附件一

行政院國家科學委員會補助專題研究計畫

□ 成 果 報 告

■ 期中進度報告(精簡版)

新渾沌系統與新渾沌控制及同步方法(第二年)

計畫類別：■ 個別型計畫　　□ 整合型計畫
計畫編號：NSC 96 －2221－ E －009 －145－ MY3
執行期間：97 年 8 月 1 日　至　98 年 7 月 31 日

計畫主持人：戈正銘
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計畫參與人員：李仕宇，徐瑜韓

成果報告類型(依經費核定清單規定繳交)：■期中進度精簡報告
□完整報告

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□出席國際學術會議心得報告及發表之論文各一份
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執行單位：國立交通大學機械工程學系

中　華　民　國　98　年　5　月　18　日

中文摘要：

關鍵詞：新 Duffing-van der Pol 系統，新 Mathieu-Duffing 系統，Duffing 系統，Mathieu 系統，Van der Pol 系統，實用適應混沌同步新方法。

渾沌系統之研究在物理、化學、生理學、各種工程等方面皆有日益重要之廣泛應用。Duffing 系統、van der Pol 系統、非線性 Mathieu 系統都是最重要的典型的渾沌系統。本計畫採取適當的耦合方式構成新創的 Duffing-van der Pol 系統、Mathieu-Duffing 系統，從而擴大了各原來單純系統的研究範圍也深化了研究內容。

本計畫提出不同系統實用渾沌適應控制新方法。傳統的渾沌控制侷限於將同一系統之渾沌運動控制至同一系統之週期解或平衡點。此法不僅可將本系統之渾沌控制到另一系統之週期解或平衡點，也可將本系統之週期解或混沌解反控制到另一更複雜系統之渾沌運動。而且涉及的各系統參數可以都是未知參數。大大地擴展了渾沌控制反控制的能力

研究重點為：

1. Duffing-van der Pol 系統與 Mathieu-Duffing 系統之渾沌研究。用相圖、分歧圖、功率譜圖、李氏指數分析渾沌之行為，包括奇異吸引子之範圍及形狀、超渾沌之行為等。
2. 實用渾沌適應控制新方法。將原渾沌系統控制到不同系統之周期解或平衡點，此為渾沌控制。這些系統之參數一般皆未知，所以再加上適應控制法以達同步，再引用具概率觀念之實用穩定理論嚴格證明參數估計值趨近於未知值。

英文摘要：

key words: new Duffing-van der Pol system, new Mathieu-Duffing system, Duffing system, Mathieu system, van der Pol system, Pragmatical adaptive chaos control method.

Chaos systems have obtained wide applications in physics, chemistry, physiology, biology and various engineerings. Duffing system, van der Pol system and nonlinear Mothieu system all are paradigmatic chaotic systems in chaotic dynamics. In this project, by suitable coupling, four new systems, namely, Duffing-van der Pol system, Mathieu-Duffing system are given.

Pragmatical adaptive chaos control for different systems is proposed in this project (the second year). Traditional chaos control and anticontrol only work for the same system. The new method extends the chaos control and anticontrol to other different systems, greatly increases its effectiveness.

The point of research:

1. The study of chaos of Duffing-van der Pol system and Mathieu-Duffing system: By phase portraits, bifurcation diagrams, power spectra, Lyapunov exponents, the various chaotic behaviors of these systems will be studied. The regions and shapes of the strange attractors, hyperchaos, ect will also be studied.
2. New pragmatical adaptive chaos control method for different systems. pragmatical asymptotical stability theory by probability concept is used to prove the estimated parameters must approach the unknown parameters.

報告內容：

（一）前言及研究目的：

渾沌系統之研究除了在理論上的重要價值外，在物理、化學、生理學及各種工程等方面皆有廣泛之應用。Duffing系統、van der Pol系統與非線性Mathieu系統都是重要的典型渾沌系統。對於這些重要系統的渾沌現象及渾沌同步都已有豐富的研究成果[1-53]。本計畫(第二年)為了對這些著名系統，擴大其研究範圍並深化其研究內容，特首先提出混合的新系統，即Duffing-van der Pol系統及Mathieu-Duffing系統。極具實用價值，其渾沌現象值得仔細研究。對上述二種新系統，首先證明其為渾沌系統，其次研究其渾沌行為。

渾沌同步、控制與反控制在物理系統、化學系統、生物系統、各種工程系統、秘密通訊、神經網路、自我組織系統等方面有長足之應用[36-86]。本計畫(第二年)提出一種新渾沌同步方法，對這些新系統加以研究。

（二）研究方法及文獻探討：

（a）Duffing-van der Pol系統及Mathieu-Duffing系統的渾沌行為研究

經典的Duffing系統是非自治系統：

$$\ddot{x}+a\dot{x}+bx+cx^3=d\cos\omega t$$

或

$$\dot{x}=y$$
$$\dot{y}=-ay-bx-cx^3+d\cos\omega t$$

（1）

其中 a,b,c,d 為常數，$d\cos\omega t$ 為外加激勵項。經典的van der Pol系統是非自治系統：

$$\ddot{x}+dx+e\dot{x}(x^2-1)+f\sin\omega t=0$$

或

$$\dot{x}=y$$
$$\dot{y}=-dx+e(1-x^2)y-f\sin\omega t$$

（2）

其中 d,e,f 為常數，$f\sin\omega t$ 為外加激勵項。現將（1）式中及（2）式中的兩個激勵項 $\cos\omega t$ 及 $\sin\omega t$ 交替換成對方的狀態變量，即得到本計畫新創的混合新自治系統（autonomous）的Duffing-van der Pol系統

$$\dot{x}=y$$
$$\dot{y}=-ay-bx-cx^3+du$$
$$\dot{u}=v$$
$$\dot{v}=-du+e(1-u^2)v-fx$$

經典非線性Mathieu系統為

$$\ddot{x}+a(1+\sin\omega t)x+(1+\sin\omega t)x^3+a\dot{x}=0$$

或

$$\dot{x}=y$$
$$\dot{y}=-a(1+\sin\omega t)x-(1+\sin\omega t)x^3-ay$$

（3）

其中 a 為常數，將（3）式中及（1）式中之激勵項 $\sin\omega t$ 及 $\cos\omega t$ 交替換成對方的狀態變量，

即得到本
計畫新創的混合新自治的Mathieu-Duffing系統

$$\dot{x}=y$$
$$\dot{y}=-a(1+u)x-(1+u)x^3-au$$
$$\dot{u}=v$$
$$\dot{v}=-av-bu-cu^3+dx$$

對此系統的研究，不僅是對單Duffing系統、單van der Pol系統行為研究之延伸與深化，此系統比兩個單系統有更複雜的渾沌行為，當可預期。本計畫將研究其周期運動、準周期運動、渾沌運動及超渾沌
運動。

（b）不同系統實用渾沌適應控制反控制新方法及應用

目前文獻中的渾沌控制及反控制皆限於同一系統，即在同一系統中將原來的渾沌運動控制到周期運動或平衡點，謂之渾沌控制。反之，在同一系統中將原來的平衡點或周期運動控制到渾沌運動，謂之渾沌反控制。本方法突破此範圍，將原本系統之渾沌控制到另一任意指定系統之周期運動或平衡點，謂之渾沌控制。將原來系統的平衡點，周期運動控制為另一任意指定系統的渾沌，謂之渾沌反控制。將原來系統的渾沌控制到另一任意指定系統的更複雜渾沌運動，也謂之渾沌反控制。如此一來，渾沌控制與反控制之能力大為增強。另外，由於多數系統之參數皆未得其精確值而屬未知，故加用適應控制法使估計參數趨近於未知參數值。但目前文獻中之適應控制對此種趨近並未提出證明[91-96]，故本計畫採用申請人所提出之實用漸近穩定理論(pragmatical assymptotical stability theorem) [97-98]，引用機率(probability)的概念嚴格證明估計參數值必然趨近於未知參數值。以上說明本方法名稱之由來。下面概述此新方法之要點。

原系統為渾沌或非渾沌系統皆為：

$$\dot{x}=f(x,\hat{A})+u(t)$$

其中 $x=[x_1,.....,x_n]^T\in R^n$ 為狀態向量，$\hat{A}$ 為 f 中之估計參數向量，f 為一非線性向量函數，

$u(t)=[u_1(t),u_2(t),....,u_n(t)]^T\in R^n$ 為輸入控制向量。

目的系統為渾沌或非渾沌系統皆可：

$$\dot{y}=g(y,\hat{B})$$

其中 $y=[y_1,.....,y_n]^T\in R^n$ 為狀態向量，$\hat{B}$ 為 g 中之估計參數向量，g 為一非線性向量函數。

我們的目的是設計一個適應控制方法及控制器 $u(t)$ 使原系統之狀態變量漸近趨於目的系統之對應狀態變量。定義誤差為 $e=y-x$。當

$$\lim_{t\to\infty}e=0$$

則同步完成。由上面二系統方程可得誤差微分方程

$$\dot{e}=\dot{y}-\dot{x}$$

$$\dot{e}=g(y,\hat{B})-f(x,\hat{A})-u(t)$$

選一最簡單的實正 Lyapunov 函數 V

$$V(e,\widetilde{A},\widetilde{B})=\frac{1}{2}e^{T}e+\frac{1}{2}\widetilde{A}^{T}\widetilde{A}+\frac{1}{2}\widetilde{B}^{T}\widetilde{B}$$

其中 $\widetilde{A}=A-\hat{A}$, $\widetilde{B}=B-\hat{B}$, $\hat{A}$, $\hat{B}$ 是給定參數向量或估計參數向量，A 、B 為目的參數向量或不確定參數向量。其沿誤差方程式及參數動態方程之任一解的全導數為

$$\dot{V}(e)=e^{T}[g(y,\hat{B})-f(x,\hat{A})-u(t)]+\widetilde{A}\dot{\widetilde{A}}+\widetilde{B}\dot{\widetilde{B}}$$

其中將 $u(t)$，$\dot{\widetilde{A}}$ 及 $\dot{\widetilde{B}}$ 選成使 $\dot{V}=e^{T}Ce$，C 為對角負定矩陣，則 $\dot{V}$ 為關於 $e,\widetilde{A},\widetilde{B}$ 之負半定函數。故未能

證明 $e,\widetilde{A},\widetilde{B}$ 會趨於零。根據目前文獻[91-96]的做法，由Babalat引理可證明 e 趨於零，但卻沒

有 $\widetilde{A},\widetilde{B}$ 會趨於零之證明。本計畫根據申請人提出之引用概率概念之實用漸近穩定定理嚴格

證明，只要符合一寬鬆的條件，$\widetilde{A},\widetilde{B}$ 一定會趨於零，糾正了目前文獻之錯誤。並以本計畫提出之諸新系統為例，用理論及數值計算證明這新方法之有效性。

結果與討論：

Duffing-van der Pol 系統與 Mathieu-Duffing 系統之渾沌行為與實用渾沌適應控制反控制新方法之研究，及對此二系統的應用所得之結果如下：

1. 採用諸多相圖、分歧圖、功率譜圖、參數圖、李亞普諾夫指數及碎形維度等研究 Duffing-van der Pol 系統及 Mathieu-Duffing 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。

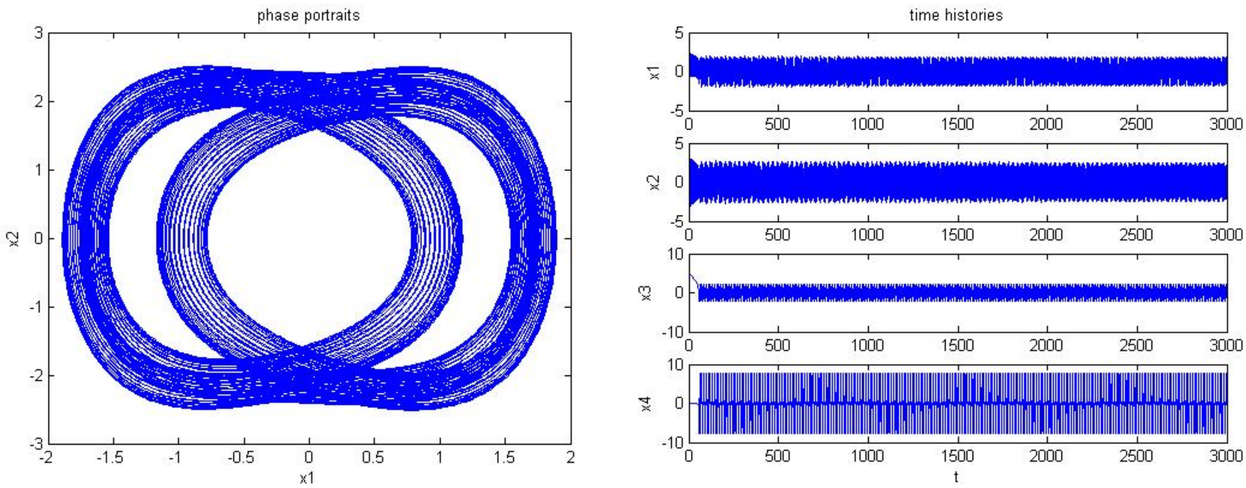


Fig. 1 Chaos of new Duffing-van der Pol system (a) Phase portrait (b) Time histories.

