附件一

行政院國家科學委員會補助專題研究計畫

□ 成 果 報 告

■ 期中進度報告(精簡版)

新渾沌系統與新渾沌控制及同步方法(第一年)

計畫類別：■ 個別型計畫　　□ 整合型計畫
計畫編號：NSC 96 －2221－ E －009 －145－ MY3
執行期間：96 年 8 月 1 日　至　97 年 7 月 31 日

計畫主持人：戈正銘
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成果報告類型(依經費核定清單規定繳交)：■期中進度精簡報告
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□出席國際學術會議心得報告及發表之論文各一份
□國際合作研究計畫國外研究報告書一份

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執行單位：國立交通大學機械工程學系

中　華　民　國　97　年　5　月　22　日

中文摘要：

關鍵詞：新 Mathieu-van der Pol 系統，新 Ikeda- Mackey-Glass 系統，Mathieu 系統，Van der Pol 系統，Ikeda 系統，Mackey-Glass 系統，非耦合渾沌同步。

渾沌系統之研究在物理、化學、生理學、各種工程等方面皆有日益重要之廣泛應用。Van der Pol 系統、非線性 Mathieu 系統都是最重要的典型的渾沌系統。Ikeda 系統及 Mackey-Glass 系統則是最重要的典型的時滯系統。本計畫(第一年)採取適當的耦合方式構成新創的 Mathieu-van der Pol 系統和 Ikeda- Mackey-Glass 系統，從而擴大了各原來單純系統的研究範圍也深化了研究內容。

目前渾沌同步皆取李氏函數為V正定、$\dot{V}$負定，同步達成之時間較長控制品質不夠好。現採指數漸進穩定理論研究誤差系統零解，使同步完成時間大為減小。再結合後步(backstepping)設計可得三優點 1. 同步完成時間大減 2. 逐步選擇V函數，減少選V函數之難度 3. 減少控制器的數目。

研究重點為：

1. Mathieu-van der Pol 系統與 Ikeda- Mackey-Glass 系統的渾沌研究。用相圖、分歧圖、功率譜圖、李雅普諾夫指數分析渾沌之行為。
2. 新式非耦合渾沌同步新方法。

英文摘要：

key words: New Mathieu-van der Pol system, new Ikeda- Mackey-Glass system, Mathieu system, Van der Pol system, Ikeda system, Mackey-Glass system, uncoupled chaos synchronization.

In this project (first year), multichanneled various excitations(various time function, chaotic function, noise, etc)are used to increase the reliability of synchronization in the accident of interruption of part of the channels.

The point of research:

1. The study of chaos of Mathieu-van der Pol system and Ikeda- Mackey-Glass system: By phase portraits, bifurcation diagrams, power spectra, Lyapunov exponents, the various chaotic behaviors of these systems will be studied.
2. New uncoupled synchronization method.

報告內容：

（一）前言及研究目的：

渾沌系統之研究除了在理論上的重要價值外，在物理、化學、生理學及各種工程等方面皆有廣泛之應用。Van der Pol系統與非線性Mathieu系統都是重要的典型渾沌系統。Ikeda系統是重要的典型光電或生理時滯系統，而Mackey-Glass系統則是著名的典型生理時滯系統。對於這些重要系統的渾沌現象及渾沌同步都已有豐富的研究成果[1-35]。本計畫(第一年)為了對這些著名系統，擴大其研究範圍並深化其研究內容，特提出混合的新系統，即新Mathieu-van der Pol系統及新Ikeda- Mackey-Glass系統。極具實用價值，其渾沌現象值得仔細研究。對上述二種新系統，首先證明其為渾沌系統，其次研究其渾沌行為。

渾沌同步在物理系統、化學系統、生物系統、各種工程系統、秘密通訊、神經網路、自我組織系統等方面有長足之應用[36-86]。本計畫(第一年)提出一種新渾沌同步方法，對這些新系統加以研究。

（二）研究方法及文獻探討：

（a）Mathieu-van der Pol系統及Ikeda- Mackey-Glass系統的渾沌研究

經典的非線性Mathieu系統為：

$$\ddot{x} + a(1+\sin\omega t)x + (1+\sin\omega t)x^3 + a\dot{x} = 0$$

或

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -a(1+\sin\omega t)x_1 - (1+\sin\omega t)x_1^3 - ax_2$$

其中 a 為常數。

經典的van der Pol系統為：

$$\ddot{x} + bx + c\dot{x}(1-x^2) - d\sin\omega t = 0$$

或

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -bx_3 + c\dot{x}_4(1-x_3^2) + d\sin\omega t$$

其中 b, c, d 為常數。現將第一系統中之 $\sin\omega t$ 與第二系統中的 $d\sin\omega t$ 中的 $\sin\omega t$ 交替換成對方的狀態變量，即得本計劃新創造之混合的Mathieu-van der Pol系統：

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -a(1+x_3)x_1 - (1+x_3)x_1^3 - ax_3$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -bx_3 + c\dot{x}_4(1-x_3^2) + dx_1$$

經典的Ikeda系統為：

$$\dot{x} + ax + b\sin x_\tau = 0$$

其中 a, b 為常數，$x_\tau = x(t-\tau)$，τ 為常數。此系統可用以表示有回授之B級雷射系統，其中B級之典型代表為固態、半導體及低壓 CO_2 雷射[21-22]，也可表示生理血液系統。

經典的Mackey-Glass系統為：

$$\dot{x}=\frac{bx\tau}{1+x_\tau^n}-rx$$

其中b,r為常數，n為正整數，$x_\tau=x(t-\tau)$，τ為常數。此系統可表示造血系統，其中x為t時刻之血液濃度，τ為造血所須之延遲時間。白血患者之τ值增加即引起血液濃度產生渾沌變化[25-26]。Ikeda- Mackey-Glass系統為：

$$\dot{x}=-\alpha x+m\sin x_\tau+\kappa y_\tau$$

$$\dot{y}=-\beta y+\frac{cy_\tau}{1+y_\tau^b}$$

對此新混合系統之研究，可說是對各單系統渾沌行為研究之延伸與深化，此混合系統比各單系統顯然有更複雜之渾沌行為，值得本計劃加以研究。

（b）非耦合渾沌同步新方法及應用

目前文獻絕大部分渾沌同步皆採耦合同步或控制器中出現主從系統之狀態變量，實際上也是耦合。以秘密通訊而言，耦合所須的主從系統狀態變量之傳送會造成失密及時滯之不良後果。非耦合同步可消除這些缺點，目前國際文獻多採以第三系統之渾沌變量或噪音對主從系統參數作用時激勵而獲同步。本計畫採多管道下之各種形式之激勵（各種時間週期函數、渾沌函數、各種噪音）以保證同步之可靠性。

以Mathieu-van der Pol系統為例。兩個Mathieu-van der Pol系統：

$$\dot{x}_1=x_2$$

$$\dot{x}_2=-a(1+x_4)x_1-(1+x_4)x_1^3-ax_2-bx_3$$

$$\dot{x}_3=x_4$$

$$\dot{x}_4=-x_3+c(1-dx_3^2)x_4+dx_1$$

和

$$\dot{y}_1=y_2$$

$$\dot{y}_2=-a(1+y_4)y_1-(1+y_4)y_1^3-ay_2-by_3$$

$$\dot{y}_3=x_4$$

$$\dot{y}_4=-y_3+c(1-dy_3^2)y_4+dy_1$$

設有第三任何渾沌系統，為簡單計，即採用第三個Mathieu-van der Pol系統：

$$\dot{z}_1=z_2$$

$$\dot{z}_2=-a(1+z_4)z_1-(1+z_4)z_1^3-az_2-bz_3$$

$$\dot{z}_3=z_4$$

$$\dot{z}_4=-z_3+c(1-dz_3^2)z_4+dz_1$$

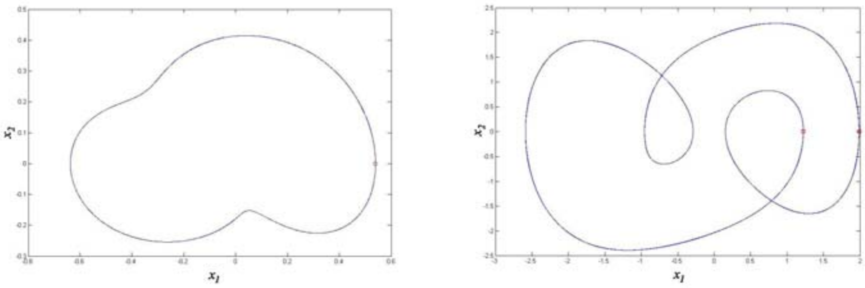
用其任一渾沌變量，z_1之任意給定函數kz_1或$f(z_1)$、$F(t)z_1$（$F(t)$為任意給定函數）等等取代第一及第二系統中對應的第四式末項中dx_1、dy_1中之d_1而成為kz_1x_1及kz_1y_1。調節k為

