行政院國家科學委員會補助專題研究計畫期中進度報告

廣義的機會式通訊：無線行動網路中之競爭、合作與感知–子計畫四：
感知無線行動網路之合作式媒體存取控制協定設計與用戶/基地台
選取研究

計畫類別：整合型計畫
計畫編號：NSC 96－2628－E－009－004－MY3
執行期間：96 年 8 月 1 日至 97 年 7 月 31 日

計畫主持人：王蒞君教授
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中 華 民 國 97 年 5 月 29 日

中文摘要

在這一部份的研究中，我們考慮了結合解碼轉送 (decode-and-forward, DF) 與網路編碼 (network coding) 的合作式通訊系統 (cooperative communication systems)。對於我們提出的合作式網路編碼 (cooperative network coding, CNC) 協定，我們推導了中斷機率 (outage probability) 和分集-多工權衡 (diversity-multiplexing tradeoff, DMT)。我們的結果顯示中繼節點不但可以提供合作式的分集增益 (diversity gain)，也可以提供合作式的多工增益 (multiplexing gain)。

關鍵詞：合作式通訊，解碼轉送，分集多工增益，網路編碼，中斷機率

Abstract

In this work we consider a decode-and-forward (DF) cooperative communications system combined with the network coding. We derive the outage probability and diversity-multiplexing tradeoff (DMT) for the proposed cooperative network coding (CNC) protocol. Our results show that the relay nodes not only can provide cooperative diversity gain, but also cooperative multiplexing gain.

Keywords: Cooperative communications, decode-and-forward, diversity-multiplexing tradeoff, network coding, outage probability.

1 Introduction

Cooperative communication attracts a great deal of interests recently. Relay terminals in a cooperative communication system can help the transmitter send information to the receiver. This is similar to a virtual multiple-input multiple-output (MIMO) system because the terminals in a cooperative network form a virtual antenna array. Clearly, cooperative communication systems can provide diversity gains similar to the MIMO techniques.

Many cooperative communication protocols were proposed to improve diversity gain, such as orthogonal amplify and forward (OAF) [11], nonorthogonal amplify and forward (NAF) [2], space-time coded (STC) cooperative diversity protocols [12, 14, 16], dynamic decode-and-forward (DDF) [2], enhanced static decode-and-forward (ESDF), and enhanced dynamic decode-and-forward (EDDF) [17].

However, how to provide multiplexing gain by taking advantage of relays has not received much attention so far. Combining the network coding with the cooperative communications, or called the cooperative network coding (CNC) [1, 3-10, 13, 15, 18], have a potential to exploit the multiplexing gain in many relay nodes (virtual antennas). The diversity-multiplexing tradeoff (DMT) analysis of CNC has not been seen in the literature. Thus, it motivates us to derive the diversity-multiplexing tradeoff of the cooperative network coding.

The rest of our paper is organized as follows. In Section 2, we describe our system model and introduce the cooperative network coding protocol. We analyze the outage probability and diversity-multiplexing tradeoff of the cooperative network coding protocol in Section 3. Numerical results are shown in Section 4. We give our conclusions in Section 5.

2 System Model and CNC Protocol

Figure 1 shows the system model for the cooperative network coding with one relay node. Terminals A and B transmit and receive users' data, and relay R forward data. Denote the channel gains between nodes X and Y as h_{XY}, where $X,Y \in \{A,B,R\}$. In addition to additive white Gaussian noise, the radio channel effect h_{XY} experienced at each terminal is assumed to be independent and identically distributed (i.i.d.) complex normal random variables with zero mean and unit variance.

Consider the half-duplex terminals which cannot transmit and receive data simultaneously. As shown in the figure, terminals A and B can directly communicate with each other.

In the figure the cooperative network coding protocols are illustrated for the case with one relay node. In phase (1) and (2), A and B transmit information a and b, respectively. Then R decodes out a and b in the binary form and compute $a \oplus b$, where $\oplus$ is the bitwise exclusive or (XOR) operator. In phase (3), terminal R broadcasts the mixed information $a \oplus b$ to A and B. Then A can obtain information b via the operation $(a \oplus b) \oplus a = b$ and terminal B can obtain information a via the operation $(a \oplus b) \oplus b = a$. Thus the relay node here play a decode-and-forward (DF) [11] role.

3 Diversity-Multiplexing Tradeoff of CNC Protocol

3.1 Equivalent Signal Models

To begin with, the signals received by B and R in the first phase are modeled as

$$y_{B1}[n] = h_{AB} x_a[n] + z_{B1}[n]$$

and

$$y_{R1}[n] = h_{AR} x_a[n] + z_{R1}[n],$$

respectively, where $x_a[n]$ is the transmitted signal which contains information a from A. Similarly, in the second phase, the received signals at A and R is represented as

$$y_{A2}[n] = h_{AB} x_b[n] + z_{A2}[n]$$

and

$$y_{R2}[n] = h_{BR} x_b[n] + z_{R2}[n]$$

respectively, where $x_b[n]$ is the transmitted signal which contains information b from B. In the third phase, the received signals at A and B are

$$y_{A3}[n] = h_{AR} x_c[n] + z_{A3}[n]$$

and

$$y_{B3}[n] = h_{BR} x_c[n] + z_{B3}[n],$$

respectively, where $x_c[n]$ is the signal which contains information $c = a \oplus b$ from R. We model $z_{Xi}[n]$ as zero-mean mutually independent, circularly symmetric, complex Gaussian random sequences with variance N_0, where $X \in \{A, B, R\}$ and $i \in \{1,2,3\}$.

3.2 Parameterizations

In this subsection, we define signal to noise ratio (SNR), multiplexing gain r, and diversity gain d for the proposed cooperative network coding system. The SNR is defined as

$$\mathrm{SNR} := \frac{\mathrm{E}\{|h_{XY} x_k[n]|^2\}}{N_0},$$

where $X, Y \in \{A, B, R\}$, and $k \in \{a, b, c\}$. $\mathrm{E}\{Z\}$ is the expectation of a random variable Z.

Denote R as the data rate on each channel, where R can be a function of SNR if the communication system applies the channel-driven rate adaptation scheme. The multiplexing gain r is defined as

$$r := \lim_{\mathrm{SNR}\to\infty} \frac{R(\mathrm{SNR})}{\log \mathrm{SNR}}. \quad (1)$$

Note that the base of the $\log$ function is e in this paper. (2)

Let $P_{\mathrm{out}}(\mathrm{SNR})$ be the system outage probability as a function of SNR, which is defined as the probability of the maximum average mutual information I between input and output being less than the data rate R, i.e., (3)

$$P_{\mathrm{out}}(\mathrm{SNR}) := \mathrm{P}[I < R],$$

where $\mathrm{P}[E]$ denotes the probability of an event E. The diversity gain d is then defined as

$$d := -\lim_{\mathrm{SNR}\to\infty} \frac{\log[P_{\mathrm{out}}(\mathrm{SNR})]}{\log \mathrm{SNR}}. \qquad (1)$$

3.3 Diversity-Multiplexing Tradeoff Analysis

The analysis of the outage probability of the cooperative network coding protocol with one relay node at high SNR regime is given by the following theorem and proof:

Theorem 1 *The outage probability of the CNC protocol with one relay node at high SNR regime is characterized by*

$$P_{\mathrm{out}}^{\mathrm{CNC}} = \frac{1}{2}\Gamma(0,s)s^2 - K_1(2s)s - \frac{1}{2}e^{-s}(s+1)+1,$$

where $s = e^{3R/2}/\mathrm{SNR}$, $\Gamma(a,z)=\int_z^\infty t^{a-1}e^{-t}dt$ is the plica function, $K_1(z)$ is the modified Bessel function of the second kind and the first order.