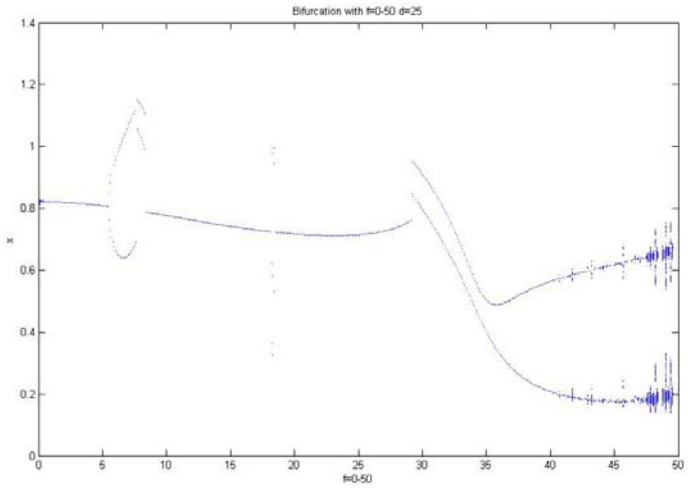
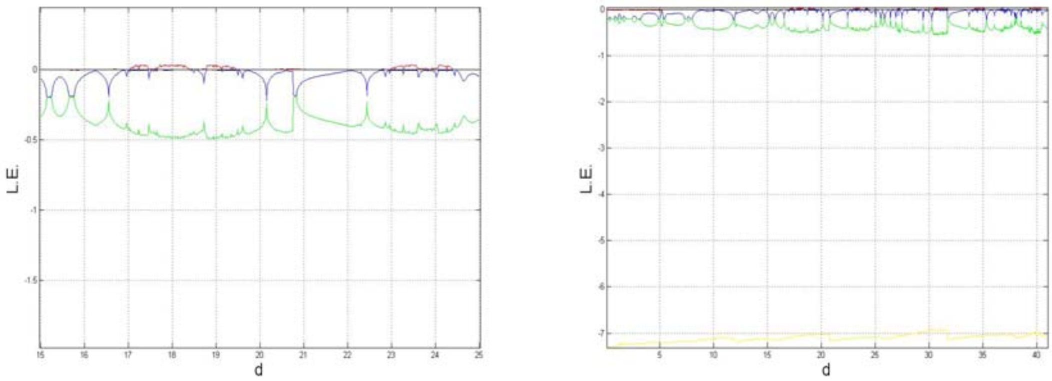


Fig. 2 Bifurcation diagram for autonomous new Mathieu-van der Pol system f=0~50.



(a) (b)

Fig. 3 Lyapunov exponents for autonomous new Duffing-van der Pol system:
(a) d=15~25 (b) d=0~45.

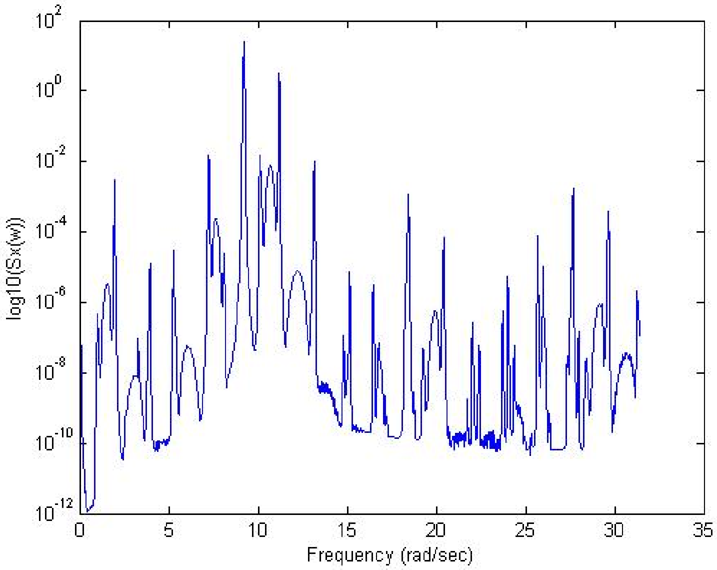
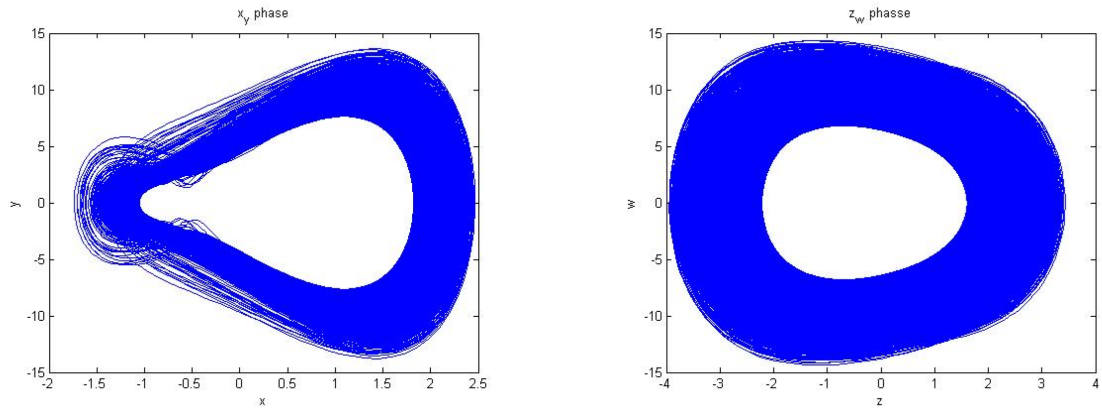


Fig. 4 Power spectrum for autonomous new Duffing-van der Pol system.



(a) (b)

Fig. 5 Phase portraits for chaotic autonomous new Mathieu –Duffing system.

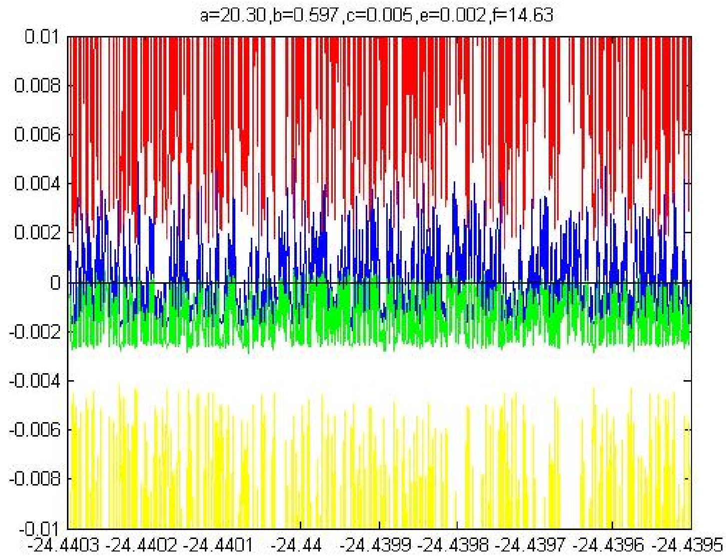


Fig. 6 The Lyapunov exponents of new Mathieu –Duffing system

2. 完成研究實用渾沌適應控制反控制新方法對雙 Duffing-van der Pol 系統之應用。
3. 完成研究實用渾沌適應控制反控制新方法對雙 Mathieu-Duffing 系統之應用。

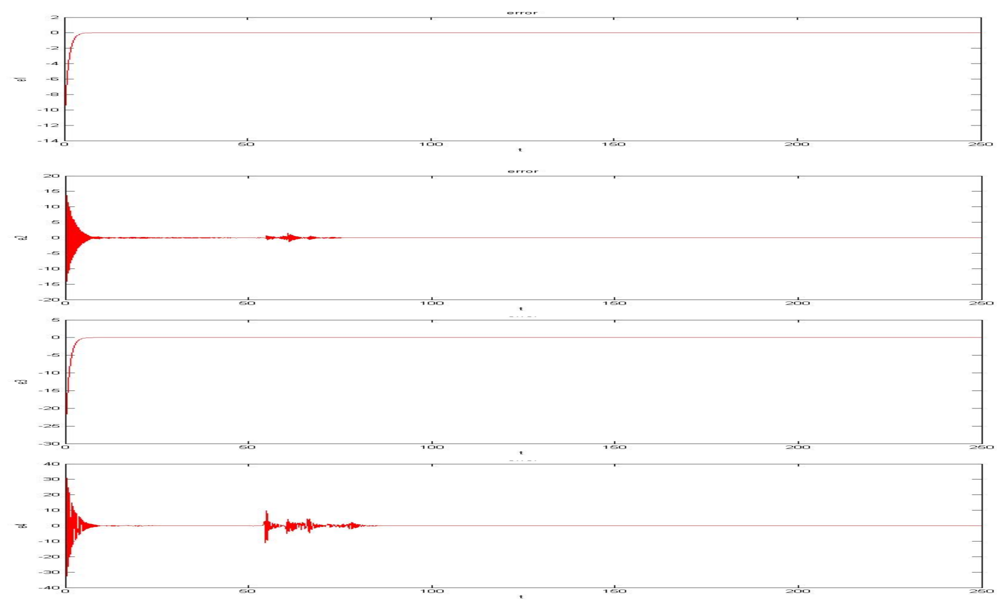


Fig. 7 Time history of errors for autonomous new Duffing-van der Pol system by pragmatical asymptotical stability theorem is obtained.

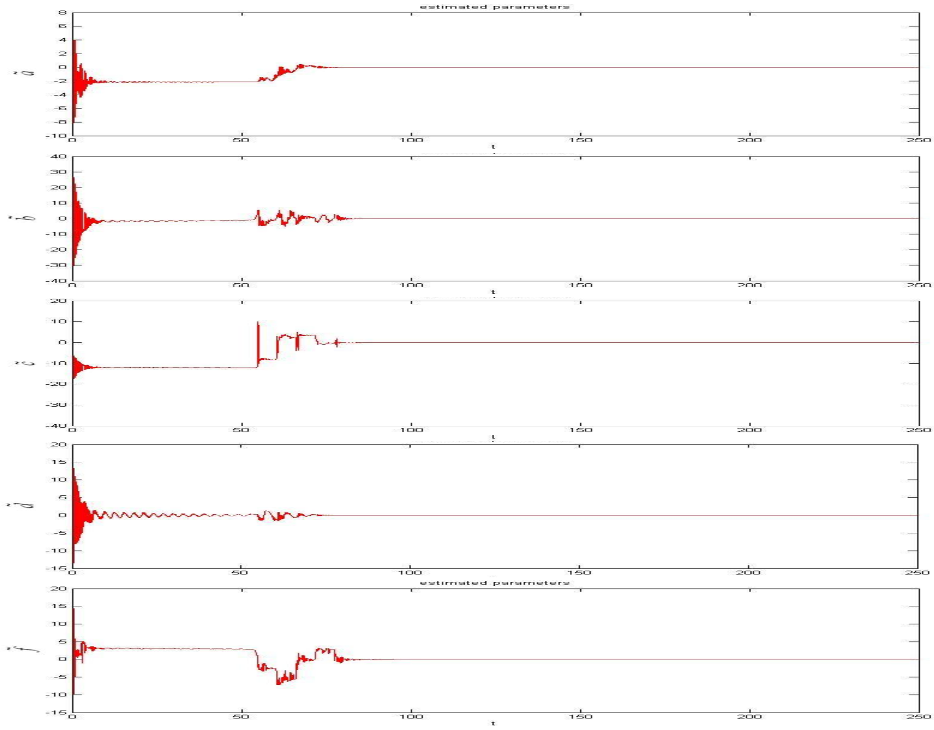


Fig. 8 Time histories of the differences of uncertain parameters and Estimated parameters for autonomous new Duffing-van der Pol system by pragmatical asymptotical stability theorem is obtained.

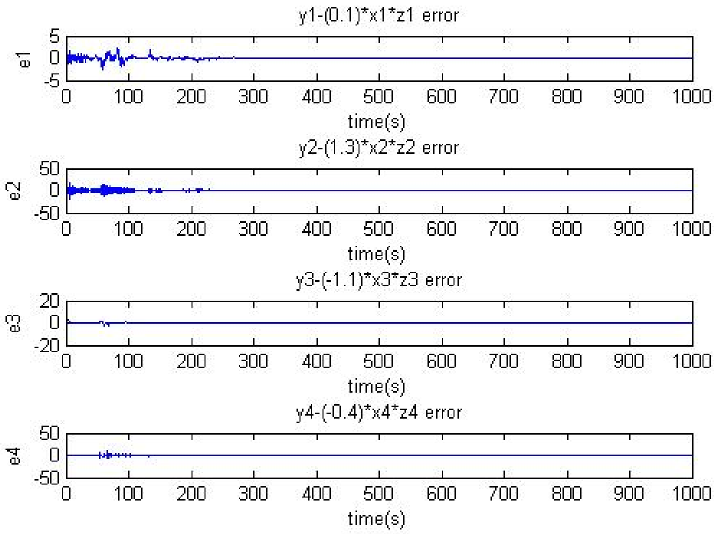


Fig. 9 Time history of errors for autonomous new Mathieu-Duffing system by pragmatical asymptotical stability theorem is obtained.

