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新渾沌系統與新渾沌控制及同步方法

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目錄

中英文摘要及關鍵詞……………………..………….………….…………VIII

前言與研究目的……………………..……………….…………….. …..1

研究內容………………………..…………….…………………….....9

Chapter 1 Introduction………………………………….…………...10

References………………………………………...………...………..……..20

Chapter 2 Hyperchaos of New Mathieu-van der Pol system with Three Positive Lyapunov Exponents……………………29

2.1 Preliminaries……………………………………………………...…29

2.2 Differential equations for Mathieu-van der Pol system and phasePortraits………………………………………………………29

2.3 Power spectrum……………………..……………………………30

2.4 Lyapunov exponents……………………………….………………30

2.5 Parameter diagrams……………………………….……….…….31

2.6 Phase portraits and its implementation of electronic circuits…………31

Figures…..…………………………………...………...……………....31

Chapter 3 Chaos Control of New Mathieu-van der Pol Systems with New Mathieu -Duffing Systems as Goal System by GYC Partial Region Stability Theory…………….................35

3.1 Preliminaries..……………….……………...………...……………... 35

3.2 Chaos Control Scheme……………………...………...…………….…...35

3.3 New Chaotic Mathieu- Duffing System……………...…..………...…38

3.4 Numerical Simulations…....…………………………………………… 41

Figures…..…………………………………...………...…………..…..45

Chapter 4 Generalized Chaos Synchronization of New Mathieu-van der Pol Systems with New Duffing-van der Pol systems as Functional system by GYC Partial Region Stability Theory………………….49

4.1 Preliminaries..……………….……………...………...……………....49

4.2 Generalized Chaos Synchronization Strategy...………………………49

4.3 New Chaotic Duffing-van der Pol System……..........…...………..…..52
Figures…..………………………………………...………...……………....63
Chapter 5 Chaos Generalized Synchronization of New Mathieu-Duffing Systems by GYC Partial Region Stability Theory……………………….....…..……………….....…. 66
5.1 Preliminaries..………………..……………...………...………….….. 66
5.2 Chaos Generalized Synchronization Strategy by GYC Partial Region Stability Theory...……...………..……………………………………66
5.3 Numerical Simulations..............…...……..……………………………...70
Figures…..………………………………………...………...……………....73
Chapter 6 Chaos Control of New Mathieu- Duffing Systems by GYC Partial Region Stability Theory……….................78
6.1 Preliminaries..………………..……………...………...……………....78
6.2 GYC Partial Region Stability Theory………...…...…...……………..... 78
6.3 Chaos Control Scheme………………………………………………....79
6.4 Chaos of new Mathieu – Duffing system……………………………...80
6.5 Numerical Simulations………………………………………………....81
Figures…..………………………………………...………...……………....82
Chapter 7 Chaos Control of New Duffing-Van der Pol System by GYC Partial Region Stability Theory…………………..86
7.1 Preliminaries..………………..……………...………...……….…….…..86
7.2 Chaos of New Duffing-Van der Pol system…….….…….….….….…. 87
7.3 Numerical Simulations…..………………………………………………90
Figures…..………………………………………...………...……………....96
Chapter 8 Chaos Generalized Synchronization of New Duffing-Van der Pol System by GYC Partial Region Stability Theory……………………..……………………..…...103
8.1 Preliminaries..………………..……………...………...……………..103
8.2 Chaos of New Duffing-Van der Pol system……………….......………103
8.3 Numerical Simulations………………………..………...…………..105
Figures…..………………………………………...………...……………..110
Chapter 9 Chaos Generalized Synchronization of New

Ikeda-Lorenz Systems by GYC Partial Region Stability Theory……………………………………………………...115
9.1 Preliminaries..……………….……………...……...……………….…115
9.2 Chaos Generalized Synchronization Strategy...……………………..115
9.3 Chaos of A New Ikeda-Lorenz System…………...…...……….........116
9.4 Simulation results………………..…………………………………..…...117
Figures…..…………………………………...……….....……………..119
Chapter 10 Chaos Control of New Ikeda-Lorenz Systems by GYC Partial Region Stability Theory………..………………128
10.1 Preliminaries..……………….……………...……...……………. 128
10.2 Chaos of A New Ikeda-Lorenz System………..……..........................128
10.3 Simulation results………………..…………………………………..…...129
10.4 Comparison between New Strategy and Traditional Method….…130
Figures…..…………………………………...……….....………..…....131
Chapter 11 Chaos Control of Tachometer System by GYC Partial Region Stability Theory…………………………….………..137
11.1 Preliminaries..……………….……………...……...……………….…137
11.2 Chaos Control Scheme………...……………….………………………...137
11.3 Hyperchaotic Tachometer System and New Hyperchaotic Mathieu-Duffing System……………………………………………137
11.4 Chaos Control of Tachometer System by GYC Partial Region Stability Theory……........………………...………...………..…..138
Figures…..…………………………………...……….....………..…....142
Chapter 12 Chaos Control of An Inertial Tachometer Systems by GYC Partial Region Stability Theory…...……………..150
12.1 Preliminaries..……………….……………...……...……………….…150
12.2 Chaos of An Inertial Tachometer system and of A New Mathieu-Van der Pol system………………………………………………………150
12.3 Numerical Simulations……………………………………………….. 157
Figures…..…………………………………...……….....……………..162
Chapter 13 Chaos Generalized Synchronization of An Inertial Tachometer wih New Mathieu-Van der Pol Systems as

Functional system by GYC Partial Region Stability Theory…...……………..……..………………………166
13.1 Preliminaries..……………..……………...………...……………..166
13.2 Chaos of An Inertial Tachometer system and New Mathieu-Van der Pol System……………...166
13.3 Simulation results………………..………………………..………...168
Figures…..……………………………………...………...…..………....173
Chapter 14 Pragmatical Hybrid Projective Chaotic Generalized Synchronization of Chaotic Systems by Adaptive Backstepping Control…….…………………….……..187
14.1 Preliminaries..……………..……………...………...……………..187
14.2 Synchronization scheme.....………... 187
14.3 Mathieu – Duffing system and Duffing – Van der Pol system……...188
14.4 Numerical results of PHPGS by adaptive backstepping control…...188
Figures…..……………………………………...………...……………......191
Chapter 15 Pragmatical Adaptive Synchronization of Different Orders Chaotic Systems with All Uncertain Parameters via Nonlinear Control…………………………………..197
15.1 Preliminaries..……………..……………...………....……….…. 197
15.2 Adaptive Synchronization Scheme……………………………....... 197
15.3 Simulation results………………..…………………………………....199
Figures…..……………………………………...………...………….......202
Chapter 16 Symplectic Hybrid Projective Synchronization of Different Order Systems with New Control Lyapunov Function by Adaptive Backstepping Control……..…..209
16.1 Preliminaries..……………..……….…...………...…………….209
16.2 Symplectic Synchronization Scheme……………..……………….......209
16.3 Mathieu – Duffing system and Lü system.………….………….......213
16.4 Numerical results of CHPS by adaptive backstepping control……..213
Figures…..……………………………………...………...…………......220
Chapter 17 Pragmatical Chaotic Symplectic Synchronization of New

Duffing-Van der Pol Systems with Different Order System as Functional System by New Dynamic Surface Control and Adaptive Control……….……………..…..226
17.1 Preliminaries..……………….……………...………...………...….. 226
17.2 Pragmatical Chaotic Symplectic Synchronization Scheme………... 227
17.3 Numerical Results for the PCSS by New Dynamic Surface Control and Adaptive Control……………………………………..……...... 228
17.4 Simulation results………………..…………………………………....229
Figures…..……………………………………...………...…………..... 230
Chapter 18 Hybrid Projective Synchronization of Hyperchaotic Tachometer systems by Backstepping Control….…… 242
18.1 Preliminaries..……………….……………...………...…………….…242
18.2 Chaos of tachometer system……..…………………...………….....242
18.3 Hybrid projective synchronization scheme……………………….......243
18.4 Hybrid projective synchronization of hyperchaotic tachometer system by backstepping control…………………………………………….243
Figures…..……………………………………...………...…………......245
Chapter 19 Chaos Synchronization of the Two Identical Ikeda-Mackey-Glass Systems without Any Controller.254
19.1 Preliminaries..……………….……………...………...…………….…254
19.2 Ikeda-Mackey Glass system…………………………..…………..... 255
19.3 Synchronization Scheme…………………………..………….......255
19.4 The lag or anticipated anti-synchronization of two identical double Ikeda systems……………..………………………..………....257
Figures…..……………………………………...………...……………..259
Chapter 20 Parameter estimation of Ikeda-Mackey Glass system through synchronization in presence of disturbance…268
20.1 Preliminaries..……………….…………….…...………...…………….268
20.2 Ikeda-Mackey-Glass system……………………………………….…....269
20.3 Synchronization scheme……………………….….…………...... 270
20.4 Estimation of parameter of Ikeda-Mackey Glass system without disturbance……….….………………………………………….......270

20.5 Estimation of parameter of Ikeda-Mackey Glass system with disturbance……………..…………………………….….……………......270
Figures…...……………………………………...……….....…………….273
Chapter 21 Chaos Synchronization of the Two Identical Ikeda-Mackey-Glass Systems without Any Controller.288
21.1 Preliminaries..……………..……………...………...……………..288
21.2 Ikeda-Mackey Glass system...... ……………...………...…………….288
21.3 Synchronization Scheme………………………………….….......289
Figures…...……………………………………...……….....…………......293
Chapter 22 Unbounded Chaos………………………….………...305
22.1 Preliminaries..……………..……………...………...……….….305
22.2 Equation……………………………………………………….….....306
Figures…...……………………………………...……….....…………......307
Chapter 23 Conclusions……………………………………...……319
Reference……………………………………………………….....325
計畫成果自評………………………………………..……...……...325

中英文摘要及關鍵詞

中文摘要

渾沌系統之研究在物理、化學、生理學、各種工程等方面皆有日益重要之廣泛應用。Duffing 系統、van der Pol 系統、非線性 Mathieu 系統都是最重要的典型的渾沌系統。Ikeda 系統及 Mackey-Glass 系統則是最重要的典型的時滯系統。兩種慣性測速器新系統也是重要的機械系統。本計畫採取適當的耦合方式構成新創的 Duffing-van der Pol 系統、Mathieu-Duffing 系統、Mathieu-van der Pol 系統、Ikeda- Mackey-Glass 系統，從而擴大了各原來單純系統的研究範圍也深化了研究內容。兩種慣性測速器系統也是創新的，有實用價值的機械系統，值得研究。

渾沌控制與同步在物理系統、化學系統、生物系統、各種工程系統、秘密通訊、神經網路、自組織等方面的廣泛應用一日千里。本計畫提出一種新渾沌控制反控制方法、兩種新渾沌同步方法，具有重要的理論與實際意義：一、不同系統實用渾沌適應控制反控制新方法。傳統的渾沌控制侷限於將同一系統之渾沌運動控制至同一系統之週期解或平衡點。此法不僅可將本系統之渾沌控制到另一系統之週期解或平衡點，也可將本系統之週期解或混沌解反控制到另一更複雜系統之渾沌運動。而且涉及的各系統參數可以都是未知參數。大大地擴展了渾沌控制反控制的能力。二、非耦合渾沌同步新方法。目前之文獻絕大部分渾沌同步皆採耦合同步。以秘密通訊而言，耦合所需的主從系統狀態變量之傳送會造成失密及時滯之不良後果。非耦合同步可消除這些缺點。目前國際文獻多採以第三系統之渾沌變量或噪音對主從系統參數作同時激勵而獲同步。本計劃採（a）多管道下之各種形式之激勵（各種時間週期函數、渾沌函數、各種噪音）以保證同步之可靠性。（b）綜合性激勵（周期調制的渾沌函數、渾沌調制之噪音等）。（c）（a）、（b）同時進行以增加保密程度。（d）對分數階系統及時滯系統進行（a）、（b）、（c）三種激勵。三、指數後步渾沌同步新方法。目前渾沌同步皆取李氏函數為V正定、$\dot{V}$負定，同步達成之時間較長控制品質不夠好。現採指數漸進穩定理論研究誤差系統零解，使同步完成時間大為減小。再結合後步(backstepping)設計可得三優點 1. 同步完成時間大減 2. 逐步選擇V函數，減少選V函數之難度 3. 減少控制器的數目。

研究重點為

1. Mathieu-van der Pol 系統與 Ikeda- Mackey-Glass 系統的渾沌研究。用相圖、分歧圖、功率譜圖、李雅普諾夫指數分析渾沌之行為。
2. 新式非耦合渾沌同步新方法。多管道、綜合性、分數階系統及時滯系統渾沌同步研究。
3. Duffing-van der Pol 系統與 Mathieu-Duffing 系統之渾沌研究。用相圖、分歧圖、功率譜圖、李氏指數分析渾沌之行為，包括奇異吸引子之範圍及形狀、超渾沌之行為等。
4. 實用渾沌適應控制反控制新方法。將原渾沌系統控制到不同系統之周期解或平衡點，此為渾沌控制。將原系統之周期解反控制為不同系統之渾沌解，或原系統之渾沌解反控制為不同系統更複雜渾沌解。此為混沌反控制。這些系統之參數一般皆未知，所以再加上適應控制法以達同步，再引用具概率觀念之實用穩定理論嚴格證明參數估計值趨近於未知值。
5. 兩種慣性測速器新系統的渾沌行為研究，用相圖、分歧圖、功率譜圖、李雅普諾夫指數

分析渾沌之行為。

6.指數渾沌同步新方法。使同步完成時間大為減少，結合後步（backstepping）設計使V函數易於選擇，減少控制器數目。

關鍵詞： Duffing-van der Pol 系統、Mathieu-Duffing 系統、Mathieu-van der Pol 系統、Ikeda-Mackey-Glass 系統、慣性測速器新系統、實用適應混沌同步與反同步新方法、多管道、綜合性激勵的非耦合渾沌同步新方法、指數後步渾沌同步新方法

Abstract

Chaos systems have obtained wide applications in physics, chemistry, physiology, biology and various engineerings. Duffing system, van der Pol system and nonlinear Mothieu system all are paradigmatic chaotic systems in chaotic dynamics. Ikeda system and Mackey-Glass system are paradigmatic electro-optical and physiological system. Two kinds of inertial tachometer are also important mechanical systems. In this project, by suitable coupling, four new systems, namely, Duffing-van der Pol system, Mathieu-Duffing system, Mathieu-van der Pol systems and Ikeda-Mackey-Glass system are given. Two kinds of inertial tachometer are also new important systems.

Chaos control and synchronization have rapidly extended their application for physical, chemical, biological systems, secure communication, neural networks, self-organization etc. In this project, a new chaos control and anticontrol method, two new chaos synchronization methods are proposed. They all deserve significant both theoretical and practical importance：1.pragmatical adaptive chaos control and anticontrol for different systems. Traditional chaos control and anticontrol only work for the same system. The new method extends the chaos control and anticontrol to other different systems, greatly increases its effectiveness. 2.new chaos synchronization method for uncoupled systems. For traditional synchronization by coupling, there exist defections of losing secret and lagging of signals, which can by eliminated by uncoupled synchronization. Traditional uncoupled synchronization are obtained by exciting two corresponding parameters of the systems to be synchronized by the same chaotic or noise signal. In this project, (a)multichanneled various excitations(various time function, chaotic function, noise, etc)are used to increase the reliability of synchronization in the accident of interruption of part of the channels.(b)Synthetic excitations(e.g. periodically modulated chaotic signal, chaos modulated noise, etc)are used to ensure the security. (c)(a)(b)are used simultaneously to ensure more security. (d)(a),(b),(c) are used for fractional order systems and time-delay systems. 3.New exponential backstepping synchronization method. In traditional chaos synchronization method, Lyapunov function V is positive definite, V is negative definite, the settling time of synchronization is rather long, the control quality is not satisfactory. In this project, exponential synchronizations used to increase the control quality greatly. Combined with backstepping design, three advantages are obtained: 1. Settling time is decreased greatly. 2.V functions are chosen step by step, which becomes more easy. 3.The number of controller is decreased. The main parts of our study are:

1. The study of chaos of Mathieu-van der Pol system and Ikeda- Mackey-Glass system: By phase portraits, bifurcation diagrams, power spectra, Lyapunov exponents, the various chaotic behaviors of these systems will be studied.
2. New uncoupled synchronization method. Multichanneledly synthetically excited

synchronizations for integral and fractional ordered, time-delay systems are studied.

3. The study of chaos of Duffing-van der Pol system and Mathieu-Duffing system: By phase portraits, bifurcation diagrams, power spectra, Lyapunov exponents, the various chaotic behaviors of these systems will be studied. The regions and shapes of the strange attractors, hyperchaos, ect will also be studied.
4. New pragmatical adaptive chaos control and anticontrol method for different systems. pragmatical asymptotical stability theory by probability concept is used to prove the estimated parameters must approach the unknown parameters.
5. The study of chaos of two kinds of inertial tachometer：By phase portraits, bifurcation diagrams, power spectra, Lyapunov exponents, the various chaotic behaviors of these systems will be studied.
6. New exponential backstepping synchronization method.

key words: Duffing-van der Pol system, Mathieu-Duffing system, Mathieu-van der Pol system, Ikeda- Mackey-Glass system, Inertial tachometer system, Pragmatical adaptive chaos control and anticontrol method, Multichannelly an Synthetically excited uncoupled chaos synchronization, Exponential backstepping chaos synchronization.

前言與研究目的

渾沌系統之研究除了在理論上的重要價值外，在物理、化學、生理學及各種工程等方面皆有廣泛之應用。Duffing系統、van der Pol系統與非線性Mathieu系統都是重要的典型渾沌系統。Ikeda系統是重要的典型光電或生理時滯系統，而Mackey-Glass系統則是著名的典型生理時滯系統。對於這些重要系
統的渾沌現象及渾沌同步都已有豐富的研究成果[1-49]。本計畫為了對這些著名系統，擴大其研究範圍並深化其研究內容，特首先提出混合的新系統，即Duffing-van der Pol系統、Mathieu-Duffing系
統、Mathieu-van der Pol系統及Ikeda- Mackey-Glass系統。然後又提出兩種新創的慣性測速器系統，極具實用價值，其渾沌現象值得仔細研究。對上述六種新系統，首先證明其為渾沌系統，其次研究其渾沌行為。

渾沌控制與反控制及同步在物理系統、化學系統、生物系統、各種工程系統、秘密通訊、神經網路、自我組織系統等方面有長足之應用[50-100]。本計畫提出一種新的渾沌系統控制及反控制方法，兩種新渾沌同步方法，對這些新系統加以研究。

（一）Mathieu-van der Pol系統與Ikeda- Mackey-Glass系統的渾沌行為及非耦合渾沌同步新方法

（a）Mathieu-van der Pol系統及Ikeda- Mackey-Glass系統的渾沌研究

經典的非線性Mathieu系統為：

$$\ddot{x}+a(1+\sin\omega t)x+(1+\sin\omega t)x^3+a\dot{x}=0$$

或

$$\dot{x}_1=x_2$$

$$\dot{x}_2=-a(1+\sin\omega t)x_1-(1+\sin\omega t)x_1^3-ax_2$$

其中 a 為常數。

經典的van der Pol系統為：

$$\ddot{x}+bx+c\dot{x}(1-x^2)-d\sin\omega t=0$$

或

$$\dot{x}_3=x_4$$

$$\dot{x}_4=-bx_3+c\dot{x}_4(1-x_3^2)+d\sin\omega t$$

其中 b,c,d 為常數。現將第一系統中之 $\sin\omega t$ 與第二系統中的 $d\sin\omega t$ 中的 $\sin\omega t$ 交替換成對方的狀態變量，即得本計劃新創造之混合的Mathieu-van der Pol系統：

$$\dot{x}_1=x_2$$

$$\dot{x}_2 = -a(1+x_3)x_1 - (1+x_3)x_1^3 - ax_3$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -bx_3 + c\dot{x}_4(1-x_3^2) + dx_1$$

經典的Ikeda系統為：

$$\dot{x} + ax + b\sin x_\tau = 0$$

其中a, b為常數，$x_\tau = x(t-\tau)$，τ為常數。此系統可用以表示有回授之B級雷射系統，其中B級之典型

代表為固態、半導體及低壓CO_2雷射[35-36]，也可表示生理血液系統。

經典的Mackey-Glass系統為：

$$\dot{x} = \frac{bx\tau}{1+x_\tau^n} - rx$$

其中b, r為常數，n為正整數，$x_\tau = x(t-\tau)$，τ為常數。此系統可表示造血系統，其中x為t時刻之血液濃度，τ為造血所須之延遲時間。白血患者之τ值增加即引起血液濃度產生渾沌變化[39-40]。Ikeda-Mackey-Glass系統為：

$$\dot{x} = -\alpha x + m\sin x_\tau + \kappa y_\tau$$

$$\dot{y} = -\beta y + \frac{cy_\tau}{1+y_\tau^b}$$

對此新混合系統之研究，可說是對各單系統渾沌行為研究之延伸與深化，此混合系統比各單系統顯然有更複雜之渾沌行為，值得本計劃加以研究。

（b）非耦合渾沌同步新方法及應用

目前文獻絕大部分渾沌同步皆採耦合同步或控制器中出現主從系統之狀態變量，實際上也是耦合。以秘密通訊而言，耦合所須的主從系統狀態變量之傳送會造成失密及時滯之不良後果。非耦合同步可消除這些缺點，目前國際文獻多採以第三系統之渾沌變量或噪音對主從系統參數作用時激勵而獲同步。本計畫採（a）多管道下之各種形式之激勵（各種時間週期函數、渾沌函數、各種噪音）以保證同步之可靠性。（b）綜合性激勵（周期調制的渾沌函數、渾沌調制之噪音等）。（c）（a）、（b）同時進行以增加保密程度。（d）對分數階系統及時滯系統進行（a）、（b）、（c）三種激勵。

以Mathieu-van der Pol系統為例。兩個Mathieu-van der Pol系統：

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -a(1+x_4)x_1 - (1+x_4)x_1^3 - ax_2 - bx_3$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -x_3 + c(1-dx_3^2)x_4 + dx_1$$

和

$$\dot{y}_1 = y_2$$

$$\dot{y}_2 = -a(1+y_4)y_1 - (1+y_4)y_1^3 - ay_2 - by_3$$

$$\dot{y}_3 = x_4$$

$$\dot{y}_4 = -y_3 + c(1-dy_3^2)y_4 + dy_1$$

設有第三任何渾沌系統，為簡單計，即採用第三個Mathieu-van der Pol系統：

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = -a(1+z_4)z_1 - (1+z_4)z_1^3 - az_2 - bz_3$$

$$\dot{z}_3 = z_4$$

$$\dot{z}_4 = -z_3 + c(1-dz_3^2)z_4 + dz_1$$

用其任一渾沌變量，z_1之任意給定函數kz_1或$f(z_1)$、$F(t)z_1$（$F(t)$為任意給定函數）等等取代第一及第二系統中對應的第四式末項中dx_1、dy_1中之d_1而成為kz_1x_1及kz_1y_1。調節k為適當值時第一、二系統即達成同步。將kz_1換為以上（a)，(b)，(c）中之各項乃可得許多豐富的同步之結果，本計畫將對眾多之激勵項用量化方法比較其優劣，預期將發現許多意外之新現象。

（d）項所指分數階系統為近年來渾沌系統之發展，例如仍以Mathieu-van der Pol系統為例：

$$\frac{d^{q_1}x_1}{dt} = x_2$$

$$\frac{d^{q_2}x_2}{dt} = -a(1+x_4)x_1 - (1+x_4)x_1^3 - ax_2 + bx_3$$

$$\frac{d^{q_3}x_3}{dt} = x_4$$

$$\frac{d^{q_4}x_4}{dt} = -x_3 + c(1-dx_3^2)x_4 + dx_1$$

其中q_1, q_2, q_3, q_4皆為分數。以前未發現分數階渾沌系統之前，就整數階自治系統而言，總階數至少為三方有可能出現渾沌。此鐵律已被分數階渾沌系統所打破。其總階數遠小於3

時皆可能有渾沌出現。由於分數階方程之多樣化，作為秘密通訊之用時，其渾沌更難破解，增加了保密度，故分數階系統之渾沌同步極有實用價值。

（二）Duffing-van der Pol系統及Mathieu-Duffing系統的渾沌行為及不同系統實用渾沌適應控制反控制新方法。

（a）Duffing-van der Pol系統及Mathieu-Duffing系統的渾沌行為研究

經典的Duffing系統是非自治系統：

$$\ddot{x}+a\dot{x}+bx+cx^3=d\cos\omega t$$

或

$$\dot{x}=y$$

$$\dot{y}=-ay-bx-cx^3+d\cos\omega t$$

（1）

其中 a,b,c,d 為常數，$d\cos\omega t$ 為外加激勵項。經典的van der Pol系統是非自治系統：

$$\ddot{x}+dx+e\dot{x}(x^2-1)+f\sin\omega t=0$$

或

$$\dot{x}=y$$

$$\dot{y}=-dx+e(1-x^2)y-f\sin\omega t$$

（2）

其中 d,e,f 為常數，$f\sin\omega t$ 為外加激勵項。現將（1）式中及（2）式中的兩個激勵項 $\cos\omega t$ 及 $\sin\omega t$ 交替換成對方的狀態變量，即得到本計畫新創的混合新自治系統（autonomous）的Duffing-van der Pol系統

$$\dot{x}=y$$

$$\dot{y}=-ay-bx-cx^3+du$$

$$\dot{u}=v$$

$$\dot{v}=-du+e(1-u^2)v-fx$$

經典非線性Mathieu系統為

$$\ddot{x}+a(1+\sin\omega t)x+(1+\sin\omega t)x^3+a\dot{x}=0$$

或

$$\dot{x}=y$$

$$\dot{y}=-a(1+\sin\omega t)x-(1+\sin\omega t)x^3-ay$$

（3）

其中 a 為常數，將（3）式中及（1）式中之激勵項 $\sin\omega t$ 及 $\cos\omega t$ 交替換成對方的狀態變量，即得到本計畫新創的混合新自治的Mathieu-Duffing系統

$$\dot{x} = y$$

$$\dot{y} = -a(1+u)x - (1+u)x^3 - au$$

$$\dot{u} = v$$

$$\dot{v} = -av - bu - cu^3 + dx$$

對此系統的研究，不僅是對單Duffing系統、單van der Pol系統行為研究之延伸與深化，此系統比兩個單系統有更複雜的渾沌行為，當可預期。本計畫將研究其周期運動、準周期運動、渾沌運動及超渾沌
運動。

（b）不同系統實用渾沌適應控制反控制新方法及應用

目前文獻中的渾沌控制及反控制皆限於同一系統，即在同一系統中將原來的渾沌運動控制到周期運動或平衡點，謂之渾沌控制。反之，在同一系統中將原來的平衡點或周期運動控制到渾沌運動，謂之渾沌反控制。本方法突破此範圍，將原本系統之渾沌控制到另一任意指定系統之周期運動或平衡點，謂之渾沌控制。將原來系統的平衡點，周期運動控制為另一任意指定系統的渾沌，謂之渾沌反控制。將原來系統的渾沌控制到另一任意指定系統的更複雜渾沌運動，也謂之渾沌反控制。如此一來，渾沌控制與反控制之能力大為增強。另外，由於多數系統之參數皆未得其精確值而屬未知，故加用適應控制法使估計參數趨近於未知參數值。但目前文獻中之適應控制對此種趨近並未提出證明[101-106]，故本計畫採用申請人所提出之實用漸近穩定理論（pragmatical assymptotical stability theorem）[119-120]，引用機率（probability）的概念嚴格證明估計參數值必然趨近於未知參數值。以上說明本方法名稱之由來。下面概述此新方法之要點。

原系統為渾沌或非渾沌系統皆為：

$$\dot{x} = f(x, \hat{A}) + u(t)$$

其中 $x = [x_1, \ldots\ldots, x_n]^T \in R^n$ 為狀態向量，$\hat{A}$ 為 f 中之估計參數向量，f 為一非線性向量函數，

$u(t) = [u_1(t), u_2(t), \ldots., u_n(t)]^T \in R^n$ 為輸入控制向量。

目的系統為渾沌或非渾沌系統皆可：

$$\dot{y} = g(y, \hat{B})$$

其中 $y = [y_1, \ldots\ldots, y_n]^T \in R^n$ 為狀態向量，$\hat{B}$ 為 g 中之估計參數向量，g 為一非線性向量函數。

我們的目的是設計一個適應控制方法及控制器 $u(t)$ 使原系統之狀態變量漸近趨於目的系統之對應狀態變量。定義誤差為 $e = y - x$。當

$$\lim_{t \to \infty} e = 0$$

則同步完成。由上面二系統方程可得誤差微分方程

$$\dot{e} = \dot{y} - \dot{x}$$

$$\dot{e} = g(y,\hat{B}) - f(x,\hat{A}) - u(t)$$

選一最簡單的實正 Lyapunov 函數V

$$V(e,\widetilde{A},\widetilde{B}) = \frac{1}{2}e^T e + \frac{1}{2}\widetilde{A}^T\widetilde{A} + \frac{1}{2}\widetilde{B}^T\widetilde{B}$$

其中 $\widetilde{A} = A - \hat{A}$, $\widetilde{B} = B - \hat{B}$, $\hat{A}$, $\hat{B}$ 是給定參數向量或估計參數向量，A、B 為目的參數向量或不確定參數向量。其沿誤差方程式及參數動態方程之任一解的全導數為

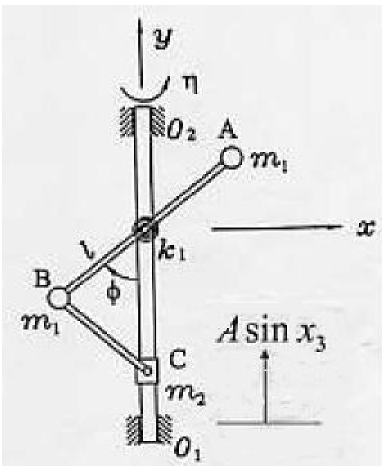
$$\dot{V}(e) = e^T[g(y,\hat{B}) - f(x,\hat{A}) - u(t)] + \widetilde{A}\dot{\widetilde{A}} + \widetilde{B}\dot{\widetilde{B}}$$

其中將$u(t)$，$\dot{\widetilde{A}}$ 及 $\dot{\widetilde{B}}$ 選成使$\dot{V} = e^T Ce$，C 為對角負定矩陣，則$\dot{V}$ 為關於$e,\widetilde{A},\widetilde{B}$ 之負半定函數。故未能證明$e,\widetilde{A},\widetilde{B}$ 會趨於零。根據目前文獻[102-104]的做法，由 Babalat 引理可證明e趨於零，但卻沒有 $\widetilde{A},\widetilde{B}$ 會趨於零之證明。本計畫根據申請人提出之引用概率概念之實用漸近穩定定理嚴格證明，只要符合一寬鬆的條件，$\widetilde{A},\widetilde{B}$ 一定會趨於零，糾正了目前文獻之錯誤。並以本計畫提出之諸新系統為例，用理論及數值計算證明這新方法之有效性。

（三）兩種慣性測速儀新系統與指數後步渾沌同步新方法

（a）兩種慣性測速儀系統的渾沌行為研究

慣性彈簧測速儀系統是用以測量轉速的主要機械系統[107]，見圖一。



圖一

A 球質量為m_1，B 球質量為m_2。B 處為鉸鍊連接。C 為滑塊，質量不計，可沿鉛垂軸上下滑動。AB 桿長度為$2l$，BC 桿長度為l，D 為 AB 桿之中點。在 D 處有一根螺捲彈簧，其彈簧常數為k_1。AB 桿與鉛垂軸在 D 處由鉸鍊連結。鉸鍊黏性摩擦係數為k_2。AB 桿、BC 桿及鉛垂轉軸之質量皆不計。AB 桿與鉛垂軸之夾角為ϕ，鉛垂轉軸之轉速為η。當鉛垂軸等速轉動時，兩球之離心力矩與螺捲彈簧之力矩平衡。故不同之轉速η對應於不同之ϕ角。

由ϕ角可知轉速η。這就是此測速儀的工作原理。今設鉛垂軸底端有外加之振動$A\sin x_3$：

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -A\sin x_3$$

此振動相當於單擺之振動。令$x_1 = \phi$，$x_2 = \dot{\phi}$，可得此慣性彈簧測速儀系統之運動微分方程為

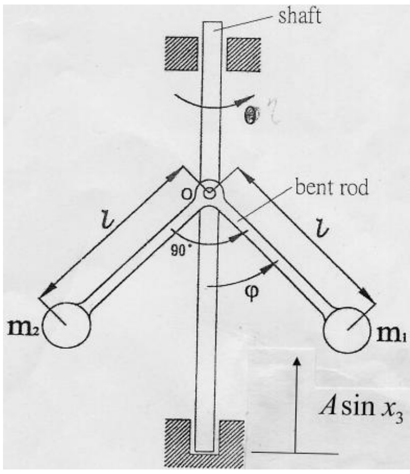
$$\dot{x}_1 = x_2$$

$$x_2 = \frac{1}{2m_1 + 4m_2 \sin^2 x_1}\left(\frac{-2m_2 g \sin x_1}{l} + \frac{2m_2 A\sin x_3 \sin x_1}{l}\right.$$
$$\left. - 4m_2 x_2^2 \sin x_1 \cos x_1 + 2m_1 \eta^2 \sin x_1 \cos x_1 - \frac{k_1}{l^2} x_1 - \frac{k_2}{l^2} x_2\right)$$

$$\dot{x}_3 = x_4$$

$$x_4 = -A\sin x_3$$

類似地直角慣性測速器系統（見圖二）之運動微分方程為



圖二

$$\dot{x}_1 = x_2$$

$$x_2 = \frac{1}{m_1 + m_2}[(m_1 - m_2)\eta^2 \cos x_1 \sin x_1$$
$$-\frac{1}{l}(g + A\eta^2 \sin x_3)(m_1 \sin x_1 - m_2 \cos x_1) - \frac{k}{l^2} x^2]$$

$$\dot{x}_3 = x_4$$

$$x_4 = -A\sin x_3$$

這兩種測速儀系統是有實用價值的新的重要系統，其渾沌行為很值得仔細研究。

（b）指數後步渾沌同步新方法

目前文獻[108-118]中，渾沌同步穩定理論皆採取 Lyapunov 漸近穩定理論中李氏函數V定為實正，其導數$\dot{V}$為負定之條件。所得之同步達成之時間較長，故自控制理論角度而言控制品質不夠好。本計畫採取指數漸近穩定理論研究誤差系統之零解漸近穩定性，使同步完成時間大為減少。再結合後步（backstepping）設計法，可以一步一步地設計李氏函數，大大降低了選擇適當V函數之難度。現在將指數漸近穩定理論描述如下。

指數穩定性理論

這是一種最好的漸近穩定性，解收斂於平衡位置的速度快，這個速度在控制論中被稱為過渡過程的品質指標[121]。

定義：設$\varphi_1,\varphi_2 \in K$，若存在$\mu > 0$，$\forall r \in [0,\mu]$，∃ $k_1 > 0, k_2 > 0$，使得

$$k_1\varphi_1(r) \le \varphi_2(r) \le k_2\varphi(r)$$

稱φ_1,φ_2具有局部同級增勢，若$\forall r \in [0,+\infty)$，上式成立，稱$\varphi_1,\varphi_2$具有全局同級增勢。

定理 1：若在含原點之區域G_H上存在$V(t,x)$與$\|x\|^a$具有同級增勢，$a > 0$及具有局部同級增勢的$\varphi_1(r),\varphi_2(r),\varphi_3(r) \in K$，使得

$$\varphi_1(\|x\|) \le V(t,x) \le \varphi_2(\|x\|)$$

$$\frac{dV}{dt} \le -\varphi_3(\|x\|)$$

則微分方程$\dot{x} = f(x,t)$的零解$x = 0$是指數穩定的。

注：將定理 1 中的G_H改為$I \times R^n, \varphi_1,\varphi_2 \in K$改為$\varphi_1,\varphi_2 \in KR$，其他條件不變，則$x = 0$是全局指數穩定的。

定理 2：若在G_H上存在$V(t,x)$，及常數$c_1 \ge 0$，$c_2 > 0$，使得

$$\|x\| \le V(t,x) \le c_1\|x\|$$

$$\frac{dV}{dt} \le -c_2(\|x\|)$$

則零解$x = 0$指數穩定。

總之，此新方法有三優點：1. 同步完成時間大為減少。2. 逐步選擇V函數，使選擇V函數之難度大為減低。3.可使控制器之數目降至最少，降低控制器設計成本。更以本計畫提出之諸新系統為例，證明此新方法之有效性。

重要性：

Duffing 系統，van der Pol 系統與線性 Mathieu 系統原為振動學科之最重要最典型的系

統。自渾沌動力學興起後，Duffing 系統，van der Pol 系統由於其為非線性系統故亦沿習成為渾沌動力學學科中最重要最典型的系統，四十年來對此二系統的渾沌研究之文獻可謂汗牛充棟，至今方興未艾。而線性
Mathieu 系統，則由於其為線性方程，不具渾沌性質，故在渾沌動力學學科中乃不再提及。人們忽視了非線性 Mathieu 系統實為 Duffing 系統中參數由常數轉為時間週期函數之推廣，實亦應成為渾沌動力學科之最重要最典型之系統。本計畫主持人率先研究非線性 Mathieu 系統之渾沌行為[6]，可謂遲來之補求。本計畫今研究三種能混合型之 Duffing-van der Pol 系統、Mathieu-Duffing 系統及 Mathieu-van der Pol 系統，不僅對渾沌動力學學科中最重要最典型的三種渾沌系統的研究的拓廣與深化，更重要的是它們本身顯然具有更複雜的，未經發現的複雜渾沌行為，本研究對渾沌動力學學科具重大意義。其應用於機械、電機、物理、化學、生科、奈米之耦合系統，具有重要的實用價值。至於 Ikeda 與 Mackey-Glass 系統同樣地是經典的、最重要的時滯系統，故其混合型的 Ikeda- Mackey-Glass 系統同樣具重要研究價值。兩種慣性測速器系統也是重要而極具實用價值之機械系統。此兩種新創之自治系統同樣極具研究價值。

本計畫提出一種新渾沌控制反控制方法，兩種新渾沌同步方法，皆具有重要理論與實用價值。不同系統實用渾沌適應控制反控制方法大大擴大的渾沌控制反控制之能力，使目前文獻中侷限於同一系統中之渾沌控制反控制擴展到可控制反控制到任意系統之任意非渾沌或渾沌運動。同時以申請人提出之實用漸近穩定理論糾正文獻中對估值參數趨近於未知參數未加證明之缺點。新的非耦合渾沌同步方法增加保密程度，增加了保證達成同步之可靠性。對分數階系統及時滯系統而言，則同樣增加了保密程度。新的指數後步同步新方法則有三優點：1. 同步完成時間大為減少。2. 逐步選擇 V 函數，使選擇 V 函數之難度大為減低。3.可使控制器之數目降至最少，降低控制器設計成本。更以本計畫提出之諸新系統為例，證明此新方法之有效性。

研究內容

Chapter 1
Introduction

Chaotic phenomena have been observed in physics, chemistry, physiology, and many disciplines [1-3]. In contrast with the famous chaotic systems, such as Lorenz system, Duffing system, and Rössler system, nonlinear Mathieu system is less mentioned [4-9]. However, nonlinear Mathieu system is important and can be applied in analysis of the resonant micro electro mechanical systems [10-12]. In this report, the new autonomous and new nonautonomous chaotic systems constructed by mutual linear coupling of two non-identical nonlinear damped Mathieu systems are studied.

Nonlinear dynamics, commonly called the chaos theory, changes the scientific way of looking at the dynamics of natural and social systems, which has been intensively studied over the past several decades. The phenomenon of chaos has attracted widespread attention amongst mathematicians, physicists and engineers. Chaos has also been extensively studied in many fields, such as chemical reactions, power converters, biological systems, information processing, secure communications, etc. [1-6]. Whilst many researchers analyze complicated, physically motivated configurations, there is also a need to investigate simple equations which may capture the essence of chaos in a less involved setting, thereby aiding the understanding of essential characteristics. The original investigation of an extraordinary three-dimensional nonlinear system by the mathematical meteorologist E.N. Lorenz who discovered chaos in a simple system of three autonomous ordinary differential equations in order to describe the simplified Rayleigh–Benard problem [7] (which is called *Yang* Lorenz system in this paper) is the most popular system for studying.

There are tremendous amount of articles in studying *Yang* Lorenz and other systems [8-12]. Although these systems have been analyzed in detail, there are no articles in looking into these systems, such as Lorenz system with $x(-t), y(-t), z(-t)$ and $-t$ (which is called *Yin* Lorenz system in this article). Since Lorenz discovered chaos on 1963, all studies of chaos concentrated when time went forward i.e. $t: 0 \rightarrow \infty$ in the last 47 years. Physically backward time, $t: 0 \rightarrow -\infty$, has not discovered up to now, but mathematically it can be easily performed and must be studied for complete understanding of the property of chaos. In this Chapter, we find out that there are rich dynamics in such *Yin* Lorenz system.

In Chinese philosophy, *Yin* is the negative, historical or feminine category in nature, while *Yang* is the positive, comtemporary or masculine category in nature. *Yin* and *Yan* are two fundamental opposites in Chinese philosophy. In Chapter 2, the *Yin* Lorenz system is introduced and the chaotic behavior with *Yin* parameters is investigated by phase portrait, Lyapunov exponents and bifurcation in the following simulation results. We use positive, i.e. *Yang*, parameters for the *Yang* Lorenz system, and negative, i.e. *Yin*, parameters for the *Yin* Lorenz system.

Chaotic systems are characterized by one positive Lyapunov exponent (PLE) in the Lyapunov spectrum [2-9]. The one PLE just indicates that the dynamics of the underlying chaotic attractor expands only in one direction. If a chaotic attractor is characterized by more than one positive Lyapunov exponent, it is termed hyperchaos. In this case, the dynamics of the chaotic attractor expands in more than one direction giving rise to a ‘‘thick’’ chaotic attractor [10-14]. There are both theoretical and practical interests in hyperchaos. Hyperchaos was first reported from computer simulations of hypothetical ordinary differential equations in [15-17]. The first observation of hyperchaos from a real physical system, a fourth-order electrical circuit, was later reported in [18]. Very few hyperchaos generators have been reported since then [19-22].

As the numerical example, recently developed new Mathieu-van der pol autonomous oscillator with four state variables is used. For this new system four Lyapunov exponents are not zero. Although by traditional theory [23], for four-dimensional continuous-time systems, there must be a zero Lyapunov exponent, however, on the history of science, as said by T. S. Kuhn in his book ‘‘The Structure of Scientific Revolution’’, the unexpected discovery or anomality (counterinstance) is not simply factual in its import and the scientist’s world is qualitatively transformed as well as quantitatively enriched by fundamental novelties of either fact or theory. “Conversion as a feature of revolutions in science” is the conclusion of the book “Revolution in Science” written by I. B. Cohen [24]. One of the patterns of the evolution of science is: current paradigm → normal science → anomality (counterinstance) → crisis → emergence of scientific theories → new paradigm.

Recently, Ott and Yorke [25] show that the existence of Lyapunov exponents is a subtle question for systems that are not conservative. They describe a simple continuous-time flow such that Lyapunov exponents fail to exist at nearly every point in the phase space. Ge and Yang [26] firstly find out the simulation results of 3PLES in Quantum Cellular Neuro Network autonomous system with four state variables. As a consequence, in Chapter 3, Mathieu-van der pol autonomous system with four state variables is introduced, and the hyperchaos for 3PLEs are investigated by phase portrait, power spectrum, Lyapunov exponents and parameter diagram in the following simulation results.

In our natural world, plenty of chaotic systems describing natural phenomenon are found that they have some states always positive. It means these states are always in the first quadrant. Such as the three species prey-predator system [36], double Mackey-Glass systems [37-38], energy communication system in biological research [39] and virus-immune system [40]. In Chapter 4, a new strategy to achieve chaos control by GYC partial region stability theory is proposed [32-33]. Via using the GYC partial region stability theory, the new Lyapunov function is a simple linear homogeneous function of error states and the lower order controllers are much more simple and introduce less simulation error.

In Chapter 5, a new chaos generalized synchronization strategy by GYC partial region stability theory is proposed [20-21]. It means that there exists a given functional relationship between the states of the master and that of the slave. Via using the GYC partial region stability theory, the new Lyapunov function is a simple linear homogeneous function of states and the lower order controllers are much more simple and introduce less simulation error.

In current scheme of adaptive synchronization, traditional Lyapunov stability theorem and

Barbalat lemma are used to prove that the error vector approaches zero as time approaches infinity, but the question that why those estimated parameters also approach the uncertain values remains no answer. In this article, pragmatical asymptotically stability theorem and an assumption of equal probability for ergodic initial conditions [50-51] are used to prove strictly that those estimated parameters approach the uncertain values. Moreover, traditional adaptive chaos synchronization in general is limited for the same system. Therefore, In Chapter 6, a new adaptive synchronizing strategy - pragmatical adaptive synchronization by GYC partial region stability theory is proposed as well. Via using this new approach, the new Lyapunov function is a simple linear homogeneous function of states and the lower order controllers and parametric update laws are much simpler and introduce less simulation error.

There are various types of synchronization, such as complete synchronization [48], generalized synchronization [49], phase synchronization [50], lag synchronization [51], and so on. Among these types of synchronization, generalized synchronization is one of the most interesting topics. Generalized synchronization refers to a functional relation between the state vectors of master and slave, i.e. $\mathbf{y}=\mathbf{F}(\mathbf{x},t)$, where $\mathbf{x}$ and $\mathbf{y}$ are the state vectors of master and slave. In the work of Ref. [52], the generalized synchronization is extended to a more general form, $\mathbf{y}=\mathbf{F}(\mathbf{x},\mathbf{y},t)$, where the "slave" $\mathbf{y}$ is not a traditional pure slave obeying the "master" $\mathbf{x}$ completely but plays a role to determine the final desired state of the "slave". Since the "slave" $\mathbf{y}$ plays an "interwined" role, this type of synchronization is called "symplectic synchronization"[1], the master is called "partner A", and the slave is called "partner B". In this report, we propose two types of new chaos synchronization, "non-simultaneous symplectic synchronization" and "double symplectic synchronization".

We propose the "non-simultaneous symplectic synchronization", $\mathbf{y}(t)=\mathbf{F}(\mathbf{x}(\tau),\mathbf{y}(t),t)$, where τ is a given function of time t, so-called variable scale time. The synchronization is achieved at "different time" for "partner A" $\mathbf{x}(\tau)$ and "partner B" $\mathbf{y}(t)$, therefore we call this type of synchronization "non-simultaneous symplectic synchronization". When $\tau=t$, non-simultaneous symplectic synchronization reduces to symplectic synchronization. When applying the non-simultaneous symplectic synchronization in secret communication, since the functional relation of the non-simultaneous symplectic synchronization is more complex than that of the traditional generalized synchronization, and cracking the variable scale time τ is an extra task for the attackers in addition to cracking the system model and cracking the functional relation, the message is harder to be detected by applying the non-simultaneous symplectic synchronization than by applying traditional generalized synchronization. Therefore, the non-simultaneous symplectic synchronization may be applied to increase the security of secret communication. In order to achieve non-simultaneous symplectic synchronization, nonlinear control [53] and adaptive control are applied. In the work of Ref. [53], the induced matrix norm and the Lipschitz constant are obtained by auxiliary numerical simulation. However, they can be estimated theoretically by using the property of induced matrix norms [54a] and by applying

[1] The term "**symplectic**" comes from the Greek for "interwined". H. Weyl first introduced the term in 1939 in his book "The Classical Groups" (p. 165 in both the first edition, 1939, and second edition, 1946, Princeton University Press).

adaptive control. Furthermore, in our case, non-simultaneous symplectic synchronization, the complexity of the functional relation $\mathbf{F}(\mathbf{x}(\tau), \mathbf{y}(t), t)$ is greater than that studied in Ref. [53], thus the Lipschitz constant may be enormous. However, by applying adaptive control, the estimated Lipschitz constant is much less than the Lipschitz constant obtained by applying nonlinear control. This result in the reduction of the gain of the controller, i.e. the cost of controller is reduced. The proposed scheme is effective and feasible for both autonomous and nonautonomous chaotic systems, whether the dimensions of $\mathbf{x}(\tau)$ and $\mathbf{y}(t)$ systems are the same or not.

In practice, some or all of the system parameters are uncertain. Moreover, these parameters change from time to time. Many researchers solve this problem by adaptive synchronization [122-127]. In current scheme of adaptive synchronization, traditional Lyapunov asymptotical stability theorem and Babalat lemma are used to prove the errors of synchronizing states approach zero. But the question that why the estimated parameters also approach the uncertain values, has still remained without answer. By the pragmatical asymptotical stability theorem [128-129] and an assumption of equal probability for ergodic initial conditions, the answer can be given.

Among many kinds of synchronizations, the generalized synchronization is investigated [130-142]. It means there exists a given functional relationship between the states of the master and that of the slave $y = G(x)$, where x, y are the states vector of master system and slave system respectively. In this report, a special kind of generalized synchronizations $y = G(x) = x + F(t)$ is studied, where $F(t)$ is a given vector function of time which may take various forms, either regular or chaotic function of time. When $F(t) = 0$, it reduces to a complete synchronization [143-144]. As a numerical example, two identical double Duffing chaotic systems [145] and a double van der Pol chaotic system [146-147] are used as master system, slave system, and goal system, respectively. The goal system gives chaotic $F(t)$. Next, the robustness of the generalized synchronization is also studied [148-154].

The contents of this report are as follows. Chapter 2 contains the dynamics of new autonomous and nonautonomous chaotic systems. The system models are described and the numerical results of regular and chaotic behaviors are presented. In Chapter 3, generalized synchronization of new chaotic systems is achieved by applying pure error dynamics and elaborate Lyapunov function. The methods of designing Lyapunov function are presented, and both new autonomous and new nonautonomous chaotic systems are illustrated in examples. By applying pure error dynamics and elaborate nondiagonal Lyapunov function, nonlinear generalized synchronization of new chaotic systems is obtained in Chapter 4. We propose the methods of designing Lyapunov function, and illustrate them by both new autonomous and new nonautonomous chaotic systems in examples. In Chapter 5, the dynamics of nonholonomic systems is studied by applying the fundamental nonholonomic form of Lagrange's equations. Two types of external nonholonomic constraints are studied for moving target pursuit problems: a straightly oscillating target and a circularly rotating target. Numerical results show that chaos exists in each case. By applying the nonlinear nonholonomic form of Lagrange's equations, the dynamics of nonlinear nonholonomic system is studied in Chapter 6. We investigate external nonlinear nonholonomic constraint: the magnitude of velocity keeping constant. Chaos is proved to exist in each case by numerical results. Furthermore, Feigenbaum number rule still holds for nonlinear nonholonomic system. In Chapter 7, the non-simultaneous symplectic synchronization is proposed, and it is achieved by applying adaptive control. The synchronization scheme is presented, and chaotic systems with the same or different dimensions are illustrated in examples. We investigate the double symplectic synchronization by applying active control in Chapter 8. The synchronization scheme is derived, and both autonomous and nonautonomous chaotic systems are illustrated in examples. In Chapter 9 the fractional derivative and its approximation are introduced. And then gives the dynamic equation of double Duffing system. The system under study is described both in its integer and fractional forms. Numerical simulation results are presented. In Chapter 10, a brief description of synchronization scheme based on the substitution of the strengths of the mutual coupling term of two identical chaotic double Duffing systems by the chaotic variable of a third double Duffing system are presented. And numerical simulations are given for illustration. It is found that one can obtain CS or AS by adjusting the driving strength and initial conditions. In Chapter 11, chaos synchronization and antisynchronization are obtained by replacing two corresponding parameters of two uncoupled identical double Duffing chaotic dynamical systems by a white noise, a Rayleigh noise, a Rician noise or a uniform noise respectively. It is found that one can obtain CS or AS by adjusting the driving strength. In Chapter 12, theoretical analyses of the pragmatical asymptotical stability are quoted. Adaptive controllers are designed for the pragmatical generalized synchronization of two double Duffing chaotic oscillators with a double van der Pol chaotic system as a goal system. High robustness of the generalized synchronization is also obtained in Chapter 12. In Chapter 13, chaotic behaviors of a fractional order double van der Pol system are studied by phase portraits and Poincaré maps. It is found that chaos exists in this system with order from 3.9 down to 0.4 much less than the number of states of the system. Linear transfer function approximations of the fractional integrator block are calculated for a set of fractional orders in [0.1, 0.9] based on frequency domain arguments. In Chapter 14, the variable of a third double van der Pol system substituted for the strength of two corresponding mutual coupling term of two identical chaotic double van der Pol system, give rise to their complete synchronization (CS) or anti-synchronization (AS). Numerical simulations show that either CS or AS depends on initial conditions and on the strengths of the substituted variable. In Chapter 15, we focus on the synchronization and antisynchronization of two identical double Duffing systems whose corresponding parameters are replaced by a white noise, a Rayleigh noise, a Rician noise or a uniform noise respectively. It is noted that whether CS or AS appear depends on the driving strength. In Chapter 16, based on a pragmatical theorem of asymptotical stability using the concept of probability, an adaptive control law is derived such that it can be proved strictly that the zero solution of error dynamics and of parameter dynamics is asymptotically stable. Numerical results are given for a chaotic double van der Pol system controlled to a double Duffing system. In Chapter 17, chaos in new integral and fractional order double Ikeda delay systems is studied. A double Ikeda delay system consists of two traditional Ikeda delay systems which are coupled together. Numerical simulations display the chaotic behaviors of the integral and fractional order delay systems by phase portraits, Poincaré maps and bifurcation diagrams. In Chapter 18, the chaotic behaviors of double Ikeda systems are obtained by replacing the original constant delay time by a function of chaotic state variable of a second chaotic double Ikeda system. The method is named delay time excited chaos synchronization which can be successfully obtained for some cases. Numerical simulations are illustrated by phase portraits. Phase portrait is expressed by numerical analysis. In Chapter 19, it is discovered that lag synchronization and lag anti-synchronization appear for two identical double Ikeda systems, without any control scheme or coupling terms, but with different initial conditions. In Chapter 20, the chaotic behaviors of double Ikeda systems are obtained by replacing the parameters by different chaotic state variables of a third chaotic double Ikeda system. The method is named parameter excited method for synchronization which will be successfully used for uncoupled synchronization. Numerical simulations are illustrated by phase portraits and time histories. In Chapter 21, a new double Mackey-Glass delay system, which consists of two coupled

Mackey-Glass systems, is studied. Numerical simulations display the chaotic behaviors of the integral and fractional order delay systems by phase portraits and bifurcation diagrams. In Chapter 22, a control method called parameter excited method is applied to control a double Mackey-Glass chaotic system and to synchronize two uncoupled double Mackey-Glass systems. By replacing a parameter of the chaotic system by a noise signal, its chaotic motion can be eliminated. By replacing the corresponding parameters of two identical chaotic systems by a noise signal, these two chaotic systems with different initial conditions can be synchronized. For some chaotic systems, such as physical and electrical systems, which are difficult or even impossible to couple, this method is effective and potential in practice. In Chapter 23, it is discovered that TLS, TAS and TALS, TAAS appear for two identical double Mackey-Glass systems, without any control scheme or coupling terms, but with different initial conditions. In Chapter 24, the lag synchronization of two uncoupled double Mackey-Glass systems is achieved via the parameter excited method. This method is accomplished by replacing the corresponding parameters of the systems with two lag noise signals. By means of the difference of the timing between two replacements for the first system and the second system, the lag synchronization can be obtained. The parameter of the first system is substituted by a noise at $t = 0\sec$, and the parameter of the second system is substituted by the noise at $t = d\sec$. In other words, the control schemes do not work synchronously for these two systems. Parameter excited method is effective and potential in practice for some chaotic systems which are difficult or even impossible to be coupled. Temporary lag synchronization, partial lag synchronization, chaos control and robustness of lag synchronization are also obtained by this method. Finally, the conclusions of the whole report are drawn in Chapter 25.

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Chapter 2

Hyperchaos of New Mathieu-van der Pol system with Three Positive Lyapunov Exponents

2.1 Preliminaries

This Chapter gives another illustration of three positive Lyapunov exponents (3PLES) in numerical simulations of a new system, Mathieu-van der pol autonomous system, with four state variables. As we know, two positive Lyapunov exponents confirm hyperchaotic nature of its dynamics and means that system can present more complicated behavior than ordinary chaos. We further generate three positive Lyapunov exponents in a new coupled nonlinear system and anticipate the advanced application in secure communication. Not only a new chaotic system with three Lyapunov exponents is proposed, but also its implementation of electronic circuit is putting into practice in this article. The phase portrait, electronic circuit, power spectrum, Lyapunov exponents and 2-D and 3-D parameter diagram with three positive Lyapunov exponents of the new system will be showed in this Chapter.

2.2 Differential equations for Mathieu-van der Pol system and phase protraits

Mathieu equation and van der Pol equation are two typical nonlinear non-autonomous systems:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -(a + b\sin\omega t)x_1 - (a + b\sin\omega t){x_1}^3 - cx_2 + d\sin\omega t \end{cases} \tag{2-2-1}$$

$$\begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -ex_3 + f(1 - x_3^2)x_4 + g\sin\omega t \end{cases} \tag{2-2-2}$$

Exchanging sinωt in Eq. (2-2-1) by x_3 and sinωt in Eq. (2-2-2) by x_1, we obtain the autonomous new Mathieu -van der Pol system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -(a+bx_3)x_1 - (a+bx_3)x_1{}^3 - cx_2 + dx_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -ex_3 + f(1-x_3{}^2)x_4 + gx_1 \end{cases} \quad (2\text{-}2\text{-}3)$$

where x, y, z and w are four stats of the system, a, b, c, d, e, f and g are parameters of the Mathieu-van der Pol system.

It is well-known that the phase portrait presents the evolution of a set of trajectories emanating from various initial conditions. When the solution becomes stable, the asymptotic behaviors of the phase trajectories are particularly interested and the transient behaviors in the system are neglected. As a result, the phase portrait projections of the Mathieu-van der Pol system, Eq. (2-2-3), is plotted in Fig. 2-1. In this numerical studies, the parametric values are taken to be *a=91.7, b=5.023, c=-0.001, d=91, e=87.001, f=0.0180 and g=9.5072* for plotting the hyperchaotic phase portrait projections.

2.3 Power spectrum

The power spectrum analysis of the nonlinear dynamical system, Eq. (2-2-3) is shown in Fig. 2-2. The noise-like spectrum is the characteristics of chaotic dynamical system.

2.4 Lyapunov exponents

The Lyapunov exponents of Mathieu-van der Pol system with 3PLEs are plotted in Figs. 2-3~2-8. These figures show that there exits at least one PLE in the Lyapunov spectrum for our new system, and the Lyapunov exponents of Mathieu-van der Pol system are varied with parameters a, b, d and e.