適當值時第一、二系統即達成同步。將 kz_1 換為以上各項乃可得許多豐富的同步之結果，本計畫對眾多之激勵項用量化方法比較其優劣。

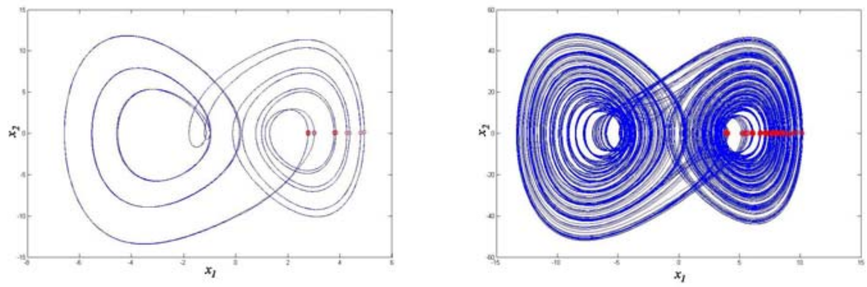
結果與討論：

Mathieu-van der Pol 系統與 Ikeda- Mackey-Glass 系統的渾沌行為與新式非耦合渾沌同步新方法所得之結果如下：

1. 採用諸多相圖、分歧圖、功率譜圖、參數圖及李亞普諾夫指數及碎形維度等研究而獲得自治的新 Mathieu-van der Pol 系統與新 Ikeda- Mackey-Glass 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。

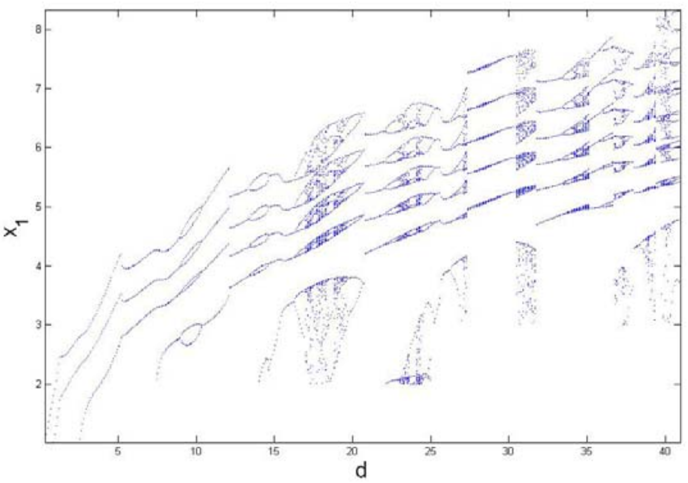


(a) (b)

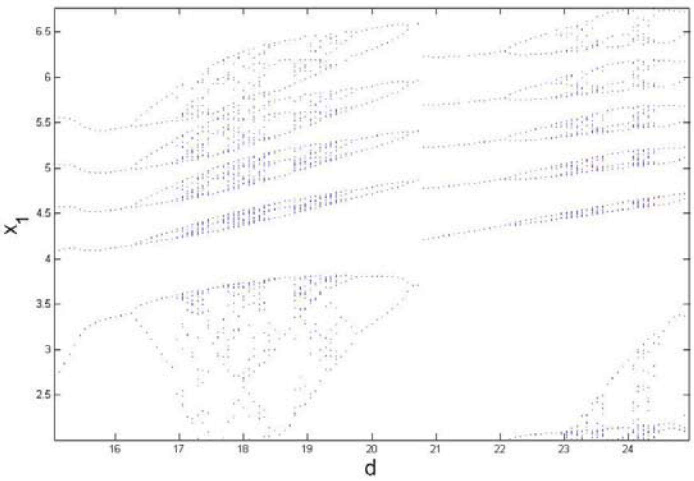


(c) (d)

Fig. 1 Phase portraits and Poincaré maps for autonomous new Mathieu-van der Pol system: (a) period 1 for $d = 0.1$, (b) period 2 for $d = 1$, (c) period 8 for $d = 10$, (d) chaotic for $d = 40$.

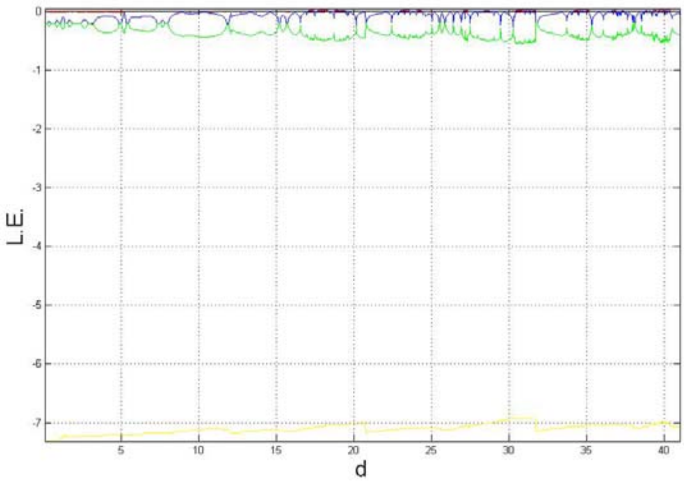


(a)

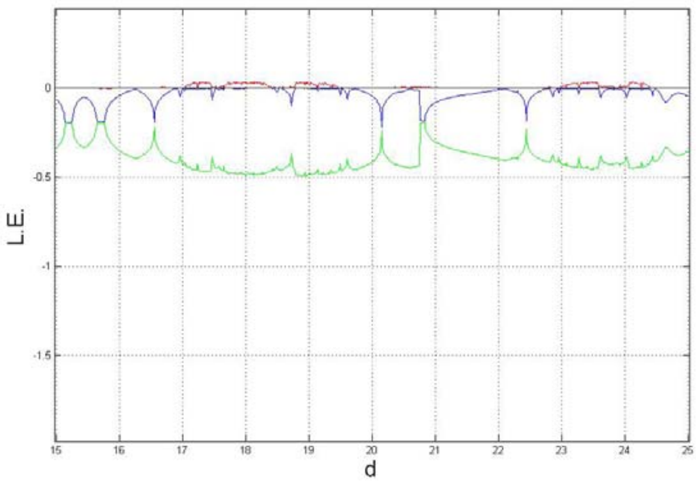


(b)

Fig. 2 Bifurcation diagram for autonomous new Mathieu-van der Pol system: (a) d=0~45 (b) d=15~25.

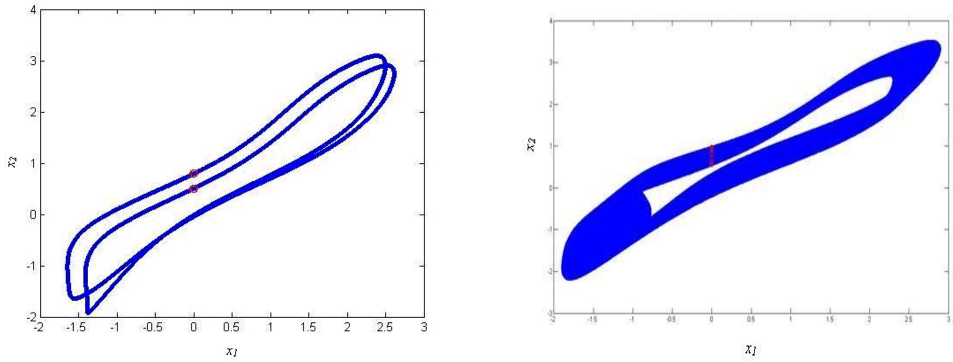


(a)



(b)

Fig. 3 Lyapunov exponents for autonomous new Mathieu-van der Pol system: (a) d=0~45 (b) d=15~25.



