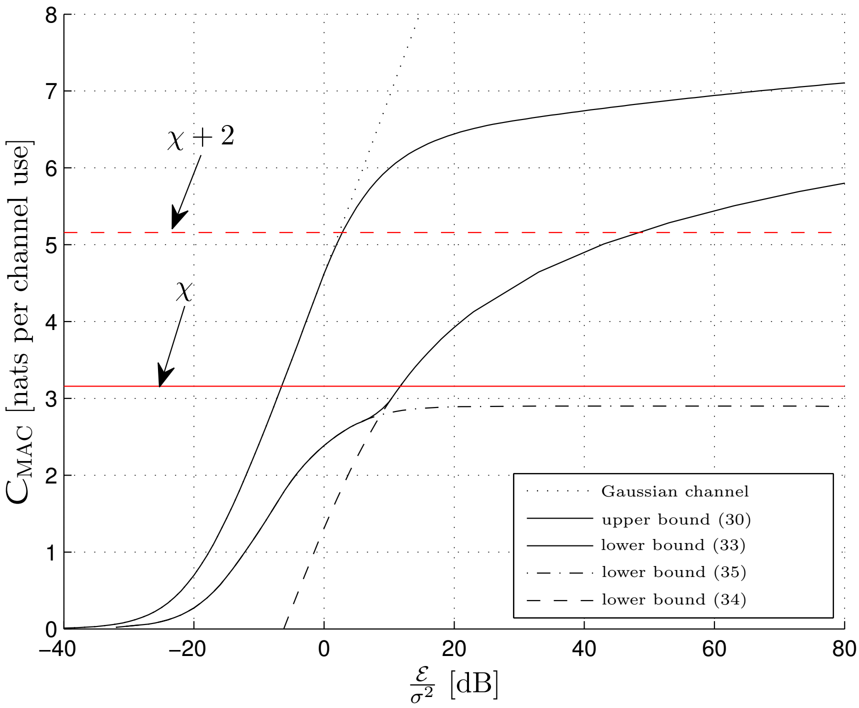

Figure 1: Nonasymptotic bounds (29) on the sum-rate of a two-user multiple-access Rician fading channel. The dotted line shows the capacity of an additive Gaussian noise channel with equivalent received SNR. The red horizontal line corresponds to the fading number χ as derived in Section 5, and the dashed red line is the approximate threshold $\chi + 2$ nats between the efficient low-SNR and the highly inefficient high-SNR behavior.

3.3 Discussion

Proposition 2 and 3 can be applied directly to Theorem 1 to get bounds on the sum-rate capacity. Figure 1 depicts an example with two users $m = 2$, each of them having the same power constraint, i.e., $\kappa_1 = \kappa_2 = \bar{\kappa} = 1$. The LOS components are assumed to be $\|\mathbf{d}_1\| = 6$ and $\|\mathbf{d}_2\| = 8$, such that $\|\mathbf{d}\| = 10$. Note that the exact choice of the vectors $\mathbf{d}_1$ and $\mathbf{d}_2$ including their dimensions n_1 and n_2 is irrelevant for the given bounds. The LOS components influence the expressions only via their magnitudes.

We clearly see that there exist two distinct regimes: for SNR values below around 10 dB (or a rate of about $C_{\mathrm{MAC}} \approx 5$ nats) the sum-rate capacity grows logarithmically in the SNR, while above the threshold the growth changes dramatically and becomes very slowly growing. We will show in the next section that this high-SNR growth is double-logarithmic.

We conclude that one should not use this channel at high SNR, and we ask for more insight about this threshold between the efficient low-SNR regime and the highly inefficient high-SNR regime. As described in [4, Sec. I.B] it turns out that an asymptotic capacity analysis is the clue to such an investigation. This might seem strange at first sight as we just have concluded that we are not interested in this channel at high SNR. However, it is important to realize that around the threshold, the sum-rate capacity is dominated by the second (constant) term of the asymptotic high-SNR expansion of the sum-rate capacity (and not by the double-logarithmic term!). Indeed, we note that

$$\log(1 + \log(1 + \Upsilon)) \approx 2 \text{ nats} \tag{37}$$

for $\Upsilon \in [20 \text{ dB}, 80 \text{ dB}]$, and therefore conclude that as a rule of thumb the threshold will be around $\mathsf{C}_{\text{MAC}} \approx \chi + 2$ nats.

Hence, in deriving the asymptotic expansion of capacity one gains important understanding of the behavior of the channel at a reasonable and finite SNR. In the remainder of this paper we will investigate the asymptotic behavior of the sum-rate capacity and in particular compute its exact asymptotic expansion.

4 The Asymptotic Sum-Rate Capacity

We will now consider the asymptotic case, i.e., the situation when the available power $\mathcal{E}$ tends to infinity. We know that for the MISO Rician fading case[2] [1, Theorem 4.27]

$$\begin{aligned} \mathsf{C}_{\text{MISO}}(\mathcal{E}) &= \mathsf{C}_{\text{MISO,av}}(\mathcal{E}) = \mathsf{C}_{\text{MISO,pp}}(\mathcal{E}) \\ &= \log\log\left(\frac{\mathcal{E}}{\sigma^2}\right) + \chi_{\text{MISO},\mathbf{d}} + o(1) \end{aligned} \tag{38}$$

where $o(1)$ denotes terms that tend to zero as $\mathcal{E}$ tends to infinity and where $\chi_{\text{MISO},\mathbf{d}}$ is a constant denoted *MISO fading number*. Note that the value of $\chi_{\text{MISO},\mathbf{d}}$ is independent of whether we have assumed an average-power or a peak-power constraint and is given by [1, Corollary 4.28]

$$\chi_{\text{MISO},\mathbf{d}} = \log\left(\|\mathbf{d}\|^2\right) - \text{Ei}\left(-\|\mathbf{d}\|^2\right) - 1 \tag{39}$$

where $\text{Ei}(\cdot)$ is defined in (31) and where $\mathbf{d}$ denotes the LOS vector of the MISO Rician fading channel.

We further note that for any constant factor β

$$\lim_{\mathcal{E}\uparrow\infty} \left\{ \log\log(\beta\mathcal{E}) - \log\log\mathcal{E} \right\} = 0, \qquad \beta > 0 \tag{40}$$

i.e., the double-logarithmic growth is not influenced by the factors $\frac{\kappa_{\min}}{m}$ or $\bar{\kappa}$ in Theorem 1. Therefore, we directly get from (29), (38), and (40) the following result.

Corollary 4. *The sum-rate capacity* (15) *of the multiple-access Rician fading channel* (3) *under any one of the three power constraints* (11), (12), *or* (13), *and irrespective of the values of* $\kappa_1, \ldots, \kappa_m$, *grows double-logarithmically in the power at high power:*

$$\varlimsup_{\mathcal{E}\uparrow\infty} \left\{ \mathsf{C}_{\text{MAC}}(\mathcal{E}) - \log\log\left(\frac{\mathcal{E}}{\sigma^2}\right) \right\} < \infty. \tag{41}$$

We next step out to analyze the second term of the high-SNR expansion of the sum-rate capacity: the MAC fading number.

Definition 5. *The* MAC fading number *is defined as*

$$\chi_{\text{MAC}} \triangleq \varlimsup_{\mathcal{E}\uparrow\infty} \left\{ \mathsf{C}_{\text{MAC}}(\mathcal{E}) - \log\left(1 + \log\left(1 + \frac{\mathcal{E}}{\sigma^2}\right)\right) \right\}. \tag{42}$$

[2] Note that asymptotically for $\mathcal{E} \uparrow \infty$, $\log\left(1 + \log\left(1 + \frac{\mathcal{E}}{\sigma^2}\right)\right) = \log\log\left(\frac{\mathcal{E}}{\sigma^2}\right) + o(1)$.

A priori χ_{MAC} depends on the type of power constraint (11), (12), or (13) that is imposed on the input. However, it will turn out that the value of the MAC fading number is identical for all three cases. We therefore take the liberty to use a slightly sloppy notation that does not specify the used power constraint.

From (29), (38), and (40) we realize that

$$\max_i \chi_{\text{MISO},\mathbf{d}_i} \leq \chi_{\text{MAC}} \leq \chi_{\text{MISO},\mathbf{d}} \tag{43}$$

or explicitly (by (39))

$$\max_i \left\{\log\left(\|\mathbf{d}_i\|^2\right) - \text{Ei}\left(-\|\mathbf{d}_i\|^2\right) - 1\right\} \leq \chi_{\text{MAC}} \leq \log\left(\|\mathbf{d}\|^2\right) - \text{Ei}\left(-\|\mathbf{d}\|^2\right) - 1 \tag{44}$$

where we remind the reader that $\mathbf{d}_i$ is the LOS vector of user i and $\mathbf{d} \triangleq (\mathbf{d}_1^{\mathsf{T}}, \ldots, \mathbf{d}_m^{\mathsf{T}})^{\mathsf{T}}$ is the stacked LOS vector of all users.

Using the monotonicity of $\xi \mapsto \log(\xi) - \text{Ei}(-\xi) - 1$ we now define $d_{\text{MAC}} \geq 0$ such that

$$\chi_{\text{MAC}} = \log\left(d_{\text{MAC}}^2\right) - \text{Ei}\left(-d_{\text{MAC}}^2\right) - 1. \tag{45}$$

From (44) we know that

$$\max\left\{\|\mathbf{d}_1\|^2, \ldots, \|\mathbf{d}_m\|^2\right\} \leq d_{\text{MAC}}^2 \leq \|\mathbf{d}\|^2 = \|\mathbf{d}_1\|^2 + \cdots + \|\mathbf{d}_m\|^2. \tag{46}$$

In the remainder we will derive the exact value of d_{MAC}.