Proof. First, the maximum average mutual information of the CNC protocol with one relay node can be seen as the sum of that of two different decode-and-forward protocols [11]:

$$I_{\mathrm{CNC}} = \frac{1}{3}\min\{\log(1+\mathrm{SNR}\,|h_{BR}|^2),$$

$$\log[1+\mathrm{SNR}(|h_{AR}|^2+|h_{AB}|^2)]\}$$

$$+\frac{1}{3}\min\{\log(1+\mathrm{SNR}\,|h_{AR}|^2),$$

$$\log[1+\mathrm{SNR}(|h_{BR}|^2+|h_{AB}|^2)]\}.$$

To ease the notation, let $x=|h_{AR}|^2, y=|h_{BR}|^2$, and $z=|h_{AB}|^2$. Then x, y, and z are i.i.d. exponential random variables with unit mean. The outage probability can be computed as

$$\begin{aligned}
&P_{\mathrm{out}}^{\mathrm{CNC}}\\
&= \mathrm{P}[I_{\mathrm{CNC}} < R]\\
&= \mathrm{P}[I_{\mathrm{CNC}} < R \mid y \le x+z, x \le y+z]\mathrm{P}[y \le x+z, x \le y+z]\\
&+ \mathrm{P}[I_{\mathrm{CNC}} < R \mid y > x+z, x \le y+z]\mathrm{P}[y > x+z, x \le y+z]\\
&+ \mathrm{P}[I_{\mathrm{CNC}} < R \mid y \le x+z, x > y+z]\mathrm{P}[y \le x+z, x > y+z]\\
&+ \mathrm{P}[I_{\mathrm{CNC}} < R \mid y > x+z, x > y+z]\mathrm{P}[y > x+z, x > y+z]\\
&=: \sum_{i=1}^{4} p_i q_i. \qquad (2)
\end{aligned}$$

Then

$$\begin{aligned}
p_1 &= \mathrm{P}\left[\frac{1}{3}\log(1+\mathrm{SNR}y)+\frac{1}{3}\log(1+\mathrm{SNR}x) < R\right]\\
&= \mathrm{P}[\log[(1+\mathrm{SNR}y)(1+\mathrm{SNR}x)] < 3R]\\
&= \mathrm{P}[(1+\mathrm{SNR}y)(1+\mathrm{SNR}x) < e^{3R}].
\end{aligned}$$

When SNR is high, $1+\mathrm{SNR}y$ and $1+\mathrm{SNR}x$ can be approximated as $\mathrm{SNR}y$ and $\mathrm{SNR}x$, respectively. Then (11)

$$\begin{aligned}
p_1 &= \mathrm{P}[\mathrm{SNR}^2 xy < e^{3R}]\\
&= \mathrm{P}[xy < e^{3R}/\mathrm{SNR}^2]\\
&= \int_0^\infty\int_0^\infty 1_{xy<s^2}\cdot e^{-x-y}dxdy\\
&= 1-2sK_1(2s),
\end{aligned}$$

where

$$1_E = \begin{cases} 1, & \text{if the expression } E \text{ is true,}\\ 0, & \text{if the expression } E \text{ is false.}\end{cases}$$

On the other hand,

$$q_1 = \int_0^\infty\int_0^\infty\int_0^\infty 1_{y\le x+z, x\le y+z}\cdot e^{-x-y-z}dxdydz = \frac{1}{2}.$$

Similarly, we have

$$p_2 = p_3 = 1-(1+s)e^{-s}+s^2\Gamma(0,s),$$

$$q_2 = q_3 = \frac{1}{4},$$

and (12)

$$q_4 = 0.$$

Combining the above equations into (2), we get the desired result.

According to the definition of the diversity gain in (1), we can find that the diversity-multiplexing tradeoff achieved by the cooperative network coding protocol with one relay node is characterized by

$$d_{\text{CNC}}(r) = 2 - 3r \qquad (21)$$

for $0 < r < \frac{2}{3}$.

4 Numerical Results

Figure 2 illustrates the diversity-multiplexing tradeoff comparison of the upper bound (UB), the CNC protocol for one relay node, selection decode-and-forward (SDF) [11], and DF. The upper bound is for a 2×1 MISO system, which is the best situation that an one-relay cooperative communications system could achieve.

From this figure, we can see that the CNC protocol improves both diversity gain and multiplexing gain compared with the DF protocol. The maximum diversity and multiplexing gain that the CNC protocol can achieve are 2 and 2/3, respectively, while the maximum diversity and multiplexing gain of the DF protocol are 1 and 1/2, respectively. Furthermore, the CNC protocol also outperforms the SDF protocol, which is an enhanced version of DF. Hence, we can conclude that using network coding at the relay node can improve not only diversity gain but also multiplexing gain.

5 Conclusions

In this paper, we investigate the diversity-multiplexing tradeoff for the cooperative network coding protocol which integrates the concept of DF relay transmission of cooperative communications with the information mixing of network coding. The proposed CNC protocol is suitable for two users which can transmit information to each other. We give a theorem to show our outage probability analytical result with proof and DMT comparison for our CNC protocol with upper bound, SDF, and DF. We find that the CNC protocol improves both diversity and multiplexing gain compared with the DF protocol.

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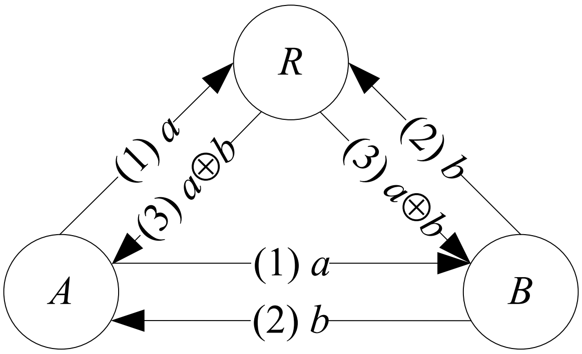

Figure 1: The system model and proposed CNC protocol, where phase (1): A sends a to B and R; phase (2): B sends b to A and R; phase (3): R broadcasts $a \oplus b$ to A and B.

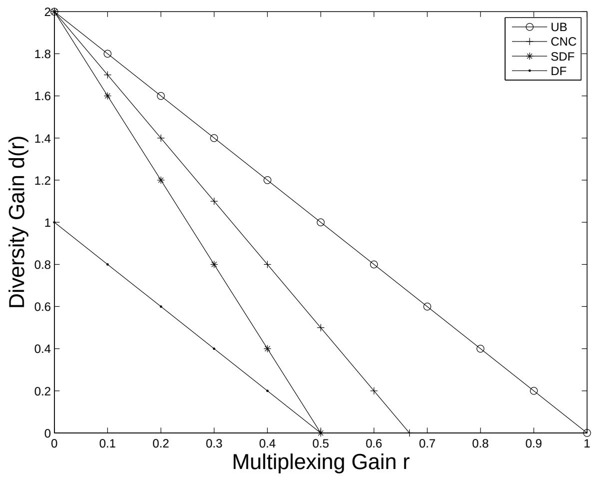

Figure 2: Diversity-multiplexing tradeoff comparison of the upper bound (UB), cooperative network coding (CNC), selection decode-and-forward (SDF), and decode-and-forward (DF).

中文摘要

我們推導出多重天線廣播系統在使用強制歸零 (zero-forcing) 前處理技術下的系統容量分析。近年來，多重天線廣播系統是一種廣泛討論的多重天線技術。其好處為透過前處理技術，可以同時傳送多筆不同資訊給不同的使用者。其中，強制歸零前處理則是一種簡單可實現卻擁有很好效能的一種技術。而過去已有許多對於多重天線廣播系統容量方面的研究，但大多着重於當使用者數目趨近無限大的容量 scaling。也就是去探討不同前處理技術在使用者數目趨近無限大時，其容量與理想的容量上界是否擁有相同的容量上升斜率(當訊雜比增加時)。在我們的研究中，我們探討在有限使用者下，使用強制歸零前處理技術之多重天線廣播系統的系統容量。我們探討了沒有考慮使用者排程(或等同於隨機排程)以及考慮了使用者排程的情況。其中，針對使用者排程的情況，我們發展了一種虛擬使用者方式的容量近似分析。而分析的結果也很接近模擬的結果。

關鍵詞：多重天線系統、強制歸零前處理、排程技術、多重天線廣播系統

Abstract

Besides delivering high data rates in a point-to-point scenario, multi-input multi-output (MIMO) antenna techniques can *broadcast* personalized data to multiple users in the point-to-multipoint scenario. Zero-forcing beamforming (ZFB) is a suboptimal but simple MIMO broadcast technique, which basically decouple the MIMO channel into many parallel single-input single-output (SISO) channels. In this article, we first derive the closed-form expression for the sum rate of the ZFB MIMO broadcast system with random user selection. Secondly, under the condition with finite users, we develop a virtual user approach approximation method for estimating the sum rate of the ZFB MIMO broadcast system with exhaustive user selection. Our results indicate that the proposed analysis method can accurately estimate the optimal sum-rate throughput of ZFB.

Keywords: MIMO systems, zero-forcing beamforming, scheduling, MIMO broadcast channels.

1. Introduction

In addition to enhancing the data rates in the point-to-point communication environment [1], multiple-input-multiple-output (MIMO) techniques play an important role in the point-to-multiple multiuser environment. More specifically, a MIMO system can transmit personalized data streams to multiple users concurrently. This kind of MIMO transmission is sometimes called the MIMO broadcasting technique [2]. However, it is a bit misleading to use the term broadcasting since broadcasting is used to send the *same* data to all the users in the system. To clarify, the term MIMO broadcast here implies that *different* personalized data streams are transmitted to a group of selected users.