(a) (b)

Fig. 4 Phase portraits and Poincaré maps for autonomous new Ikeda- Mackey-Glass system: (a) period 2 for $K_1 = 13.41$, (b) chaotic for $K_1 = 14.1$.

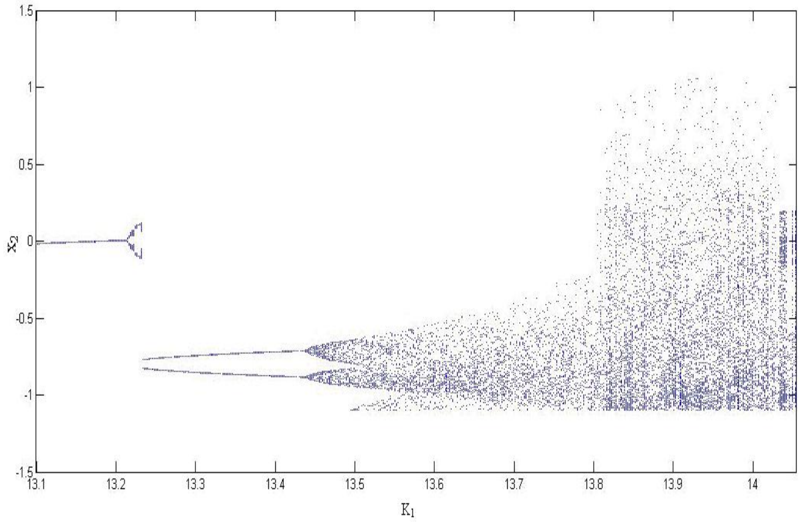


Fig. 5 Bifurcation diagram for autonomous new Ikeda- Mackey-Glass system.

2. 採用諸多相圖、分歧圖、功率譜圖、參數圖及李亞普諾夫指數及碎形維度等研究而獲得非自治的新 Mathieu-van der Pol 系統與新 Ikeda- Mackey-Glass 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。
3. 獲得新式非耦合渾沌同步新方法對自治的新 Mathieu-van der Pol 系統與新 Ikeda- Mackey-Glass 系統之應用。

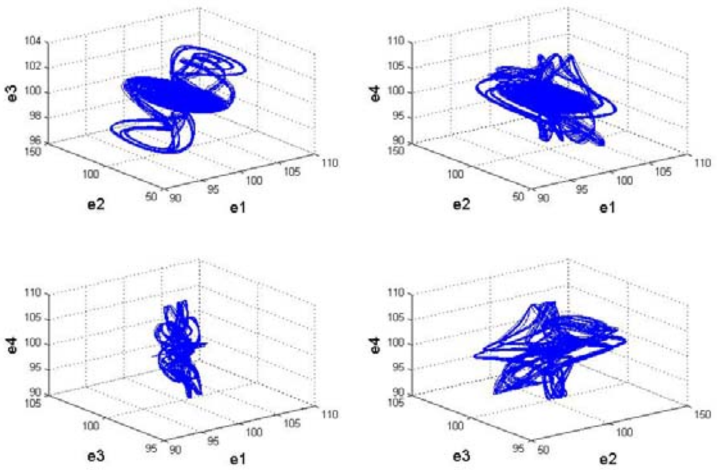


Fig. 6 Phase portraits of error dynamics for autonomous new Mathieu-van der Pol system by GYC Partial Region Stability Theory.

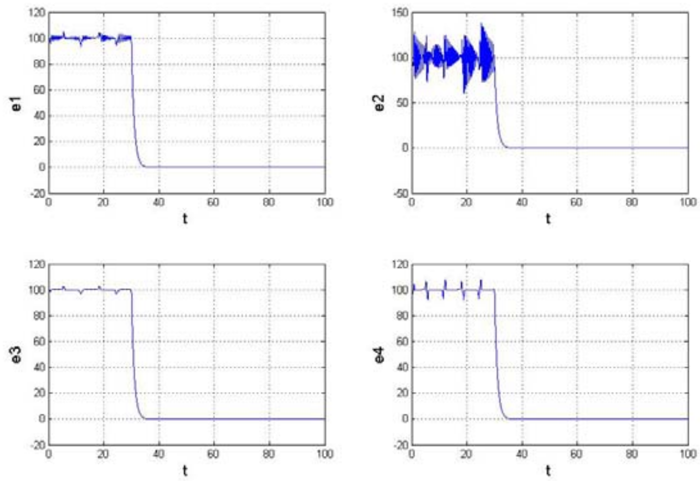


Fig. 7 Time histories of errors for autonomous new Mathieu-van der Pol system when generalized synchronization is obtained.

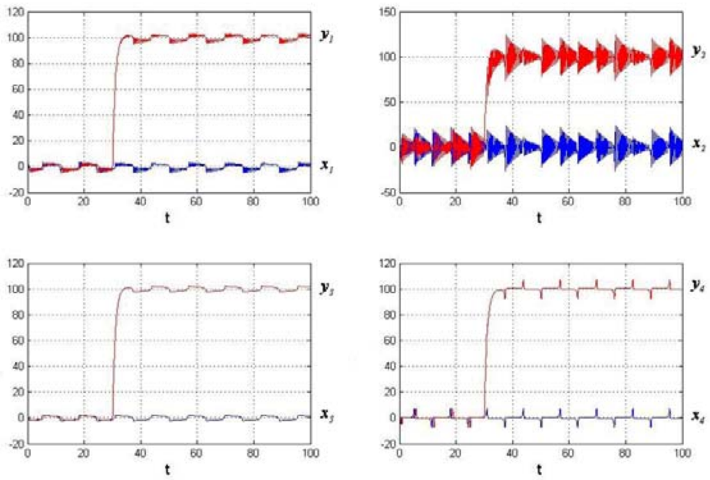


Fig. 8 Time histories of x_1, x_2, x_3, y_1, y_2, y_3 for autonomous new Mathieu-van der Pol system

when generalized synchronization is obtained.

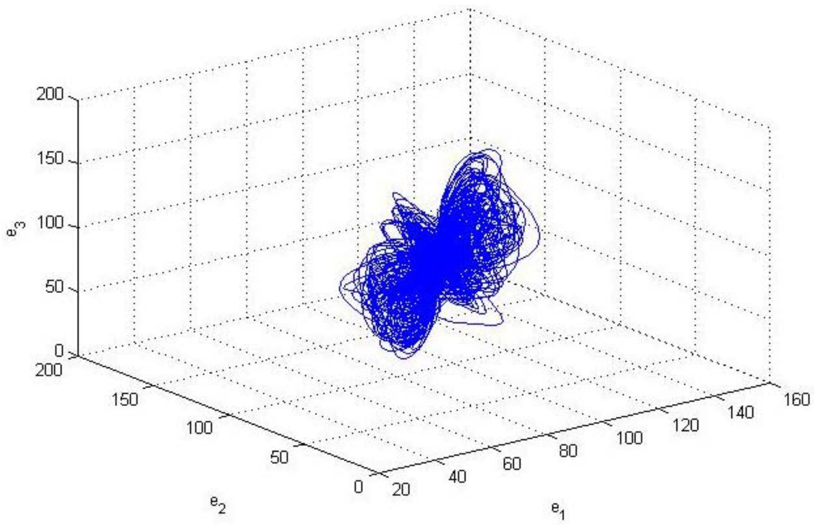


Fig. 9 Phase portraits of error dynamics for autonomous new Ikeda- Mackey-Glass system by GYC Partial Region Stability Theory.

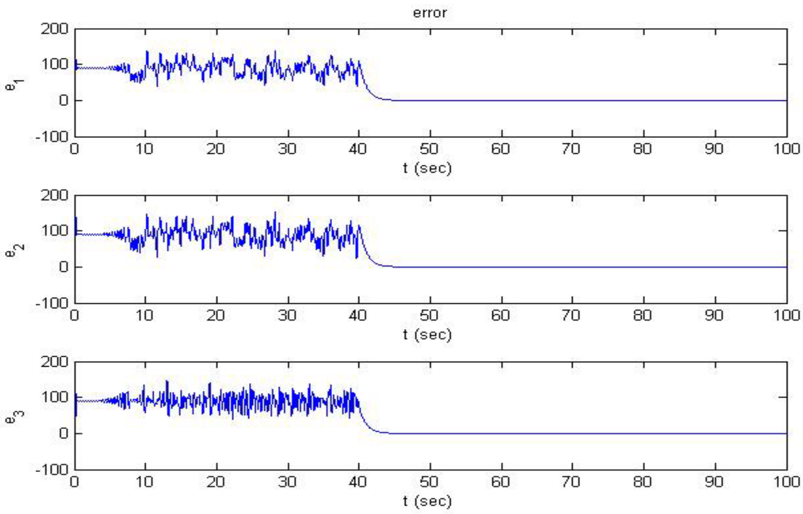


Fig. 10 Time histories of errors for autonomous new Ikeda- Mackey-Glass system when generalized synchronization is obtained.