We would like to point out that in [5] it has been proven that for the two-user case $m = 2$ with $n_1 = n_2 = 1$ (and with $\kappa_1 = \kappa_2 = 1$) the upper bound in (46) cannot be achieved, i.e.,

$$d_{\text{MAC}}^2 < \|\mathbf{d}\|^2 \tag{47}$$

with strict inequality.

5 Main Result: The MAC Fading Number

Theorem 6. *Consider a multiple-access Rician fading channel as defined in* (3)*. Then, irrespective of which power constraint* (11), (12)*, or* (13) *is imposed on the input and irrespective of the values of* $\kappa_1, \ldots, \kappa_m$*, the MAC fading number* χ_{MAC} (42) *is given by*

$$\chi_{\text{MAC}} = \log\left(d_{\text{MAC}}^2\right) - \text{Ei}\left(-d_{\text{MAC}}^2\right) - 1 \tag{48}$$

with

$$d_{\text{MAC}}^2 \triangleq \max\left\{\|\mathbf{d}_1\|^2, \ldots, \|\mathbf{d}_m\|^2\right\}. \tag{49}$$

This shows that the lower bound in (46) is tight, which is a rather pessimistic result. It means that if the magnitude of the LOS vector of one user is strictly larger than the LOS vectors of the other users, then the asymptotic sum-rate capacity can only be achieved if all but this strongest user are switched off at all times. If there are several users with LOS vectors of identical largest magnitude, the sum-rate capacity can also be achieved by time sharing among those best users.

Note that the result holds even if we allow for power sharing among the users.

6 Proof of Main Result

The proof of Theorem 6 consists of two parts. The first part is given already in Section 4: it is shown in (46) that $\max_i \|\mathbf{d}_i\|^2$ is a lower bound to d^2_{MAC}. Note that this lower bound can be achieved using an input that satisfies the strictest constraint, i.e., the peak-power constraint (11).

The second part will be to prove that $\max_i \|\mathbf{d}_i\|^2$ also is an upper bound to d^2_{MAC}. We will prove this under the assumption of the slackest constraint, i.e., the power-sharing average-power constraint (13). Since the peak-power constraint (11) and the average-power constraint (12) are more stringent than the power-sharing average-power constraint (13), the result will follow.

Before we start with the actual derivation of this upper bound, we need to generalize a concept that has been introduced in [1] and [3].

Proposition 7 (Input Distributions that Escape to Infinity). *Let $\{Q_{\mathcal{E}}\}_{\mathcal{E}>0}$ be a family of joint input distributions of the multiple-access Rician fading channel* (3)*, parametrized by the available power $\mathcal{E} > 0$, satisfying the power-sharing average-power constraint* (13)*, and satisfying*

$$\lim_{\mathcal{E}\uparrow\infty} \frac{I(Q_{\mathcal{E}})}{\log\log\mathcal{E}} = 1 \tag{50}$$

where $I(Q)$ denotes the mutual information between input and output of this channel induced by the input distribution Q.

Then at least one user's input distribution must escape to infinity*, i.e., for any fixed $\mathcal{E}_0 > 0$,*

$$\lim_{\mathcal{E}\uparrow\infty} Q_{\mathcal{E}}\left(\left\{\|\mathbf{X}_1\|^2 \geq \frac{\mathcal{E}_0}{m}\right\} \cup \cdots \cup \left\{\|\mathbf{X}_m\|^2 \geq \frac{\mathcal{E}_0}{m}\right\}\right) = 1. \tag{51}$$

Proof. See Appendix C. □

To put it in an engineering way, Proposition 7 says that in the limit when the available power tends to infinity, at least one user must use a coding scheme where every used symbol uses infinite energy. Or, if all users use one (or more) symbol with finite energy, the asymptotic growth rate of the sum-rate capacity cannot be achieved.

Definition 8. *We define $\mathcal{A}$ to be the set of families of joint input distributions of all users such that the users are independent* (7)*, such that the power-sharing average-power constraint* (13) *is satisfied, and such that the input distribution of at least one user escapes to infinity when the available power $\mathcal{E}$ tends to infinity* (51)*, i.e.,*

$$\mathcal{A} \triangleq \bigl\{\{Q_{\mathbf{X}}\}_{\mathcal{E}>0} \colon \text{(7), (13), and (51) are satisfied}\bigr\}. \tag{52}$$

We are now ready for the derivation of an upper bound on the MAC fading number. The following bound is derived from a duality-based bound on mutual information.

Lemma 9. *The MAC fading number* (42) *is upper-bounded as follows:*

$$\chi_{\text{MAC}} \leq \varlimsup_{\mathcal{E}\uparrow\infty} \sup_{Q_{\mathcal{E}}\in\mathcal{A}} \left\{ \log\left(\mathsf{E}\left[\frac{|\mathbf{d}^{\mathsf{T}}\mathbf{X}|^2}{\|\mathbf{X}\|^2}\right]\right) - \text{Ei}\left(-\mathsf{E}\left[\frac{|\mathbf{d}^{\mathsf{T}}\mathbf{X}|^2}{\|\mathbf{X}\|^2}\right]\right) - 1 \right\}. \tag{53}$$

Proof. See Appendix D. □

Noting that $\xi \mapsto \log(\xi) - \mathrm{Ei}(-\xi) - 1$ is a monotonically increasing function and using our definition of d_{MAC} in (45) we hence conclude that

$$d_{\mathrm{MAC}}^2 \leq \varlimsup_{\mathcal{E}\uparrow\infty} \sup_{Q_{\mathcal{E}}\in\mathcal{A}} \mathsf{E}\left[\frac{|\mathbf{d}^\mathsf{T}\mathbf{X}|^2}{\|\mathbf{X}\|^2}\right]. \tag{54}$$

We would like to point out that without the constraint (51) the right-hand side (RHS) of (54) actually equals to $\|\mathbf{d}\|^2$, i.e., to the RHS of (46), from which we already know that it is (at least in some cases) strictly loose. So we see that the presented generalization of the concept of input distributions that escape to infinity (Proposition 7) is crucial to this proof.

We now continue as follows:

$$\sup_{Q_{\mathcal{E}}\in\mathcal{A}} \mathsf{E}\left[\frac{|\mathbf{d}^\mathsf{T}\mathbf{X}|^2}{\|\mathbf{X}\|^2}\right] = \sup_{Q_{\mathcal{E}}\in\mathcal{A}} \mathsf{E}\left[\frac{\left|\mathbf{d}_1^\mathsf{T}\mathbf{X}_1 + \cdots + \mathbf{d}_m^\mathsf{T}\mathbf{X}_m\right|^2}{\|\mathbf{X}_1\|^2 + \cdots + \|\mathbf{X}_m\|^2}\right] \tag{55}$$

$$\leq \sup_{Q_{\mathcal{E}}\in\mathcal{A}} \mathsf{E}\left[\frac{|\mathbf{d}_1^\mathsf{T}\mathbf{X}_1|^2 + \cdots + |\mathbf{d}_m^\mathsf{T}\mathbf{X}_m|^2}{\|\mathbf{X}_1\|^2 + \cdots + \|\mathbf{X}_m\|^2}\right] + \sum_{i=1}^m \sum_{\substack{j=1\\j\neq i}}^m \sup_{Q_{\mathcal{E}}\in\mathcal{A}} \mathsf{E}\left[\frac{|\mathbf{d}_i^\mathsf{T}\mathbf{X}_i|\cdot|\mathbf{d}_j^\mathsf{T}\mathbf{X}_j|}{\|\mathbf{X}_1\|^2 + \cdots + \|\mathbf{X}_m\|^2}\right] \tag{56}$$

$$\leq \sup_{Q_{\mathcal{E}}\in\mathcal{A}} \mathsf{E}\left[\frac{\|\mathbf{d}_1\|^2\|\mathbf{X}_1\|^2 + \cdots + \|\mathbf{d}_m\|^2\|\mathbf{X}_m\|^2}{\|\mathbf{X}_1\|^2 + \cdots + \|\mathbf{X}_m\|^2}\right] + \sum_{i=1}^m \sum_{\substack{j=1\\j\neq i}}^m \sup_{Q_{\mathcal{E}}\in\mathcal{A}} \mathsf{E}\left[\frac{\|\mathbf{d}_i\|\|\mathbf{X}_i\|\|\mathbf{d}_j\|\|\mathbf{X}_j\|}{\|\mathbf{X}_1\|^2 + \cdots + \|\mathbf{X}_m\|^2}\right] \tag{57}$$

where in (56) we split the supremum into many separate suprema, and where (57) follows from the Cauchy-Schwarz inequality

$$|\mathbf{d}_i^\mathsf{T}\mathbf{X}_i|^2 \leq \|\mathbf{d}_i\|^2\|\mathbf{X}_i\|^2. \tag{58}$$

We next upper-bound the first term in (57) as follows:

$$\sup_{Q_{\mathcal{E}}\in\mathcal{A}} \mathsf{E}\left[\frac{\|\mathbf{d}_1\|^2\|\mathbf{X}_1\|^2 + \cdots + \|\mathbf{d}_m\|^2\|\mathbf{X}_m\|^2}{\|\mathbf{X}_1\|^2 + \cdots + \|\mathbf{X}_m\|^2}\right]$$

$$\leq \sup_{r_1,\ldots,r_m} \left\{\frac{\|\mathbf{d}_1\|^2 r_1^2 + \cdots + \|\mathbf{d}_m\|^2 r_m^2}{r_1^2 + \cdots + r_m^2}\right\} \tag{59}$$

$$= \sup_{\mathbf{r}} \frac{\mathbf{r}^\mathsf{T}\tilde{\mathsf{D}}\mathbf{r}}{\|\mathbf{r}\|^2} \tag{60}$$

$$= \lambda_{\max}(\tilde{\mathsf{D}}) \tag{61}$$

$$= \max\left\{\|\mathbf{d}_1\|^2, \ldots, \|\mathbf{d}_m\|^2\right\} \tag{62}$$

where we have defined the vector $\mathbf{r} \triangleq (r_1, \ldots, r_m)^\mathsf{T}$ and the matrix

$$\tilde{\mathsf{D}} \triangleq \operatorname{diag}\left(\|\mathbf{d}_1\|^2, \ldots, \|\mathbf{d}_m\|^2\right). \tag{63}$$

The equality (61) follows from the Rayleigh-Ritz Theorem [6, Theorem 4.2.2].