Scheduling plays a key role in the multiuser MIMO system. Taking advantage of independent statistics in fading channels of multiple users, scheduling techniques can provide another form of diversity -- *multiuser diversity* [3]. If the system selects one user at the time, this kind of selection principle is called time division multiple access ZFB. (TDMA)-based scheduling. Unlike the TDMA principle, the MIMO broadcast system select a group of users and thus can achieve higher data rates, but it requires a huge amount of feedback information during scheduling.

Capacity analysis of multiuser MIMO broadcast channels with independent information is a very hot research area [2, 4-8]. The capacity region of the MIMO broadcast channel was derived in [4] [9] [10]. When the complete channel state information (CSI) is available at the transmitter, the sum rate of MIMO broadcast systems can be maximized by resorting to dirty paper coding (DPC) [2] [4] [9]. Although DPC is the optimal rate-achieving scheme, the complexity issue and the huge requirement of feedback information motivate a new line of research for other suboptimal MIMO broadcast transmission strategies, such as zero-forcing dirty-paper coding, orthogonal random beamforming and zero-forcing beamforming (ZFB) [6-8].

According to [6], ZFB with optimal user scheduling can achieve the same slope of throughput against SNR in dB as that for the capacity-achieving DPC strategy. In [8], it was shown that ZFB combined with multiuser scheduling can achieve the capacity asymptotically when the number of users approaches to the infinity. Because it is simpler than DPC and its asymptotical sum rate is the same as DPC, ZFB becomes an attractive MIMO broadcast technique. However, the performance analysis of ZFB MIMO broadcast system is still an open issue.

In this paper, we aim to evaluate the sum-rate performance of ZFB MIMO broadcast system with a finite number of users, rather than study the scalability of sum rate with an extremely large number of users as other existing work. We first develop analytical expressions for the sum rate of the ZFB MIMO broadcast system with random user selection. We also provide a virtual user approach approximation technique for evaluating the sum rate of the ZFB MIMO broadcast system with exhaustive user selection (the optimal selection policy).

The rest of this paper is organized as follows. In Section II, we describe the system model of MIMO downlink system and review ZFB scheme briefly. In Section III, we evaluate the sum rate of ZFB with random user selection first. Then, we propose an approximate analysis for the exhaustive user selection at both low and high SNRs in Section IV. Numerical results are presented in Section V. Finally, we give our concluding remarks in Section VI.

2. Background

2.1 System Model

Consider a MIMO downlink channel with a single base station and K users. The base station and each user terminal is equipped with M_T transmit antennas and single receive antennas, respectively. The base station can transmit M_T different data streams up to M_T users simultaneously. Let $x \in \mathrm{X}^{M_T \times 1}$ be the transmitted signal vector, $y \in \mathrm{X}^{M_T \times 1}$ the received signal vector, and H the $M_T \times M_T$ channel matrix. Denote $n \in \mathrm{X}^{M_T \times 1}$ as the circular complex additive white Gaussian noise vector with covariance matrix $E[nn^T] = \sigma^2 I_{M_T}$, where $(\cdot)^T$ represents the transpose conjugate operation and it is assumed $\sigma^2 = 1$ for simplicity. Then the received signal can be expressed as

$$y = Hx + n, \quad (1)$$

where the entries of $H \in \mathrm{X}^{M_T \times M_T}$ are Rayleigh fading channel element. Assume that all users experience independent fading and the transmit power is constrained by $E[x^T x] = P_T$.

2.2 Zero-Forcing Beamforming

The ZFB scheme aims to invert the channel matrix to create orthogonal channels between the transmitter and the receivers without the receiver's cooperation. The transmitted signal vector with beamforming weights can be written as $\mathbf{x} = \mathbf{W}\mathbf{u}$, where $\mathbf{W}$ is the $M_T \times M_T$ beamforming matrix and $\mathbf{u} \in \mathrm{X}^{M_T \times 1}$ is the input signal vector. Denote

$\Sigma \subset \{1,\ldots,K\}$, $|\Sigma| = M_T$ a subset of user indices to which a base station intends to transmit information and $\mathbf{H}(\Sigma)$ the channel matrix corresponding to Σ. Then the beamforming matrix $\mathbf{W}$ becomes

$$\mathbf{W} = \mathbf{H}(\Sigma)^T (\mathbf{H}(\Sigma)\mathbf{H}(\Sigma)^T)^{-1}. \quad (2)$$

In [2], under the assumption of perfect CSI at the transmitter side, the sum rate of the MIMO system with ZFB is given by

$$R_{ZFB}(\Sigma) = \sum_{i\in\Sigma} [\log(\mu b_i)]_+, \quad (3)$$

where $[x]_+$ represents $\max\{x,0\}$ and the effective channel gain b_i of i-th subchannel is

$$b_i = \frac{1}{\left[\left(\mathbf{H}(\Sigma)\mathbf{H}(\Sigma)^T\right)^{-1}\right]_{ii}}. \quad (4)$$

Note that $[A]_{ij}$ represents the (i,j)-th entry of the matrix A and μ is the water level satisfies the following criterion

$$\sum_{i\in\Sigma} [\mu - \frac{1}{b_i}]_+ = P_T. \quad (5)$$

2.3 Sum Rate with Long-term Power Constraint

Consider the sum rate performance of ZFB subject to a long-term power constraint. The average throughput is

$$\begin{aligned} R_{ZFB}(\Sigma) &= E\left[\sum_{i\in\Sigma} [\log(\mu_0 b_i)]_+\right] \\ &= \sum_{i\in\Sigma} E\left[\log(\mu_0 b_i)\right]_+ \\ &= \sum_{i\in\Sigma} \int_{1/\mu_0}^{\infty} \log(\mu_0 z) f_{b_i}(z) dz, \end{aligned} \quad (6)$$

where $f_{b_i}(z)$ is the probability density function (PDF) of b_i and μ_0 is the solution of the water-filling equation with respect to the long-term power constraint

$$E\left[\sum_{i\in\Sigma} [\mu_0 - \frac{1}{b_i}]_+\right] = \sum_{i\in\Sigma} E\left[\mu_0 - \frac{1}{b_i}\right]_+ = P_T. \quad (7)$$

2.4 Problem Formulation

In general, the number of users is greater than the number of transmit antenna, i.e., $K > M_T$. Thus, it is necessary to combine ZFB with scheduling in a multiuser MIMO system. It is shown that ZFB can achieve optimal throughput asymptotically for large K when the selected users are searched by an exhaustive method [8]. Denote the maximal sum rate with exhaustive user selection as

$$R_{ZFB}^{max} = \max_{\Sigma\subset\{1,\ldots,K\}:|\Sigma|=M_T} R_{ZFB}(\Sigma). \quad (8)$$

This is a combinatorial optimization problem to select the best one from $\binom{K}{M_T}$ combinations so that the explicit performance closed-form is difficult to be found. The goal of this paper is to develop an analytical approach for evaluating the sum rate of downlink ZFB MIMO broadcast systems. We consider user selection approaches: random user selection and exhaustive user selection.

3. Sum Rate Analysis With Random User Selection

Consider a simple round-robin (RR) scheduling policy, which selects users in turns and does not exploit the multiuser diversity gain. In this case, it is just like there are $K = M_T$ users with the Rayleigh fading channel vector. From observation on a point-to-point $M_T \times M_T$ MIMO system with ZF receiver, the effective channel gain in (4) has the same form as the ZF receiver's substream effective channel gain. Due to the same statistics we can see the system as a virtual $M_T \times M_T$ MIMO system with ZF receiver. Therefroe, the PDF of effective channel gain b_i in (4) can be obtained through the PDF of the ZF receiver's substream SNR. According to [11] [12], the substream SNRs $\{\gamma_i\}_{i=1}^{M_T}$ for an $M_T \times M_R$

MIMO system with ZF receiver under the equal power allocation principle are independent and identically distributed (i.i.d.) χ^2 distributed random variables with $2(M_R - M_T + 1)$ degrees of freedom, i.e.