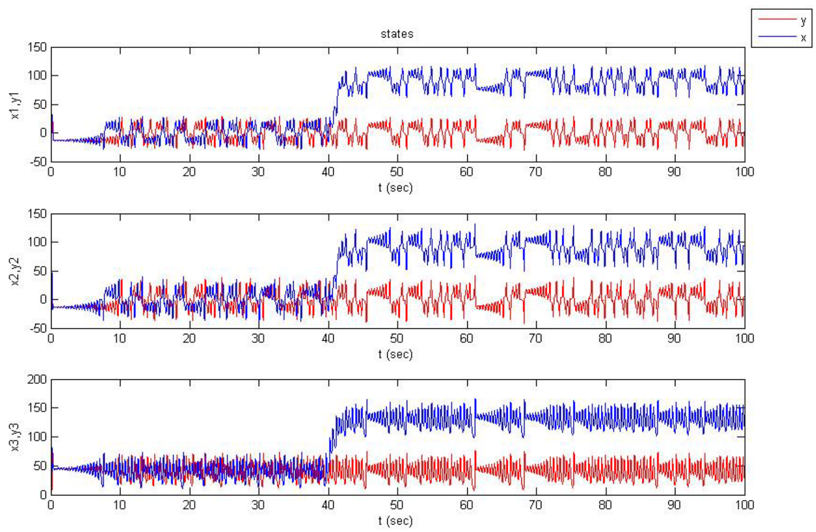


Fig. 8 Time histories of x_1, x_2, x_3, y_1, y_2, y_3 for autonomous new Ikeda- Mackey-Glass system when generalized synchronization is obtained.

4. 獲得新式非耦合渾沌同步新方法對非自治的 Mathieu-van der Pol 系統與 Ikeda-Mackey-Glass 系統之應用。

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計畫(第一年)成果自評：

Mathieu系統、van der Pol系統、Ikeda系統及Mackey-Glass系統皆為經典之系統。經巧妙之取代與結合既可獲得自治新Mathieu−van der Pol系統，自治新Ikeda-Mackey Glass系統亦可得非自治之新Mathieu−van der Pol系統與非自治新Ikeda-Mackey Glass系統大大擴大了經典渾沌系統的範圍及研究領域。而新的非耦合渾沌同步方法具有很大的機動性及發展潛力。取代之參數可以變化多端，採用之取代函數更是取之不盡。對用於秘密通訊而言，機密性大為增加。已投出之國際著名期刊論文已達七篇。

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以本計劃經費資助出版之國際著名期刊論文一篇，見附錄。

附錄

1. Zheng-Ming Ge "Necessary and Sufficient Condition of the Stability of a Sleeping Top Described by Three Forms of Dynamic Equations", PHYSICAL REVIEW E (2008).

Necessary and sufficient conditions for the stability of a sleeping top described by three forms of dynamic equations

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Necessary and sufficient conditions for the stability of a sleeping top described by dynamic equations of six state variables, Euler equations, and Poisson equations, by a two-degree-of-freedom system, Krylov equations, and by a one-degree-of-freedom system, nutation angle equation, is obtained by the Lyapunov direct method, Ge-Liu second instability theorem, an instability theorem, and a Ge-Yao-Chen partial region stability theorem without using the first approximation theory altogether.

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I. INTRODUCTION

The stability of a sleeping top is a classical problem which appears in the standard courses of classical mechanics [1,2]. Routh [3], Klein [4], and Grammel [5] studied this problem from various heuristic points of view. In 1946, Chetaev [6,9] first strictly studied the problem described by a two-degree-of-freedom system, Krylov equations [7]. By the Lyapunov direct method, he obtained the sufficient condition of conditional direction stability :

$$C^2\omega^2 > 4Amga, \tag{1}$$

where C is the axial moment of inertia of the top about a fixed point, A is the equatorial moment of inertia of the top about the fixed point, ω is the angular velocity of the top about the symmetric axis of the vertical sleeping top, m is the mass of the top, a is the distance between the center of gravity of the top and the fixed point, and g is the gravity acceleration. In 1954, Chetaev [8,9] studied the same problem by Euler equations and Poisson equations for six state variables and obtained the same sufficient condition of unconditional stability by the Lyapunov direct method. In 1979 using the same equations, Ge [10,11] obtained the necessary and sufficient condition of unconditional stability of a sleeping top

$$C^2\omega^2 \geq 4Amga \tag{2}$$

by the Lyapunov direct method and first approximation theory, and corrected the error of Loitsyanskii and Lurie [12], Rumjantsev [13], and Magnus [14]. They declared that the necessary and sufficient condition for stability of a sleeping top is $C^2\omega^2 > 4Amga$.

In this paper, the necessary and sufficient condition for unconditional stability and conditional direction stability of the sleeping top is obtained by using Euler equations and Poisson equations, and by using Krylov equations by the Lyapunov direct method, Ge-Liu second instability theorem [15], Ge theorem for determining the definiteness of functions [16], and an instability theorem. The necessary and sufficient condition of conditional nutation angle stability is obtain by using the nutation angle equation by the Lyapunov direct method and Ge-Yao-Chen (GYC) partial region stability theorem [17,18]. In this paper, the first approximation theory has not been used altogether.

This paper is organized as follows. In Sec. II, a necessary and sufficient condition for unconditional stability of a sleeping top by using Euler equations and Poisson equations is obtained by the Lyapunov direct method and Ge-Liu second instability theorem. In Sec. III, the same condition of conditional direction stability is obtained by using Krylov equations by the Lyapunov direct method and an instability theorem. In Sec. IV, the same condition of conditional nutation angle stability is obtained by using a nutation angle equation by the Lyapunov direct method and GYC partial region stability theorem. In Sec. V, conclusions are drawn.

II. STABILITY OF A SLEEPING TOP DESCRIBED BY EULER EQUATIONS AND POISSON EQUATIONS

A. Euler equations and Poisson equations

In Fig 1, O is the fixed point of a symmetric top. $Ox_1y_1z_1$ is an inertial frame with vertical axis z_1. $Oxyz$ is a body frame fixed with the symmetric top and coincides with the principal axes of inertia of the top. A, B, and C are the principal moments of inertia of the top about the Ox, Oy, and Oz axes, respectively. The conditions for a Lagrange top are

$$A=B,\ \ x=o,\ \ y=0,\ \ z=a>0, \tag{3}$$

where x, y, and z are the coordinates of the center of gravity of the Lagrange top in the $Oxyz$ frame. Rigid body motion about a fixed point with condition (3) is called a Lagrange case. Let p, q, and r be the projections of the angular velocity vector of the Lagrange top on three principal axes Ox, Oy, and Oz, respectively, $\gamma_1,\gamma_2,\gamma_3$ be the direction cosines between Ox,Oy,Oz and the vertical axis Oz_1, respectively. The

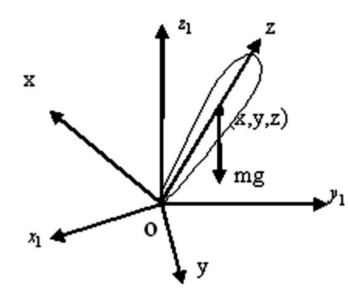

FIG. 1. Rigid body motion in the Lagrange case.

dynamic equations for a Lagrange top are the combination of Euler equations and Poisson equations:

$$A\frac{dp}{dt}+(C-A)qr=mga\gamma_2,$$

$$A\frac{dq}{dt}+(A-C)rp=-mga\gamma_1,$$

$$C\frac{dr}{dt}=0,$$

$$\frac{d\gamma_1}{dt}=r\gamma_2-q\gamma_3,$$

$$\frac{d\gamma_2}{dt}=p\gamma_3-r\gamma_1,$$

$$\frac{d\gamma_3}{dt}=q\gamma_1-p\gamma_2. \tag{4}$$

We shall study the stability of a solution, a vertical permanent rotation:

$$p=0,\ \ q=0,\ \ r=\omega,\ \ \gamma_1=0,$$

$$\gamma_2=0,\ \ \gamma_3=1 \tag{5}$$

of system (4). A Lagrange top with condition (5) is called a sleeping top. Let

$$p=\xi,\ \ q=\eta,\ \ r=\omega+\varsigma,$$

$$\gamma_1=\alpha,\ \ \gamma_2=\beta,\ \ \gamma_3=1+\delta, \tag{6}$$

where ξ, η, ς, α, β, and δ are the disturbances of six state variables in Eq (4).