To address the remaining terms in (57) we note that by definition of $\mathcal{A}$ in (52) at least one user's input must escape to infinity. Without loss of generality assume that $\mathbf{X}_1$ is among them. Then we can separate the remaining terms in (57) into two kinds:

$$\sup_{Q_{\mathcal{E}}\in\mathcal{A}} \mathsf{E}\left[\frac{\|\mathbf{d}_1\|\|\mathbf{d}_i\|\|\mathbf{X}_1\|\|\mathbf{X}_i\|}{\|\mathbf{X}_1\|^2+\cdots+\|\mathbf{X}_m\|^2}\right], \qquad i\in\{2,\ldots,m\} \tag{64}$$

and

$$\sup_{Q_{\mathcal{E}}\in\mathcal{A}} \mathsf{E}\left[\frac{\|\mathbf{d}_i\|\|\mathbf{d}_j\|\|\mathbf{X}_i\|\|\mathbf{X}_j\|}{\|\mathbf{X}_1\|^2+\cdots+\|\mathbf{X}_m\|^2}\right], \qquad i,j\in\{2,\ldots,m\},\ i\neq j. \tag{65}$$

Our proof is concluded once we can show that

$$\varlimsup_{\mathcal{E}\uparrow\infty}\sup_{Q_{\mathcal{E}}\in\mathcal{A}} \mathsf{E}\left[\frac{\|\mathbf{d}_1\|\|\mathbf{d}_i\|\|\mathbf{X}_1\|\|\mathbf{X}_i\|}{\|\mathbf{X}_1\|^2+\cdots+\|\mathbf{X}_m\|^2}\right]=0 \tag{66}$$

$$\varlimsup_{\mathcal{E}\uparrow\infty}\sup_{Q_{\mathcal{E}}\in\mathcal{A}} \mathsf{E}\left[\frac{\|\mathbf{d}_i\|\|\mathbf{d}_j\|\|\mathbf{X}_i\|\|\mathbf{X}_j\|}{\|\mathbf{X}_1\|^2+\cdots+\|\mathbf{X}_m\|^2}\right]=0 \tag{67}$$

for $i,j\in\{2,\ldots,m\}$, $i\neq j$.

We start with (66) and note that by dropping some terms in the denominator we have

$$\varlimsup_{\mathcal{E}\uparrow\infty}\sup_{Q_{\mathcal{E}}\in\mathcal{A}} \mathsf{E}\left[\frac{\|\mathbf{d}_1\|\|\mathbf{d}_i\|\|\mathbf{X}_1\|\|\mathbf{X}_i\|}{\|\mathbf{X}_1\|^2+\cdots+\|\mathbf{X}_m\|^2}\right]\leq \|\mathbf{d}_1\|\|\mathbf{d}_i\|\varlimsup_{\mathcal{E}\uparrow\infty}\sup_{Q_{\mathcal{E}}\in\mathcal{A}} \mathsf{E}\left[\frac{\|\mathbf{X}_1\|\|\mathbf{X}_i\|}{\|\mathbf{X}_1\|^2+\|\mathbf{X}_i\|^2}\right]. \tag{68}$$

Next we define

$$\mathcal{E}_1 \triangleq \mathsf{E}\left[\|\mathbf{X}_1\|^2\right] \tag{69}$$

and recall that if $\mathcal{E}\uparrow\infty$ then $\mathcal{E}_1\uparrow\infty$ by our assumption that user 1 escapes to infinity. Note further that

$$\frac{r_1 r_i}{r_1^2+r_i^2}\leq\frac{1}{2} \tag{70}$$

and that $r_1\mapsto\frac{r_1 r_i}{r_1^2+r_i^2}$ is monotonically decreasing if $r_1>r_i$.

Therefore, for an arbitrary choice of $a>1$, we define the set $\mathcal{D}$ as

$$\mathcal{D}\triangleq\left\{\mathbf{x}_1\colon 0\leq\|\mathbf{x}_1\|\leq a\|\mathbf{x}_i\|\right\} \tag{71}$$

and bound

$$\begin{aligned}
&\varlimsup_{\mathcal{E}\uparrow\infty}\sup_{Q_{\mathcal{E}}\in\mathcal{A}} \mathsf{E}\left[\frac{\|\mathbf{X}_1\|\|\mathbf{X}_i\|}{\|\mathbf{X}_1\|^2+\|\mathbf{X}_i\|^2}\right]\\
&\leq \sup_{Q_{\mathbf{X}_i}}\varlimsup_{\mathcal{E}_1\uparrow\infty}\sup_{Q_{\mathbf{X}_1}\in\mathcal{A}_1}\mathsf{E}\left[\frac{\|\mathbf{X}_1\|\|\mathbf{X}_i\|}{\|\mathbf{X}_1\|^2+\|\mathbf{X}_i\|^2}\right] &(72)\\
&= \sup_{Q_{\mathbf{X}_i}}\varlimsup_{\mathcal{E}_1\uparrow\infty}\sup_{Q_{\mathbf{X}_1}\in\mathcal{A}_1}\iint\frac{\|\mathbf{x}_1\|\|\mathbf{x}_i\|}{\|\mathbf{x}_1\|^2+\|\mathbf{x}_i\|^2}\,\mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_1)\,\mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) &(73)\\
&\leq \sup_{Q_{\mathbf{X}_i}}\varlimsup_{\mathcal{E}_1\uparrow\infty}\sup_{Q_{\mathbf{X}_1}\in\mathcal{A}_1}\iint_{\mathbf{x}_1\in\mathcal{D}}\frac{\|\mathbf{x}_1\|\|\mathbf{x}_i\|}{\|\mathbf{x}_1\|^2+\|\mathbf{x}_i\|^2}\,\mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_1)\,\mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i)\\
&\quad+\sup_{Q_{\mathbf{X}_i}}\varlimsup_{\mathcal{E}_1\uparrow\infty}\sup_{Q_{\mathbf{X}_1}\in\mathcal{A}_1}\iint_{\mathbf{x}_1\in\mathcal{D}^{\mathrm{c}}}\frac{\|\mathbf{x}_1\|\|\mathbf{x}_i\|}{\|\mathbf{x}_1\|^2+\|\mathbf{x}_i\|^2}\,\mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_1)\,\mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i). &(74)
\end{aligned}$$

Here in the first inequality (72) we define $\mathcal{A}_1$ as the set of all input distributions of the first user that escape to infinity, and take the supremum over all $Q_{\mathbf{X}_i}$ without any constraint on the average power and no dependence on $Q_{\mathbf{X}_1}$. The last inequality (74) then follows from splitting the inner integration into two parts and from the property that the supremum of a sum is always upper-bounded by the sum of the suprema.

Next, let's look at the first term in (74) and use (70):

$$\begin{aligned}
&\varlimsup_{\mathcal{E}_1\uparrow\infty} \sup_{Q_{\mathbf{X}_1}\in\mathcal{A}_1} \iint_{\mathbf{x}_1\in\mathcal{D}} \underbrace{\frac{\|\mathbf{x}_1\|\|\mathbf{x}_i\|}{\|\mathbf{x}_1\|^2+\|\mathbf{x}_i\|^2}}_{\leq\frac{1}{2}} \,\mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_1)\,\mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) \\
&\leq \varlimsup_{\mathcal{E}_1\uparrow\infty} \sup_{Q_{\mathbf{X}_1}\in\mathcal{A}_1} \iint_{\mathbf{x}_1\in\mathcal{D}} \frac{1}{2}\,\mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_1)\,\mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) && (75)\\
&\leq \varlimsup_{\mathcal{E}_1\uparrow\infty} \int \left(\sup_{Q_{\mathbf{X}_1}\in\mathcal{A}_1} \frac{1}{2} \int_{\mathbf{x}_1\in\mathcal{D}} \mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_1) \right) \mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) && (76)\\
&= \int \varlimsup_{\mathcal{E}_1\uparrow\infty} \left(\sup_{Q_{\mathbf{X}_1}\in\mathcal{A}_1} \frac{1}{2} \int_{\mathbf{x}_1\in\mathcal{D}} \mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_1) \right) \mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) && (77)\\
&= \int \left(\varlimsup_{\mathcal{E}_1\uparrow\infty} \sup_{Q_{\mathbf{X}_1}\in\mathcal{A}_1} \frac{1}{2} \Pr[\|\mathbf{X}_1\| \leq a\|\mathbf{x}_i\|] \right) \mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) && (78)\\
&= \int 0 \,\mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) = 0. && (79)
\end{aligned}$$

Here, (75) follows from (70); the subsequent inequality (76) follows by taking the supremum into the first integral which can only enlarge the expression; in (77) we exchange limit and integration which needs justification: define

$$\begin{aligned}
g_{\mathcal{E}_1}(\mathbf{x}_i) &\triangleq \sup_{Q_{\mathbf{X}_1}\in\mathcal{A}_1} \frac{1}{2} \int_{\mathbf{x}_1\in\mathcal{D}} \mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_1) && (80)\\
&\leq \sup_{Q_{\mathbf{X}_1}\in\mathcal{A}_1} \frac{1}{2} \int \mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_1) && (81)\\
&= \frac{1}{2} \triangleq g_{\text{upper}}(\mathbf{x}_i) && (82)
\end{aligned}$$

and then note that

$$\int g_{\text{upper}}(\mathbf{x}_i)\,\mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) = \int \frac{1}{2}\,\mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) = \frac{1}{2} \tag{83}$$

i.e., $g_{\text{upper}}(\cdot)$ is independent of $\mathcal{E}_1$ and integrable. Thus, by the Dominated Convergence Theorem [7] we are allowed to swap limit and integration.