$$f_\gamma(\gamma) = \frac{M_T e^{-M_T\gamma/\rho}}{\rho(M_R - M_T)!}\left(\frac{M_T}{\rho}\gamma\right)^{M_R - M_T}, \quad \gamma \geq 0, \tag{9}$$

where $\rho = P_T/\sigma^2$ represents the average received SNR and $\gamma_i = \rho b_i/M_T = \gamma$ ($i = 1,\ldots,M_T$). Note that the substream SNR performance in (9) is under the assumption of independent decoding [12]. The PDF of the effective channel gain b_i can be obtained from (9) by letting $M_R = M_T$ and be an exponential distribution with parameter one. With the unordered i.i.d. effective channel gain $\{b_i\}_{i=1}^{M_T}$, the long-term water-filling equation in (7) becomes

$$\begin{aligned}\sum_{i=1}^{M_T} E\left[\mu_0 - \frac{1}{b_i}\right]_+ &= M_T\int_{1/\mu_0}^{\infty}\left(\mu - \frac{1}{z}\right)e^{-z}dz \\ &= M_T\left(\mu_0 e^{-1/\mu} + E_i\left(-\frac{1}{\mu_0}\right)\right) \\ &= P_T,\end{aligned} \tag{10}$$

where $E_i(x) = -\int_{-x}^{\infty} e^{-t}/t dt$ is the exponential integral function. The resulting average sum rate of ZFB with random user selection is given by

$$\begin{aligned}R_{ZFB} &= \sum_{i=1}^{M_T} E\left[\log(\mu_0 b_i)\right]_+ \\ &= M_T\int_{1/\mu_0}^{\infty}\log(\mu_0 z)e^{-z}dz \\ &= M_T\Gamma\left(0, \frac{1}{\mu_0}\right),\end{aligned} \tag{11}$$

where μ_0 be the solution of the water-filling equation for the long-term power constraint in (10) and $\Gamma(a,x) = \int_x^{\infty} t^{a-1}e^{-t}dt$ is the incomplete gamma function [13]. Note that $\Gamma(0,x) = -E_i(-x)$ for $x > 0$ when x is real.

4 Sum Rate Analysis With Exhaustive User Selection

Motivated by the observation in [14], we translate the MIMO broadcast system with M_T transmit antennas at the base station and M_T users with single antenna into the MIMO TDMA-based scheduling system with M_T transmit antennas at the base station and M_T receive antennas per user. In the reformulated scenario, there are $\binom{K}{M_T} = \frac{K!}{(K-M_T)!M_T!}$ virtual users, each of which has M_T receive antennas and a ZF MIMO receiver as shown in Fig 1. The maximal sum rate can be obtained by selecting the best transmission pair from $\binom{K}{M_T}$ combinations Furthermore, the TDMA-based scheduling for the reformulated scenario adopt the scalar feedback, meaning that only a scalar value is sent back from each receiver to base station. From the result in [14], the max-max and max-min scheduling schemes can approach the maximal sum rate for a scalar feedback MIMO system at low SNR and high SNRs, respectively. The basic principles for approximating the $M_T \times 1$ antenna system for M_T users based on broadcasting by the $M_T \times M_T$ antenna system for one user based on the TDMA scheduling are described as follows:

- At the low SNR region, because the property of the logarithm function $\log_2(1+x) \approx x\log_2 e$ for $x \approx 0$, the ideal policy for achieving the maximal sum rate for the point-to-point TDMA-based scheduling is to find a user having the maximal strongest subchannel and to allocate all power only to the strongest subchannel, i.e., a user with the best effective channel gain will most likely be selected. This principle coincides with the max-max scheduling scheme.

• At the high SNR region, the property of the logarithm function is $\log_2(1+x) \approx \log_2 x$ for $x ? 1$. Therefore, the maximal sum rate will be caused by improving all subchannels with suitable scheduling gains and be allocated corresponding power. It is implies that no subchannel will be omitted in each scheduling run. From [14] and [15], the max-min scheduling scheme provides uniformly scheduling gains for all subchannels and has close rate performance compared to the maximal sum rate. Thus we use it to approach maximal sum rate in high SNR.

4.1 Low SNR Region

The max-max scheduling algorithm selects the target user with the maximal strongest subchannel among virtual $\begin{pmatrix} K \\ M_T \end{pmatrix}$ users at each time slot. Denote $\{b_i^k\}_{i=1}^{M_T}$ the set of all subchannel effective channel gains for k th virtual user ($k=1,\ldots,Z=\begin{pmatrix} K \\ M_T \end{pmatrix}$) and $b_{1:M_T}^k \le \ldots \le b_{M_T:M_T}^k$ as ordered effective channel gains. With the information of $\{b_{M_T:M_T}^k\}_{k=1}^Z$ from all users, the transmitter chooses the target user according to

$$k^* = \arg\max_k b_{M_T:M_T}^k . \tag{12}$$

After determining the target user $k*$, we have

$$\tilde{b}_{i:M_T}^{max} = b_{i:M_T}^{k^*} \ for i = 1,\ldots,M_T \tag{13}$$

where the superscript *max* denotes the max-max scheduling. Based on the order statistics analysis proposed in [11], we can obtain the PDFs of $\{\tilde{b}_{i:M_T}^{max}\}_{i=1}^{M_T}$ as follows:

$$f_{\tilde{b}_{M_T:M_T}^{max}}(b_{M_T}) = ZM_T e^{-b_{M_T}}\left(1-e^{-b_{M_T}}\right)^{ZM_T-1}, \tag{14}$$

and

$$f_{\tilde{b}_{i:M_T}^{max}}(b_i) = \frac{ZM_T(M_T-1)!}{(i-1)!(M_T-i-1)!}\sum_{a_1=0}^{i-1}\begin{pmatrix} i-1 \\ a_1 \end{pmatrix} \cdot \sum_{a_2=0}^{M_T(Z-1)}\begin{pmatrix} M_T(Z-1) \\ a_2 \end{pmatrix} e^{-(a_1+a_2+1+M_T-i)b_i} \cdot \sum_{a_3=0}^{M_T-i-1}\begin{pmatrix} M_T-i-1 \\ a_3 \end{pmatrix}\frac{(-1)^{a_1+a_2+a_3}}{a_2+a_3+1}. \tag{15}$$

As a result, from (14) and (15), we have all the PDFs of $\tilde{b}_{i:M_T}^{max}$ for $i=1,\cdots,M_T$. Applying (14) and (15) to (6) and (7), we can obtain the sum-rate throughput and long-term power constraint equation, respectively.

4.2 High SNR Region

Different from the max-max scheme, the max-min scheduling selects the target user according to the maximal weakest subchannel among the Z virtual users. Based on the information of $\{b_{1:M_T}^k\}_{k=1}^Z$, the base station arranges the transmission during each time slot according to

$$k^* = \arg\max_k b_{1:M_T}^k . \tag{16}$$

Once the target user k^* is selected, we have

$$\tilde{b}_{i:M_T}^{min} = b_{i:M_T}^{k^*} \ for i = 1, \ldots, M_T \qquad (17)$$

where the superscript *min* indicates the max-min scheduling. Similarly, we can get the PDFs of $\{\tilde{b}_{i:M_T}^{min}\}_{i=1}^{M_T}$ based on the analysis in [11] as follows:

$$f_{\tilde{b}_{1:M_T}^{min}}(b_1) = ZM_T e^{-b_1 M_T}\left(1 - e^{-b_1 M_T}\right)^{Z-1}, \qquad (18)$$

and

$$f_{\tilde{b}_{i:M_T}^{min}}(b_i) = \frac{ZM_T \prod_{j=2}^{i}(M_T - j + 1)}{(i-2)!} \sum_{a_1=0}^{Z-1}$$

$$\cdot \sum_{a_2=2}^{i} \frac{(-1)^{i+a_1+a_2}\binom{Z-1}{a_1}\binom{i-2}{a_2-2}}{(M_T + M_T a_1 + a_2 - M_T - 1)}$$

$$\cdot \left[e^{-b_i(M_T - a_2 + 1)} - e^{-b_i(M_T + a_1 M_T)}\right] \qquad (19)$$

After obtaining all the PDFs of $\{\tilde{b}_{i:M_T}^{min}\}_{i=1}^{M_T}$, we can obtain the sum rate by applying (18) and (19) to (6) and long-term power constraint (7).

5 Numerical Results

First, we verify the assumption that the PDF of the ZFB multiuser scheduling system can be approximated by the PDF of the ZF receiver according to (9). Figure 2 depicts the PDF of unordered the effective channel gain b_i for $M_T = 4$ by simulations and the analytical result from (9). For simulations, four users are selected randomly from the entire group of the users to obtain its value of b_i. We repeat this procedure 10,000 times to get the simulated PDF of b_i. As shown in the figure, the PDF of b_i can be closed matched by the PDF of the ZF receiver of (9). Figure 3 shows the sum rate of the ZFB with random user selection for transmit antennas $M_T = 3$ and 4. One can see that the sum rate obtained by simulations matches the value obtained by (11) well.