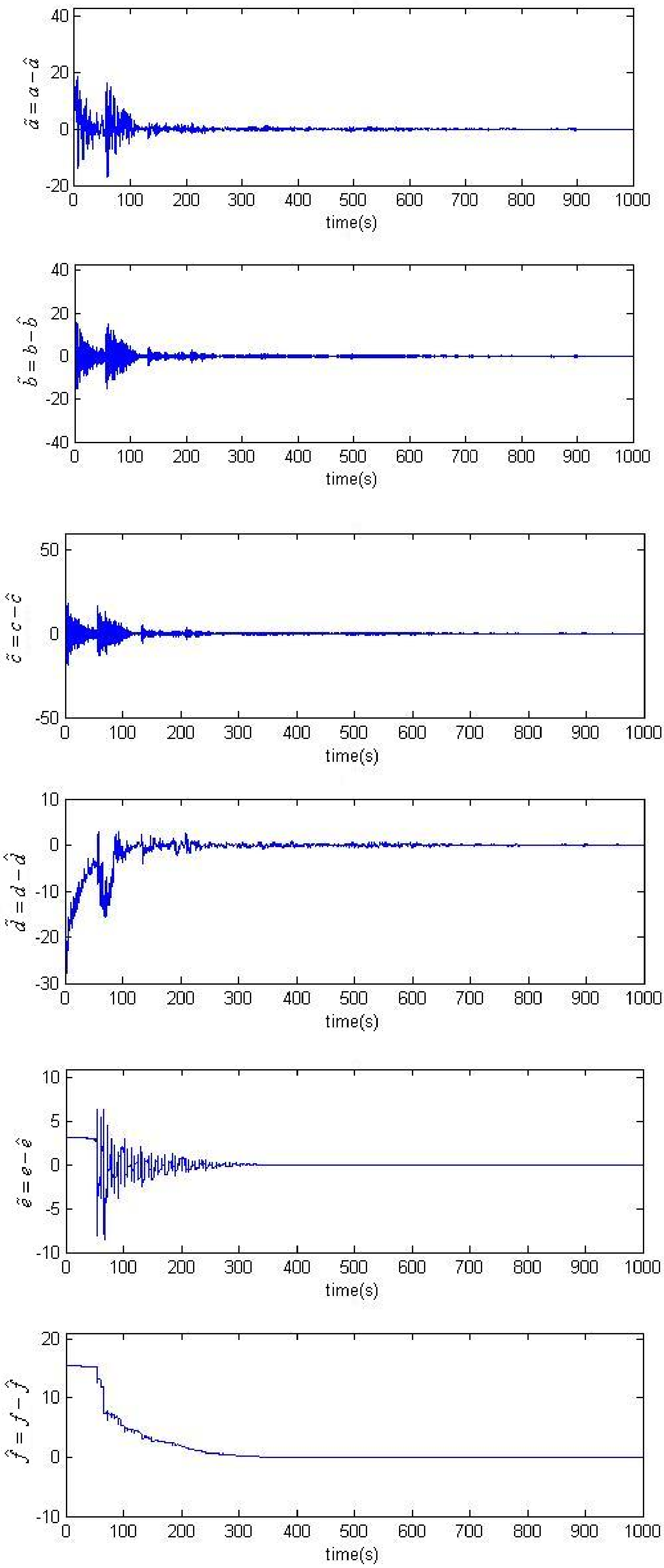


Fig. 10 Time histories of $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}$, $\tilde{e}$ and $\tilde{f}$ for autonomous new Mathieu-Duffing system

by pragmatical asymptotical stability theorem is obtained..

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計畫(第二年)成果自評：

Duffing系統、van der Pol系統及Mathieu系統皆為經典之系統。經巧妙之取代與結合既可獲得自治新Duffing−van der Pol系統，自治新Mathieu-Duffing系統亦可得非自治之新Duffing−van der Pol系統與非自治新Mathieu-Duffing系統大大擴大了經典渾沌系統的範圍及研究領域。而不同系統實用渾沌適應控制新方法具有很大的發展潛力，涉及的各系統參數可以都是未知參數。大大地擴展了渾沌控制反控制的能力。取代之參數可以變化多端，採用之取代函數更是取之不盡。對用於秘密通訊而言，機密性大為增加。已投出之國際著名期刊論文已達九篇。

1. Zheng-Ming Ge and Kai-Ming Hsu “Pragmatical Chaotic Symplectic Synchronization of New Duffing-Van der Pol Systems with Different Order System as Functional System by New Dynamic Surface Control and Adaptive Control”, submitted to Chaos, Solitons & Fractals. (SCI, Impact factor: 3.025)
2. Zheng-Ming Ge and Kai-Ming Hsu “Pragmatical Hybrid Projective Chaotic Generalized Synchronization of Chaotic System with Uncertain Parameters by Adaptive Control”, submitted to International Journal of Robust and Nonlinear Control. (SCI, Impact factor: 1.637)
3. Zheng-Ming Ge and Kai-Ming Hsu, “Chaos Control of New Duffing-Van der Pol System by GYC Partial Region Stability Theory”, submitted to Journal of Computational and Applied Mathematics. (SCI, Impact factor: 0.943)
4. Zheng-Ming Ge and Kai-Ming Hsu, “Chaos Generalized Synchronization of New Duffing-Van der Pol System by GYC Partial Region Stability Theory”, submitted to Applied Mathematical Modelling. (SCI, Impact factor: 0.572)
5. Zheng-Ming Ge, Yan-Sian Li, “Pragmatical Hybrid Projective Generalized Synchronization of New Mathieu- Duffing Systems with Bessel Function Parameters by Adaptive Control and GYC Partial Region Stability Theory”, submitted to Applied physics Letter. (SCI, Impact factor: 3.596)
6. Zheng-Ming Ge, Yan-Sian Li, “Pragmatical Hybrid Projective Chaotic Generalized Synchronization of Chaotic Systems by Adaptive Backstepping Control”, submitted to International Journal of Robust and Nonlinear Control. (SCI, Impact factor: 1.637)
7. Zheng-Ming Ge, Yan-Sian Li, “Symplectic Hybrid Projective Synchronization of Different Order Systems with New Control Lyapunov Function by Adaptive Backstepping Control”, submitted to Physica A: Statistical Mechanics and its Applications. (SCI, Impact factor: 1.430)
8. Zheng-Ming Ge, Yan-Sian Li, “Chaos Control of New Mathieu- Duffing Systems by GYC Partial Region Stability Theory”, submitted to Mechanical Systems and Signal Processing. (SCI, Impact factor: 1.333)
9. Zheng-Ming Ge, Yan-Sian Li, “Chaos Generalized Synchronization of New Mathieu- Duffing Systems by GYC Partial Region Stability Theory”, submitted to Mathematics and Computers in Simulation. (SCI, Impact factor: 0.738)

以本計劃經費資助出版之國際著名期刊論文二篇，見附錄。

附錄

1. Zheng-Ming Ge and Shih-Yu Li "Chaos Control of New Mathieu-Van der Pol Systems with New Mathieu -Duffing Systems as Functional System by GYC Partial Region Stability Theory" accepted by Nonlinear Analysis: Theory, Methods, and Applications (2009). (SCI, Impact factor: 1.097)
2. Zheng-Ming Ge, Cheng-Hsiung Yang "Chaos synchronization and chaotization of complex chaotic systems in series form by optimal control", accepted by Chaos, Solitons & Fractals (2009). (SCI, Impact factor: 3.025)

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Chaos control of new Mathieu–Van der Pol systems with new Mathieu–Duffing systems as functional system by GYC partial region stability theory

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ABSTRACT

In this paper, a new strategy by using GYC partial region stability theory is proposed to achieve chaos control. Using the GYC partial region stability theory, the new Lyapunov function used is a simple linear homogeneous function of error states and the lower order controllers are much more simple and introduce less simulation error. Numerical simulations are given for new Mathieu–Van der Pol system and new Mathieu–Duffing system to show the effectiveness of this strategy.

1. Introduction

Since Ott et al. [1] gave the famous OGY control method in 1990, the applications of the various methods to control a chaotic behavior in natural sciences and engineering are well known. For example, the adaptive control [2–5], the method of chaos control based on sampled data [6], the method of pulse feedback of systematic variable [7], the active control [8,9] and linear error feedback control [10,11]. However, when Lyapunov stability of zero solution of states is studied, the stability of solutions on the whole neighborhood region of the origin is demanded.

In this paper, a new strategy to achieve chaos control by GYC partial region stability theory is proposed [12,13]. Using the GYC partial region stability theory, the new Lyapunov function is a simple linear homogeneous function of error states and the lower order controllers are much more simple and introduce less simulation error.

The layout of the rest of the paper is as follows. In Section 2, chaos control scheme by GYC partial region stability theory is proposed. In Section 3, new Mathieu–Van der pol system and new Mathieu–Duffing system are presented. In Section 4, three simulation examples are given. In Section 5, conclusions are drawn. The partial region stability theory is enclosed in Appendix.

2. Chaos control scheme

Consider the following chaotic system

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}) \tag{2.1}$$

where $\mathbf{x} = [x_1, x_2, \dots, x_n]^T \in R^n$ is a state vector, $\mathbf{f} : R_+ \times R^n \to R^n$ is a vector function.

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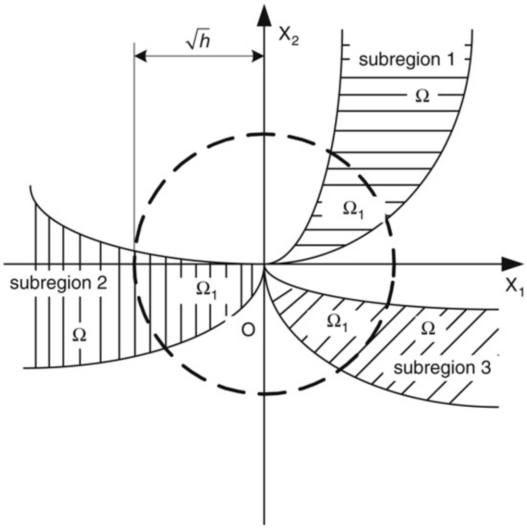

Fig. 1. Partial regions Ω and Ω_1.

The goal system which can be either chaotic or regular, is

$$\dot{\mathbf{y}} = \mathbf{g}(t, \mathbf{y}) \tag{2.2}$$

where $\mathbf{y} = [y_1, y_2, \ldots, y_n]^T \in R^n$ is a state vector, $\mathbf{g} : R_+ \times R^n \to R^n$ is a vector function.

In order to make the chaos state $\mathbf{x}$ approaching the goal state $\mathbf{y}$, define $\mathbf{e} = \mathbf{x} - \mathbf{y}$ as the state error. The chaos control is accomplished in the sense that [13–22]:

$$\lim_{t\to\infty} \mathbf{e} = \lim_{t\to\infty} (\mathbf{x} - \mathbf{y}) = 0. \tag{2.3}$$

In this paper, we will use examples in which the error dynamics always happens in the first quadrant of coordinate system and use GYC partial region stability theory which is enclosed in the Appendix. The Lyapunov function is a simple linear homogeneous function of error states and the controllers are simpler because they are in lower order than that of traditional controllers.

3. New Chaotic Mathieu–Van der pol system and new chaotic Mathieu–Duffing system

This section introduces new Mathieu–van der Pol system and new Mathieu–Duffing system, respectively.

3.1. New Mathieu–Van der Pol system

Mathieu equation and van der Pol equation are two typical nonlinear nonautonomous systems:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -(a + b\sin\omega t)x_1 - (a + b\sin\omega t)x_1^3 - cx_2 + d\sin\omega t \end{cases} \tag{3.1}$$

$$\begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -ex_3 + f(1 - x_3^2)x_4 + g\sin\omega t. \end{cases} \tag{3.2}$$

Exchanging $\sin\omega t$ in Eq. (3.1) with x_3 and $\sin\omega t$ in Eq. (3.2) with x_1, we obtain the autonomous new Mathieu–Van der Pol system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -(a + bx_3)x_1 - (a + bx_3)x_1^3 - cx_2 + dx_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -ex_3 + f(1 - x_3^2)x_4 + gx_1 \end{cases} \tag{3.3}$$

where a, b, c, d, e, f, g are uncertain parameters. This system exhibits chaos when the parameters of system are $a = 10$, $b = 3, c = 0.4, d = 70, e = 1, f = 5, g = 0.1$ and the initial states of system are $(x_{10}, x_{20}, x_{30}, x_{40}) = (0.1, -0.5, 0.1, -0.5)$. Its phase portraits are shown in Fig. 2.

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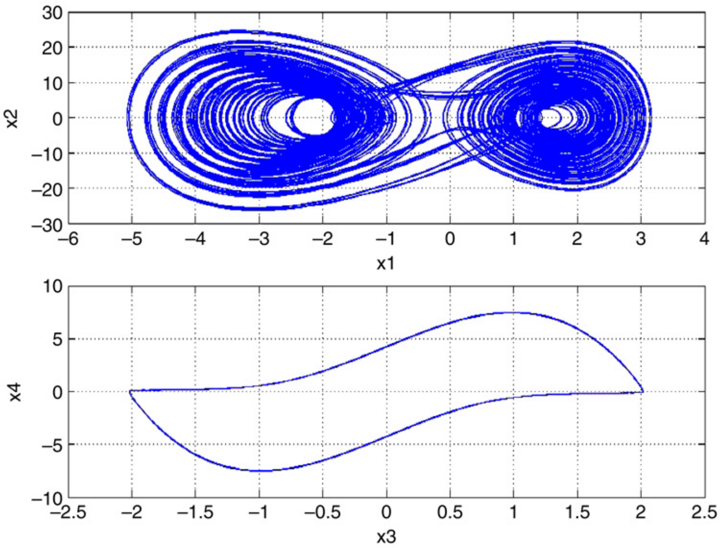

Fig. 2. Chaotic phase portraits for new Mathieu–Van der Pol system.

3.2. New Mathieu–Duffing system

Mathieu equation and Duffing equation are two typical nonlinear nonautonomous systems:

$$\begin{cases}\dot{z}_1 = z_2\\ \dot{z}_2 = -(a_1 + b_1 \sin\omega t)z_1 - (a_1 + b_1 \sin\omega t)z_1^3 - c_1 z_2 + d_1 \sin\omega t\end{cases} \tag{3.4}$$

$$\begin{cases}\dot{z}_3 = z_4\\ \dot{z}_4 = -z_3 - z_3^3 - e_1 z_4 + f_1 \sin\omega t.\end{cases} \tag{3.5}$$

Exchanging $\sin\omega t$ in Eq. (3.4) with z_3 and $\sin\omega t$ in Eq. (3.5) with z_1, we obtain the autonomous master new Mathieu–Duffing system:

$$\begin{cases}\dot{z}_1 = z_2\\ \dot{z}_2 = -(a_1 + b_1 z_3)z_1 - (a_1 + b_1 z_3)z_1^3 - c_1 z_2 + d_1 z_3\\ \dot{z}_3 = z_4\\ \dot{z}_4 = -z_3 - z_3^3 - e_1 z_4 + f_1 z_1\end{cases} \tag{3.6}$$

where a_1, b_1, c_1, d_1, e_1 and f_1 are uncertain parameters. This system exhibits chaos when the parameters of system are $a_1 = 20.30, b_1 = 0.5970, c_1 = 0.005, d_1 = -24.441, e_1 = 0.002, f_1 = 14.63$ and initial states is $(-2, 10, -2, 10)$. Its phase portraits are shown in Fig. 3.