2.5 Parameter diagrams

A system with more than one positive Lyapunov exponent can be classified as a hyperchaotic system. In this study, the parameter values, *b*, *d*, *g*, and *f*, are varied to observe the regions of chaos of our new system. The enriched information of chaotic behaviors of the system can be obtained from the Figs 2-9~2-14.

In Figs 2-9~2-14, the regions of 3PLEs are yellow, 2PLEs green and 1PLEs purple. It can be realized that the Mathieu-van der pol system is chaotic in several different region, especially hyperchaos with 3 PLEs are found in many regions between hyperchaos with 2 PLE and chaos with 1 PLE.

2.6 Phase portraits and its implementation of electronic circuits

It is well-known that the phase space can present the evolution of a set of trajectories emanating from various initial conditions. When the solution becomes stable, the asymptotic behaviors of the phase trajectories are particularly interested and the transient behaviors in the system are neglected. As a result, the phase portrait of the Mathieu-van der pol system, equation (2-2-1), is plotted in Fig. 2-1. In this numerical studies, the parametric values are taken to be $a=91.7$, $b=5.023$, $c=0.01$, $d=91$, $e=87.001$, $f=0.0180$ *and* $g=9.5072$ for plotting the tri-chaotic phase portrait. The new system can be represented as an electronic oscillator circuit and projection of phase portraits outputs shown in Figs. 2-15~16. We have implemented it using an electronics simulation package Multisim (previously called Electronic Workbench, EWB) and the approximated nonlinear electronic circuits are presented to realize the disordered behavior in the new chaotic system. The voltage outputs have been normalized to 1 V and the operational amplifiers are considered to be ideal. The phase diagrams are plotted within the time interval 500 s. The time step is 0.001 s. Due to the limit of the scope of implementation of electronic circuits, the phase portraits can be only shown in two dimensions. In Fig. 2-16, the configuration of electronic circuits is also given.

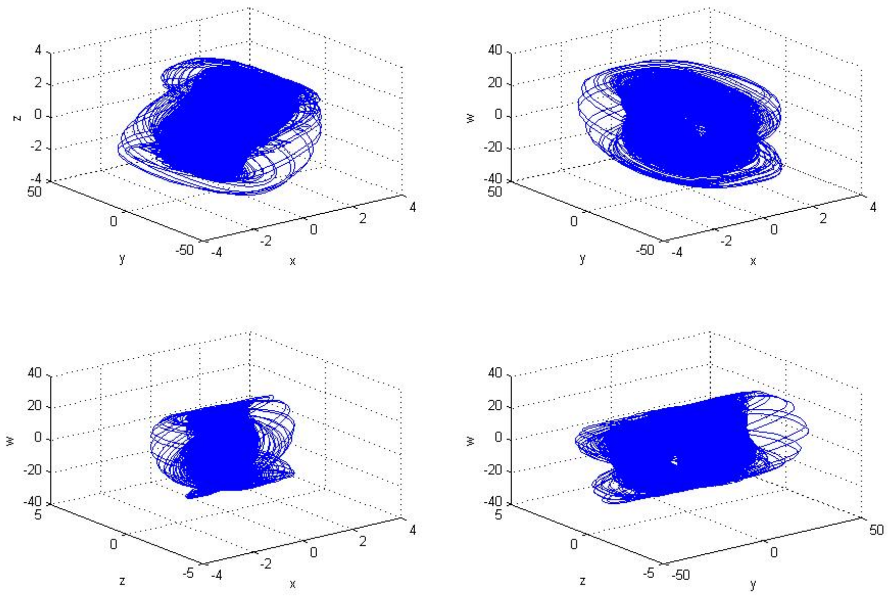


Fig.2- 1 Phase portrait projections of four state Mathieu-van der Pol system with *a*=91.17, *b*=5.023, *c*=-0.001, *d*=91, *e*=87.001, *f*=0.0180and *g*=9.5072.

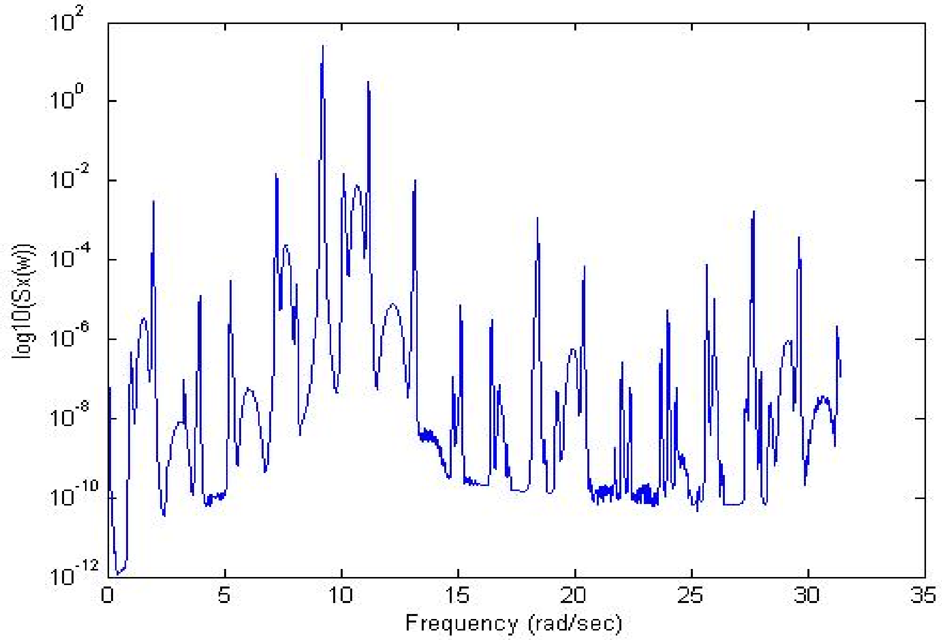


Fig.2- 2 Power spectrum of *x* for Mathieu-van der Pol system with *a*=91.17, *b*=5.023, *c*=-0.001, *d*=91, *e*=87.001, *f*=0.018and *g*=9.5072.

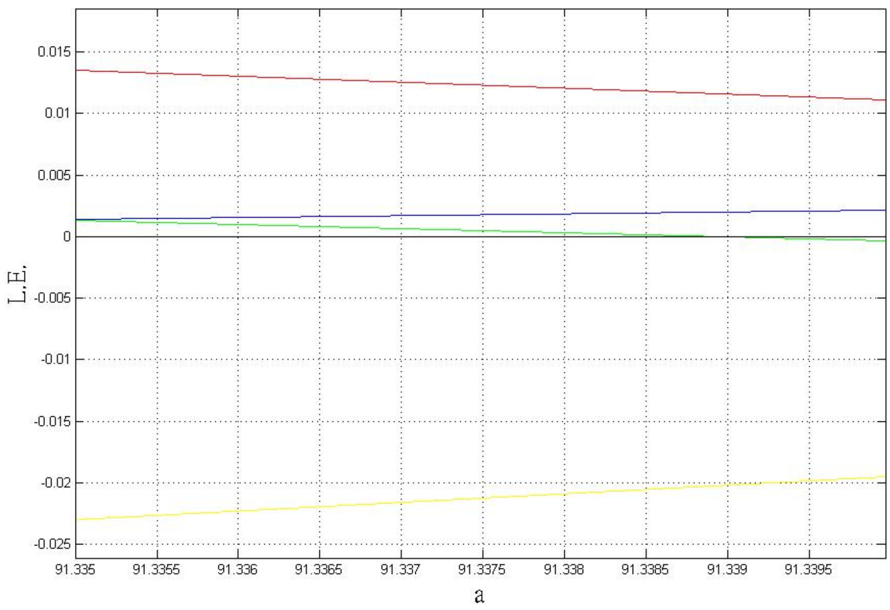


Fig.2- 3 Lyapunov exponents of Mathieu-van der Pol system with *b*=5.023, *c*=-0.001, *d*=91, *e*=87.001, *f*=0.018and *g*=9.5072.

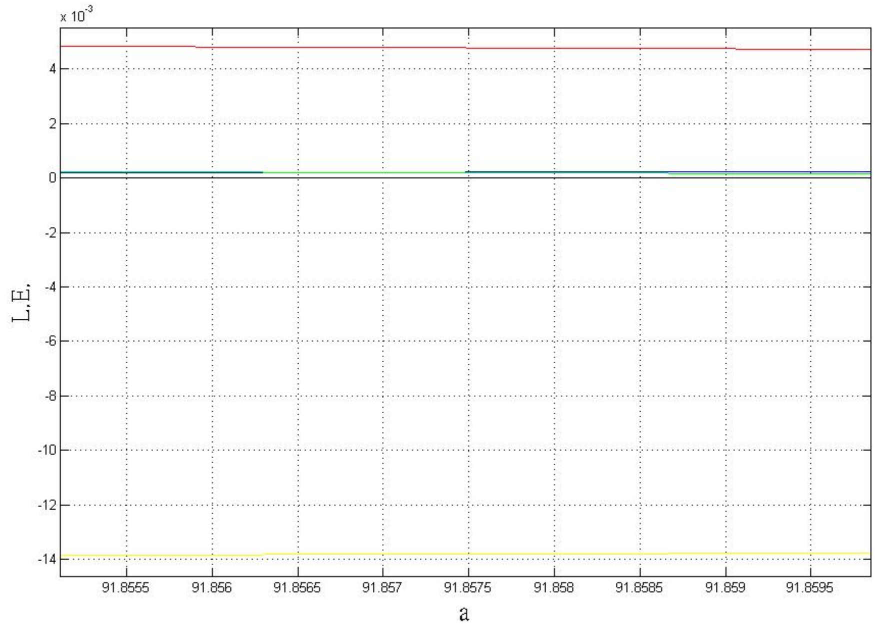


Fig.2- 4 Lyapunov exponents of Mathieu-van der Pol system with *b*=5.023, *c*=-0.001, *d*=25, *e*=87.001, *f*=0.018and *g*=9.5072.

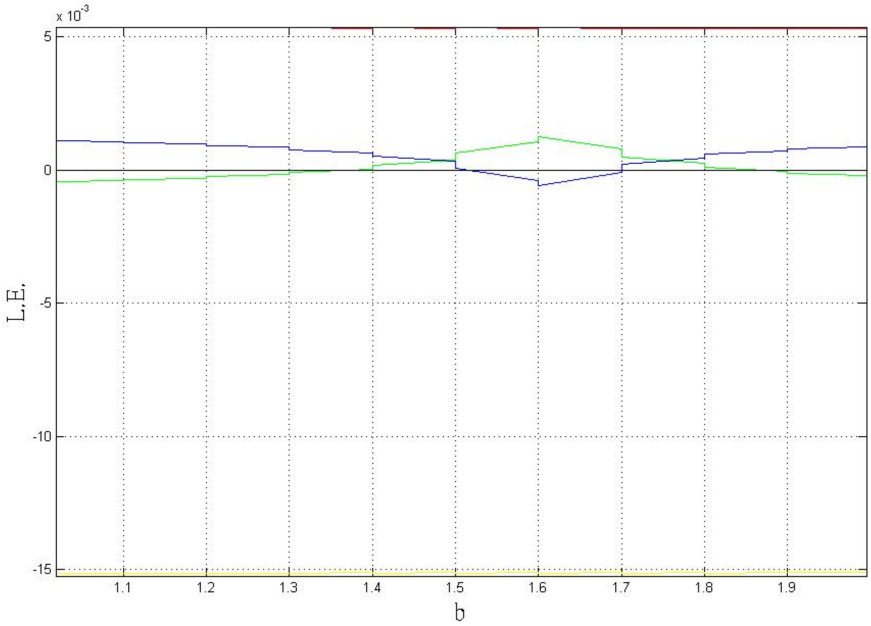


Fig.2- 5 Lyapunov exponents of Mathieu-van der Pol system with *a*=96.326680, *c*=-0.001, *d*=25, *e*=87.001, *f*=0.018and *g*=9.5072.

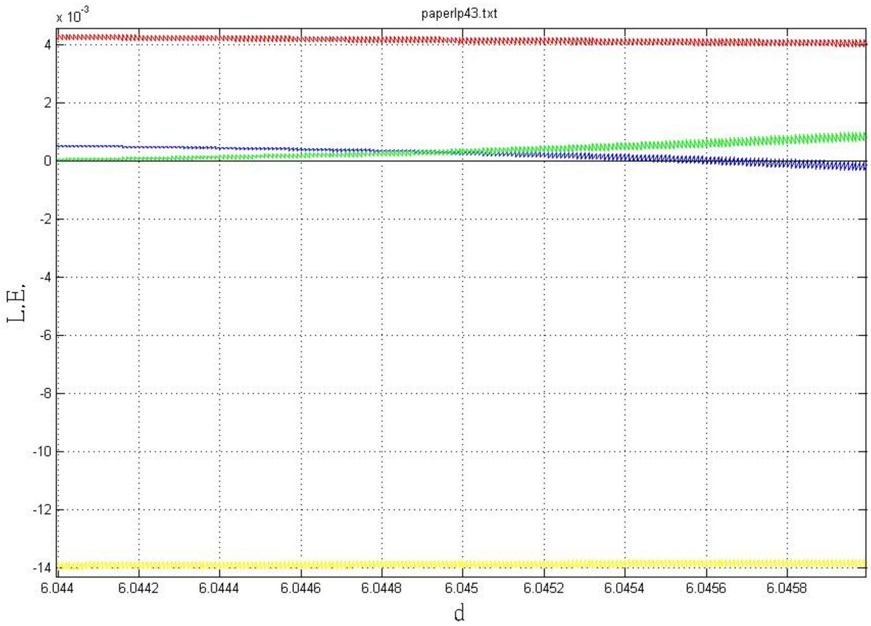


Fig.2- 6 Lyapunov exponents of Mathieu-van der Pol system with *a*=96.326680, *b*=5.023, *c*=-0.001, *e*=87.001, *f*=0.018and *g*=9.5072.

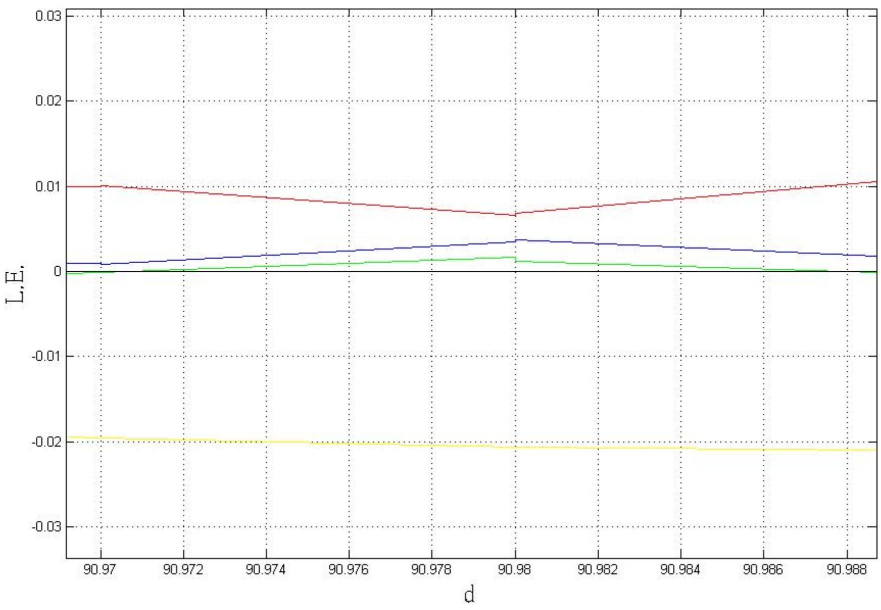


Fig.2- 7 Lyapunov exponents of Mathieu-van der Pol system with *a*=96.326680, *b*=5.023, *c*=-0.001, *e*=87.001, *f*=0.018and *g*=9.5072.

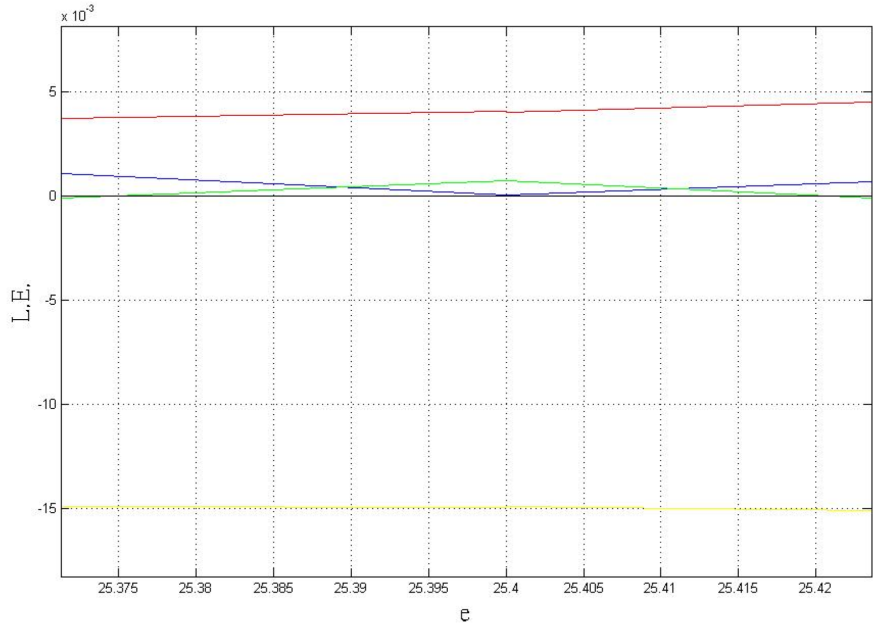


Fig.2- 8 Lyapunov exponents of Mathieu-van der Pol system with *a*=96.326680, *b*=5.023, *c*=-0.001, *d*=25, *f*=0.018and *g*=9.5072.

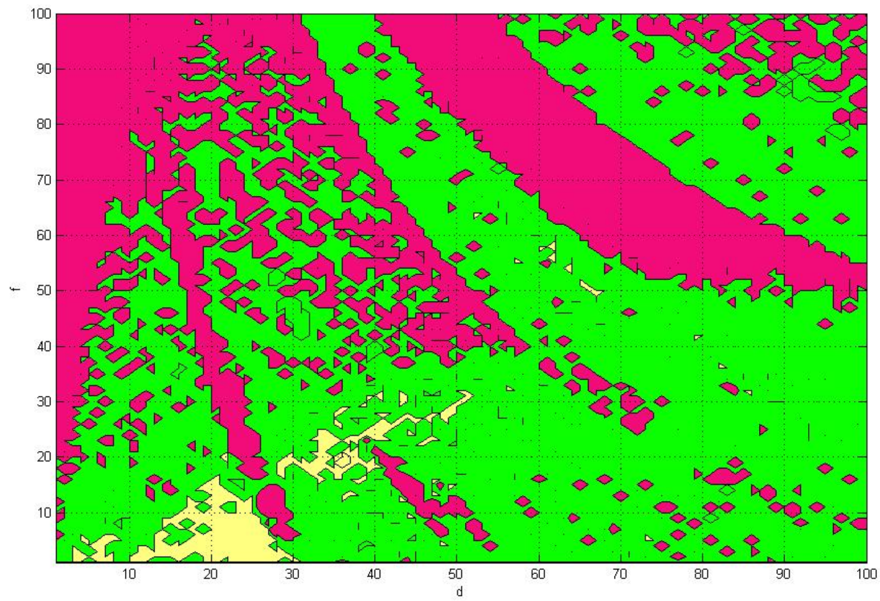
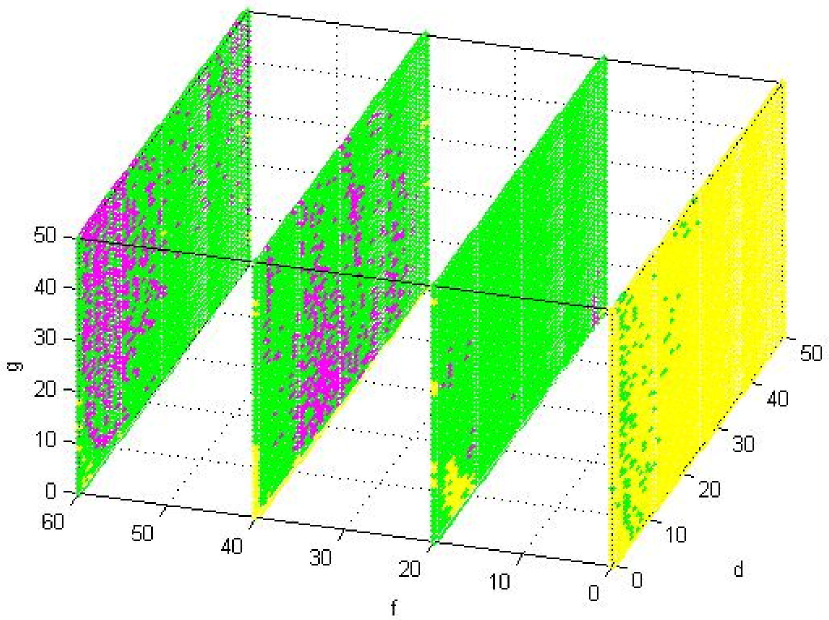


Fig.2- 9 Parameter diagrams of Mathieu-van der Pol system with *a*=96.326680, *b*=5.023, *c*=-0.001, *e*=87.001and *f*=0.018.



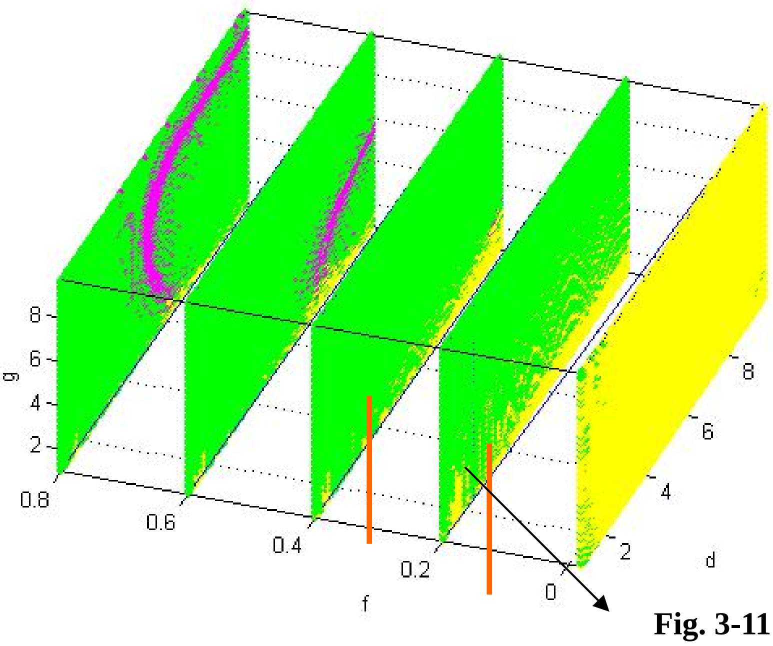


Fig.2- 10 2D Parameter diagrams varied with f. *a*=96.326680, *b*=5.023, *c*=-0.001and *e*=87.001. *Part A* and *B* are shown in Fig.7.

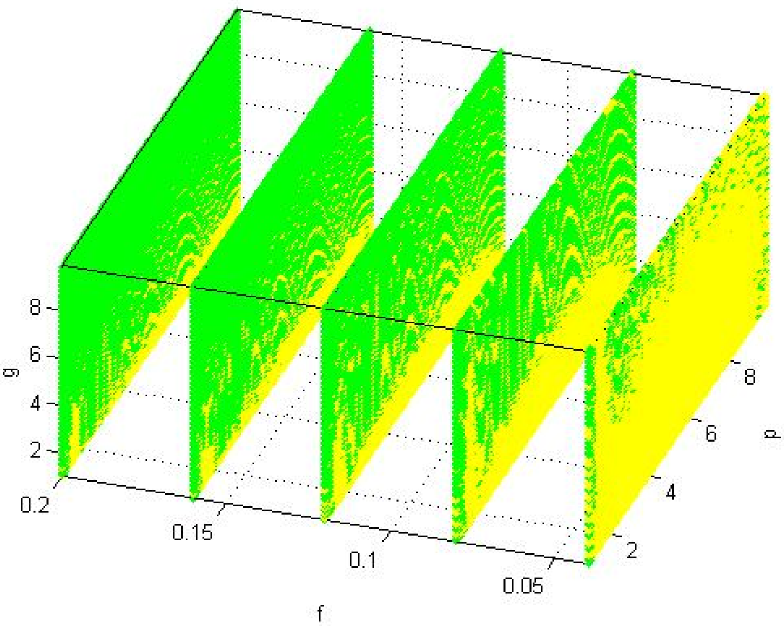


Fig.2- 11 2D Parameter diagrams varied with f. *a*=96.326680, *b*=5.023, *c*=-0.001and *e*=87.001. *Part C* are shown in Fig. 8.

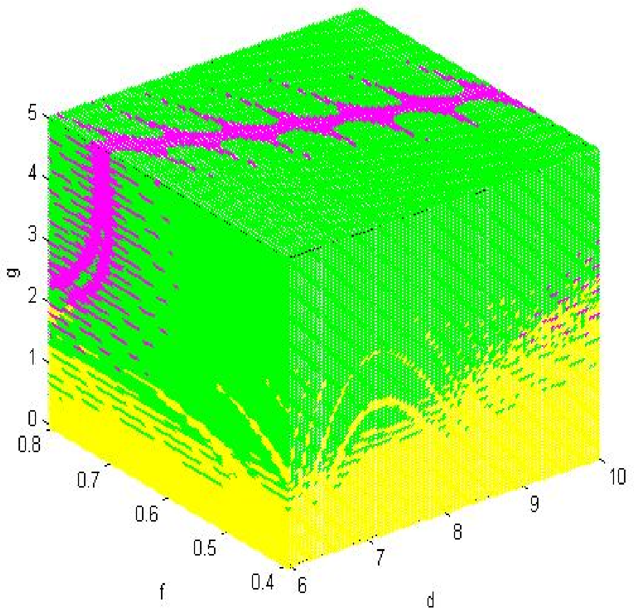


Fig.2- 12 3D Parameter diagrams of Mathieu-van der Pol system with *a*=96.326680, *b*=5.023, *c*=-0.001 and *e*=87.001.

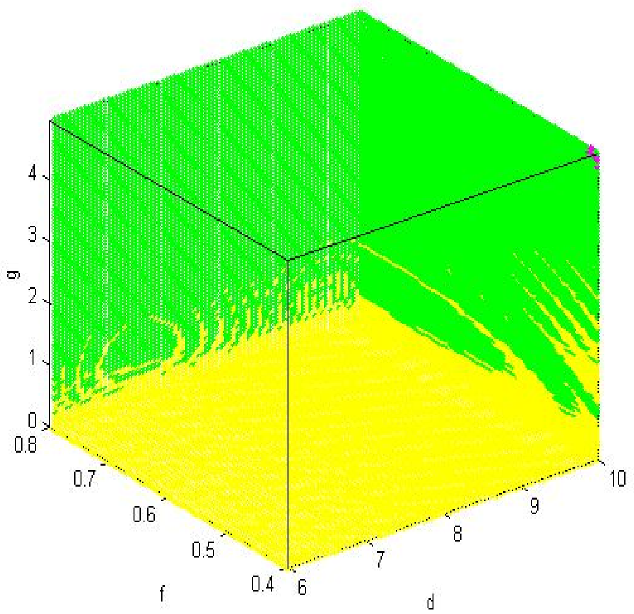


Fig.2- 13 3D Parameter diagrams of Mathieu-van der Pol system with *a*=96.326680, *b*=5.023, *c*=-0.001 and *e*=87.001.

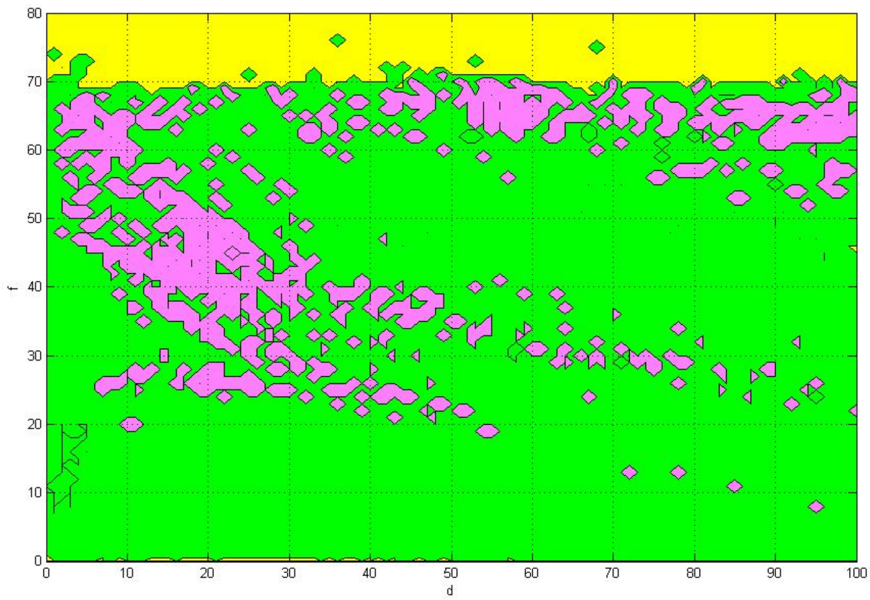


Fig.2- 14 Parameter diagrams of Mathieu-van der Pol system with *a*=96.326680, *b*=5.023, *c*=-0.001, *e*=87.001and *g*=9.5072.

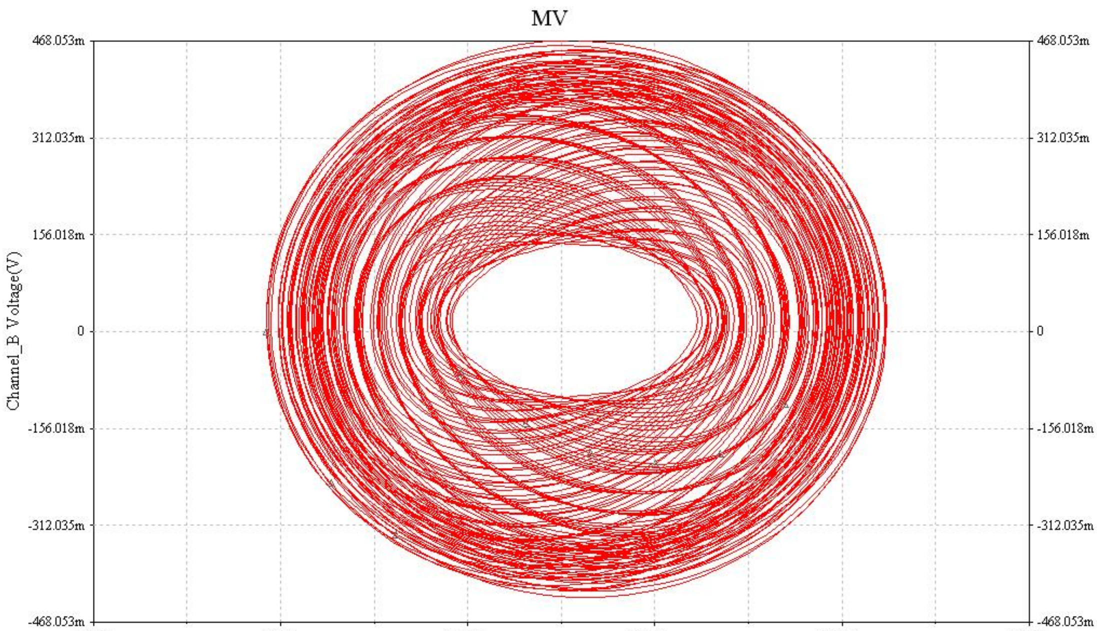


Fig.2- 15 Projection of phase portraits outputs in electronic circuit for Mathieu-van der Pol system.

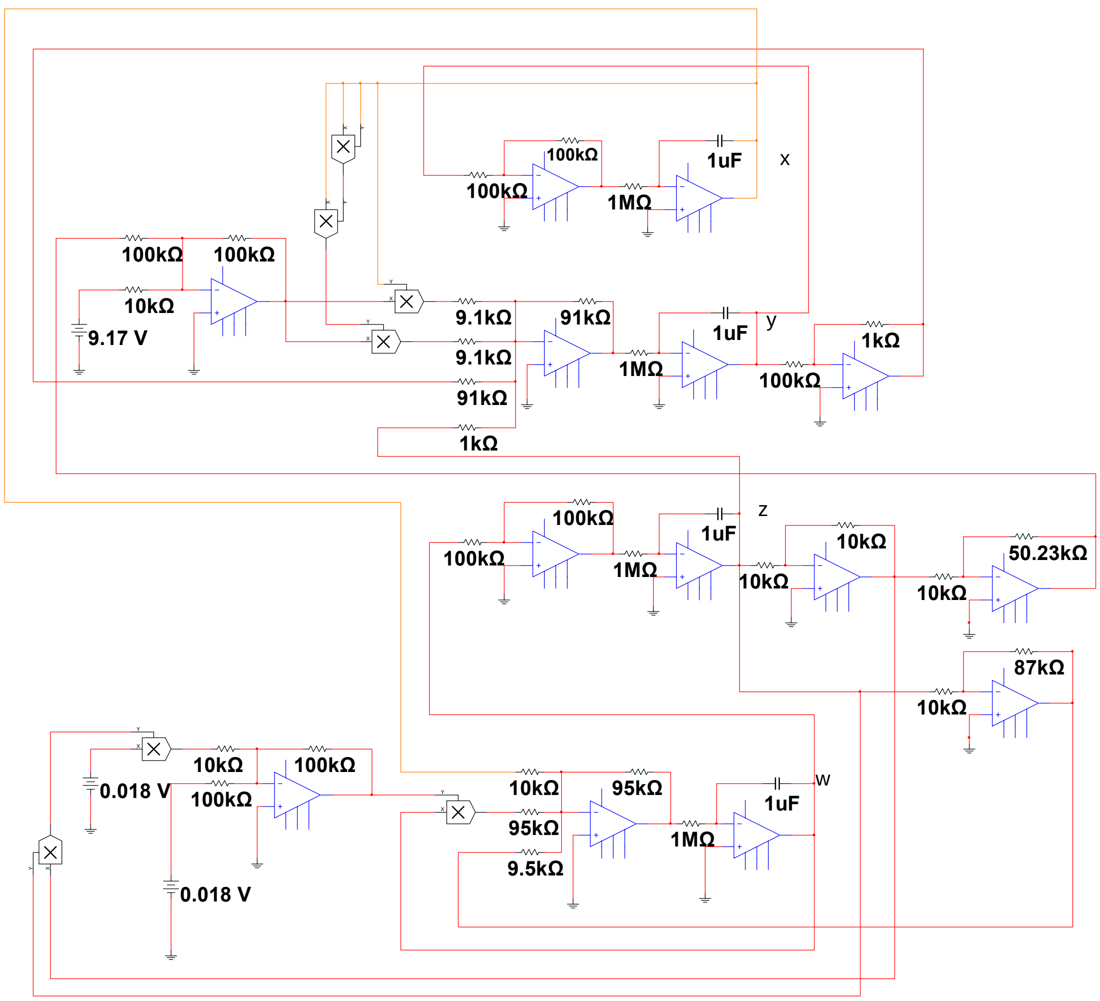


Fig.3- 16 The configuration of electronic circuit for chaotic Mathieu-van der Pol system.

Chapter 3

Chaos Control of New Mathieu-van der Pol Systems with New Mathieu -Duffing Systems as Goal System by GYC Partial Region Stability Theory

3.1 Preliminaries

In this Chapter, a new strategy by using GYC partial region stability theory is proposed to achieve chaos control. Via using the GYC partial region stability theory, the new Lyapunov function used is a simple linear homogeneous function of error states and the lower order controllers are much simpler and introduce less simulation error. Numerical simulations are given for new Mathieu-van der Pol system and new Mathieu-Duffing system to show the effectiveness of this strategy.

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3.2 Chaos Control Scheme

Consider the following chaotic system

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}) \tag{3-2-1}$$

where $\mathbf{x} = [x_1, x_2, \cdots, x_n]^T \in R^n$ is a the state vector, $\mathbf{f} : R_+ \times R^n \to R^n$ is a vector function.

The goal system which can be either chaotic or regular, is

$$\dot{\mathbf{y}} = \mathbf{g}(t, \mathbf{y}) \tag{3-2-2}$$

where $\mathbf{y} = [y_1, y_2, \cdots, y_n]^T \in R^n$ is a state vector, $\mathbf{g} : R_+ \times R^n \to R^n$ is a vector function.

In order to make the chaos state $\mathbf{x}$ approaching the goal state $\mathbf{y}$, define $\mathbf{e} = \mathbf{x} - \mathbf{y}$ as the state error. The chaos control is accomplished in the sense that [13-22]:

$$\lim_{t\to\infty} \mathbf{e} = \lim_{t\to\infty} (\mathbf{x} - \mathbf{y}) = 0 \tag{3-2-3}$$

In this Chapter, we will use examples in which the error dynamics always happens in the first quadrant of coordinate system and use GYC partial region stability theory [43-44]. The

Lyapunov function is a simple linear homogeneous function of error states and the controllers are simpler because they are in lower order than that of traditional controllers

3.3 New Chaotic Mathieu- Duffing System

Mathieu equation and Duffing equation are two typical nonlinear non-autonomous systems:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -(a_1 + b_1 \sin \omega t) z_1 - (a_1 + b_1 \sin \omega t) z_1^{\ 3} - c_1 z_2 + d_1 \sin \omega t \end{cases} \tag{3-3-4}$$

$$\begin{cases} \dot{z}_3 = z_4 \\ \dot{z}_4 = -z_3 - z_3^3 - e_1 z_4 + f_1 \sin \omega t \end{cases} \tag{3-3-5}$$

Exchanging sinωt in Eq. (3-3-4) by z_3 and sinωt in Eq. (3-3-5) by z_1, we obtain the autonomous master new Mathieu-Duffing system:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -(a_1 + b_1 z_3) z_1 - (a_1 + b_1 z_3) z_1^{\ 3} - c_1 z_2 + d_1 z_3 \\ \dot{z}_3 = z_4 \\ \dot{z}_4 = -z_3 - z_3^{\ 3} - e_1 z_4 + f_1 z_1 \end{cases} \tag{3-3-6}$$

where a_1, b_1, c_1, d_1, e_1 and f_1 are uncertain parameters. This system exhibits chaos when the parameters of system are $a_1 = 20.30$, $b_1 = 0.5970$, $c_1 = 0.005$, $d_1 =$ -24.441, $e_1 = 0.002$, $f_1 = 14.63$ and initial states is (-2, 10, -2, 10). Its phase portraits are shown in Fig. 3-1.

3.4 Numerical Simulations

The following chaotic system

$$\begin{cases} \dot{x}_1 = x_2 - 200 \\ \dot{x}_2 = -(a + b(x_3 - 200))(x_1 - 200) - (a + b(x_3 - 200))(x_1 - 200)^3 \\ \qquad - c(x_2 - 200) + d(x_3 - 200) \\ \dot{x}_3 = (x_4 - 200) \\ \dot{x}_4 = -e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) + g(x_1 - 200) \end{cases} \tag{3-4-7}$$

is the new Mathieu-van der Pol system of which the old origin is translated to $(x_1, x_2, x_3, x_4) = (200,200,200,200)$ in order that the error dynamics happens always in the first quadrant of error state coordinate system. This translated new Mathieu-van der Pol system presents chaotic motion when initial conditions is $(x_{10}, x_{20}, x_{30}, x_{40}) =$ (210.1, 209.5, 210.1, 209.5) and the parameters are $a = 10$, $b = 3$, $c = 0.4$, $d = 70$, $e = 1$, $f = 5$, $g = 0.1$.

In order to lead (x_1, x_2, x_3, x_4) to the goal, we add control terms u_1, u_2, u_3 and u_4 to each equation of Eq. (3-4-7), respectively.

$$\begin{cases} \dot{x}_1 = x_2 - 200 + u_1 \\ \dot{x}_2 = -(a + b(x_3 - 200))(x_1 - 200) - (a + b(x_3 - 200))(x_1 - 200)^3 \\ \quad - c(x_2 - 200) + d(x_3 - 200) + u_2 \\ \dot{x}_3 = (x_4 - 200) + u_3 \\ \dot{x}_4 = -e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) + g(x_1 - 200) + u_4 \end{cases} \tag{3-4-8}$$

CASE I. Control the chaotic motion to zero.

In this case we will control the chaotic motion of the new Mathieu-van der Pol system (3-4-8) to zero. The goal is $y = 0$. The state error is $e_i = x_i - y_i = x_i$, (i=1, 2, 3, 4) and error dynamics becomes

$$\begin{cases} \dot{e}_1 = \dot{x}_1 = x_2 - 200 + u_1 \\ \dot{e}_2 = \dot{x}_2 = -(a + b(x_3 - 200))(x_1 - 200) - (a + b(x_3 - 200))(x_1 - 200)^3 \\ \qquad - c(x_2 - 200) + d(x_3 - 200) + u_2 \\ \dot{e}_3 = \dot{x}_3 = (x_4 - 200) + u_3 \\ \dot{e}_4 = \dot{x}_4 = -e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) + g(x_1 - 200) + u_4 \end{cases} \tag{3-4-9}$$

In Fig. 3-2, we can see that the error dynamics always exists in first quadrant.

By GYC partial region asymptotical stability theorem, one can easily choose a Lyapunov function in the form of a positive definite function in first quadrant as:

$$V = e_1 + e_2 + e_3 + e_4 \tag{3-4-10}$$

Its time derivative through error dynamics (3-4-3) is

$$\begin{aligned} \dot{V} &= \dot{e}_1 + \dot{e}_2 + \dot{e}_3 + \dot{e}_4 \\ &= (x_2 - 200 + u_1) + (-(a + b(x_3 - 200))(x_1 - 200) \\ &\quad - (a + b(x_3 - 200))(x_1 - 200)^3 - c(x_2 - 200) + d(x_3 - 200) + u_2) \\ &\quad + (x_4 - 200 + u_3) + (-e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) \\ &\quad + g(x_1 - 200) + u_4) \end{aligned} \tag{3-4-11}$$

Choose

$$
\begin{aligned}
u_1 &= -(x_2 - 200) - e_1 \\
u_2 &= (-(a + b(x_3 - 200))(x_1 - 200) - (a + b(x_3 - 200))(x_1 - 200)^3 \\
&\quad - c(x_2 - 200) + d(x_3 - 200)) - e_2 \\
u_3 &= -(x_4 - 200) - e_3 \\
u_4 &= (-e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) \\
&\quad + g(x_1 - 200)) - e_4
\end{aligned}
\qquad (3\text{-}4\text{-}12)
$$

We obtain

$$\dot{V} = -e_1 - e_2 - e_3 - e_4 < 0$$

which is negative definite function in first quadrant. The numerical results are shown in Fig.3-3. After 10 sec, the error trajectories approach the origin.

CASE II. Control the chaotic motion to a regular function.

In this case we will control the chaotic motion of the new Mathieu-van der Pol system (3-4-8) to regular function of time. The goal is $y_i = F_i e^{\sin \omega t}$, ($i$=1, 2, 3, 4). The error equation

$$e_i = x_i - y_i = x_i - F_i e^{\sin \omega t} \quad , (i=1, 2, 3, 4) \qquad (3\text{-}4\text{-}13)$$

$$\lim_{t \to \infty} e_i = \lim_{t \to \infty} (x_i - F_i e^{\sin \omega t}) = 0 \quad , (i=1, 2, 3, 4)$$

where $F_1 = F_2 = F_3 = F_4 = F = 10$ and $\omega = 0.5$

The error dynamics is

$$
\begin{cases}
\dot{e}_1 = x_2 - 200 + u_1 - F_1 \omega e^{\sin \omega t} (\cos \omega t) \\
\dot{e}_2 = -(a + b(x_3 - 200))(x_1 - 200) - (a + b(x_3 - 200))(x_1 - 200)^3 \\
\qquad - c(x_2 - 200) + d(x_3 - 200) + u_2 - F_2 \omega e^{\sin \omega t} (\cos \omega t) \\
\dot{e}_3 = (x_4 - 200) + u_3 - F_3 \omega e^{\sin \omega t} (\cos \omega t) \\
\dot{e}_4 = -e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) + g(x_1 - 200) \\
\qquad + u_4 - F_4 \omega e^{\sin \omega t} (\cos \omega t)
\end{cases}
\qquad (3\text{-}4\text{-}14)
$$

In Fig. 4-4, the error dynamics always exists in first quadrant.

By GYC partial region asymptotical stability theorem, one can easily choose a Lyapunov function in the form of a positive definite function in first quadrant as:

$$V = e_1 + e_2 + e_3 + e_4$$

Its time derivative is

$$\begin{aligned} V &= \dot{e}_1 + \dot{e}_2 + \dot{e}_3 + \dot{e}_4 = (x_2 - 200 + u_1 - F_1 \omega e^{\sin \omega t}(\cos \omega t)) \\ &+ (-(a + b(x_3 - 200))(x_1 - 200) - (a + b(x_3 - 200))(x_1 - 200)^3 \\ &- c(x_2 - 200) + d(x_3 - 200) + u_2 - F_2 \omega e^{\sin \omega t}(\cos \omega t)) \\ &+ ((x_4 - 200) + u_3 - F_3 \omega e^{\sin \omega t}(\cos \omega t)) \\ &+ (-e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) + g(x_1 - 200) \\ &+ u_4 - F_4 \omega e^{\sin \omega t}(\cos \omega t)) \end{aligned} \tag{3-4-15}$$

Choose

$$\begin{aligned} u_1 &= -(x_2 - 200 - F_1 \omega e^{\sin \omega t}(\cos \omega t)) - e_1 \\ u_2 &= -(-(a + b(x_3 - 200))(x_1 - 200) - (a + b(x_3 - 200))(x_1 - 200)^3 \\ &\quad - c(x_2 - 200) + d(x_3 - 200) - F_2 \omega e^{\sin \omega t}(\cos \omega t)) - e_2 \\ u_3 &= -((x_4 - 200) - F_3 \omega e^{\sin \omega t}(\cos \omega t)) - e_3 \\ u_4 &= -(-e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) + g(x_1 - 200) \\ &\quad - F_4 \omega e^{\sin \omega t}(\cos \omega t)) - e_4 \end{aligned} \tag{3-4-16}$$

We obtain

$$\dot{V} = -e_1 - e_2 - e_3 - e_4 < 0$$

which is a negative definite function in first quadrant. The numerical results are shown in Fig.4-5 and Fig. 4-6. After 10 sec., the errors approach zero and the chaotic trajectories approach to regular motion.

CASE III. Control the chaotic motion of the new Mathieu-van der Pol system to chaotic motion of the new Mathieu-Duffing system.

In this case we will control chaotic motion of the new Mathieu-van der Pol system (3-4-1) to that of following goal system, i.e. the new chaotic Mathieu-Duffing system with initial states (-2, 10, -2, 10), system parameters $a_1 = 20.30$, $b_1 = 0.5970$, $c_1 = 0.005$, $d_1 = \text{-}24.441$, $e_1 = 0.002$ and $f_1 = 14.63$.

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -(a_1 + b_1 z_3) z_1 - (a_1 + b_1 z_3) z_1^{\ 3} - c_1 z_2 + d_1 z_3 \\ \dot{z}_3 = z_4 \\ \dot{z}_4 = -z_3 - z_3^{\ 3} - e_1 z_4 + f_1 z_1 \end{cases} \tag{3-4-17}$$

The error equation is $e_i = x_i - z_i$, (i=1, 2, 3, 4). Our aim is $\lim_{t\to\infty} e_i = 0$ (i=1, 2, 3, 4).

The error dynamics becomes

$$\begin{cases} \dot{e}_1 = \dot{x}_1 - \dot{z}_1 = (x_2 - 200 - z_2) + u_1 \\ \dot{e}_2 = \dot{x}_2 - \dot{z}_2 = (-(a + b(x_3 - 200))(x_1 - 200) - (a + b(x_3 - 200))(x_1 - 200)^3 \\ \qquad - c(x_2 - 200) + d(x_3 - 200) - (-(a_1 + b_1 z_3) z_1 - (a_1 + b_1 z_3) z_1^{\ 3} \\ \qquad - c_1 z_2 + d_1 z_3)) + u_2 \\ \dot{e}_3 = \dot{x}_3 - \dot{z}_3 = (x_4 - 200 - z_4) + u_3 \\ \dot{e}_4 = \dot{x}_4 - \dot{z}_4 = (-e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) + g(x_1 - 200) \\ \qquad - (-z_3 - z_3^{\ 3} - e_1 z_4 + f_1 z_1)) + u_4 \end{cases} \tag{3-4-18}$$

In Fig. 4-7, the error dynamics always exists in first quadrant.

By GYC partial region asymptotical stability theorem, one can easily choose a Lyapunov function in the form of a positive definite function in first quadrant as:

$$V = e_1 + e_2 + e_3 + e_4$$

Its time derivative is

$$\begin{aligned} \dot{V} = \dot{e}_1 + \dot{e}_2 + \dot{e}_3 + \dot{e}_4 &= ((x_2 - 200 - z_2) + u_1) \\ &+ ((-(a + b(x_3 - 200))(x_1 - 200) - (a + b(x_3 - 200))(x_1 - 200)^3 \\ &- c(x_2 - 200) + d(x_3 - 200) - (-(a_1 + b_1 z_3) z_1 - (a_1 + b_1 z_3) z_1^{\ 3} \\ &- c_1 z_2 + d_1 z_3)) + u_2) + ((x_4 - 200 - z_4) + u_3) \\ &+ ((-e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) + g(x_1 - 200) \\ &- (-z_3 - z_3^{\ 3} - e_1 z_4 + f_1 z_1)) + u_4) \end{aligned} \tag{3-4-19}$$

Choose

$$
\begin{aligned}
u_1 &= -(x_2 - 200 - z_2) - e_1 \\
u_2 &= -(-(a + b(x_3 - 200))(x_1 - 200) - (a + b(x_3 - 200))(x_1 - 200)^3 \\
&\quad - c(x_2 - 200) + d(x_3 - 200) - (-(a_1 + b_1 z_3) z_1 - (a_1 + b_1 z_3) z_1^{\ 3} \\
&\quad - c_1 z_2 + d_1 z_3)) - e_2 \\
u_3 &= -(x_4 - 200 - z_4) - e_3 \\
u_3 &= -(-e(x_3 - 200) + f(1 - (x_3 - 200)^2)(x_4 - 200) + g(x_1 - 200) \\
&\quad - (-z_3 - z_3^{\ 3} - e_1 z_4 + f_1 z_1)) - e_4
\end{aligned} \tag{3-4-20}
$$

We obtain

$$\dot{V} = -e_1 - e_2 - e_3 - e_4 < 0$$

which is negative definite function in first quadrant. The numerical results are shown in Fig.4-8 and Fig. 4-9. After 10 sec., the errors approach zero and the chaotic trajectories of the new Mathieu-van der Pol system approach to that of the new Mathieu-Duffing system.

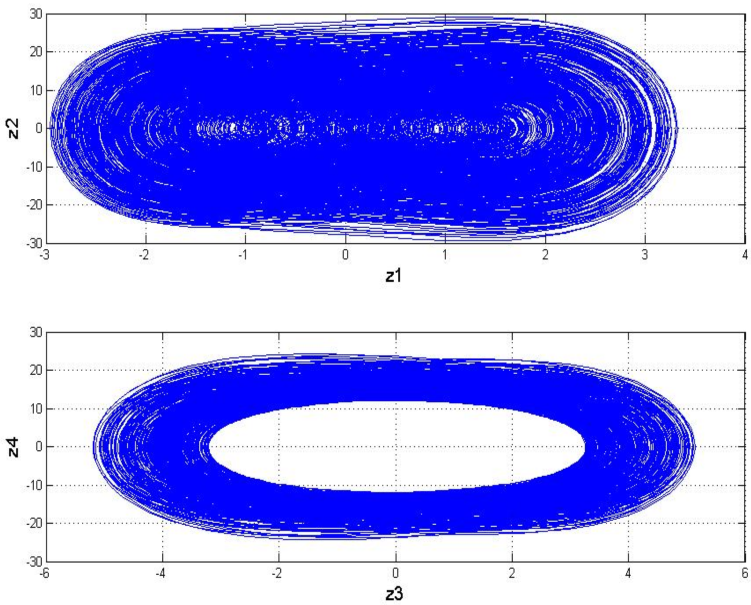


Fig. 3-1 Chaotic phase portrait projections for new Mathieu-Duffing system.

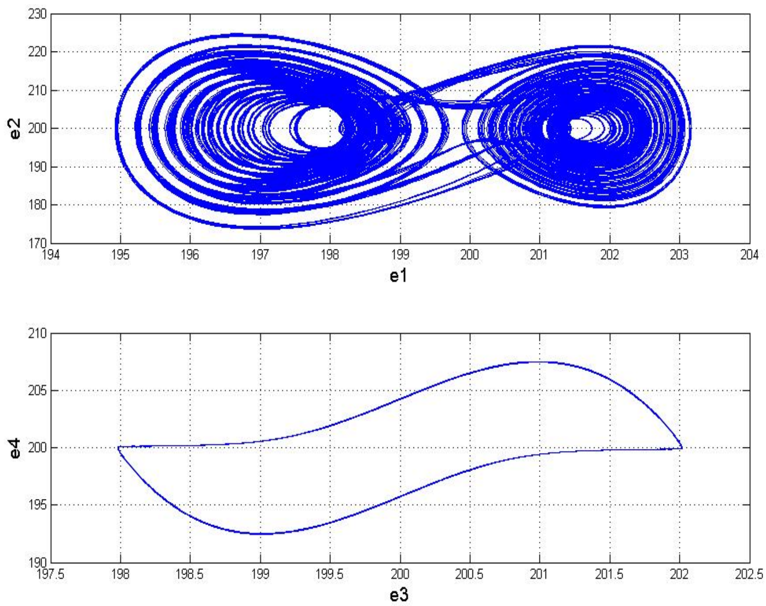


Fig. 3-2 Phase portrait projections of error dynamics for Case I.

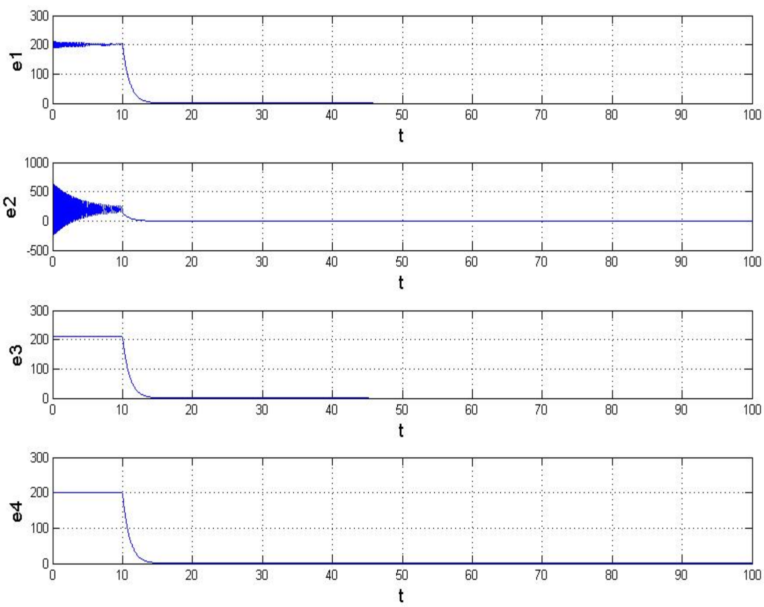


Fig. 3-3 Time histories of errors for Case I.

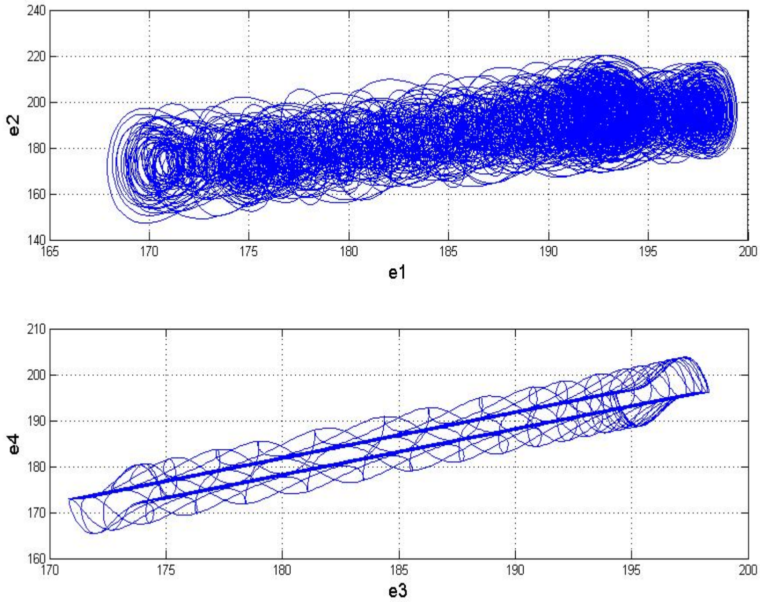


Fig. 3-4 Phase portrait projections of error dynamics for Case II.

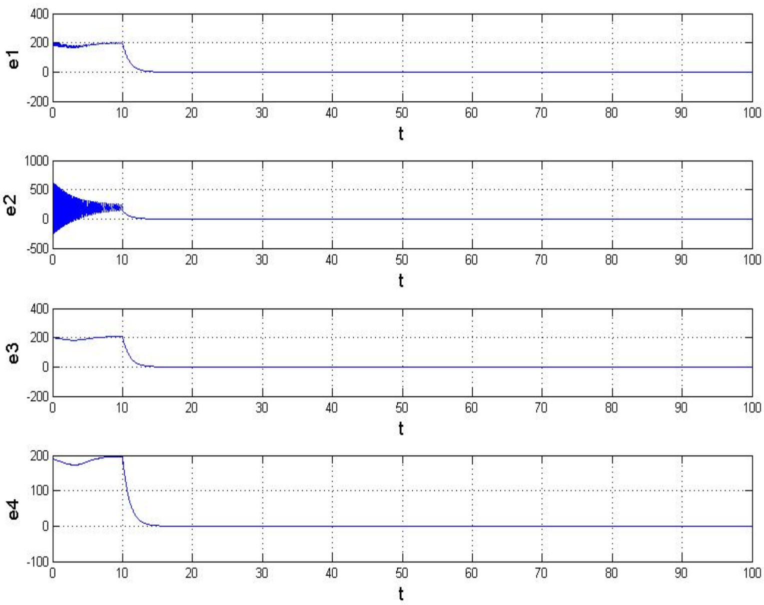


Fig. 3-5 Time histories of errors for Case II.