Figure 4 shows the sum rate of ZFB in the low SNR region $-20:\ 0$ dB by the exhaustive user selection algorithm with $K = 10$ users. The sum rate for the multiuser scheduling with the ZF-based MIMO receivers according to the max-max scheduling approach is also shown in the figure for comparison. From the figure, the sum rate of the ZF based on the max-max selection criterion is very close to that of the exhaustive search specially from -20 to -5 dB. Furthermore, for comparison, the sum rate with random user selection is also shown in the figure. For SNR $= -5$ dB, the exhaustive search can provide the sum rate of 1 nats/Hz/sec, while the random user selection can only provide 0.4 nats/Hz/sec.

By contrast, Fig. 5 compares the sum rates of the exhaustive user selection with approximately max-min approach in the high SNR region $0:\ 20$ dB. It is shown that the sum rate performance of the max-min approach match the simulation result well. For SNR $= 5$ dB, the sum rate of ZFB with exhaustive search is about 4 nats/Hz/sec and that of ZFB with random user selection is 2 nats/Hz/sec.

Basically, Figs. 4 and 5 show that the gains of multiuser diversity is significant even if the degrees of freedom is merely $K = 10$. More importantly, one can observe that at low SNRs a ``max-max" scheduling strategy is close to being optimal in an achievable sum rate sense, while at high SNRs, the ``max-min" scheduling strategy is not far from being optimal.

6. Conclusion

In this paper, we evaluated the sum rate of ZFB MIMO broadcast systems in the Rayleigh fading channel. An analytical expression for the sum rate of the ZFB MIMO broadcast systems with random user selection is presented. Since the closed-form expression for the ZFB MIMO with exhaustive user selection is difficult to obtain, we develop an approximation method based on the maximal sum rate of the MIMO TDMA-based scheduling systems with the max-max and max-min scheduling at the low SNR and high SNR regions, respectively. Our results show that the proposed analytical method can accurately estimate the maximum sum rate of the ZFB MIMO broadcast system with exhaustive user selection. Besides the sum rate issue, the relationship of link reliability and coverage performance is also importance in practice. The future research direction will explore the cell coverage performance for MIMO broadcast systems.

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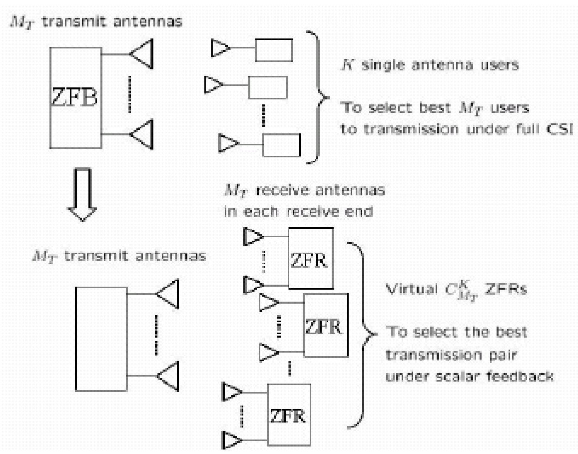

Figure 1: The modified problem model for exhaustive search.

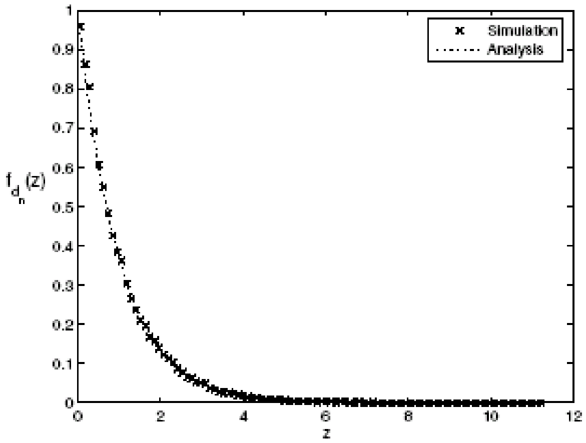

Figure 2: The PDF of unordered effective channel gain b_i

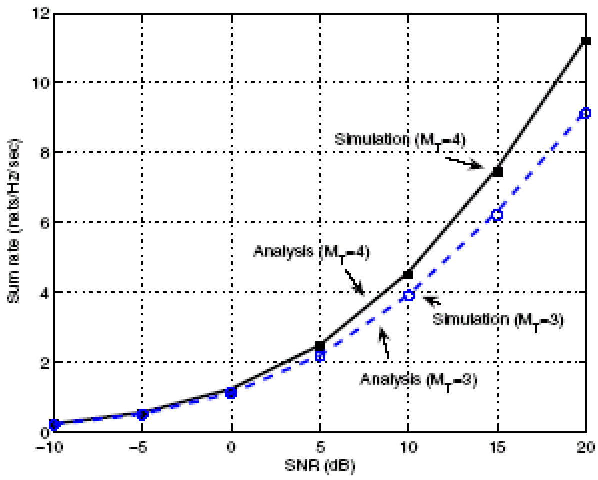

Figure 3: Sum-rate capacity of the ZFB with random user selection for the number of transmit antennas $M_T = 3$ and 4.

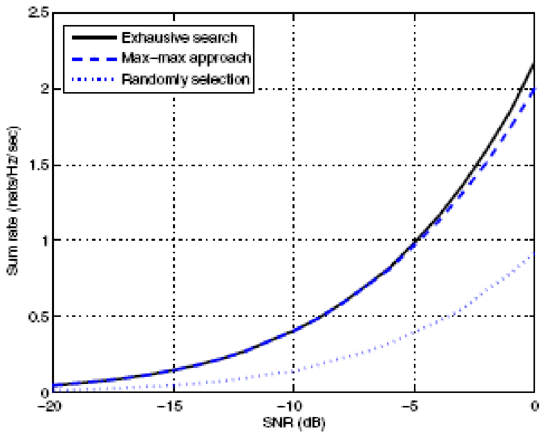

Figure 4: Sum rates of ZFB with exhaustive user selection and approximate analysis by the max-max approach for $K = 10$ users with the number of transmit antennas $M_T = 3$ in low SNR region $-20 : \ 0$ dB.

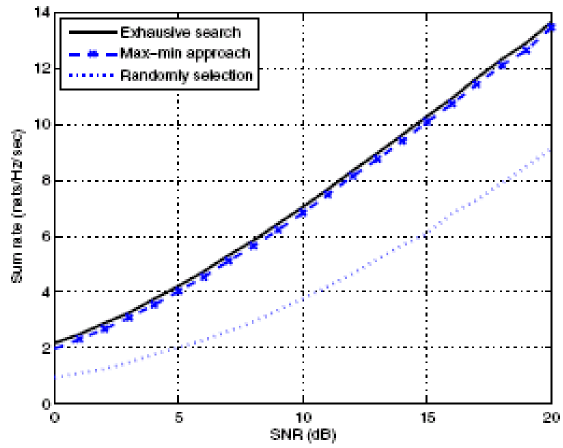

Figure 5: Sum rates of ZFB with exhaustive user selection and approximate analysis by the max-min approach for $K = 10$ users with the number of transmit antennas $M_T = 3$ in high SNR region $0 : \ 20$ dB.

中文摘要

我們從系統設計的角度來看多重天線廣播技術。不同於過去及第二部分著重於系統的容量分析，在此我們探討多重天線廣播系統的涵蓋範圍。我們從分析多重天線廣播系統的傳輸中斷機率，進而分析出傳輸連線能提供可信賴傳送的涵蓋範圍。其中，可信賴傳送代表著連線能在一定機率下，達到所需的訊雜比臨界值。在這一部份我們著重於兩種著名的傳送前處理技術：強制歸零髒紙編碼前處理及強制歸零前處理。當沒有考慮使用者排程時，我們提供了兩種前處理技術的連線中斷機率與涵蓋範圍的分析。當考慮了使用者排程，我們對強制歸零髒紙編碼前處理進行分析，並對強制歸零前處理做模擬的比較。我們也探討了不同系統參數如傳送功率與通道衰減因子對於連線中斷機率與涵蓋範圍的影響。

關鍵詞：多重天線系統、強制歸零前處理、強制歸零髒紙編碼前處理、多重天線廣播系統

Abstract

We consider the downlink of a multiuser multi-input multi-output (MIMO) broadcast channel under a single cell structure. To study the achievable link coverage performance of zero-forcing beamforming (ZFB) and zero-forcing dirty-paper coding (ZF-DPC) when the channel state information (CSI) is available to the transmitter. First we develop analytical closed-form expressions for the link outage probability and coverage reliability of baseline ZFB and ZF-DPC when no multiuser scheduling involved. We find that the coverage performance of ZFB can only approach to that of the weakest link of ZF-DPC under predetermined required SNR and outage probability. Secondly, for exploring the achievable cell coverage, we discuss the strongest link performance of both broadcast beamforming schemes under multiuser scheduling. Under a framework of analysis for ZF-DPC and simulation for ZFB, we show that a soft coverage enhancement can be easily done by using scheduling techniques without extra hardware power consumption.