Finally, (79) follows from Proposition 7 because $Q_{\mathbf{X}_1}$ escapes to infinity.

Continuing with (74) we now have:

$$\begin{aligned}
&\varlimsup_{\mathcal{E}\uparrow\infty} \sup_{Q_{\mathcal{E}}\in\mathcal{A}} \mathsf{E}\left[\frac{\|\mathbf{X}_1\|\|\mathbf{X}_i\|}{\|\mathbf{X}_1\|^2+\|\mathbf{X}_i\|^2}\right] \\
&\leq \sup_{Q_{\mathbf{X}_i}} \varlimsup_{\mathcal{E}_1\uparrow\infty} \sup_{Q_{\mathbf{X}_1}\in\mathcal{A}_1} \iint_{\mathbf{x}_1\in\mathcal{D}^{\mathrm{c}}} \frac{\|\mathbf{x}_1\|\|\mathbf{x}_i\|}{\|\mathbf{x}_1\|^2+\|\mathbf{x}_i\|^2} \,\mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_1)\,\mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) && (84)\\
&\leq \sup_{Q_{\mathbf{X}_i}} \varlimsup_{\mathcal{E}_1\uparrow\infty} \sup_{Q_{\mathbf{X}_1}\in\mathcal{A}_1} \iint_{\mathbf{x}_1\in\mathcal{D}^{\mathrm{c}}} \frac{(a\|\mathbf{x}_i\|)\,\|\mathbf{x}_i\|}{(a\|\mathbf{x}_i\|)^2+\|\mathbf{x}_i\|^2} \,\mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_1)\,\mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) && (85)\\
&= \sup_{Q_{\mathbf{X}_i}} \varlimsup_{\mathcal{E}_1\uparrow\infty} \sup_{Q_{\mathbf{X}_1}\in\mathcal{A}_1} \iint_{\mathbf{x}_1\in\mathcal{D}^{\mathrm{c}}} \frac{a}{a^2+1} \,\mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_1)\,\mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) && (86)\\
&\leq \sup_{Q_{\mathbf{X}_i}} \varlimsup_{\mathcal{E}_1\uparrow\infty} \sup_{Q_{\mathbf{X}_1}\in\mathcal{A}_1} \iint \frac{a}{a^2+1} \,\mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_1)\,\mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) && (87)
\end{aligned}$$

$$= \sup_{Q_{\mathbf{X}_i}} \int \frac{a}{a^2+1} \, \mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) \tag{88}$$

$$= \frac{a}{a^2+1} < \epsilon \tag{89}$$

for any $\epsilon > 0$ if we choose a large enough. Here (85) follows because $r_1 \mapsto \frac{r_1 r_i}{r_1^2 + r_i^2}$ is monotonically decreasing if $r_1 > r_i$. Since $a > 1$ is arbitrary, we obtain:

$$\varlimsup_{\mathcal{E}\uparrow\infty} \sup_{Q_{\mathcal{E}} \in \mathcal{A}} \mathsf{E}\left[\frac{\|\mathbf{X}_1\| \|\mathbf{X}_i\|}{\|\mathbf{X}_1\|^2 + \|\mathbf{X}_i\|^2} \right] = 0. \tag{90}$$

This proves (66).

To prove (67), we again drop some terms in the denominator:

$$\mathsf{E}\left[\frac{\|\mathbf{d}_i\| \|\mathbf{d}_j\| \|\mathbf{X}_i\| \|\mathbf{X}_j\|}{\|\mathbf{X}_1\|^2 + \cdots + \|\mathbf{X}_m\|^2} \right] \leq \|\mathbf{d}_i\| \|\mathbf{d}_j\| \mathsf{E}\left[\frac{\|\mathbf{X}_i\| \|\mathbf{X}_j\|}{\|\mathbf{X}_1\|^2 + \|\mathbf{X}_i\|^2 + \|\mathbf{X}_j\|^2} \right]. \tag{91}$$

We once more use definition (69) and note that

$$\frac{r_i r_j}{r_1^2 + r_i^2 + r_j^2} \leq \frac{r_i^2}{r_1^2 + 2r_i^2} \leq \frac{1}{2} \tag{92}$$

and that $r_1 \mapsto \frac{r_i^2}{r_1^2 + 2r_i^2}$ is monotonically decreasing.

For an arbitrary choice of $a > 1$, we use the set $\mathcal{D}$ from (71) to derive

$$\begin{aligned}
&\varlimsup_{\mathcal{E}\uparrow\infty} \sup_{Q_{\mathcal{E}} \in \mathcal{A}} \mathsf{E}\left[\frac{\|\mathbf{X}_i\| \|\mathbf{X}_j\|}{\|\mathbf{X}_1\|^2 + \|\mathbf{X}_i\|^2 + \|\mathbf{X}_j\|^2} \right] \\
&\leq \sup_{Q_{\mathbf{X}_i} \cdot Q_{\mathbf{X}_j}} \varlimsup_{\mathcal{E}_1\uparrow\infty} \sup_{Q_{\mathbf{X}_1} \in \mathcal{A}_1} \iiint \frac{\|\mathbf{x}_i\| \|\mathbf{x}_j\|}{\|\mathbf{x}_1\|^2 + \|\mathbf{x}_i\|^2 + \|\mathbf{x}_j\|^2} \, \mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_1) \, \mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) \, \mathrm{d}Q_{\mathbf{X}_j}(\mathbf{x}_j)
\end{aligned} \tag{93}$$

$$\leq \sup_{Q_{\mathbf{X}_i}} \varlimsup_{\mathcal{E}_1\uparrow\infty} \sup_{Q_{\mathbf{X}_1} \in \mathcal{A}_1} \iint \frac{\|\mathbf{x}_i\|^2}{\|\mathbf{x}_1\|^2 + 2\|\mathbf{x}_i\|^2} \, \mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_1) \, \mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) \tag{94}$$

$$\begin{aligned}
&\leq \sup_{Q_{\mathbf{X}_i}} \varlimsup_{\mathcal{E}_1\uparrow\infty} \sup_{Q_{\mathbf{X}_1} \in \mathcal{A}_1} \iint_{\mathbf{x}_1 \in \mathcal{D}} \frac{\|\mathbf{x}_i\|^2}{\|\mathbf{x}_1\|^2 + 2\|\mathbf{x}_i\|^2} \, \mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_m) \, \mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) \\
&\quad + \sup_{Q_{\mathbf{X}_i}} \varlimsup_{\mathcal{E}_1\uparrow\infty} \sup_{Q_{\mathbf{X}_1} \in \mathcal{A}_1} \iint_{\mathbf{x}_1 \in \mathcal{D}^{\mathrm{c}}} \frac{\|\mathbf{x}_i\|^2}{\|\mathbf{x}_1\|^2 + 2\|\mathbf{x}_i\|^2} \, \mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_m) \, \mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i).
\end{aligned} \tag{95}$$

Here in (93) we define $\mathcal{A}_1$ as the set of all input distributions such that the first user escapes to infinity, and take the supremum over all joint distributions of $Q_{\mathbf{X}_i}$ and $Q_{\mathbf{X}_j}$ without any restriction on the average power. In the subsequent inequality (94) we apply (92) to replace $\|\mathbf{x}_j\|$ by $\|\mathbf{x}_i\|$. In the last inequality we split the inner integration into two parts using (71).

For the first term in (95), we have

$$\begin{aligned}
&\varlimsup_{\mathcal{E}_1\uparrow\infty} \sup_{Q_{\mathbf{X}_1} \in \mathcal{A}_1} \iint_{\mathbf{x}_1 \in \mathcal{D}} \underbrace{\frac{\|\mathbf{x}_i\|^2}{\|\mathbf{x}_1\|^2 + 2\|\mathbf{x}_i\|^2}}_{\leq \frac{1}{2}} \, \mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_1) \, \mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) \\
&\leq \varlimsup_{\mathcal{E}_1\uparrow\infty} \sup_{Q_{\mathbf{X}_1} \in \mathcal{A}_1} \iint_{\mathbf{x}_1 \in \mathcal{D}} \frac{1}{2} \, \mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_1) \, \mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) && (96) \\
&= 0 && (97)
\end{aligned}$$

where (97) follows from a derivation analogous to (75)–(79).