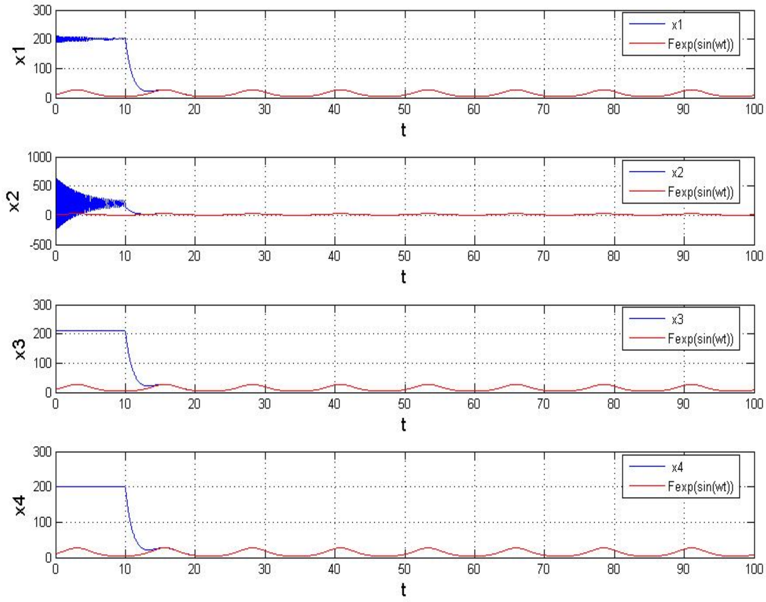


Fig. 3-6 Time histories of x_1, x_2, x_3, x_4 for Case II.

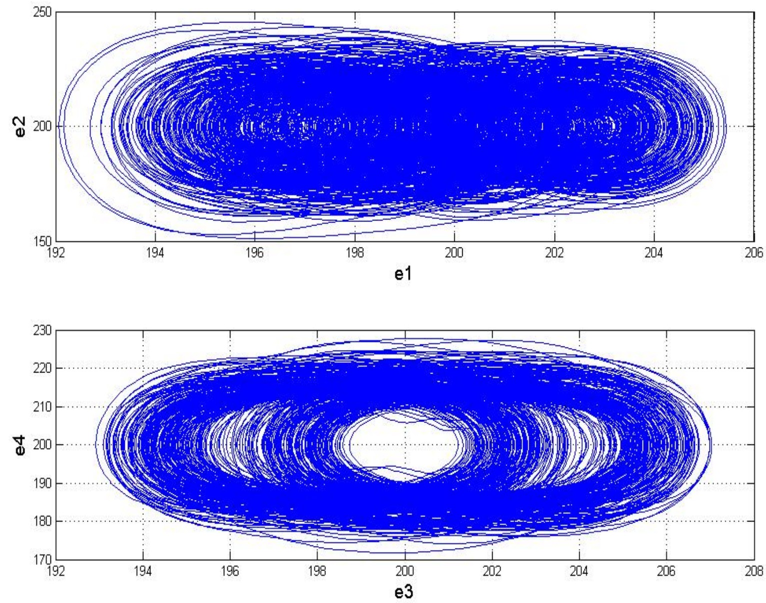


Fig. 3-7 Phase portrait projections of error dynamics for Case III.

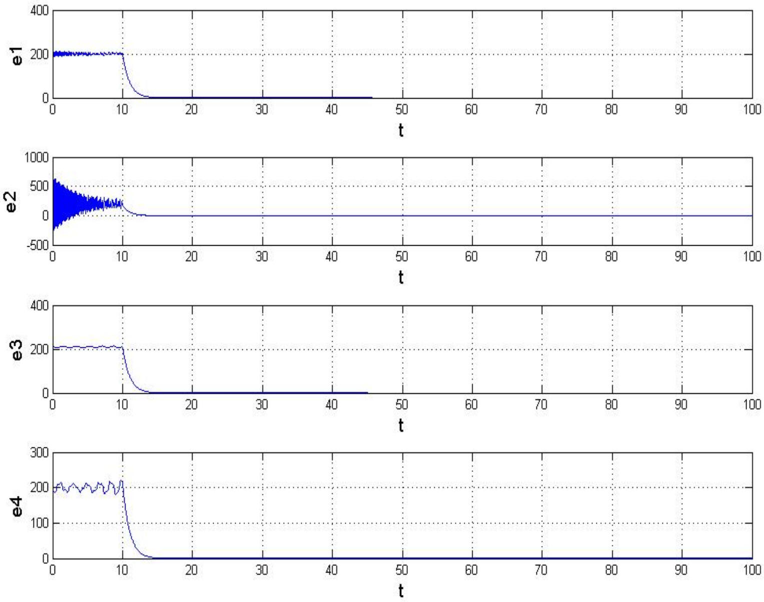


Fig. 3-8 Time histories of errors for Case III.

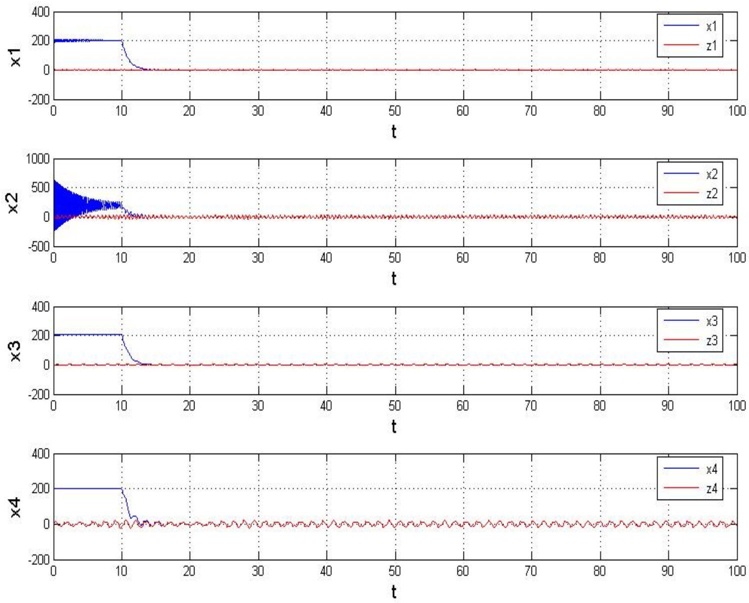


Fig. 3-9 Time histories of x_1, x_2, x_3, x_4 and z_1, z_2, z_3, $z_4$for Case III.

Chapter 4

Generalized Chaos Synchronization of New Mathieu-van der Pol Systems with New Duffing-van der Pol systems as Functional system by GYC Partial Region Stability Theory

4.1 Preliminaries

In this Chapter, a new strategy by using GYC partial region stability theory is proposed to achieve generalized chaos synchronization. Via using the GYC partial region stability theory, the new Lyapunov function used is a simple linear homogeneous function of states and the lower order controllers are much more simple and introduce less simulation error. Numerical simulations are given for new Mathieu-van der Pol system and new Duffing-van der Pol system to show the effectiveness of this strategy.

4.2 Generalized Chaos Synchronization Strategy

Consider the following unidirectional coupled chaotic systems

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(t, \mathbf{x}) \\ \dot{\mathbf{y}} &= \mathbf{h}(t, \mathbf{y}) + \mathbf{u} \end{aligned} \tag{4-2-1}$$

where $\mathbf{x} = [x_1, x_2, \cdots, x_n]^T \in R^n$, $\mathbf{y} = [y_1, y_2, \cdots, y_n]^T \in R^n$ denote the master state vector and slave state vector respectively, $\mathbf{f}$ and $\mathbf{h}$ are nonlinear vector functions, and $\mathbf{u} = [u_1, u_2, \cdots, u_n]^T \in R^n$ is a control input vector.

The generalized synchronization can be accomplished when $t \to \infty$, the limit of the error vector $\mathbf{e} = [e_1, e_2, \cdots, e_n]^T$ approaches zero:

$$\lim_{t \to \infty} \mathbf{e} = 0 \tag{4-2-2}$$

where

$$\mathbf{e} = \mathbf{G}(\mathbf{x}) - \mathbf{y} \tag{4-2-3}$$

$\boldsymbol{G(x)}$ is a given function of $\boldsymbol{x}$.

By using the partial region stability theory [50-51], the Lyapunov function is easier to find, since the linear homogenous terms of the entries of $\mathbf{e}$ can be used to construct the definite Lyapunov function and the controllers can be designed in lower order.

4.3 New Chaotic Duffing-van der Pol System

Duffing equation and van der Pol equation are two typical nonlinear non-autonomous systems:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -z_1 - z_1^3 - hz_2 + i\sin\omega t \end{cases} \tag{4-3-1}$$

$$\begin{cases} \dot{z}_3 = z_4 \\ \dot{z}_4 = -jz_3 - k(1-z_3^2)z_4 + l\sin\omega t \end{cases} \tag{4-3-2}$$

Exchanging sinωt in Eq. (4-3-1) by z_3 and sinωt in Eq. (4-3-2) by z_1, we obtain the autonomous master new Duffing-van der Pol system:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -z_1 - z_1^{\;3} - hz_2 + iz_3 \\ \dot{z}_3 = z_4 \\ \dot{z}_4 = -jz_3 + k(1-z_3^{\;2})z_4 + lz_1 \end{cases} \tag{4-3-3}$$

where *h, i, j, k, l* are uncertain parameter. This system exhibits chaos when the parameters of system are $h = 0.0006$, $j = 1$, $k = 5$, $i = 0.67$ and $l = 0.05$ and initial states is (2, 2.4, 5, 6), its phase portraits projections and Lyapunov exponents as shown in Fig. 4-1 and 4-2.

4.4 Numerical Simulations

The two unidirectional coupled new chaotic Mathieu-van der pol systems are shown as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -(a+bx_3)x_1 - (a+bx_3)x_1^{\;3} - cx_2 + dx_3 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= -ex_3 + f(1-x_3^{\;2})x_4 + gx_1 \end{aligned} \tag{4-4-1}$$

$$
\begin{aligned}
\dot{y}_1 &= y_2 + u_1 \\
\dot{y}_2 &= -(a+by_3)y_1 - (a+by_3)y_1^{\,3} - cy_2 + dy_3 + u_2 \\
\dot{y}_3 &= y_4 + u_3 \\
\dot{y}_4 &= -ey_3 + f(1-y_3^{\,2})y_4 + gy_1 + u_4
\end{aligned}
$$

CASE I. The generalized synchronization error function is $e_i = (x_i - y_i + 100)$, (i=1, 2, 3, 4.).

The addition of 100 makes the error dynamics always happens in first quadrant. Our goal is $y_i = x_i + 100$, i.e.

$$
\lim_{t\to\infty} e_i = \lim_{t\to\infty}(x_i - y_i + 100) = 0 \quad (i\text{=1, 2, 3, 4}) \tag{4-4-2}
$$

The error dynamics becomes:

$$
\begin{aligned}
\dot{e}_1 &= \dot{x}_1 - \dot{y}_1 = x_2 - y_2 - u_1 \\
\dot{e}_2 &= \dot{x}_2 - \dot{y}_2 = -((a+bx_3)x_1 - (a+by_3)y_1) - ((a+bx_3)x_1^{\,3} - (a+by_3)y_1^{\,3}) \\
&\qquad\qquad - c(x_2 - y_2) + d(x_3 - y_3) - u_2 \\
\dot{e}_3 &= \dot{x}_3 - \dot{y}_3 = x_4 - y_4 - u_3 \\
\dot{e}_4 &= \dot{x}_4 - \dot{y}_4 = -e(x_3 - y_3) + f((1-x_3^{\,2})x_4 - (1-y_3^{\,2})y_4) + g(x_1 - y_1) - u_4
\end{aligned} \tag{4-4-3}
$$

System parameters are chosen as $a=10$, $b=3$, $c=0.4$, $d=70$, $e=1$, $f=5$, $g=0.1$ and initial states are $(x_{10}, x_{20}, x_{30}, x_{40})$=(0.1, -0.5, 0.1, -0.5), $(y_{10}, y_{20}, y_{30}, y_{40})$=(0.3, -0.1, 0.3, -0.1). Before control action, the error dynamics always happens in first quadrant as shown in Fig. 4-3. By GYC partial region stability, one can choose a Lyapunov function in the form of a positive definite function in first quadrant:

$$
V = e_1 + e_2 + e_3 + e_4 \tag{4-4-4}
$$

Its time derivative through Eq. (4-4-2) is

$$
\begin{aligned}
\dot{V} &= \dot{e}_1 + \dot{e}_2 + \dot{e}_3 + \dot{e}_4 \\
&= (x_2 - y_2 - u_1) + (-((a+bx_3)x_1 - (a+by_3)y_1) - ((a+bx_3)x_1^{\,3} \\
&\quad - (a+by_3)y_1^{\,3}) - c(x_2 - y_2) + d(x_3 - y_3) - u_2) + (x_4 - y_4 - u_3) \\
&\quad + (-e(x_3 - y_3) + f((1-x_3^{\,2})x_4 - (1-y_3^{\,2})y_4) + g(x_1 - y_1) - u_4)
\end{aligned} \tag{4-4-5}
$$

Choose

$$
\begin{aligned}
u_1 &= (x_2 - y_2) + e_1 \\
u_2 &= (-((a+bx_3)x_1 - (a+by_3)y_1) - ((a+bx_3)x_1^3 - (a+by_3)y_1^3) \\
&\quad - c(x_2 - y_2) + d(x_3 - y_3)) + e_2 \\
u_3 &= (x_4 - y_4) + e_3 \\
u_4 &= (-e(x_3 - y_3) + f((1-x_3^2)x_4 - (1-y_3^2)y_4) + g(x_1 - y_1)) + e_4
\end{aligned}
\tag{4-4-6}
$$

We obtain

$$
\dot{V} = -e_1 - e_2 - e_3 - e_4 < 0 \tag{4-4-7}
$$

which is negative definite function in the first quadrant. Four state errors versus time and time histories of states are shown in Fig. 4-4 and Fig. 4-5.

CASE II. The generalized synchronization error function is $e_i = (x_i - y_i + F_i \sin \omega t + 100)$, ($i$=1, 2, 3, 4).

The addition of 100 makes the error dynamics always happens in first quadrant.

Our goal is $y_i = x_i + F_i \sin \omega t + 100$, i.e.

$$
\lim_{t\to\infty} e_i = \lim_{t\to\infty} (x_i - y_i + F_i \sin \omega t + 100) = 0 \quad \text{(i=1, 2, 3, 4).} \tag{4-4-8}
$$

where $F_1 = F_2 = F_3 = F_4 = F = 10, \ \omega = 0.5$.

The error dynamics becomes

$$
\begin{aligned}
\dot{e}_1 &= x_2 - y_2 - u_1 + F \sin \omega t \\
\dot{e}_2 &= -((a+bx_3)x_1 - (a+by_3)y_1) - ((a+bx_3)x_1^3 - (a+by_3)y_1^3) \\
&\quad - c(x_2 - y_2) + d(x_3 - y_3) - u_2 + F \sin \omega t \\
\dot{e}_3 &= x_4 - y_4 - u_3 + F \sin \omega t \\
\dot{e}_4 &= -e(x_3 - y_3) + f((1-x_3^2)x_4 - (1-y_3^2)y_4) + g(x_1 - y_1) - u_4 + F \sin \omega t
\end{aligned}
\tag{4-4-9}
$$

System parameters are chosen as $a = 10$, $b = 3$, $c = 0.4$, $d = 70$, $e = 1$, $f = 5$, $g = 0.1$ and initial states are $(x_{10}, x_{20}, x_{30}, x_{40})$=(0.1, -0.5, 0.1, -0.5), $(y_{10}, y_{20}, y_{30}, y_{40})$=(0.3, -0.1, 0.3, -0.1). Before control action, the error dynamics always happens in first quadrant as shown in Fig. 5-6. By GYC partial region stability, one can choose a Lyapunov function in the form of a positive definite function in first quadrant:

$$V = e_1 + e_2 + e_3 + e_4 \qquad (4\text{-}4\text{-}10)$$

Its time derivative through Eq. (5-4-8) is

$$\begin{aligned}\dot{V} &= \dot{e}_1 + \dot{e}_2 + \dot{e}_3 + \dot{e}_4 \\ &= (x_2 - y_2 - u_1 + F\omega\cos\omega t) + (-((a+bx_3)x_1 - (a+by_3)y_1) - ((a+bx_3)x_1^{\ 3} \\ &\quad - (a+by_3)y_1^{\ 3}) - c(x_2 - y_2) + d(x_3 - y_3) - u_2 + F\omega\cos\omega t) + (x_4 - y_4 - u_3 \\ &\quad + F\omega\cos\omega t) + (-e(x_3 - y_3) + f((1 - x_3^{\ 2})x_4 - (1 - y_3^{\ 2})y_4) + g(x_1 - y_1) \\ &\quad - u_4 + F\omega\cos\omega t)\end{aligned} \qquad (4\text{-}4\text{-}11)$$

Choose

$$\begin{aligned}u_1 &= (x_2 - y_2) + F\omega\cos\omega t + e_1 \\ u_2 &= (-((a+bx_3)x_1 - (a+by_3)y_1) - ((a+bx_3)x_1^{\ 3} - (a+by_3)y_1^{\ 3}) \\ &\quad - c(x_2 - y_2) + d(x_3 - y_3)) + F\omega\cos\omega t + e_2 \\ u_3 &= (x_4 - y_4) + F\omega\cos\omega t + e_3 \\ u_4 &= (-e(x_3 - y_3) + f((1 - x_3^{\ 2})x_4 - (1 - y_3^{\ 2})y_4) + g(x_1 - y_1)) + F\omega\cos\omega t + e_4\end{aligned} \qquad (4\text{-}4\text{-}12)$$

We obtain

$$\dot{V} = -e_1 - e_2 - e_3 - e_4 < 0 \qquad (4\text{-}4\text{-}13)$$

which is a negative definite function in the first quadrant. Four state errors versus time and time histories of $x_i - y_i + 100$ and $-F_i \sin wt$ are shown in Fig. 4-7 and Fig. 4-8.

CASE III. The generalized synchronization error function is $e_i = x_i - y_i + F_i e^{\sin \omega t} + 100$, (*i*=1, 2, 3, 4).

The addition of 100 makes the error dynamics always happens in first quadrant.

Our goal is $y_i = x_i + F_i e^{\sin \omega t} + 100$, i.e.

$$\lim_{t\to\infty} e_i = \lim_{t\to\infty}(x_i - y_i + F_i e^{\sin \omega t} + 100) = 0 \quad \text{(i=1, 2, 3, 4).} \qquad (4\text{-}4\text{-}14)$$

The error dynamics becomes

$$\begin{aligned}\dot{e}_1 &= x_2 - y_2 - u_1 + Fe^{\sin\omega t} \\ \dot{e}_2 &= -((a+bx_3)x_1 - (a+by_3)y_1) - ((a+bx_3)x_1^{\ 3} - (a+by_3)y_1^{\ 3}) \\ &\quad - c(x_2 - y_2) + d(x_3 - y_3) - u_2 + Fe^{\sin\omega t} \\ \dot{e}_3 &= x_4 - y_4 - u_3 + Fe^{\sin\omega t} \\ \dot{e}_4 &= -e(x_3 - y_3) + f((1 - x_3^{\ 2})x_4 - (1 - y_3^{\ 2})y_4) + g(x_1 - y_1) - u_4 + Fe^{\sin\omega t}\end{aligned} \qquad (4\text{-}4\text{-}15)$$

System parameters are chosen as $a = 10$, $b = 3$, $c = 0.4$, $d = 70$, $e = 1$, $f = 5$, $g = 0.1$, $F_1 = F_2 = F_3 = F_4 = F = 10$, $\omega = 0.5$ and initial states are $(x_{10}, x_{20}, x_{30}, x_{40})$ =(0.1, -0.5, 0.1, -0.5), $(y_{10}, y_{20}, y_{30}, y_{40})$ =(0.3, -0.1, 0.3, -0.1). Before control action, the error dynamics always happens in first quadrant as shown in Fig. 5-9. By GYC partial region stability, one can choose a Lyapunov function in the form of a positive definite function in first quadrant:

$$V = e_1 + e_2 + e_3 + e_4 \tag{4-4-16}$$

Its time derivative through Eq. (5-4-14) is

$$\begin{aligned}
\dot{V} &= \dot{e}_1 + \dot{e}_2 + \dot{e}_3 + \dot{e}_4 \\
&= (x_2 - y_2 - u_1 + F\omega e^{\sin\omega t}\cos\omega t) + (-((a + bx_3)x_1 - (a + by_3)y_1) - ((a + bx_3)x_1^3 \\
&\quad - (a + by_3)y_1^3) - c(x_2 - y_2) + d(x_3 - y_3) - u_2 + F\omega e^{\sin\omega t}\cos\omega t) + (x_4 - y_4 - u_3 \\
&\quad + F\omega e^{\sin\omega t}\cos\omega t) + (-e(x_3 - y_3) + f((1 - x_3^2)x_4 - (1 - y_3^2)y_4) + g(x_1 - y_1) \\
&\quad - u_4 + F\omega e^{\sin\omega t}\cos\omega t)
\end{aligned} \tag{4-4-17}$$

Choose

$$\begin{aligned}
u_1 &= (x_2 - y_2) + F\omega e^{\sin\omega t}\cos\omega t + e_1 \\
u_2 &= (-((a + bx_3)x_1 - (a + by_3)y_1) - ((a + bx_3)x_1^3 - (a + by_3)y_1^3) \\
&\qquad - c(x_2 - y_2) + d(x_3 - y_3)) + F\omega e^{\sin\omega t}\cos\omega t + e_2 \\
u_3 &= (x_4 - y_4) + F\omega e^{\sin\omega t}\cos\omega t + e_3 \\
u_4 &= (-e(x_3 - y_3) + f((1 - x_3^2)x_4 - (1 - y_3^2)y_4) + g(x_1 - y_1)) + F\omega e^{\sin\omega t}\cos\omega t + e_4
\end{aligned} \tag{4-4-18}$$

We obtain

$$\dot{V} = -e_1 - e_2 - e_3 - e_4 < 0 \tag{4-4-19}$$

which is a negative definite function in the first quadrant. Four state errors versus time and time histories of $x_i - y_i + 100$ and $-F_i e^{\sin wt}$ are shown in Fig. 4-10 and Fig. 4-11.

CASE IV. The generalized synchronization error function is $e_i = \frac{1}{2}x_i^2 - y_i + 100$, ($i$=1, 2, 3, 4).

The addition of 100 makes the error dynamics always happens in first quadrant.

Our goal is $y_i = \frac{1}{2}{x_i}^2 + 100$, i.e.

$$\lim_{t\to\infty} e_i = \lim_{t\to\infty}(\frac{1}{2}{x_i}^2 - y_i + 100) \quad \text{(i=1, 2, 3, 4)} \tag{4-4-20}$$

The error dynamics becomes

$$\begin{aligned}
\dot{e}_1 &= x_1\dot{x}_1 - \dot{y}_1 = x_1x_2 - y_2 - u_1 \\
\dot{e}_2 &= x_2\dot{x}_2 - \dot{y}_2 = -((a+bx_3)x_2x_1 - (a+by_3)y_1) - ((a+bx_3)x_2{x_1}^3 - (a+by_3){y_1}^3) \\
&\quad - c(x_2^2 - y_2) + d(x_2x_3 - y_3) - u_2 \\
\dot{e}_3 &= x_3\dot{x}_3 - \dot{y}_3 = x_3x_4 - y_4 - u_3 \\
\dot{e}_4 &= x_4\dot{x}_4 - \dot{y}_4 = -e(x_4x_3 - y_3) + f((1-{x_3}^2)x_4^2 - (1-{y_3}^2)y_4) + g(x_4x_1 - y_1) - u_4
\end{aligned} \tag{4-4-21}$$

System parameters are chosen as $a = 10$, $b = 3$, $c = 0.4$, $d = 70$, $e = 1$, $f = 5$, $g = 0.1$ and initial states are $(x_{10}, x_{20}, x_{30}, x_{40})$=(0.1, -0.5, 0.1, -0.5), $(y_{10}, y_{20}, y_{30}, y_{40})$=(0.3, -0.1, 0.3, -0.1). Before control action, the error dynamics always happens in first quadrant as shown in Fig. 4-12. By GYC partial region stability, one can choose a Lyapunov function in the form of a positive definite function in first quadrant:

$$V = e_1 + e_2 + e_3 + e_4 \tag{4-4-22}$$

Its time derivative through Eq. (5-4-20) is

$$\begin{aligned}
\dot{V} &= \dot{e}_1 + \dot{e}_2 + \dot{e}_3 + \dot{e}_4 \\
&= (x_1x_2 - y_2 - u_1) + (-((a+bx_3)x_2x_1 - (a+by_3)y_1) - ((a+bx_3)x_2{x_1}^3 \\
&\quad - (a+by_3){y_1}^3) - c(x_2^2 - y_2) + d(x_2x_3 - y_3) - u_2) + (x_3x_4 - y_4 - u_3) \\
&\quad + (-e(x_4x_3 - y_3) + f((1-{x_3}^2)x_4^2 - (1-{y_3}^2)y_4) + g(x_4x_1 - y_1) - u_4)
\end{aligned} \tag{4-4-23}$$

Choose

$$\begin{aligned}
u_1 &= x_1x_2 - y_2 + e_1 \\
u_2 &= -((a+bx_3)x_2x_1 - (a+by_3)y_1) - ((a+bx_3)x_2{x_1}^3 - (a+by_3){y_1}^3) \\
&\quad - c(x_2^2 - y_2) + d(x_2x_3 - y_3) + e_2 \\
u_3 &= x_3x_4 - y_4 + e_3 \\
u_4 &= -e(x_4x_3 - y_3) + f((1-{x_3}^2)x_4^2 - (1-{y_3}^2)y_4) + g(x_4x_1 - y_1) + e_4
\end{aligned} \tag{4-4-24}$$

We obtain

$$\dot{V} = -e_1 - e_2 - e_3 - e_4 < 0 \tag{4-4-25}$$

which is a negative definite function in the first quadrant. Three state errors versus time is shown in Fig. 4-13.

CASE V. The generalized synchronization error function is $e_i = \frac{1}{3}x_i^{\ 3} - y_i + 10000$ (i=1, 2, 3, 4).

The addition of 10000 makes the error dynamics always happens in first quadrant.

Our goal is $y_i = \frac{1}{3}x_i^{\ 3} + 10000$, i.e.

$$\lim_{t\to\infty} e_i = \lim_{t\to\infty}(\frac{1}{3}x_i^{\ 3} - y_i + 10000) \quad (i\text{=1, 2, 3, 4}) \tag{4-4-26}$$

The error dynamics becomes

$$\begin{aligned}
\dot{e}_1 &= x_1^2\dot{x}_1 - \dot{y}_1 = x_1^2 x_2 - y_2 - u_1 \\
\dot{e}_2 &= x_2^2\dot{x}_2 - \dot{y}_2 = -((a+bx_3)x_2^2 x_1 - (a+by_3)y_1) - ((a+bx_3)x_2^2 x_1^{\ 3} - (a+by_3)y_1^{\ 3}) \\
&\qquad - c(x_2^3 - y_2) + d(x_2^2 x_3 - y_3) - u_2 \\
\dot{e}_3 &= x_3^2\dot{x}_3 - \dot{y}_3 = x_3^2 x_4 - y_4 - u_3 \\
\dot{e}_4 &= x_4^2\dot{x}_4 - \dot{y}_4 = -e(x_4^2 x_3 - y_3) + f((1-x_3^{\ 2})x_4^3 - (1-y_3^{\ 2})y_4) + g(x_4^2 x_1 - y_1) - u_4
\end{aligned} \tag{4-4-27}$$

System parameters are chosen as $a=10$, $b=3$, $c=0.4$, $d=70$, $e=1$, $f=5$, $g=0.1$ and initial states are $(x_{10}, x_{20}, x_{30}, x_{40})$=(0.1, -0.5, 0.1, -0.5), $(y_{10}, y_{20}, y_{30}, y_{40})$=(0.3, -0.1, 0.3, -0.1). Before control action, the error dynamics always happens in first quadrant as shown in Fig. 4-14. By GYC partial region stability, one can choose a Lyapunov function in the form of a positive definite function in first quadrant:

$$V = e_1 + e_2 + e_3 + e_4 \tag{4-4-28}$$

Its time derivative through Eq. (4-4-26) is

$$\begin{aligned}
\dot{V} &= \dot{e}_1 + \dot{e}_2 + \dot{e}_3 + \dot{e}_4 \\
&= (x_1^2 x_2 - y_2 - u_1) + (-((a+bx_3)x_2^2 x_1 - (a+by_3)y_1) - ((a+bx_3)x_2^2 x_1^{\ 3} \\
&\quad - (a+by_3)y_1^{\ 3}) - c(x_2^3 - y_2) + d(x_2^2 x_3 - y_3) - u_2) + (x_3^2 x_4 - y_4 - u_3) \\
&\quad + (-e(x_4^2 x_3 - y_3) + f((1-x_3^{\ 2})x_4^3 - (1-y_3^{\ 2})y_4) + g(x_4^2 x_1 - y_1) - u_4))
\end{aligned} \tag{4-4-29}$$

Choose

$$
\begin{aligned}
&u_1 = x_1^2 x_2 - y_2 + e_1 \\
&u_2 = -((a+bx_3)x_2^2 x_1 - (a+by_3)y_1) - ((a+bx_3)x_2^2 {x_1}^3 - (a+by_3){y_1}^3) \\
&\qquad - c(x_2^3 - y_2) + d(x_2^2 x_3 - y_3) + e_2 \\
&u_3 = x_3^2 x_4 - y_4 + e_3 \\
&u_4 = -e(x_4^2 x_3 - y_3) + f((1-{x_3}^2)x_4^3 - (1-{y_3}^2)y_4) + g(x_4^2 x_1 - y_1) + e_4
\end{aligned}
\tag{4-4-30}
$$

We obtain

$$
\dot{V} = -e_1 - e_2 - e_3 - e_4 < 0 \tag{4-4-31}
$$

which is a negative definite function in the first quadrant. Three state errors versus time is shown in Fig. 4-15.

CASE VI. The generalized synchronization error function is $e_i = x_i - y_i + z_i + 100$, z_i (*i*=1, 2, 3, 4) is the states of new chaotic Duffing-van der Pol system.

The functional system for synchronization is a new Duffing-van der pol system and initial states is (2, 2.4, 5, 6), system parameters $h = 0.0006$, $j = 1$, $k = 5$, $i = 0.67$ and $l = 0.05$.

$$
\begin{aligned}
&\dot{z}_1 = z_2 \\
&\dot{z}_2 = -z_1 - {z_1}^3 - hz_2 + iz_3 \\
&\dot{z}_3 = z_4 \\
&\dot{z}_4 = -jz_3 + k(1-{z_3}^2)z_4 + lz_1
\end{aligned}
\tag{4-4-32}
$$

We have $\lim_{t\to\infty} e = \lim_{t\to\infty}(x_i - y_i + z_i + 100) = 0$ (i=1, 2, 3, 4) (4-4-33) The error dynamics becomes

$$
\begin{aligned}
&\dot{e}_1 = \dot{x}_1 - \dot{y}_1 = x_2 + z_2 - y_2 - u_1 \\
&\dot{e}_2 = \dot{x}_2 - \dot{y}_2 = -((a+bx_3)x_1 - (a+by_3)y_1) - ((a+bx_3){x_1}^3 - (a+by_3){y_1}^3) \\
&\qquad - c(x_2 - y_2) + d(x_3 - y_3) + (-z_1 - {z_1}^3 - hz_2 + iz_3) - u_2 \\
&\dot{e}_3 = \dot{x}_3 - \dot{y}_3 = x_4 + z_4 - y_4 - u_3 \\
&\dot{e}_4 = \dot{x}_4 - \dot{y}_4 = -e(x_3 - y_3) + f((1-{x_3}^2)x_4 - (1-{y_3}^2)y_4) + g(x_1 - y_1) - u_4 \\
&\qquad + (-jz_3 + k(1-{z_3}^2)z_4 + lz_1)
\end{aligned}
\tag{4-4-34}
$$

System parameters are chosen as $a = 10$, $b = 3$, $c = 0.4$, $d = 70$, $e = 1$, $f = 5$, $g = 0.1$ and initial states are $(x_{10}, x_{20}, x_{30}, x_{40})$=(0.1, -0.5, 0.1, -0.5), $(y_{10}, y_{20}, y_{30}, y_{40})$=(0.3, -0.1, 0.3, -0.1). Before control action, the error dynamics always happens in first quadrant as shown in Fig. 5-16. By GYC partial region stability, one can choose a Lyapunov function in the form of a positive definite function in first quadrant:

$$V = e_1 + e_2 + e_3 + e_4 \tag{4-4-35}$$

Its time derivative through Eq. (5-4-33) is

$$\begin{aligned} \dot{V} &= \dot{e}_1 + \dot{e}_2 + \dot{e}_3 + \dot{e}_4 \\ &= (x_2 + z_2 - y_2 - u_1) + (-((a + bx_3)x_1 - (a + by_3)y_1) - ((a + bx_3)x_1^{\ 3} \\ &\quad - (a + by_3)y_1^{\ 3}) - c(x_2 - y_2) + d(x_3 - y_3) + (-z_1 - z_1^{\ 3} - hz_2 + iz_3) \\ &\quad - u_2) + (x_4 + z_4 - y_4 - u_3) + (-e(x_3 - y_3) + f((1 - x_3^{\ 2})x_4 - (1 - y_3^{\ 2})y_4) \\ &\quad + g(x_1 - y_1) - u_4 + (-jz_3 + k(1 - z_3^{\ 2})z_4 + lz_1)) \end{aligned} \tag{4-4-36}$$

Choose

$$\begin{aligned} u_1 &= x_2 + z_2 - y_2 + e_1 \\ u_2 &= -((a + bx_3)x_1 - (a + by_3)y_1) - ((a + bx_3)x_1^{\ 3} - (a + by_3)y_1^{\ 3}) \\ &\quad - c(x_2 - y_2) + d(x_3 - y_3) + (-z_1 - z_1^{\ 3} - hz_2 + iz_3) + e_2 \\ u_3 &= x_4 + z_4 - y_4 + e_3 \\ u_4 &= -e(x_3 - y_3) + f((1 - x_3^{\ 2})x_4 - (1 - y_3^{\ 2})y_4) + g(x_1 - y_1) + e_4 \\ &\quad + (-jz_3 + k(1 - z_3^{\ 2})z_4 + lz_1) \end{aligned} \tag{4-4-37}$$

We obtain

$$\dot{V} = -e_1 - e_2 - e_3 - e_4 < 0 \tag{4-4-38}$$

which is a negative definite function in the first quadrant. Four state errors versus time and time histories of $x_i - y_i + 100$ and $-z_i$ are shown in Fig. 4-17 and Fig. 4-18.

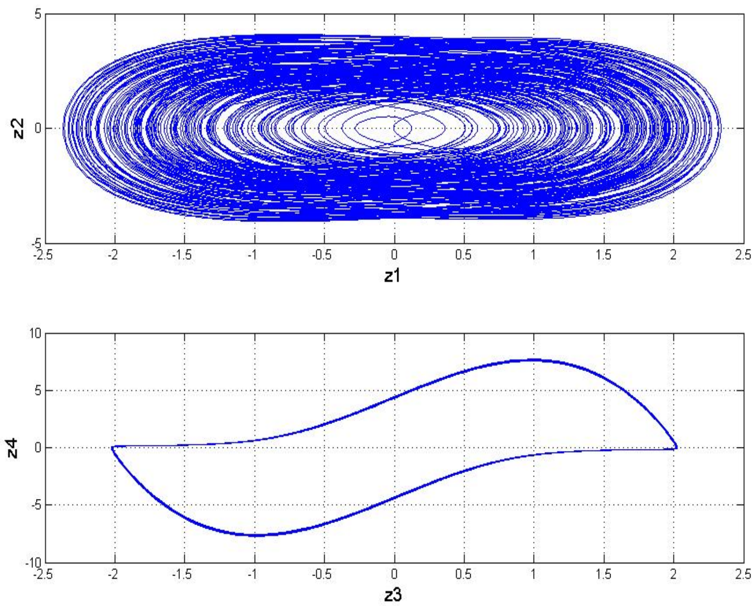


Fig. 4-1 Phase portrait projections of new chaotic Duffing-van der Pol System.

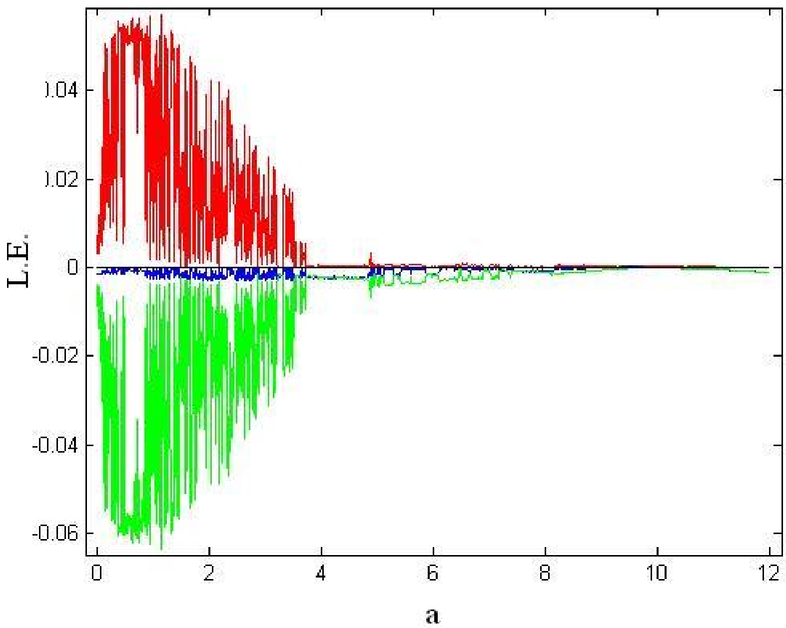


Fig. 4-2 Lyapunov exponents of new chaotic Duffing-van der Pol System.

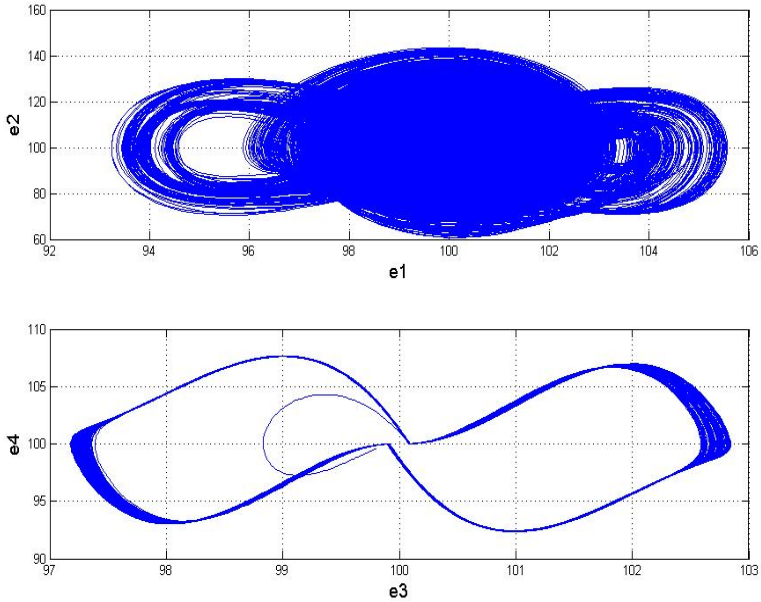


Fig. 4-3 Phase portrait projections of error dynamics for Case I.

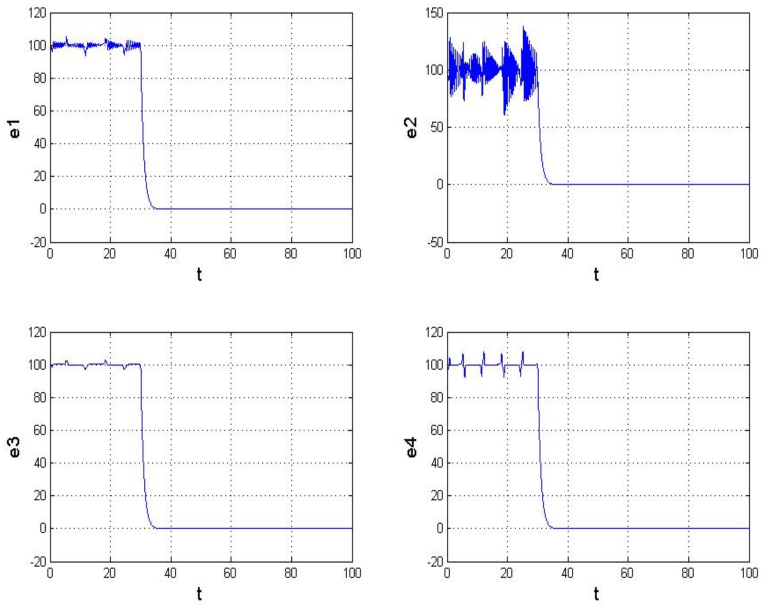


Fig. 4-4 Time histories of errors for Case I.

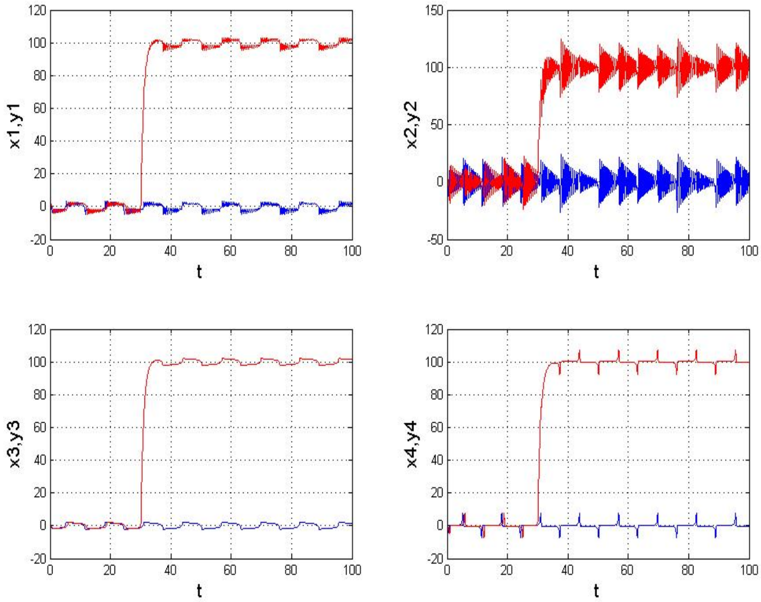


Fig. 4-5 Time histories of x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4 for Case I.

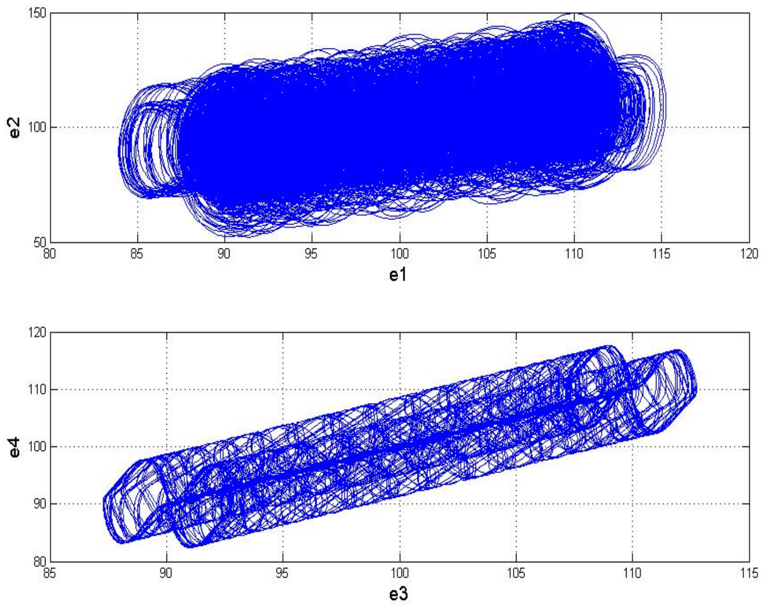


Fig. 4-6 Phase portrait projections of error dynamics for Case II.

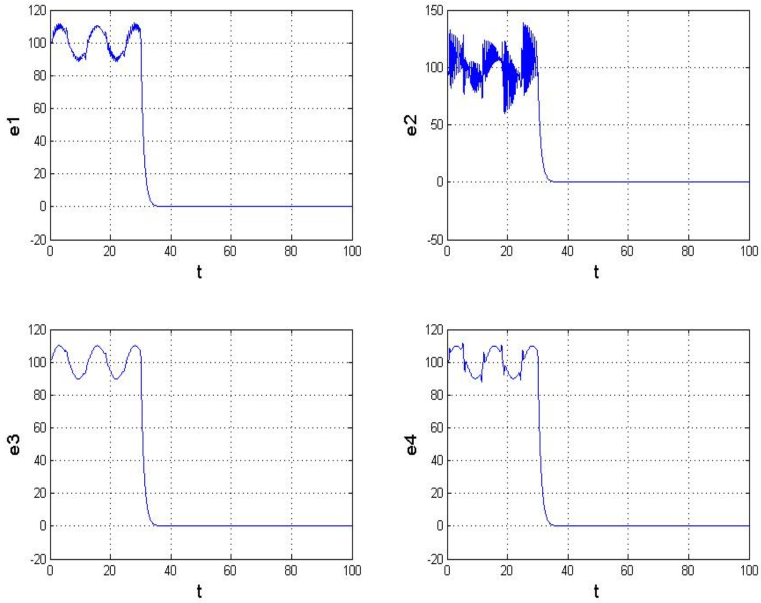


Fig. 4-7 Time histories of errors for Case II.

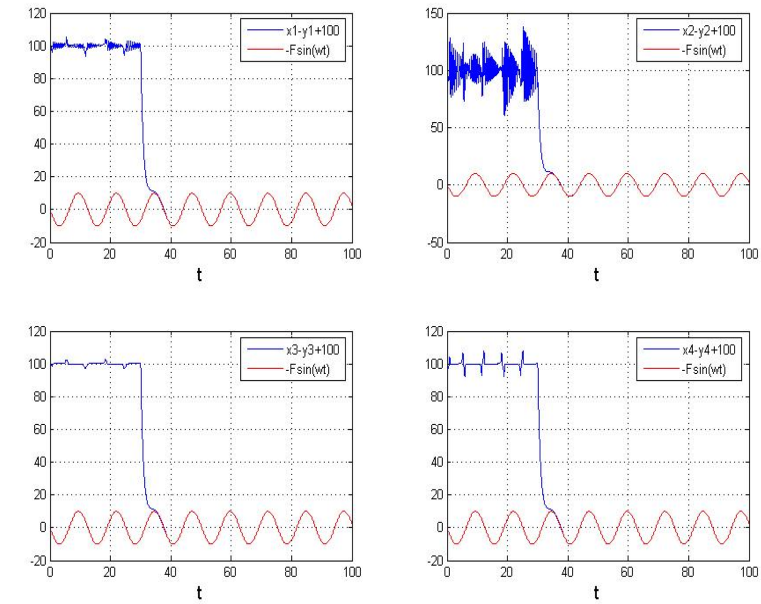


Fig. 4-8 Time histories of $x_i - y_i + 100$ and $-F \sin \omega t$ for Case II.

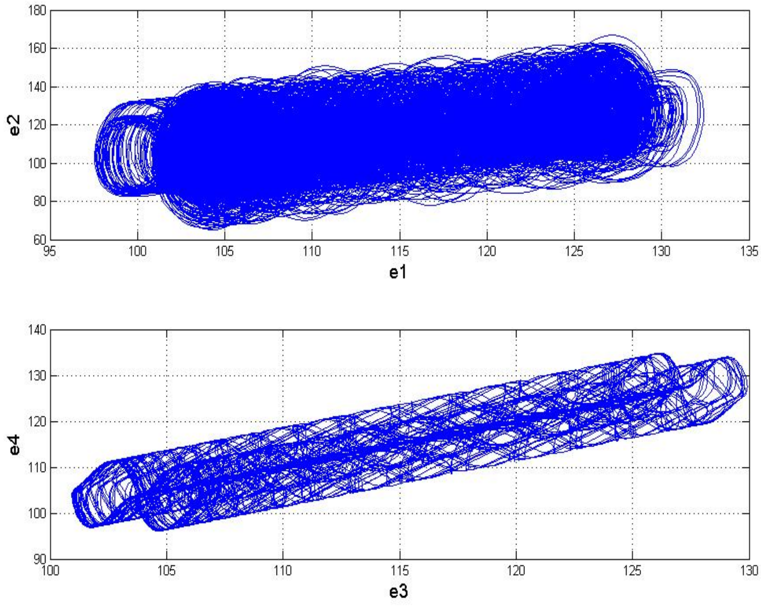


Fig. 4-9 Phase portrait projections of error dynamics for Case III.

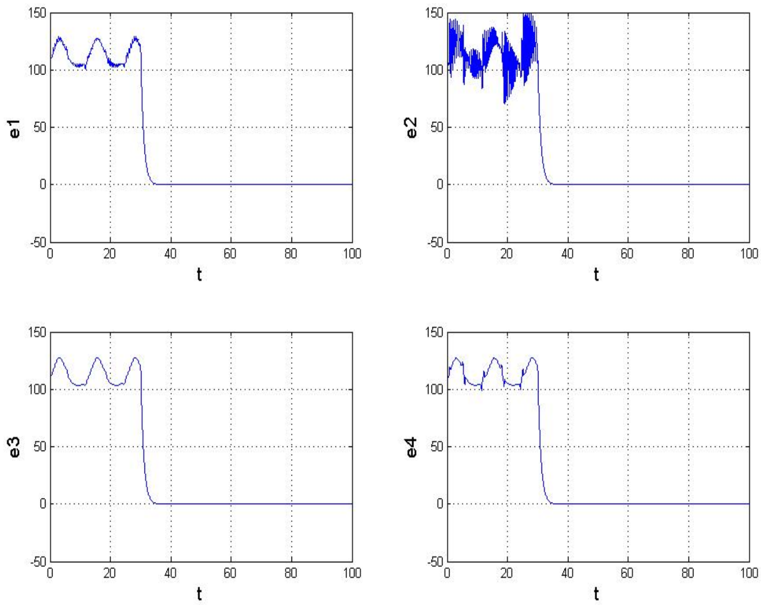


Fig. 4-10 Time histories of errors for Case III.

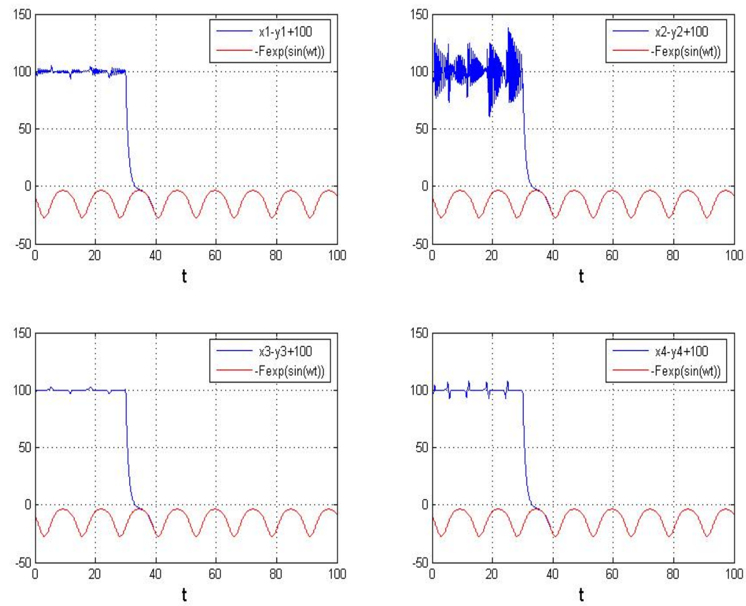


Fig. 4-11 Time histories of $x_i - y_i + 100$ and $-Fe^{\sin(wt)}$ for Case III.

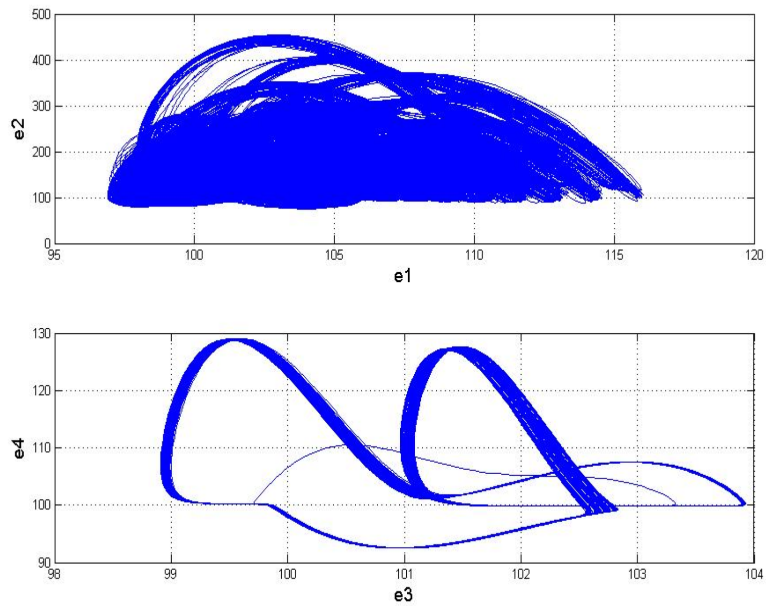


Fig. 4-12 Phase portrait projections of error dymanics for Case IV.

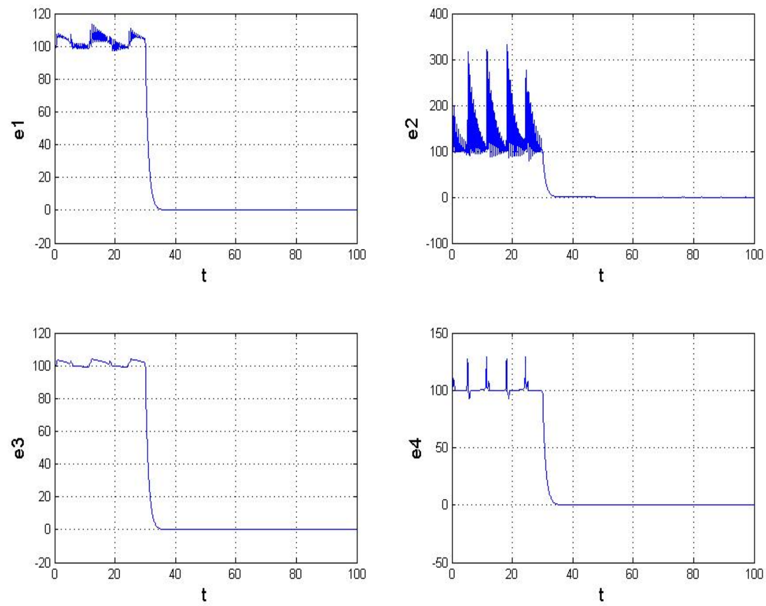


Fig. 4-13 Time histories of errors for Case IV.

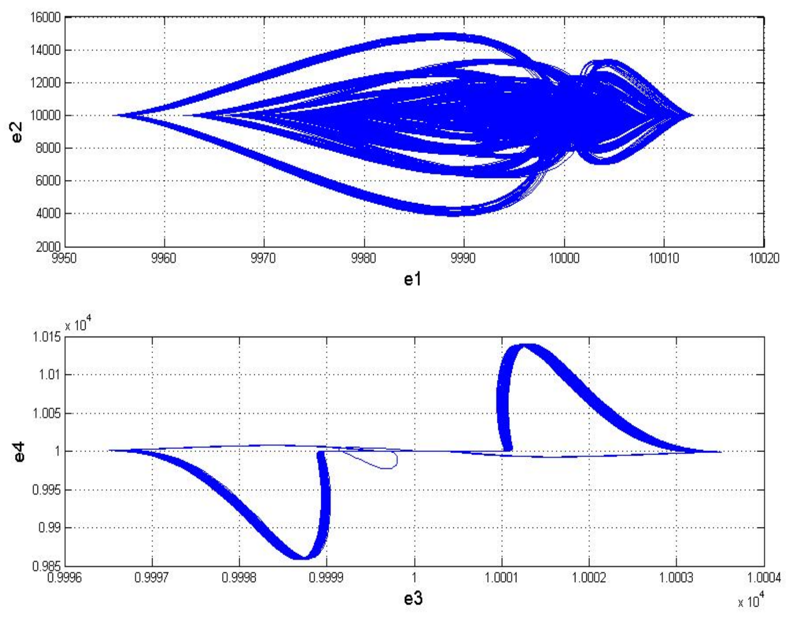


Fig. 4-14 Phase portrait projections of error dymanics for Case V.

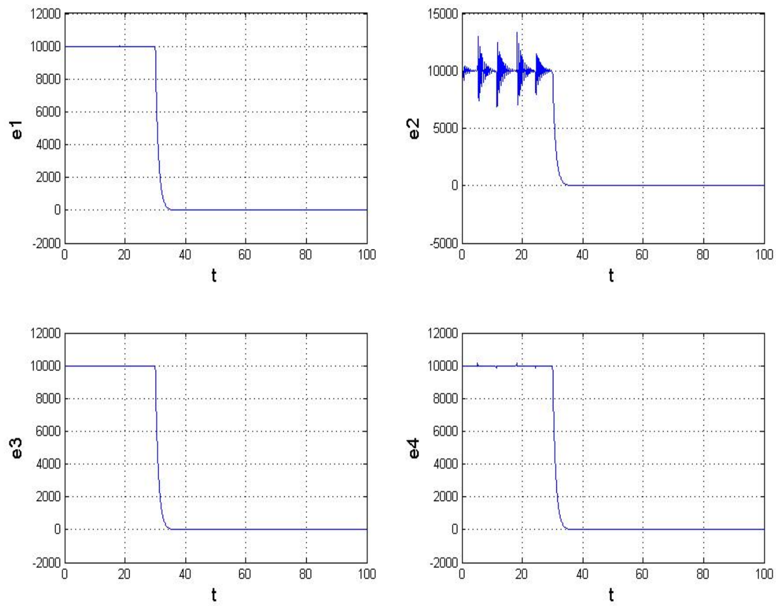


Fig. 4-15 Time histories of errors for Case V.