4. Numerical simulations

The following chaotic system

$$\begin{cases}\dot{x}_1 = x_2 - 200\\ \dot{x}_2 = -(a + b(x_3 - 200))(x_1 - 200) - (a + b(x_3 - 200))(x_1 - 200)^3 - c(x_2 - 200) + d(x_3 - 200)\\ \dot{x}_3 = (x_4 - 200)\\ \dot{x}_4 = -e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) + g(x_1 - 200)\end{cases} \tag{4.1}$$

is the new Mathieu–Van der pol system of which the old origin is translated to $(x_1, x_2, x_3, x_4) = (200, 200, 200, 200)$ in order that the error dynamics happens always in the first quadrant of error state coordinate system. This translated new Mathieu–Van der pol system presents chaotic motion when initial conditions is $(x_{10}, x_{20}, x_{30}, x_{40}) = (210.1, 209.5, 210.1, 209.5)$ and the parameters are $a = 10, b = 3, c = 0.4, d = 70, e = 1, f = 5, g = 0.1$.

In order to lead (x_1, x_2, x_3, x_4) to the goal, we add control terms u_1, u_2, u_3 and u_4 to each equation of Eq. (4.1), respectively.

$$\begin{cases}\dot{x}_1 = x_2 - 200 + u_1\\ \dot{x}_2 = -(a + b(x_3 - 200))(x_1 - 200) - (a + b(x_3 - 200))(x_1 - 200)^3 - c(x_2 - 200) + d(x_3 - 200) + u_2\\ \dot{x}_3 = (x_4 - 200) + u_3\\ \dot{x}_4 = -e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) + g(x_1 - 200) + u_4.\end{cases} \tag{4.2}$$

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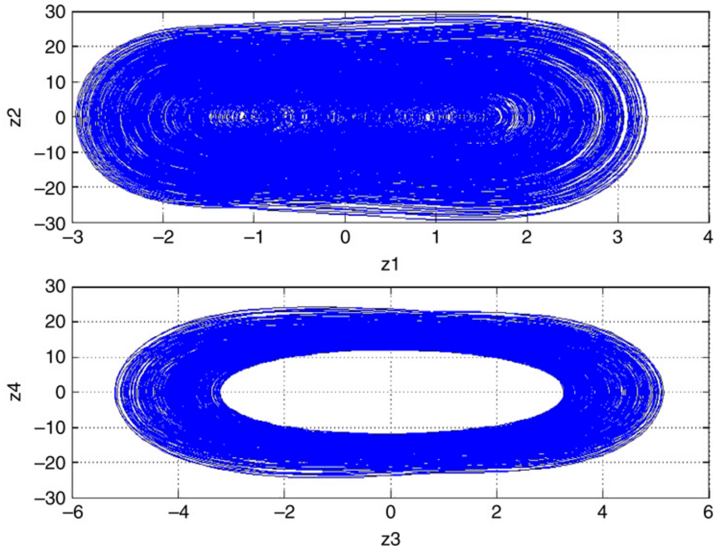

Fig. 3. Chaotic phase portraits for new Mathieu–Duffing system in the first quadrant.

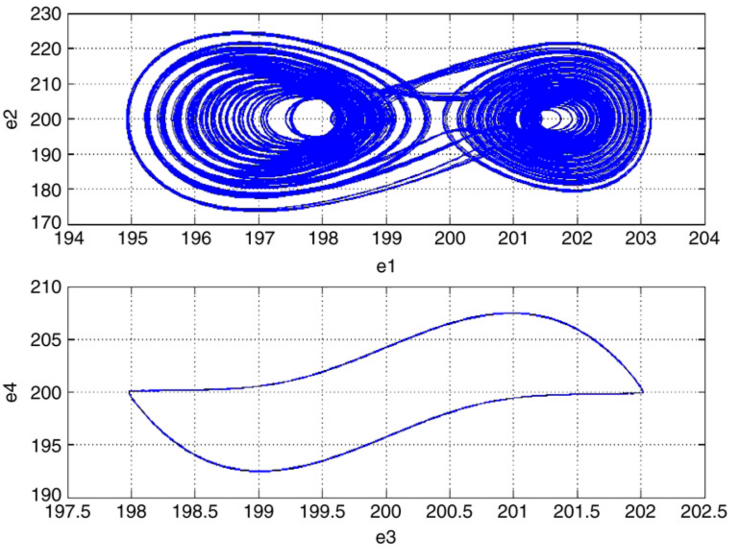

Fig. 4. Phase portrait of error dynamics for Case I.

CASE I. Control the chaotic motion to zero.

In this case we will control the chaotic motion of the new Mathieu–Van der pol system (4.1) to zero. The goal is $y = 0$. The state error is $e_i = x_i - y_i = x_i$, $(i = 1, 2, 3, 4)$ and error dynamics becomes

$$\begin{cases} \dot{e}_1 = \dot{x}_1 = x_2 - 200 + u_1 \\ \dot{e}_2 = \dot{x}_2 = -(a + b(x_3 - 200))(x_1 - 200) - (a + b(x_3 - 200))(x_1 - 200)^3 \\ \qquad - c(x_2 - 200) + d(x_3 - 200) + u_2 \\ \dot{e}_3 = \dot{x}_3 = (x_4 - 200) + u_3 \\ \dot{e}_4 = \dot{x}_4 = -e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) + g(x_1 - 200) + u_4. \end{cases} \tag{4.3}$$

In Fig. 4, we can see that the error dynamics always exists in first quadrant.

By GYC partial region asymptotical stability theorem, one can easily choose a Lyapunov function in the form of a positive definite function in first quadrant as:

$$V = e_1 + e_2 + e_3 + e_4. \tag{4.4}$$

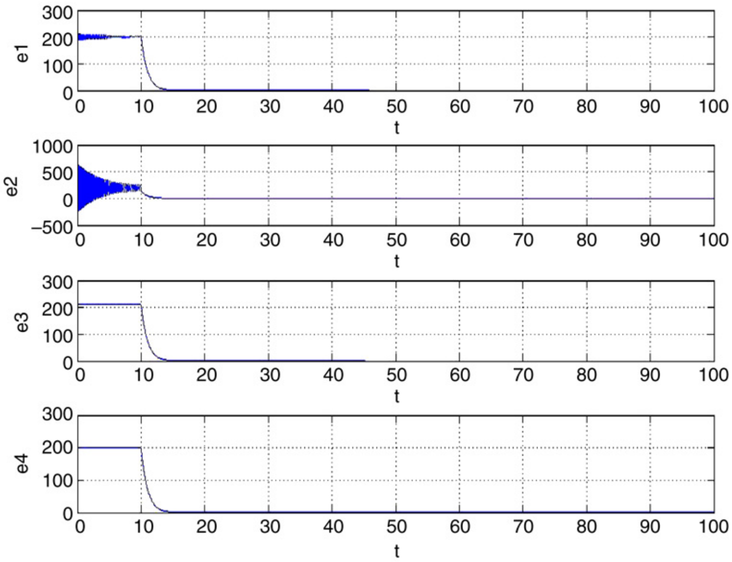

Fig. 5. Time histories of errors for Case I.

Its time derivative through error dynamics (4.3) is

$$\begin{aligned}\dot{V} &= \dot{e}_1 + \dot{e}_2 + \dot{e}_3 + \dot{e}_4 \\ &= (x_2 - 200 + u_1) + (-(a + b(x_3 - 200))(x_1 - 200) \\ &\quad - (a + b(x_3 - 200))(x_1 - 200)^3 - c(x_2 - 200) + d(x_3 - 200) + u_2) + (x_4 - 200 + u_3) \\ &\quad + (-e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) + g(x_1 - 200) + u_4).\end{aligned} \tag{4.5}$$

Choose

$$\begin{aligned} u_1 &= -(x_2 - 200) - e_1 \\ u_2 &= (-(a + b(x_3 - 200))(x_1 - 200) - (a + b(x_3 - 200))(x_1 - 200)^3 - c(x_2 - 200) + d(x_3 - 200)) - e_2 \\ u_3 &= -(x_4 - 200) - e_3 \\ u_4 &= (-e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) + g(x_1 - 200)) - e_4. \end{aligned} \tag{4.6}$$

We obtain

$$\dot{V} = \dot{e}_1 + \dot{e}_2 + \dot{e}_3 + \dot{e}_4 < 0$$

which is negative definite function in first quadrant. The numerical results are shown in Fig. 5. After 10 s, the error trajectories approach the origin.

CASE II. Control the chaotic motion to a regular function.

In this case we will control the chaotic motion of the new Mathieu–Van der pol system (4.1) to regular function of time. The goal is $y_i = F_i e^{\sin \omega t}$, $(i = 1, 2, 3, 4)$. The error equation

$$\begin{aligned} &e_i = x_i - y_i = x_i - F_i e^{\sin \omega t}, \quad (i = 1, 2, 3, 4) \\ &\lim_{t\to\infty} e_i = \lim_{t\to\infty} (x_i - F_i e^{\sin \omega t}) = 0, \quad (i = 1, 2, 3, 4) \end{aligned} \tag{4.7}$$

where $F_1 = F_2 = F_3 = F_4 = F = 10$ and $\omega = 0.5$.

The error dynamics is

$$\begin{cases} \dot{e}_1 = x_2 - 200 + u_1 - F_1 \omega e^{\sin \omega t} (\cos \omega t) \\ \dot{e}_2 = -(a + b(x_3 - 200))(x_1 - 200) - (a + b(x_3 - 200))(x_1 - 200)^3 \\ \qquad - c(x_2 - 200) + d(x_3 - 200) + u_2 - F_2 \omega e^{\sin \omega t} (\cos \omega t) \\ \dot{e}_3 = (x_4 - 200) + u_3 - F_3 \omega e^{\sin \omega t} (\cos \omega t) \\ \dot{e}_4 = -e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) + g(x_1 - 200) + u_4 - F_4 \omega e^{\sin \omega t} (\cos \omega t). \end{cases} \tag{4.8}$$

In Fig. 6, the error dynamics always exists in first quadrant.

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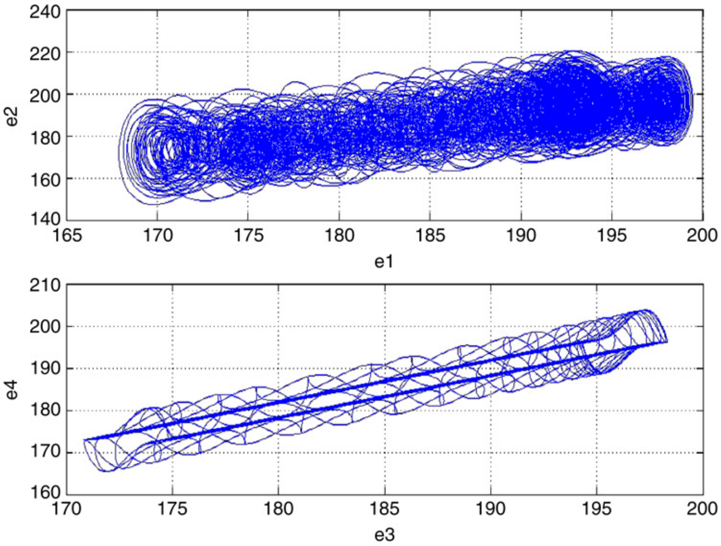

Fig. 6. Phase portraits of error dynamics for Case II.

By GYC partial region asymptotical stability theorem, one can easily choose a Lyapunov function in the form of a positive definite function in first quadrant as:

$$V = e_1 + e_2 + e_3 + e_4.$$

Its time derivative is

$$\begin{aligned} V &= \dot{e}_1 + \dot{e}_2 + \dot{e}_3 + \dot{e}_4 = (x_2 - 200 + u_1 - F_1\omega e^{\sin\omega t}(\cos\omega t)) + (-(a + b(x_3 - 200))(x_1 - 200) \\ &\quad - (a + b(x_3 - 200))(x_1 - 200)^3 - c(x_2 - 200) + d(x_3 - 200) + u_2 - F_2\omega e^{\sin\omega t}(\cos\omega t)) \\ &\quad + ((x_4 - 200) + u_3 - F_3\omega e^{\sin\omega t}(\cos\omega t)) + (-e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) \\ &\quad + g(x_1 - 200) + u_4 - F_4\omega e^{\sin\omega t}(\cos\omega t)). \end{aligned} \tag{4.9}$$

Choose

$$\begin{aligned} u_1 &= -(x_2 - 200 - F_1\omega e^{\sin\omega t}(\cos\omega t)) - e_1 \\ u_2 &= -(-(a + b(x_3 - 200))(x_1 - 200) - (a + b(x_3 - 200))(x_1 - 200)^3 \\ &\quad - c(x_2 - 200) + d(x_3 - 200) - F_2\omega e^{\sin\omega t}(\cos\omega t)) - e_2 \\ u_3 &= -((x_4 - 200) - F_3\omega e^{\sin\omega t}(\cos\omega t)) - e_3 \\ u_4 &= -(-e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) + g(x_1 - 200) - F_4\omega e^{\sin\omega t}(\cos\omega t)) - e_4. \end{aligned} \tag{4.10}$$

We obtain

$$\dot{V} = -e_1 - e_2 - e_3 - e_4 < 0$$

which is a negative definite function in first quadrant. The numerical results are shown in Figs. 7 and 8. After 10 s, the errors approach zero and the chaotic trajectories approach to regular motion.

CASE III. Control the chaotic motion of the new Mathieu–Van der pol system to chaotic motion of the new Mathieu–Duffing system.

In this case we will control chaotic motion of the new Mathieu–Van der pol system (4.1) to that of the new chaotic Mathieu–Duffing system. The goal system for control is new Mathieu–Duffing system with initial states (−2, 10, −2, 10), system parameters $a_1 = 20.30$, $b_1 = 0.5970$, $c_1 = 0.005$, $d_1 = -24.441$, $e_1 = 0.002$ and $f_1 = 14.63$.

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -(a_1 + b_1z_3)z_1 - (a_1 + b_1z_3)z_1^3 - c_1z_2 + d_1z_3 \\ \dot{z}_3 = z_4 \\ \dot{z}_4 = -z_3 - z_3^3 - e_1z_4 + f_1z_1. \end{cases} \tag{4.11}$$

The error equation is $e_i = x_i - z_i$, $(i = 1, 2, 3, 4)$. Our aim is $\lim_{t\to\infty} e_i = 0$, $(i = 1, 2, 3, 4)$.