The second term in (95) can be bounded as follows:

$$\sup_{Q_{\mathbf{X}_i}} \varlimsup_{\mathcal{E}_1\uparrow\infty} \sup_{Q_{\mathbf{X}_1}\in\mathcal{A}_1} \iint_{\mathbf{x}_1\in\mathcal{D}^c} \frac{\|\mathbf{x}_i\|^2}{\|\mathbf{x}_1\|^2+2\|\mathbf{x}_i\|^2}\,\mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_1)\,\mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i)$$

$$\leq \sup_{Q_{\mathbf{X}_i}} \varlimsup_{\mathcal{E}_1\uparrow\infty} \sup_{Q_{\mathbf{X}_1}\in\mathcal{A}_1} \iint_{\mathbf{x}_1\in\mathcal{D}^c} \frac{\|\mathbf{x}_i\|^2}{(a\|\mathbf{x}_i\|)^2+2\|\mathbf{x}_i\|^2}\,\mathrm{d}Q_{\mathbf{X}_1}(\mathbf{x}_1)\,\mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) \tag{98}$$

$$\leq \sup_{Q_{\mathbf{X}_i}} \int \frac{1}{a^2+2}\,\mathrm{d}Q_{\mathbf{X}_i}(\mathbf{x}_i) \tag{99}$$

$$= \frac{1}{a^2+2} < \epsilon \tag{100}$$

for any $\epsilon > 0$ if we choose a large enough. Here in the first inequality we use that $r_1 \mapsto \frac{r_i^2}{r_1^2+2r_i^2}$ is monotonically decreasing. Since a is arbitrary, this proves (67) and concludes the proof.

7 Conclusions

In this paper we have derived a new upper and lower bound on the sum-rate capacity of a noncoherent memoryless multiple-access Rician fading channel with m transmitters (with a different number of antennas each) and one receiver (with only one antenna). We have shown that while the sum-rate capacity at low SNR behaves normally with a logarithmic growth in the available power, at high SNR it is highly power-inefficient and only grows double-logarithmically. It is therefore advisable not to operate such a channel at high SNR. These bounds rely on novel bounds on the capacity of a single-user MISO Rician fading channel that are valid for any SNR.

In a second step we then derived the exact asymptotic high-SNR expansion of the sum-rate capacity, which has the form

$$\mathsf{C}_{\text{MAC}} = \log\log\left(\frac{\mathcal{E}}{\sigma^2}\right) + \chi_{\text{MAC}} + o(1). \tag{101}$$

We have shown that this asymptotic sum-rate capacity is limited by the asymptotic capacity of the user seeing the best channel and can only be achieved if all users with a channel that is strictly worse than the best channel are always switched off and cannot communicate. Note that this should not be confused with the idea of time sharing where at any given time only one user is allowed to communicate. In the presented setup, as long as the channel model does not change, the best user will remain the best user, i.e., all other users can never communicate.[3] This very pessimistic result fits to the already rather pessimistic double-logarithmic behavior and strengthen the conviction that these channels should not be used at high SNR, but only at low SNR where the channel will behave normally like a coherent fading channel.

At first sight our results seems very similar to a result by Knopp and Humblet [8] [9] [10], who showed that the strategy of one user at a time also is optimal for the MAC with full channel state information both at the transmitter and receiver side. In [8] a continuous-time system is considered and it is shown that if the transmitter and receiver have full knowledge of the fading, then it is best if the

[3]The only exception is if there are several users having the same best channel. In this case these equivalent best users can use time sharing to alternatively communicate.

users are assigned separate frequency and time slots corresponding to best fading realizations (orthogonal signaling). However, we would like to point out that in this setup, each user can transmit regularly and has a strictly nonzero average communication rate, while in the channel model considered here, it turns out that optimally most users have a zero transmission rate. So these two results are not properly comparable.

We remark further that from the fact that the sum-rate capacity is achieved in a corner where only one user has positive rate, one can deduce that the asymptotic capacity region has the shape of an m-dimensional simplex.

In the analysis of the channel we have allowed for many different types of power constraints. We grouped them into three categories: an individual peak-power constraint for each user, an individual average-power constraint for each user, and a combined power-sharing average-power constraint among all users. The power-sharing constraint does not make sense in a practical setup as it requires the users to share a common battery, while their signals still are restricted to be independent. However, the inclusion of this case helps with the analysis. Moreover, it turns out that the pessimistic results described above even hold if we allow for such power sharing.

Within a category of constraints, we do allow for different power settings for different users as long as the constraints scale linearly (see the constants κ_i and $\bar{\kappa}$ in (11)–(13)). It would be possible to extend the shown results to situations where the power constraints among the different user differ exponentially, i.e., if every user i is allowed to use a power of at most

$$\frac{\kappa_i}{m}\mathcal{E}^{\vartheta_i}$$

for some $\kappa_i, \vartheta_i > 0$. In this case, however, ϑ_i will influence the MAC fading number[4] via an additive term $\log \vartheta_i$. This then means that in the evaluation of the MAC fading number (48) not only $\|\mathbf{d}_i\|$ is important, but also this additive term $\log \vartheta_i$ has to be taken into account.

While in this paper we have restricted the channel model to be memoryless, a generalization to a fading process with memory is possible. Again, one has to be careful as the memory will influence the MAC fading number and thereby affect the search for the best channel.

As already discussed in Section 3.3, we would like to emphasize once more that the analysis of the asymptotic sum-rate capacity of this channel is of practical interest in spite of the fact that we will not use the channel at high SNR. The reason is that the MAC fading number χ_{MAC} is a good indicator for the threshold between the efficient low-SNR and the highly inefficient high-SNR regime. As a rule of thumb, the MAC Rician fading channel can be used up to a sum-rate of about $\chi + 2$ nats (see Figure 1 for an example). It is ominous that the fading number—and ergo also the threshold—does not vary with the type of the used power constraint. This means that once a sum rate of around $\chi + 2$ nats is achieved, the channel behavior will become very poor and cannot be improved by any optimization of the power allocation. Instead the system has to be changed in a more fundamental fashion in order to achieve a change in the channel model.

An important clue to the derivations is a generalization of the concept of *input distributions that escape to infinity.* To put it in engineering words, the concept says that one cannot achieve the sum-rate capacity asymptotically unless at least one of the users always uses input symbols of infinite power. Note that while we have

[4] The double-logarithmic term in the asymptotic expansion will remain unchanged.

stated this concept here specifically for the multiple-access Rician fading channel at hand, it can be further extended to more general multiple-user channels.

A Upper Bound on MISO Rician Fading Capacity

The upper bound (30) on the MISO Rician fading channel (20) is a generalization of an upper bound on the SISO Rician fading channel presented in [1, Eq. (166)]. It is based on a duality-based upper bound on the mutual information taken from [1, Eq. (25)]:

$$\begin{aligned} I(\mathbf{X};Y) \le -h(Y|\mathbf{X}) + \log\pi + \alpha\log\beta + \log\Gamma\left(\alpha, \frac{\nu}{\beta}\right) \\ + (1-\alpha)\mathsf{E}\big[\log\big(|Y|^2+\nu\big)\big] + \frac{1}{\beta}\mathsf{E}\big[|Y|^2\big] + \frac{\nu}{\beta} \end{aligned} \tag{102}$$

where $\alpha, \beta > 0$ and $\nu \ge 0$ are free parameters.

We start with an upper bound on the fifth term on the RHS of (102). To that goal we assume $0 < \alpha < 1$ such that $1-\alpha > 0$ and define

$$\epsilon_\nu \triangleq \sup_{\mathbf{x}} \Big\{ \mathsf{E}\big[\log\big(|Y|^2+\nu\big) \bigm| \mathbf{X}=\mathbf{x}\big] - \mathsf{E}\big[\log|Y|^2 \bigm| \mathbf{X}=\mathbf{x}\big] \Big\} \tag{103}$$

such that

$$\begin{aligned} &(1-\alpha)\mathsf{E}\big[\log\big(|Y|^2+\nu\big)\big] \\ &\quad = (1-\alpha)\mathsf{E}\big[\log|Y|^2\big] + (1-\alpha)\big(\mathsf{E}\big[\log\big(|Y|^2+\nu\big)\big] - \mathsf{E}\big[\log|Y|^2\big]\big) && (104) \\ &\quad \le (1-\alpha)\mathsf{E}\big[\log|Y|^2\big] \\ &\qquad + (1-\alpha)\sup_{\mathbf{x}}\Big\{ \mathsf{E}\big[\log\big(|Y|^2+\nu\big) \bigm| \mathbf{X}=\mathbf{x}\big] - \mathsf{E}\big[\log|Y|^2 \bigm| \mathbf{X}=\mathbf{x}\big] \Big\} && (105) \\ &\quad = (1-\alpha)\mathsf{E}\big[\log|Y|^2\big] + (1-\alpha)\epsilon_\nu. && (106) \end{aligned}$$

Next we apply (102) to the MISO Rician fading channel (20). We note that conditional on $\mathbf{X}=\mathbf{x}$

$$Y \sim \mathcal{N}_{\mathbb{C}}\big(\mathbf{d}^\mathsf{T}\mathbf{x}, \|\mathbf{x}\|^2+\sigma^2\big) \tag{107}$$

and compute

$$\begin{aligned} h(Y|\mathbf{X}=\mathbf{x}) &= \log\pi + 1 + \log\big(\|\mathbf{x}\|^2+\sigma^2\big) && (108) \\ \mathsf{E}\big[|Y|^2 \bigm| \mathbf{X}=\mathbf{x}\big] &= |\mathbf{d}^\mathsf{T}\mathbf{x}|^2 + \|\mathbf{x}\|^2 + \sigma^2 && (109) \end{aligned}$$

and

$$\begin{aligned} \mathsf{E}\big[\log|Y|^2 \bigm| \mathbf{X}=\mathbf{x}\big] &= \log\left(\frac{|\mathbf{d}^\mathsf{T}\mathbf{x}|^2}{\|\mathbf{x}\|^2+\sigma^2}\right) - \mathrm{Ei}\left(-\frac{|\mathbf{d}^\mathsf{T}\mathbf{x}|^2}{\|\mathbf{x}\|^2+\sigma^2}\right) + \log(\|\mathbf{x}\|^2+\sigma^2) && (110) \\ \epsilon_\nu &= \sup_{\mathbf{x}} \mathsf{E}\left[\log\left(1+\frac{\nu}{|Y|^2}\right) \middle| \mathbf{X}=\mathbf{x}\right] && (111) \\ &= \mathsf{E}\left[\log\left(1+\frac{\nu}{|Y|^2}\right) \middle| \mathbf{X}=\mathbf{0}\right] && (112) \\ &= \log\left(\frac{\nu}{\sigma^2}\right) - e^{\nu/\sigma^2}\,\mathrm{Ei}\left(-\frac{\nu}{\sigma^2}\right) + \gamma. && (113) \end{aligned}$$