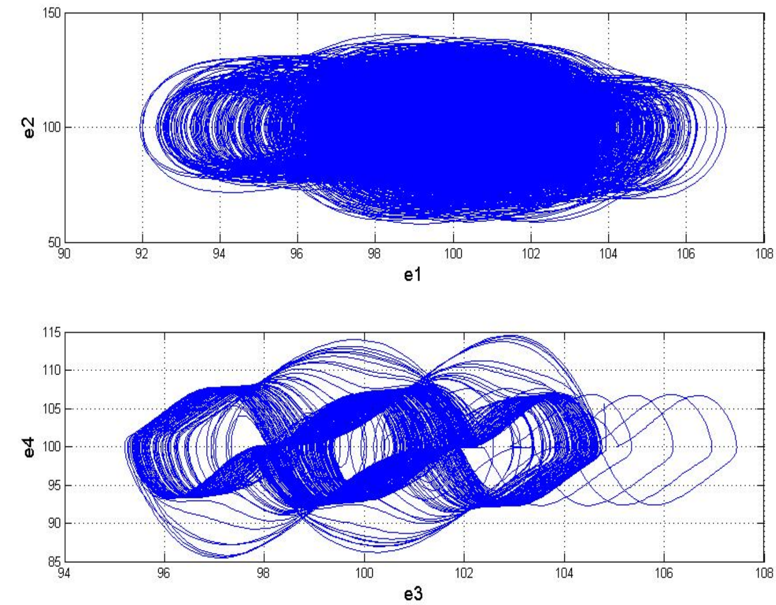


Fig. 4-16 Phase portrait projections of error dymanics for Case VI.

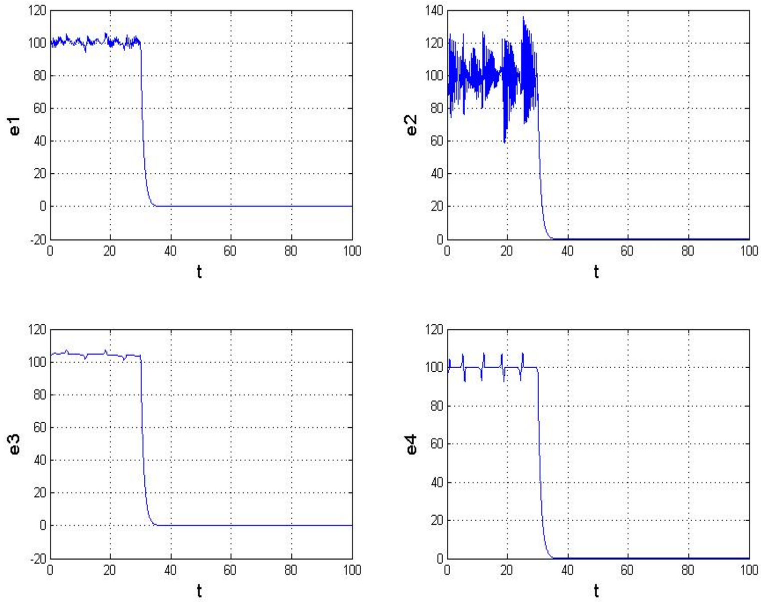


Fig. 4-17 Time histories of errors for Case VI.

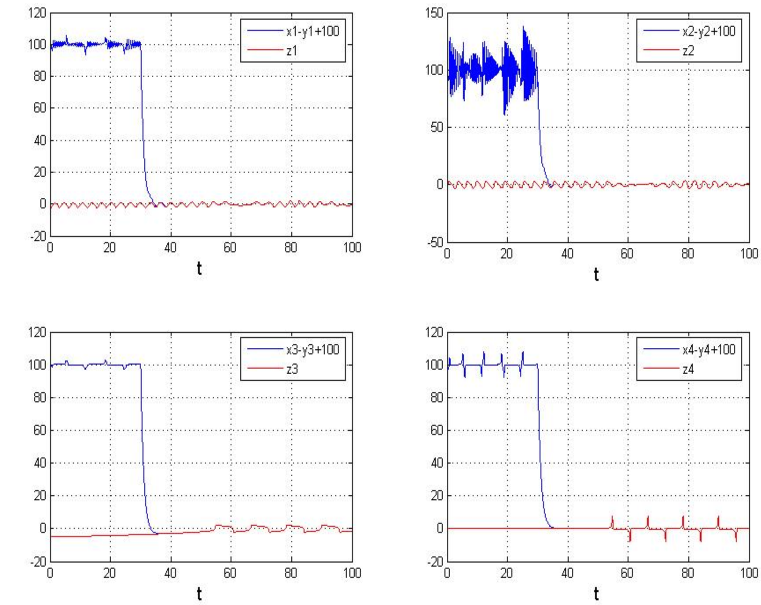


Fig. 4-18 Time histories of $x_i - y_i + 100$ and $-z_i$ for Case VI.

Chapter 5

Chaos Generalized Synchronization of New Mathieu- Duffing Systems by GYC Partial Region Stability Theory

5.1 Preliminaries

In this paper, a new method of achieving chaos generalized synchronization by GYC partial region stability is proposed. Using the GYC partial region stability theory, the Lyapunov function becomes a simple linear homogeneous function of states and the controllers are simpler then traditional controllers and the error values of simulation can be reduced because they are in lower order than that of traditional controllers. A new Mathieu – Duffing system is used in numerical simulations to prove the effectiveness of the scheme.

5.2 Chaos Generalized Synchronization Strategy by GYC Partial Region Stability Theory

5.2.1 GYC Partial Region Stability Theory

Consider the differential equations of disturbed motion of a nonautonomous system in the normal form

$$\frac{dx_s}{dt} = X_s(t, x_1, \cdots, x_n), \qquad (s = 1, \cdots, n) \tag{5-2-1}$$

where the function X_s is defined on the intersection of the partial region Ω (shown in Fig. 1) and

$$\sum_s x_s^2 \le H \tag{5-2-2}$$

and $t > t_0$, where t_0 and H are certain positive constants. X_s which vanishes when the variables x_s are all zero, is a real valued function of t, $x_1, \cdots, x_n$. It is assumed that X_s is smooth enough to ensure the existence, uniqueness of the solution of the initial value problem. When X_s does not contain *t* explicitly, the system is autonomous.

Obviously, $x_s = 0 \quad (s = 1, \cdots n)$ is a solution of Eq.(2.1). We are interested to the asymptotical stability of this zero solution on partial region Ω (including the boundary) of the neighborhood of the origin which in general may consist of several subregions (Fig. 5-1).

Definition 1:

For any given number $\varepsilon > 0$, if there exists a $\delta > 0$, such that on the closed given partial region Ω when

$$\sum_s x_{s0}^2 \le \delta, \quad (s = 1, \cdots, n) \tag{5-2-3}$$

for all $t \ge t_0$, the inequality

$$\sum_s x_s^2 < \varepsilon, \quad (s = 1, \cdots, n) \tag{5-2-4}$$

is satisfied for the solutions of Eq.(2.27) on Ω, then the disturbed motion $x_s = 0 \quad (s = 1, \cdots n)$ is stable on the partial region Ω.

Definition 2:

If the undisturbed motion is stable on the partial region Ω, and there exists a $\delta' > 0$, so that on the given partial region Ω when

$$\sum_s x_{s0}^2 \le \delta', \quad (s = 1, \cdots, n) \tag{5-2-5}$$

The equality

$$\lim_{t \to \infty} \left(\sum_s x_s^2 \right) = 0 \tag{5-2-6}$$

is satisfied for the solutions of Eq.(5-2-1) on Ω, then the undisturbed motion $x_s = 0 \quad (s = 1, \cdots n)$ is asymptotically stable on the partial region Ω.

The intersection of Ω and region defined by Eq.(5-2-2) is called the region of attraction.

Definition of Functions $V(t, x_1, \cdots, x_n)$:

Let us consider the functions $V(t, x_1, \cdots, x_n)$ given on the intersection Ω_1 of the partial region Ω and the region

$$\sum_s x_s^2 \le h, \quad (s=1,\cdots,n) \tag{5-2-7}$$

for $t \ge t_0 > 0$, where t_0 and h are positive constants. We suppose that the functions are single-valued and have continuous partial derivatives and become zero when $x_1 = \cdots = x_n = 0$.

Definition 3:

If there exists $t_0 > 0$ and a sufficiently small $h > 0$, so that on partial region Ω_1 and $t \ge t_0$, $V \ge 0$ (or ≤ 0), then V is a positive (or negative) semidefinite, in general semidefinite, function on the Ω_1 and $t \ge t_0$.

Definition 4:

If there exists a positive (negative) definitive function $W(x_1 \dots x_n)$ on Ω_1, so that on the partial region Ω_1 and $t \ge t_0$

$$V - W \ge 0 \, (or\ -V - W \ge 0), \tag{5-2-8}$$

then $V(t, x_1, \dots, x_n)$ is a positive definite function on the partial region Ω_1 and $t \ge t_0$.

Definition 5:

If $V(t, x_1, \dots, x_n)$ is neither definite nor semidefinite on Ω_1 and $t \ge t_0$, then $V(t, x_1, \dots, x_n)$ is an indefinite function on partial region Ω_1 and $t \ge t_0$. That is, for any small $h > 0$ and any large $t_0 > 0$, $V(t, x_1, \dots, x_n)$ can take either positive or negative value on the partial region Ω_1 and $t \ge t_0$.

Definition 6: Bounded function V

If there exist $t_0 > 0$, $h > 0$, so that on the partial region Ω_1, we have

$$\left|V(t, x_1, \dots, x_n)\right| < L \tag{5-2-9}$$

where L is a positive constant, then V is said to be bounded on Ω_1.

Definition 7: Function with infinitesimal upper bound

If V is bounded, and for any $\lambda > 0$, there exists $\mu > 0$, so that on Ω_1 when $\sum_s x_s^2 \le \mu$, and $t \ge t_0$, we have

$$|V(t, x_1, \dots, x_n)| \le \lambda \tag{5-2-10}$$

then V admits an infinitesimal upper bound on Ω_1.

Theorem 1 [20, 21]

If there can be found for the differential equations of the disturbed motion (Eq.(5-2-27)) a definite function $V(t, x_1, \dots, x_n)$ on the partial region, and for which the derivative with respect to time based on these equations as given by the following :

$$\frac{dV}{dt} = \frac{\partial V}{\partial t} + \sum_{s=1}^{n} \frac{\partial V}{\partial x_s} X_s \tag{5-2-11}$$

is a semidefinite function on the paritial region whose sense is opposite to that of V, or if it becomes zero identically, then the undisturbed motion is stable on the partial region.

Proof:

Let us assume for the sake of definiteness that V is a positive definite function. Consequently, there exists a sufficiently large number t_0 and a sufficiently small number h < H, such that on the intersection Ω_1 of partial region Ω and

$$\sum_s x_s^2 \le h, \quad (s = 1, \dots, n) \tag{5-2-12}$$

and $t \ge t_0$, the following inequality is satisfied

$$V(t, x_1, \dots, x_n) \ge W(x_1, \dots, x_n) \tag{5-2-13}$$

where W is a certain positive definite function which does not depend on t. Besides that, Eq. (5-2-7) may assume only negative or zero value in this region.

Let ε be an arbitrarily small positive number. We shall suppose that in any case $\varepsilon < h$.

Let us consider the aggregation of all possible values of the quantities $x_1,\dots,x_n$, which are on the intersection ω_2 of Ω_1 and

$$\sum_s x_s^2 = \varepsilon, \tag{5-2-14}$$

and let us designate by $l > 0$ the precise lower limit of the function W under this condition. by virtue of Eq. (5-2-5), we shall have

$$V(t, x_1,\dots,x_n) \geq l \quad \text{for} \quad (x_1,\dots,x_n) \quad \text{on} \quad \omega_2 . \tag{5-2-15}$$

We shall now consider the quantities x_s as functions of time which satisfy the differential equations of disturbed motion. We shall assume that the initial values x_{s0} of these functions for $t = t_0$ lie on the intersection Ω_2 of Ω_1 and the region

(5-2-16)

z

where δ is so small that

$$V(t_0, x_{10},\dots,x_{n0}) < l \tag{5-2-17}$$

By virtue of the fact that $V(t_0, 0,\dots,0) = 0$, such a selection of the number δ is obviously possible. We shall suppose that in any case the number δ is smaller than ε .Then the inequality

$$\sum_s x_s^2 < \varepsilon, \tag{5-2-18}$$

being satisfied at the initial instant will be satisfied, in the very least, for a sufficiently small $t - t_0$, since the functions $x_s(t)$ very continuously with time. We shall show that these inequalities will be satisfied for all values $t > t_0$. Indeed, if these inequalities were not satisfied at some time, there would have to exist such an instant *t=T* for which this inequality would become an equality. In other words, we would have

$$\sum_s x_s^2(T) = \varepsilon, \tag{5-2-19}$$

and consequently, on the basis of Eq. (5-2-9)

$$V(T, x_1(T), \ldots, x_n(T)) \geq l \tag{5-2-20}$$

On the other hand, since $\varepsilon < h$, the inequality (Eq.(5-2-4)) is satisfied in the entire interval of time [t_0, T], and consequently, in this entire time interval $\frac{dV}{dt} \leq 0$. This yields

$$V(T, x_1(T), \ldots, x_n(T)) \leq V(t_0, x_{10}, \ldots, x_{n0}), \tag{5-2-21}$$

which contradicts Eq. (5-2-12) on the basis of Eq. (5-2-11). Thus, the inequality (Eq.(5-2-1)) must be satisfied for all values of $t > t_0$, hence follows that the motion is stable.

Finally, we must point out that from the view-point of mathenatics, the stability on partial region in general does not be related logically to the stability on whole region. If an undisturbed solution is stable on a partial region, it may be either stable or unstable on the whole region and vice versa. From the viewpoint of dynamics, we wre not interesting to the solution starting from Ω_2 and going out of Ω.

Theorem 2 [20, 21]

If in satisfying the conditions of theorem 1, the derivative $\frac{dV}{dt}$ is a definite function on the partial region with opposite sign to that of V and the function V itself permits an infinitesimal upper limit, then the undisturbed motion is asymptotically stable on the partial region.

Proof:

Let us suppose that V is a positive definite function on the partial region and that consequently, $\frac{dV}{dt}$ is negative definite. Thus on the intersection Ω_1 of Ω and the region defined by Eq. (5-2-4) and $t \geq t_0$ there will be satisfied not only the inequality (Eq.(5-2-5)), but the following inequality as will:

$$\frac{dV}{dt} \leq -W_1(x_1, \ldots x_n), \tag{5-2-22}$$

where W_1 is a positive definite function on the partial region independent of t.

Let us consider the quantities x_s as functions of time which satisfy the differential equations of disturbed motion assuming that the initial values $x_{s0} = x_s(t_0)$ of these quantities satisfy the inequalities (Eq. (5-2-10)). Since the undisturbed motion is stable in any case, the magnitude δ may be selected so small that for all values of $t \geq t_0$ the quantities x_s remain within Ω_1. Then, on the basis of Eq. (5-2-13) the derivative of function $V(t, x_1(t), \ldots, x_n(t))$ will be negative at all times and, consequently, this function will approach a certain limit, as t increases without limit, remaining larger than this limit at all times. We shall show that this limit is equal to some positive quantity different from zero. Then for all values of $t \geq t_0$ the following inequality will be satisfied:

$$V(t, x_1(t), \ldots, x_n(t)) > \alpha \tag{5-2-23}$$

where $\alpha > 0$.

Since V permits an infinitesimal upper limit, it follows from this inequality that

$$\sum_s x_s^2(t) \geq \lambda, \quad (s = 1, \ldots, n), \tag{5-2-24}$$

where λ is a certain sufficiently small positive number. Indeed, if such a number λ did not exist, that is , if the quantity $\sum_s x_s(t)$ were smaller than any preassigned number no matter how small, then the magnitude $V(t, x_1(t), \ldots, x_n(t))$, as follows from the definition of an infinitesimal upper limit, would also be arbitrarily small, which contradicts (5-2-14).

If for all values of $t \geq t_0$ the inequality (Eq. (5-2-15)) is satisfied, then Eq. (5-2-13) shows that the following inequality will be satisfied at all times:

$$\frac{dV}{dt} \leq -l_1, \tag{5-2-25}$$

where l_1 is positive number different from zero which constitutes the precise lower limit of the function $W_1(t, x_1(t), \ldots, x_n(t))$ under condition (Eq. (5-2-15)). Consequently, for all values of $t \geq t_0$ we shall have:

$$V(t,x_1(t),\ldots,x_n(t)) = V(t_0,x_{10},\ldots,x_{n0}) + \int_{t_0}^{t} \frac{dV}{dt}dt \le V(t_0,x_{10},\ldots,x_{n0}) - l_1(t-t_0),$$

which is, obviously, in contradiction with Eq.(5-2-14). The contradiction thus obtained shows that the function $V(t,x_1(t),\ldots,x_n(t))$ approached zero as t increase without limit. Consequently, the same will be true for the function $W(x_1(t),\ldots,x_n(t))$ as well, from which it follows directly that

$$\lim_{t\to\infty} x_s(t) = 0, \quad (s = 1,\ldots,n), \tag{5-2-26}$$

which proves the theorem.

5.2.2 Chaos Generalized Synchronization Strategy

Consider the following unidirectional coupled chaotic systems

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(t,\mathbf{x}) \\ \dot{\mathbf{y}} &= \mathbf{h}(t,\mathbf{y}) + \mathbf{u} \end{aligned} \tag{5-2-27}$$

where $\mathbf{x} = [x_1, x_2, \cdots, x_n]^T \in R^n$, $\mathbf{y} = [y_1, y_2, \cdots, y_n]^T \in R^n$ denote two state vectors, $\mathbf{f}$ and $\mathbf{h}$ are nonlinear vector functions, and $\mathbf{u} = [u_1, u_2, \cdots, u_n]^T \in R^n$ is a control input vector.

The generalized synchronization can be accomplished when $t \to \infty$, the limit of the error vector $\mathbf{e} = [e_1, e_2, \cdots, e_n]^T$ approaches zero:

$$\lim_{t\to\infty} \mathbf{e} = 0 \tag{5-2-28}$$

where

$$\mathbf{e} = \mathbf{G}(\mathbf{x}) - \mathbf{y} \tag{5-2-29}$$

By using the GYC partial region stability theory, the Lyapunov function is easier to find, a homogeneous function of first degree can be used to construct a positive definite Lyapunov function and the controllers can be designed in lower order.

5.3 Numerical Simulations

Two new Mathieu – Duffing systems, with the unidirectional coupling appear as

$$\begin{cases} \frac{d}{dt}x_1 = x_2 \\ \frac{d}{dt}x_2 = -(a+bx_3)x_1 - (a+bx_3)x_1^3 - cx_2 + dx_3 \\ \frac{d}{dt}x_3 = x_4 \\ \frac{d}{dt}x_4 = -x_3 - x_3^3 - ex_4 + fx_1 \end{cases} \quad (5\text{-}3\text{-}1)$$

$$\begin{cases} \frac{d}{dt}y_1 = y_2 + u_1 \\ \frac{d}{dt}y_2 = -(a+by_3)y_1 - (a+by_3)y_1^3 - cy_2 + dy_3 + u_2 \\ \frac{d}{dt}y_3 = y_4 + u_3 \\ \frac{d}{dt}y_4 = -y_3 - y_3^3 - ey_4 + fy_1 + u_4 \end{cases} \quad (5\text{-}3\text{-}2)$$

CASE I. The generalized synchronization error function is $e_i = x_i - y_i + k_i \quad ,(i = 1,2,3,4)$.

$$\begin{cases} e_1 = x_1 - y_1 + k_1 \\ e_2 = x_2 - y_2 + k_2 \\ e_3 = x_3 - y_3 + k_3 \\ e_4 = x_4 - y_4 + k_4 \end{cases} \quad (5\text{-}3\text{-}3)$$

where k_i, $(i = 1,2,3,4)$ is positive constants, we choose $k_1=10$, $k_2=75$, $k_3=15$, $k_4=60$ in order that the error dynamics always happens in first quadrant.

Our goal is $\mathbf{y} = \mathbf{x} + k$, i.e. the controlling goal is that

$$\lim_{t\to\infty} e_i = \lim_{t\to\infty}(x_i - y_i + k_i) = 0 \text{ , } (i = 1,2,3,4) \quad (5\text{-}3\text{-}4)$$

The error dynamics becomes

$$\begin{cases} \dot{e}_1 = x_2 - y_2 - u_2 \\ \dot{e}_2 = -(a+bx_3)x_1 - (a+bx_3)x_1^3 - cx_2 + dx_3 \\ \qquad -(-(a+by_3)y_1 - (a+by_3)y_1^3 - cy_2 + dy_3) - u_2 \\ \dot{e}_3 = x_4 - y_4 - u_3 \\ \dot{e}_4 = -x_3 - x_3^3 - ex_4 + fx_1 - (-y_3 - y_3^3 - ey_4 + fy_1) - u_4 \end{cases} \quad (5\text{-}3\text{-}5)$$

where

$$\dot{e}_i = \dot{x}_i - \dot{y}_i, \quad (i = 1,2,3,4) \quad (5\text{-}3\text{-}6)$$

Let initial states be (x_1, x_2, x_3, x_4) = (-2, 10, -2, 10), (y_1, y_2, y_3, y_4) = (-1, 11, -1, 11), we find the error dynamics always exists in first quadrant as shown in Fig. 5-3. By GYC partial region asymptotical stability theorem, one can choose a Lyapunov function in the form of a positive definite function in first quadrant:

$$V = e_1 + e_2 + e_3 + e_4 \tag{5-3-7}$$

Its time derivative is

$$\begin{aligned}\dot{V} &= \dot{e}_1 + \dot{e}_2 + \dot{e}_3 + \dot{e}_4 \\ &= \left(x_2 - y_2 - u_1\right) \\ &\quad + \Big(-(a+bx_3)x_1 - (a+bx_3)x_1^3 - cx_2 + dx_3 \\ &\quad - (-(a+by_3)y_1 - (a+by_3)y_1^3 - cy_2 + dy_3) - u_2\Big) \\ &\quad + \left(x_4 - y_4 - u_3\right) \\ &\quad + (-x_3 - x_3^3 - ex_4 + fx_1 - (-y_3 - y_3^3 - ey_4 + fy_1) - u_4)\end{aligned} \tag{5-3-8}$$

Choose

$$\begin{aligned}u_1 &= x_2 - y_2 + e_1 \\ u_2 &= -(a+bx_3)x_1 - (a+bx_3)x_1^3 - cx_2 + dx_3 \\ &\quad - (-(a+by_3)y_1 - (a+by_3)y_1^3 - cy_2 + dy_3) + e_2 \\ u_3 &= x_4 - y_4 + e_3 \\ u_4 &= -x_3 - x_3^3 - ex_4 + fx_1 - (-y_3 - y_3^3 - ey_4 + fy_1) + e_4\end{aligned} \tag{5-3-9}$$

We obtain

$$\dot{V} = -e_1 - e_2 - e_3 - e_4 < 0 \tag{5-3-10}$$

which is negative definite function in first quadrant. Four error states versus time and time histories of states are shown in Figs.5-4~5-5.

CASE II. The generalized synchronization error function is $e_i = x_i - y_i + m\sin wt + k_i$, , $(i = 1,2,3,4)$.

Our goal is $y_i = x_i + m\sin wt + k_i$, i.e. $\lim_{t\to\infty} e_i = \lim_{t\to\infty}(x_i - y_i + m\sin wt + k_i) = 0$, , $(i = 1,2,3,4)$

The error dynamics become

$$\begin{cases} \dot{e}_1 = x_2 - y_2 + mw\cos wt - u_2 \\ \dot{e}_2 = -(a+bx_3)x_1 - (a+bx_3)x_1^3 - cx_2 + dx_3 \\ \quad -(-(a+by_3)y_1 - (a+by_3)y_1^3 - cy_2 + dy_3) + mw\cos wt - u_2 \\ \dot{e}_3 = x_4 - y_4 + mw\cos wt - u_3 \\ \dot{e}_4 = -x_3 - x_3^3 - ex_4 + fx_1 - (-y_3 - y_3^3 - ey_4 + fy_1) + mw\cos wt - u_4 \end{cases} \tag{5-3-11}$$

where

$$\dot{e}_i = \dot{x}_i + mw\cos wt - \dot{y}_i\,, \quad (i = 1,2,3,4) \tag{5-3-12}$$

Let initial states be (x_1, x_2, x_3, x_4) = (-2, 10, -2, 10), (y_1, y_2, y_3, y_4) = (-1, 11, -1, 11), and w=1, m=2, $k_1 = 100, k_2 = 100, k_3 = 100, k_4 = 100$, we find the error dynamic always exists in first quadrant as shown in Fig. 5-6. By GYC partial region asymptoical stability theorem, one can choose a Lyapunov function in the form of a positive definite function in first quadrant:

$$V = e_1 + e_2 + e_3 + e_4 \tag{5-3-13}$$

Its time derivative is

$$\begin{aligned} \dot{V} &= \left(x_2 - y_2 - u_1 + mw\cos wt\right) \\ &+ \left(-(a+bx_3)x_1 - (a+bx_3)x_1^3 - cx_2 + dx_3\right. \\ &\left. -(-(a+by_3)y_1 - (a+by_3)y_1^3 - cy_2 + dy_3) - u_2 + mw\cos wt\right) \\ &+ \left(x_4 - y_4 - u_3 + mw\cos wt\right) \\ &+ (-x_3 - x_3^3 - ex_4 + fx_1 - (-y_3 - y_3^3 - ey_4 + fy_1) - u_4 + mw\cos wt) \end{aligned} \tag{5-3-14}$$

Choose

$$\begin{aligned} u_1 &= x_2 - y_2 + mw\cos wt + e_1 \\ u_2 &= -(a+bx_3)x_1 - (a+bx_3)x_1^3 - cx_2 + dx_3 \\ &\quad -(-(a+by_3)y_1 - (a+by_3)y_1^3 - cy_2 + dy_3) + mw\cos wt + e_2 \\ u_3 &= x_4 - y_4 + mw\cos wt + e_3 \\ u_4 &= -x_3 - x_3^3 - ex_4 + fx_1 - (-y_3 - y_3^3 - ey_4 + fy_1) + mw\cos wt + e_4 \end{aligned} \tag{5-3-15}$$

We obtain

$$\dot{V} = -e_1 - e_2 - e_3 - e_4 < 0 \tag{5-3-16}$$

which is negative definite function in first quadrant. Four state errors versus time and time histories of $x_i - y_i + k_i$ are shown in Figs. 5-8.

CASE III. The generalized synchronization error function is $e_i = \frac{1}{2}x_i^2 - y_i + k_i$, , $(i = 1,2,3,4)$

where $k_1 = 10, k_2 = 700, k_3 = 20, k_4 = 350$.

Our goal is $y = \frac{1}{2}x^2 + k$, i.e. $\lim_{t\to\infty} \mathbf{e} = \lim_{t\to\infty}(\frac{1}{2}x_i^2 - y_i + k_i) = 0$, $(i = 1,2,3,4)$

The error dynamics become

$$\begin{cases} \dot{e}_1 = x_1 x_2 - y_2 - u_2 \\ \dot{e}_2 = x_2(-(a + bx_3)x_1 - (a + bx_3)x_1^3 - cx_2 + dx_3) \\ \qquad -(-(a + by_3)y_1 - (a + by_3)y_1^3 - cy_2 + dy_3) - u_2 \\ \dot{e}_3 = x_3 x_4 - y_4 - u_3 \\ \dot{e}_4 = x_4(-x_3 - x_3^3 - ex_4 + fx_1) - (-y_3 - y_3^3 - ey_4 + fy_1) - u_4 \end{cases} \tag{5-3-17}$$

where

$$\dot{e}_i = x_i \dot{x}_i - \dot{y}_i, \quad (i = 1,2,3,4)$$

Let initial states be (x_1, x_2, x_3, x_4)= (-2, 10, -2, 10), (y_1, y_2, y_3, y_4)= (-1, 11, -1, 11), we find that the error dynamics always exists in first quadrant as shown in Fig. 5-9. By GYC partial region asymptotial stability theorem, one can choose a Lyapunov function in the form of a positive definite function in first quadrant:

$$V = e_1 + e_2 + e_3 + e_4 \tag{5-3-18}$$

Its time derivative is

$$\begin{aligned} \dot{V} &= \dot{e}_1 + \dot{e}_2 + \dot{e}_3 + \dot{e}_4 \\ &= x_1 x_2 - y_2 - u_1 \\ &\quad + \left(x_2(-(a + bx_3)x_1 - (a + bx_3)x_1^3 - cx_2 + dx_3)\right. \\ &\quad \left. -(-(a + by_3)y_1 - (a + by_3)y_1^3 - cy_2 + dy_3) - u_2\right) \\ &\quad + x_3 x_4 - y_4 - u_3 \\ &\quad + (x_4(-x_3 - x_3^3 - ex_4 + fx_1) - (-y_3 - y_3^3 - ey_4 + fy_1) - u_4) \end{aligned} \tag{5-3-19}$$

Choose

$$
\begin{aligned}
u_1 &= x_1x_2 - y_2 + e_1 \\
u_2 &= x_2(-(a+bx_3)x_1 - (a+bx_3)x_1^3 - cx_2 + dx_3) \\
&\quad -(-(a+by_3)y_1 - (a+by_3)y_1^3 - cy_2 + dy_3) + e_2 \\
u_3 &= x_3x_4 - y_4 + e_3 \\
u_4 &= x_4(-x_3 - x_3^3 - ex_4 + fx_1) - (-y_3 - y_3^3 - ey_4 + fy_1) + e_4
\end{aligned}
\tag{5-3-20}
$$

We obtain

$$\dot{V} = -e_1 - e_2 - e_3 - e_4 < 0 \tag{5-3-21}$$

which is negative definite function in first quadrant. Four state errors versus time and time histories of $\frac{1}{2}x_i^2 - y_i + k$ are shown in Figs. 5-10~5-11.

CASE IV. The generalized synchronization error function is $e = x - y + z + k$, $\mathbf{z}$ is the state vector of generalized Lorenz system.

The goal system for synchronization is generalized chaotic Lorenz system [22] and initial states are (0.1, 0.1, 0.1, 0.1), system parameters $a_1 = 1$, $b_1 = 26$, $c_1 = 0.7$, $d_1 = 1.5$.

$$
\begin{cases}
\frac{d}{dt}z_1 = a_1(z_2 - z_1) + dz_4 \\
\frac{d}{dt}z_2 = b_1z_1 - z_1z_3 - z_2 \\
\frac{d}{dt}z_3 = z_1z_2 - c_1z_3 \\
\frac{d}{dt}z_4 = -z_1 - a_1z_4
\end{cases}
\tag{5-3-22}
$$

We have $\lim_{t\to\infty} \mathbf{e} = \lim_{t\to\infty}(x - y + z + k) = 0$, where $k = [70\ 70\ 70\ 70]^{\mathrm{T}}$.

The error dynamics becomes

(5-3-23)

s

Let initial states be (x_1, x_2, x_3, x_4) = (-2, 10, -2, 10), (y_1, y_2, y_3, y_4) = (-1, 11, -1, 11), we find the error dynamics always exists in first quadrant as shown in Fig. 5-12. By GYC partial region asymptotical stability theorem, one can choose a Lyapunov function in the form of a positive definite function in first quadrant:

$$V = e_1 + e_2 + e_3 + e_4 \tag{5-3-24}$$

Its time derivative is

$$\begin{aligned}
\dot{V} &= \left(x_2 - y_2 + a_1(z_2 - z_1) + dz_4 - u_2\right) \\
&\quad + (-(a + bx_3)x_1 - (a + bx_3)x_1^3 - cx_2 + dx_3 + b_1 z_1 - z_1 z_3 - z_2 \\
&\quad - (-(a + by_3)y_1 - (a + by_3)y_1^3 - cy_2 + dy_3) - u_2) \\
&\quad + x_4 - y_4 + z_1 z_2 - c_1 z_3 - u_3 \\
&\quad + \left(-x_3 - x_3^3 - ex_4 + fx_1 - (-y_3 - y_3^3 - ey_4 + fy_1) - z_1 - a_1 z_4 - u_4\right)
\end{aligned} \tag{5-3-25}$$

Choose

$$\begin{aligned}
u_1 &= x_2 - y_2 + a_1(z_2 - z_1) + dz_4 + e_1 \\
u_2 &= -(a + bx_3)x_1 - (a + bx_3)x_1^3 - cx_2 + dx_3 + b_1 z_1 - z_1 z_3 - z_2 \\
&\qquad - (-(a + by_3)y_1 - (a + by_3)y_1^3 - cy_2 + dy_3) + e_2 \\
u_3 &= x_4 - y_4 + z_1 z_2 - c_1 z_3 + e_3 \\
u_4 &= -x_3 - x_3^3 - ex_4 + fx_1 - (-y_3 - y_3^3 - ey_4 + fy_1) - z_1 - a_1 z_4 + e_4
\end{aligned} \tag{5-3-26}$$

We obtain

$$\dot{V} = -e_1 - e_2 - e_3 - e_4 < 0 \tag{5-3-27}$$

which is negative definite function in first quadrant. Four state errors versus time and time histories of $x_i - y_i + k_i$ are shown in Figs. 5-13~5-14.

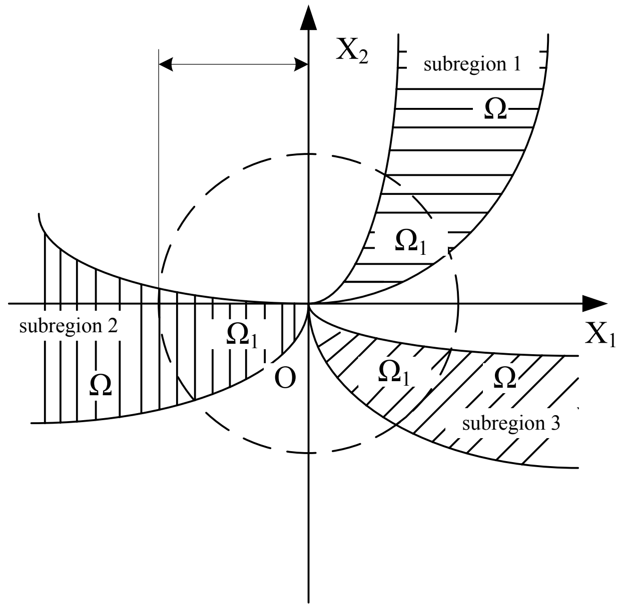


Fig. 5-1. Partial regions Ω and Ω_1.

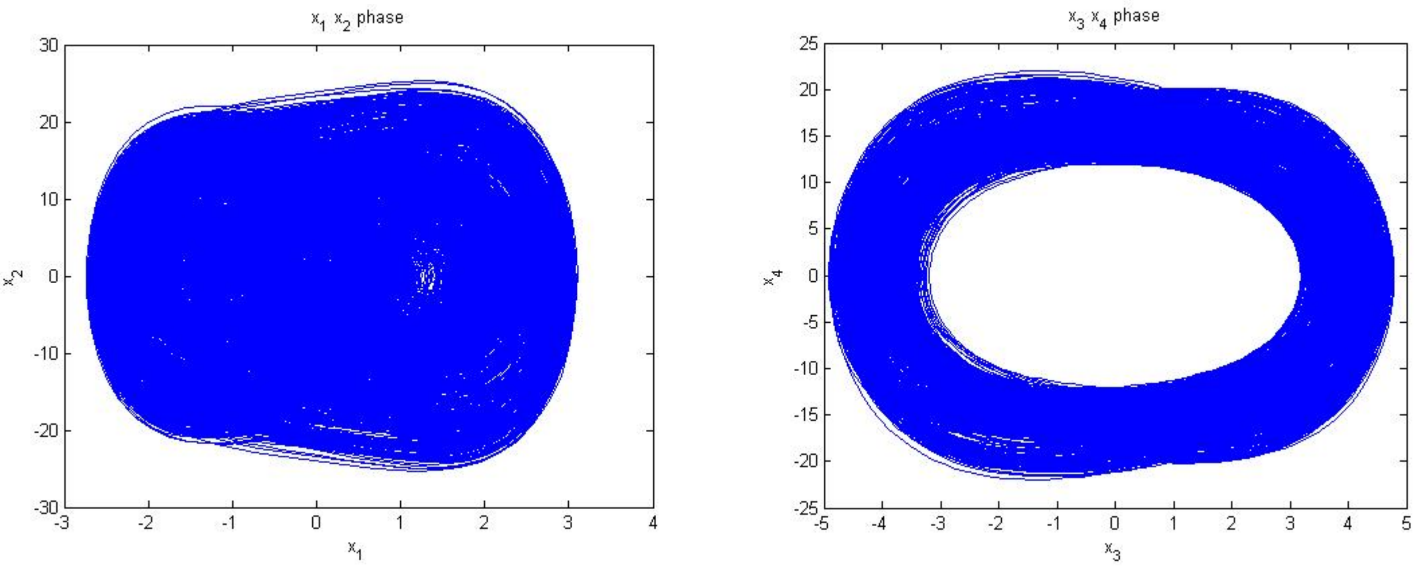


Fig. 5-2 Chaotic phase portraits for new Mathieu-Duffing system in the first quadrant.

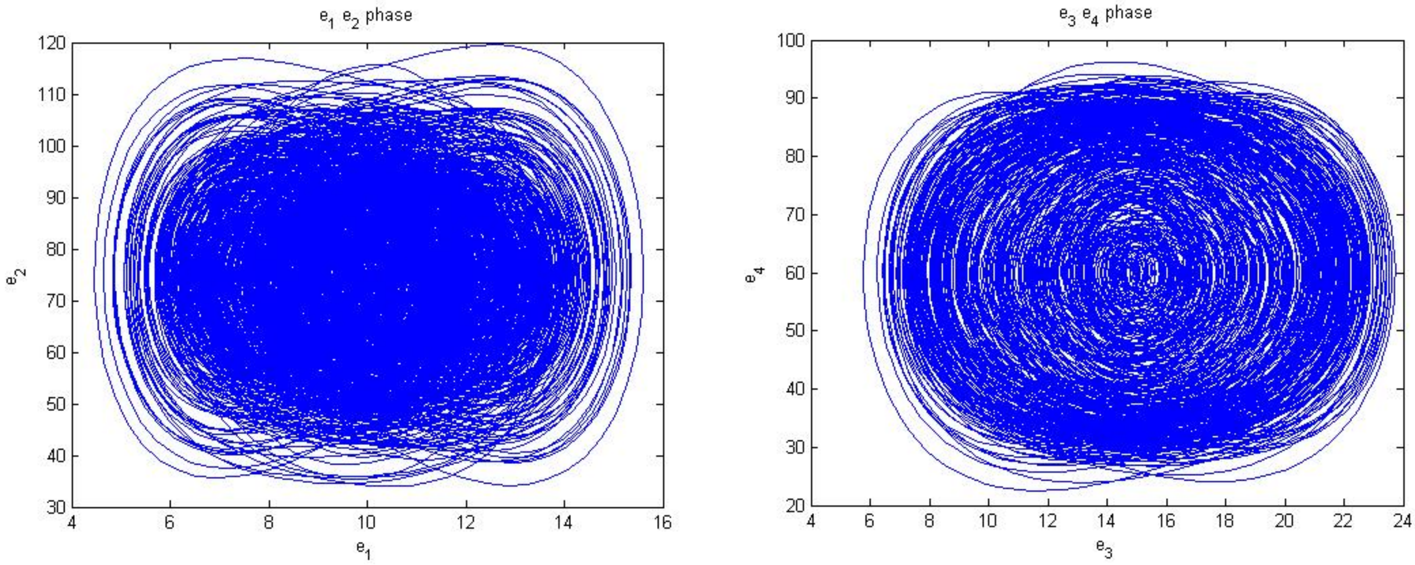


Fig. 5-3.   Phase portrait of four errors dynamics for Case I.

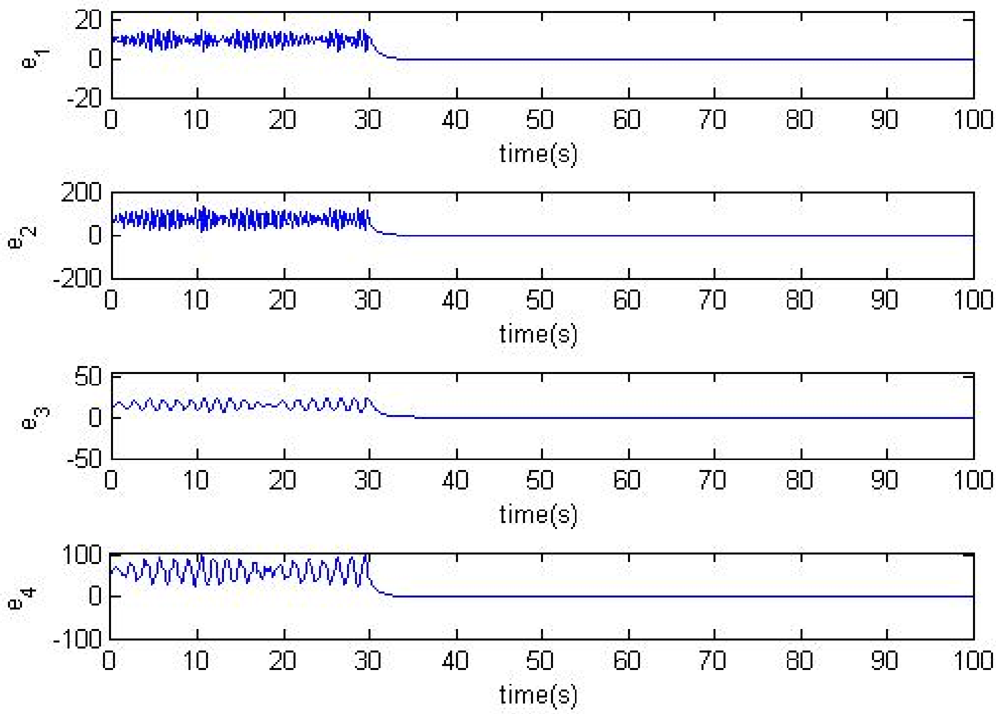


Fig. 5-4. Time histories of errors for Case I.

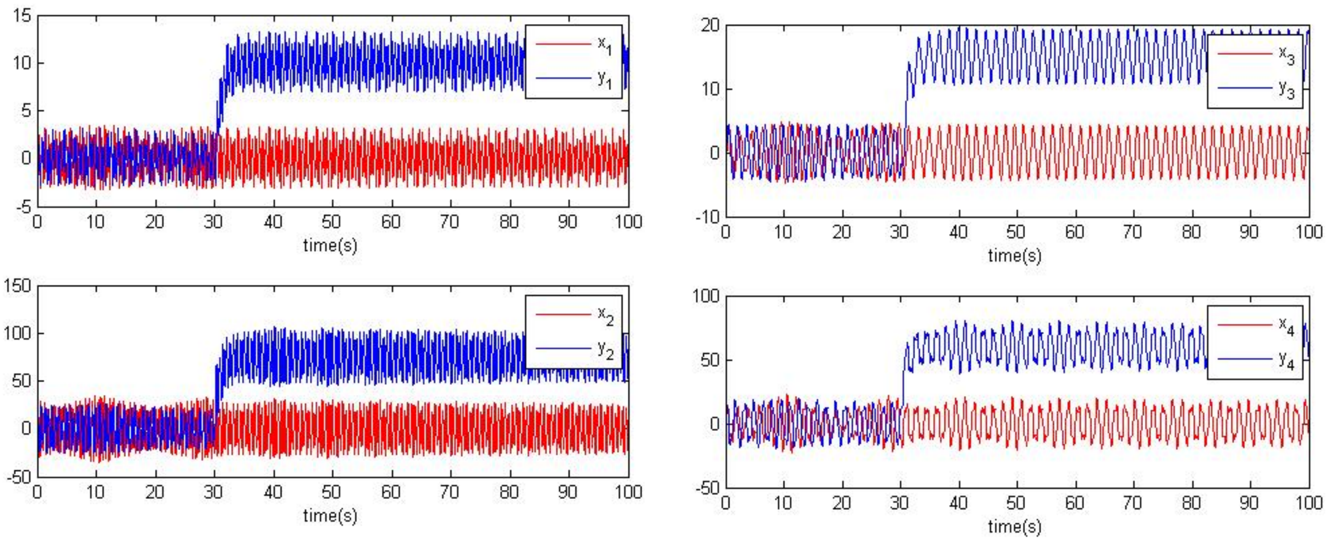


Fig. 5-5. Time histories of $x_{1,}\ x_{2,}\ x_{3,}\ x_{4,}\ y_{1,}\ y_{2,}\ y_{3,}\ y_4$ for Case I.

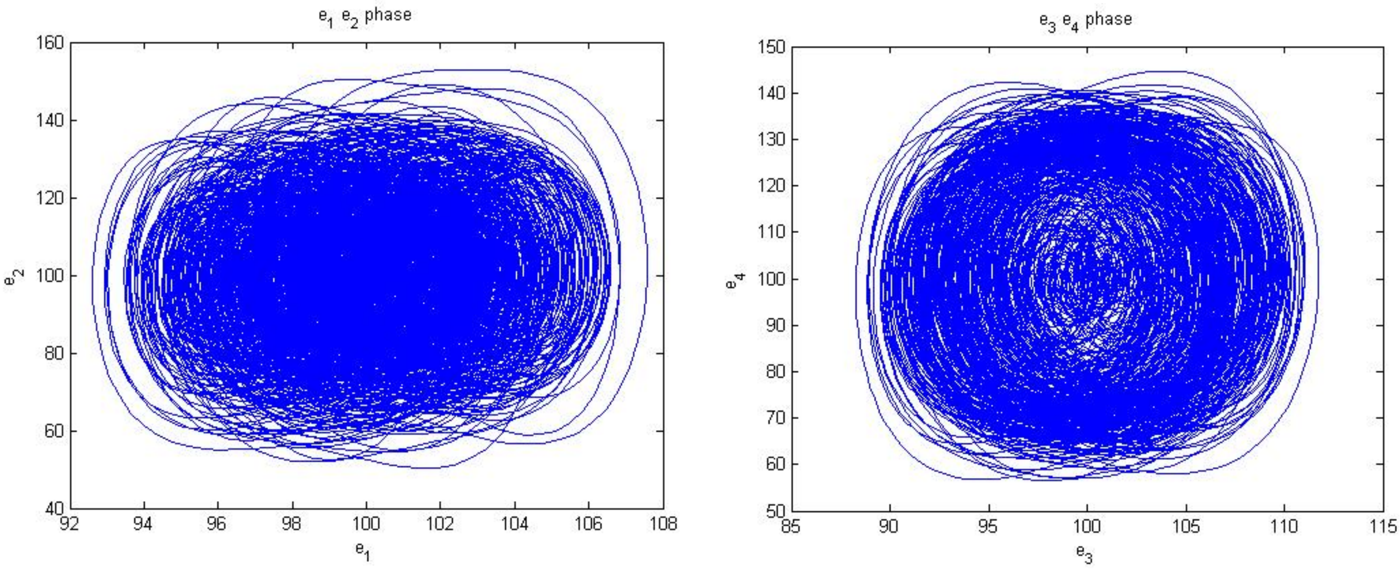


Fig. 5-6. Phase portrait of error dynamics for Case II.

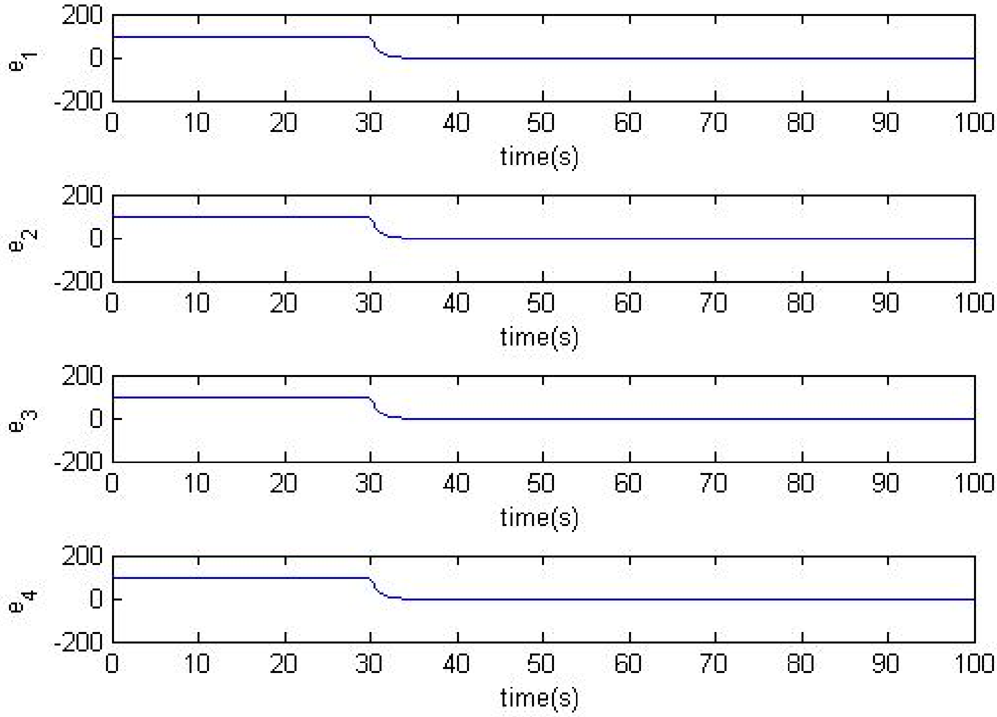


Fig. 5-7. Time histories of errors for Case II.

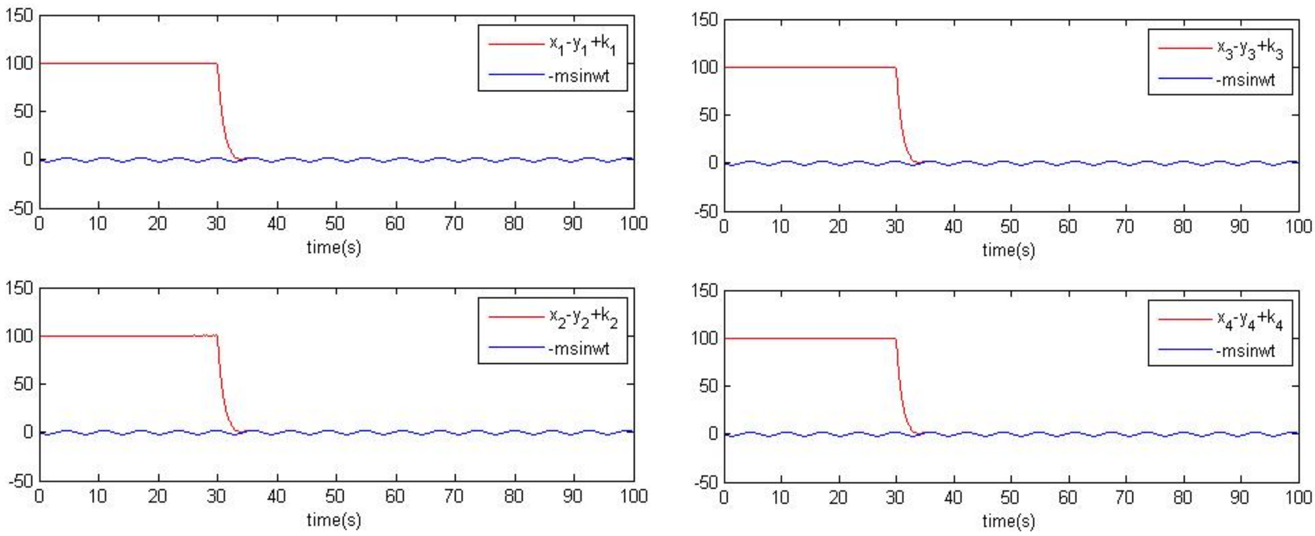


Fig. 5-8. Time histories of $x_i - y_i + k_i$ and $-m\sin wt$ for Case II.

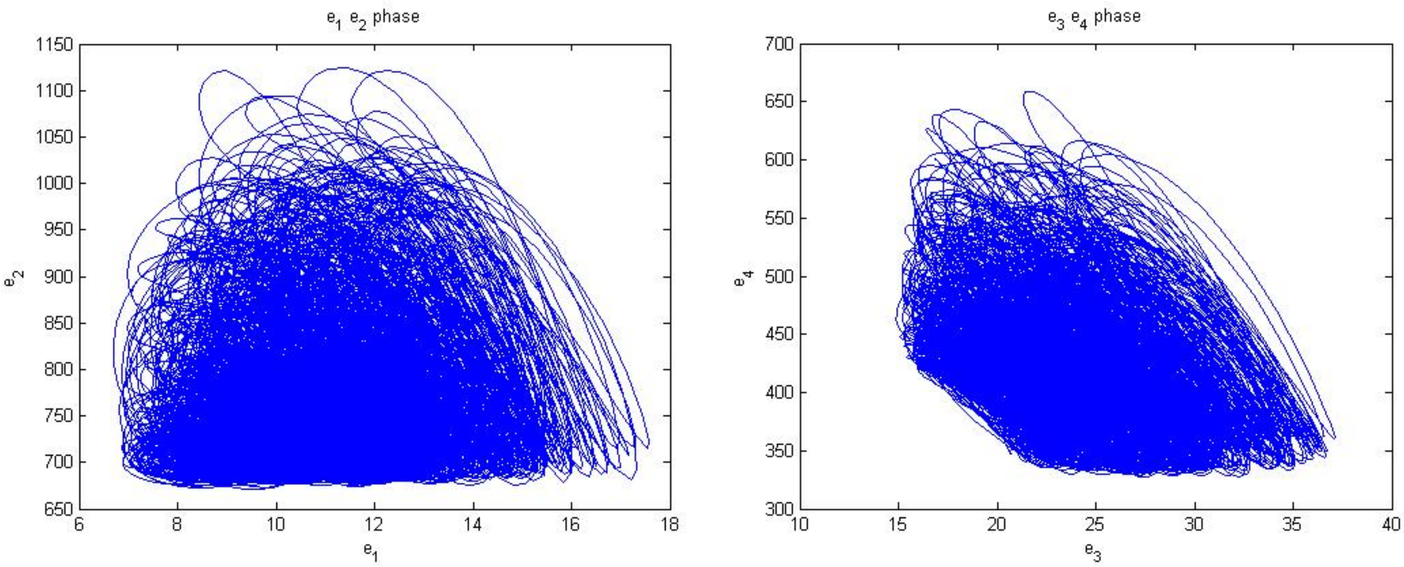


Fig. 5-9. Phase portrait of error dynamics for Case III.

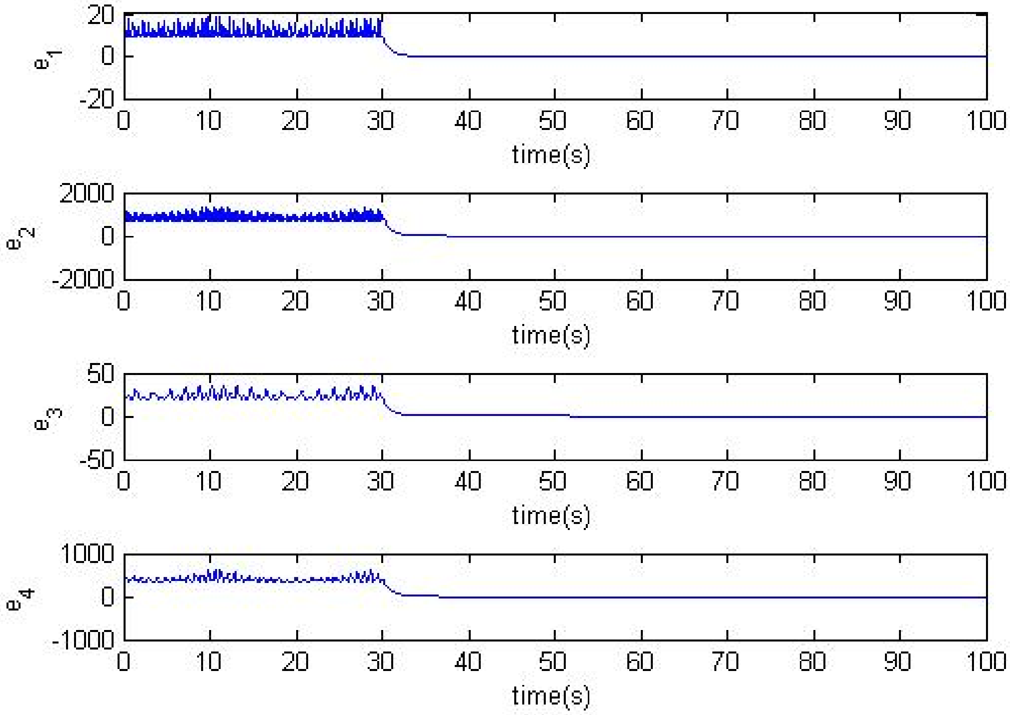


Fig. 5-10. Time histories of errors for Case III.

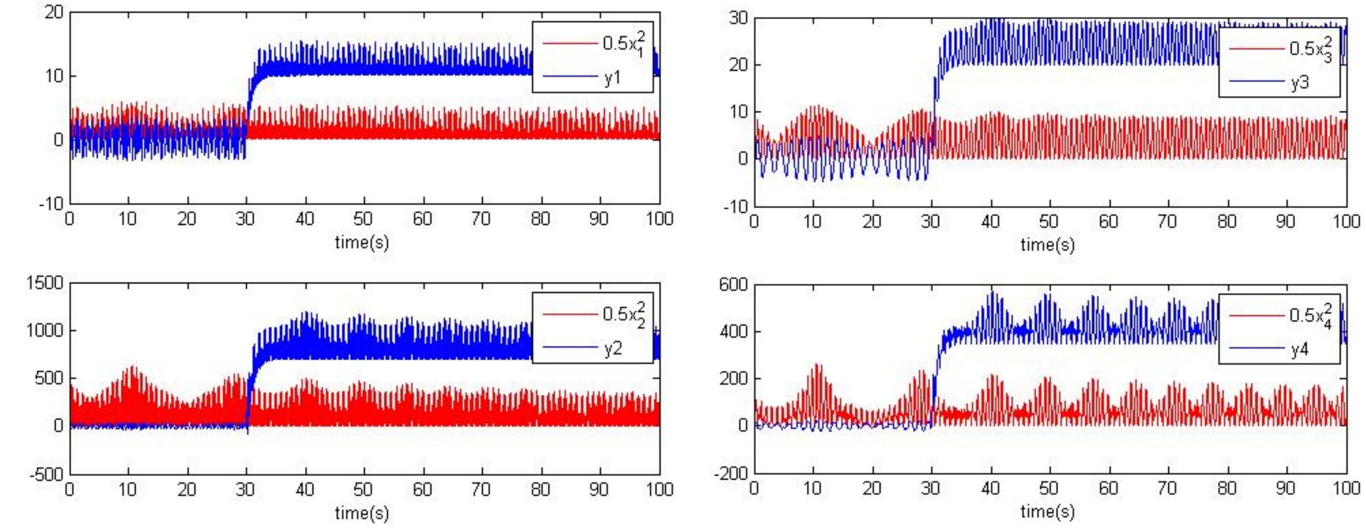


Fig. 5-11. Time histories of $\frac{1}{2}x_i^2$ and y_i for Case III.