B. Sufficient condition of unconditional stability

In the Lagrange case, the first integrals of Eq. (4) are

$$A(p^2+q^2)+Cr^2+2mga\gamma_3=h,$$

$$A(p\gamma_1+q\gamma_2)+Cr\gamma_3=k,$$

$$\gamma_1^2+\gamma_2^2+\gamma_3^2=1,$$

$$r=\omega, \tag{7}$$

where h, k, and ω are constants determined by initial conditions of Eq. (4). For the differential equations of disturbances ξ, η, ς, α, β, and δ, the corresponding first integrals are

$$V_1=A(\xi^2+\eta^2)+C(\varsigma^2+2\omega\xi)+2mag\delta,$$

$$V_2=A(\xi\alpha+\eta\beta)+C(\delta\varsigma+\omega\delta+\varsigma),$$

$$V_3=\alpha^2+\beta^2+\delta^2+2\delta(=0),$$

$$V_4=\varsigma. \tag{8}$$

The positive definite Lyapunov function given by Chetaev when $C^2\omega^2>4Amga$ is

$$V_{\mathrm{I}}=V_1+2\lambda V_2-(mga+C\omega\lambda)V_3+\mu V_4^2,$$

$$-2(C\omega+C\lambda)V_4=A\xi^2+2\lambda A\xi\alpha,$$

$$-(mga+C\omega\lambda)\alpha^2+A\eta^2+2\lambda A\eta B,$$

$$-(mga+C\omega\lambda)\beta^2+(C+\mu)\varsigma^2+2\lambda C\delta\varsigma,$$

$$-(mga+C\omega\lambda)\delta^2, \tag{9}$$

where $\mu=C(C-A)/A$, $\lambda=-C\omega/2A$.

We have $\dot{V}_{\mathrm{I}}=0$. By Lyapunov stability theorem, the null solution of $\xi,\eta,\varsigma,\alpha,\beta$, and δ is stable, i.e., the solution (5) is stable. Equation (1) is the sufficient condition of stability for a sleeping top, which is given by Chetaev [8,9]. Since six disturbances correspond to the whole six states of dynamic equations, Euler equations, and Possion equations, we call this stability unconditional stability. In this case, both the magnitude and the direction of the angular velocity vector are stable.

When $C^2\omega^2\geq 4Amga$, Ge [10,11] gave another positive definite Lyapunov function

$$V=V_{\mathrm{I}}+V_{\mathrm{II}}, \tag{10}$$

where

$$V_{\mathrm{II}}=C\omega\left(V_3-\frac{4}{\omega}V_4\right)^2=C\omega\left(\alpha^2+\beta^2+\delta^2+2\delta-\frac{4}{\omega}\varsigma\right)^2 \tag{11}$$

is a positive semidefinite function of ξ, η, ς, α, β, and δ. $\dot{V}=0$ also. The Lyapunov stability theorem is satisfied, the sufficient condition for stability of a sleeping top is now

$$C^2\omega^2\geq 4Amga. \tag{12}$$

When $C^2\omega^2<4Amga$, by first approximation theory, the sleeping top is unstable. In this paper, instead of using first approximation theory, the Ge-Liu second instability theorem is used to prove that when $C^2\omega^2<4Amga$, the sleeping top is unstable.

C. Ge-Liu second instability theorem

In 1999, Ge and Liu [15] gave two instability theorems. The second of them is as follows.

Consider a nonautonomous vector differential equation

$$\dot{x}=f(t,x(t))\qquad \forall\, t\geq 0, \tag{13}$$

where $x\in R^n$ and $f:R_+\times R^n\to R^n$ is continuous. Let $x=0$ be an equilibrium point for the system described by Eq. (13). Then $f(t,0)=0, \forall t\geq 0$. We can prove the following [15].

Theorem. If there exists a C^n function $V:R_+\times R^n\to R$, a ball $B_r=\{x\in R^n,\|x\|<r\}$, an open set $\Omega\subset B_r$, such that

(i) $0<V(t,x)\leq L<\infty$, for some L, $\forall t\geq t_0$, $\forall x\in\Omega$.

(ii) $0 \in \partial\Omega$ (the boundary of Ω).

(iii) $\dot{V}(t,x) > 0, \dot{V}(t,x)$ is uniformly continuous in t, $\forall t \geq t_0$, $\forall x \in \Omega$.

(iv) (a) There exists an even $n \geq 2$, such that for some nonempty set $G \subset \partial\Omega \cap B_r$,

$$V^i(t,x) = 0 \quad \text{for } 1 \leq i \leq n-1,$$

$$\exists \gamma > 0, \quad V^{(n)}(t,x) \geq \gamma, \quad \forall t \geq t_0, \quad \forall x \in G.$$

(b) $V(t,x) = 0$, $\forall t \geq t_0$, $\forall x \in \partial\Omega \cap B_r - G$,

Here $V^{(*)}(t,x)$ denotes the $(*)$th time derivative of V with respect to time. Then the equilibrium point 0 of the system (13) is unstable.

D. Sufficient condition for instability and necessary and sufficient condition for stability

Since $C^2\omega^2 < 4Amga$, choose

$$\lambda = -\frac{C\omega}{4A} \tag{14}$$

and let

$$\frac{b}{c} = \frac{C\omega \pm \sqrt{C^2\omega^2 + 4(4Amga - C^2\omega^2)}}{4A},$$

$$\frac{d}{e} = \frac{C\omega \pm \sqrt{C^2\omega^2 + 4(4Amga - C^2\omega^2)}}{4C}. \tag{15}$$

Since $4Amga > C^2\omega^2, b, d$ are positive, and c, e, are negative. Now

$$V_{\mathrm{I}} = A(\xi - b\alpha)(\xi - c\alpha) + A(\eta - b\beta)(\eta - c\beta) + \frac{C^2}{A}(\varsigma - d\delta)(\varsigma - e\delta)$$

is an indefinite function.

The positive definite Lyapunov function V is chosen as

$$V = V_{\mathrm{I}}^2 + \alpha^2 + \beta^2 + \delta^2, \tag{16}$$

$$\dot{V} = \frac{d}{dt}(\alpha^2 + \beta^2 + \delta^2), \tag{17}$$

since V_{I} is a first integral. By the third equation of Eq. (8), the sixth equation of Eq. (4), and Eq. (6)

$$\dot{V} = -2\frac{d\delta}{dt} = -2(\eta\alpha - \xi\beta) = 2(\xi\beta - \eta\alpha) \tag{18}$$

is indefinite. There exists Ω in which $\xi\beta > \eta\alpha$, $V > 0$, and $\dot{V} > 0$, where $0 \in \partial\Omega$. Since V does not contain t explicitly, conditions (i)–(iii) in the above theorem are satisfied. We shall prove that (iv, a) is also satisfied. $\ddot{V}$ can be found as

$$\ddot{V} = 2(\xi\dot{\beta} + \beta\dot{\xi} - \eta\dot{\alpha} - \alpha\dot{\eta}) = 2\left(\xi^2 - \frac{C}{A}\omega\xi\alpha - \frac{C}{A}\omega\beta\eta + \frac{mga}{A}\beta^2 + \eta^2 + \frac{mga}{A}\alpha^2\right) + 2\left(\xi^2\delta - \xi\alpha\beta - \frac{C}{A}\beta\eta\varsigma + \eta^2\delta - \frac{(C-A)}{A}\alpha\xi\varsigma\right). \tag{19}$$

On the boundary of Ω, $\partial\Omega$, $\dot{V}=0$, i.e.,

$$\xi\beta - \eta\alpha = 0. \tag{20}$$

We can prove that $\ddot{V} > 0$ on $\partial\Omega$. There are many cases satisfying Eq. (20).