Here, in (110) we evaluate the expected logarithm of a noncentral chi-square random variable as derived in [11], [1, Lemma 10.1], [12, Lemma A.6]; and (112) follows from a stochastic ordering argument by noting that the function $\xi \mapsto \log\left(1+\frac{1}{\xi}\right)$ is monotonically decreasing and that the distribution of Y conditional on $\mathbf{X}=\mathbf{x}$ is stochastically larger than the distribution of Y conditional on $\mathbf{X}=\mathbf{0}$ [1, Sec. IV.B]. The final step (113) follows by a direct calculation.

Plugging (106) and (108)–(113) into (102) then yields

$$
\begin{aligned}
I(\mathbf{X};Y) \leq{} & -1+\alpha\log\beta-\alpha\mathsf{E}\left[\log\left(\|\mathbf{X}\|^2+\sigma^2\right)\right]+\log\Gamma\left(\alpha,\frac{\nu}{\beta}\right) \\
& +\frac{\mathsf{E}\left[\|\mathbf{X}\|^2+\sigma^2+|\mathbf{d}^{\mathsf{T}}\mathbf{X}|^2\right]}{\beta}+\frac{\nu}{\beta} \\
& +(1-\alpha)\left(\mathsf{E}\left[\log\left(\frac{|\mathbf{d}^{\mathsf{T}}\mathbf{X}|^2}{\|\mathbf{X}\|^2+\sigma^2}\right)-\mathrm{Ei}\left(-\frac{|\mathbf{d}^{\mathsf{T}}\mathbf{X}|^2}{\|\mathbf{X}\|^2+\sigma^2}\right)\right]+\epsilon_\nu\right) \qquad (114)\\
\leq{} & -1+\alpha\log\left(\frac{\beta}{\sigma^2}\right)+\log\Gamma\left(\alpha,\frac{\nu}{\beta}\right) \\
& +\frac{\mathsf{E}\left[\|\mathbf{X}\|^2\right]+\sigma^2+\|\mathbf{d}\|^2\mathsf{E}\left[\|\mathbf{X}\|^2\right]}{\beta}+\frac{\nu}{\beta} \\
& +(1-\alpha)\left(\log\left(\frac{\|\mathbf{d}\|^2\mathsf{E}\left[\|\mathbf{X}\|^2\right]}{\mathsf{E}[\|\mathbf{X}\|^2]+\sigma^2}\right)-\mathrm{Ei}\left(-\frac{\|\mathbf{d}\|^2\mathsf{E}\left[\|\mathbf{X}\|^2\right]}{\mathsf{E}[\|\mathbf{X}\|^2]+\sigma^2}\right)+\epsilon_\nu\right) \qquad (115)
\end{aligned}
$$

where for the last step we have lower-bounded $\mathsf{E}\left[\log\left(\|\mathbf{X}\|^2+\sigma^2\right)\right]\geq\log\sigma^2$; used the monotonicity of $\xi\mapsto\log(\xi)-\mathrm{Ei}\left(-\xi\right)$ together with the Cauchy-Schwarz inequality (58); and applied Jensen's inequality to the concave function

$$\xi\mapsto\log\left(\frac{\|\mathbf{d}\|^2\xi}{\xi+\sigma^2}\right)-\mathrm{Ei}\left(-\frac{\|\mathbf{d}\|^2\xi}{\xi+\sigma^2}\right). \qquad (116)$$

The upper bound (30) now follows from the average-power constraint (21).

B Lower Bound on MISO Rician Fading Capacity

The first lower bound (34) on the capacity of the MISO Rician fading channel (26) with peak-power constraint (27) is based on the following lemma that has been proven in [1, Lemma 4.9].

Lemma 10 ([1]). *Let the random vector* $\mathbf{X}$ *take value in* $\mathbb{C}^{n_{\mathrm{T}}}$ *and satisfy*

$$\Pr\left[\|\mathbf{X}\|^2\geq x_{\min}^2\right]=1 \qquad (117)$$

for some $x_{\min}>0$*. Let* $\mathbb{H}$ *be a random* $n_{\mathrm{R}}\times n_{\mathrm{T}}$ *matrix having finite entropy* $h(\mathbb{H})>-\infty$ *and finite expected squared Frobenius norm* $\mathsf{E}\left[\|\mathbb{H}\|_{\mathrm{F}}^2\right]<\infty$*. Let* $\mathbf{Z}\sim\mathcal{N}_{\mathbb{C}}\left(\mathbf{0},\sigma^2\mathsf{I}\right)$ *and assume that* $\mathbf{X}$, $\mathbb{H}$*, and* $\mathbf{Z}$ *are independent. Then*

$$I(\mathbf{X};\mathbb{H}\mathbf{X}+\mathbf{Z})\geq I(\mathbf{X};\mathbb{H}\mathbf{X})-\sup_{\|\hat{\mathbf{x}}\|=1}\left\{h\left(\mathbb{H}\hat{\mathbf{x}}+\frac{\mathbf{Z}}{x_{\min}}\right)-h(\mathbb{H}\hat{\mathbf{x}})\right\}. \qquad (118)$$

We apply this lemma to the situation of the MISO Rician fading channel (26) and choose the following distribution on $\mathbf{X}_i$:

$$\mathbf{X}_i\triangleq R\cdot\frac{\mathbf{d}_i}{\|\mathbf{d}_i\|}e^{\mathrm{i}\Phi} \qquad (119)$$

where Φ and R are statistically independent, Φ is uniform between 0 and 2π, $\Phi \sim \mathcal{U}([0, 2\pi))$, and R is such that

$$\log R^2 \sim \mathcal{U}([\log \Upsilon_0, \log \Upsilon]) \tag{120}$$

for some fixed Υ_0. This choice satisfies the peak-power constraint (27) and also

$$\Pr\bigl[\|\mathbf{X}_i\|^2 \geq \Upsilon_0\bigr] = 1. \tag{121}$$

Hence by Lemma 10, we get

$$\begin{aligned}
\mathsf{C}_{\text{MISO,pp},n_i}(\Upsilon) &\geq I(\mathbf{X}_i; \mathbf{d}_i^{\mathsf{T}}\mathbf{X}_i + \mathbf{H}_i^{\mathsf{T}}\mathbf{X}_i + Z) && (122)\\
&\geq I(\mathbf{X}_i; \mathbf{d}_i^{\mathsf{T}}\mathbf{X}_i + \mathbf{H}_i^{\mathsf{T}}\mathbf{X}_i) \\
&\quad - \sup_{\|\hat{\mathbf{x}}\|=1} \left\{ h\left(\mathbf{d}_i^{\mathsf{T}}\hat{\mathbf{x}} + \mathbf{H}_i^{\mathsf{T}}\hat{\mathbf{x}} + \frac{Z}{\sqrt{\Upsilon_0}}\right) - h(\mathbf{d}_i^{\mathsf{T}}\hat{\mathbf{x}} + \mathbf{H}_i^{\mathsf{T}}\hat{\mathbf{x}}) \right\} && (123)\\
&= I(\mathbf{X}_i; \mathbf{d}_i^{\mathsf{T}}\mathbf{X}_i + \mathbf{H}_i^{\mathsf{T}}\mathbf{X}_i) - \log\left(1 + \frac{\sigma^2}{\Upsilon_0}\right). && (124)
\end{aligned}$$

We introduce a random variable

$$\tilde{H} \triangleq \mathbf{H}_i^{\mathsf{T}} \cdot \frac{\mathbf{d}_i}{\|\mathbf{d}_i\|} e^{\mathrm{i}\Phi} \sim \mathcal{N}_{\mathbb{C}}(0, 1) \tag{125}$$

and rewrite the first term on the RHS of (124) as follows:

$$\begin{aligned}
&I(\mathbf{X}_i; \mathbf{d}_i^{\mathsf{T}}\mathbf{X}_i + \mathbf{H}_i^{\mathsf{T}}\mathbf{X}_i) \\
&\quad = h(\mathbf{d}_i^{\mathsf{T}}\mathbf{X}_i + \mathbf{H}_i^{\mathsf{T}}\mathbf{X}_i) - h(\mathbf{d}_i^{\mathsf{T}}\mathbf{X}_i + \mathbf{H}_i^{\mathsf{T}}\mathbf{X}_i | \mathbf{X}_i) && (126)\\
&\quad = h(\|\mathbf{d}_i\| e^{\mathrm{i}\Phi} R + \tilde{H} R) - \mathsf{E}\bigl[\log\bigl(\pi e \|\mathbf{X}_i\|^2\bigr)\bigr] && (127)\\
&\quad = h\left(\bigl|\|\mathbf{d}_i\| e^{\mathrm{i}\Phi} R + \tilde{H} R\bigr|^2\right) - 1 - \mathsf{E}\bigl[\log R^2\bigr] && (128)\\
&\quad \geq h\left(R^2 \cdot \bigl|\|\mathbf{d}_i\| e^{\mathrm{i}\Phi} + \tilde{H}\bigr|^2 \,\middle|\, \Phi, \tilde{H}\right) - 1 - \mathsf{E}\bigl[\log R^2\bigr] && (129)\\
&\quad = h\left(R^2\right) + \mathsf{E}\left[\log \bigl|\|\mathbf{d}_i\| e^{\mathrm{i}\Phi} + \tilde{H}\bigr|^2\right] - 1 - \mathsf{E}\bigl[\log R^2\bigr]. && (130)
\end{aligned}$$

Here, (128) follows from the fact that for a circularly symmetric random variable U we have [1, Lemma 6.16]

$$h(U) = h\left(|U|^2\right) + \log \pi. \tag{131}$$

In (129) we condition the differential entropy which cannot increase its value; and (130) follows from the scaling property of differential entropy [13, Th. 8.6.4].