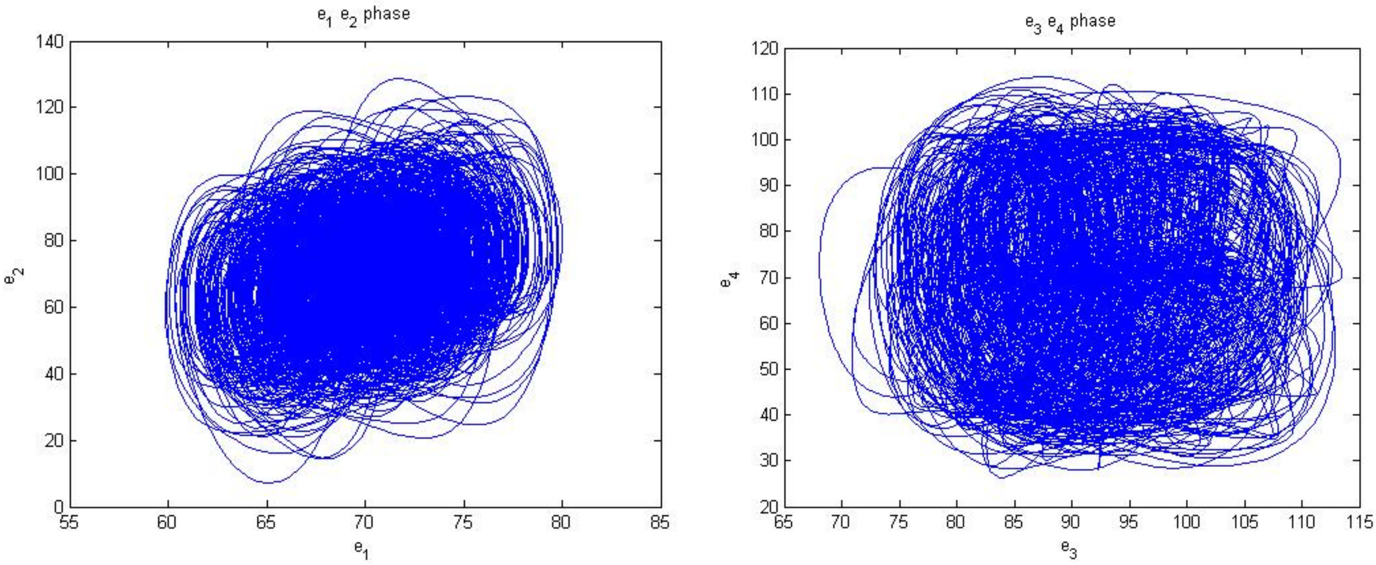


Fig. 5-12. Phase portrait of error dymanics for Case IV.

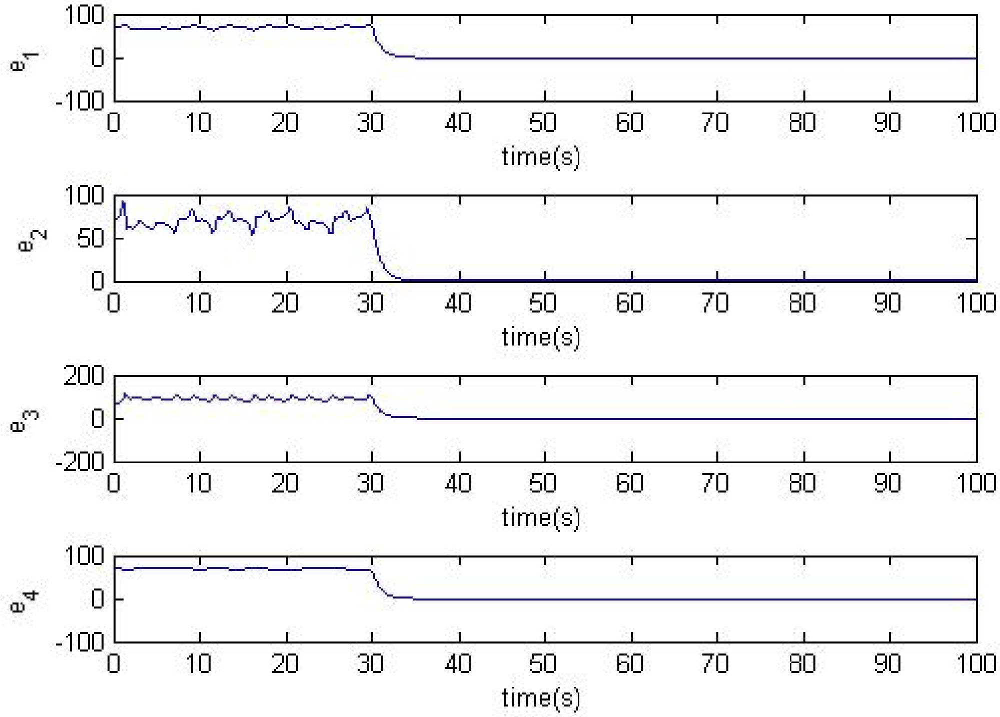


Fig. 5-13. Time histories of errors for Case IV.

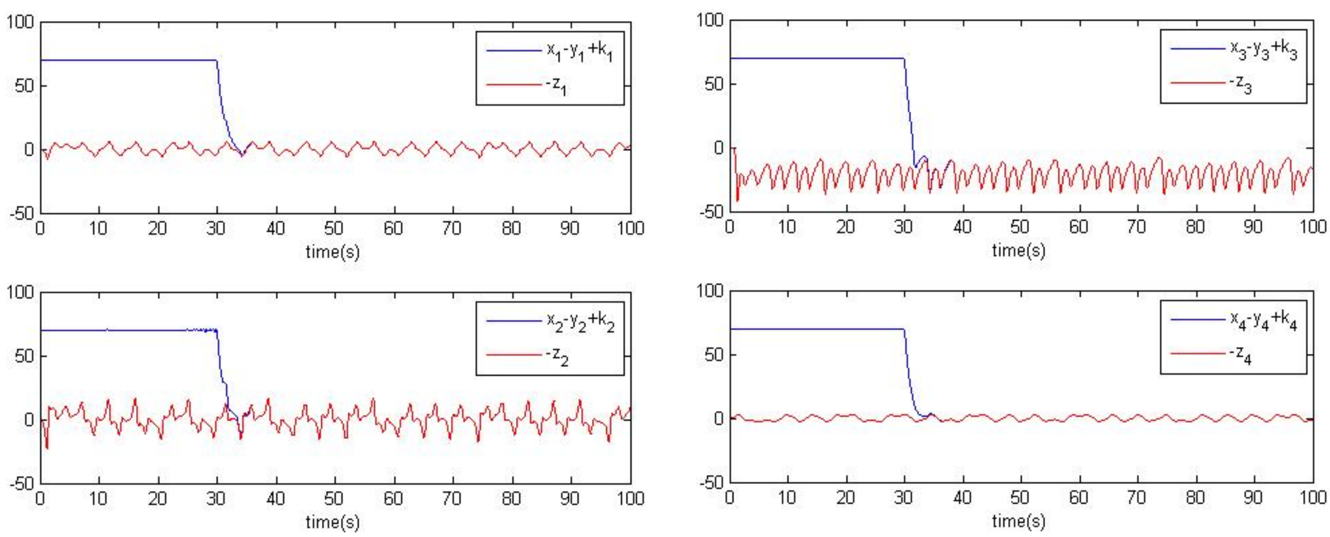


Fig. 5-14. Time histories of $x_i - y_i + k_i$ and $-z_i$ for Case IV.

Chapter 17

Pragmatical Chaotic Symplectic Synchronization of New Duffing-Van der Pol Systems with Different Order System as Functional System by New Dynamic Surface Control and Adaptive Control

17.1 Preliminaries

A new type of chaotic synchronization, *pragmatical chaotic symplectic synchronization* (PCSS), is obtained with the state variables of another different order system as a constituent of the functional relation between 〝master〞 and 〝slave〞. A new Duffing-Van der Pol system is used as 〝master〞 system and 〝slave〞 system. Especially, the traditional generalized synchronizations are special cases of the 〝symplectic synchronization〞. Based on the GYC pragmatical asymptotical stability theorem, new dynamic surface control (NDSC), and adaptive control, the synchronization is achieved. Numerical simulations are provided to verify the effectiveness of the proposed scheme.

17.2 Pragmatical Chaotic Symplectic Synchronization Scheme

There are two identical nonlinear chaotic dynamical systems, and the 〝master〞 system controls the 〝slave〞 system partly. In symplectic synchronization, the 〝master〞 system is called partner A:

*The term **"Symplectic"** comes from the Greek for ''intertwined''. H. Weyl first introduced the term in 1939 in his book "The Classical Groups"(P. 165 in both the first edition, 1939, and second edition, 1946, Princeton University Press)

$$\dot{x} = Ax + f(x, B) \tag{17-2-1}$$

where $x = [x_1, x_2, \cdots x_n]^T \in R^n$ denotes a state vector, *A* is an $n \times n$ uncertain constant coefficient matrix, *f* is a nonlinear vector function, and *B* is a vector of uncertain constant coefficients in *f*. The 〝slave〞 system is called partner B:

$$\dot{y} = \hat{A}y + f(y, \hat{B}) \tag{17-2-2}$$

where $y = [y_1, y_2, \cdots y_n]^T \in R^n$ denotes a state vector, $\hat{A}$ is an $n \times n$ estimated coefficient matrix, $\hat{B}$ is a vector of estimated coefficients in *f*. With controllers, partner B becomes

$$\dot{y} = \hat{A}y + f(y, \hat{B}) + u(t) \tag{17-2-3}$$

where $u(t) = [u_1(t), u_2(t), \cdots u_n(t)]^T \in R^n$ is a control input vector. The chaotic system which affords chaotic *F(t)* vector, is called functional system. However, the PCSS also can be achieved even the order of functional system is different from that of partners A and B. Now we choose the order of the former is less than the latter. The augmented functional system can be easily obtained as shown in Section 3. The augmented functional system becomes

$$\dot{F} = CF + g(F) \tag{17-2-4}$$

where $F = [F_1, F_2, \cdots F_n]^T \in R^n$ denotes a state vector, *C* is an $n \times n$ constant coefficient matrix, *g* is a nonlinear vector function. PCSS demands:

$$y = H(x, y, t) + F(t) \tag{17-2-5}$$

where $H(x, y, t)$ consists of state vector *x* of partner A and state vector *y* of partner B. Our goal is to accomplish Eq. (6) via controller *u(t)*, parameter update dynamics, and new dynamic surface control (NDSC). Define the error vector *e*:

$$e = H(x, y, t) - y + F(t) \tag{17-2-6}$$

The synchronization is achieved when

$$\lim_{t \to \infty} e_i = 0 \qquad (i = 1, 2, ..., n) \tag{17-2-7}$$

The error dynamics is

$$\dot{e} = \frac{\partial H}{\partial x}\dot{x} + \frac{\partial H}{\partial y}\dot{y} + \frac{\partial H}{\partial t} - \dot{y} + \dot{F}(t) \tag{17-2-8}$$

By Eqs (17-2-2) ~ (17-2-5), Eq. (17-2-9) becomes

$$\begin{aligned}\dot{e} = &\frac{\partial H}{\partial x}\left[Ax + f\left(x,B\right)\right] + \frac{\partial H}{\partial y}\left[\hat{A}y + f\left(y,B\right)\right] + \frac{\partial H}{\partial t} \\ &- \hat{A}y - f\left(y,\hat{B}\right) - u\left(t\right) + CF + g\left(F\right)\end{aligned} \tag{17-2-9}$$

In order to reduce terms of the u(t), NDSC is used which makes u(t) more simple. This method extends the traditional dynamic surface control [30]. A virtual controller $\overline{W}$ is chosen as follows

$$\overline{W} = H(x, y, t) + F(t) \tag{17-2-10}$$

Then

$$m\dot{W} + W = \overline{W}, \qquad \lim_{t\to\infty} W\left(t\right) = \lim_{t\to\infty} \overline{W}\left(t\right) \tag{17-2-11}$$

where *m(0)* is a constant. The *m(t)* is a bounded function of time and approaches to zero as follows

$$\lim_{t\to\infty} m\left(t\right) = 0 \tag{17-2-12}$$

Define the boundary layer errors as

$$s = W - \overline{W} \tag{17-2-13}$$

Its derivative is

$$\dot{s} = \frac{-s}{m} - \dot{\overline{W}} \tag{17-2-14}$$

By NDSC, Eq. (17-2-6) and Eq. (17-2-9) becomes

$$e = W - y \tag{17-2-15}$$

$$\dot{e} = \dot{W} - \hat{A}y - f\left(y,\hat{B}\right) - u\left(t\right) \tag{17-2-16}$$

A Lyapunov function $V(e, s, \tilde{A}_c, \tilde{B}_c)$ is chosen as a positive definite function of e, s, $\tilde{A}_c$, $\tilde{B}_c$:

$$V(e, s, \tilde{A}_c, \tilde{B}_c) = \frac{1}{2}e^T e + \frac{1}{2}s^T s + \frac{1}{2}\tilde{A}_c^{\,T}\tilde{A}_c + \frac{1}{2}\tilde{B}_c^{\,T}\tilde{B}_c \tag{17-2-17}$$

where $\tilde{A}=A-\hat{A}$, $\tilde{B}=B-\hat{B}$, $\tilde{A}_c$ and $\tilde{B}_c$ are two vectors whose elements are all the elements of matrix $\tilde{A}$ and that of matrix $\tilde{B}$, respectively. Its time derivative along any solution of Eq. (15), Eq. (17), and update parameter differential equations for $\tilde{A}_c$ and $\tilde{B}_c$ is $\dot{V}$. Choose *u(t)*, *m(t)*, $\dot{\tilde{A}}_c$ and $\dot{\tilde{B}}_c$ so that

$$\dot{V} = e^T Pe + s^T Qs \tag{17-2-18}$$

where *P* and *Q* are diagonal negative definite matrixs, and $\dot{V}$ is a negative semi-definite function of $e, s, \tilde{A}_c, \tilde{B}_c$. In the current scheme of adaptive synchronization [21-25], the traditional Lyapunov stability theorem and Babalat lemma are used to prove that the error vector approaches zero, as time approaches infinity. But the question of why the estimated parameters also approach uncertain parameters remains unanswered. By the GYC pragmatical asymptotical stability theorem, the question can be answered strictly. The equilibrium point $e = s = \tilde{A} = \tilde{B} = 0$ is pragmatically asymptotically stable (see Appendix). Under the assumption of equal probability, it is actually asymptotically stable. Hence, the PCSS can be achieved.

17.3 Numerical Results for the PCSS by New Dynamic Surface Control and Adaptive Control

Since the partner A, a new Duffing-Van der Pol system, is described as

$$\begin{cases} \dfrac{dx_1}{dt} = x_2 \\ \dfrac{dx_2}{dt} = -x_1 - x_1^3 - ax_2 + dx_3 \\ \dfrac{dx_3}{dt} = x_4 \\ \dfrac{dx_4}{dt} = -bx_3 + c\left(1 - x_3^2\right)x_4 + fx_1 \end{cases} \tag{17-2-19}$$

where *a, b, c, d, f* are uncertain parameters. When *a=0.01, b=1, c=5, d=0.67, f=0.05*, chaotic dynamics of this new system is shown in Fig 17-1.The partner B is described as

$$\begin{cases} \frac{dy_1}{dt} = y_2 \\ \frac{dy_2}{dt} = -y_1 - y_1^3 - \hat{a}y_2 + \hat{d}y_3 \\ \frac{dy_3}{dt} = y_4 \\ \frac{dy_4}{dt} = -\hat{b}y_3 + \hat{c}\left(1 - y_3^2\right)y_4 + \hat{f}y_1 \end{cases} \tag{17-2-20}$$

where $\hat{a}$, $\hat{b}$, $\hat{c}$, $\hat{d}$ and $\hat{f}$ are estimated parameters.

For this scheme, u_1, u_2, u_3 and u_4 are added to the partner B then becomes controlled partner B:

$$\begin{cases} \frac{dy_1}{dt} = y_2 + u_1 \\ \frac{dy_2}{dt} = -y_1 - y_1^3 - \hat{a}y_2 + \hat{d}y_3 + u_2 \\ \frac{dy_3}{dt} = y_4 + u_3 \\ \frac{dy_4}{dt} = -\hat{b}y_3 + \hat{c}\left(1 - y_3^2\right)y_4 + \hat{f}y_1 + u_4 \end{cases} \tag{17-2-21}$$

The chaotic Lü system is chosen as functional system [31] and the augmented state variable is $z_4 = z_1^2$:

$$\begin{cases} \frac{dz_1}{dt} = g\left(z_2 - z_1\right) \\ \frac{dz_2}{dt} = -z_1 z_3 + h z_2 \\ \frac{dz_3}{dt} = z_1 z_2 - k z_3 \\ \frac{dz_4}{dt} = 2g z_1\left(z_2 - z_1\right) \end{cases} \tag{17-2-22}$$

In the PCSS, we select the

$$H_i\left(x, y, t\right) = \left(-x_i\right)^j y_i + z_i^j \qquad \begin{cases} i = 1, 2, \cdots, n \\ j = \begin{cases} 2, & i = even \\ 3, & i = odd \end{cases} \end{cases} \tag{17-2-23}$$

Now n=4. By dynamic surface control, the error dynamics Eq. (17-2-16)becomes:

$$\begin{cases} \dot{e}_1 = \dot{W}_1 - y_2 - u_1 \\ \dot{e}_2 = \dot{W}_2 + y_1 + y_1^3 + \hat{a}y_2 - \hat{d}y_3 - u_2 \\ \dot{e}_3 = \dot{W}_3 - y_4 - u_3 \\ \dot{e}_4 = \dot{W}_4 + \hat{b}y_3 - \hat{c}\left(1 - y_3^2\right)y_4 - \hat{f}y_1 - u_4 \end{cases} \tag{17-2-24}$$

and the boundary layer error dynamics Eq. (17-2-14) becomes:

$$\begin{cases} \dot{s}_1 = \dfrac{-s_1}{m_1} - \left[-3x_1^2 x_2 y_1 - x_1^3 y_2 + 3gz_1^2\left(z_2 - z_1\right)\right] \\ \dot{s}_2 = \dfrac{-s_2}{m_2} - \Big[-2x_2 y_2\left(-x_1 - x_1^3 - ax_2 + dx_3\right) - x_2^2\left(-y_1 - y_1^3 - \hat{a}y_2 + \hat{d}y_3\right) \\ \qquad +2z_2\left(-z_1 z_3 + hz_2\right)\Big] \\ \dot{s}_3 = \dfrac{-s_3}{m_3} - \left[-3x_3^2 x_4 y_3 - x_3^3 y_4 + 3z_3^2\left(z_1 z_2 - kz_3\right)\right] \\ \dot{s}_4 = \dfrac{-s_4}{m_4} - \Big[-2x_4 y_4\left(-bx_3 + c\left(1 - x_3^2\right)x_4 + fx_1\right) - x_4^2\left(-\hat{b}y_3 + \hat{c}\left(1 - y_3^2\right)y_4 + \hat{f}y_1\right) \\ \qquad +4gz_1 z_4\left(z_2 - z_1\right)\Big] \end{cases} \tag{17-2-25}$$

Choose a positive definite Lyapunov function for e_1, e_2, e_3, e_4, s_1, s_2, s_3, s_4, $\tilde{a}$, $\tilde{b}$, $\tilde{c}$, $\tilde{d}$, $\tilde{f}$:

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + s_1^2 + s_2^2 + s_3^2 + s_4^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2 + \tilde{f}^2) \tag{17-2-26}$$

where $\tilde{a} = (a - \hat{a})$, $\tilde{b} = (b - \hat{b})$, $\tilde{c} = (c - \hat{c})$, $\tilde{d} = (d - \hat{d})$, and $\tilde{f} = (f - \hat{f})$. We select controllers, estimated parameter dynamics, and *m(t)* as:

$$\begin{cases} u_1 = \dot{W}_1 - y_2 + e_1 \\ u_2 = \dot{W}_2 + y_1 + y_1^3 + \hat{a}y_2 - \hat{d}y_3 + e_2 \\ u_3 = \dot{W}_3 - y_4 + e_3 \\ u_4 = \dot{W}_4 + \hat{b}y_3 - \hat{c}\left(1 - y_3^2\right)y_4 - \hat{f}y_1 + e_4 \end{cases} \tag{17-2-27}$$

$$\begin{cases} \dot{\hat{a}} = -2x_2^2 y_2 s_2 \\ \dot{\hat{b}} = -2x_3 x_4 y_4 s_4 \\ \dot{\hat{c}} = 2x_4^2 y_4\left(1 - x_3^2\right)s_4 \\ \dot{\hat{d}} = 2x_2 x_3 y_2 s_2 \\ \dot{\hat{f}} = 2x_1 x_4 y_4 s_4 \end{cases} \tag{17-2-28}$$

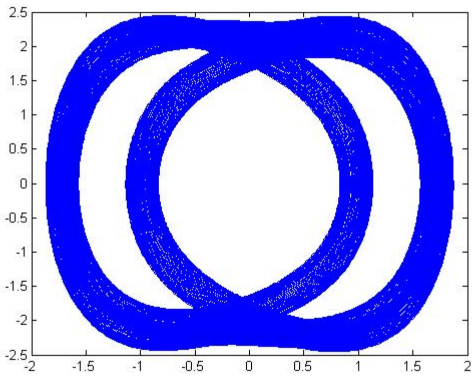
$$\begin{cases} m_1 = \dfrac{-s_1}{-s_1 - 3x_1^2 x_2 y_1 - x_1^3 y_2 + 3gz_1^2\left(z_2 - z_1\right)} \\ m_2 = \dfrac{-s_2}{-s_2 - 2x_2 y_2\left(-x_1 - x_1^3 - \hat{a}x_2 + \hat{d}x_3\right) - x_2^2\left(-y_1 - y_1^3 - \hat{a}y_2 + \hat{d}y_3\right) + 2z_2\left(-z_1 z_3 + hz_2\right)} \\ m_3 = \dfrac{-s_3}{-s_3 - 3x_3^2 x_4 y_3 - x_3^3 y_4 + 3z_3^2\left(z_1 z_2 - kz_3\right)} \\ m_4 = \dfrac{-s_4}{-s_4 - 2x_4 y_4\left(-\hat{b}x_3 + \hat{c}\left(1 - x_3^2\right)x_4 + \hat{f}x_1\right) - x_4^2\left(-\hat{b}y_3 + \hat{c}\left(1 - y_3^2\right)y_4 + \hat{f}y_1\right) + 4gz_1 z_4\left(z_2 - z_1\right)} \end{cases} \tag{17-2-29}$$

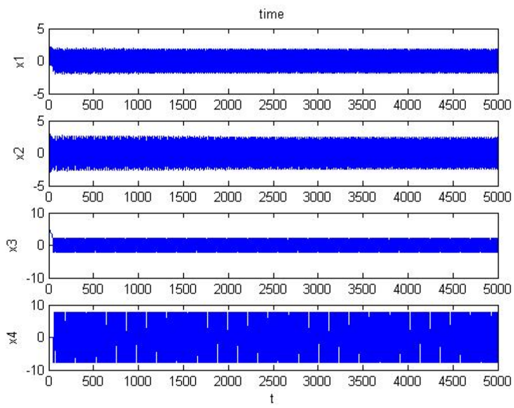
The time derivative of V is

$$\dot{V} = -e_1^2 - e_2^2 - e_3^2 - e_4^2 - s_1^2 - s_2^2 - s_3^2 - s_4^2 \qquad \le 0 \tag{17-2-30}$$

which is negative semi-definite function for $e_1, e_2, e_3, e_4, s_1, s_2, s_3, s_4, \tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}, \tilde{f}$. The Lyapunov asymptotical stability theorem cannot be satisfied in this case. The common origin of error dynamics, parameter update dynamics, and boundary layer error dynamics cannot be concluded to be asymptotically stable. By GYC pragmatical asymptotical stability theorem, *D* is a 13-manifold, *n* = 13 and the number of error state variables *p* = 8. When $e_1 = e_2 = e_3 = e_4 = s_1 = s_2 = s_3 = s_4 = 0$ and $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}, \tilde{f}$ take arbitrary values , $\dot{V} = 0$, so *X* is a 5-manifold, *m* = *n* - *p* = 13 – 8 = 5. $m + 1 \le n$ are satisfied. By the GYC pragmatical asymptotical stability theorem, the common origin of error dynamics (25), boundary layer error dynamics (26), and parameter dynamics (29) are asymptotically stable. The equilibrium point $e_1 = e_2 = e_3 = e_4 = s_1 = s_2 = s_3 = s_4 = \tilde{a} = \tilde{b} = \tilde{c} = \tilde{d} = \tilde{f} = 0$ is pragmatically asymptotically stable. The PCSS is achieved under this scheme.

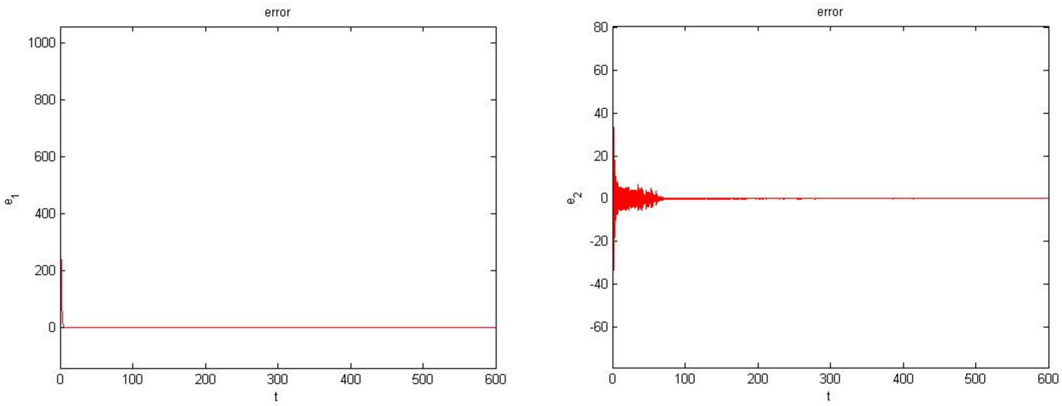
In this numerical simulation, we select the “unknown” parameter and initial states of the partner A and of functional system as *a=0.01, b=1, c=5, d=0.67, f=0.05, g=36, h=20, k=3* to ensure the chaotic behavior. The initial states of those system are $x_1(0) = 2,\ x_2(0) = 2.4,\ x_3(0) = 5,\ x_4(0) = 6,\ y_1(0) = 5,\ y_2(0) = 5,\ y_3(0) = 10,$ $y_4(0) = 10,\ z_1(0) = z_2(0) = z_3(0) = z_4(0) = 10$. The estimated parameters have initial conditions $\hat{a}(0) = \hat{b}(0) = \hat{c}(0) = \hat{d}(0) = \hat{f}(0) = 0$. The numerical results are shown in Fig. 17-2 ~ Fig. 17-5.



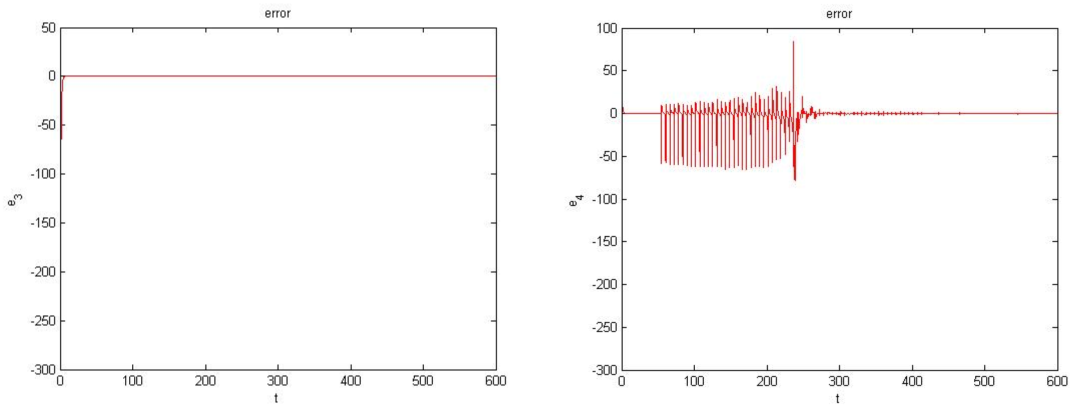


(b)

Fig. 17-1 Duffing-Van der Pol system (a) Phase portraits (b) time histories

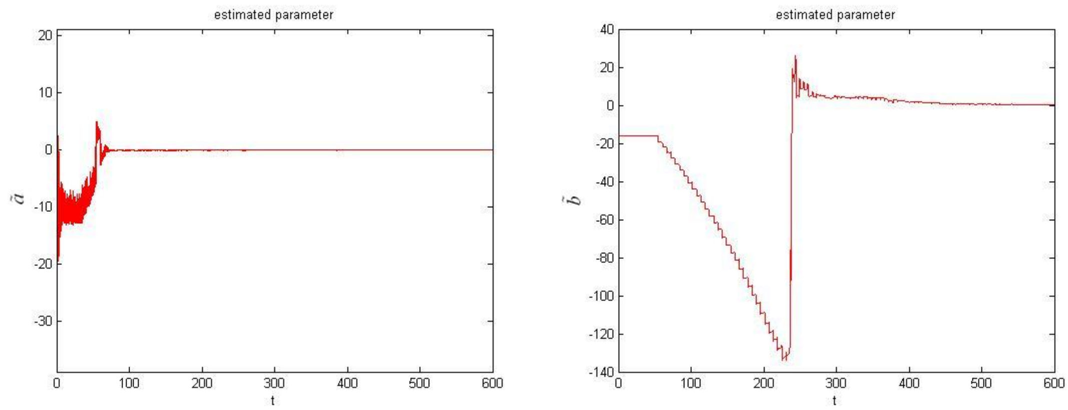


(b)

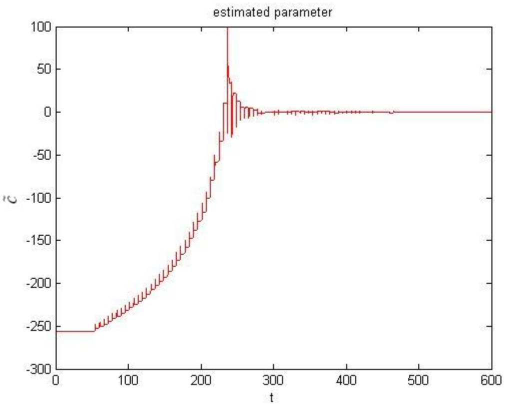


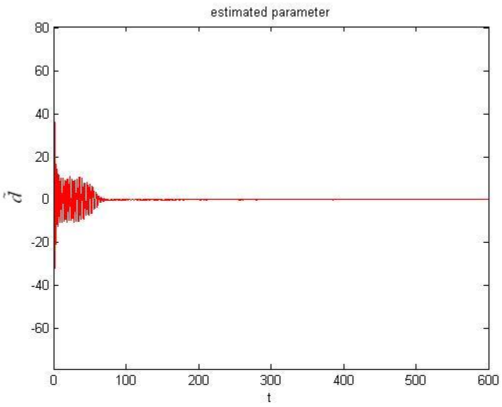
(c) (d)

Fig. 17-2 Time histories of errors.

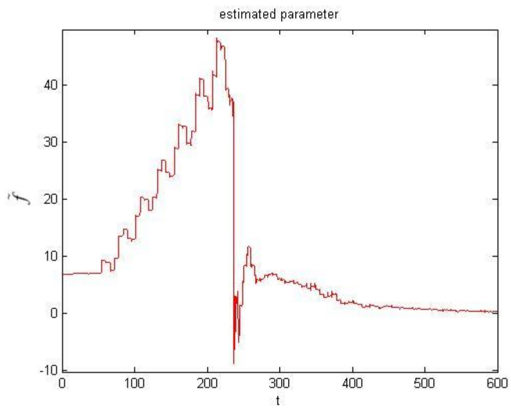


(a) (b)



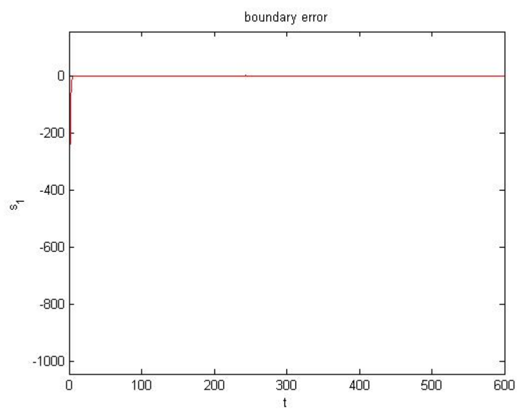


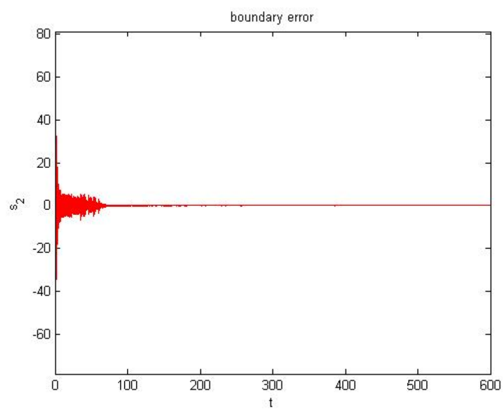
(c) (d)



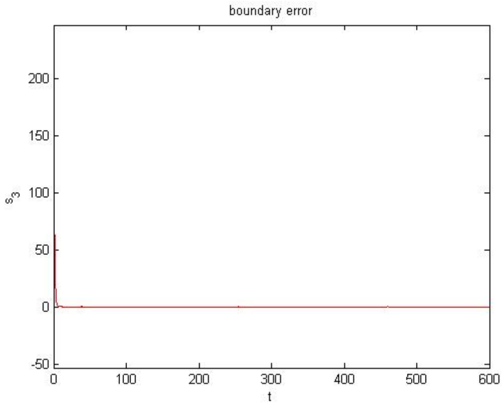
(e)

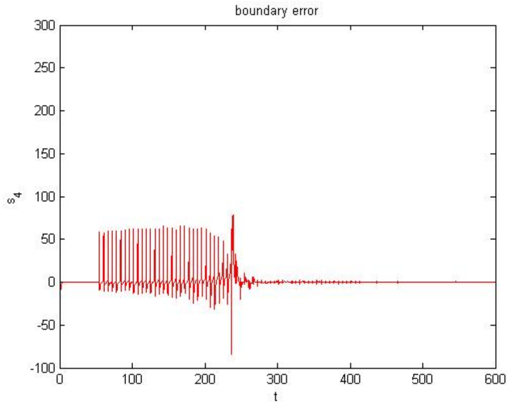
Fig. 17-3 Time histories of the differences of uncertain parameters and estimated parameters.





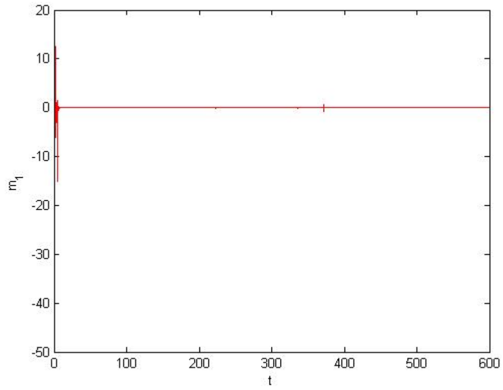
(a) (b)

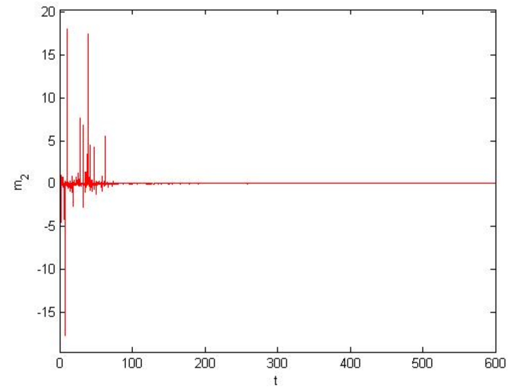




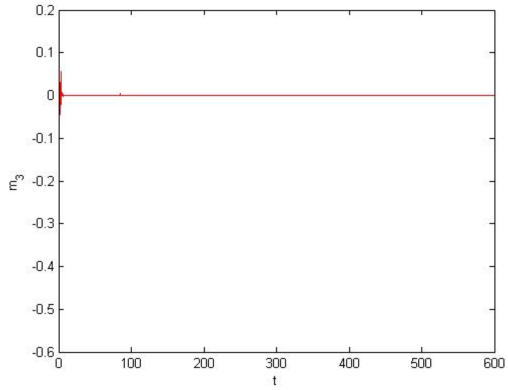
(c) (d)

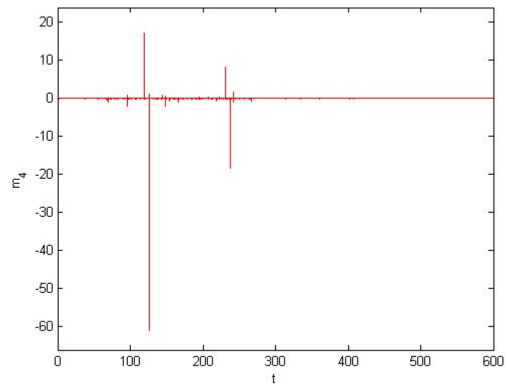
Fig. 17-4 Time histories of boundary layer errors.





(a) (b)





(c) (d)

Fig. 17-5 Time histories of *m(t)* which is a bounded function of time and approaches to zero.

Chapter 18

Hybrid Projective Synchronization of Hyperchaotic Tachometer systems by Backstepping Control

18.1 Preliminaries

In this paper, the hyperchaotic dynamics of a tachometer system is studied by means of phase portraits, Poincare maps, bifurcation diagram, power spectra and Lyapunov exponents. Next, backstepping control is used to obtaine the pragmatical hybrid hyperchaotic generalized synchronization of two hyperchaotic tachometer systems. Numerical simulations show that this method works very well.

18.2. Chaos of tachometer system

The tachometer system considered is shown in Fig 1 [17]. The masses of the rods and vertical axis O_1O_2 are neglected, and ball A and B are assumed as particles with equal mass m_1. The vertical axis rotates with constant speed η and is subjected to a vertical vibration $A\sin x_3$ where x_3 is state variable, A is the amplitude of vibration. m_2 is the mass of the sleeve C, l is the length of rod BC, 2l is the length of AB. ϕ is the angle between rod AB and vertical axis O_1O_2, k_1 is the spring constant of a restoring spiral spring which is used to restrain the angle ϕ caused by centrifugal forces of A and B, k_2 is the viscous damping coefficient caused by friction in the bearings. Let $x_1=\phi,\ x_2=\dot{\phi},\ x_4=\dot{x}_3$.

By Lagrange equation, the state equations for the autonomous tachometer system are

$$\begin{cases} \frac{dx_1}{dt} = x_2 \\ \frac{dx_2}{dt} = \frac{1}{2m_1 + 4m_2 \sin^2 x_1} \left(\frac{-2m_2 g \sin x_1}{l} + \frac{2m_2 A \sin x_3 \sin x_1}{l} - 4m_2 x_2^2 \sin x_1 \cos x_1 \right. \\ \quad \left. + 2m_1 \sin x_1 \cos x_1 \eta^2 - \frac{k_1 x_1}{l^2} - \frac{k_2 x_2}{l^2} \right) \\ \frac{dx_3}{dt} = x_4 \\ \frac{dx_4}{dt} = -A \sin x_3 \end{cases} \tag{18-2-1}$$

The third and fourth equation of system (1) give a simple harmonic vibration system. When A=0, at steady state, a given constant η corresponds to a definite ϕ, therefore this system can be used as a tachometer.

Choose m_1=3, m_2=3, g=9.8, l=1.5, A=5, k_1=4, k_2=1. η is used as a variable parameter. η=1 gives period 1 motion, η=1.5 gives period 3 motion, η=4 gives chaotic motion. Taking η as abscissa, the Lyapunov exponents diagram is shown as Fig 2 . Hyperchaos [18] with two positive LE is found. Bifurcation diagram, Phase portraits & Poincare maps, time histories, and power spectra are presented in Fig 3~6.

18.3. Hybrid projective synchronization scheme

The projective synchronization means that the drive and response vectors synchronize up to a scaling factor vector α, i.e. the former two vectors become proportional. When the elements of scaling vector take both positive and negative constants, we have hybrid projective synchronization.

Consider the chaotic systems:

$$\begin{cases} \dot{x}_d = f(x_d) = Ax_d \\ \dot{x}_r = f(x_r) = \hat{A}x_r + u(t) \end{cases} \tag{18-3-1}$$

where n-dimensional state vector x_d, $x_r \in R^n$. The subscripts '*d*' and '*r*' stand for the drive and response systems, respectively. $f : R^n \to R^n$ is vector fields in n-dimensional space. If there exists a constant vector α ($\alpha \neq 0$) such that $\lim_{t\to\infty} \|x_r - \alpha x_d\| = 0$, then hybrid projective synchronization of the system is accomplished, and we call α scaling factor.

Define error vector

$$e = x_r - \alpha x_d \tag{18-3-2}$$

where $e = [e_1, e_2, \cdots, e_n]^T$. Our goal is to design a controller u(t) by backstepping method so that

$$\lim_{t\to\infty} e = 0 \tag{18-3-3}$$

18.4. Hybrid projective synchronization of hyperchaotic tachometer system by backstepping control

The tachometer system is the master system:

$$\begin{cases} \dfrac{d}{dt}x_1 = x_2 \\ \dfrac{d}{dt}x_2 = \dfrac{-k_1x_1}{2m_1l^2} + \cdots + \dfrac{1}{2m_1 + 4m_2\sin^2 x_1}\Big(\dfrac{-2m_2 g\sin x_1}{l} + \dfrac{2m_2 A\sin x_3 \sin x_1}{l} \\ \qquad -4m_2x_2^2 \sin x_1 \cos x_1 + 2m_1 \sin x_1 \cos x_1 \eta^2 - \dfrac{k_2x_2}{l^2}\Big) \\ \dfrac{d}{dt}x_3 = x_4 \\ \dfrac{d}{dt}x_4 = -A\sin x_3 \end{cases} \tag{18-4-1}$$

where x_1, x_2, x_3, x_4 are state variables and k_1, k_2, A, l, g, m_1, m_2 are constants, when $k_1 = 4$, $k_2 = 1$, $m_2 = 3$, m_1=3, A=5, η=4, g=9.8, l=1.5, the system exhibits chaotic behavior .

The slave system is

$$\begin{cases} \dfrac{d}{dt}y_1 = y_2 \\ \dfrac{d}{dt}y_2 = \dfrac{-k_1y_1}{2m_1l^2} + \cdots + \dfrac{1}{2m_1 + 4m_2\sin^2 y_1}\Big(\dfrac{-2m_2 g\sin y_1}{l} + \dfrac{2m_2 A\sin y_3 \sin y_1}{l} \\ \qquad -4m_2y_2^2 \sin y_1 \cos y_1 + 2m_1 \sin y_1 \cos y_1 \eta^2 - \dfrac{k_2y_2}{l^2}\Big) \\ \dfrac{d}{dt}y_3 = y_4 \\ \dfrac{d}{dt}y_4 = -A\sin y_3 \end{cases} \tag{18-4-2}$$

where y_1, y_2, y_3, y_4 are state variables.

In order to lead (y_1, y_2, y_3, y_4) to (a_1x_1, b_1x_2, c_1x_3, d_1x_4), add u_1, u_2, u_3, u_4 as controllers in Eq (18-2-6):

$$\begin{cases} \frac{d}{dt}y_1 = y_2 + u_1 \\ \frac{d}{dt}y_2 = \frac{-k_1y_1}{2m_1l^2} + \cdots + \frac{1}{2m_1 + 4m_2\sin^2 y_1}(\frac{-2m_2g\sin y_1}{l} + \frac{2m_2A\sin y_3 \sin y_1}{l} \\ \qquad -4m_2y_2^2\sin y_1\cos y_1 + 2m_1\sin y_1\cos y_1\eta^2 - \frac{k_2y_2}{l^2}) + u_2 \\ \frac{d}{dt}y_3 = y_4 + u_3 \\ \frac{d}{dt}y_4 = -A\sin y_3 + u_4 \end{cases} \quad (18\text{-}4\text{-}3)$$

Define error state vectors as follows:

$$\begin{cases} e_1 = y_1 - a_1x_1 \\ e_2 = y_2 - b_1x_2 \\ e_3 = y_3 - c_1x_3 \\ e_4 = y_4 - d_1x_4 \end{cases} \quad (18\text{-}4\text{-}4)$$

where a_1, b_1, c_1, d_1 are scaling constants, and we choose a_1 =3, b_1=-4, c_1 =6, d_1=-2 in order to get hybrid projective synchronization.

Differentiate Eg (18-2-8) with respect to time, error dynamics is

$$\begin{aligned} \dot{e}_1 &= e_2 - b_1x_2 - a_1x_2 + u_1 \\ \dot{e}_2 &= -\frac{k_1e_1}{2m_1l^2} - \frac{k_1a_1x_1}{2m_1l^2} - \frac{k_1b_1x_1}{2m_1l^2} + \cdots \\ &\quad + \frac{1}{2m_1 + 4m_2\sin^2 y_1}(-\frac{2m_2g\sin y_1}{l} + \frac{2m_2A\sin y_3\sin y_1}{l} - 4m_2y_2^2\sin y_1\cos y_1 \\ &\quad + 2m_1\sin y_1\cos y_1\eta^2 - \frac{k_2y_2}{l^2}) \\ &\quad + \frac{b_1}{2m_1 + 4m_2\sin^2 x_1}(-\frac{2m_2g\sin x_1}{l} + \frac{2m_2A\sin x_3\sin x_1}{l} - 4m_2x_2^2\sin x_1\cos x_1 \\ &\quad + 2m_1\sin x_1\cos x_1\eta^2 - \frac{k_2x_2}{l^2}) + u_2 \\ \dot{e}_3 &= e_4 - d_1x_4 - c_1x_4 + u_3 \\ \dot{e}_4 &= -A\sin y_3 - d_1A\sin x_3 + u_4 \end{aligned} \quad (18\text{-}4\text{-}5)$$

Choose a positive definite Lyapunov function

$$V_1 = \frac{1}{2}e_1^2 \qquad (18\text{-}4\text{-}6)$$

Differentiate Eq (18-2-10) with respect to time, we have:

$$\dot{V}_1 = e_1(e_2 - b_1x_2 - a_1x_2 + u_1) \qquad (18\text{-}4\text{-}7)$$

Choose $u_1 = b_1x_2 + a_1x_2$, and $e_2 = \alpha_1(e_1) = -e_1$, Eq (18-2-11) becomes

$$\dot{V}_1 = -e_1^2 < 0 \qquad (18\text{-}4\text{-}8)$$

e_1=0 is asymptotically stable. When e_2 is considered as a controller, $\alpha_1(e_1)$ is an estimative function, define $W_2 = e_2 - \alpha_1(e_1) = e_2 + e_1$ and its derivative is

$$\dot{W}_2 = \dot{e}_2 + \dot{e}_1 . \qquad (18\text{-}4\text{-}9)$$

Choose a positive definite Lyapunov function

$$V_2 = V_1 + \frac{1}{2}W_2^2 \qquad (18\text{-}4\text{-}10)$$

Then:

$$\dot{V}_2 = \dot{V}_1 + W_2\dot{W}_2 \qquad (18\text{-}4\text{-}11)$$

$$\begin{aligned}
\dot{V}_2 = {} & -e_1^2 + W_2(W_2 - e_1 - \frac{k_1e_1}{2m_1l^2} - \frac{k_1a_1x_1}{2m_1l^2} - \frac{k_1b_1x_1}{2m_1l^2} + \cdots \\
& + \frac{1}{2m_1 + 4m_2\sin^2 y_1}(-\frac{2m_2g\sin y_1}{l} + \frac{2m_2A\sin y_3 \sin y_1}{l} - 4m_2y_2^2\sin y_1\cos y_1 \\
& + 2m_1\sin y_1\cos y_1\eta^2 - \frac{k_2y_2}{l^2}) \\
& + \frac{b_1}{2m_1 + 4m_2\sin^2 x_1}(-\frac{2m_2g\sin x_1}{l} + \frac{2m_2A\sin x_3\sin x_1}{l} \\
& - 4m_2x_2^2\sin x_1\cos x_1 + 2m_1\sin x_1\cos x_1\eta^2 - \frac{k_2x_2}{l^2}) + u_2)
\end{aligned}$$

Choose:

$$\begin{aligned}
u_2 = {} & -2W_2 + e_1 + \frac{k_1e_1}{2m_1l^2} + \frac{k_1a_1x_1}{2m_1l^2} + \frac{k_1b_1x_1}{2m_1l^2} \\
& - \frac{1}{2m_1 + 4m_2\sin^2 y_1}(-\frac{2m_2g\sin y_1}{l} + \frac{2m_2A\sin y_3\sin y_1}{l} - 4m_2y_2^2\sin y_1\cos y_1 \\
& + 2m_1\sin y_1\cos y_1\eta^2 - \frac{k_2y_2}{l^2}) \\
& - \frac{b_1}{2m_1 + 4m_2\sin^2 x_1}(-\frac{2m_2g\sin x_1}{l} + \frac{2m_2A\sin x_3\sin x_1}{l} - 4m2x_2^2\sin x_1\cos x_1 \\
& + 2m_1\sin x_1\cos x_1\eta^2 - \frac{k_2x_2}{l^2})
\end{aligned}$$

Eq (18-2-15) becomes:

$$\dot{V}_2 = -e_1^2 - W_2^2 < 0 \tag{18-4-12}$$

e_2=0 is asymptotically stable.

Choose a positive definite Lyapunov function

$$V_3 = V_1 + V_2 + \frac{1}{2}e_3^2 \tag{18-4-13}$$

Then by the third equation of Eq (18-2-9), we have

$$\begin{aligned}\dot{V}_3 &= \dot{V}_1 + \dot{V}_2 + e_3\dot{e}_3 \\ &= -e_1^2 - W_2^2 + e_3(e_4 - d_1x_4 - c_1x_4 + u_3)\end{aligned} \tag{18-4-14}$$

Choose $u_3 = d_1x_4 + c_1x_4$, and put $e_4 = \alpha_1(e_3) = -e_3$, Eq (18-2-18) becomes

$$\dot{V}_3 = -e_1^2 - W_2^2 - e_3^2 < 0 \tag{18-4-15}$$

e_3=0 is asymptotically stable. When e_4 is considered as a controller, $\alpha_1(e_3)$ is an estimative function, define

$W_4 = e_4 - \alpha_1(e_3) = e_4 + e_3$ and

$$\dot{W}_4 = \dot{e}_4 + \dot{e}_3 \tag{18-4-16}$$

Choose a positive definite Lyapunov function

$$V_4 = V_1 + V_2 + V_3 + \frac{1}{2}W_4^2 \tag{18-4-17}$$

Then by the fourth equation of Eq (18-2-9), we have

$$\begin{aligned}\dot{V}_4 &= \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + W_4\dot{W}_4 \\ &= -e_1^2 - W_2^2 - e_3^2 + W_4(W_4 - e_3 - A\sin y_3 - d_1A\sin x_3 + u_4)\end{aligned} \tag{18-4-18}$$

Choose $u_4 = -2W_4 + e_3 + A\sin y_3 + d_1A\sin x_3$, Eq (18-2-22) becomes :

$$\dot{V}_4 = -e_1^2 - W_2^2 - e_3^2 - W_4^2 < 0 \tag{18-4-19}$$

e_4=0 is asymptotically stable.

Numerical simulations show that the result is satisfactory as shown in Figs 7, 8, 9, and 10.

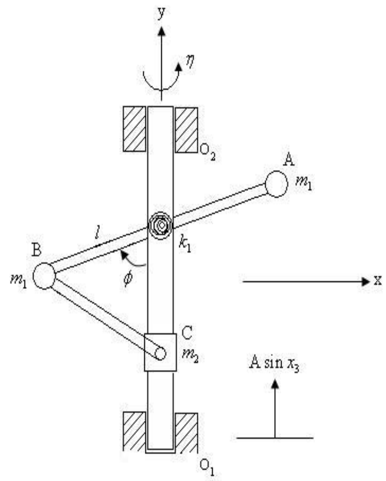


Fig 1 Sketch of a tachometer with vibrating base.

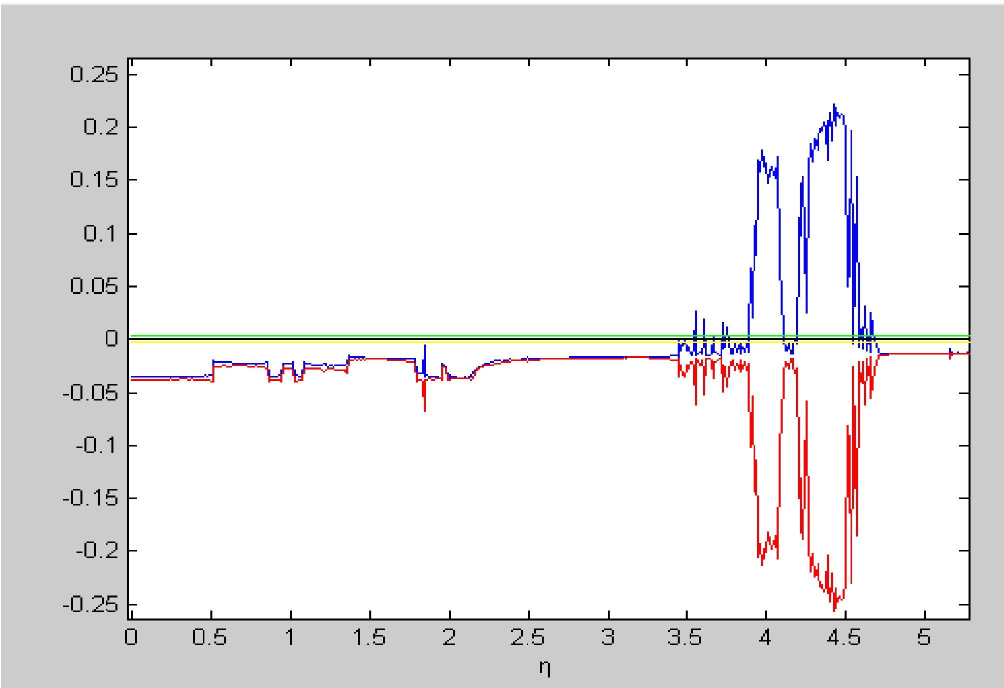


Fig 2 Lyapunov exponents for tachometer system.

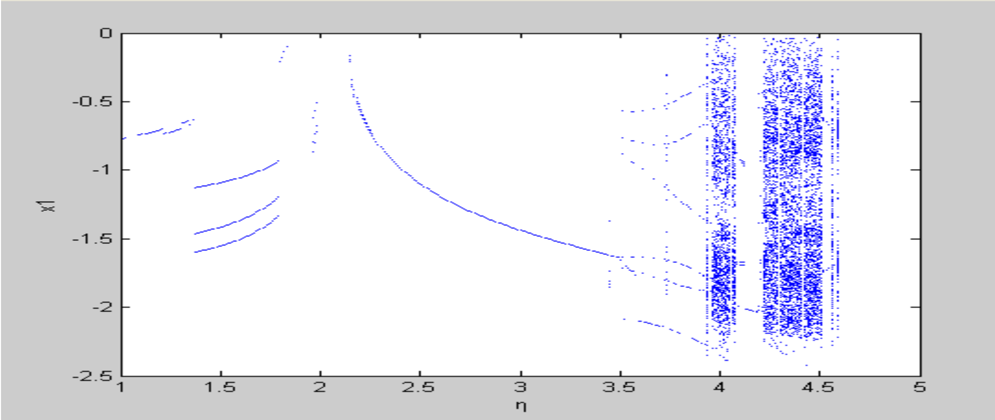
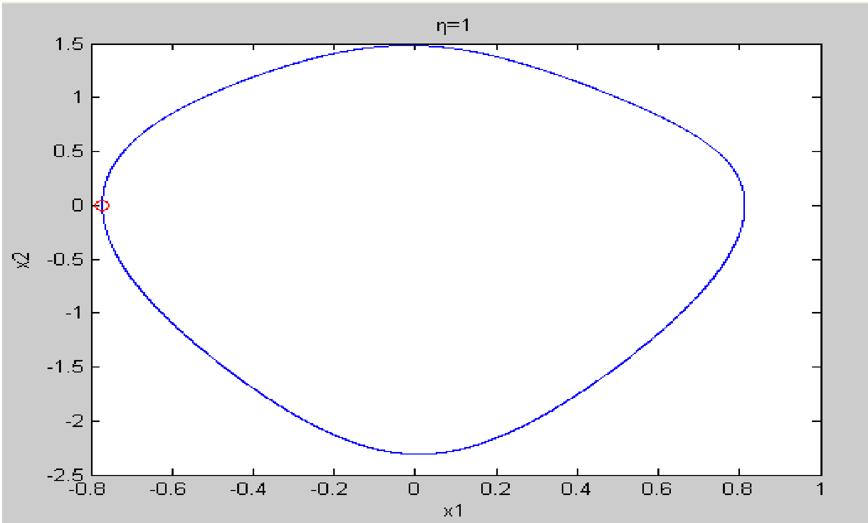
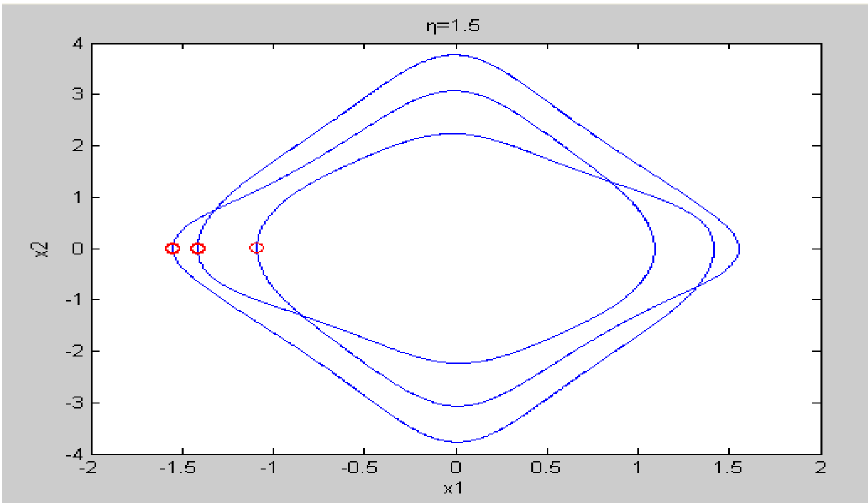


Fig 3 Bifurcation diagram.