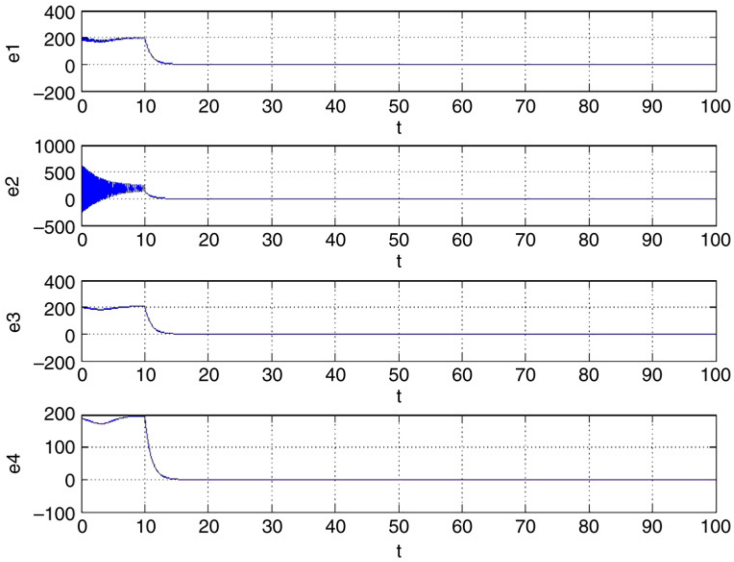

Fig. 7. Time histories of errors for Case II.

The error dynamics becomes

$$\begin{cases} \dot{e}_1 = \dot{x}_1 - \dot{z}_1 = (x_2 - 200 - z_2) + u_1 \\ \dot{e}_2 = \dot{x}_2 - \dot{z}_2 = (-(a + b(x_3 - 200))(x_1 - 200) - (a + b(x_3 - 200))(x_1 - 200)^3 \\ \qquad - c(x_2 - 200) + d(x_3 - 200) - (-(a_1 + b_1 z_3)z_1 - (a_1 + b_1 z_3)z_1^3 - c_1 z_2 + d_1 z_3)) + u_2 \\ \dot{e}_3 = \dot{x}_3 - \dot{z}_3 = (x_4 - 200 - z_4) + u_3 \\ \dot{e}_4 = \dot{x}_4 - \dot{z}_4 = (-e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) \\ \qquad + g(x_1 - 200) - (-z_3 - z_3^3 - e_1 z_4 + f_1 z_1)) + u_4. \end{cases} \tag{4.12}$$

In Fig. 9, the error dynamics always exists in first quadrant.

By GYC partial region asymptotical stability theorem, one can easily choose a Lyapunov function in the form of a positive definite function in first quadrant as:

$$V = e_1 + e_2 + e_3 + e_4.$$

Its time derivative is

$$\begin{aligned} \dot{V} &= \dot{e}_1 + \dot{e}_2 + \dot{e}_3 + \dot{e}_4 = ((x_2 - 200 - z_2) + u_1) + ((-(a + b(x_3 - 200))(x_1 - 200) \\ &\quad - (a + b(x_3 - 200))(x_1 - 200)^3 - c(x_2 - 200) + d(x_3 - 200) - (-(a_1 + b_1 z_3)z_1 - (a_1 + b_1 z_3)z_1^3 \\ &\quad - c_1 z_2 + d_1 z_3)) + u_2) + ((x_4 - 200 - z_4) + u_3) + ((-e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) \\ &\quad + g(x_1 - 200) - (-z_3 - z_3^3 - e_1 z_4 + f_1 z_1)) + u_4). \end{aligned} \tag{4.13}$$

Choose

$$\begin{aligned} u_1 &= -(x_2 - 200 - z_2) - e_1 \\ u_2 &= -(-(a + b(x_3 - 200))(x_1 - 200) - (a + b(x_3 - 200))(x_1 - 200)^3 - c(x_2 - 200) \\ &\quad + d(x_3 - 200) - (-(a_1 + b_1 z_3)z_1 - (a_1 + b_1 z_3)z_1^3 - c_1 z_2 + d_1 z_3)) - e_2 \\ u_3 &= -(x_4 - 200 - z_4) - e_3 \\ u_3 &= -(-e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) + g(x_1 - 200) - (-z_3 - z_3^3 - e_1 z_4 + f_1 z_1)) - e_4. \end{aligned} \tag{4.14}$$

We obtain

$$\dot{V} = -e_1 - e_2 - e_3 - e_4 < 0$$

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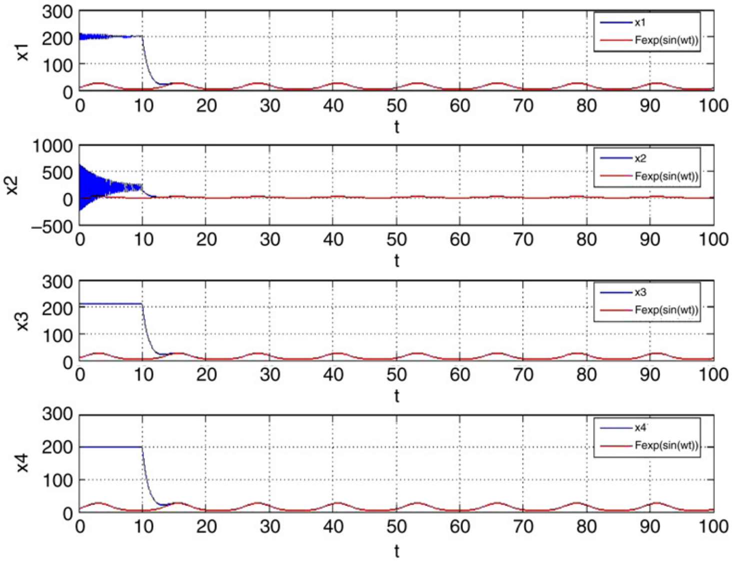

Fig. 8. Time histories of x_1, x_2, x_3, x_4 for Case II.

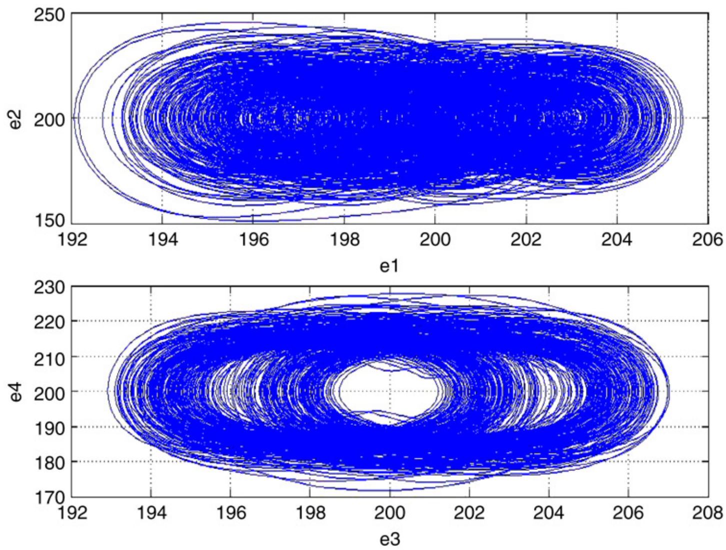

Fig. 9. Phase portraits of error dynamics for Case III.

which is negative definite function in first quadrant. The numerical results are shown in Figs. 10 and 11. After 10 s, the errors approach zero and the chaotic trajectories of the new Mathieu–Van der pol system approach to that of the new Mathieu–Duffing system.

5. Conclusions

In this paper, a new strategy by using GYC partial region stability theory is proposed to achieve chaos control. Using the GYC partial region stability theory, the new Lyapunov function used is a simple linear homogeneous function of states and the lower order controllers are much more simple and introduce less simulation error. The new chaotic Mathieu–Van der pol system and new chaotic Mathieu–Duffing system system are used as simulation examples which confirm the scheme effectively.

Acknowledgment

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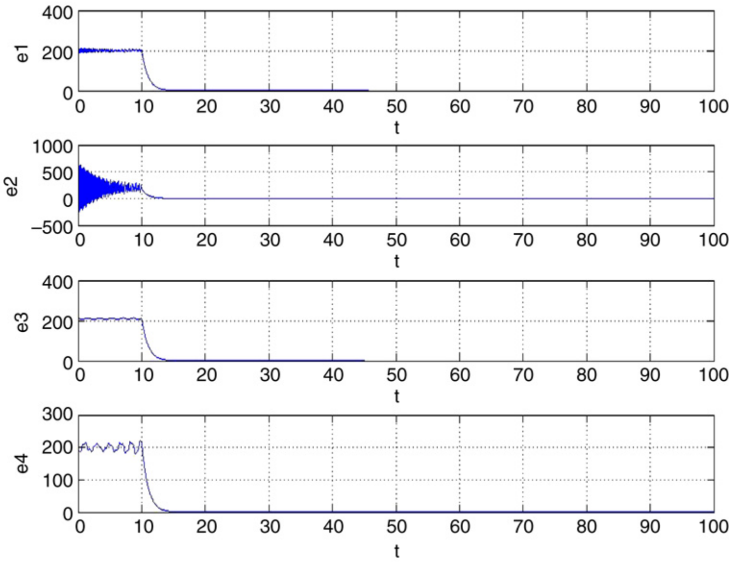

Fig. 10. Time histories of errors for Case III.

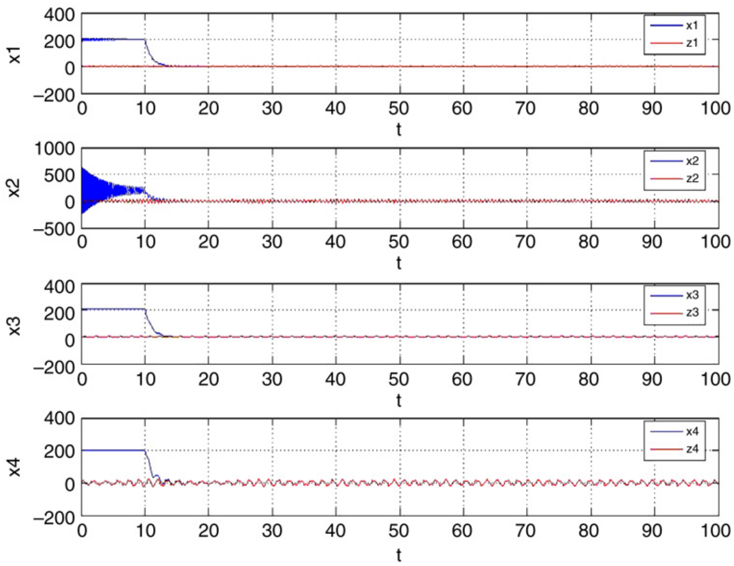

Fig. 11. Time histories of x_1, x_2, x_3, x_4 and z_1, z_2, z_3, z_4 for Case III.

Appendix. GYC partial region stability theory

A.1. Definition of the stability on partial region

Consider the differential equations of disturbed motion of a nonautonomous system in the normal form

$$\frac{dx_s}{dt} = X_s(t, x_1, \ldots, x_n), \quad (s = 1, \ldots, n) \tag{A.1}$$

where the function X_s is defined on the intersection of the partial region Ω (shown in Fig. 1) and

$$\sum_s x_s^2 \leq H \tag{A.2}$$

and $t > t_0$, where t_0 and H are certain positive constants. X_s which vanishes when the variables x_s are all zero, is a real-valued function of $t, x_1, \ldots, x_n$. It is assumed that X_s is smooth enough to ensure the existence, uniqueness of the solution of the initial value problem. When X_s does not contain t explicitly, the system is autonomous.

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Obviously, $x_s = 0$ ($s = 1, \ldots, n$) is a solution of Eq. (A.1). We are interested to the asymptotical stability of this zero solution on partial region Ω (including the boundary) of the neighborhood of the origin which in general may consist of several subregions (Fig. 1).

Definition 1. For any given number $\varepsilon > 0$, if there exists a $\delta > 0$, such that on the closed given partial region Ω when

$$\sum_s x_{s0}^2 \le \delta, \quad (s = 1, \ldots, n) \tag{A.3}$$

for all $t \ge t_0$, the inequality

$$\sum_s x_s^2 < \varepsilon, \quad (s = 1, \ldots, n) \tag{A.4}$$

is satisfied for the solutions of Eq. (A.1) on Ω, then the zero solution $x_s = 0$ ($s = 1, \ldots, n$) is stable on the partial region Ω.

Definition 2. If the undisturbed motion is stable on the partial region Ω, and there exists a $\delta' > 0$, so that on the given partial region Ω when

$$\sum_s x_{s0}^2 \le \delta', \quad (s = 1, \ldots, n). \tag{A.5}$$

The equality

$$\lim_{t\to\infty} \left(\sum_s x_s^2 \right) = 0 \tag{A.6}$$

is satisfied for the solutions of Eq. (A.1) on Ω, then the zero solution $x_s = 0$ ($s = 1, \ldots, n$) is asymptotically stable on the partial region Ω.

The intersection of Ω and region defined by Eq. (A.5) is called the region of attraction.

Definition of functions $V(t, x_1, \ldots, x_n)$: Let us consider the functions $V(t, x_1, \ldots, x_n)$ given on the intersection Ω_1 of the partial region Ω and the region

$$\sum_s x_s^2 \le h, \quad (s = 1, \ldots, n) \tag{A.7}$$

for $t \ge t_0 > 0$, where t_0 and h are positive constants. We suppose that the functions are single-valued and have continuous partial derivatives and become zero when $x_1 = \cdots = x_n = 0$.

Definition 3. If there exist $t_0 > 0$ and a sufficiently small $h > 0$, so that on partial region Ω_1 and $t \ge t_0$, $V \ge 0$ (or ≤ 0), then V is a positive (or negative) semidefinite, in general semidefinite, function on the Ω_1 and $t \ge t_0$.