Next, we again evaluate the expected logarithm of a noncentral chi-square random variable [11], [1, Lemma 10.1], [12, Lemma A.6]:

$$\begin{aligned}
\mathsf{E}\left[\log\left(\bigl|\|\mathbf{d}_i\| e^{\mathrm{i}\Phi} + \tilde{H}\bigr|^2\right)\right] &= \mathsf{E}\left[\log\left(\bigl|\|\mathbf{d}_i\| + \tilde{H}\bigr|^2\right)\right] && (132)\\
&= \log\left(\|\mathbf{d}_i\|^2\right) - \mathrm{Ei}\left(-\|\mathbf{d}_i\|^2\right) && (133)
\end{aligned}$$

(where the first equality follows because $\tilde{H}$ is circularly symmetric) and use the following identity [1, Lemma 6.15]:

$$h\left(\log R^2\right) = h\left(R^2\right) - \mathsf{E}\bigl[\log R^2\bigr]. \tag{134}$$

The lower bound (34) now follows by plugging (130), (133), and (134) into (124) and noting that because of (120)

$$h\left(\log R^2\right) = \log\log\left(\frac{\Upsilon}{\Upsilon_0}\right). \tag{135}$$

The second lower bound (35) follows from (122) with the choice (32):

$$\mathsf{C}_{\text{MISO,pp},n_i}(\Upsilon) \geq h(\mathbf{d}_i^{\mathsf{T}}\mathbf{X}_i + \mathbf{H}_i^{\mathsf{T}}\mathbf{X}_i + Z) - h(\mathbf{d}_i^{\mathsf{T}}\mathbf{X}_i + \mathbf{H}_i^{\mathsf{T}}\mathbf{X}_i + Z|\mathbf{X}_i) \tag{136}$$
$$= h(\mathbf{d}_i^{\mathsf{T}}\mathbf{X}_i + \mathbf{H}_i^{\mathsf{T}}\mathbf{X}_i + Z) - \mathsf{E}\left[\log\left(\pi e(\|\mathbf{X}\|^2 + \sigma^2)\right)\right] \tag{137}$$
$$= h(R^2) - 1 - p\log(\Upsilon + \sigma^2) - (1-p)\log\sigma^2. \tag{138}$$

Here in the last step we have used (131) together with the fact that

$$\mathbf{d}_i^{\mathsf{T}}\mathbf{X}_i + \mathbf{H}_i^{\mathsf{T}}\mathbf{X}_i + Z \overset{\mathscr{L}}{=} \|\mathbf{d}_i\|\sqrt{\Upsilon}e^{\mathrm{i}\Phi}\Xi + \sqrt{\Upsilon}\tilde{H}\Xi + Z \tag{139}$$
$$\overset{\mathscr{L}}{=} \left(\|\mathbf{d}_i\|\sqrt{\Upsilon}\Xi + \sqrt{\Upsilon}\tilde{H}\Xi + Z\right)e^{\mathrm{i}\Phi} \tag{140}$$
$$\overset{\mathscr{L}}{=} \underbrace{\left|\|\mathbf{d}_i\|\sqrt{\Upsilon}\Xi + \sqrt{\Upsilon}\tilde{H}\Xi + Z\right|}_{\triangleq R_i}e^{\mathrm{i}\Phi} \tag{141}$$

where "$\overset{\mathscr{L}}{=}$" denotes "equal in probability law." Hence, we see that $\mathbf{d}_i^{\mathsf{T}}\mathbf{X}_i + \mathbf{H}_i^{\mathsf{T}}\mathbf{X}_i + Z$ is circularly symmetric with a magnitude R_i that, conditional on $\Xi = 0$, is Rayleigh and, conditional on $\Xi = 1$, Rician distributed. The probability density function of R_i^2 is given by (36).

C Proof of Proposition 7

Fix some $\mathcal{E}_0 > 0$ and let

$$U \triangleq \begin{cases} 1 & \text{if } \exists\, i\colon \|\mathbf{X}_i\|^2 \geq \frac{\mathcal{E}_0}{m} \\ 0 & \text{if } \|\mathbf{X}_i\|^2 < \frac{\mathcal{E}_0}{m},\ \forall\, i. \end{cases} \tag{142}$$

Further we define

$$\mu \triangleq \Pr[U = 1]. \tag{143}$$

To prove Proposition 7, we need to show that

$$\varliminf_{\mathcal{E}\uparrow\infty} \mu = 1. \tag{144}$$

To that goal, note the following:

$$I(Q_{\mathcal{E}}) = I(\mathbf{X}_1, \ldots, \mathbf{X}_m; Y) \tag{145}$$
$$= I(\mathbf{X}_1, \ldots, \mathbf{X}_m, U; Y) \tag{146}$$
$$= I(U;Y) + I(\mathbf{X}_1, \ldots, \mathbf{X}_m; Y \,|\, U) \tag{147}$$
$$= I(U;Y) + I(\mathbf{X}_1, \ldots, \mathbf{X}_m; Y \,|\, U = 0)\Pr[U = 0] + I(\mathbf{X}_1, \ldots, \mathbf{X}_m; Y \,|\, U = 1)\Pr[U = 1] \tag{148}$$
$$\leq \log 2 + I(\mathbf{X}_1, \ldots, \mathbf{X}_m; Y \,|\, U = 0) + \mu I(\mathbf{X}_1, \ldots, \mathbf{X}_m; Y \,|\, U = 1) \tag{149}$$
$$\leq \log 2 + \mathsf{C}_{\text{MISO,av},n_{\text{T}}}(\mathcal{E}_0) + \mu\mathsf{C}_{\text{MISO,av},n_{\text{T}}}\left(\frac{\mathcal{E}}{\mu}\right). \tag{150}$$

Here (149) follows because U is a binary random variable and because $\Pr[U = 0] \leq 1$. To justify the subsequent inequality (150), we note that because conditional on $U = 0$, $\|\mathbf{X}_i\|^2 < \frac{\mathcal{E}_0}{m}$ for all i, i.e.,

$$\mathsf{E}\left[\sum_{i=1}^{m}\|\mathbf{X}_i\|^2 \,\middle|\, U = 0\right] \leq \mathcal{E}_0 \tag{151}$$

we can upper-bound the MAC situation by full-cooperation MISO:

$$I(\mathbf{X}_1,\ldots,\mathbf{X}_m;Y\,|\,U=0)\le \mathsf{C}_{\text{MISO,av},n_\text{T}}(\mathcal{E}_0). \tag{152}$$

Moreover, by total expectation,

$$\mathsf{E}\left[\sum_{i=1}^m \|\mathbf{X}_i\|^2\right] = \mu\mathsf{E}\left[\sum_{i=1}^m \|\mathbf{X}_i\|^2\,\middle|\, U=1\right] + \underbrace{(1-\mu)\mathsf{E}\left[\sum_{i=1}^m \|\mathbf{X}_i\|^2\,\middle|\, U=0\right]}_{\ge 0} \tag{153}$$

$$\ge \mu\mathsf{E}\left[\sum_{i=1}^m \|\mathbf{X}_i\|^2\,\middle|\, U=1\right] \tag{154}$$

from which follows that

$$\mathsf{E}\left[\sum_{i=1}^m \|\mathbf{X}_i\|^2\,\middle|\, U=1\right] \le \frac{\mathsf{E}\left[\sum_{i=1}^m \|\mathbf{X}_i\|^2\right]}{\mu} \le \frac{\mathcal{E}}{\mu} \tag{155}$$

and hence, again allowing full-cooperation,

$$I(\mathbf{X}_1,\ldots,\mathbf{X}_m;Y\,|\,U=1)\le \mathsf{C}_{\text{MISO,av},n_\text{T}}\left(\frac{\mathcal{E}}{\mu}\right). \tag{156}$$

Next, let $\mathcal{E}_n$ be a sequence with $\mathcal{E}_n\uparrow\infty$, let $\{Q_{\mathcal{E}_n}\}_n$ be a family of joint input distributions for the multiple-access Rician fading channel (3) such that

$$\lim_{n\uparrow\infty}\frac{I(Q_{\mathcal{E}_n})}{\log\log\mathcal{E}_n}=1 \tag{157}$$

and define

$$\mu_n \triangleq Q_{\mathcal{E}_n}\left(\left\{\|\mathbf{X}_1\|^2\ge\frac{\mathcal{E}_0}{m}\right\}\cup\cdots\cup\left\{\|\mathbf{X}_m\|^2\ge\frac{\mathcal{E}_0}{m}\right\}\right). \tag{158}$$