(a) $\eta = f\xi, \beta = f\alpha$, where f can take an arbitrary positive value except zero. From Eq. (19),

$$\ddot{V} = 2(1+f^2)\left(\xi^2 - \frac{C}{A}\omega\xi\alpha + \frac{mga}{A}\alpha^2\right) + 2(1+f^2)\left(\xi^2\delta - \frac{C}{A}\xi\alpha\varsigma\right). \tag{21}$$

Since $4Amga > C^2\omega^2$, the second order terms of $\ddot{V}$ are a positive definite function of ξ, α, while the third order terms of $\ddot{V}$ have no influence on the definiteness of $\ddot{V}$. Therefore $\ddot{V} > 0$. When $\eta = f\beta$, $\xi = f\alpha$, or $\xi = f\beta$, $\eta = f\alpha$, $\beta = \alpha$ it can be proved similarly that $\ddot{V} > 0$.

(b) $\beta = \eta = 0$. Now

$$\ddot{V} = 2\left(\xi^2 - \frac{C\omega}{A}\xi\alpha + \frac{mga}{A}\alpha^2\right) + 2\left(\xi^2\delta - \frac{C}{A}\alpha\xi\delta\right) \tag{22}$$

is a positive definite function of ξ, α, i.e., $\ddot{V} > 0$.

When $\beta = \alpha = 0$; $\xi = \eta = 0$ or $\xi = \alpha = 0$, it can also be easily obtained that $\ddot{V} > 0$.

(c) $\beta = \alpha = \eta = 0$. Now

$$\ddot{V} = 2\xi^2 + 2\xi^2\delta \tag{23}$$

is a positive definite function of $\xi, \ddot{V} > 0$. When $\alpha = \beta = \xi = 0$, $\beta = \xi = \eta = 0$, or $\alpha = \xi = \eta = 0$ it can also be easily obtained that $\ddot{V} > 0$. By the above results, (iv,a) of the theorem is proved.

Since V is positive definite, $\partial\Omega \cap B_r - G = 0$, (iv,b) need not be proved. V satisfies the Ge-Liu second instability theorem, the sufficient condition of instability for the sleeping top is

$$C^2\omega^2 < 4Amga. \tag{24}$$

Together with the above result of Sec. II B, we conclude that the necessary and sufficient condition for unconditional stability of the sleeping top is

$$C^2\omega^2 \geq 4Amga. \tag{25}$$

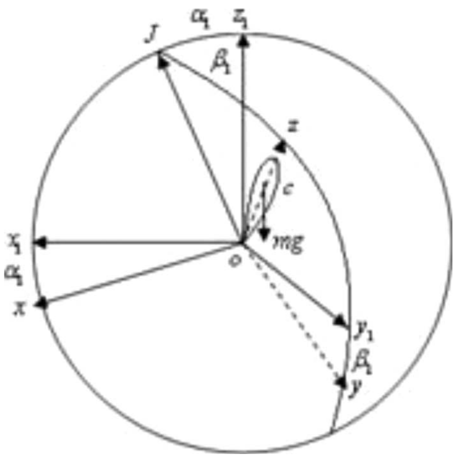

FIG. 2. Sleeping top described by Krylov equations.

III. CONDITIONAL DIRECTION STABILITY OF A SLEEPING TOP DESCRIBED BY KRYLOV EQUATIONS

A. Krylov equations

In Fig. 2, $Ox_1y_1z_1$ is the inertial frame, where Oz_1 is the vertical axis. Oz is the dynamic symmetrical axis of the sleeping top with center of gravity c, where $Oc=a$. The direction of Oz is determined by two angles: α_1, the angle between the projection of Oz on the Ox_1z_1 vertical plane, OJ, and vertical axis Oz_1; β_1 is the angle between OJ and Oz_1. The moving frame $Oxyz$ does not participate in the spin motion of the sleeping top. The motion of Oz is described by $\dot{\alpha}_1$ and $\dot{\beta}_1$. From Fig. 2, $(x,x_1)=\alpha_1$, $(y,y_1)=\beta_1$. ϕ is the spin angle of the sleeping top about Oz. The angular velocity of the sleeping top Ω is

$$\Omega = \omega_1 + \omega_2 + \omega_3, \tag{26}$$

where $\omega_1=\dot{\alpha}_1$, $\omega_2=\dot{\beta}_1$, and $\omega_3=\dot{\phi}$. The projections of Ω on the principal axes Oz,Ox,Oy are, respectively,

$$p = \dot{\phi} + \dot{\alpha}_1 \sin \beta,$$

$$q = -\dot{\beta}_1, r = \dot{\alpha}_1 \cos \beta_1. \tag{27}$$

The kinetic and potential energies of the top are

$$T = [C(\dot{\phi} + \dot{\alpha}_1 \sin \beta_1)^2 + A(\dot{\beta}_1^2 + \dot{\alpha}_1^2 \cos^2 \beta_1)]/2, \tag{28}$$

$$\Pi = mga \cos \alpha_1 \cos \beta_1, \tag{29}$$

where $\gamma=(z,z_1)$. By spherical trigonometry, $\cos \gamma = \cos \alpha_1 \cos \beta_1$. A first integral corresponding to cyclic coordinate ϕ is

$$G_Z = Ap = A(\dot{\phi} + \dot{\alpha}_1 \sin \beta_1) = \text{const}, \tag{30}$$

where G_z is the projection of the angular momentum $\mathbf{G}$ of the top of the Oz axis. Only under condition (30), two-degree-of-freedom Lagrange equations for α_1,β_1, Krylov equations can be obtained:

$$A\ddot{\alpha}_1 \cos \beta_1 - 2A\dot{\alpha}_1\dot{\beta}_1 \sin \beta_1 + C\omega\dot{\beta}_1 = mga \sin \alpha_1,$$

$$A\ddot{\beta}_1 + A\dot{\alpha}_1^2 \sin \beta_1 \cos \beta_1 - C\omega\dot{\alpha}_1 \cos \beta_1 = mga \sin \beta_1 \cos \alpha_1. \tag{31}$$

They correspond to four first order differential equations of four state variables α_1, β_1, $\dot{\alpha}_1$, and $\dot{\beta}_1$, which have zero solution $\alpha_1=\beta_1=\dot{\alpha}_1=\dot{\beta}_1=0$. This solution corresponds to undisturbed sleeping top motion. Therefore Eq. (31) is the differential equation of disturbances α_1, β_1, $\dot{\alpha}_1$, and $\dot{\beta}_1$.

B. Sufficient condition of conditional direction stability

There exist two other first integrals

$$T + \Pi = \frac{1}{2}[C(\dot{\phi} + \dot{\alpha}_1 \sin \beta_1)^2 + A(\dot{\beta}_1^2 + \dot{\alpha}_1^2 \cos^2 \beta_1)] + mga \cos \alpha_1 \cos \beta_1 = \text{const}, \tag{32}$$

$$G_{z_1} = C\omega \cos \alpha_1 \cos \beta_1 + A(\dot{\beta} \sin \alpha_1 - \dot{\alpha} \cos \alpha_1 \cos \beta_1 \sin \beta_1), \tag{33}$$

where G_{z_1} is the projection of $\mathbf{G}$ on the Oz_1 axis. Form the other two first integrals by Eqs. (30), (32), and (33):

$$W_1 = T + \Pi - \frac{G_z^2}{C} = \frac{1}{2}C(\dot{\alpha}_1^2 \cos {}^2\beta_1 + \dot{\beta}_1^2) + mga(\cos \alpha_1 \cos \beta_1 - 1) = \text{const}, \tag{34}$$

$$W_2 = G_{z_1} = A(\dot{\beta}_1 \sin \alpha_1 - \dot{\alpha}_1 \cos \alpha_1 \cos \beta_1 \sin \beta_1) + C\omega(\cos \alpha_1 \cos \beta_1 - 1) = \text{const}. \tag{35}$$

They become zero when the $\alpha_1=\beta_1=\dot{\alpha}_1=\dot{\beta}_1=0$. Lyapunov function is chosen as

$$V = W_1 - \lambda W_2, \tag{36}$$

where λ is a constant to be determined to make V positive definite. Express V in series:

$$V = \frac{1}{2}[A\dot{\alpha}_1^2 + 2A\lambda\dot{\alpha}_1\beta_1 + (C\omega\lambda - mga)\beta_1^2] + \frac{1}{2}[A\dot{\beta}_1^2 - 2A\lambda\dot{\beta}_1\alpha_1 + (C\omega\lambda - mga)\alpha_1^2] + \text{H.O.T.} \tag{37}$$