Keywords: MIMO systems, zero-forcing beamforming, zero-forcing dirty paper coding, coverage, MIMO broadcast channels

1. Introduction

Multiple-input multiple-output (MIMO) systems can significantly increase the spectral efficiency by exploiting the spatial degrees of freedom created by multiple antennas. In point-to-point MIMO system, it is well known that the capacity increases linearly with the minimum of the number of transmit and receive antennas [1] [2]. In the MIMO broadcast channels, [3] shows that higher capacity can be obtained by exploiting the spatial multiplexing of transmit antennas to multiple users simultaneously rather than to maximize the capacity of a single-user link.

Capacity analysis of multiuser MIMO broadcast channels is a very hot research area [3-9]. When the complete channel state information (CSI) is available at the transmitter, the maximum sum rate of MIMO broadcast systems can be achieved by dirty paper coding (DPC) [3] [4]. Although DPC is the optimal rate-achieving scheme, the applicability is limited due to its computational complexity and the need for full CSI at the transmitter (CSIT). It motivates a new line of research for other suboptimal MIMO broadcast transmission strategies, such as zero-forcing dirty-paper coding (ZF-DPC), zero-forcing beamforming (ZFB), orthogonal random beamforming and orthogonal linear beamforming [6-9]. Those suboptimal schemes can achieve the same throughput of DPC asymptotically when the number of users approaches to the infinity. However, The capacity gain of multiuser MIMO broadcast system is highly dependent on the availability of CSIT.

Due to practicality, finite rate feedback have become a popular research area recently. Limited feedback was first considered for point-to-point MIMO system in [10] [11]. While in point-to-point case, CSIT contributes little to achieving the multiplexing gain but it is crucial for broadcast channels. For MIMO broadcast channels, the feedback load per user must be scaled with both the number of transmit antennas as well as the system SNR in order to achieve the full multiplexing gain with near-perfect CSI performance [12].

Besides dealing with practical feedback problems, some references begin to apply broadcast transmission strategies from a downlink single-cell setup to multi-cell scenarios [13] [14]. The main goal of using broadcast techniques combining with base station cooperation for multi-cell environment is to mitigate inter-cell interference for improving spectral efficiency. In [14] [15], a network coordination conception is proposed based on ZFB and ZF-DPC schemes. [16] analyzed the sum-rate performance of multi-cell cooperative ZFB under a Wyner downlink channel setup which is a simplified cellular model proposed in [17].

From both limited feedback transmission and coordinated network researches, we know that broadcast transmission techniques may play an important role in future increasingly high-speed wireless environment. However, most of the works focus on sum-rate sense performance. Based on development of coordinated network, we find that some research directions begin to move from single-cell to multi-cell setup. The link quality and achievable link coverage of broadcast transmission techniques are still open issues for baseline single-cell setup. In this paper, we aim to derive analytical closed-form expressions for the link outage probability and coverage reliability of the single-cell multiuser MIMO broadcast system. We focus on the two popular schemes: ZF-DPC and ZFB.

2. Background

2.1 System Model

We consider a single-cell multiuser MIMO broadcast system with a base station and K user. The base station is equipped with N_t transmit antennas, but all K user terminals with single receive antenna. Since the base station has N_t transmit antenna, N_t users can be selected among K users for simultaneous transmission with different data streams. Denote $\Sigma \subset \{1,\ldots,K\}$, $|\Sigma| = N_t$ a subset of user indices to which a base station intends to transmit different information.

Using linear spatial processing at the transmitter. Denote $\mathbf{W} = [\mathbf{w}_1 \ldots \mathbf{w}_{N_t}]$ as the linear beamforming weight matrix and $\mathbf{u} = [\sqrt{P_1}u_1,\ldots,\sqrt{P_{N_t}}u_{N_t}]^T$ the input signal vector, where P_i represents the power allocated for i th antenna, u_i represents uncorrelated unit-power signal symbol and $(\cdot)^T$ represents the transpose conjugate operation. Then the transmitted signal vector $\mathbf{x} = \mathbf{W}\mathbf{u} = \sum_{i=1}^{N_t}\sqrt{P_i}\mathbf{w_i}u_i$. Let $y \in \mathrm{X}^{N_t \times 1}$ the received signal vector, and $\mathbf{G}(\Sigma)$ the $N_t \times N_t$ channel matrix corresponding to Σ. Denote $n \in \mathrm{X}^{N_t \times 1}$ as the circular complex additive white Gaussian noise vector with covariance matrix $E[nn^T] = \sigma^2 I_{N_t}$. Then the received signal can be expressed as

$$y = \mathbf{G}(\Sigma)x + n = g\mathbf{H}(\Sigma)x + n, \qquad (1)$$

where

$$g = \begin{bmatrix} \sqrt{g_1} & 0 & \cdots & 0 \\ 0 & \sqrt{g_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sqrt{g_{N_t}} \end{bmatrix}, \qquad (2)$$

and g_i depicts the large-scale slowly-varying behavior of the local average channel gain between the i th transmit antenna and the corresponding user terminal. For a user at a distance of R from the base station, g_i can be written as [18]

$$10\log_{10} g_i = -10\mu \log_{10} R + g_0[dB], \quad (3)$$

where μ is the path loss exponent and g_0 is a constant subject to certain path loss models. Assume that all users experience independent flat fading and the entries of $\mathbf{H}(\Sigma) \in X^{N_t \times N_t}$ are Rayleigh fading channel element. The transmit power is constrained by $E[x^T x] = P_T$.

2.2 Zero-Forcing Dirty-Paper Coding

In [3], an intuitive suboptimal solution of the weight matrix $\mathbf{W}$ based on QR-type decomposition have been proposed. Let $\mathbf{H}(\Sigma) = \mathbf{LQ}$ be the QR-type decomposition obtained by applying Gram-Schmidt orthogonalization to the rows of $\mathbf{H}(\Sigma)$, where $\mathbf{L}$ is a lower triangular matrix and $\mathbf{Q}$ has orthonormal rows. By letting $\mathbf{W} = \mathbf{Q}^T$, the corresponding system model in (1) is given by

$$y_i = l_{i,i}\sqrt{g_i P_i}u_i + \sum_{j<i} l_{i,j}\sqrt{g_j P_j}u_j + n_i, = 1, \ldots, N_t \quad (4)$$

Note that the particular choice of the weight matrix $\mathbf{W} = \mathbf{Q}^T$ nulls out the interference from users with indices $j > i$. The remaining interference terms from terminals with indices $j < i$ are taken care of by successive application of DPC. For simplicity, we consider equal power allocation, that is, $P_i = P_T/N_t$ for $i = 1, \ldots, N_t$. Therefore, the rate of i th link for ZF-DPC is $\log(1 + |l_{i,i}|^2 \rho_i) = \log(1 + \gamma_i)$, where ρ_i is the average receive SNR shown as follows

$$\rho_i = \frac{P_T g_i}{N_t \sigma^2} = \frac{P_T 10^{g_0/10}}{N_t \sigma^2 R_i^{\mu}}, \qquad (5)$$

and γ_i is the effective receive SNR. The term $|l_{i,i}|^2$ can be viewed as effective channel gain of i th stream link.

2.3 Zero-Forcing Beamforming

The ZFB scheme [3] aims to invert the channel matrix to create orthogonal channels between the transmitter and the receivers without the receiver's cooperation. The beamforming weight matrix is

$$\mathbf{W} = \mathbf{H}(\Sigma)^T (\mathbf{H}(\Sigma)\mathbf{H}(\Sigma)^T)^{-1}. \qquad (6)$$

Then, the corresponding system model in (1) is given by

$$\begin{aligned} \mathbf{y} &= \mathbf{gH}(\Sigma)\mathbf{H}(\Sigma)^{\mathbf{T}}(\mathbf{H}(\Sigma)\mathbf{H}(\Sigma)^{\mathbf{T}})^{-1}\mathbf{u} + \mathbf{n} \\ &= \mathbf{gu} + \mathbf{n}, \end{aligned} \qquad (7)$$

and the i th receive signal $y_i = \sqrt{g_i P_i}u_i + n_i$.