Definition 4. If there exists a positive (negative) definite function $W(x_1 \ldots x_n)$ on Ω_1, so that on the partial region Ω_1 and $t \ge t_0$

$$V - W \ge 0 \text{ (or } -V - W \ge 0), \tag{A.8}$$

then $V(t, x_1, \ldots, x_n)$ is a positive definite function on the partial region Ω_1 and $t \ge t_0$.

Definition 5. If $V(t, x_1, \ldots, x_n)$ is neither definite nor semidefinite on Ω_1 and $t \ge t_0$, then $V(t, x_1, \ldots, x_n)$ is an indefinite function on partial region Ω_1 and $t \ge t_0$. That is, for any small $h > 0$ and any large $t_0 > 0$, $V(t, x_1, \ldots, x_n)$ can take either positive or negative value on the partial region Ω_1 and $t \ge t_0$.

Definition 6. Bounded function V.

If there exist $t_0 > 0$, $h > 0$, so that on the partial region Ω_1, we have

$$|V(t, x_1, \ldots, x_n)| < L$$

where L is a positive constant, then V is said to be bounded on Ω_1.

Definition 7. Function with infinitesimal upper bound.

If V is bounded, and for any $\lambda > 0$, there exists $\mu > 0$, so that on Ω_1 when $\sum_s x_s^2 \le \mu$, and $t \ge t_0$, we have

$$|V(t, x_1, \ldots, x_n)| \le \lambda$$

then V admits an infinitesimal upper bound on Ω_1.

A.2. GYC theorem of stability and asymptotical stability on partial region

Theorem 1. *If there can be found a definite function $V(t, x_1, \ldots, x_n)$ on the partial region for Eq. (A.1), and the derivative with respect to time based on these equations are:*

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\partial V}{\partial t} + \sum_{s=1}^{n} \frac{\partial V}{\partial x_s} X_s. \tag{A.9}$$

Then, it is a semidefinite function on the partial region whose sense is opposite to that of V, or if it becomes zero identically, then the undisturbed motion is stable on the partial region.

Proof. Let us assume for the sake of definiteness that V is a positive definite function. Consequently, there exists a sufficiently large number t_0 and a sufficiently small number $h < H$, such that on the intersection Ω_1 of partial region Ω and

$$\sum_s x_s^2 \le h, \quad (s = 1, \ldots, n)$$

and $t \ge t_0$, the following inequality is satisfied

$$V(t, x_1, \ldots, x_n) \ge W(x_1, \ldots, x_n),$$

where W is a certain positive definite function which does not depend on t. Besides that, Eq. (A.9) may assume only negative or zero value in this region. □

Let ε be an arbitrarily small positive number. We shall suppose that in any case $\varepsilon < h$. Let us consider the aggregation of all possible values of the quantities $x_1, \ldots, x_n$, which are on the intersection ω_2 of Ω_1 and

$$\sum_s x_s^2 = \varepsilon, \tag{A.10}$$

and let us designate by $l > 0$ the precise lower limit of the function W under this condition. By virtue of Eq. (A.8), we shall have

$$V(t, x_1, \ldots, x_n) \ge l \quad \text{for } (x_1, \ldots, x_n) \text{ on } \omega_2. \tag{A.11}$$

We shall now consider the quantities x_s as functions of time which satisfy the differential equations of disturbed motion. We shall assume that the initial values x_{s0} of these functions for $t = t_0$ lie on the intersection Ω_2 of Ω_1 and the region

$$\sum_s x_s^2 \le \delta, \tag{A.12}$$

where δ is so small that

$$V(t_0, x_{10}, \ldots, x_{n0}) < l. \tag{A.13}$$

By virtue of the fact that $V(t_0, 0, \ldots, 0) = 0$, such a selection of the number δ is obviously possible. We shall suppose that in any case the number δ is smaller than ε. Then the inequality

$$\sum_s x_s^2 < \varepsilon, \tag{A.14}$$

being satisfied at the initial instant will be satisfied, in the very least, for a sufficiently small $t - t_0$, since the functions $x_s(t)$ very continuously with time. We shall show that these inequalities will be satisfied for all values $t > t_0$. Indeed, if these inequalities were not satisfied at some time, there would have to exist such an instant $t = T$ for which this inequality would become an equality. In other words, we would have

$$\sum_s x_s^2(T) = \varepsilon,$$

and consequently, on the basis of Eq. (A.11)

$$V(T, x_1(T), \ldots, x_n(T)) \ge l. \tag{A.15}$$

On the other hand, since $\varepsilon < h$, the inequality (Eq. (A.7)) is satisfied in the entire interval of time $[t_0, T]$, and consequently, in this entire time interval $\frac{\mathrm{d}V}{\mathrm{d}t} \le 0$. This yields

$$V(T, x_1(T), \ldots, x_n(T)) \le V(t_0, x_{10}, \ldots, x_{n0}),$$

which contradicts Eq. (A.14) on the basis of Eq. (A.13). Thus, the inequality (Eq. (A.4)) must be satisfied for all values of $t > t_0$, hence follows that the motion is stable.

Finally, we must point out that from the view-point of mathematics, the stability on partial region in general does not relate logically to the stability on the whole region. If an undisturbed solution is stable on a partial region, it may be either stable or unstable on the whole region and vice versa. In specific practical problems, we do not study the solution starting within Ω_2 and running out of Ω.

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Theorem 2. *If in satisfying the conditions of Theorem 1, the derivative $\frac{dV}{dt}$ is a definite function on the partial region with opposite sign to that of V and the function V itself permits an infinitesimal upper limit, then the undisturbed motion is asymptotically stable on the partial region.*

Proof. Let us suppose that V is a positive definite function on the partial region and that consequently, $\frac{dV}{dt}$ is negative definite. Thus on the intersection Ω_1 of Ω and the region defined by Eq. (A.7) and $t \geq t_0$ there will be satisfied not only the inequality (Eq. (A.8)), but the following inequality as well:

$$\frac{dV}{dt} \leq -W_1(x_1, \ldots, x_n), \tag{A.16}$$

where W_1 is a positive definite function on the partial region independent of t.

Let us consider the quantities x_s as functions of time which satisfy the differential equations of disturbed motion assuming that the initial values $x_{s0} = x_s(t_0)$ of these quantities satisfy the inequalities (Eq. (A.12)). Since the undisturbed motion is stable in any case, the magnitude δ may be selected so small that for all values of $t \geq t_0$ the quantities x_s remain within Ω_1. Then, on the basis of Eq. (A.16) the derivative of function $V(t, x_1(t), \ldots, x_n(t))$ will be negative at all times and, consequently, this function will approach a certain limit, as t increases without limit, remaining larger than this limit at all times. We shall show that this limit is equal to some positive quantities different from zero. Then for all values of $t \geq t_0$ the following inequality will be satisfied:

$$V(t, x_1(t), \ldots, x_n(t)) > \alpha \tag{A.17}$$

where $\alpha > 0$.

Since V permits an infinitesimal upper limit, it follows from this inequality that

$$\sum_s x_s^2(t) \geq \lambda, \quad (s = 1, \ldots, n), \tag{A.18}$$

where λ is a certain sufficiently small positive number. Indeed, if such a number λ did not exist, that is, if the quantity $\sum_s x_s(t)$ is smaller than any preassigned number no matter how small, then the magnitude $V(t, x_1(t), \ldots, x_n(t))$, as follows from the definition of an infinitesimal upper limit, would also be arbitrarily small, which contradicts Eq. (A.17).

If for all values of $t \geq t_0$ the inequality (Eq. (A.18)) is satisfied, then Eq. (A.16) shows that the following inequality will be satisfied at all times:

$$\frac{dV}{dt} \leq -l_1,$$

where l_1 is a positive number different from zero which constitutes the precise lower limit of the function $W_1(t, x_1(t), \ldots, x_n(t))$ under condition (Eq. (A.18)). Consequently, for all values of $t \geq t_0$ we shall have:

$$V(t, x_1(t), \ldots, x_n(t)) = V(t_0, x_{10}, \ldots, x_{n0}) + \int_{t_0}^{t} \frac{dV}{dt} dt \leq V(t_0, x_{10}, \ldots, x_{n0}) - l_1(t - t_0),$$

which is, obviously, in contradiction with Eq. (A.17). The contradiction thus obtained shows that the function $V(t, x_1(t), \ldots, x_n(t))$ approaches zero as t increases without limit. Consequently, the same will be true for the function $W(x_1(t), \ldots, x_n(t))$ as well, from which it follows directly that

$$\lim_{t \to \infty} x_s(t) = 0, \quad (s = 1, \ldots, n),$$

which proves the theorem. □

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Chaos synchronization and chaotization of complex chaotic systems in series form by optimal control

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ABSTRACT

By the method of quadratic optimum control, a quadratic optimal regulator is used for synchronizing two complex chaotic systems in series form. By this method the least error with less control energy is achieved, and the optimization on both energy and error is realized synthetically. The simulation results of two Quantum-CNN chaos systems in series form prove the effectiveness of this method. Finally, chaotization of the system is given by optimal control.

1. Introduction

Chaos synchronization has been widely investigated and many effective methods have been presented recently. Thus, as a key technique of secret communication, chaos synchronization has become a very important goal. Since Pecora and Corrall discovered the synchronization of chaotic systems [1–5], many synchronization methods have been developed [6–9]. For chaos synchronization of practical engineering systems, the control cost must be taken into account. Optimal control method is preferable in such cases [10–13].

In this paper, a quadratic optimal regulator is used for chaos synchronization. In practical system, it is difficult to obtain the precise mathematical model, so in practical applications the investigators would like to employ simple and efficient controllers. Therefore, how to design a simple controller with limited information of a chaotic system is still an open problem [20-26].

As numerical example, recently developed Quantum Cellular Neural Network (Quantum-CNN) chaotic oscillator in series form is used. Quantum-CNN oscillator equations are derived from a Schrödinger equation taking account of quantum dots cellular automata structures to which in the last decade a wide interest has been devoted, with particular attention towards quantum computing [19].

Furthermore, chaotization is studied. Chaotization aims at creating or enhancing the system complexity. Chaotization of Quantum-CNN system is accomplished by an optimal control method.

This paper is organized as follows. In Section 2, a linearly coupled chaos synchronization scheme by optimum control is given. In Section 3, numerical results of the synchronization of two Quantum-CNN oscillator systems by unidirectional and by mutual linear coupling are presented, respectively. In Section 4, chaotization of Quantum-CNN chaotic system and simulation results are obtained. Finally, conclusions are given in Section 5.

2. Linearly coupled chaos synchronization scheme by optimum control

The optimum control is defined as a method by which the specified performance index of a system has optimum value when the desired control assignment is fulfilled.

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The state equation of a linear system is

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

where $x(t)$ is an n-dimensional state variable of the system, A is an $n \times n$ dimensional constant matrix and B is an appropriate $n \times r$ dimensional constant matrix. The matrix $[A\ B]$ is controllable entirely and $u(t)$ is an r-dimensional control input of the system. Assuming that $u(t)$ has no restriction and $u(0) = 0$, the performance index is

$$J = \int_0^{\infty} (x^T Q x + u^T R u) dt. \tag{2}$$

In Eq. (2), Q is an $n \times n$ dimensional positive semidefinite real symmetric constant matrix; R is an $r \times r$ dimensional positive definite real symmetric constant matrix. The choice of the weighting matrix Q or R is based on eclectic considerations which can enhance the control performance and reduce the control energy consumption. The aim of the optimum control is to get $u(t) = Kx(t)$ and then make the performance index Eq. (2) to be minimum, where Kalman gain K is an $r \times n$ dimensional matrix.

So the design of the optimum control system is simplified to get the elements of matrix K. By stability theory, the optimization of the quadratic performance index indicated by Eq. (2) can be solved.

The feedback gain matrix K of the quadratic optimal regulator is obtained as follows [29]:

$$K = R^{-1} B^T S. \tag{3}$$

The matrix S in Eq. (3) is a positive definite matrix and must satisfy the following Riccati equation [9]:

$$A^T S + SA - SBR^{-1} B^T S + Q = 0. \tag{4}$$

Then the following nonlinear chaotic system is considered:

$$\dot{x}(t) = Ax(t) + F(t,x) + Bu_1(t), \tag{5}$$

where A is an $n \times n$ dimensional constant matrix, $x = (x_1, x_2, \ldots, x_n) \in R^n$ is the state variable of the system, $F(x) = (F_1, F_2, \ldots, F_n)^T$ is the nonlinear terms of the chaotic system and $u_1(t) = k_a(y(t) - x(t))$ is an r-dimensional control input where k_a is a constant vector. The second chaotic system is

$$\dot{y}(t) = Ay(t) + F(t,y) + Bu_2(t), \tag{6}$$

where B is an appropriate constant matrix, $u_2(t) = k_s(x(t) - y(t))$ is an r-dimensional control input where k_s is also a constant vector.

Define error vector $e = x - y$. From Eqs. (5) and (6), the error system is

$$\dot{e}(t) = [A - B(k_s + k_a)]e + F(t,x) - F(t,y). \tag{7}$$

In current schemes of chaos synchronization, maximum values of states must be determined by simulation [15–18]. They are half analytic method but not pure analytic method. In [14] $F(t,x) - F(t,y)$ nonlinear item is ignored. This is incorrect since there exist linear terms of e in $F(t,x) - F(t,y)$, which cannot be ignored. In this paper, the series expansion analysis offers a correct method.

The series expansion form of Eq. (7) is

$$\dot{e} = [A + M(x(t), y(t)) - B(k_s + k_a)]e + H(x(t), y(t), e), \tag{8}$$

where $M(x(t),y(t))e + H(x(t),y(t),e) = F(t,x) - F(t,y)$. The elements of $M(x(t),y(t))$ depend on state vectors x, y, and all of them are bounded convergent infinite series of x, y. $H(x(t),y(t),e)$ contains higher degree terms of e only.

If we choose appropriate k_a and k_s to make $A + M(x(t),y(t)) - B(k_s + k_a)$ asymptotically stable, then by first approximation theory, the zero solution $e = 0$ of Eq. (8) is asymptotically stable, i.e., systems (5) and (6) are synchronized.