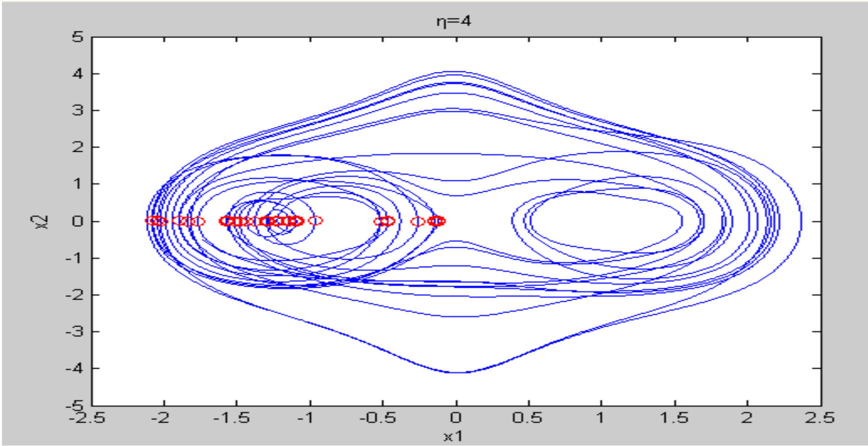
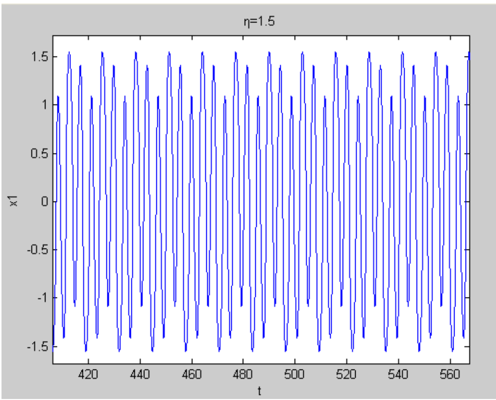
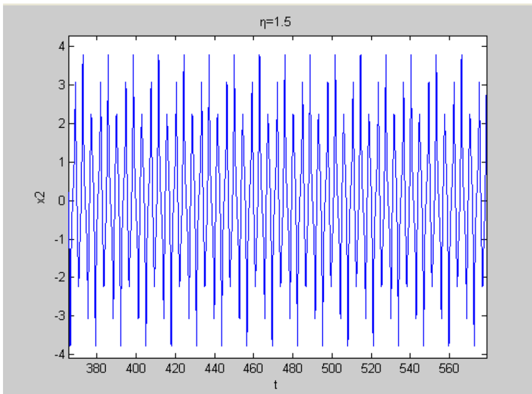
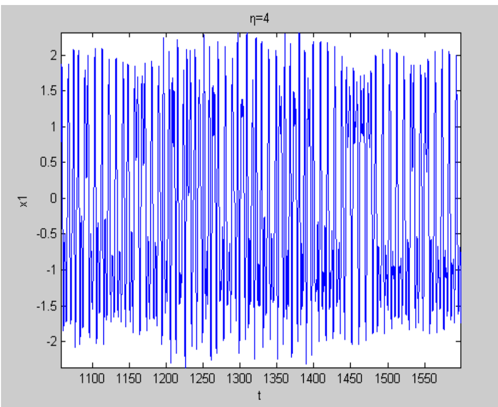


Fig 4 Phase portraits of η =1, η =1.5, η =4, respectively.







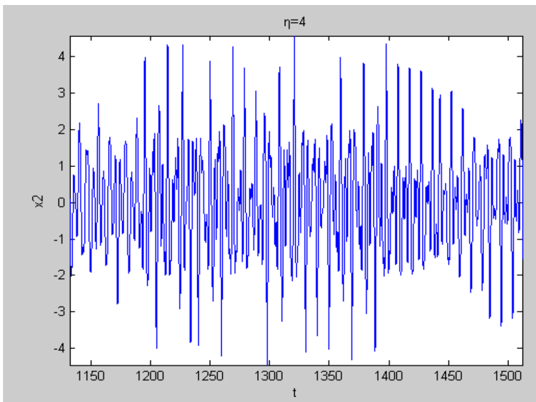


Fig 5 Time histories of state variables for η =1.5, η =4, respectively.

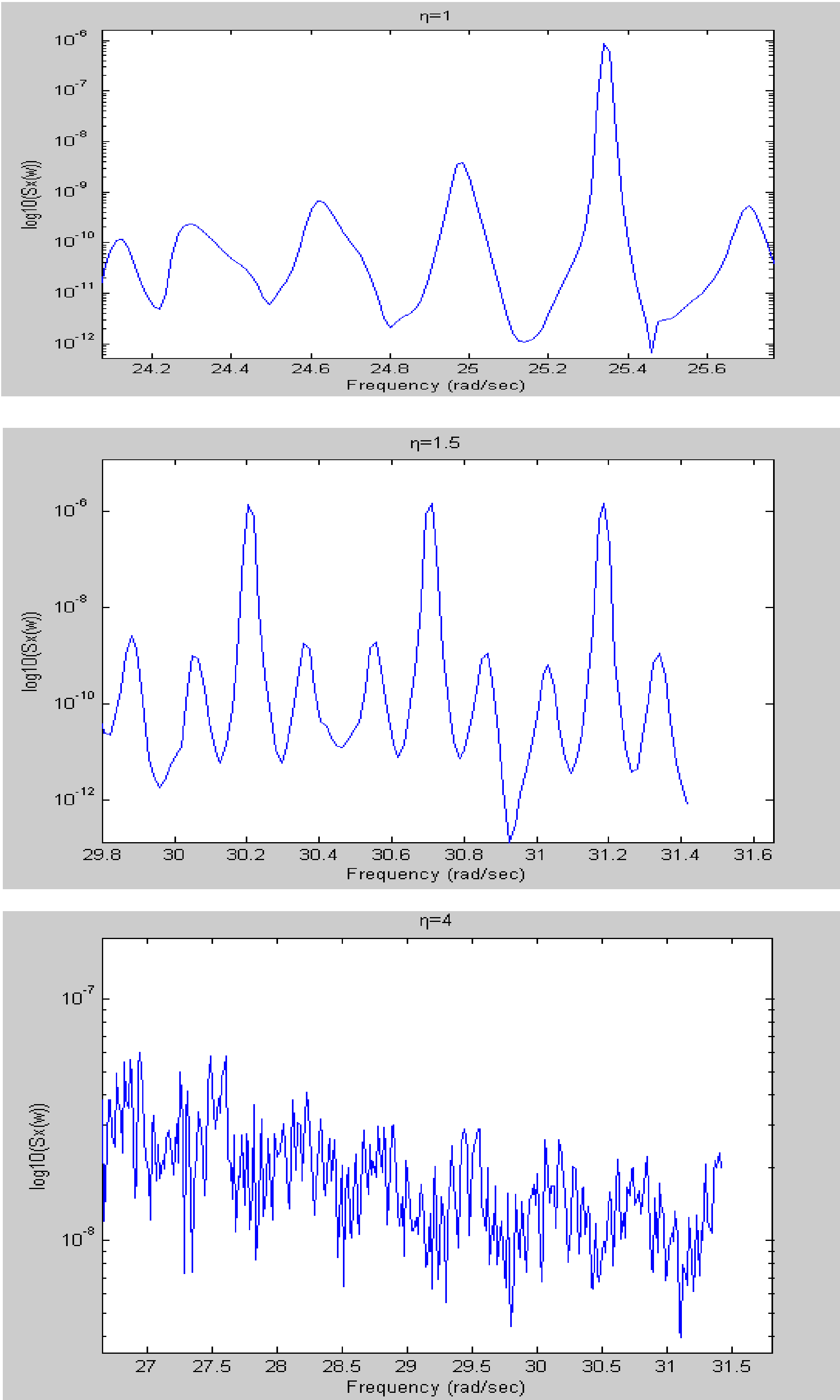


Fig 6 Power spectra for $\eta=1$, $\eta=1.5$, $\eta=4$.

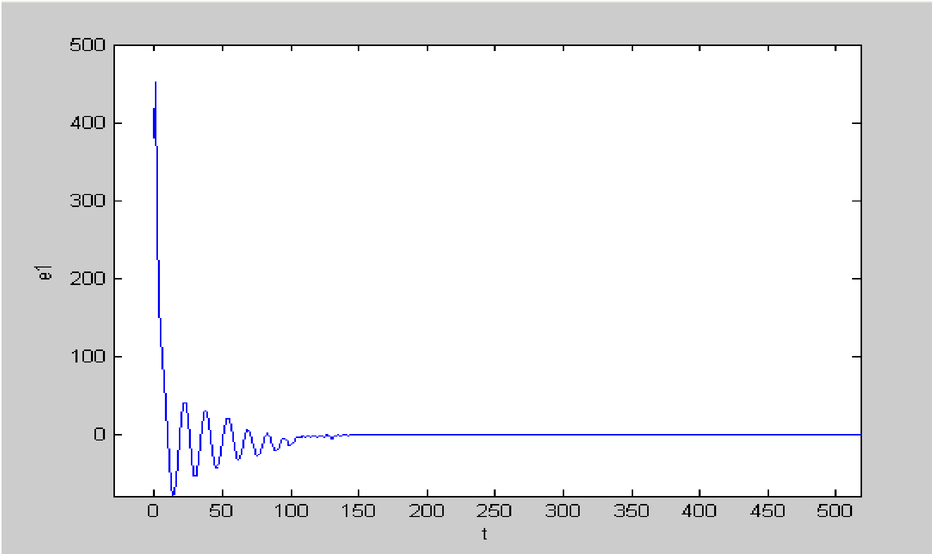


Fig 7 Time history of e_1 when e_{10} is 381.3.

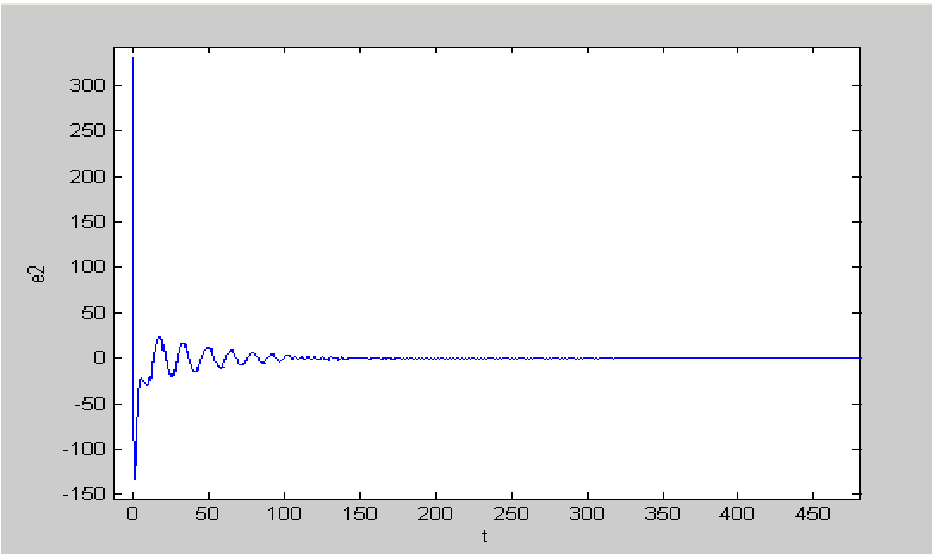


Fig 8 Time history of e_2 when e_{20} is 330.

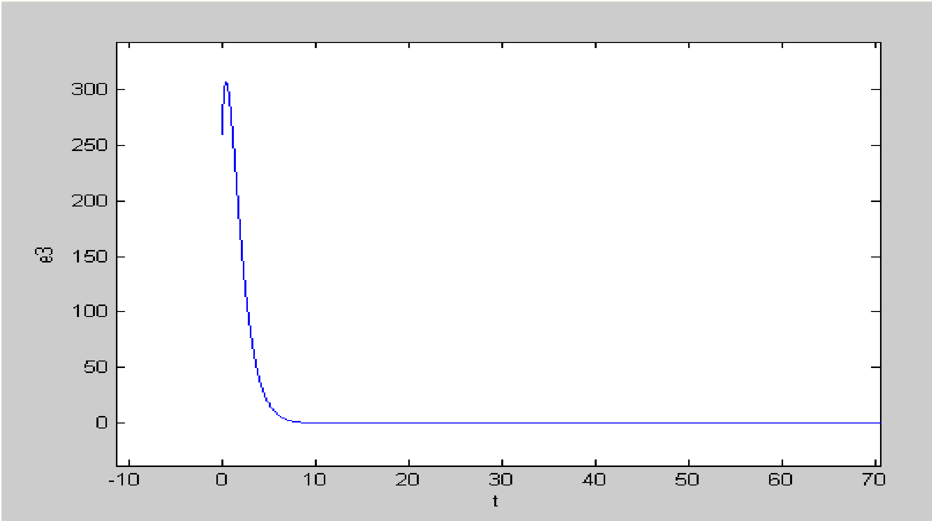


Fig 9　　Time history of e_3 when e_{30} is 260.

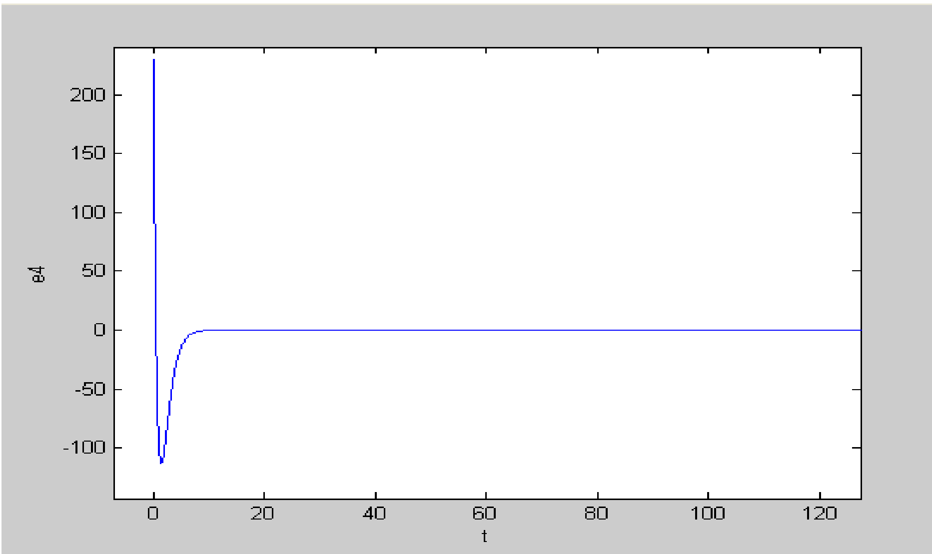


Fig 10　　Time history of e_4 when e_{40} is 230.

Chapter 19

Chaos Synchronization of the Two Identical Ikeda-Mackey-Glass Systems without Any Controller

19.1 Preliminaries

In 1990, Pecora and Carroll [1], showed the possibility of chaotic synchronization and started a new research interest. Synchronization phenomena in coupled chaotic systems have been extensively studied in laser dynamics [2], electronic circuits [3], chemical and biological systems [4], and secure communication [5].

Time-delay chaotic systems widely occur in everywhere, such as nature, technology, and society[6], and they are typical of high dimensional chaotic systems. Mackey-Glass system has been introduced as a model of blood generation for patients with leukemia[7]. The Ikeda model has been introduced to describe the dynamics of an optical bistable resonator[8-10]. A new Ikeda-Mackey Glass system is studied in this paper.

There are different types of synchronization in interacting chaotic systems, such as complete synchronization [1,11], generalized synchronization [12], phase synchronization [13,14], lag synchronization[11,15,16], anticipating synchronization [9,17] and so on.

To achieve synchronization, different schemes, such as the Pecora and Carroll (PC) method [1], unidirectional coupling [11], bidirectional coupling [17], adaptive control [18,19] and impulsive control [20–22] are proposed.

This paper is organized as follows. In Section 2, the phase portraits, bifurcation diagram of a new chaotic Ikeda-Mackey-Glass system are presented. In Section 3, synchronization scheme is given. In Section 3.1, No synchronization of the two identical IMG systems with slightly different initial conditions are presented when one of delay time τ_2 is zero. In Section 3.2, generalized synchronization, anti-synchronization and generalized lag-synchronization of the two identical IMG systems with slightly different conditions are presented when one of delay time τ_2 is positive. In Section 4, conclusions are drawn.

19.2. Ikeda-Mackey Glass system

A new IMG system is described by the following differential equations:

$$\dot{x}_1(t) = -\alpha_1 x_1(t) - \beta \sin x_1(t-\tau_1) + K_1 x_2(t-\tau_2)$$

$$\dot{x}_2(t) = -\alpha_2 x_2(t) + b\frac{x_2(t-\tau_1)}{1+\{x_2(t-\tau_1)\}^c} + K_2 x_1(t-\tau_2) \qquad (19\text{-}2\text{-}1)$$

where the Ikeda model x_1 is the phase lag of the electric field across the resonator; α_1 is the relaxation coefficient for the driving x_1 dynamical variable; β is the laser intensity injected into the driving system. τ_1, τ_2 are the delay time in the new IMG system, and the dynamical variable in the Mackey Glass model is the concentration of the mature cells in blood at time t and the delay time is the time between the initiation of cellular production in the bone marrow and release of mature cells into the blood[8]. α_2 is the relaxation coefficient for the driven x_2 dynamical variable, b is the feedback rate for the driven system, and K_1, K_2 is the coupling rate between the driver system x_1 and the response system x_2.

This system has a chaotic attractor shown in Fig.1. Fig.2 shows the bifurcation diagram, where α_1=25, β=24.8 , k_1 = 14.1 , α_2=4.7, b=1.2348,c=10, K_2=8, τ_1=5 and τ_2=1.

If the delay time τ_2 is zero, also it is found that there is also a chaotic behavior for Ikeda-Mackey Glass system. Fig.3 show that the chaotic attractor of this system where α_1=25, β=24.8 , k_1 = 14.1 , α_2=4.7, b=1.2348, c=10, K_2=8, τ_1=5 and τ_2=0.

19.3. Synchronization Scheme

Consider the time-delayed system:

$$\dot{x}(t) = f(x(t), x(t-\tau)) \qquad (19\text{-}3\text{-}1)$$

where $x \in R$ represents the state of the system, and $\dot{x}(t) = dx(t)/dt$.

To synchronize system (3), the form of the other system is

$$\dot{y}(t) = f(y(t), y(t-\tau)) + u \qquad (19\text{-}3\text{-}2)$$

where u is the controlling term.

In this paper, we find that these two Ikeda-Mackey Glass system can be synchronized without any controller, only by changing the delay time τ_1 and τ_2 .

Consider synchronization between the Ikeda-Mackey-Glass system

$$\begin{cases} \dot{x}_1(t) = -\alpha_1 x_1(t) - \beta \sin x_1(t-\tau_1) + K_1 x_2(t-\tau_2) \\ \dot{x}_2(t) = -\alpha_2 x_2(t) + b\dfrac{x_2(t-\tau_1)}{1+\{x_2(t-\tau_1)\}^c} + K_2 x_1(t-\tau_2) \end{cases} \tag{19-3-3}$$

$$\begin{cases} \dot{y}_1(t) = -\alpha_1 y_1(t) - \beta \sin y_1(t-\tau_1) + K_1 y_2(t-\tau_2) + u_1 \\ \dot{y}_2(t) = -\alpha_2 y_2(t) + b\dfrac{y_2(t-\tau_1)}{1+\{y_2(t-\tau_1)\}^c} + K_2 y_1(t-\tau_2) + u_2 \end{cases} \tag{19-3-4}$$

where the controlling term $u_1 = u_2 = 0$.

19.3.1. Case1: If the delay time τ_2 =0

In this subsection it is shown that if the delay time τ_2 is zero, no synchronization can be obtained. Simulation results are shown in Fig.4 and Fig.5.

19.3.2. Case2: If the delay time τ_2 =1

In this subsection it is shown that if the delay time τ_2 is not zero, different types of synchronization can be obtained.

Fig.6 and Fig.7 show that the generalized synchronization of the two identical IMG systems can be obtained, where error

$$e_{1,2}(t)=x_{1,2}(t)-y_{1,2}(t)+R(t) \tag{19-3-5}$$

R(t) is a periodic function of time.

Fig.8 and Fig.9 show that the time response of the two identical IMG systems. It is verified that the anti-synchronization can be obtained by Fig.10 and Fig.11, where error

$$e_{1,2}(t)=x_{1,2}(t)+y_{1,2}(t) \tag{19-3-6}$$

Fig.12 and Fig.13 show that the time response of the two identical IMG systems. It is verified that the generalized lag-synchronization can be obtained by Fig.14 and Fig.15, where error

$$e_1(t) = x_1(t-\mu_1) - y_1(t) + F(t)\,, e_2(t) = x_2(t) - y_2(t-\mu_2) + F^{'}(t) \tag{19-3-7}$$

$\mu_1 = 1.2427$ sec, $\mu_2 = 1.08$ sec, $F(t)$ and $F^{'}(t)$ are periodic function of time.

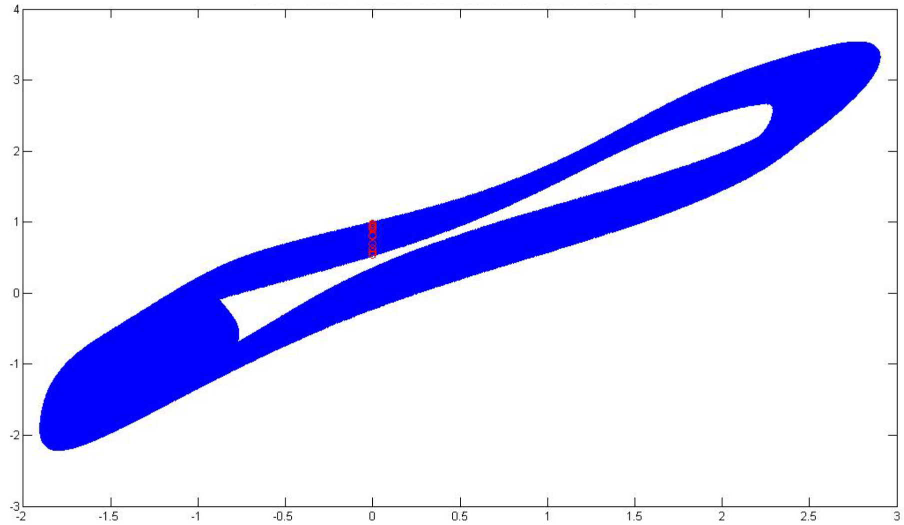


Fig. 1. An Ikeda-Mackey Glass chaotic attractor when the delay times τ_1 =5, τ_2 =1

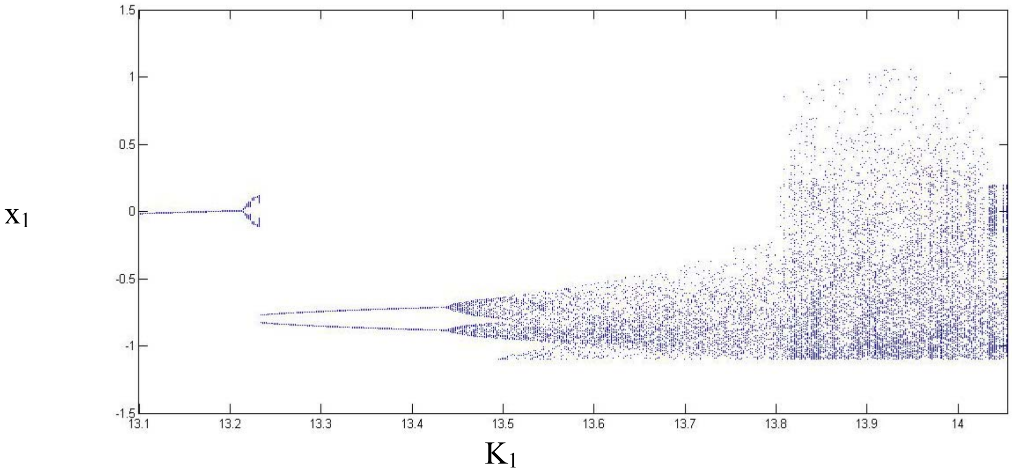


Fig.2. The bifurcation diagram of the IMG system when the delay times τ_1 =5, τ_2 =1.

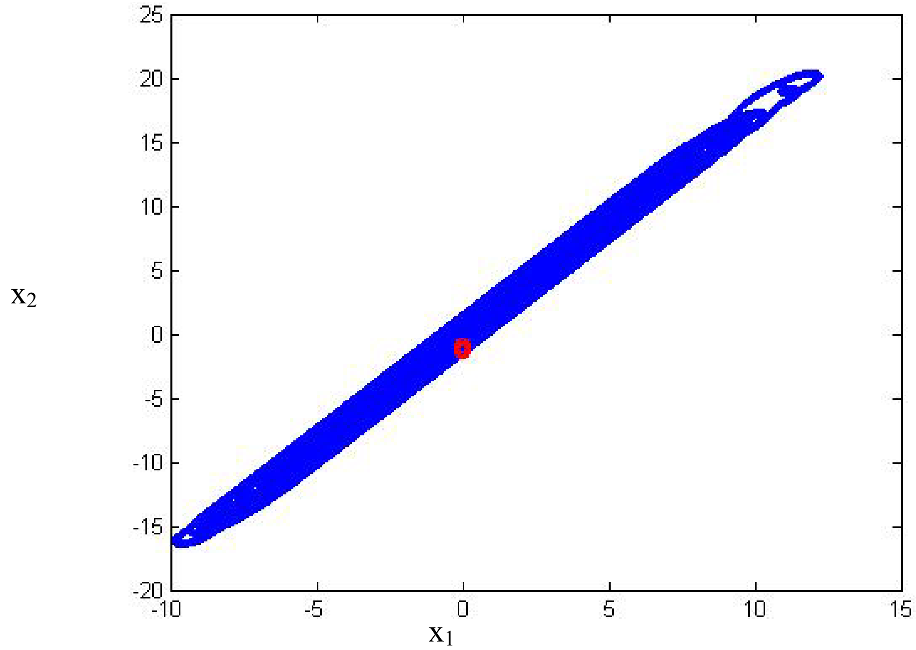


Fig.3. An IMG chaotic attractor when the delay times $\tau_1 = 5, \tau_2 = 0$

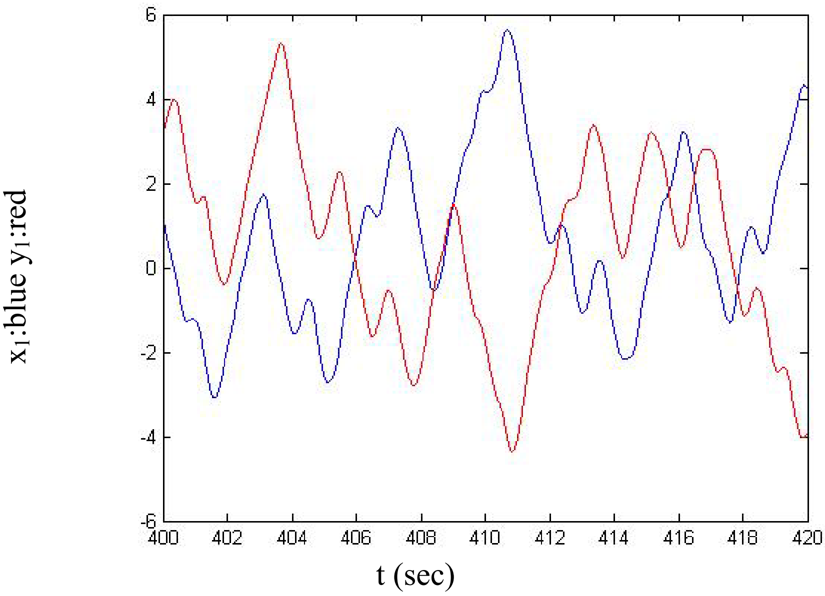


Fig.4.Time response of the two identical IMG systems with $x_1(0)=1$, $x_2(0)=0$,$y_1(0)=-1$ and $y_2(0)=0.5$, when $\tau_2 = 0$.

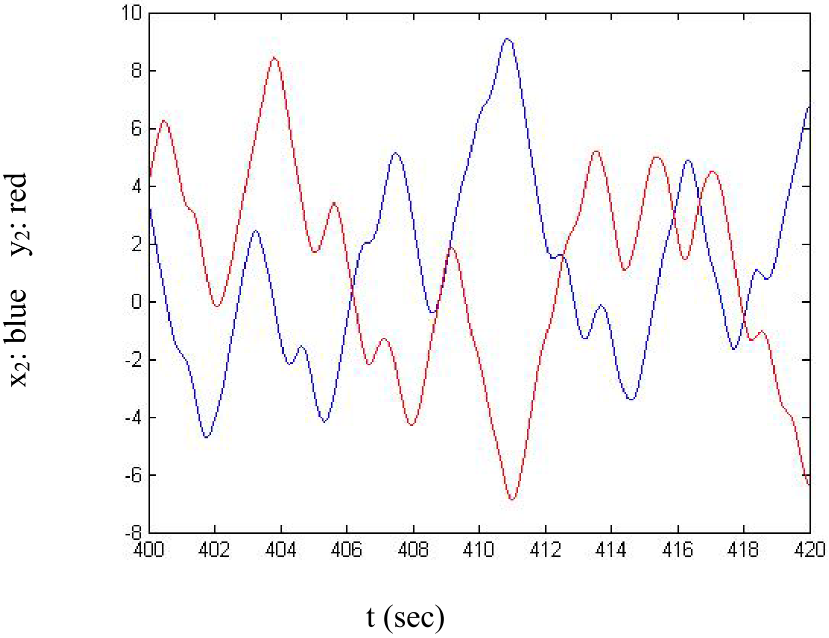


Fig.5.Time response of the two identical IMG systems with $x_1(0)=1$, $x_2(0)=0$,$y_1(0)=-1$ and $y_2(0)=0.5$, when $\tau_2=0$.

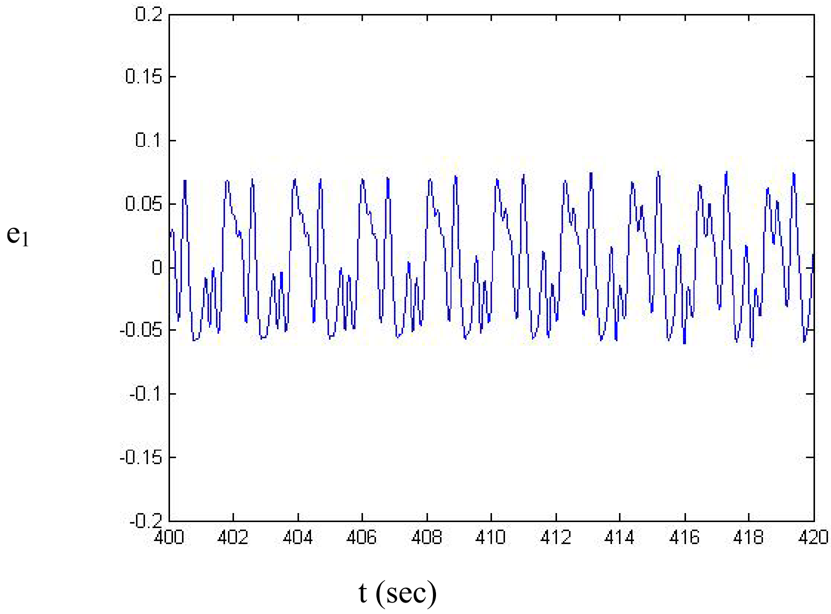


Fig.6. Error dynamics of the two identical IMG systems with $x_1(0)=100$, $x_2(0)=10$, $y_1(0)=101$ and $y_2(0)=10.001$, when $\tau_2=1$.

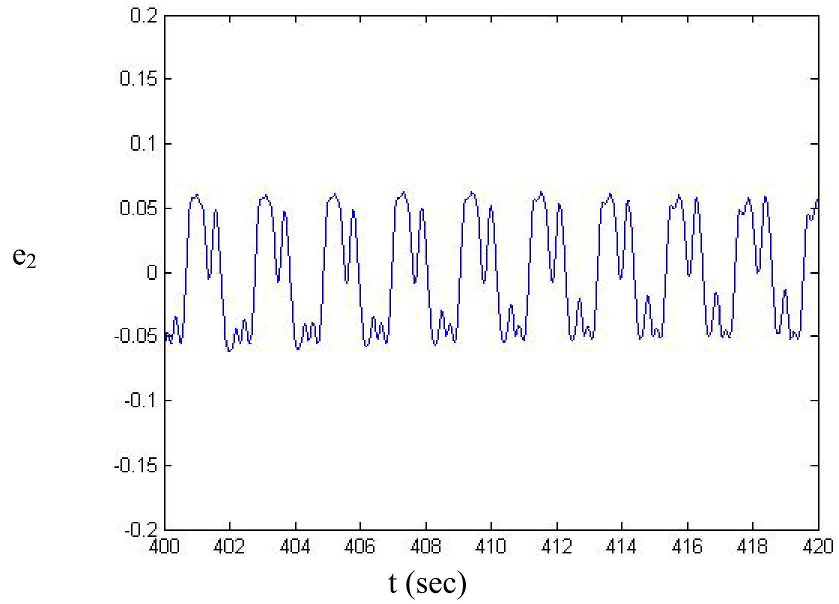


Fig.7. Error dynamics of the two identical IMG systems with $x_1(0)$=100, $x_2(0)$=10, $y_1(0)$=101 and $y_2(0)$=10.001, when $\tau_2 = 1$.

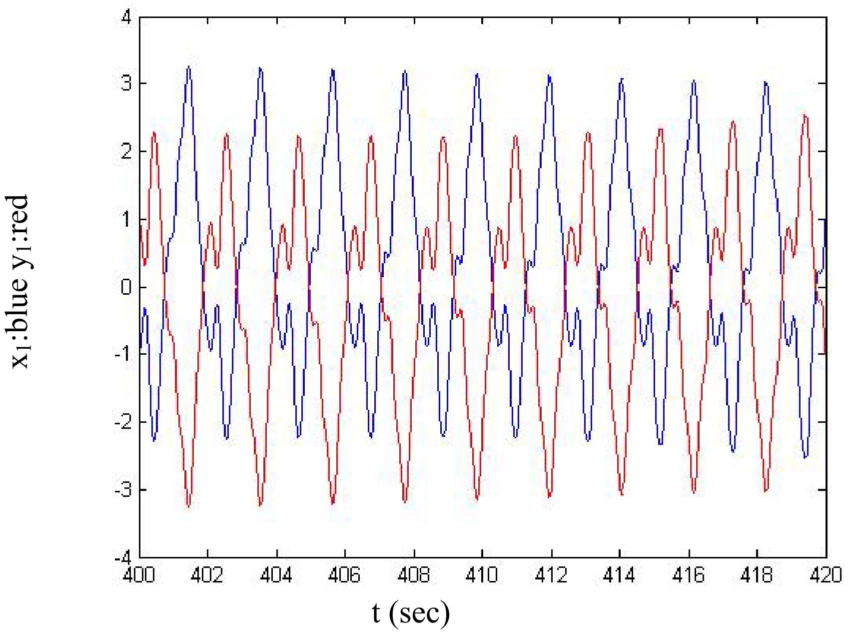


Fig.8. Time response of the two identical Ikeda-Mackey Glass systems with $x_1(0)$=1, $x_2(0)$=0, $y_1(0)$=-1 and $y_2(0)$=0, when $\tau_2 = 1$.

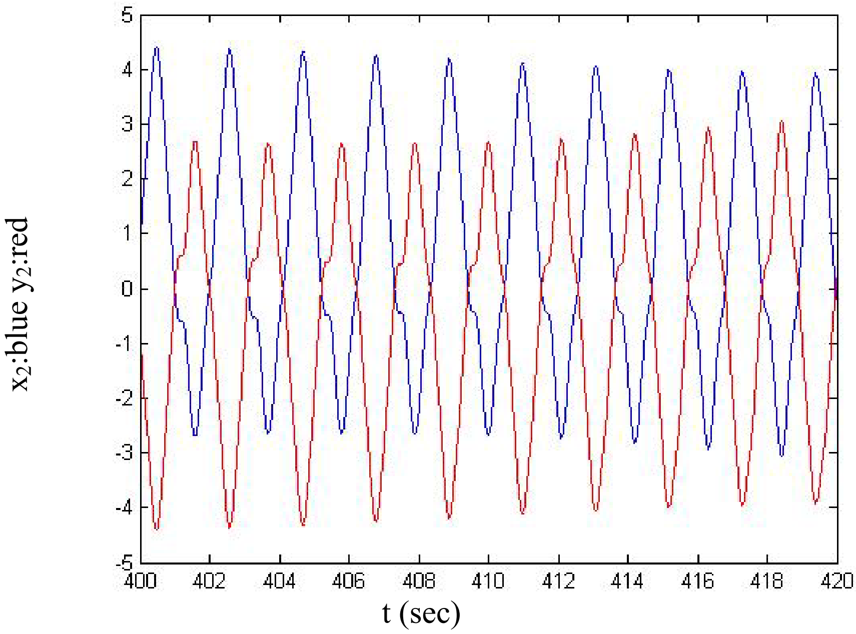


Fig.9. Time response of the two identical Ikeda-Mackey Glass systems with $x_1(0)=1$, $x_2(0)=0$, $y_1(0)=-1$ and $y_2(0)=0$, when $\tau_2 = 1$.

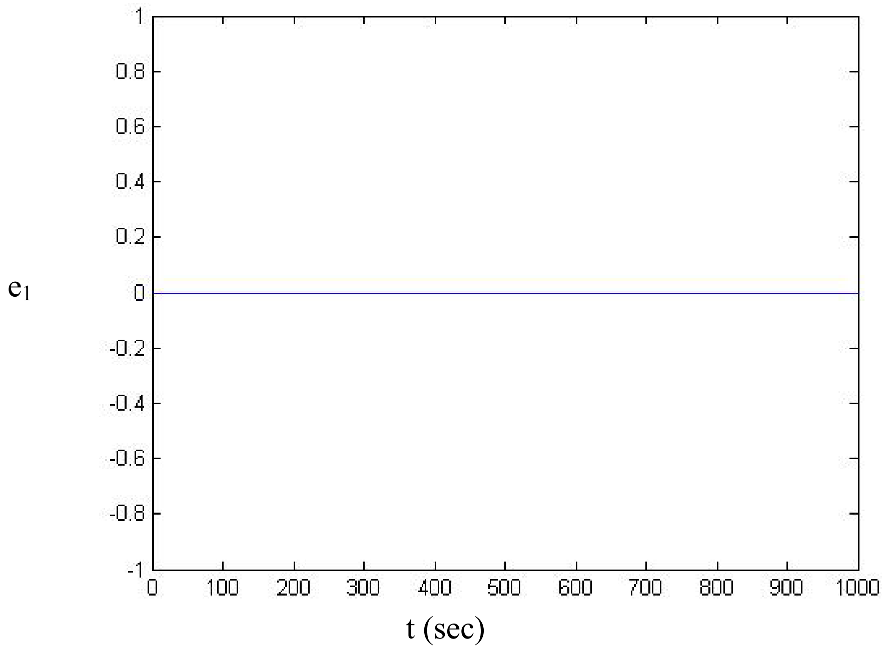


Fig.10. Error dynamics of the two identical IMG systems with $x_1(0)=1$, $x_2(0)=0$, $y_1(0)=-1$ and $y_2(0)=0$, when $\tau_2 = 1$.

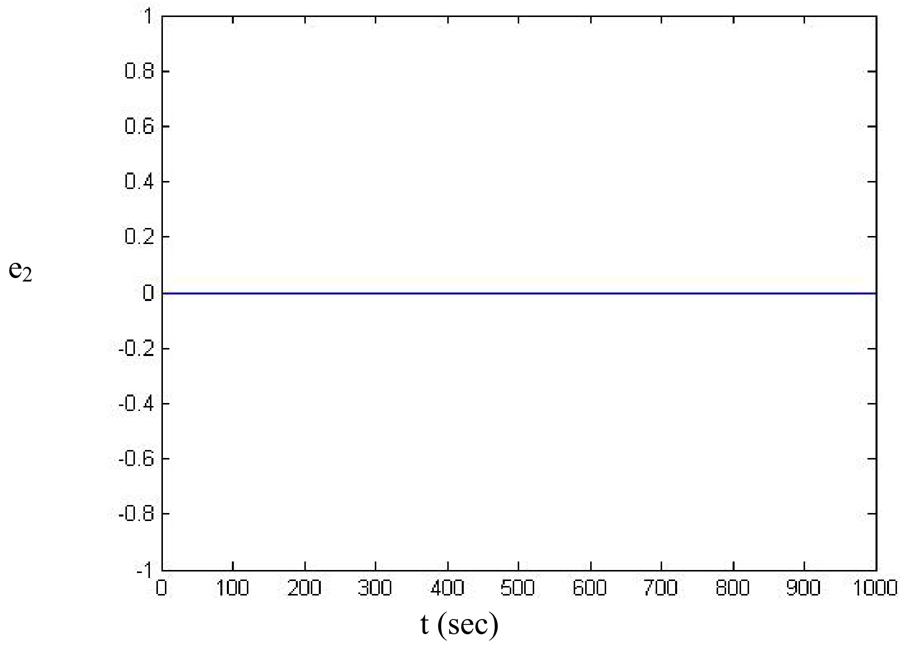


Fig.11. Error dynamics of the two identical IMG systems with
$x_1(0)=1$, $x_2(0)=0$, $y_1(0)=-1$ and $y_2(0)=0$, when $\tau_2 = 1$.

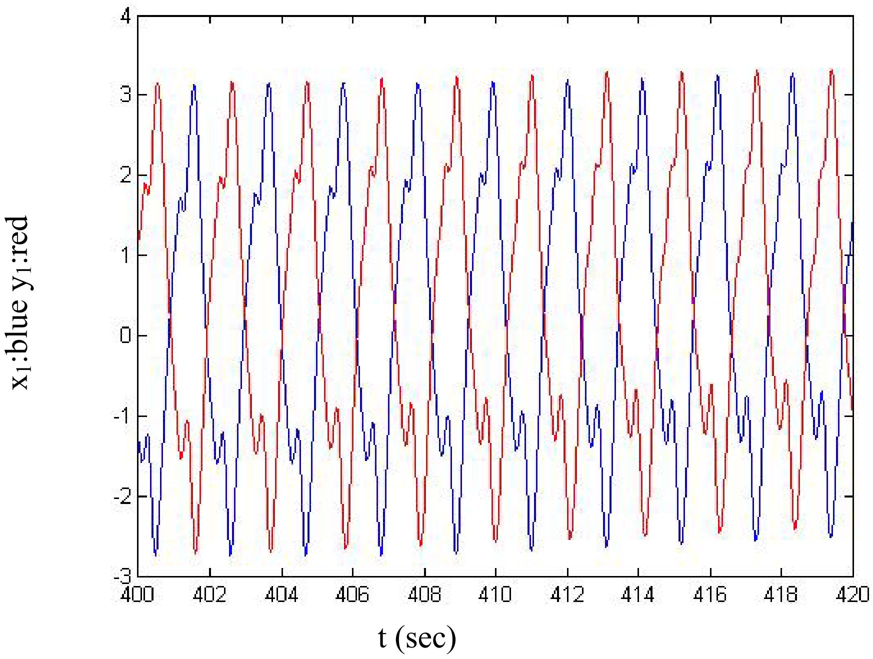


Fig.12. Time response of the two identical IMG systems with $x_1(0)=1$, $x_2(0)=0.1$, $y_1(0)=-1$ and $y_2(0)=0.5$, when $\tau_2 = 1$, $\mu_1 = 1.2427$ sec.

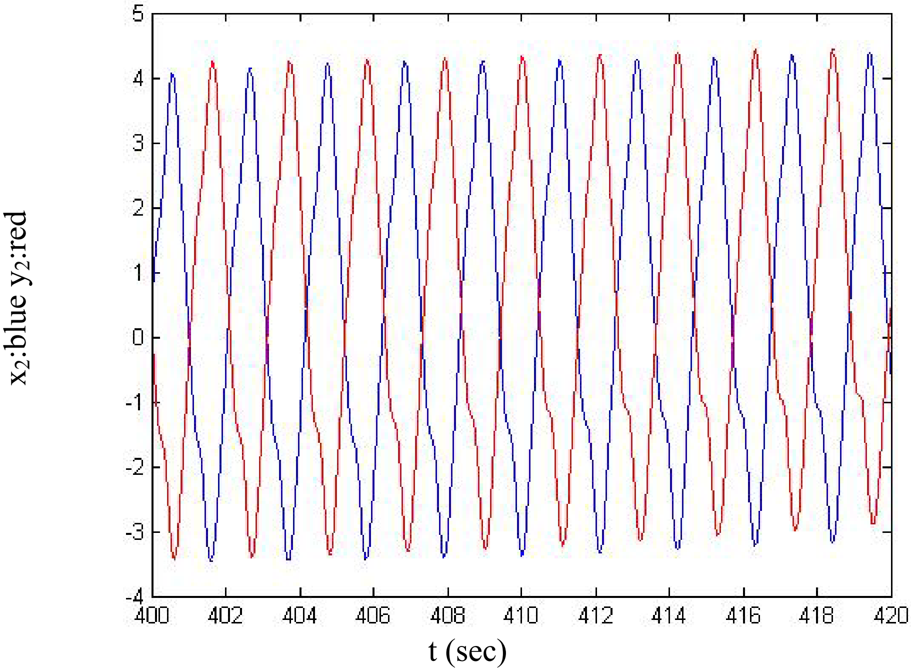


Fig.13. Time response of the two identical IMG systems with $x_1(0)=1$, $x_2(0)=0.1$, $y_1(0)=-1$ and $y_2(0)=0.5$, when $\tau_2 = 1$, $\mu_2 = 1.08$ sec.

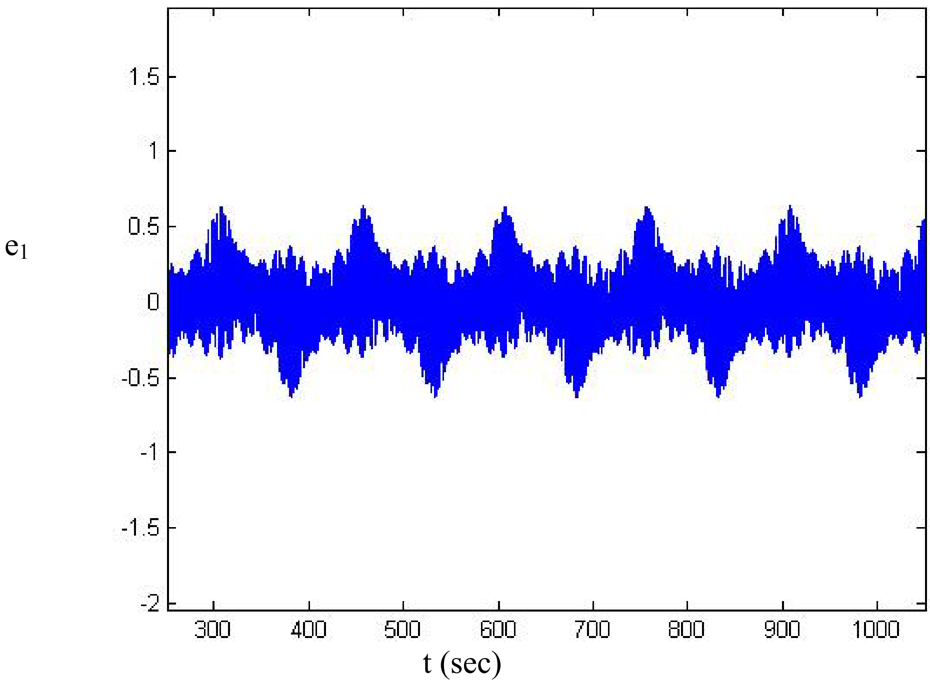


Fig.14. Error dynamics of the two identical IMG systems with $x_1(0)=1$, $x_2(0)=0.1$, $y_1(0)=-1$ and $y_2(0)=0.5$, when $\tau_2 = 1$.

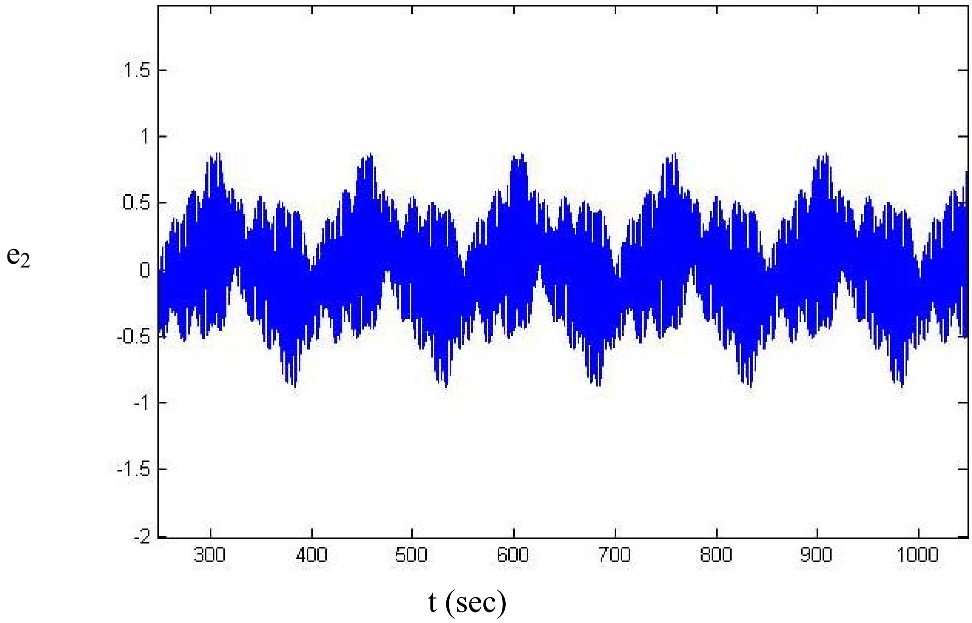


Fig.15. Error dynamics of the two identical IMG systems with $x_1(0)$=1, $x_2(0)$=0.1, $y_1(0)$=-1 and $y_2(0)$=0.5, when $\tau_2 = 1$.

Chapter 20

Parameter estimation of Ikeda-Mackey Glass system through synchronization in presence of disturbance

20.1 Preliminaries

In this paper, parameter estimation of a new Ikeda-Mackey Glass system through synchronization is studied. Parameter estimation of this system by using a least square approach to derive the time delay system equations. Two time delay Ikeda-Mackey Glass systems are synchronized and their corresponding parameters converge to same value. When the Ikeda-Mackey Glass system is disturbed by Rayleigh noise, the parameters are very sensitive to its initial conditions. Numerical simulations are presented to verify these results.

20.2 Ikeda-Mackey-Glass system

Ikeda-Mackey Glass system is described by the following differential equations:

$$\begin{aligned} \dot{x}_1(t) &= -\alpha_1 x_1(t) - \beta \sin x_1(t-\tau) \\ \dot{x}_2(t) &= -\alpha_2 x_2(t) + k_1 \frac{x_2(t-\tau)}{1+\{x_2(t-\tau)\}^b} + K x_1(t) \end{aligned} \qquad (20\text{-}2\text{-}1)$$

where the Ikeda model x_1 is the phase lag of the electric field across the resonator; α_1 is the relaxation coefficient for the driving x_1 dynamical variable; β is the laser intensity injected into the driving system. τ is the delay time in the coupled systems or the round trip time of the light in the resonator, and the dynamical variable in the Mackey Glass model is the concentration of the mature cells in blood at time t and the delay time is the time between the initiation of cellular production in the bone marrow and release of mature cells into the blood[8]. α_2 is the relaxation coefficient for the driven x_2 dynamical variable, k_1 is the feedback rate for the driven system, and K is the coupling rate between the driver system x1 and the response system x2.

This system has a chaotic attractor shown in Fig.1. Fig.2 show the bifurcation diagram, where α_1=25, β=24.8985, α_2=4.7, k_1=1.2348,b=10, K=8. Phase portrait of period 1 is shown in Fig.3 when K=23.5.

20.3 Synchronization scheme

Differential equations of a general delay system are described as follows:

$$\dot{x} = f(x(t), x(t-\tau), \beta) \tag{20-3-1}$$

where $x \in R^n$ is the state vector, $\beta \in R^m$ is the parameter vector which is to be estimated.

Let Eq.(20-3-1) be coupled to Eq.(1) which is described as follows:

$$\dot{y} = g(y(t), y(t-\tau), \hat{\beta}) \tag{20-3-2}$$

where $y \in R^n$ is the state vector, $\hat{\beta} \in R^m$ is the parameter vector. The state x asymptotically synchronized with y when $\beta = \hat{\beta}$.

The general representation of the feedback coupling is

$$\dot{y} = g(y(t), y(t-\tau), \hat{\beta}) - CG^T[y-x] \tag{20-3-3}$$

where C is a constant vector and G is the gain vector. Next we construct a mechanism that drives the measured synchronization error y-x to zero so that $(y, \hat{\beta}) \to (x, \beta)$ as $t \to \infty$. For that we consider the following minimization problem to construct a system of differential equation governing the evolution of the model system parameter $\hat{\beta}$,

$$F(\hat{\beta}) = \min\{[y-x]^2\} \tag{20-3-4}$$

The parameter vector $\hat{\beta}$ is to be suitably tuned so that the system (20-3-1) asymptotically synchronizes with the system (1) through the choice of function given by Eq.(4). The measure output is one of the state variables of the system. Then the minimization problem can be rewritten as the following system of differential equation.

$$\dot{\hat{\beta}}_j = -\frac{\partial F}{\partial \hat{\beta}_j} = -2(y_i - x_i)\frac{\partial y_i}{\partial \hat{\beta}_j} \quad , j=1,\ldots,m \tag{20-3-5}$$

The variational derivatives $\frac{\partial y_i}{\partial \hat{\beta}_j}$ for i=1,…,n and j=1,…,m are to be known for solving this system of equations. These derivatives are given by

$$\frac{d}{dt}\left(\frac{\partial F}{\partial \hat{\beta}_j}\right) = \sum_{k=1}^{n} \frac{\partial g_i}{\partial y_k} \cdot \frac{\partial y_k}{\partial \hat{\beta}_j} + \frac{\partial g_i}{\partial \hat{\beta}_j} - CG^T \frac{\partial y_k}{\partial \hat{\beta}_j} \quad \text{for i=1,…,n and j=1,…,m} \tag{20-3-6}$$

So we have to solve the original system (1), the coupling system (3), where the equations

$$\dot{\hat{\beta}}_j = -\varepsilon_j \frac{\partial F}{\partial \hat{\beta}_j} = -2\varepsilon_j (y_i - x_i)\frac{\partial y_i}{\partial \hat{\beta}_j}, \text{ j=1,…m} \tag{20-3-7}$$

which governs the evolution of the parameters with a vector of additional parameters ε needed for controlling the stability of the overall system and rate of synchronization, and the equations corresponding to the evolution of the variational derivatives.

So an extended system consisting of (n+m+nm) equations is to be solved in order to estimate m parameters and simultaneous synchronization of the n-dimensional system.

20.4 Estimation of parameter of Ikeda-Mackey Glass system without disturbance

By using above formulation, the equations are described as follows:

$$\dot{x}_1(t) = -25x_1(t) - 24.8985 \sin x_1(t-\tau) \tag{20-4-1}$$

$$\dot{y}_1(t) = -25y_1(t) - \hat{\beta}_1(t) \cdot \sin y_1(t-\tau) - (y_1(t) - x_1(t)) \tag{20-4-2}$$

$$\dot{\hat{\beta}}_1(t) = -2\varepsilon\left(y_1(t) - x_1(t)\right)v_1 \tag{20-4-3}$$

$$\dot{v}_1(t) = \left(-25 - \frac{\hat{\beta}_1(t) \cdot \cos y_1(t-\tau) \cdot \dot{y}_1(t-\tau)}{\dot{y}_1(t)}\right)v_1(t) - v_1(t) - \sin y_1(t-\tau) \tag{20-4-4}$$

$$\dot{x}_2(t) = -4.7x_2(t) - 1.2348\frac{x_2(t-\tau)}{1+x_2(t-\tau)^{10}} + 8x_1(t) \tag{20-4-5}$$

$$\dot{y}_2(t) = -\hat{\beta}_2 \cdot y_2(t) - 1.2348\frac{y_2(t-\tau)}{1+y_2(t-\tau)^{10}} + 8y_1(t) - [y_2(t) - x_2(t)] \tag{20-4-6}$$

$$\dot{\hat{\beta}}_2(t) = -2\varepsilon\left(y_2(t) - x_2(t)\right)v_2 \tag{20-4-7}$$

$$\begin{aligned} \dot{v}_1(t) = &\left(-\hat{\beta}_2 - \frac{1.2348\cdot \dot{y}_2(t-\tau)\{1-9y_2(t-\tau)^{10}\}}{\dot{y}_2(t)\{1+y_2(t-\tau)^{10}\}^2} + 8\frac{\dot{y}_1(t)}{\dot{y}_2(t)} \right) v_2(t) \\ &+ \frac{y_2(t-\tau)}{1+y_2(t-\tau)^{10}} - v_2(t) \end{aligned} \tag{20-4-8}$$

where

$$\dot{y}_1(t-\tau) = -25y_1(t-\tau) - \hat{\beta}_1(t)\cdot \sin y_1(t-2\tau) - [y_1(t-\tau) - x_1(t-\tau)] \tag{20-4-9}$$

$$\begin{aligned} \dot{y}_2(t-\tau) = &-\hat{\beta}_2(t)\cdot y_2(t-\tau) - 1.2348\frac{y_2(t-2\tau)}{1+y_2(t-2\tau)^{10}} \\ &+ 8y_1(t-1) - [y_2(t-\tau) - x_2(t-\tau)] \end{aligned} \tag{20-4-10}$$

In this paper, we tried to estimate the parameters $\hat{\beta}_1$ and $\hat{\beta}_2$ of the system. The results show that the parameter $\hat{\beta}_1$ converges to its actual value when the master and response systems are synchronized. Although the parameter $\hat{\beta}_2$ cannot converge to its actual value, the response system made parameter $\hat{\beta}_1$ converge to its actual value rapidly. The results are shown in Fig.4.

20.5. Estimation of parameter of Ikeda-Mackey Glass system with disturbance

Parameter estimation is studied when there exists disturbance. The equations of the system with disturbance are given by

$$\dot{x}_1(t) = -25x_1(t) - 24.8985\sin x_1(t-\tau) \tag{20-5-1}$$

$$\dot{y}_1(t) = -25y_1(t) - \hat{\beta}_1(t)\cdot \sin y_1(t-\tau) - (y_1(t) - x_1(t) - \varphi(t)) \tag{20-5-2}$$

$$\dot{\hat{\beta}}_1(t) = -2[y_1(t) - x_1(t) - \varphi(t)]v_1 \tag{20-5-3}$$

$$\dot{v}_1(t) = \left(-25 - \frac{\hat{\beta}_1(t)\cdot \cos y_1(t-\tau)\cdot \dot{y}_1(t-\tau)}{\dot{y}_1(t)}\right)v_1(t) - v_1(t) - \sin y_1(t-\tau) \tag{20-5-4}$$

where

$$\dot{y}_1(t-\tau) = -25y_1(t-\tau) - \hat{\beta}_1(t)\cdot \sin y_1(t-2\tau) - [y_1(t-\tau) - x_1(t-\tau) - \varphi(t)] \tag{20-5-5}$$

$$\dot{y}_2(t-\tau) = -\hat{\beta}_2(t)\cdot y_2(t-\tau) - 1.2348\frac{y_2(t-2\tau)}{1+y_2(t-2\tau)^{10}} + 8y_1(t-1) - [y_2(t-\tau) - x_2(t-\tau)] \tag{20-5-6}$$

where $\varphi(t)$ is the Rayleigh noise. The results show that the parameter $\hat{\beta}_1$ is very sensitive to its initial conditions when disturbance exists in Ikeda-Mackey Glass system. By changing the initial conditions of $\hat{\beta}_1$ which can be found that the parameter $\hat{\beta}_1$ also converges to its actual value. The results are shown in Fig.20-5.