Due to the transmit power constraint $E[\mathbf{x}^T\mathbf{x}] \le P_T$, we have the following relation:

$$\|\mathbf{w}_1\|^2 P_1 + \ldots + \|\mathbf{w}_{N_t}\|^2 P_{N_t} \le P_T, \ldots \qquad (8)$$

where $\mathbf{w}_i$ is the i th column of $\mathbf{W}$ and $\|\mathbf{w}_i\|^2 = [(\mathbf{HH}^T)^{-1}]_{i,i}$. Equation (8) implies that ZFB incurs an excess transmission power penalty due to the required interference cancellation power of weight matrix $\mathbf{W}$. Note that we can express data rate of i th link of ZFB as

$$\begin{aligned} \log(1 + \frac{g_i P_i}{\sigma^2}) &= \log(1 + \frac{g_i b_i \hat{P}_i}{\sigma^2}) \\ &= \log(1 + \gamma_i), \end{aligned} \qquad (9)$$

where $\hat{P}_i = \|\mathbf{w}_i\|^2 P_i$ is the transmit power allocated to the i th stream link so that P_i

becomes effective transmit power loading. Hence, $b_i = 1/\|\mathbf{w}_i\|^2$ can be the effective channel gain. Under the assumption of equal power allocation, the transmit power $\hat{P}_i = P_T/N_t$ so that the average receive SNR $\rho_i = g_i\hat{P}_i/\sigma^2$

3. Definition

3.1 link Outage Probability

To begin with, we first define the link outage probability which reflects how reliable a system can support the corresponding link quality. For a single-input single-output (SISO) system in flat fading channel, link outage is usually defined as the probability that the effective received SNR is less than a predetermined value γ_{th}, i.e. $P_{out} = P_r\{\gamma < \gamma_{th}\}$ [19]. The link outage for a point-to-point spatial multiplexing MIMO system in a flat fading channel is defined as the event when the effective receive SNR of any substream is less than γ_{th} [20] [21]. As for a point-to-multipoint MIMO broadcast system, we can define link outage probability of individual stream link as same as SISO case , i.e. $P_{out}^i = P_r\{\gamma_i < \gamma_{th}\}$.

3.2 Link Coverage Reliability

With P_{out} being the link outage probability, we define $(1-P_{out})$ to be the link coverage reliability for its corresponding link radius associated with the required SNR. That is, for a user terminal at the link radius with link coverage reliability $(1-P_{out})$, the probability of the effective received SNR being higher than the required threshold γ_{th} is no less than $(1-P_{out})$. Note that we concern the link reliability likely of high percentile, such as 90% or even higher, in this paper. For a point-to-point MIMO system, the data stream with the lowest SNR will dominate the cell coverage performance due to the more likely outage link. As for point-to-multipoint MIMO broadcast system, all stream links represent different individual users so that the cell coverage will be determined by the strongest link.

4. Link Outage and Coverage Analysis

4.1 ZF-DPC without scheduling (or random selection)

In this section, we analyze the baseline performance of ZF-DPC without user selection. To begin with, we first analyze the effective received SNR of individual stream link i (denoted by γ_i) with the help of following lemma shown in [3]:

Lemma 1 *Let* $\mathbf{H} \in \mathbb{C}^{r\times t}$ *have i.i.d. entries* $\sim \mathcal{CN}(0,1)$, *and let* $l_{i,i}$ *be the* i*th diagonal element of* $\mathbf{L}$ *in the* **QR** *decomposition* $\mathbf{H} = \mathbf{LQ}$. *Then, the random variables* $d_i = |l_{i,i}|^2$ *are statistically independent and* $d_i \sim \chi^2_{2(t-i+1)}$, *where* χ^2_{2a} *denotes the central Chi-squared distribution with* $2a$ *degrees of freedom, whose probability density function (PDF) is* $f(z) = z^{a-1}e^{-z}/(a-1)!$.

So that the PDF of effective channel gain d_i is

$$f_{d_i}(z) = \frac{z^{N_t-i}e^{-z}}{(N_t-i)!}\; i = 1,\ldots,N_t, \ldots.. \quad (10)$$

and the cumulative distribution function (CDF) of d_i can be written as

$$\begin{aligned} F_{d_i}(z) &= \int_0^z \frac{x^{N_t-i}e^{-x}}{(N_t-i)!}dx \\ &= 1-\int_z^\infty \frac{x^{Nt-i}e^{-x}}{\Gamma(N_t-i+1)}dx \\ &= 1-\frac{\Gamma(N_t-i+1,z)}{\Gamma(N_t-i+1)} \\ &= 1-Q(N_t-i+1,z), \end{aligned} \quad (11)$$

where $\Gamma(x)=\int_0^{\infty} t^{x-1}e^{-t}dt$ is the complete gamma function, $\Gamma(a,x)=\int_x^{\infty} t^{a-1}e^{-t}dt$ is the upper incomplete gamma function and $Q(a,x)=\frac{\Gamma(a,x)}{\Gamma(a)}$ is the regularized gamma function. Note that $\Gamma(n)=(n-1)!$ for a positive integer n. Form (10), we know that $d_1 \ge d_2 \ge \ldots \ge d_{N_t}$ so that the first channel row vector results in the strongest link.

The CDF of i th link's effective receive SNR $\gamma_i = d_i \rho_i$ is

$$F_{\gamma_i}(\gamma)=F_{d_i}(\frac{\gamma}{\rho_i})=1-Q(N_t-i+1,\frac{\gamma}{\rho_i}). \quad (12)$$

Thus, for a given threshold $\gamma_{th}>0$, the link outage probability of i th link of ZF-DPC is

$$P_{out}^i = P_r\{\gamma_i < \gamma_{th}\}$$

$$= F_{r_i}(\gamma_{th})$$

$$= 1-Q(N_t-i+1,\frac{\gamma_{th}}{\rho_i}).$$

(13)To derive cell coverage R_{ZFDPC}^i from (13), we first introduce the inverse of the regularized incomplete gamma function which is shown as follows

$$x=Q(a,z)\Rightarrow z=Q^{-1}(a,x). \quad (14)$$

By substituting (5) and (14) into (13), the link coverage is given by

$$R_{ZFDPC}^i=\left[\frac{P_t 10^{g_0/10}}{N_t\gamma_{th}\sigma^2}Q^{-1}(N_t-i+1,1-P_{out}^i)\right]^{\frac{1}{\mu}} \quad (15)$$

for $i=1,\ldots,N_t$

4. 2 ZFB without scheduling (or random selection)

Alternately, we analyze the baseline performance of ZFB without user selection in which selects users randomly and does not exploit the multiuser diversity gain. In this case, it is just like there are $K=N_t$ users with the Rayleigh fading channel vector. From observation on a point-to-point $N_t \times N_t$ MIMO system with ZF receiver, the effective channel gain b_i has the same form as the ZF receiver's substream effective channel gain. Due to the same statistics we can see the system as a virtual $N_t \times N_t$ MIMO system with ZF receiver. According to [22], the substream SNRs $\{\gamma_i\}_{i=1}^{N_t}$ for an $N_t \times N_r$ MIMO system with ZF receiver under the equal power allocation principle are i.i.d. Chi-squared distributed random variables with $2(N_r-N_t+1)$ degrees of freedom, i.e.

$$f_{\gamma}(\gamma)=\frac{e^{-\gamma/\rho_i}}{\rho_i(N_r-N_t)!}\left(\frac{\gamma}{\rho_i}\right)^{N_r-N_t}, \quad \gamma\ge 0, \quad (16)$$

where $\gamma_i = b_i\rho_i$. The PDF of unordered $\{b_i\}_{i=1}^{N_t}$ are i.i.d. exponential distribution with parameter one from (16) in the $N_t=N_r$ case. (13) Therefore, the link outage probability of i th stream link for ZFB is

$$P_{out}^i = F_{r_i}(\gamma_{th})$$

$$= F_{b_i}(\frac{\gamma_{th}}{\rho_i})$$

$$= 1-\exp(-\frac{\gamma_{th}}{\rho_i}). \quad (17)$$

As a result, the link coverage R_{ZFB}^i can be written as follows:

$$R_{ZFB}^i=\left[\frac{P_t 10^{g_0/10}}{N_t\gamma_{th}\sigma^2}\log\left(\frac{1}{1-P_{out}^i}\right)\right]^{\frac{1}{\mu}} \quad i=1,\ldots,N_t. \quad (18)$$

Note that the link coverage of all ZFB stream links is equal to that of ZF-DPC's weakest link as shown in (15) for $i=N_t$ under the same link outage probability constraint.

4.3 ZF-DPC with greedy scheduling

For considering scheduling, we focus on the strongest stream link which has the largest radius will determines the cell range. In [5], the authors have proposed a greedy scheduling algorithm for selecting N_t out of K users to form $\mathbf{H}(\Sigma)$ and ordering those selected channel row vectors in the Gram-Schmidt orthogonalization, aiming to maximize the throughput.