Now we construct an optimal regulator, which is used to synchronize chaotic systems according to the theory of the quadratic optimal regulator, respectively, and the aim is to get the feedback gain matrices k_a and k_s of system (5) and of system (6), respectively. The steps to get matrices k_a and k_s are: (a) selecting an $n \times n$ dimensional positive semidefinite real symmetric constant matrix Q, an $r \times r$ dimensional positive definite real symmetric constant matrix R and a constant matrix B, with the constant matrix A we can get a Riccati equation as shown in Eq. (4). Then, we should solve this equation to get matrix S. If the positive definite matrix S exists, the matrix $A + M(x(t),y(t)) - B(k_s + k_a)$ is asymptotically stable and the design of control for the synchronization of two systems is successful. Otherwise we should reselect Q, R and B and calculate again. (b) Putting the matrix S in Eq. (3), we can get the gain matrices k_a and k_s of the regulators. After getting the matrices k_a and k_s according to the above steps, we put k_a, k_s and the matrix B in Eqs. (5) and (6). Then we get two synchronized systems.

3. Numerical results of the synchronization of two Quantum-CNN oscillator systems by unidirectional and by mutual linear coupling

Case I. The synchronization by unidirectional linear coupling.

For a two-cell Quantum-CNN, the following differential equations are obtained [19]:

$$\begin{cases} \dot{x}_1 = -2a_1\sqrt{1-x_1^2}\sin x_2, \\ \dot{x}_2 = -\omega_1(x_1-x_3) + 2a_1\frac{x_1}{\sqrt{1-x_1^2}}\cos x_2, \\ \dot{x}_3 = -2a_2\sqrt{1-x_3^2}\sin x_4, \\ \dot{x}_4 = -\omega_2(x_3-x_1) + 2a_2\frac{x_3}{\sqrt{1-x_3^2}}\cos x_4, \end{cases} \tag{9}$$

where x_1 and x_3 are polarizations, x_2 and x_4 are quantum phase displacements, a_1 and a_2 are proportional to the inter-dot energy inside each cell and ω_1 and ω_2 are parameters that weigh effects on the cell of the difference of the polarization of neighboring cells, like the cloning templates in traditional CNNs. Let $a_1 = a_2 = 2.47$, $\omega_1 = 1$, $\omega_2 = 1$, chaos is obtained for this system [20,23,24].

Two Quantum-CNN chaotic systems using the unidirectional linear coupling can be written as

$$\begin{cases} \dot{x}_1 = -2a_1\sqrt{1-x_1^2}\sin x_2, \\ \dot{x}_2 = -\omega_1(x_1-x_3) + 2a_1\frac{x_1}{\sqrt{1-x_1^2}}\cos x_2, \\ \dot{x}_3 = -2a_2\sqrt{1-x_3^2}\sin x_4, \\ \dot{x}_4 = -\omega_2(x_3-x_1) + 2a_2\frac{x_3}{\sqrt{1-x_3^2}}\cos x_4 \end{cases} \tag{10}$$

and

$$\begin{cases} \dot{y}_1 = -2a_1\sqrt{1-y_1^2}\sin y_2 + k_1(x_1-y_1), \\ \dot{y}_2 = -\omega_1(y_1-y_3) + 2a_1\frac{y_1}{\sqrt{1-y_1^2}}\cos y_2 + k_2(x_2-y_2), \\ \dot{y}_3 = -2a_2\sqrt{1-y_3^2}\sin y_4 + k_3(x_3-y_3), \\ \dot{y}_4 = -\omega_2(y_3-y_1) + 2a_2\frac{y_3}{\sqrt{1-y_3^2}}\cos y_4 + k_4(x_4-y_4). \end{cases} \tag{11}$$

The initial values for these linearly coupled Quantum-CNN systems are taken as $x_1(0) = 0.8$, $x_2(0) = -0.77$, $x_3(0) = -0.72$, $x_4(0) = 0.57$, $y_1(0) = -0.2$, $y_2(0) = 0.41$, $y_3(0) = 0.25$ and $y_4(0) = -0.81$.

Expand the right hand sides of Eqs. (10) and (11) into power series:

$$\begin{cases} \dot{x}_1 = -2a_1\left(-\frac{1}{2}x_1^2x_2 + \frac{1}{12}x_1^2x_2^3 - \frac{1}{8}x_1^4x_2 + x_2 - \frac{1}{6}x_2^3 + \frac{1}{120}x_2^5 + \cdots\right), \\ \dot{x}_2 = -\omega_1(x_1-x_3) + 2a_1\left(x_1 - \frac{1}{2}x_1x_2^2 + \frac{1}{24}x_1x_2^4 + \frac{1}{2}x_1^3 - \frac{1}{4}x_1^3x_2^2 + \frac{5}{8}x_1^5 + \cdots\right), \\ \dot{x}_3 = -2a_2\left(-\frac{1}{2}x_3^2x_4 + \frac{1}{12}x_3^2x_4^3 - \frac{1}{8}x_3^4x_4 + x_4 - \frac{1}{6}x_4^3 + \frac{1}{120}x_4^5 + \cdots\right), \\ \dot{x}_4 = -\omega_2(x_3-x_1) + 2a_2\left(x_3 - \frac{1}{2}x_3x_4^2 + \frac{1}{24}x_3x_4^4 + \frac{1}{2}x_3^3 - \frac{1}{4}x_3^3x_4^2 + \frac{5}{8}x_3^5 + \cdots\right) \end{cases} \tag{12}$$

and

$$\begin{cases} \dot{y}_1 = -2a_1\left(-\frac{1}{2}y_1^2y_2 + \frac{1}{12}y_1^2y_2^3 - \frac{1}{8}y_1^4y_2 + y_2 - \frac{1}{6}y_2^3 + \frac{1}{120}y_2^5 + \cdots\right) + k_1(x_1-y_1), \\ \dot{y}_2 = -\omega_1(y_1-y_3) + 2a_1\left(y_1 - \frac{1}{2}y_1y_2^2 + \frac{1}{24}y_1y_2^4 + \frac{1}{2}y_1^3 - \frac{1}{4}y_1^3y_2^2 + \frac{5}{8}y_1^5 + \cdots\right) + k_2(x_2-y_2), \\ \dot{y}_3 = -2a_2\left(-\frac{1}{2}y_3^2y_4 + \frac{1}{12}y_3^2y_4^3 - \frac{1}{8}y_3^4y_4 + y_4 - \frac{1}{6}y_4^3 + \frac{1}{120}y_4^5 + \cdots\right) + k_3(x_3-y_3), \\ \dot{y}_4 = -\omega_2(y_3-y_1) + 2a_2\left(y_3 - \frac{1}{2}y_3y_4^2 + \frac{1}{24}y_3y_4^4 + \frac{1}{2}y_3^3 - \frac{1}{4}y_3^3y_4^2 + \frac{5}{8}y_3^5 + \cdots\right) + k_4(x_4-y_4). \end{cases} \tag{13}$$

From Eqs. (12) and (13), the error dynamics is:

$$\dot{e} = [A + M(x(t), y(t)) - Bk_s]e + H(x, y, e), \tag{14}$$

where $e = (y_1 - x_1, y_2 - x_2, y_3 - x_3, y_4 - x_4)^T$ and

$$M(x(t), y(t)) = \begin{pmatrix} M_{11} & -2a_1 + M_{21} & 0 & 0 \\ 2a_1 + M_{12} & M_{22} & 0 & 0 \\ 0 & 0 & M_{33} & -2a_2 + M_{43} \\ 0 & 0 & 2a_2 + M_{34} & M_{44} \end{pmatrix}$$

in which

$$M_{11} = a_1\left[2x_1y_2 - \frac{1}{6}x_1y_2^3 + \frac{1}{4}(x_1y_1^2y_2 + 3x_1^2y_1y_2) + \cdots\right]$$

$$\ldots$$

and $H(x,y,e)$ contains higher degree terms of e only.

The infinite power series of the first element of M, i.e., M_{11} is

$$2x_1y_2 - \frac{1}{6}x_1y_2^3 + \frac{1}{4}(x_1y_1^2y_2 + 3x_1^2y_1y_2) + \cdots \tag{15}$$

It is well-known [28] that a necessary and sufficient condition for the convergence of the infinite series

$$u_1 + u_2 + \cdots + u_n + \cdots$$

is that for any previously assigned positive ε there exists an N such that, for any $n > N$ and for positive p,

$$\left|u_{n+1} + u_{n+2} + \cdots u_{n+p}\right| < \varepsilon. \tag{16}$$

From Fig. 1, we know that

$$|x_i| < 1, \quad |y_i| < 1 \qquad (i = 1, 2, 3, 4). \tag{17}$$

Therefore, M_{11} and series contained in other elements of $M(x(t), y(t))$ are convergent series and they have bounded sums.

We can get the optimum gain $k_s = [k_1, k_2, k_3, k_4]^T$ by the method of constructing a quadratic optimal regulator. With

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -\omega_1 & 0 & \omega_1 & 0 \\ 0 & 0 & 0 & 0 \\ \omega_2 & 0 & -\omega_2 & 0 \end{bmatrix}$$

we choose

$$B = [0 \quad 0 \quad 0 \quad 1]^T; \quad R = [1]; \quad Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}. \tag{18}$$

After solving the corresponding Riccati equation, we get the gain matrix $k_s = [k_1, k_2, k_3, k_4]^T = [0, 1, 0, 1]^T$.

From the simulation results of Fig. 1, it is shown that master system and slave system reach the synchronization state after they are controlled by the quadratic optimal regulator. It is noticed that the synchronization effect is good.

Case II. The synchronization by mutual linear coupling.

Two Quantum-CNN systems with mutual linear coupling are given:

$$\begin{cases} \dot{x}_1 = -2a_1\sqrt{1 - x_1^2}\sin x_2 + k_{11}(y_1 - x_1), \\ \dot{x}_2 = -\omega_1(x_1 - x_3) + 2a_1\frac{x_1}{\sqrt{1-x_1^2}}\cos x_2 + k_{12}(y_2 - x_2), \\ \dot{x}_3 = -2a_2\sqrt{1 - x_3^2}\sin x_4 + k_{13}(y_3 - x_3), \\ \dot{x}_4 = -\omega_2(x_3 - x_1) + 2a_2\frac{x_3}{\sqrt{1-x_3^2}}\cos x_4 + k_{14}(y_4 - x_4) \end{cases} \tag{19}$$

and

$$\begin{cases} \dot{y}_1 = -2a_1\sqrt{1 - y_1^2}\sin y_2 + k_{21}(x_1 - y_1), \\ \dot{y}_2 = -\omega_1(y_1 - y_3) + 2a_1\frac{y_1}{\sqrt{1-y_1^2}}\cos y_2 + k_{22}(x_2 - y_2), \\ \dot{y}_3 = -2a_2\sqrt{1 - y_3^2}\sin y_4 + k_{23}(x_3 - y_3), \\ \dot{y}_4 = -\omega_2(y_3 - y_1) + 2a_2\frac{y_3}{\sqrt{1-y_3^2}}\cos y_4 + k_{24}(x_4 - y_4). \end{cases} \tag{20}$$

Expand the right hand sides of Eqs. (19) and (20) into power series:

$$\begin{cases} \dot{x}_1 = -2a_1\left(-\frac{1}{2}x_1^2x_2 + \frac{1}{12}x_1^2x_2^3 - \frac{1}{8}x_1^4x_2 + x_2 - \frac{1}{6}x_2^3 + \frac{1}{120}x_2^5 + \cdots\right) + k_{11}(y_1 - x_1), \\ \dot{x}_2 = -\omega_1(x_1 - x_3) + 2a_1\left(x_1 - \frac{1}{2}x_1x_2^2 + \frac{1}{24}x_1x_2^4 + \frac{1}{2}x_1^3 - \frac{1}{4}x_1^3x_2^2 + \frac{5}{8}x_1^5 + \cdots\right) + k_{12}(y_2 - x_2), \\ \dot{x}_3 = -2a_2\left(-\frac{1}{2}x_3^2x_4 + \frac{1}{12}x_3^2x_4^3 - \frac{1}{8}x_3^4x_4 + x_4 - \frac{1}{6}x_4^3 + \frac{1}{120}x_4^5 + \cdots\right) + k_{13}(y_3 - x_3), \\ \dot{x}_4 = -\omega_2(x_3 - x_1) + 2a_2\left(x_3 - \frac{1}{2}x_3x_4^2 + \frac{1}{24}x_3x_4^4 + \frac{1}{2}x_3^3 - \frac{1}{4}x_3^3x_4^2 + \frac{5}{8}x_3^5 + \cdots\right) + k_{14}(y_4 - x_4) \end{cases} \tag{21}$$

and

$$\begin{cases} \dot{y}_1 = -2a_1\left(-\frac{1}{2}y_1^2y_2 + \frac{1}{12}y_1^2y_2^3 - \frac{1}{8}y_1^4y_2 + y_2 - \frac{1}{6}y_2^3 + \frac{1}{120}y_2^5 + \cdots\right) + k_{21}(x_1 - y_1), \\ \dot{y}_2 = -\omega_1(y_1 - y_3) + 2a_1\left(y_1 - \frac{1}{2}y_1y_2^2 + \frac{1}{24}y_1y_2^4 + \frac{1}{2}y_1^3 - \frac{1}{4}y_1^3y_2^2 + \frac{5}{8}y_1^5 + \cdots\right) + k_{22}(x_2 - y_2), \\ \dot{y}_3 = -2a_2\left(-\frac{1}{2}y_3^2y_4 + \frac{1}{12}y_3^2y_4^3 - \frac{1}{8}y_3^4y_4 + y_4 - \frac{1}{6}y_4^3 + \frac{1}{120}y_4^5 + \cdots\right) + k_{23}(x_3 - y_3), \\ \dot{y}_4 = -\omega_2(y_3 - y_1) + 2a_2\left(y_3 - \frac{1}{2}y_3y_4^2 + \frac{1}{24}y_3y_4^4 + \frac{1}{2}y_3^3 - \frac{1}{4}y_3^3y_4^2 + \frac{5}{8}y_3^5 + \cdots\right) + k_{24}(x_4 - y_4). \end{cases} \tag{22}$$