The degrees of higher order terms (H.O.T.) are no less than four. When λ is chosen as

$$\lambda = C\omega/2A \tag{38}$$

V is a positive definite function of α_1, β_1, $\dot{\alpha}_1$, $\dot{\beta}_1$, and $\dot{V}=0$. Lyapunov stability theorem is satisfied. Therefore when $C^2\omega^2 > 4Amga$, the sleeping top is conditionally stable. Since four disturbances α_1, β_1, $\dot{\alpha}_1$, and $\dot{\beta}_1$ are not all arbitrary, condition (30)must be satisfied, so we call this stability conditional stability. In this case only the direction of the angular velocity vector of the sleeping top is proven to be stable. When

$$C^2\omega^2 = 4Amga \tag{39}$$

V becomes

$$V = V_2 + V_4 + \text{H.O.T.} = \frac{1}{2}A(\dot{\alpha}_1 + C\omega\beta_1/2A)^2 + \frac{1}{2}A\left(\dot{\beta}_1 - \frac{C\omega}{2A}\alpha_1\right)^2 + V_4 + \text{H.O.T.}, \quad (40)$$

where V_2 is a second degree positive semidefinite function and V_4 is a fourth degree function. When

$$\dot{\alpha}_1 = -\lambda_2\beta_1, \quad \dot{\beta}_1 = \lambda_2\alpha_1 \quad (41)$$

$V_2=0$. Substituting Eq. (41) in V_4, after a complicated calculation we obtain

$$V_4 = \frac{mga}{8}(\alpha_1^2 + \beta_1^2)^2. \quad (42)$$

Now

$$V = mga(\alpha_1^2 + \beta_1^2)^2/8 + \text{H.O.T.}, \quad (43)$$

where H.O.T. are terms of α_1, β_1 of degree no less than six. V_4 in Eq. (42) is positive definite for α_1, β_1. We can prove that V in Eq. (40) is positive definite for $\alpha_1, \beta_1, \dot{\alpha}_1, \dot{\beta}_1$ [16]. Lyapunov stability theorem is satisfied. Therefore when $C^2\omega^2=4Amga$, the sleeping top is stable. It is concluded that the sufficient condition of conditional direction stability is $C^2\omega^2 \geq 4Amga$.

When $C^2\omega^2 < 4Amga$, by Lyapunov first approximation theory, the sleeping top is unstable. In this paper, instead of using first approximation theory, a different instability theorem is used to prove that when $C^2\omega^2 < 4Amga$, the sleeping top is unstable.

C. Instability theorem

Consider an autonomous vector differential equation

$$\dot{x} = f(x(t)) \quad \forall\, t \geq 0, \quad (44)$$

where $x \in R^n$, and $f: R^n \to R^n$ is continuous. Let $x=0$ be an equilibrium point for the system described by Eq. (44). Then $f(0)=0$, $\forall t \geq 0$.

Theorem. If there exists a C^n positive definite function $V: R^n \to R$, a ball $B_r = \{x \in R^n \|x\| \leq r\}$, and

(i) There exists an open set $\Omega \subset B_r$ in which $\dot{V}(x) = |(O|x|^2)| > 0$.

(ii) $O \subset \partial\Omega$ (the boundary Ω).

(iii) For $\partial\Omega \cap B_r$, $\dot{V}(x) = O(|x|^4)$, $\ddot{V}(x) = |O(|x|^2)|$,

then the equilibrium O of Eq. (24) is unstable.

Proof. For any trajectory initiated in Ω, we assume that it can escape Ω by moving across $\partial\Omega$. When a trajectory approaches and touches $\partial\Omega$, $\dot{V}$ diminishes from $|O|x|^2|$ to $\dot{V} = O(|x|^4)$ by (i) and (iii), i.e., $\ddot{V}$ is negative; but by (iii), $\ddot{V} = |O(|x|^2)| > 0$. This shows that it is not true that $x(t)$ leaves Ω through $\partial\Omega$.

Next we prove that $x(t)$ must leave B_r through the sphere $\|x\| = r$. The initial point x_0 is in the interior of Ω and $V(x_0) = a > 0$. The trajectory $x(t)$ starting from $x(0)=x_0$ must leave Ω. To prove this fact, we notice that as long as $x(t)$ is inside Ω, $V(x) \geq a$, since $\dot{V}(x) > 0$ in Ω. Let $\gamma = \min\{\dot{V}(x) | x \in \Omega \cup \partial\Omega \quad \text{and} \quad V(x) \geq a\}$ which exists since the continuous function $\dot{V}(x)$ has a minimum over the compact set $\{x \in \Omega \cup \partial\Omega \quad \text{and} \quad V(x) \geq a\}$. Then $\gamma > 0$ and

$$V[x(t)] = V(x_0) + \int_0^t \dot{V}[x(s)]ds \geq a + \int_0^t \gamma ds = a + \gamma. \quad (45)$$

This inequality shows that $x(t)$ cannot stay forever in Ω because V is bounded on Ω. Hence $x(t)$ must leave Ω through the sphere $\|x\| = r$. The origin is unstable.

D. Sufficient condition for instability and the necessary and sufficient condition for conditional direction stability

When $C^2\omega^2 < 4Amga$, choose $\lambda = -C\omega/4A$ and use b, c, d, and e in Eq. (15), where b, d are positive, while c, e are negative. Now

$$V_{\text{I}} = A(\xi - b\alpha)(\xi - c\alpha) + A(\eta - b\beta)(\eta - c\beta) + \frac{C^2}{A}(\varsigma - d\delta)(\varsigma - e\delta).$$

The positive definite Lyapunov function is chosen as

$$V = \dot{\alpha}_1^2 + \dot{\beta}_1^2 + \alpha_1^2 + \beta_1^2. \quad (46)$$

Through Eq. (31), indefinite

$$\dot{V} = 2\frac{(mga + A)}{A}(\alpha_1\dot{\alpha}_1 + \beta_1\dot{\beta}_1) + O(|\alpha^4|) \quad (47)$$

is obtained. There exists Ω in which $\dot{V} = |O(|\alpha_1|^2)| > 0$. We have

$$\ddot{V} = 2\frac{(mga + A)}{A}\left[\frac{mga(\alpha_1^2 + \beta_1^2)}{A} + \frac{C\omega}{A}(\beta_1\dot{\alpha}_1 - \alpha_1\dot{\beta}_1) + \dot{\alpha}_1^2 + \dot{\beta}_1^2\right]. \quad (48)$$

In Ω, when $\alpha_1 + \xi\beta_1 > 0$, $\dot{\alpha}_1 - \dot{\beta}_1/\xi > 0$, where ξ takes any positive value except 0, then $\dot{V} = |O(|\alpha_1|^2)| > 0$. On $\partial\Omega$, $\alpha_1 + \xi\beta_1 = 0$, $\dot{\alpha}_1 - \dot{\beta}_1/\xi = 0$, $\dot{V} = O(|\alpha_1|^4)$, and $\ddot{V}$ becomes

$$\ddot{V} = 2\frac{(mga + A)}{A}\left[\frac{mga(\xi^2 + 1)}{A}\beta_1^2 + \frac{C\omega}{A}\left(\xi + \frac{1}{\xi}\right)\beta_1\dot{\beta}_1 + \left(\frac{1}{\xi^2} + 1\right)\dot{\beta}_1^2\right]. \quad (49)$$

By Sylvester theorem, since $4Amga > C^2\omega^2$, $\ddot{V}$ is positive definite, i.e., $\ddot{V} > 0$ for any ξ. Similarly, when $\alpha_1 - \xi\beta_1 > 0$, $\dot{\alpha}_1 + \dot{\beta}_1/\xi > 0$; $\alpha_1 + \xi\dot{\beta}_1 > 0$, $\dot{\alpha}_1 - \beta_1/\xi > 0$; $\alpha_1 - \xi\dot{\beta}_1 > 0$, $\dot{\alpha}_1 + \beta_1/\xi > 0$, we can also prove that $\ddot{V} > 0$. The above theorem is satisfied. When $C^2\omega^2 < 4amg$, the motion is unstable. It is concluded that the necessary and sufficient condition for conditional direction stability of a sleeping top is also $C^2\omega^2 \geq 4Amga$.