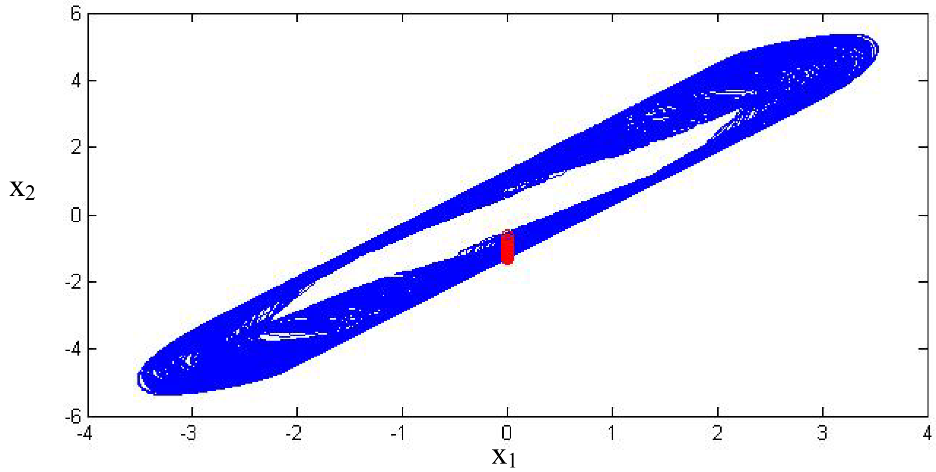


Fig. 20-1. An Ikeda-Mackey Glass chaotic attractor.

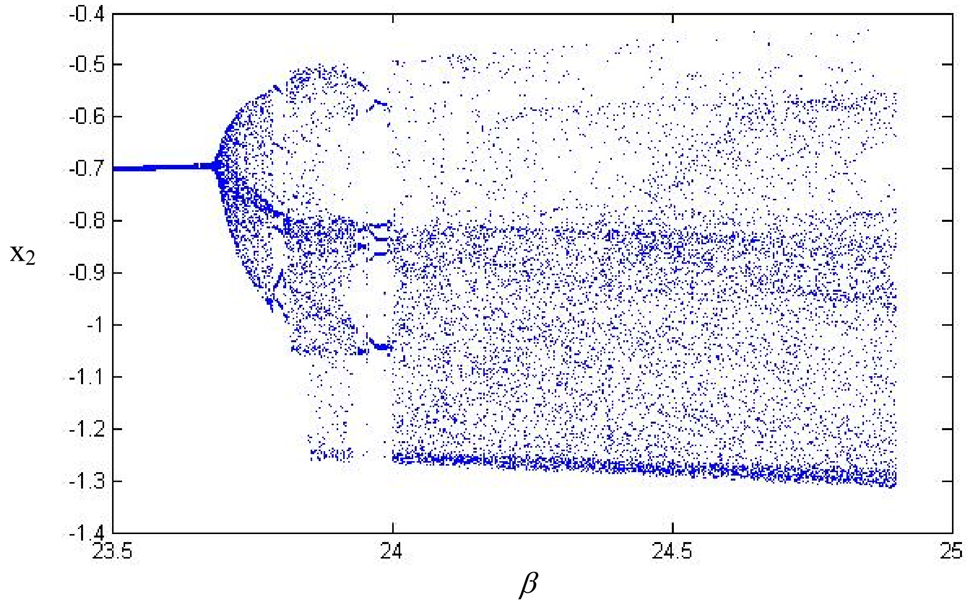


Fig. 20-2. The bifurcation diagram of the Ikeda-Mackey Glass system.

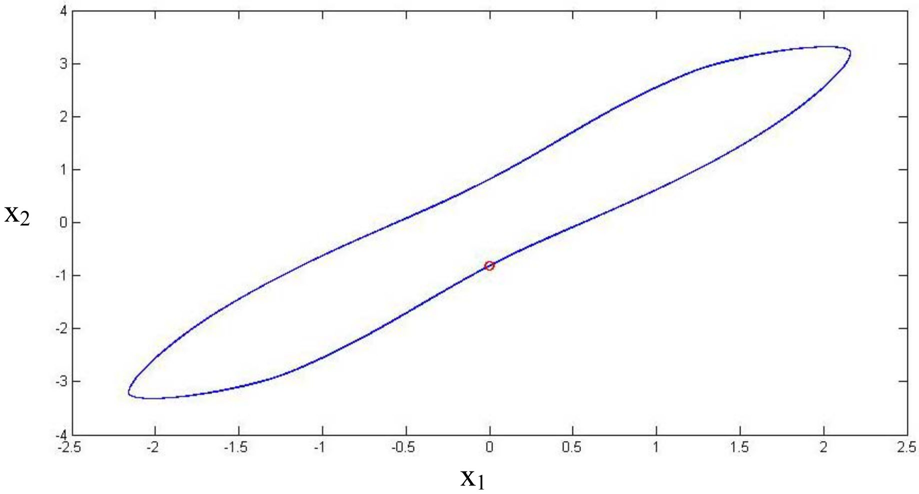


Fig. 20-3 Phase portrait of period 1.

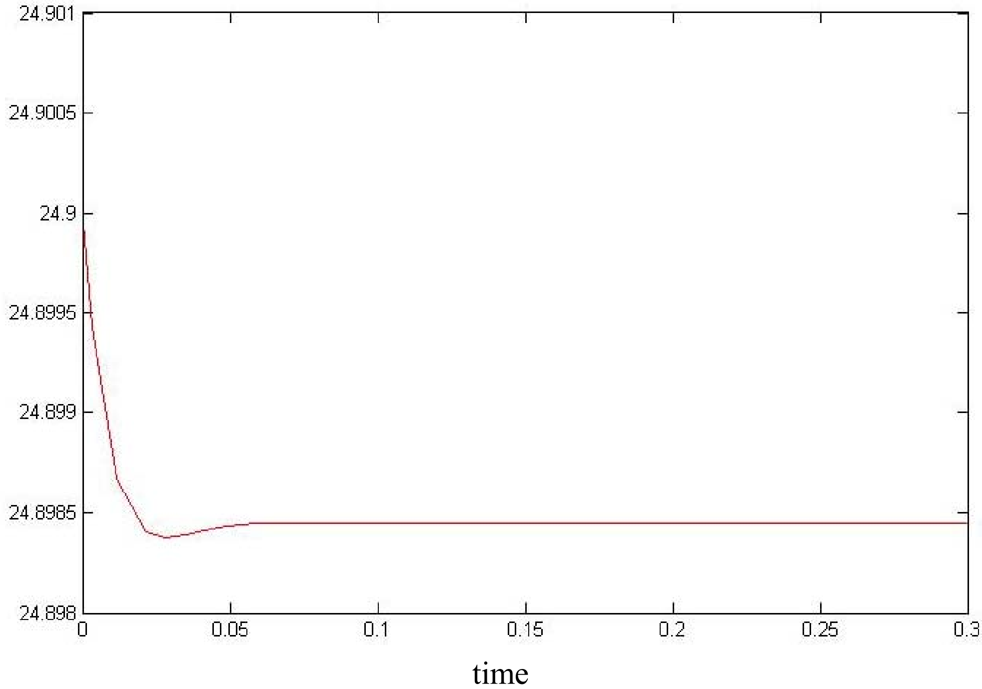


Fig. 20-4. Convergence of the estimated parameter to its actual value without disturbance in the system.

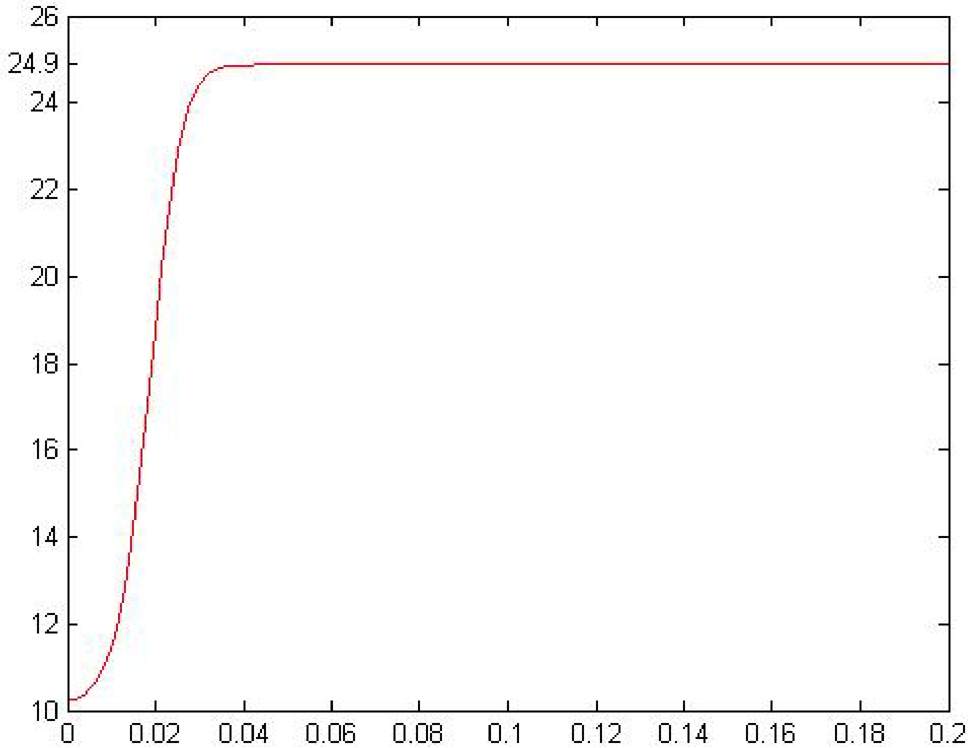


Fig. 20-5. Convergence of the estimated parameter to its actual value with disturbance in the system

Chapter 21

Chaos Synchronization of the Two Identical Ikeda-Mackey-Glass Systems without Any Controller

21.1 Preliminaries

In this paper, synchronization of the two identical new Ikeda-Mackey Glass(IMG) systems without any control is studied. When one of delay time is zero, two identical IMG system cannot be synchronized with slightly different conditions. It is found that when one of delay time is positive, different types of synchronization can be obtained with slightly different initial conditions, such as generalized synchronization, anti-synchronization, and generalized lag-synchronization. Numerical simulations are presented to verify these results.

21. 2. Ikeda-Mackey Glass system

A new IMG system is described by the following differential equations:

$$\begin{aligned} \dot{x}_1(t) &= -\alpha_1 x_1(t) - \beta \sin x_1(t-\tau_1) + K_1 x_2(t-\tau_2) \\ \dot{x}_2(t) &= -\alpha_2 x_2(t) + b\frac{x_2(t-\tau_1)}{1+\{x_2(t-\tau_1)\}^c} + K_2 x_1(t-\tau_2) \end{aligned} \tag{21-2-1}$$

where the Ikeda model x_1 is the phase lag of the electric field across the resonator; α_1 is the relaxation coefficient for the driving x_1 dynamical variable; β is the laser intensity injected into the driving system. τ_1, τ_2 are the delay time in the new IMG system, and the dynamical variable in the Mackey Glass model is the concentration of the mature cells in blood at time t and the delay time is the time between the initiation of cellular production in the bone marrow and release of mature cells into the blood[8]. α_2 is the relaxation coefficient for the driven x_2 dynamical variable, b is the feedback rate for the driven system, and K_1, K_2 is the coupling rate between the driver system x_1 and the response system x_2.

This system has a chaotic attractor shown in Fig.1. Fig.2 shows the bifurcation diagram, where α_1=25, β=24.8 , k_1= 14.1 , α_2=4.7, b=1.2348,c=10, K_2=8, τ_1=5 and τ_2=1.

If the delay time τ_2 is zero, also it is found that there is also a chaotic behavior for Ikeda-Mackey Glass system. Fig.3 show that the chaotic attractor of this system where α_1=25, β=24.8 , k_1= 14.1 , α_2=4.7, b=1.2348, c=10, K_2=8, τ_1=5 and τ_2=0.

21. 3. Synchronization Scheme

Consider the time-delayed system:

$$\dot{x}(t) = f(x(t), x(t-\tau)) \qquad (21\text{-}3\text{-}1)$$

where $x \in R$ represents the state of the system, and $\dot{x}(t) = dx(t)/dt$.

To synchronize system (3), the form of the other system is

$$\dot{y}(t) = f(y(t), y(t-\tau)) + u \qquad (21\text{-}3\text{-}2)$$

where u is the controlling term.

In this paper, we find that these two Ikeda-Mackey Glass system can be synchronized without any controller, only by changing the delay time τ_1 and τ_2.

Consider synchronization between the Ikeda-Mackey-Glass system

$$\begin{cases} \dot{x}_1(t) = -\alpha_1 x_1(t) - \beta \sin x_1(t-\tau_1) + K_1 x_2(t-\tau_2) \\ \dot{x}_2(t) = -\alpha_2 x_2(t) + b\dfrac{x_2(t-\tau_1)}{1+\{x_2(t-\tau_1)\}^c} + K_2 x_1(t-\tau_2) \end{cases} \qquad (21\text{-}3\text{-}3)$$

$$\begin{cases} \dot{y}_1(t) = -\alpha_1 y_1(t) - \beta \sin y_1(t-\tau_1) + K_1 y_2(t-\tau_2) + u_1 \\ \dot{y}_2(t) = -\alpha_2 y_2(t) + b\dfrac{y_2(t-\tau_1)}{1+\{y_2(t-\tau_1)\}^c} + K_2 y_1(t-\tau_2) + u_2 \end{cases} \qquad (21\text{-}3\text{-}4)$$

where the controlling term $u_1 = u_2 = 0$.

21.3.1. Case1: If the delay time τ_2 =0

In this subsection it is shown that if the delay time τ_2 is zero, no synchronization can be obtained. Simulation results are shown in Fig.4 and Fig.5.

21.3.2. Case2: If the delay time τ_2 =1

In this subsection it is shown that if the delay time τ_2 is not zero, different types of synchronization can be obtained.

Fig.21-6 and Fig.21-7 show that the generalized synchronization of the two identical IMG systems can be obtained, where error

$$e_{1,2}(t)=x_{1,2}(t)-y_{1,2}(t)+R(t) \qquad (21\text{-}3\text{-}5)$$

R(t) is a periodic function of time.

Fig.8 and Fig.9 show that the time response of the two identical IMG systems. It is verified that the anti-synchronization can be obtained by Fig.21-10 and Fig.21-11, where error

$$e_{1,2}(t)=x_{1,2}(t)+y_{1,2}(t) \qquad (21\text{-}3\text{-}6)$$

Fig.12 and Fig.13 show that the time response of the two identical IMG systems. It is verified that the generalized lag-synchronization can be obtained by Fig.21-14 and Fig.21-15, where error

$$e_1(t)=x_1(t-\mu_1)-y_1(t)+F(t)\,, e_2(t)=x_2(t)-y_2(t-\mu_2)+F^{'}(t) \qquad (21\text{-}3\text{-}7)$$

$\mu_1=1.2427$ sec, $\mu_2=1.08$ sec, $F(t)$ and $F^{'}(t)$ are periodic function of time.

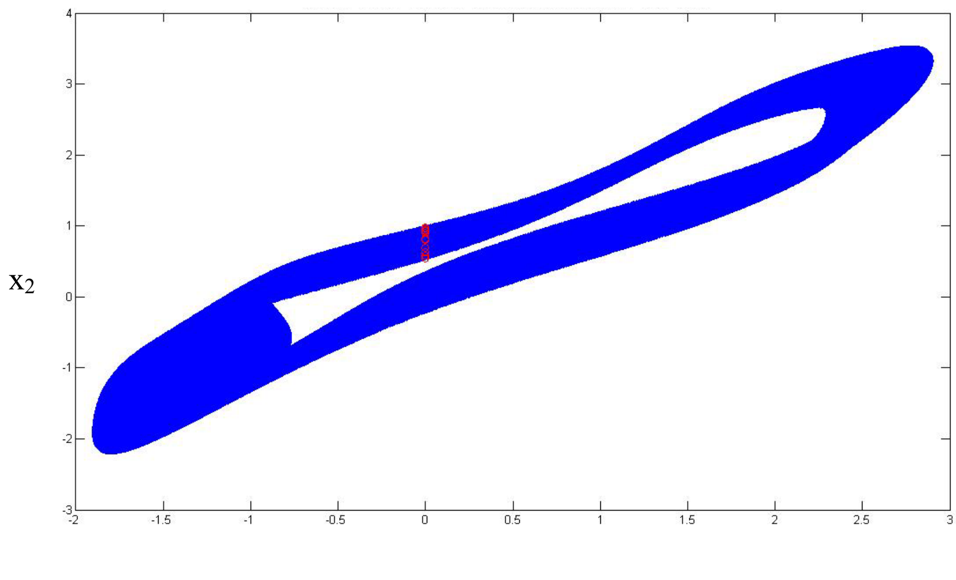


Fig. 21-1. An Ikeda-Mackey Glass chaotic attractor when the delay times $\tau_1=5, \tau_2=1$

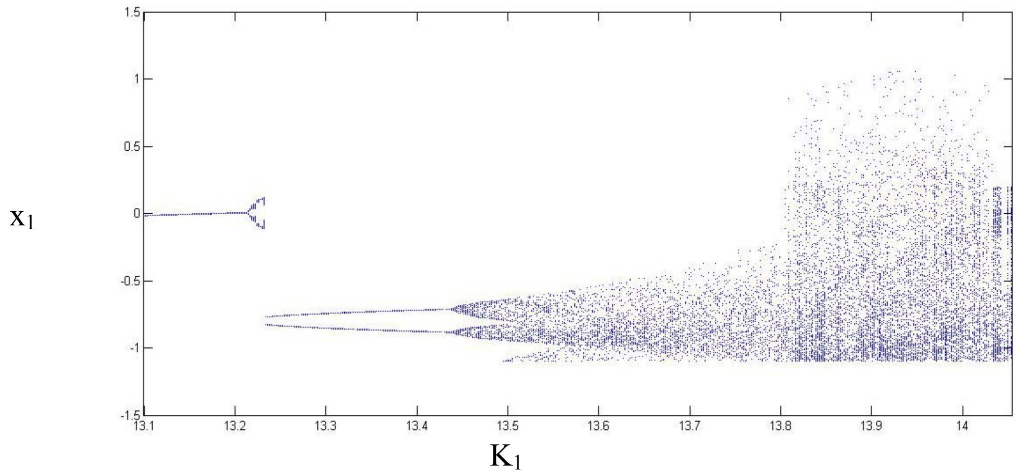


Fig. 21-2. The bifurcation diagram of the IMG system when the delay times τ_1=5, τ_2=1.

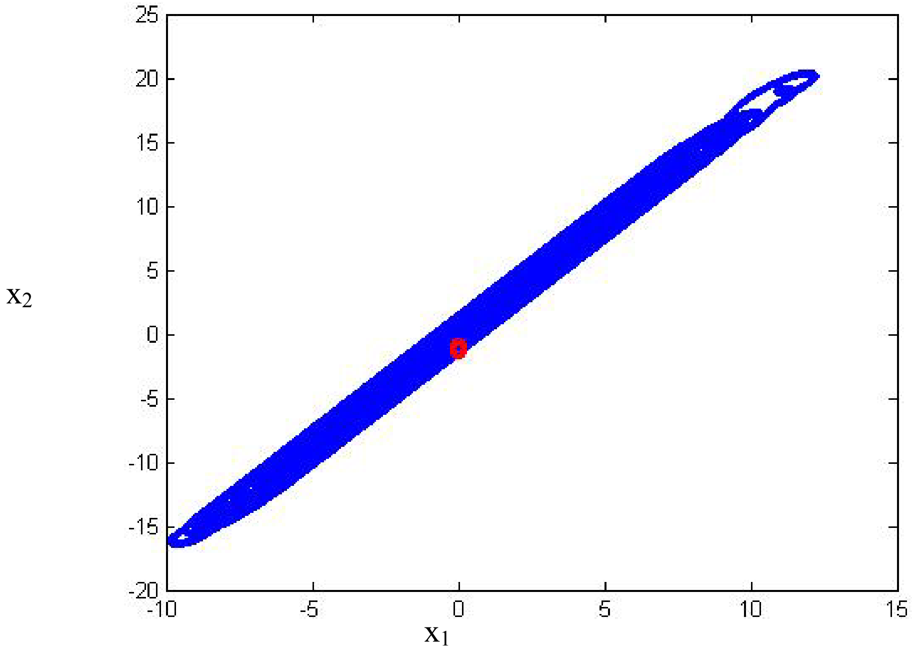


Fig. 21-3. An IMG chaotic attractor when the delay times τ_1=5, τ_2=0

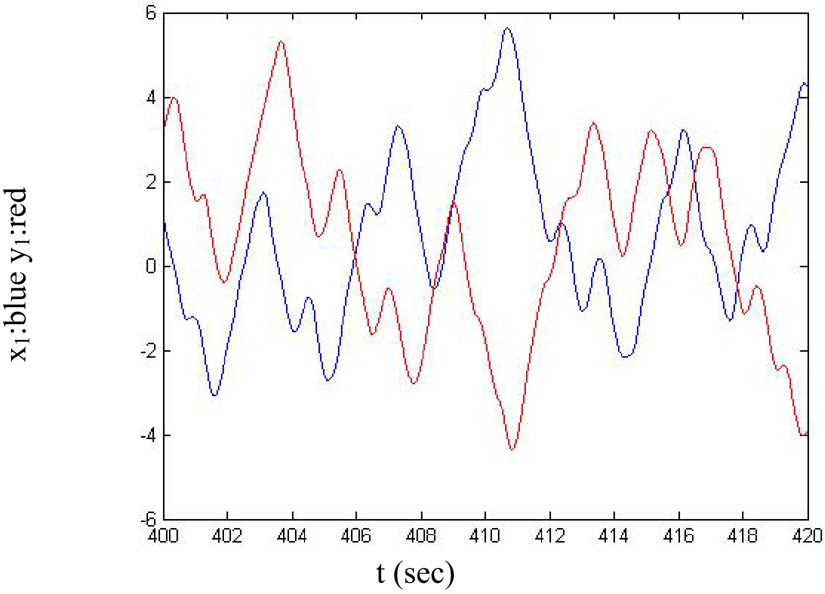


Fig. 21-4.Time response of the two identical IMG systems with $x_1(0)=1$, $x_2(0)=0$,$y_1(0)=-1$ and $y_2(0)=0.5$, when $\tau_2=0$.

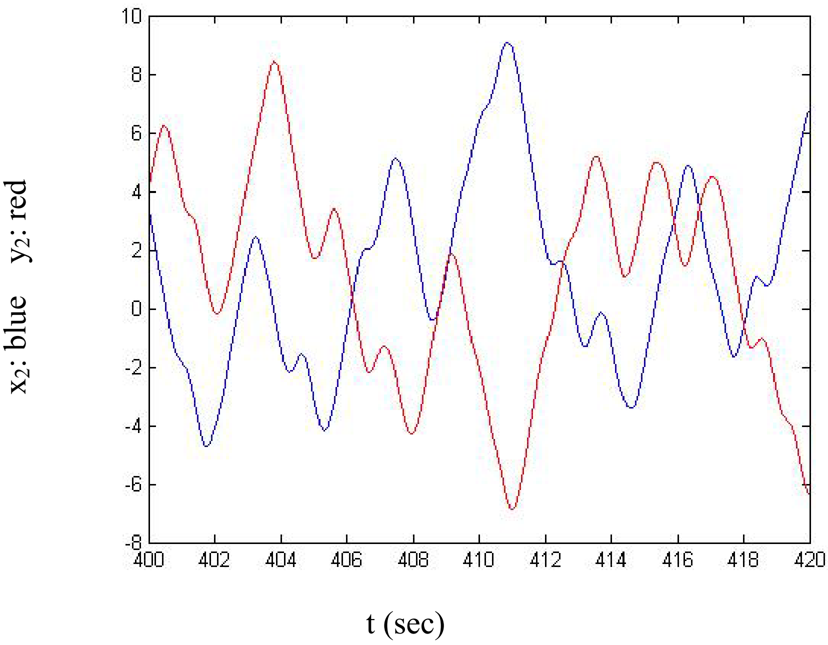
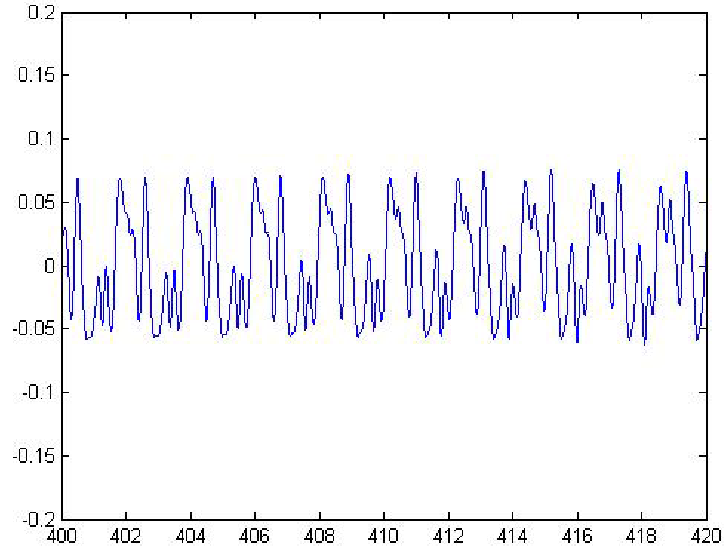


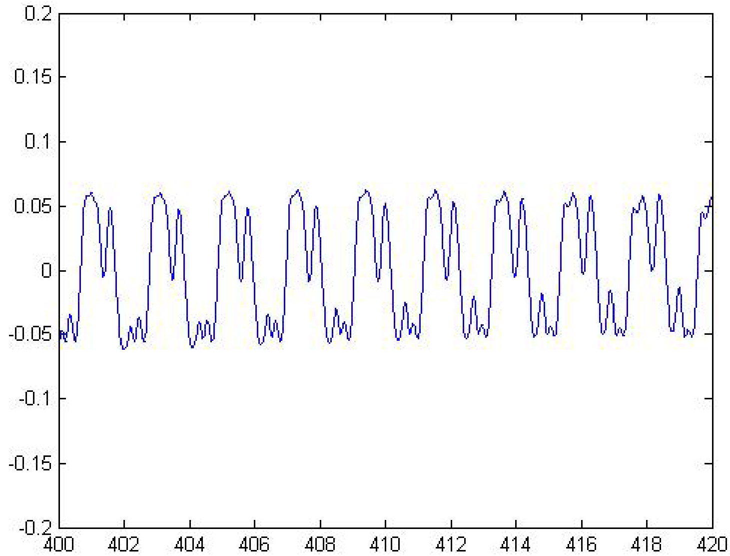
Fig. 21-5.Time response of the two identical IMG systems with $x_1(0)=1$, $x_2(0)=0$,$y_1(0)=-1$ and $y_2(0)=0.5$, when $\tau_2=0$.



e_1

t (sec)

Fig. 21-6. Error dynamics of the two identical IMG systems with $x_1(0)$=100, $x_2(0)$=10, $y_1(0)$=101 and $y_2(0)$=10.001, when $\tau_2 = 1$.



e_2

t (sec)

Fig. 21-7. Error dynamics of the two identical IMG systems with $x_1(0)$=100, $x_2(0)$=10, $y_1(0)$=101 and $y_2(0)$=10.001, when $\tau_2 = 1$.

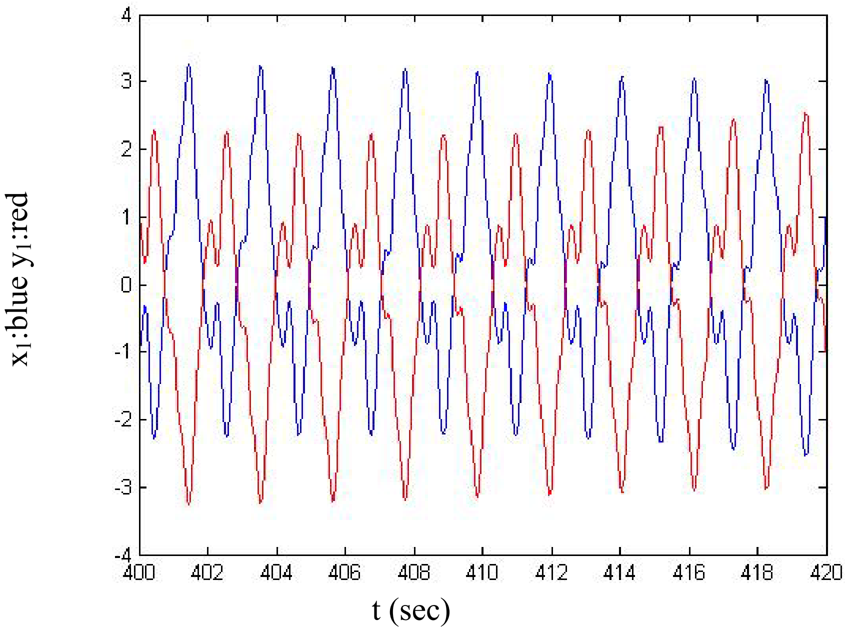


Fig. 21-8. Time response of the two identical Ikeda-Mackey Glass systems with $x_1(0)=1$, $x_2(0)=0$, $y_1(0)=-1$ and $y_2(0)=0$, when $\tau_2 = 1$.

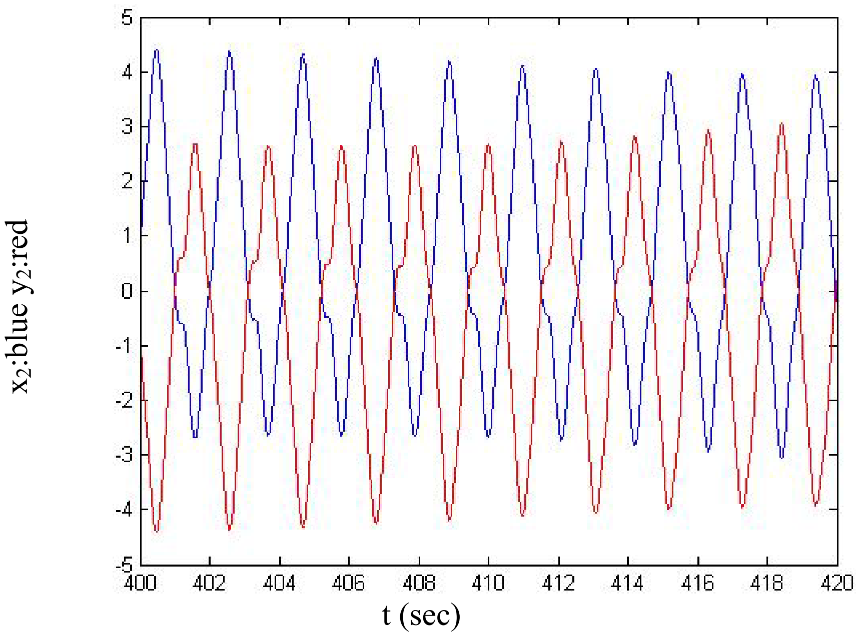


Fig. 21-9. Time response of the two identical Ikeda-Mackey Glass systems with $x_1(0)=1$, $x_2(0)=0$, $y_1(0)=-1$ and $y_2(0)=0$, when $\tau_2 = 1$.

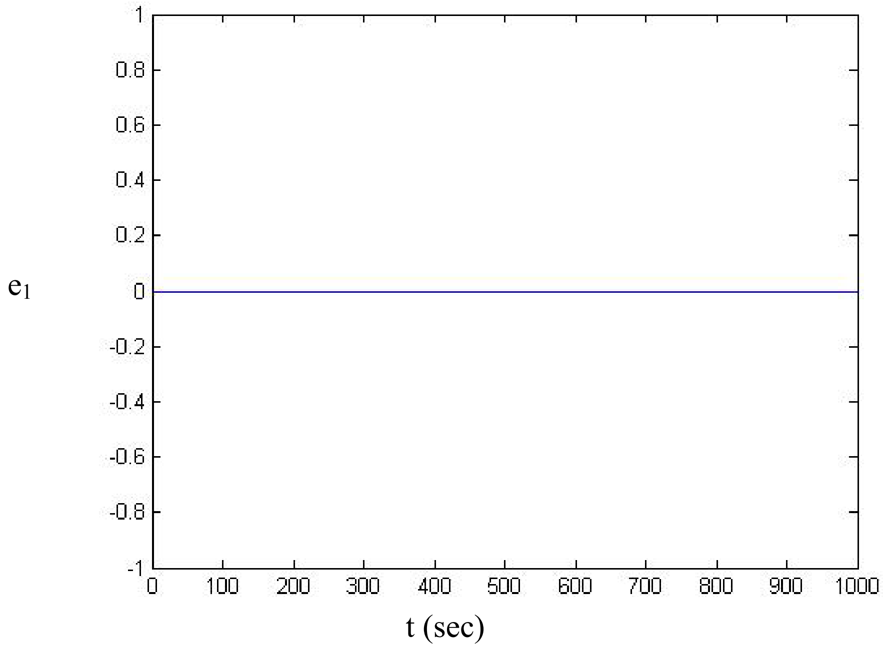


Fig. 21-10. Error dynamics of the two identical IMG systems with $x_1(0)=1$, $x_2(0)=0$, $y_1(0)=-1$ and $y_2(0)=0$, when $\tau_2 = 1$.

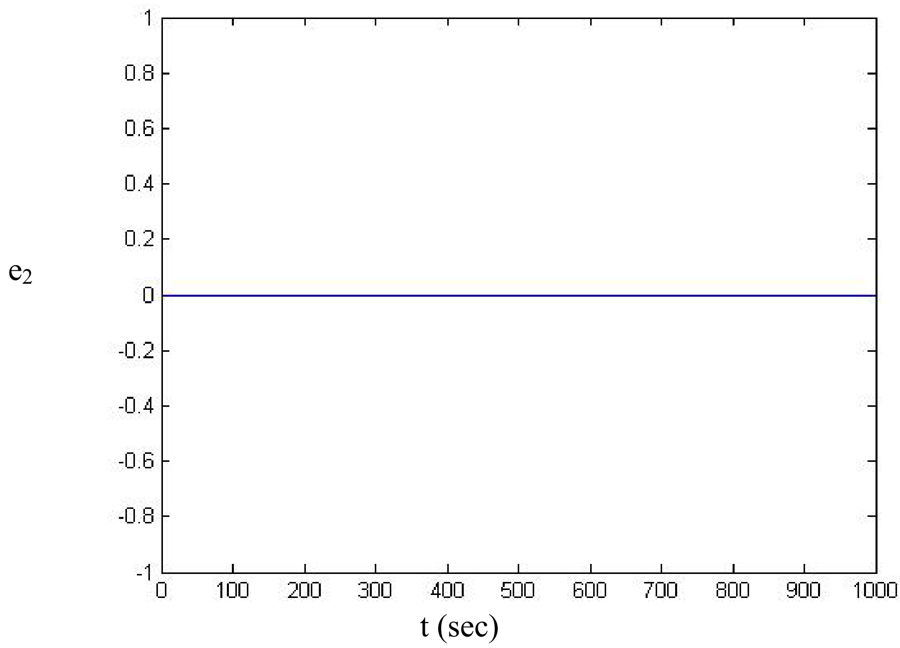


Fig. 21-11. Error dynamics of the two identical IMG systems with $x_1(0)=1$, $x_2(0)=0$, $y_1(0)=-1$ and $y_2(0)=0$, when $\tau_2 = 1$.

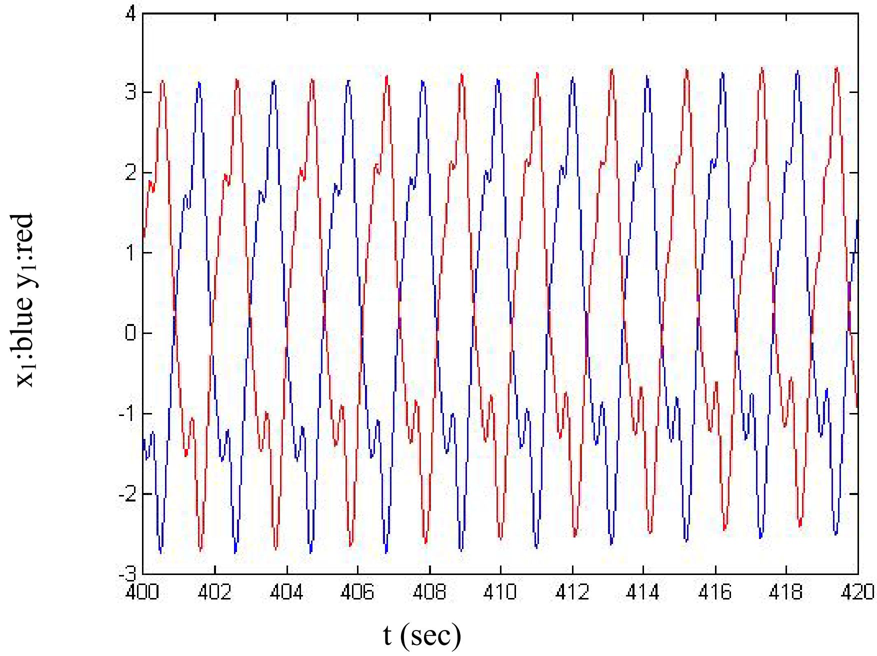


Fig. 21-12. Time response of the two identical IMG systems with $x_1(0)=1$, $x_2(0)=0.1$, $y_1(0)=-1$ and $y_2(0)=0.5$, when $\tau_2 = 1$, $\mu_1 = 1.2427$ sec.

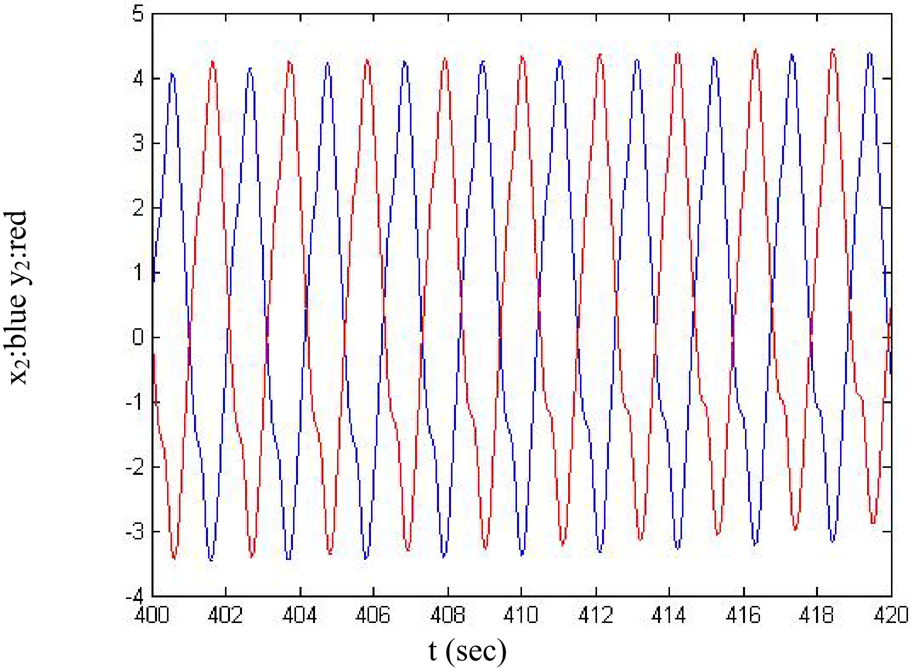


Fig. 21-13. Time response of the two identical IMG systems with $x_1(0)=1$, $x_2(0)=0.1$, $y_1(0)=-1$ and $y_2(0)=0.5$, when $\tau_2 = 1$, $\mu_2 = 1.08$ sec.

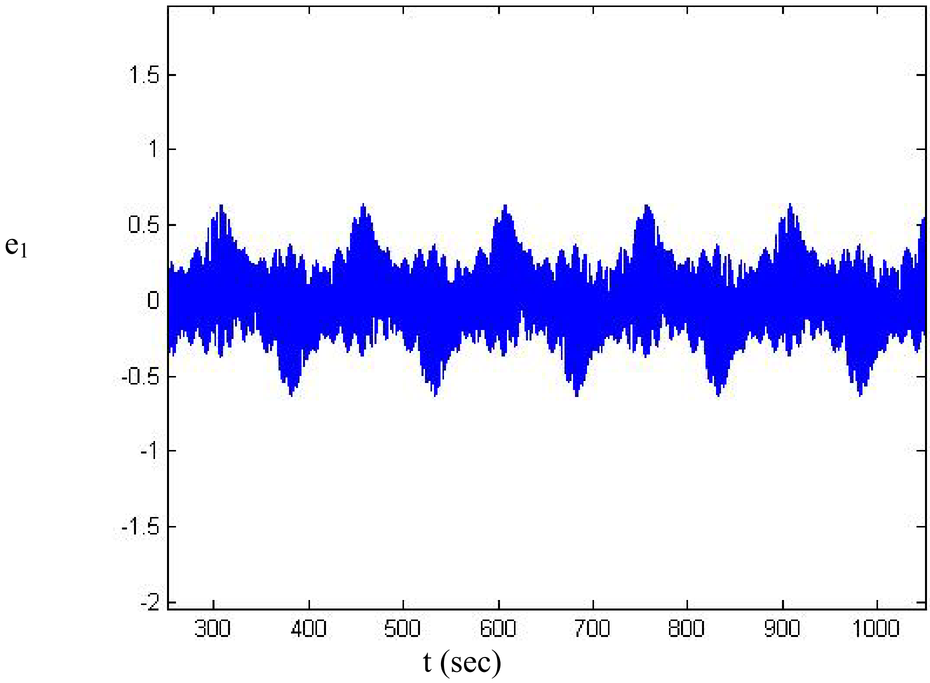


Fig. 21-14. Error dynamics of the two identical IMG systems with $x_1(0)=1$, $x_2(0)=0.1$, $y_1(0)=-1$ and $y_2(0)=0.5$, when $\tau_2 = 1$.

Chapter 22

Unbounded Chaos

22.1 Preliminaries

Unbounded chaos with three positive Lyapunov exponents in four dimensional phase space is found for a modified mechanical tachometer.

Chaos is defined as the phenomenon of occurence of *bounded* nonperiodic evolution in completely deterministic nonlinear dynamical system with high sensitive dependence on initial conditional [1]. Other definitions are, for instance: (a) Chaos is recurrent motion in simple systems or low-dimensional behaviour that has some random aspect as well as a certain order. Exponential divergence from adjacent starts while remaining in a *bounded* region of phase space is a signature of chaotic motion. [2] (b) An asymptotic motion that is not an equilibrium point, periodic, or quasi- periodic is often called chaotic. · · · Additionally, we require that a chaotic motion is a *bounded* asymptotic solution that possesses sensitive dependence on initial conditions. [3]

Unbounded chaos is found for a modified mechanical tachometer system. The tachometer system considered is shown in Fig.1 [4]. The masses of the rods and vertical axis O_1O_2 are neglected, and ball A and B are assumed as particles with equal mass m_1. The vertical axis rotates with constant speed η and is subjected to a vertical vibration $A\sin x_3$ where x_3 is state variable , A is the amplitude of vibration. m_2 is the mass of the sleeve C, l is the length of rod BC, 2l is the length of AB. ϕ is the angle between rod AB and vertical axis O_1O_2, k_1 is the spring constant of a restoring spiral spring which is used to restrain the angle ϕ caused by centrifugal forces of A and B, k_2 is the viscous damping coefficient caused by friction in the bearings. Let $x_1=\phi$, $x_2=\dot{\phi}$, $x_4=\dot{x}_3$.

22.2 Equation

By Lagrange equation, we obtain the state equations for the autonomous tachometer system:

$$\begin{cases} \frac{dx_1}{dt} = x_2 \\ \frac{dx_2}{dt} = \frac{1}{2m_1 + 4m_2 \sin^2 x_1} \left(\frac{-2m_2 g \sin x_1}{l} + \frac{2m_2 A \sin x_3 \sin x_1}{l} - 4m_2 x_2^2 \sin x_1 \cos x_1 \right. \\ \qquad \left. + 2m_1 \sin x_1 \cos x_1 \eta^2 - \frac{k_1 x_1}{l^2} - \frac{k_2 x_2}{l^2} \right) \\ \frac{dx_3}{dt} = x_4 \\ \frac{dx_4}{dt} = -A \sin x_3 \end{cases} \qquad (22\text{-}2\text{-}1)$$

The third and fourth equation of system (1) give a vibration system. When A=0, at steady state, given η corresponds to a definite η, therefore this system can be used as a tachometer. Up to now, the tachometer system has clear mechanical explanation. Now we modify the third equation of system (1) and the system becomes:

$$\begin{cases} \frac{dx_1}{dt} = x_2 \\ \frac{dx_2}{dt} = \frac{1}{2m_1 + 4m_2 \sin^2 x_1} \left(\frac{-2m_2 g \sin x_1}{l} + \frac{2m_2 A \sin x_3 \sin x_1}{l} - 4m_2 x_2^2 \sin x_1 \cos x_1 \right. \\ \qquad \left. + 2m_1 \sin x_1 \cos x_1 \eta^2 - \frac{k_1 x_1}{l^2} - \frac{k_2 x_2}{l^2} \right) \\ \frac{dx_3}{dt} = x_4 + kx_1 \\ \frac{dx_4}{dt} = -A \sin x_3 \end{cases} \qquad (22\text{-}2\text{-}2)$$

The term kx_1 would be an unusual driving term forcing the velocity $\frac{dx_3}{dt}$. However it might be realized in an electrical system. We choose m_1=3, m_2=3, g=9.8, l=1.5, A=5, k_1=4, k_2=1, k=8, η=18.81. Taking η as abscissa, the Lyapunov exponents diagram is shown as Fig 2 . Hyperchaos [5] with two positive LE is found. Uncommon hyperchaos with three positive LE is also found for this four state autonomous system. Phase portraits, time history, bifurcation diagram and power spectrum are presented in Fig 3~7. In Fig 5(c), it is noted that x_3 is unbounded, while in Fig 5(a),(b),(d), x_1, x_2, x_4 are bounded. Therefore a unbounded chaos is found in the four dimensional state space. Our conclusion is that in the definitions of chaos quoted at the beginning of this paper, the word "*bounded*" should be deleted.

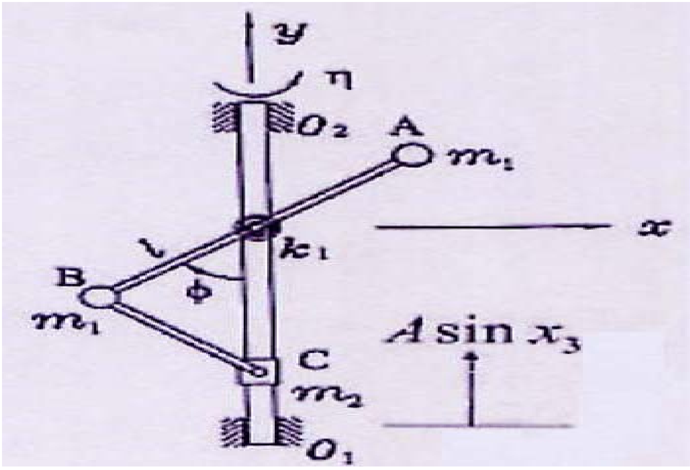
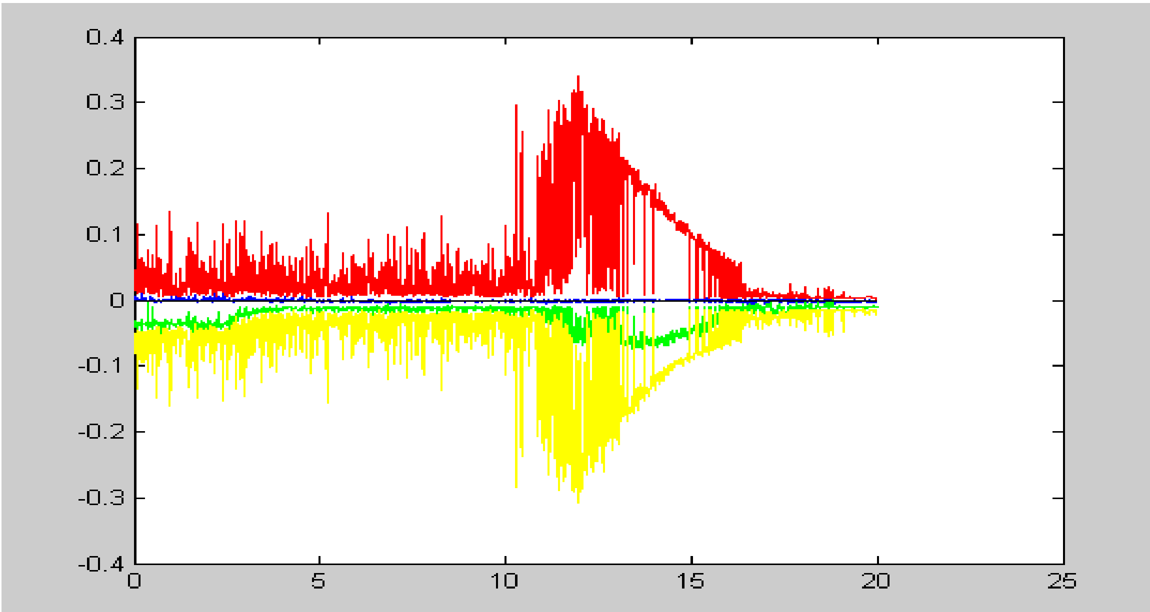
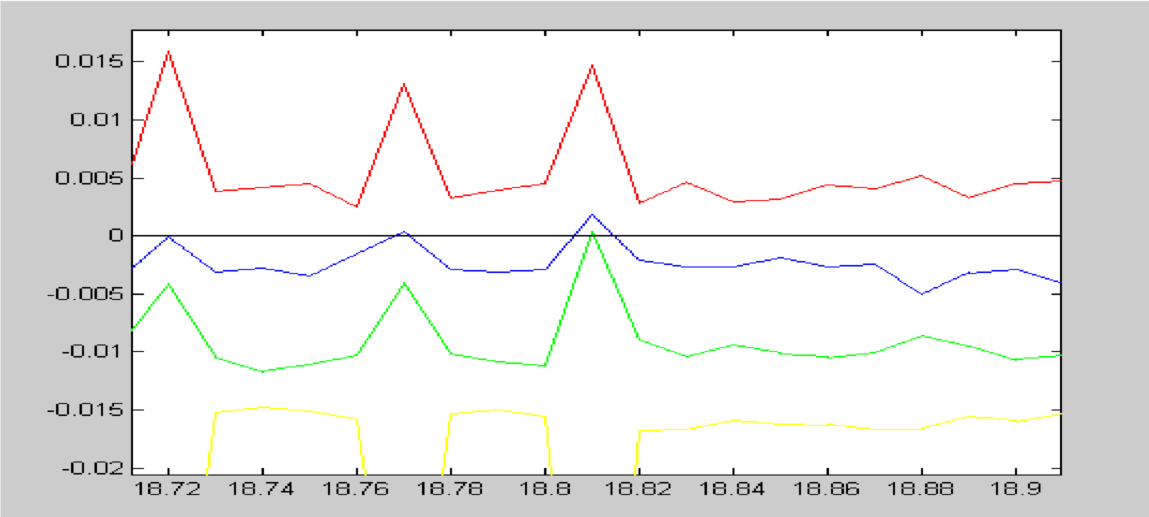


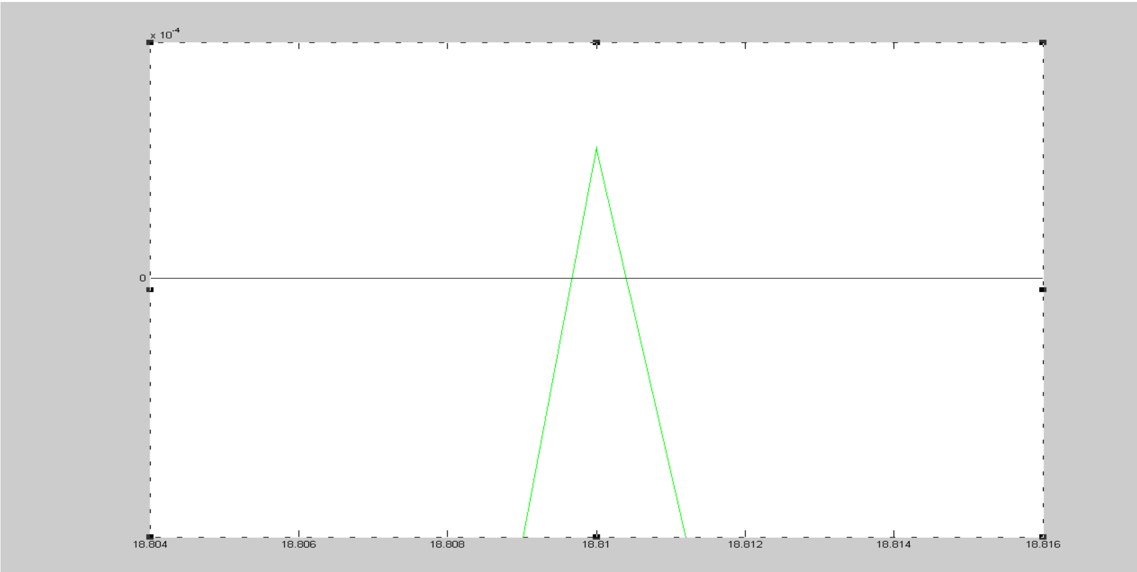
Fig 22-1 ϕ is the angle between rod AB and vertical axis O_1O_2, k_1 is the spring constant of a restoring spiral spring which is used to restrain the angle ϕ caused by centrifugal forces of A and B, k_2 isthe viscous damping coefficient caused by friction in the bearings. Let $x_1 = \phi$, $x_2 = \dot{\phi}$, $x_4 = \dot{x}_3$.