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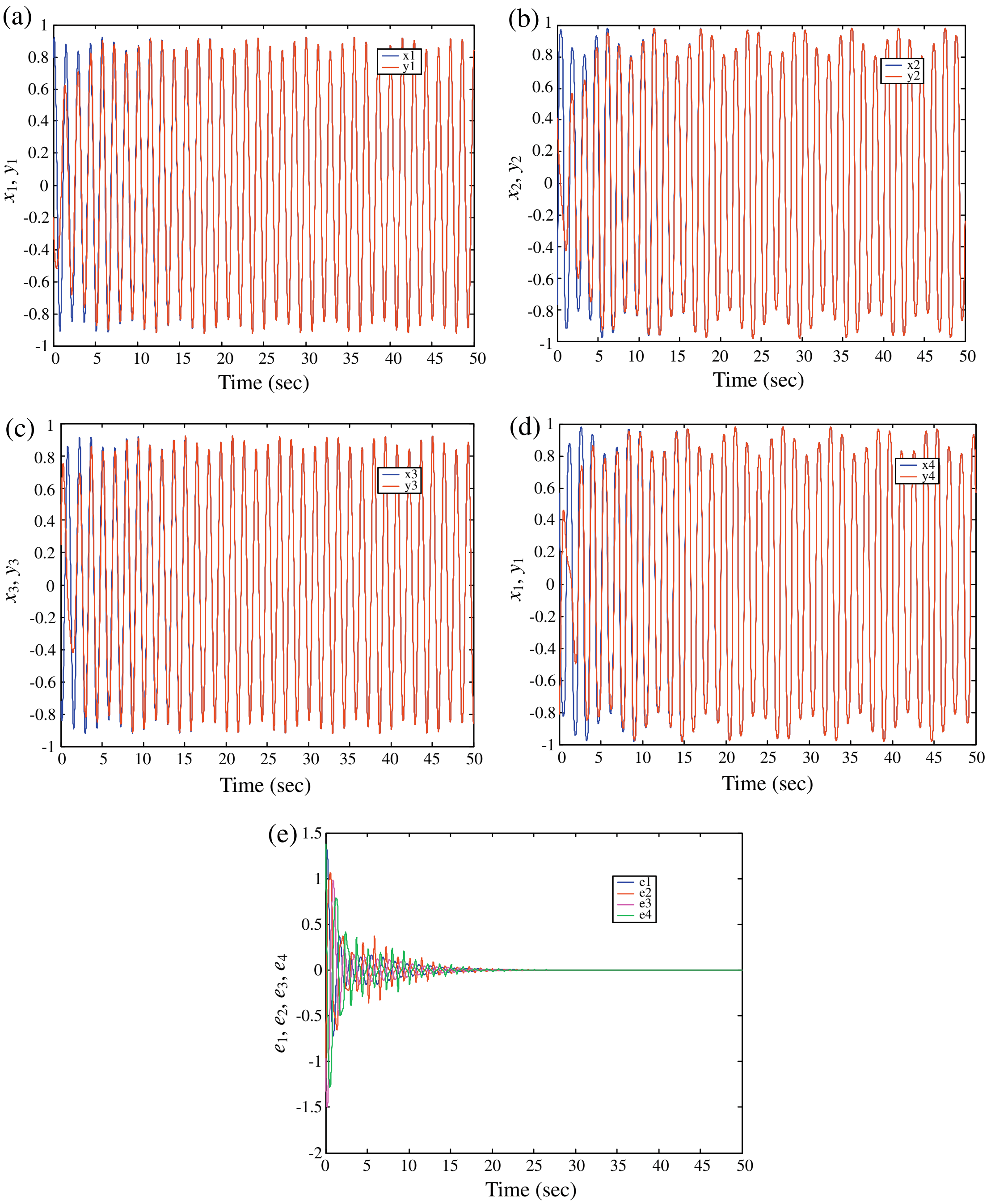

Fig. 1. Time histories of states, state errors for unidirectional linear coupling case.

From Eqs. (21) and (22), the error dynamics is:

$$\dot{e} = [A + M(x(t), y(t) - B(k_s + k_a))]e + H(x, y, e), \tag{23}$$

where $e = (y_1 - x_1, y_2 - x_2, y_3 - x_3, y_4 - x_4)^T$ and

$$M(x(t), y(t)) = \begin{pmatrix} M_{11} & -2a_1 + M_{21} & 0 & 0 \\ 2a_1 + M_{12} & M_{22} & 0 & 0 \\ 0 & 0 & M_{33} & -2a_2 + M_{43} \\ 0 & 0 & 2a_2 + M_{34} & M_{44} \end{pmatrix}$$

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in which

$$M_{11} = a_1\left[2x_1y_2 - \frac{1}{6}x_1y_2^3 + \frac{1}{4}(x_1y_1^2y_2 + 3x_1^2y_1y_2) + \cdots\right]$$
$$\cdots$$

Similar to Case I, from Fig. 2, $|x_i| < 1$, $|y_i| < 1$ $(i = 1,2,3,4)$, the infinite power series elements of $M(x(t), y(t))$ are all convergent and have bounded sums [27,28].

The optimum gains $k_a = [k_{11}, k_{12}, k_{13}, k_{14}]^T$ and $k_s = [k_{21}, k_{22}, k_{23}, k_{24}]^T$ can be obtained by the method of constructing a quadratic optimal regulator. With

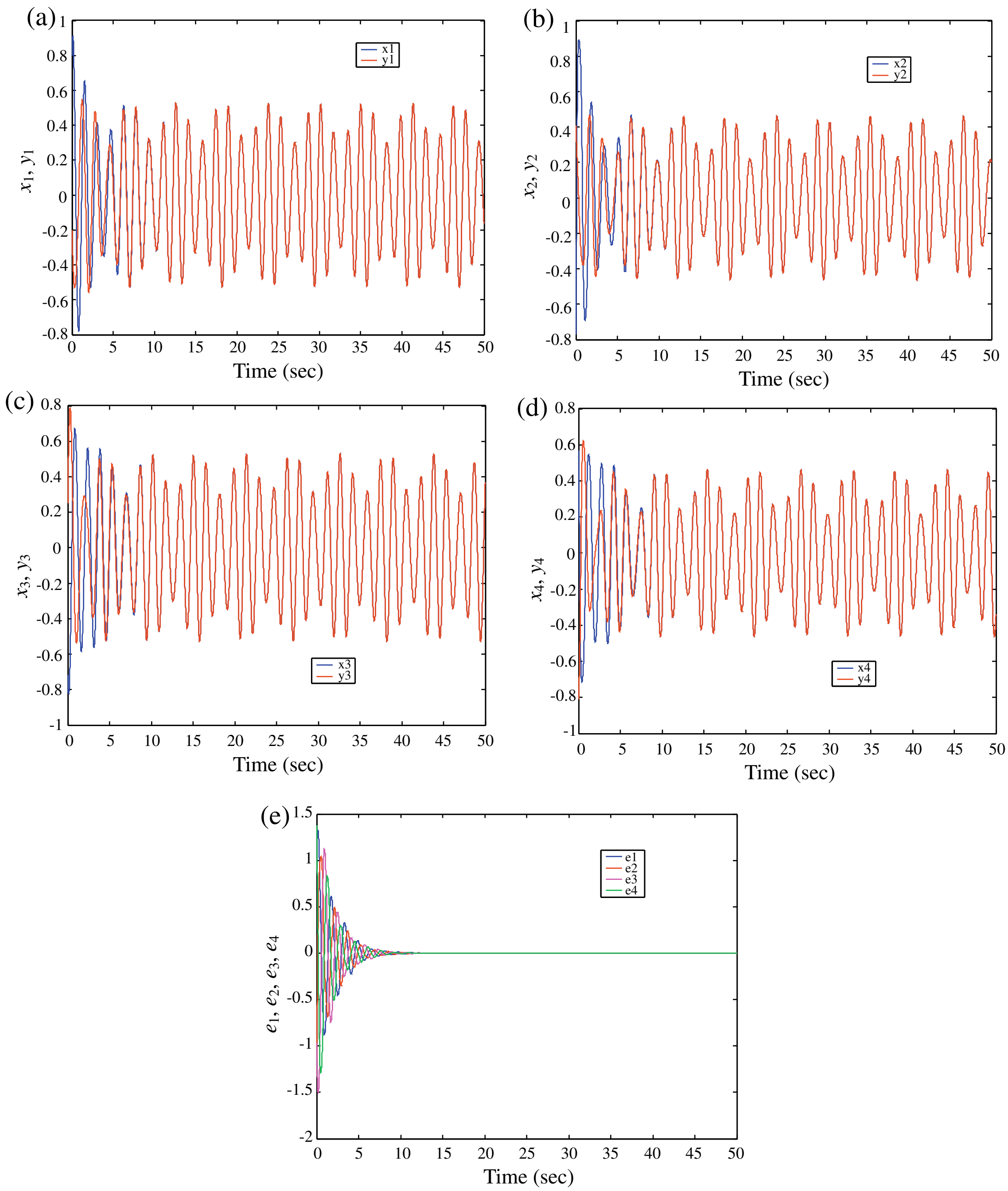

Fig. 2. Time histories of states, state errors for mutual linear coupling case.

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -\omega_1 & 0 & \omega_1 & 0 \\ 0 & 0 & 0 & 0 \\ \omega_2 & 0 & -\omega_2 & 0 \end{bmatrix}$$

we choose

$$B = [0 \quad 0 \quad 0 \quad 1]^T; \quad R = [1]; \quad Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}. \tag{24}$$

After solving the corresponding Riccati equation, we then get two gain matrices $k_a = [k_{11}, k_{12}, k_{13}, k_{14}]^T = [0, 0.5, 0, 0.5]^T$ and $k_s = [k_{21}, k_{22}, k_{23}, k_{24}]^T = [0, 0.5, 0, 0.5]^T$.

From the simulation results of Fig. 2 two systems reach the synchronization state after they are controlled by the quadratic optimal regulator. It is noticed that the synchronization effect is also satisfactory.

4. Chaotization of Quantum-CNN chaotic system scheme and simulation

Optimal control is a well-established engineering control strategy, and is useful for both linear and nonlinear system with linear or nonlinear controllers [3]. Now, we use a typical optimal control for the chaotization of Quantum-CNN system. Consider the system (9) with a controller u and define the Hamilton function:

$$\begin{aligned} &H(x_1, x_2, x_3, x_4, u, p) = \boldsymbol{p}^T \boldsymbol{F}(x_1, x_2, x_3, x_4, u, p); \\ &\boldsymbol{p}^T = [p_1 \ p_2 \ p_3 \ p_4], \end{aligned} \tag{25}$$

where $\boldsymbol{p}$ is a Lagrange multiplier, called a co-state vector, $\boldsymbol{F}$ is the right hand side of Eq. (9). Following the variational principle of optimal control, we can obtain

$$p_2\left(-\omega_1(x_1 - x_3) + 2a_1 \frac{x_1}{\sqrt{1 - x_1^2}} \cos x_2\right) + p_3\left(-2a_2\sqrt{1 - x_3^2} \sin x_4\right) + p_4\left(-\omega_2(x_3 - x_1) + 2a_2 \frac{x_3}{\sqrt{1 - x_3^2}} \cos x_4\right) = 0, \tag{26}$$

$$p_2 \frac{-2a_1}{\sqrt{1 - x_1^2}} \sin x_2 = 0. \tag{27}$$

This yield a non-trivial solution for (p_2, p_3, p_4) if and only if

$$\frac{-2a_1}{\sqrt{1 - x_1^2}} \sin x_2 = 0. \tag{28}$$

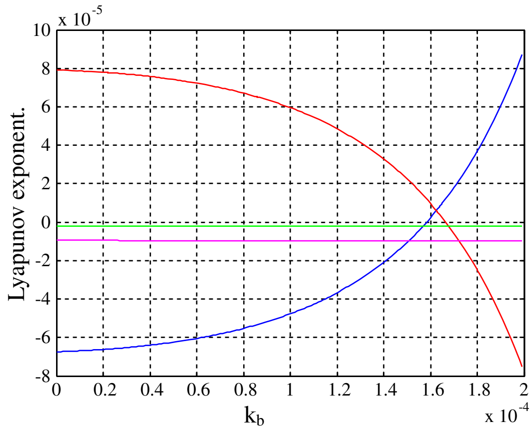

Fig. 3. Lyapunov exponents of controlled Quantum-CNN system.

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It gives an optimal surface singularly in the state space. This type of control assumes values on the two allowable boundaries (27) and (28) alternatively according to a switching surface. Locating system trajectories on the surface, a typical feedback control in the form

$$u = -k_b \operatorname{sgn}\left[\frac{-2a_1}{\sqrt{1-x_1^2}}\sin x_2\right] \tag{29}$$

can be used. By adjusting the value of k_b from zero initial value to $k_b = 1.6 \times 10^{-4}$ in the above controller with the signum function

$$\operatorname{sgn}[v] = \begin{cases} 1 & \text{if } v > 0, \\ 0 & \text{if } v = 0, \\ -1 & \text{if } v < 0 \end{cases} \tag{30}$$

the chaotic motion with one positive Lyapunov exponent can be controlled to chaotic motion with two positive Lyapunov exponents as shown by the simulation result in Fig. 3.

5. Conclusions

Two chaotic Quantum-CNN systems are synchronized in two cases: unidirectional linear coupling by optimum control, mutual linear coupling by optimum control. The number of controllers for optimum control is less than that for synchronization only by linear couplings. This results in lower cost. In chaos synchronization cases, by a theorem of convergent series, we prove that all infinite power series elements of $A + M(x(t), y(t)) - B(k_s + k_a)$ are convergent and have bounded sums. This synchronization of chaos systems can be used to increase the security of communication. Next, the optimum control is used for chaotization, i.e., to enhance original chaotic state to more complex chaotic state. Numerical simulations are used to verify the effectiveness of the proposed scheme.

Acknowledgments

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