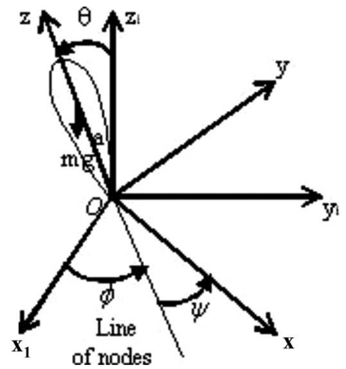

FIG. 3. Three generalized coordinates ϕ, ψ, and θ.

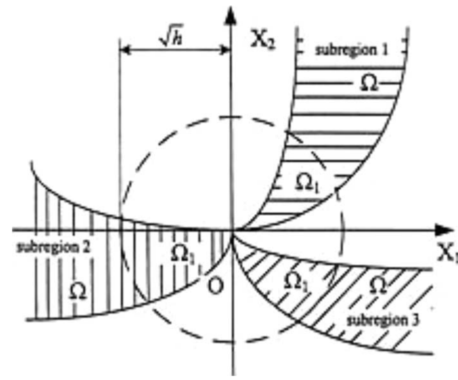

FIG. 4. Partial regions Ω and Ω_1.

IV. CONDITIONAL NUTATION ANGLE STABILITY FOR A SLEEPING TOP DESCRIBED BY NUTATION ANGLE EQUATION

A. Nutation angle equation

In Fig. 3, the symmetric top motion can be described by the Lagrange equation of three generalized coordinates, precession angle ϕ, spin angle ψ, and nutation angle θ. Since ϕ, ψ are cyclic coordinates, there are two corresponding first integrals:

$$(I_1 \sin^2 \theta + I_3 \cos^2 \theta)\dot{\phi} + I_3\dot{\psi}\cos\theta = \text{const},$$

$$I_3(\dot{\psi} + \dot{\phi}\cos\theta) = \text{const}, \tag{50}$$

where I_1 is the equatorial principal moment of inertia, and I_3 is the axial principal moment of inertia. By Eq. (50), $\phi, \dot{\phi}, \psi, \dot{\psi}$ are absent in the only dynamic equation, nutation angle equation. For the sleeping top case, the energy equation is [1]

$$\dot{u}^2 = (1-u^2)[\beta(1+u-a^2)], \tag{51}$$

where $u=\cos\theta$, $\beta=2mga/A$, $a=C\omega/A$. Taking the time derivative of Eq. (51), the nutation angle equation is obtained:

$$\ddot{u} = -(a^2+\beta)u + \frac{3}{2}\beta u^2 + a^2 - \frac{\beta}{2}. \tag{52}$$

For the sleeping top, $u=\cos\theta=\cos 0=1$. Let $u=1+u'$, where u' is disturbance, the standard form equations of disturbances become

$$\dot{u}' = v',$$

$$\dot{v}' = (2\beta - a^2)u' + \frac{3}{2}\beta u'^2. \tag{53}$$

B. GYC partial region stability theorem

Ge, Yao, and Chen [17,18] gave a stability theorem on the partial region of the neighborhood (whole space for global stability) of the origin.

Consider an autonomous differential equation

$$\dot{x} = f(x), \tag{54}$$

where $x \in R^n$, and $f: R^n \to R^n$ is continuous and satisfies the Lipschitz condition. Let $x=0$ be an equilibrium point for the system described by Eq. (56), and $f(0)=0$.

We are only interested in stability of this zero solution on the partial region Ω (including the boundary) of the neighborhood of the origin which in general may consist of several subregions as shown in Fig. 4. It is stipulated that the state point cannot go out of Ω.

Definition. The equilibrium point $x=0$ of Eq. (54) is stable on Ω if for each $\varepsilon>0$ there is $\delta=\delta(\varepsilon)>0$ such that

$$\|x(0)\| < \delta \Rightarrow \|x(t)\| < \varepsilon \quad \forall\, t \geq 0, \tag{55}$$

where $x \neq 0$ is any point in Ω.

Let us consider a continuously differentiable function $V(x)$ given on $\Omega_1 = \Omega \cap H$ where H is the region $\|x\| \leq h > 0$. If $V(x)>0$ in Ω_1 and $V(0)=0$ expect at origin, $V(x)$ is positive definite. If $V(x) \geq$ in Ω_1 and $V(0)=0$, $V(x)$ is positive semidefinite.

Theorem. If $V(x)$ is positive definite, $\dot{V}(x)$ through Eq. (54) is negative semidefinite, $x=0$ is stable in Ω.

The proof of this theorem is similar to that of the Lyapunov stability theorem [19].

C. Necessary and sufficient condition for conditional nutation angle stability

There are three cases.

(a) $\alpha^2 - 2\beta > 0$.

The positive definite Lyapunov function is chosen as

$$V = \frac{v'^2}{2} + \frac{(\alpha^2-2\beta)u'^2}{2} - \frac{\beta u'^3}{2}. \tag{56}$$

The time derivative of $\dot{V}$ through any solution of Eq. (53) is

$$\dot{V} = v'\left[(2\beta-\alpha^2)u' + \frac{3\beta u'^2}{2}\right] - \left[(2\beta-\alpha^2)u' + \frac{3\beta u'^2}{2}\right]v'$$
$$= 0.$$

By Lyapunov stability theorem, the motion is stable.

(b) $\alpha^2 - 2\beta = 0$.

Equation (53) becomes

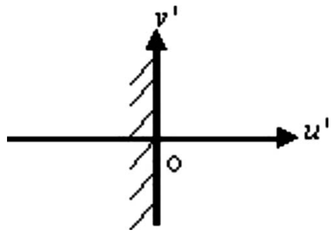

FIG. 5. Partial region.

$$\dot{u}' = v',$$

$$\dot{v}' = \frac{3\beta u'^2}{2}. \tag{57}$$

Since $u=\cos 0=1$ is the maximum value of $\cos\theta$, $u'=u-1$ is always negative. The partial region is the left half plane of the $u'v'$ plane as shown in Fig. 5.

The partial region positive definite Lyapunov function is chosen as

$$V = \frac{v'^2}{2} - \frac{\beta u'^3}{2}. \tag{58}$$

The time derivative of V through any solution of Eq. (57) is

$$\dot{V} = \frac{3v'\beta u'^2}{2} - \frac{3\beta u'^2 v'}{2} = 0. \tag{59}$$

By GYC partial region stability theorem, the motion is stable.

(c) $\alpha^2 - 2\beta < 0$.

The indefinite Lyapunov function is chosen as

$$V = u'v'. \tag{60}$$

The time derivation of V through any solution of Eq. (53) is

$$\dot{V} = v'\dot{u}' + \dot{v}'u' = v'^2 + u'\left[(2\beta - \alpha^2)u' + \frac{3\beta u'^2}{2}\right] = v'^2 + (2\beta - \alpha^2)u'^2 + \frac{3\beta u'^3}{2}, \tag{61}$$

which is positive definite. By Lyapunov first instability theorem, the motion is unstable.

From above results, we obtain that the necessary and sufficient condition for conditional nutation angle stability is also

$$C^2\omega^2 \geq 4Amga. \tag{62}$$

Since two conditions in Eq. (50) must be satisfied, the stability is called the conditional nutation angle stability.

V. CONCLUSIONS

The necessary and sufficient condition for the stability of a sleeping top described by three forms of dynamic equations is obtained. For dynamic equations of six stable variables, Euler equations, and Poisson equations, unconditional stability is obtained by the Lyapunov direct method and the Ge-Liu second instability theorem. For dynamic equations of a two-degree-of-freedom system, Krylov equations, conditional direction stability is obtained by the Lyapunov direct method and a different instability theorem. For dynamic equations of a one-degree-of-freedom system, a nutation angle equation, conditional nutation angle stability is obtained by the Lyapunov direct method and GYC partial region stability theorem. The necessary and sufficient condition for a sleeping top obtained from the above three cases is the same:

$$C^2\omega^2 \geq 4Amga. \tag{63}$$

By using the direct method, unconditional instability, conditional direction stability for $C^2\omega^2 \geq 4Amga$, conditional direction instability, and three cases for conditional nutation angle stability and instability, the results were obtained in this paper.

The classical problem of classical mechanics has been studied for more than 100 years and is solved in this paper by the direct method only without the use of first approximation theory.

ACKNOWLEDGMENTS

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