The strongest link will be determined by the first selected user's channel row vector. The effective channel gain $d_{1,k} = \mathbf{h}_k \mathbf{h}_k^*$, where $\mathbf{h}_k \in X^{1\times N_t}$ represents the channel row vector of kth user. Note that $d_{1,k}$ is a sum of N_t squared magnitudes of circularly symmetric, zero-mean, unit-variance complex Gaussian random variables. Therefore, $d_{1,k} : \Xi^{2}_{2N_t}$ with PDF

$$f_{d_{1,k}}(z) = \frac{z^{N_t-1}e^{-z}}{(N_t-1)!}. \tag{19}$$

According to the greedy selection algorithm, the selected user k^* is

$$k^* = \arg\max_{k\in\{1,\ldots,K\}} d_{1,k}. \tag{20}$$

Thus, the effective channel gain $\tilde{d}_1$ of the strongest link for greedy scheduling algorithm is

$$\tilde{d}_1 = d_{1,k^*} = \max_{k\in\{1,\ldots,K\}} d_{1,k}. \tag{21}$$

From order statistics, the PDF of $\tilde{d}_1$ is

$$f_{\tilde{d}_1}(z) = K[F_{d_{1,k}}(z)]^{K-1} f_{d_{1,k}}(z), \tag{22}$$

and the CDF of $\tilde{d}_1$ can be written as

$$F_{\tilde{d}_1}(z) = [F_{d_{1,k}}(z)]^K$$

$$= [1 - Q(N_t, z)]^K, \tag{23}$$

Therefore, the CDF of the strongest link's effective receive SNR $\tilde{\gamma}_1$ is

$$F_{\tilde{\gamma}_1}(\gamma) = F_{\tilde{d}_1}(\frac{\gamma}{\rho_1}) = [1 - Q(N_t, \frac{\gamma}{\rho_1})]^K, \tag{24}$$

and the link outage probability is

$$\tilde{P}^1_{out} = [1 - Q(N_t, \frac{\gamma_{th}}{\rho_1})]^K. \tag{25}$$

To derive link coverage $\tilde{R}^1_{ZFDPC}$ of the strongest link from (25), we use again the inverse of the regularized incomplete gamma function and get the following closed form:

$$\tilde{R}^1_{ZFDPC} = \left[\frac{P_t 10^{g_0/10}}{N_t \gamma_{th} \sigma^2} Q^{-1}(N_t, 1 - [\tilde{P}^1_{out}]^{\frac{1}{K}})\right]^{\frac{1}{\mu}}. \tag{26}$$

5 Numerical Results

In this section, we present numerical examples to illustrate achievable link outage and link coverage performances of both ZF-DPC and ZFB in multiuser MIMO broadcast systems. For ZFB with optimal user selection policy, exhaustive users search, the explicit performance closed-form is difficult to be found due to an optimization problem involved in itself. Therefore, we use simulation results to show the performance of strongest stream link for ZFB with exhaustive search. We first assume a predetermined value $\gamma_{th} = 2$ dB, $noisepower = -103$ dBm, $g_0 = -32$, $\mu = 3$ and $N_t = 3$. Figure 1 shows the simulative and analytical link outage performances of both ZF-DPC and ZFB without scheduling for user terminals at distance $R = 1$ km from the base station. Similar to the analytic results shown before, the link outage of all ZFB stream links performs equally to that of ZF-DPC's weakest link under certain radius. Especially, we can find there is a diversity-like performance behavior between stream links of ZF-DPC.

For example, the strongest link d_1 has diversity three, d_2 has diversity two but the weakest link d_3 has only diversity one in link outage performance.

Figure 2 shows link outage performances for the strongest links of both ZF-DPC and ZFB with and without scheduling for user terminals at distance $R = 2$ km from the base station and $K = 5$. Form the improvement of link outage performance, one can see that the gains of multiuser diversity is significant even if the degrees of freedom is merely $K = 5$. The corresponding link coverage performances can be found from fig. 3 in which we set the link reliability as 0.9 under γ_{th}. From fig. 3, ZF-DPC has better link coverage than ZFB in arbitrary stream link. In addition, an obvious coverage extension exists in both ZF-DPC and ZFB even if $K = 5$. For example, it can only maintain 0.9 link reliability as far as about 2 km radius without scheduling but extend to 3 km with scheduling in $P_T = 4$ dBW for ZF-DPC. It tells us a soft coverage enhancement can be easily done by using scheduling techniques no need to increase extra transmit power.

6. Conclusion

In the paper, we have analyzed the link outage and link coverage performance of the multiuser MIMO broadcast systems over ZF-DPC and ZFB schemes with full CSI at transmitter. We present analytical formulas that can evaluate the link outage probability and link coverage radius for both ZF-DPC and ZFB without scheduling. We also provide closed-form expressions for ZF-DPC and show simulation results for ZFB to discuss both link outage and cell coverage performances when considering scheduling. From our numerical results, we validate the accuracy of the analytical model by simulation and present a soft coverage enhancement conception for system design. In the future, we will consider the both performance criteria under practical limited feedback scenario and see the degradation in both link outage and link coverage radius. In fact, our analysis provides performance upper bound corresponding to MIMO broadcast systems with practical finite rate feedback.

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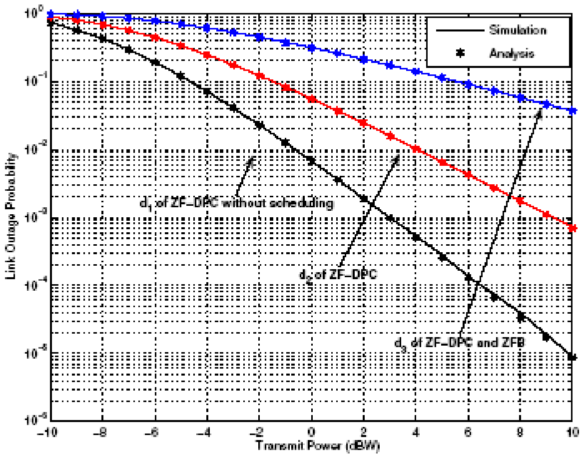

Figure 1：Link outage probability performance v.s. transmit power P_T for different stream links of both ZF-DPC and ZFB when $N_t = 3$, noise power $= -103$ dBm, $\mu = 3$, $R = 1$ km and $\gamma_{th} = 2$ dB.

中文摘要

在無線通道的環境中,訊號衰弱 (fading) 是主要面臨的問題,利用協力式通訊 (cooperative communication) 的技術可以增加分集增益 (diversity gain) ,也可以藉此克服訊號衰退所造成的困擾,但是協力式通訊跟直接傳輸 (direct transmission) 相比較,傳輸率 (throughput) 的表現會相對的較差,因此這個研究將著力於以傳輸率為導向,設計出一套中繼站選擇方法(relay selection scheme),並且同時保留協力式通訊在中斷機率 (outage probability) 方面的優點. 這個研究做了下列的假設: 考慮二步通訊 (two-hop communication) 、瑞利衰退 (Rayleigh fading) 、路徑損耗指數 (path loss exponent) 等於四、接收端使用最大比例組合 (maximal ratio combing) 技術、以及當接收端接受從多中繼站發出的多重訊號時, 這些訊號是完美的同步. 這個研究提出了兩種以傳輸率為導向的中繼站選擇方法, 兩種方法的傳輸率表現相較於文獻探討中的方法有大幅的提升, 而在中斷機率的表現上第二種方法比第一種好, 也更為接近文獻討論中的方法中的中斷機率表現最好的方法. 未來的研究方向是將現有的兩種方法從選擇一個中繼站推廣至選擇多個中繼站, 以及傳輸功率的配置上。

關鍵詞：協力式通訊，中繼站選擇方法

Abstract

In the environment of wireless channel, fading is the main challenge. Utilizing the techniques of cooperative communication can increase the diversity gain to mitigate fading. Comparing the cooperative communication to direct transmission, the transmission rate of the former one is lower. Therefore this study is aiming at design a throughput-oriented relay selection rule while maintain reasonable outage probability. There are a few assumptions: Considering two-hop communication、 Rayleigh fading、 path loss exponent equals to four、 maximal ratio combing are applied at the receiver、 perfect synchronization of signals arrive at the destination receiver. This study proposed two throughput-oriented relay selection rules. Throughput of these two methods is significant higher than methods in the literature. For the outage probability, the second rule is better than the first rule, approaching the best outage performance of a method in the literature. Extending form single relay to multiple relays and distributing transmission power to the two phases is our future works.

Keywords: cooperative communication, relay selection scheme