By contradiction, assume $\mu_n\to\mu^*<1$. Then there must exist some $\mu_0<1$ such that

$$\mu_n<\mu_0,\quad n \text{ sufficiently large.} \tag{159}$$

From (150) we have

$$\underbrace{\frac{I(Q_{\mathcal{E}_n})}{\log\log\mathcal{E}_n}}_{\to 1} \le \underbrace{\frac{\log 2+\mathsf{C}_{\text{MISO,av},n_\text{T}}(\mathcal{E}_0)}{\log\log\mathcal{E}_n}}_{\to 0} + \underbrace{\frac{\mathsf{C}_{\text{MISO,av},n_\text{T}}\left(\frac{\mathcal{E}_n}{\mu_n}\right)}{\log\log\left(\frac{\mathcal{E}_n}{\mu_n}\right)}}_{\to 1}\cdot\frac{\mu_n\log\log\left(\frac{\mathcal{E}_n}{\mu_n}\right)}{\log\log\mathcal{E}_n}. \tag{160}$$

Here the limiting behavior of the left-hand side (LHS) follows from (157); the limiting behavior of the first term on the RHS is because $\mathsf{C}_{\text{MISO,av},n_\text{T}}(\mathcal{E}_0)<\infty$; the second term on the RHS tends to one because $\mathcal{E}_n\uparrow\infty$ implies $\mathcal{E}_n/\mu_n\uparrow\infty$ and because of (38). Hence, when $n\uparrow\infty$ we obtain

$$1\le \varliminf_{n\uparrow\infty}\frac{\mu_n\log\log\left(\frac{\mathcal{E}_n}{\mu_n}\right)}{\log\log\mathcal{E}_n} \tag{161}$$

$$\le \varliminf_{\mathcal{E}\uparrow\infty}\left\{\sup_{\mu\in(0,\mu_0]}\frac{\mu\log\log\left(\frac{\mathcal{E}}{\mu}\right)}{\log\log\mathcal{E}}\right\} \tag{162}$$

where the first inequality follows from (160) and the second inequality follows from (159). This, however, is a contradiction to the fact that

$$\varlimsup_{\mathcal{E}\uparrow\infty}\left\{\sup_{\mu\in(0,\mu_0]}\frac{\mu\log\log\left(\frac{\mathcal{E}}{\mu}\right)}{\log\log\mathcal{E}}\right\}<1,\quad\forall\,0<\mu_0<1. \tag{163}$$

Hence, we must have that $\mu_n \to 1$, which proves the claim.

D Proof of Lemma 9

The derivation of this result is based on (114). We start by bounding the following expressions:

$$\mathsf{E}\left[\log\left(\|\mathbf{X}\|^2+\sigma^2\right)\right]\geq\log\sigma^2 \tag{164}$$

$$\mathsf{E}\left[\|\mathbf{X}\|^2+\sigma^2+|\mathbf{d}^\mathsf{T}\mathbf{X}|^2\right]\leq\mathcal{E}+\sigma^2+\|\mathbf{d}\|^2\mathcal{E} \tag{165}$$

$$(1-\alpha)\epsilon_\nu\leq\epsilon_\nu \tag{166}$$

and

$$\mathsf{E}\left[\log\left(\frac{|\mathbf{d}^\mathsf{T}\mathbf{X}|^2}{\|\mathbf{X}\|^2+\sigma^2}\right)-\mathrm{Ei}\left(-\frac{|\mathbf{d}^\mathsf{T}\mathbf{X}|^2}{\|\mathbf{X}\|^2+\sigma^2}\right)\right]\geq-\gamma. \tag{167}$$

Here (165) follows from Cauchy-Schwarz (58) and the fact that the input needs to satisfy the power-sharing average-power constraint (13); and (167) follows because $\log\xi-\mathrm{Ei}\left(-\xi\right)\geq-\gamma$ where $\gamma\approx 0.57$ denotes Euler's constant.

Plugging these bounds into (114) and applying once more Jensen's inequality to $\xi\mapsto\log\xi-\mathrm{Ei}\left(-\xi\right)$ now yields

$$\begin{aligned}I(\mathbf{X};Y)\leq{}&\log\left(\mathsf{E}\left[\frac{|\mathbf{d}^\mathsf{T}\mathbf{X}|^2}{\|\mathbf{X}\|^2}\right]\right)-\mathrm{Ei}\left(-\mathsf{E}\left[\frac{|\mathbf{d}^\mathsf{T}\mathbf{X}|^2}{\|\mathbf{X}\|^2}\right]\right)-1\\&+\alpha\bigl(\log\beta-\log\sigma^2+\gamma\bigr)+\log\Gamma\left(\alpha,\frac{\nu}{\beta}\right)+\epsilon_\nu\\&+\frac{\nu}{\beta}+\frac{1}{\beta}\left((1+\|\mathbf{d}\|^2)\mathcal{E}+\sigma^2\right).\end{aligned} \tag{168}$$

We will now make the following choices of the free parameters α and β:

$$\alpha\triangleq\frac{\nu}{\log\left((1+\|\mathbf{d}\|^2)\mathcal{E}+\sigma^2\right)} \tag{169}$$

$$\beta\triangleq\frac{1}{\alpha}e^{\frac{\nu}{\alpha}} \tag{170}$$

for some constant $\nu\geq 0$. This leads to the following asymptotic behavior:

$$\varlimsup_{\mathcal{E}\uparrow\infty}\left\{\log\Gamma\left(\alpha,\frac{\nu}{\beta}\right)-\log\left(\frac{1}{\alpha}\right)\right\}=\log\left(1-e^{-\nu}\right) \tag{171}$$

$$\varlimsup_{\mathcal{E}\uparrow\infty}\alpha\bigl(\log\beta-\log\sigma^2+\gamma\bigr)=\nu \tag{172}$$

$$\varlimsup_{\mathcal{E}\uparrow\infty}\left\{\frac{1}{\beta}\bigl((1+\|\mathbf{d}\|^2)\mathcal{E}+\sigma^2\bigr)+\frac{\nu}{\beta}\right\}=0 \tag{173}$$

and

$$\varlimsup_{\mathcal{E}\uparrow\infty}\left\{\log\left(\frac{1}{\alpha}\right)-\log\left(1+\log\left(1+\frac{\mathcal{E}}{\sigma^2}\right)\right)\right\}=-\log\nu. \tag{174}$$

(Compare with [1, Appendix VII], [12, Sec. B.5.9].)

Hence, using the definition of the MAC fading number (42) and the definition of the sum-rate capacity (15), we have derived the following upper bound on the MAC fading number:

$$\begin{aligned}
\chi_{\text{MAC}} &= \varlimsup_{\mathcal{E}\uparrow\infty} \left\{ \mathsf{C}_{\text{MAC}}(\mathcal{E}) - \log\left(1+\log\left(1+\frac{\mathcal{E}}{\sigma^2}\right)\right)\right\} && (175)\\
&= \varlimsup_{\mathcal{E}\uparrow\infty} \left\{ \sup_{\substack{Q_{\mathbf{X}} \\ \text{independent users} \\ \text{power constraint (13)}}} I(\mathbf{X};Y) - \log\left(1+\log\left(1+\frac{\mathcal{E}}{\sigma^2}\right)\right)\right\} && (176)\\
&= \varlimsup_{\mathcal{E}\uparrow\infty} \left\{ \sup_{Q_{\mathcal{E}}\in\mathcal{A}} I(\mathbf{X};Y) - \log\left(1+\log\left(1+\frac{\mathcal{E}}{\sigma^2}\right)\right)\right\} && (177)\\
&\leq \varlimsup_{\mathcal{E}\uparrow\infty} \left\{ \sup_{Q_{\mathcal{E}}\in\mathcal{A}} \left\{ \log\left(\mathsf{E}\left[\frac{|\mathbf{d}^{\mathsf{T}}\mathbf{X}|^2}{\|\mathbf{X}\|^2}\right]\right) - \operatorname{Ei}\left(-\mathsf{E}\left[\frac{|\mathbf{d}^{\mathsf{T}}\mathbf{X}|^2}{\|\mathbf{X}\|^2}\right]\right) - 1\right\}\right.\\
&\qquad + \alpha\bigl(\log\beta - \log\sigma^2 + \gamma\bigr) + \log\Gamma\left(\alpha, \frac{\nu}{\beta}\right) + \epsilon_\nu + \frac{\nu}{\beta}\\
&\qquad \left. + \frac{1}{\beta}\bigl((1+\|\mathbf{d}\|^2)\mathcal{E} + \sigma^2\bigr) - \log\left(1+\log\left(1+\frac{\mathcal{E}}{\sigma^2}\right)\right)\right\} && (178)\\
&= \varlimsup_{\mathcal{E}\uparrow\infty} \sup_{Q_{\mathcal{E}}\in\mathcal{A}} \left\{ \log\left(\mathsf{E}\left[\frac{|\mathbf{d}^{\mathsf{T}}\mathbf{X}|^2}{\|\mathbf{X}\|^2}\right]\right) - \operatorname{Ei}\left(-\mathsf{E}\left[\frac{|\mathbf{d}^{\mathsf{T}}\mathbf{X}|^2}{\|\mathbf{X}\|^2}\right]\right) - 1\right\}\\
&\quad + \epsilon_\nu + \nu + \log\left(1-e^{-\nu}\right) - \log\nu. && (179)
\end{aligned}$$

Here, in (177) we make use of Proposition 7; (178) follows from (168); and the last equality is due to (171)–(174).

By letting ν tend to zero which makes sure that $\epsilon_\nu \to 0$ (as can be seen from (103) and (113)) the claim follows.

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