(a)

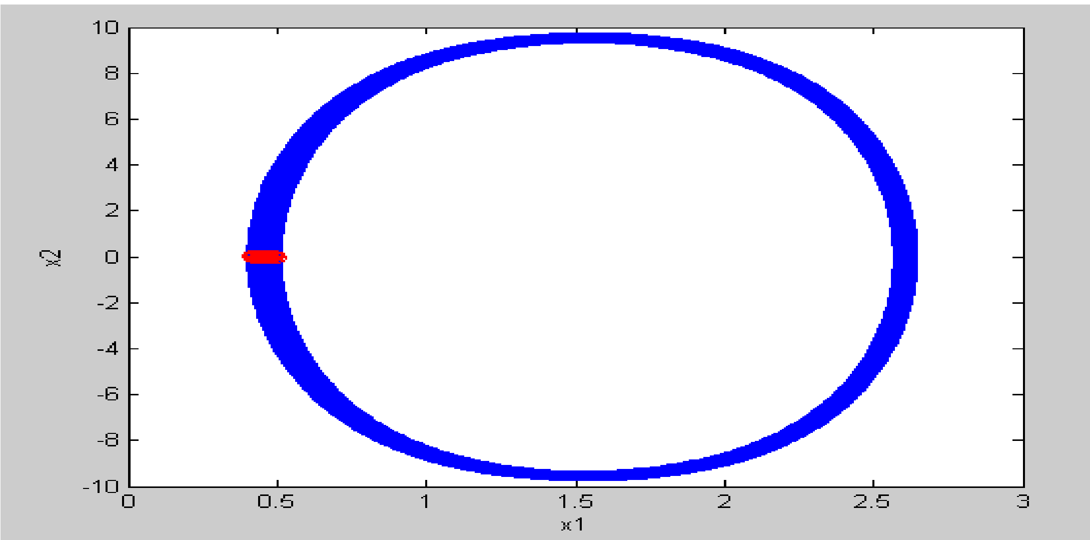


(b)

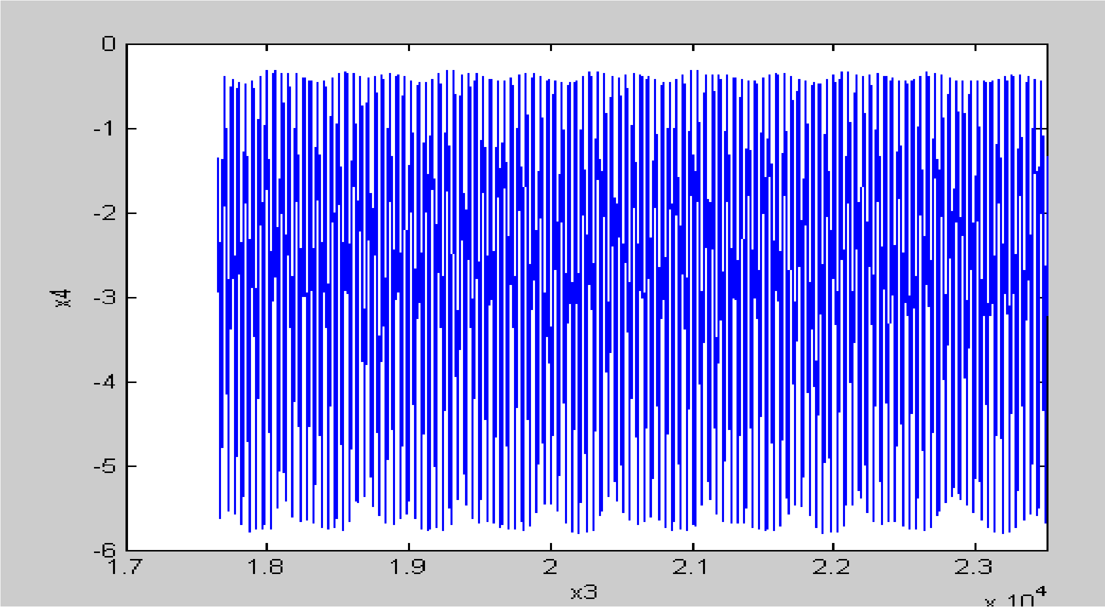


(c)

Fig 22-2 --(a)Lyapunov exponents, (b)(c) Enlarged parts

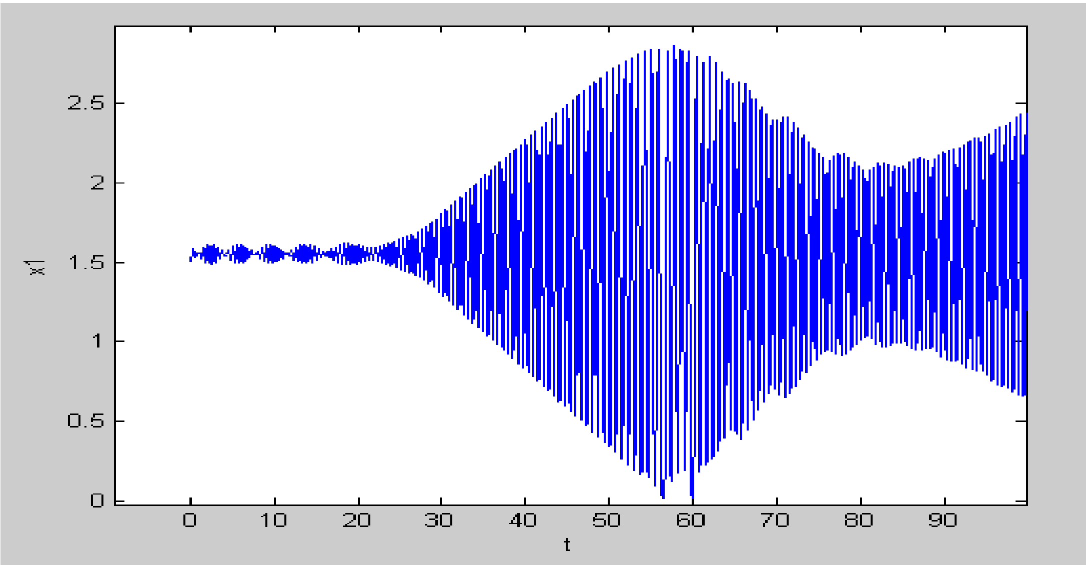


(a)

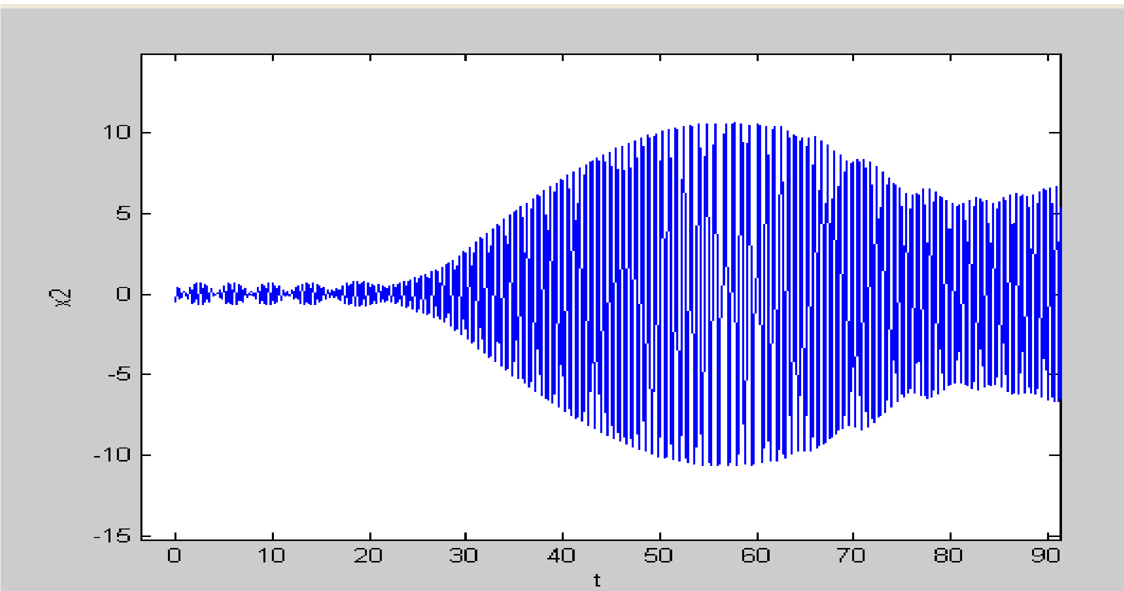


(b)

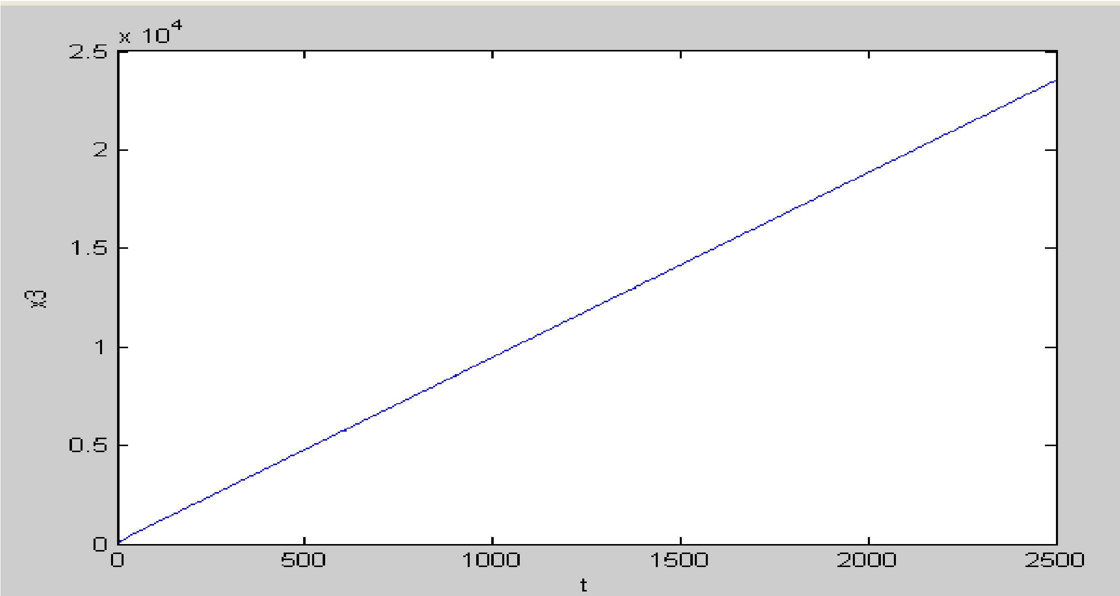
Fig 22-3 Phase portraits with η=18.81



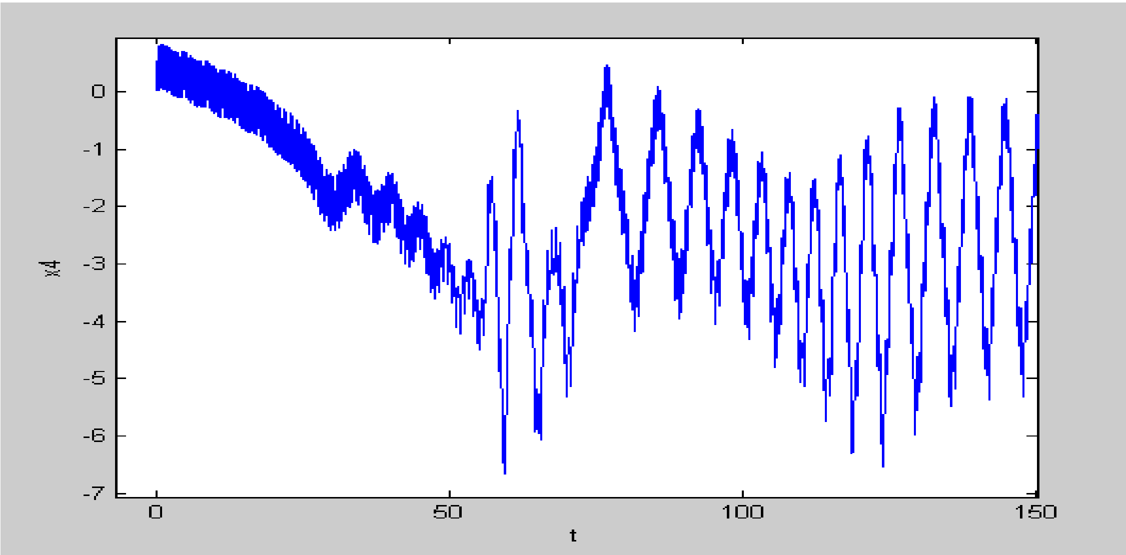
(a)



(b)



(c)



(d)

Fig 22-5 Time histories with η=18.81

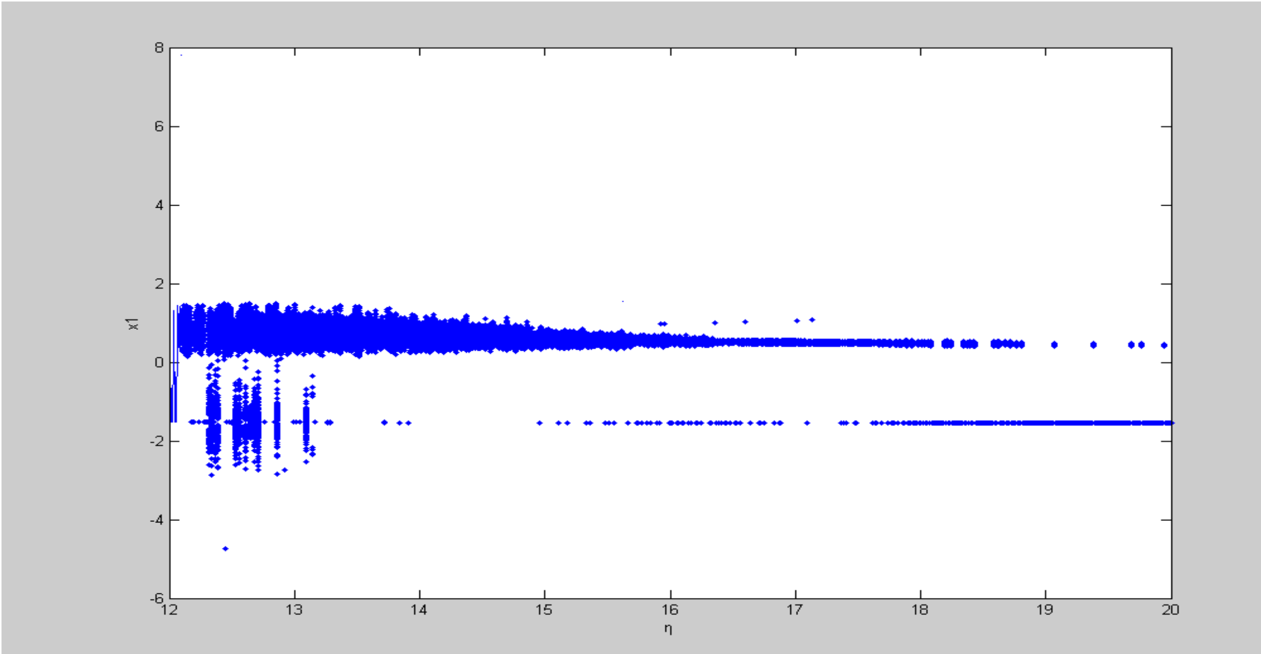


Fig 22-6　Bifurcation diagram with ηfrom 12 to 20

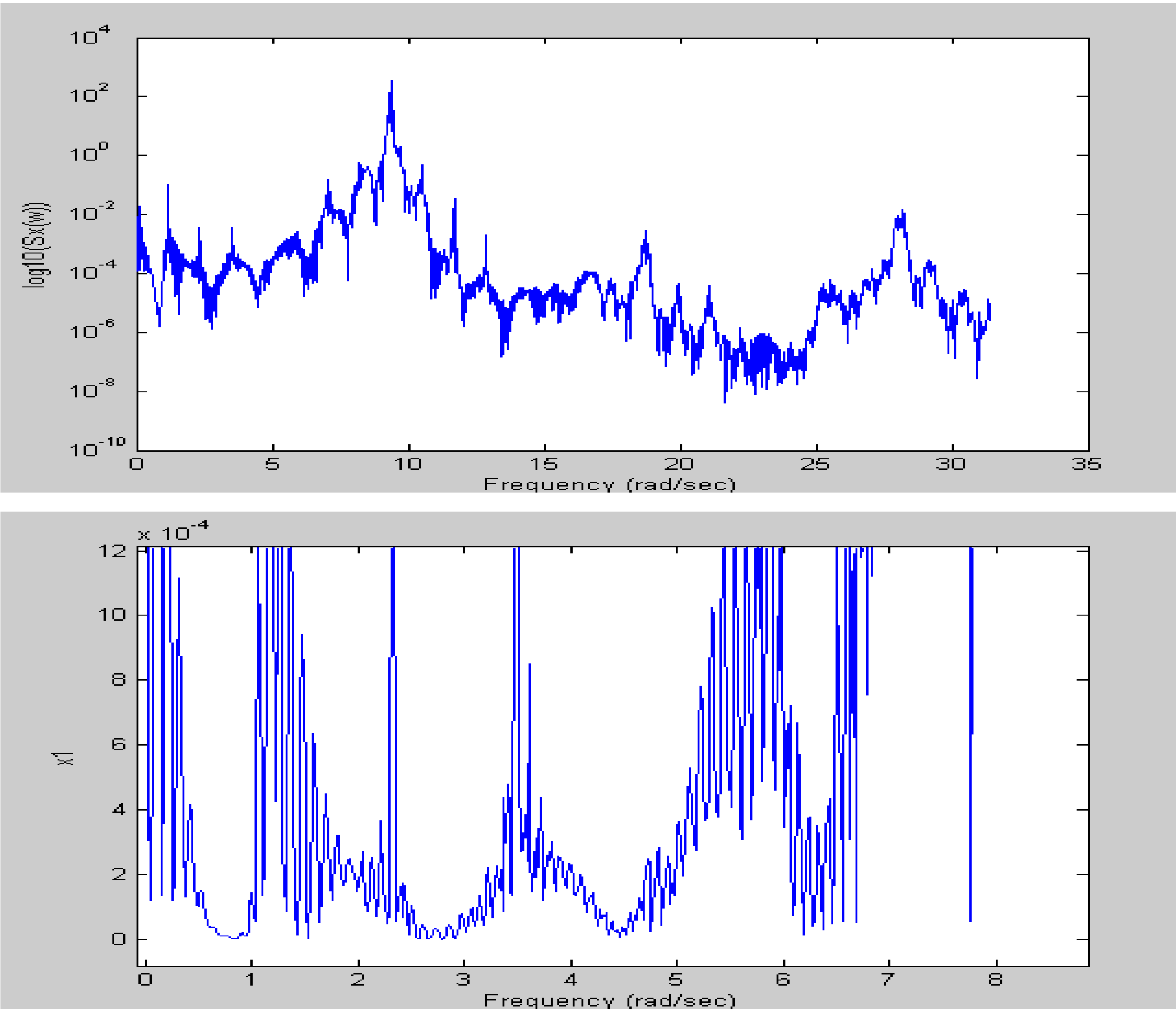


Fig 22-7　Power spectrum with η=18.81

Chapter 23
Conclusions

In this report, the generalized synchronization of new chaotic systems by pure error dynamics and elaborate Lyapunov function, chaos of nonholonomic systems, non-simultaneous symplectic synchronization of different chaotic systems with variable scale time, double symplectic synchronization of different chaotic systems, chaos and chaos synchronization of double Duffing system, chaos and chaos synchronization of double van der Pol system, chaos and chaos synchronization of double Ikeda system, chaos and chaos synchronization of double Macky Glass system, are studied.

Chapter 2 contains the dynamics of new autonomous and new nonautonomous chaotic systems. The system model and the numerical results of regular and chaotic phenomena are presented.

In Chapter 3, the generalized synchronization is studied by applying pure error dynamics and elaborate Lyapunov function. In Chapter 4, by applying pure error dynamics and elaborate nondiagonal Lyapunov function, the nonlinear generalized synchronization is achieved. The methods give rigorous theories for generalized synchronization and nonlinear generalized synchronization and greatly extend the use of various forms of Lyapunov function while current method only gives semi-simulation theory for generalized synchronization, in which the maximum values of state variables must be given by simulation, and monotonous square sum Lyapunov function is used. By the systematic procedures, the complexity of designing suitable elaborate Lyapunov function and elaborate nondiagonal Lyapunov function is reduced greatly. The proposed methods are effectively applied to both new autonomous and new nonautonomous chaotic systems.

Complete identification of chaos in nonholonomic systems and nonlinear nonholonomic systems is firstly presented in Chapter 5 and Chapter 6. The scope of chaos study has been extended to nonholonomic systems and nonlinear nonholonomic system. By applying the fundamental nonholonomic form of Lagrange's equations, the chaos of two nonholonomic moving target pursuit systems is studied in Chapter 5, in which nonholonomic pursuit system with a straightly oscillating target and nonholonomic pursuit system with a circularly rotating target are investigated. In Chapter 6, chaos of nonlinear nonholonomic problem, the magnitude of velocity keeping constant, is studied by applying the nonlinear nonholonomic form of Lagrange's equations. Complete identification of chaotic phenomena is obtained in nonlinear nonholonomic system by Lyapunov exponents, phase portraits, Poincaré maps, and bifurcation diagrams. Futhermore, the Feigenbaum number rule still holds for nonlinear nonholonomic system.

In Chapter 7, the non-simultaneous symplectic synchronization with variable scale time, $\mathbf{y}(t) = \mathbf{F}(\mathbf{x}(\tau), \mathbf{y}(t), t)$, is studied. By applying adaptive control, the non-simultaneous symplectic synchronization is achieved and the estimated Lipschitz constant is much less than the Lipschitz constant obtained by applying nonlinear control. This result in the reduction of the gain of the controller, i.e. the cost of controller is reduced. The simulation results show that the proposed scheme is feasible for both autonomous and nonautonomous chaotic systems, whether the dimensions of $\mathbf{x}(\tau)$ and $\mathbf{y}(t)$ are the same or not. Furthermore, when applying the non-simultaneous symplectic synchronization in secret communication, since the functional relation of the non-simultaneous symplectic synchronization is more complex than that of traditional generalized synchronization, and cracking the variable scale time τ is an extra task for the attackers in addition to cracking the system model and cracking the functional relation, the non-simultaneous symplectic synchronization may be applied to increase the security of secret communication.

In Chapter 8, the double symplectic synchronization, $\mathbf{G}(\mathbf{x}, \mathbf{y}) = \mathbf{F}(\mathbf{x}, \mathbf{y}, t)$, is studied. It is an extension of symplectic synchronization, $\mathbf{y} = \mathbf{F}(\mathbf{x}, \mathbf{y}, t)$. By applying active control, the double symplectic synchronization is achieved. By simulation results, it is shown that the proposed scheme is effective and feasible for both autonomous and nonautonomous chaotic systems. Furthermore, the double symplectic synchronization may be applied to increase the security of secret communication due to the complexity of its synchronization form.

In Chapter 9, we have studied the chaos in the integral order and fractional order double Duffing system by phase portraits, Poincaré maps and bifurcation diagrams. The total orders of the system for the existence of chaos are 0.1 to 0.7 and 1.

In Chapter 10, parameter excited chaos synchronizations of two identical double Duffing systems are studied by adjusting the strengths of the substituting state variables. Numerical simulations are illustrated for CS or AS of which the occurrence depends on initial conditions and driving strength. Besides, alternative CS and AS is also discovered with same initial conditions and same driving strengths.

In Chapter 11, synchronization and antisynchronization scheme based on the substitution of the corresponding parameters in two identical chaotic double Duffing systems by a white noise, a Rayleigh noise, a Rician noise or a uniform noise respectively. For the white noise case, neither CS and AS are found. For the Rayleigh noise case, CS and AS are obtained for different noise strengths. For the Rician noise case and the uniform noise case, only AS is obtained. Numerical simulations show that whether CS or AS occurs is sensitive to the noise strength.

In Chapter 12, a new scheme to achieve the pragmatical generalized synchronization of adaptive control via the pragmatical asymptotical stability theorem is gavien. By the procedure of the proposed scheme, two double Duffing systems and a double van der Pol system are used as master system, slave system, and goal system, respectively. The validity of this approach is verified theoretically and numerically. Based on pragmatical asymptotical stability theorem, using this theorem, we can obtain the generalized synchronization of chaotic systems and prove that the estimated parameters approach the uncertain values.

In Chapter 13, chaos in double van der Pol system and in its fractional order systems is studied. It is found that with reducing the total derivative order $\alpha_1+\beta_1+\alpha_2+\beta_2$ the ranges of the chaotic phase portraits of the system decrease and its shape changes differently for different choices of parameters.Twenty-one chaotic cases for $0.4 \le (\alpha_1+\beta_1+\alpha_2+\beta_2) \le 4.0$ are studied, and the lowest total order for chaos existence in the system is found to be 0.4. Thirty nonchaotic cases are found.

In Chapter 14, the variable with adjustable strength of a third double van der Pol system substituted for the strength of two corresponding mutual coupling terms of two uncoupled identical chaotic double van der Pol system, gives rise to their synchronization or anti-synchronization. Both CS and AS can be achived by adjusting the strength of the substituted variable and the initial conditions.

In Chapter 15, complete synchronization and antisynchronization scheme based on the substitution of two same parameters in two identical chaotic double van der Pol systems by a white noise, a Rayleigh noise respectively. For the white noise case and Rayleigh noise case, CS and AS are obtained for different noise strengths and initial conditions. Numerical simulations show that whether CS or AS occurs is sensitive to the noise strength.

In Chapter 16, controling chaotic systems to different systems is studied by new pragmatical adaptive control method. The pragmatical asymptotical stability theorem fills the vacancy between the actual asymptotical stability and mathematical asymptotical stability, the conditions of the Lyapunov function for pragmatical asymptotical stability are lower than that for traditional asymptotical stability. By using this theorem, with the same conditions for Lyapunov function, $V > 0$, $\dot{V} \le 0$, as that in current scheme of adaptive chaos control, we not only obtain the adaptive control of chaotic systems but so prove that the estimated parameters approach the uncertain values. Traditional chaos control is limited for the same system. This method enlarges the function of chaos control. We can control a chaotic system to a given chaotic system. The method also downhill simplex the controllers and reduce their cost.

In Chapter 17, the chaos in integral and fractional order double Ikeda systems with total order of derivatives from 2 to 0.2 are studied by phase portraits, Poincaré maps and bifurcation diagrams. It is found that chaos exists in all cases.

In Chapter 18, the chaotic behaviors of double Ikeda systems are obtained by replacing their delay time by a function of chaotic state variables of a second chaotic system. It is found that chaos exists for Case 1, 3, 5, 6. The chaotization of a double Ikeda system is studied by using a function of state variable of a second identical system to replace a parameter of the first system. It is found that in Case 9, 10, 11, chaotization exists.

In Chapter 19, lag or anticipated synchronization and the lag or anticipated anti-synchronization of two double Ikeda systems with different initial conditions are discovered. There are two situations in all possible initial conditions. Cases 1~8 are the lag or anticipated synchronizations. Cases 9~16 are the lag or anticipated anti-synchronizations.

In Chapter 20, robust lag chaos synchronization, lag quasi-synchronization and chaos control of two uncoupled double Ikeda system, are achieved by replacing the corresponding parameters of two systems by different chaotic state variables of a third chaotic system. Robustness of synchronization is studied by addition of various noises. The results are satisfactory.

In Chapter 21, first, we introduce the definition and approximation of fractional order operator briefly. Then the double Mackey-Glass delay systems in integral and fractional forms are described. We find the chaos which exists in the integral system and in fractional systems with orders 0.9, 0.8, 0.1 by phase portraits and the bifurcation diagrams.

In Chapter 22, we apply the parameter excited method to control the double Mackey-Glass system and to synchronize two uncoupled double Mackey-Glass systems. By replacing the corresponding parameters of chaotic system with noise, chaos control and chaos synchronization can be accomplished. This method is effective to synchronize two systems, for which coupling method of synchronization is difficult or even impossible. Finally, numerical simulations show the proposed method is effective to suppress the chaotic behavior and drag the trajectories to the origin. Also, chaos synchronizations are successfully achieved in many cases with Rayleigh noise Rician noise, and uniform noise respectively.

In Chapter 23, temporary lag or anticipated synchronization and the lag or anticipated anti-synchronization of double Mackey-Glass systems with small and similar initial conditions are discovered. For the first interval of TLS, when all initial values are positive, temporary lag synchronizations are found. The trajectory will be reversed if the initial condition of x_1 or y_1 is negative. In these cases, the lag or anticipated anti-synchronization exists. From the results of simulation, we find six temporary lag (anticipated) synchronization intervals in 30000seconds. Although the numerical simulations of temporary lag and anticipated synchronization and anti-synchronization are showed in this . However, the theoretical analysis and its applications should be open for further work in the future.

In Chapter 24, the parameter excited method is applied to synchronize two uncoupled double Mackey-Glass systems. By replacing the corresponding parameters with a Rayleigh noise and choose the appropriate noise strength, the lag synchronization can be successfully obtained. Temporary lag synchronization, partial lag synchronization, chaos control and robustness of lag synchronization are also obtained. The abundance of various phenomena fully exhibits the potential application of this method.

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計畫成果自評

研究成果之學術或應用價值：

Mathieu系統、van der Pol系統、Ikeda系統及Mackey-Glass系統皆為經典之系統。經巧妙之取代與結合既可獲得自治新Mathieu−van der Pol系統，自治新Ikeda-Mackey Glass系統亦可得非自治之新Mathieu−van der Pol系統與非自治新Ikeda-Mackey Glass系統大大擴大了經典渾沌系統的範圍及研究領域。而新的非耦合渾沌同步方法具有很大的機動性及發展潛力。取代之參數可以變化多端，採用之取代函數更是取之不盡。對用於秘密通訊而言，機密性大為增加。Duffing系統、van der Pol系統及Mathieu系統皆為經典之系統。經巧妙之取代與結合既可獲得自治新Duffing−van der Pol系統，自治新Mathieu-Duffing系統亦可得非自治之新Duffing−van der Pol系統與非自治新Mathieu-Duffing系統大大擴大了經典渾沌系統的範圍及研究領域。而不同系統實用渾沌適應控制新方法具有很大的發展潛力，涉及的各系統參數可以都是未知參數。大大地擴展了渾沌控制反控制的能力。取代之參數可以變化多端，採用之取代函數更是取之不盡。對用於秘密通訊而言，機密性大為增加。

本計畫之研究不僅對渾沌動力學學科中最重要最典型的四種渾沌系統的研究的拓廣與深化，更重要的是它們本身顯然具有更複雜的，未經發現的複雜渾沌行為，本研究對渾沌動力學學科具重大意義。其應用於機械、電機、物理、化學、生科、奈米之耦合系統，具有重要的實用價值。

渾沌同步除本身之重要理論價值外，其研究在秘密通訊、神經網路、自我組織等方面有日益廣泛之應用。廣義渾沌同步則為渾沌同步之進一步發展，其應用亦方興未艾。本計畫提出三種新的渾沌同步。實用適應廣義同步法糾正了目前國際文獻中未經証明即認為估值參數趨於為之參數之錯誤，首次在渾沌同步中引入概率概念，具重大理論及實用意義。由於$\dot{V}$之要求降低，實際應用亦較易實現。純誤差穩定的廣義同步，則彌補了國際文獻中需用數值計算結果為條件之理論，即有缺陷之理論。在理論與實用上有重要意義。不同起始條件的延遲同步等多種渾沌同步則為新發現的渾沌運動之現象，特別是 Ikeda 系統的永遠性延遲同步或反同步，不同於傳統理論，尤具重大意義。

達成預期目標情況：

第一年：Mathieu-van der Pol 系統與 Ikeda- Mackey-Glass 系統的渾沌行為與新式非耦合渾沌同步新方法之研究，並探討對此二系統之應用。

1. 完成採用諸多相圖、分歧圖、功率譜圖、參數圖及李亞普諾夫指數及碎形維度等研究自治的 Mathieu-van der Pol 系統與 Ikeda- Mackey-Glass 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。(二個月)
2. 完成採用諸多相圖、分歧圖、功率譜圖、參數圖及李亞普諾夫指數及碎形維度等研究非自治的 Mathieu-van der Pol 系統與 Ikeda- Mackey-Glass 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。(二個月)
3. 完成研究新式非耦合渾沌同步新方法之理論。(三個月)
4. 完成研究新式非耦合渾沌同步新方法對自治的 Mathieu-van der Pol 系統與 Ikeda-Mackey-Glass 系統之應用。(二個月)
5. 完成研究新式非耦合渾沌同步新方法對非自治的 Mathieu-van der Pol 系統與 Ikeda-Mackey-Glass 系統之應用。(二個月)
6. 完成撰寫年度報告書。(一個月)

第二年：Duffing-van der Pol 系統與 Mathieu-Duffing 系統之渾沌行為與實用渾沌適應控制反控制新方法之研究，及對此二系統的應用

1. 完成採用諸多相圖、分歧圖、功率譜圖、參數圖、李亞普諾夫指數及碎形維度等研究 Duffing-van der Pol 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。(二個月)
2. 完成採用諸多相圖、分歧圖、功率譜圖、參數圖、李亞普諾夫指數及碎形維度等研究 Mathieu-Duffing 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。(二個月)
3. 完成研究實用渾沌適應控制反控制新方法之理論。(三個月)
4. 完成研究實用渾沌適應控制反控制新方法對雙 Duffing-van der Pol 系統之應用。(二個月)
5. 完成研究實用渾沌適應控制反控制新方法對雙 Mathieu-Duffing 系統之應用。(二個月)

6. 完成撰寫年度報告書。(一個月)

第三年：兩種慣性測速器新系統的渾沌行為與指數渾沌同步新方法之研究。

1. 完成採用諸多相圖、分歧圖、功率譜圖、參數圖研究第一種種慣性測速器新系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。(二個月)
2. 完成採用諸多相圖、分歧圖、功率譜圖、參數圖研究第二種慣性測速器新系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。(二個月)
3. 完成研究指數渾沌同步新方法之理論。(一個月)
3. 完成研究在各種不同起始條件下，第一種慣性測速器新系統之延遲或預期同步。(二個月)
4. 完成研究在各種不同起始條件下，第二種慣性測速器新系統之延遲或預期同步。(二個月)
5. 完成研究在各種不同起始條件下，第一種慣性測速器新系統之延遲或預期反同步。(一個月)
6. 完成研究在各種不同起始條件下，第二種慣性測速器新系統之延遲或預期反同步。(一個月)
8. 完成撰寫年度報告書。(一個月)

具體成果：

1. 本計畫完成自治新 Mathieu−van der Pol 系統，自治新 Ikeda-Mackey Glass 系統，自治新 Duffing−van der Pol 系統，自治新 Mathieu-Duffing 系統共四種新型系統的渾沌行為之完整而詳盡之資料。本計畫更發展出六種新型的渾沌同步成果：(1)運用部分區域穩定定理更有效率的達成混沌系統同步。(2)運用部分區域穩定定理完成更有效率的渾沌控制。(3)運用實用漸進穩定定理達成渾沌系統的適應性同步。(4)運用部分區域穩定定理搭配實用漸進穩定定理發展高效率適應性控制方法，使渾沌系統控制更精準更快速的達到目標系統。(5) 導入 Besel function 於渾沌系統，激發更為複雜的渾沌現象，透過部份穩定定理及後進式控制方法完成系統同步。(6) 交織同步的發展及同步研究，增強秘密通訊的複雜度及實用性。
2. 所得結果在渾沌系統研究與應用方面具重要學術及實用價值。
3. 培養研究生科學研究能力以及透過觀念與推理來撰寫程式的能力。

研究內容寫成之期刊論文已有 31 篇，其中已有 7 篇被接受：

1. Zheng-Ming Ge and Shih-Yu Li, 2009, "Chaos Control of New Mathieu-Van der Pol Systems with New Mathieu -Duffing Systems as Functional System by GYC Partial Region Stability Theory", Nonlinear Analysis: Theory, Methods, and Applications, Vol. 71, pp. 4047-4059. (SCI, Impact Factor: 1.295)
2. Shih-Yu Li and Zheng-Ming Ge, 2009, "A Novel Study of Parity and Attractor in the Time Reversed Lorentz System", Physics Letter A, Vol. 373, pp, 4053-4059. (SCI, Impact Factor: 2.174).
3. Zheng-Ming Ge, Chun-Yen Ho, Shih-Yu Li and Ching Ming Chang, 2009 ,"Chaos control of new Ikeda–Lorenz systems by GYC partial region stability theory" Mathematical Methods in the Applied Sciences Vol. 32, pp. 1564-1584. (SCI, Impact Factor: 0.717)
4. Zheng-Ming Ge and Shih-Yu Li, 2010, "Fuzzy Modeling and Synchronization of Chaotic Quantum Cellular Neural Networks Nano System via A Novel Fuzzy Model and Its Implementation on Electronic Circuits" Journal of Computational and Theoretical Nanoscience, Vol. 7, pp. 1-10. (SCI, Impact Factor: 1.256)
5. Zheng-Ming Ge, Shih-Chung Li, Shih-Yu Li and Ching-Ming Chang, 2008, "Pragmatical adaptive chaos control from a new double van der Pol system to a new double Duffing system", Applied Mathematics and Computation 203 (2008) 513–522 (SCI, Impact factor: 1.778).
6. Zheng-Ming Ge and Shih-Yu Li, 2010, "Yang and Yin parameters in the Lorenz system", accepted by Nonlinear Dynamics. (SCI, Impact factor: 1.295).
7. Zheng-Ming Ge and Chun-Yu Chiang, 2008, "Chaos Control and Anticontrol of Tachometer System by GYC Partial Region Stability Theory", Proceedings of the Institution of Mechanical Engineering, Part C, Journal of Mechanical Engineering Science, Vol. 223, pp. 1069-1082. (SCI, Impact factor: 0.416)
8. Zheng-Ming Ge and Shih-Yu Li, 2009, "Pragmatical Adaptive Synchronization of Different Orders Chaotic Systems with All Uncertain Parameters via Nonlinear Control" submitted to Nonlinear Dynamics. (SCI, Impact Factor: 1.295)
9. Zheng-Ming Ge and Shih-Yu Li, 2009, "Generating Tri-Chaos Attractors with Three Positive Lyapunov Exponents in New Four Order System via Linear Coupling" submitted to Nonlinear Dynamics. (SCI, Impact Factor: 1.295)
10. Zheng-Ming Ge and Shih-Yu Li, 2009, "Chaos Generalized Synchronization of New Chaotic Systems by GYC Partial Region Stability Theory" submitted to International Journal of Control. (SCI, Impact Factor: 1.130)
11. Zheng-Ming Ge and Shih-Yu Li, 2008, "Chaos Generalized Synchronization of fuzzy chaotic systems by GYC Partial Region Stability Theory" submitted to Journal of the franklin institute. (SCI, Impact Factor: 0.616)
12. Zheng-Ming Ge and Shih-Yu Li, 2009, "Chaotic Motion on Electronic Circuits of Historical Lorenz System and Its Pragmatical Adaptive P-N Synchronization" submitted to Applied

Mathematics and Computation. (SCI, Impact Factor: 0.961)

13. Zheng-Ming Ge and Shih-Yu Li, 2009, “Chaotic Motions in the Real Fuzzy Electronic Circuits and Its synchronizing Circuits” submitted to IEEE Transactions on Circuits and Systems Part I: Regular Papers. (SCI, Impact Factor: 2.043)
14. Zheng-Ming Ge and Shih-Yu Li, 2010, “Fuzzy Modeling and Synchronization of Two Totally Different Chaotic Systems via Novel Fuzzy Model” submitted to IEEE Transactions on Systems Man and Cybernetics Part B- Cybernetics. (SCI, Impact Factor: 2.361)
15. Zheng-Ming Ge, Kai-Ming Hsu, Shih-Yu Li and Ching-Ming Chang, 2010, “Chaos Control of New Duffing-Van der Pol System by GYC Partial Region Stability Theory” revision submitted to Journal of Computational and Applied Mathematics. (SCI, Impact Factor: 1.048)
16. Zheng-Ming Ge and Kai-Ming Hsu, 2009, “Pragmatical Chaotic Symplectic Synchronization of New Duffing-Van der Pol Systems with Different Order System as Functional System by New Dynamic Surface Control and Adaptive Control” submitted to Optics Communications. (SCI, Impact Factor: 1.552)
17. Zheng-Ming Ge and Kai-Ming Hsu, 2008, “Pragmatical Hybrid Projective Chaotic Generalized Synchronization of Chaotic System with Uncertain Parameters by Adaptive Control” submitted to Nonlinear Analysis: Theory, Methods & Applications. (SCI, Impact Factor: 1.295)
18. Zheng-Ming Ge and Kai-Ming Hsu, 2009, “Chaos Generalized Synchronization of New Duffing-Van der Pol System by GYC Partial Region Stability Theory” submitted to Journal of Mathematical Analysis and Applications. (SCI, Impact Factor: 1.046)
19. Zheng-Ming Ge and Yan-Sian Li, 2008, “Symplectic Hybrid Projective Synchronization of Different Order Systems with New Control Lyapunov Function by Adaptive Backstepping Control” submitted to Physica D. (SCI, Impact Factor: 1.926)
20. Zheng-Ming Ge and Yan-Sian Li, 2008, “Chaos Generalized Synchronization of New Mathieu- Duffing Systems by GYC Partial Region Stability Theory” submitted to Journal of Sound and Vibration. (SCI, Impact Factor: 1.364)
21. Zheng-Ming Ge and Yan-Sian Li, 2008, “Chaos Generalized Synchronization of New Mathieu- Duffing Systems by GYC Partial Region Stability Theory” submitted to International Journal of Engineering Science. (SCI, Impact Factor: 1.366)
22. Zheng-Ming Ge and Yan-Sian Li, 2009, “Pragmatical Hybrid Projective Chaotic Generalized Synchronization of Chaotic Systems by Adaptive Backstepping Control” submitted to Mathematics and Computers in Simulation. (SCI, Impact Factor: 0.930)
23. Zheng-Ming Ge and Yan-Sian Li, 2009, “Pragmatical Hybrid Projective Chaotic Generalized Synchronization of Chaotic Systems by Adaptive Backstepping Control” submitted to Mathematics and Computers in Simulation. (SCI, Impact Factor: 0.930)
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25. Zheng-Ming Ge, Chun-Yen Ho, 2009, "Chaos Synchronization of the Two Identical Ikeda-Mackey-Glass Systems without Any Controller" submitted to Signal Processing. (SCI, Impact Factor: 1.256)
26. Zheng-Ming Ge, Chun-Yen Ho, 2009, "Chaos Generalized Synchronization of New Ikeda-Lorenz Systems by GYC Partial Region Stability Theory" submitted to Physica A: Statistical Mechanics and its Applications. (SCI, Impact Factor: 1.441)
27. Zheng-Ming Ge, Chun-Yen Ho, 2010, "Parameter estimation of Ikeda-Mackey Glass system through synchronization in presence of disturbance" submitted to Mathematical Methods in the Applied Sciences. (SCI, Impact Factor: 0.821)
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29. Zheng-Ming Ge and Chun-Yu Chiang, 2008, "Hybrid Projective Synchronization of Hyperchaotic Tachometer systems by Backstepping Control", submitted to Mathematics and Computers in Simulation. (SCI, Impact Factor: 0.738)
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31. Zheng-Ming Ge and Chun-Yu Chiang, 2009, "Pragmatical Hybrid Projective Hyperchaotic Symplectic Synchronization of Hyperchaotic Tachometer Systems with Different Order System by Adaptive Backstepping Control", submitted to Philosophical Transactions of the Royal Society A: Mathematical, Physical & Engineering Sciences. (SCI, Impact Factor: 2.282)

本計畫經費贊助之已出版及接受之期刊論文7篇：

1. Zheng-Ming Ge and Shih-Yu Li, 2009, “Chaos Control of New Mathieu-Van der Pol Systems with New Mathieu-Duffing Systems by GYC Partial Region Stability Theory”, Nonlinear Analysis: Theory, Methods, and Applications, Vol. 71, pp. 4047-4059. (SCI, Impact factor: 1.295).
2. Zheng-Ming Ge and Shih-Yu Li, 2009, “A novel study of parity and attractor in the time reversed Lorentz system”, Physics Letter A, Vol. 373, pp. 4053-4059. (SCI, Impact factor: 2.174).
3. Zheng-Ming Ge, Chun-Yen Ho, Shih-Yu Li and Ching Ming Chang, 2009 ,“Chaos control of new Ikeda–Lorenz systems by GYC partial region stability theory” Mathematical Methods in the Applied Sciences Vol. 32, pp. 1564-1584. (SCI, Impact Factor: 0.717)
4. Zheng-Ming Ge, Shih-Chung Li, Shih-Yu Li and Ching-Ming Chang, 2008, “Pragmatical adaptive chaos control from a new double van der Pol system to a new double Duffing system”, Applied Mathematics and Computation 203 (2008) 513–522 (SCI, Impact factor: 1.778).
5. Zheng-Ming Ge and Shih-Yu Li, 2010, “Fuzzy Modeling and Synchronization of Chaotic Quantum Cellular Neural Networks Nano System via A Novel Fuzzy Model and Its Implementation on Electronic Circuits” Journal of Computational and Theoretical Nanoscience, Vol. 7, pp. 1-10. (SCI, Impact Factor: 1.256)
6. Zheng-Ming Ge and Shih-Yu Li, 2010, “Yang and Yin parameters in the Lorenz system”, accepted by Nonlinear Dynamics. (SCI, Impact factor: 1.295).
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Chaos control of new Mathieu–Van der Pol systems with new Mathieu–Duffing systems as functional system by GYC partial region stability theory

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ABSTRACT

In this paper, a new strategy by using GYC partial region stability theory is proposed to achieve chaos control. Using the GYC partial region stability theory, the new Lyapunov function used is a simple linear homogeneous function of error states and the lower order controllers are much more simple and introduce less simulation error. Numerical simulations are given for new Mathieu–Van der Pol system and new Mathieu–Duffing system to show the effectiveness of this strategy.

1. Introduction

Since Ott et al. [1] gave the famous OGY control method in 1990, the applications of the various methods to control a chaotic behavior in natural sciences and engineering are well known. For example, the adaptive control [2–5], the method of chaos control based on sampled data [6], the method of pulse feedback of systematic variable [7], the active control [8,9] and linear error feedback control [10,11]. However, when Lyapunov stability of zero solution of states is studied, the stability of solutions on the whole neighborhood region of the origin is demanded.

In this paper, a new strategy to achieve chaos control by GYC partial region stability theory is proposed [12,13]. Using the GYC partial region stability theory, the new Lyapunov function is a simple linear homogeneous function of error states and the lower order controllers are much more simple and introduce less simulation error.

The layout of the rest of the paper is as follows. In Section 2, chaos control scheme by GYC partial region stability theory is proposed. In Section 3, new Mathieu–Van der pol system and new Mathieu–Duffing system are presented. In Section 4, three simulation examples are given. In Section 5, conclusions are drawn. The partial region stability theory is enclosed in Appendix.

2. Chaos control scheme

Consider the following chaotic system

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}) \tag{2.1}$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T \in R^n$ is a state vector, $\mathbf{f} : R_+ \times R^n \to R^n$ is a vector function.

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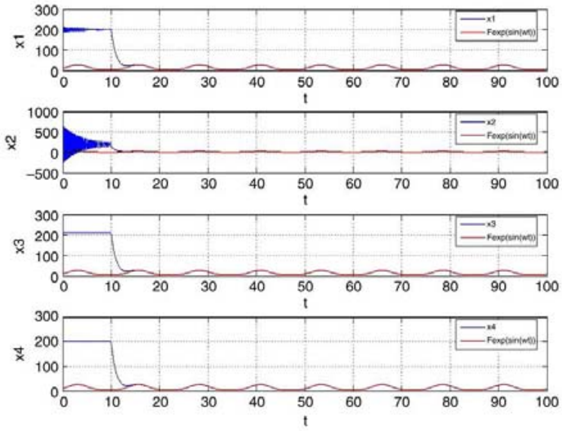

Fig. 8. Time histories of x_1, x_2, x_3, x_4 for Case II.

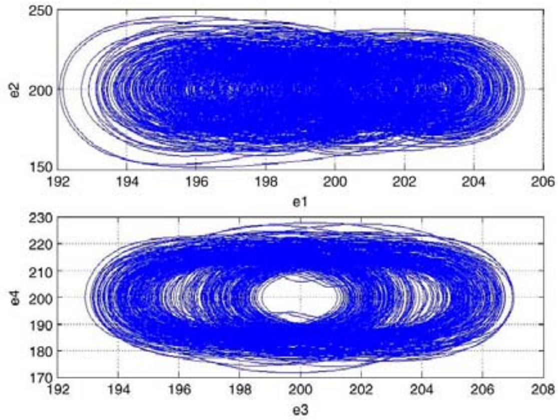

Fig. 9. Phase portraits of error dynamics for Case III.

which is negative definite function in first quadrant. The numerical results are shown in Figs. 10 and 11. After 10 s, the errors approach zero and the chaotic trajectories of the new Mathieu–Van der pol system approach to that of the new Mathieu–Duffing system.

5. Conclusions

In this paper, a new strategy by using GYC partial region stability theory is proposed to achieve chaos control. Using the GYC partial region stability theory, the new Lyapunov function used is a simple linear homogeneous function of states and the lower order controllers are much more simple and introduce less simulation error. The new chaotic Mathieu–Van der pol system and new chaotic Mathieu–Duffing system system are used as simulation examples which confirm the scheme effectively.

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A novel study of parity and attractor in the time reversed Lorentz system

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ABSTRACT

In this Letter, a new effective approach to achieve adaptive synchronization is proposed. Via using Ge–Yao–Chen (GYC) partial region stability theory and pragmatical asymptotically stability theorem, the numerical simulation results show that the states errors and parameter errors approach to zero much more exactly and efficiently than traditional method. The time reversed Lorentz system (called historical Lorenz system in this Letter) is introduced and used for example in this Letter. The simulation results are given in figures and tables for comparison between the new approach and traditional one to show the effectiveness and feasibility of our new strategy.

1. Introduction

Nonlinear dynamics, commonly called the chaos theory, changes the scientific way of looking at the dynamics of natural and social systems, which has been intensively studied over the past several decades. The phenomenon of chaos has attracted widespread attention amongst mathematicians, physicists and engineers. Chaos has also been extensively studied in many fields, such as chemical reactions, power converters, biological systems, information processing, secure communications, etc. [1–9].

Since E.N. Lorenz [10] discovered chaos in a simple system of three autonomous ordinary differential equations in 1963 (called contemporary Lorenz system in this Letter), there are lots of articles in studying contemporary Lorenz system [11–15]. Although the contemporary Lorenz system has been analyzed in detail, there are no articles in looking into the historical Lorenz system. According to modern physics, there are many virtual parity of particles, such as electron (negative) and positron (positive), proton (positive) and anti-proton (negative), etc. Therefore, there is a positive chaotic Lorenz system. Is there a negative chaotic Lorenz system? In [16], we use positive parameters (P-parameters) for the contemporary Lorenz system, negative parameters (N-parameters) for the historical Lorenz system and give a complete report in studying historical Lorenz system.

In this Letter, a new adaptive synchronizing strategy – pragmatical [17,18] adaptive synchronization by GYC partial region stability theory (which is proposed by Ge, Yao and Chen [19–21]) is proposed. Via using this new approach, the new Lyapunov function is a simple linear homogeneous function of states and the lower order controllers and parametric update laws are much simpler and introduce less simulation error.

The layout of the rest of the Letter is as follows. In Section 2, GYC pragmatical adaptive synchronization scheme is presented. In Section 3, the contemporary and historical Lorenz system is discussed. In Section 4 and 5, simulation results are given for comparing and observation. In Section 6, conclusions are given.

2. GYC pragmatical adaptive synchronization scheme

There are two identical nonlinear dynamical systems, and the master system controls the slave system. The master system is given by

$$\dot{x} = Ax + f(x, B) \tag{2.1}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$ denotes a state vector, A is an $n \times n$ uncertain constant coefficient matrix, f is a nonlinear vector function, and B is a vector of uncertain constant coefficients in f.

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the corresponding N-parameters. This study in historical chaos explores another half battle field for chaos study, will prove to have epoch-making significance in the future.

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Pragmatical adaptive chaos control from a new double van der Pol system to a new double Duffing system

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Uncoupled chaotic system

ABSTRACT

A new pragmatical adaptive control method for different chaotic systems is proposed. Traditional chaos control is limited to decrease chaos of one chaotic system. This method enlarges the effective scope of chaos control. We can control a chaotic system, e.g. a new chaotic double van der Pol system, to a given chaotic or regular system, e.g. a new chaotic double Duffing system or to a damped simple harmonic system. By a pragmatical theorem of asymptotical stability based on an assumption of equal probability of initial point, an adaptive control law is derived such that it can be proved strictly that the common zero solution of error dynamics and of parameter dynamics is asymptotically stable. Numerical simulations are given to show the effectiveness of the proposed scheme.

1. Introduction

Since chaos control was firstly used by Ott et al. [1], it has been studied extensively. Many control methods have been employed to control chaos [2–6]. Simple linear feedback control was proposed [7–9]. Time delay feedback control [10–13], sliding mode control [14–17], backstepping method [18] and adaptive control [19–22] were widely used. However, traditional adaptive chaos control is limited to control the chaotic motion of one chaotic system to regular motion or to fixed point. Proposed pragmatical adaptive control method enlarges the scope of chaos control. We can control a chaotic system to a given simple unchaotic system or to a more complex chaotic system. In current scheme of adaptive control of chaotic motion [23–25], traditional Lyapunov stability theorem and Babalat lemma are used to prove the error vector approaches zero, as time approaches infinity. But the question, why the estimated or given parameters also approach to the uncertain or goal parameters, remains no answer. By a pragmatical theorem of asymptotical stability [29–31] based on an assumption of equal probability of initial points, an adaptive control law is derived such that it can be proved strictly that the common zero solution of error dynamics and of parameter dynamics is asymptotically stable. Numerical results are given for a chaotic double van der Pol system to be controlled to a chaotic double Duffing system and to a regular damped simple harmonic system.

This paper is organized as follows: In Section 2, a pragmatical adaptive control scheme is given. In Section 3 numerical results of chaos control are given. A chaotic double van der Pol system is controlled to a chaotic double Duffing system and to a regular damped simple harmonic system. Finally, conclusions are given in Section 4.

2. Pragmatical adaptive control scheme

Consider the following chaotic system

$$\dot{x} = f(x, A) + u(t), \tag{1}$$

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Eq. (30) is the parameter dynamics. Substituting Eqs. (29) and (30) into Eq. (28), we obtain

$$\dot{V} = E_1^2 - E_2^2 - E_3^2 - E_4^2 < 0$$

which is negative semi-definite function of E_1, E_2, E_3, E_4. The Lyapunov asymptotical stability theorem is not satisfied. We cannot obtain that the common origin of error dynamics (26) and parameter dynamics (27) is asymptotically stable. Now, D is an 8-manifold, $n = 12$ and the number of error state variables $p = 4$. When $E_1 = E_2 = E_3 = E_4 = 0$ and $\tilde{k}_1, \tilde{a}_1, \tilde{b}_1, \tilde{c}_1, \tilde{d}_1, \tilde{j}_1, \tilde{f}_1, \tilde{g}_1, \tilde{h}_1, \tilde{\lambda}_1$, take arbitrary values, $\dot{V} = 0$, so X is 4-manifold, $m = n - p = 12 - 4 = 8$. $m + 1 < n$ is satisfied. By pragmatical asymptotical stability theorem, error vector e approaches zero and the estimated parameters also approach the uncertain parameters. The pragmatical generalized synchronization is obtained. Under the assumption of equal probability, it is actually asymptotically stable. This means that the chaos control for different systems, from a double van der Pol system to a exponentially damped-simple harmonic system, can be achieved. The simulation results are shown in Figs. 5 and 6.

4. Conclusions

To control chaotic systems to different systems is study by new pragmatical adaptive control method. The pragmatical asymptotical stability theorem fills the vacancy between the actual asymptotical stability and mathematical asymptotical stability. The conditions of the Lyapunov function for pragmatical asymptotical stability are lower than that for traditional asymptotical stability. By using this theorem, with the same conditions for Lyapunov function, $V > 0$, $\dot{V} \leqslant 0$, as that in current scheme of adaptive chaos control, we not only obtain the adaptive control of chaotic systems but also prove that the estimated parameters approach the uncertain values. Traditional chaos control is limited to decrease chaos of one chaotic system. This method enlarges the effective scope of chaos control. We can control a chaotic system to a given chaotic system or to a given regular system.

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Chaos control and anticontrol of tachometer system by Ge–Yao–Chen partial region stability theory

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Q1 **Abstract:** In this paper, chaos control and anticontrol of a tachometer system by Ge–Yao–Chen (GYC) partial region stability are proposed. The Lyapunov function becomes a simple linear homogeneous function and the controllers become simpler by using the GYC partial region stability theory. The simulation results are more precise because the controllers are in lower degree than that of traditional controllers. Finally, chaos control and anticontrol of tachometer system by GYC partial region stability are obtained and verified by numerical simulations.

Keywords: tachometer system, Lyapunov exponent, hyperchaos, Ge–Yao–Chen partial region stability theorem, chaos control, anticontrol

1 INTRODUCTION

Since the chaos control problem was first considered by Ott *et al.* [1, 2], it has been studied extensively. Chaos control is used to suppress or eliminate the chaotic dynamical behaviour in non-linear systems. There are many control methods for chaos control, such as feedback and non-feedback control [3–6], adaptive control [7, 8], observer-based control [9], inverse optimal control [10], and active control [11]. However, when Lyapunov asymptotical stability of zero solution of states is used, the asymptotical stability of solutions on the whole neighbourhood region of the origin is demanded.

Anticontrol [12–18] is an interesting, new, and challenging phenomenon. As a reverse process of suppressing or eliminating chaotic behaviours in order to reduce the complexity of an individual system or a coupled system, anticontrol of chaos aims at creating or enhancing the system complexity for some special applications. More precisely, anticontrolling chaos is to generate some chaotic behaviours from a given system, which is non-chaotic or even stable originally. By fully exploiting the intrinsic non-linearity, this control technique provides another dimension for feedback systems design. Its potential applications can be easily found in many fields, including typically physics, biology, engineering, and medical as well as social sciences.

In this paper, a new method to achieve chaos control and anticontrol by the Ge–Yao–Chen (GYC) the partial region stability theory [19–21] is proposed. In this theory, when the asymptotical stability of zero solution of states is studied, the asymptotical stability of solution only on the partial neighbourhood region of the origin is demanded. Using this stability theory, the Lyapunov function becomes a simple linear homogeneous function of error states, and every terms of the controllers are of lower degree than that of the controllers when the traditional Lyapunov asymptotical stability theory is used. The simulation results are more precise since the controllers are in lower degree than that of traditional controllers.

This paper is organized as follows. In section 2, chaos control and anticontrol scheme by the GYC partial region stability theory is proposed. In section 3, a tachometer system and a new hyperchaotic Mathieu–Duffing is introduced. In section 4, chaos control and anticontrol of the tachometer system by the GYC partial region stability theory are presented in three examples by simulations. Finally, conclusions are drawn in section 5. The partial region stability theory is introduced in the Appendix.

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Choose

$$
\begin{aligned}
u_1 &= -x_2 + g_1 + z_2 - e_1 \\
u_2 &= -\frac{1}{2m_1 + 4m_2 \sin^2(x_1 - g_2)} \left(\frac{-2m_2 g \sin(x_1 - g_2)}{l} \right. \\
&\quad + \frac{2m_2 A \sin(x_3 - g_2) \sin(x_1 - g_2)}{l} \\
&\quad - 4m_2 (x_2 - g_2)^2 \sin(x_1 - g_2) \cos(x_1 - g_2) \\
&\quad + 2m_1 \sin(x_1 - g_2) \cos(x_1 - g_2) \eta^2 \\
&\quad \left. - \frac{k_1(x_1 - g_2)}{l^2} - \frac{k_2(x_2 - g_2)}{l^2} \right) \\
&\quad - az_1 - bz_3 z_1 - az_1^3 - bz_3 z_1^3 - cz_2 + dz_3 - e_2 \\
u_3 &= -x_4 + g_3 + z_4 - e_3 \\
u_4 &= +A \sin(x_3 - g_4) - z_3 - z_3^3 - tz_4 + fz_1 - e_4
\end{aligned} \tag{24}
$$

One can obtain

$$\dot{V} = -e_1 - e_2 - e_3 - e_4 < 0 \tag{25}$$

which is a negative definite function in the first quadrant. By the asymptotical stability theorem for the partial region, $\lim_{t\to\infty} e = 0$. The numerical results are shown in Figs. 13 and 14. When the controllers act, the errors approach zero and the chaotic motion of tachometer system approaches to the hyperchaotic motion of the new Mathieu–Duffing system within 5 s.

5 CONCLUSIONS

In this paper, chaos control and anticontrol of a chaotic tachometer system by GYC partial region stability are studied. The simulation results are more precise because the controllers are in lower degree than that of traditional controllers. Three simulation examples are given. When controllers are in action, the error states rapidly approach zero within 5 s in all three examples, which means this chaos control and anticontrol scheme is very effective. Simulations show that for other continuous systems, this method is also effective. Besides, traditional anticontrol of a chaotic system is limited to increase the chaos or hyperchaos
Q5 of the chaotic system itself. In this paper, chaos anticontrol is extended to make the chaos of a chaotic system increase to the hyperchaos of any other system.

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ORIGINAL PAPER

Yang and Yin parameters in the Lorenz system

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Abstract The history of the Lorenz system is firstly discussed in this paper. In Chinese philosophy, *Yin* is the negative, historical, or feminine principle in nature, while *Yang* is the positive, contemporary, or masculine principle in nature. *Yin* and *Yan* are two fundamental opposites in Chinese philosophy (therefore, in this paper, these words "Yin parameter," "Yang parameter," "historical system," and "contemporary system" are used to represent the "positive parameter," "negative parameter," "time reversed $(-t)$ system," and "time forward (t) system," respectively). Simulation results show that chaos of historical Lorenz system can be generated when using "*Yin*" parameters. To our best knowledge, most characters of contemporary Lorenz system are studied in detail, but there are no articles in making a thorough inquiry about the history of Lorenz system. As a result, the chaos of historical Lorenz system with "*Yin* parameters" is introduced in this paper and various kinds of phenomena in the historical Lorenz system are investigated by Lyapunov exponents, phase portraits, and bifurcation diagrams.

Keywords Time reversed Lorenz system · Yin parameters · Lyapunov exponent · Chaos

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1 Introduction

Nonlinear dynamics, commonly called the chaos theory, changes the scientific way of looking at the dynamics of natural and social systems, which has been intensively studied over the past several decades. The phenomenon of chaos has attracted widespread attention amongst mathematicians, physicists, and engineers. Chaos has also been extensively studied in many fields, such as chemical reactions, power converters, biological systems, information processing, secure communications, etc. [1–9]. While many researchers analyze complicated, physically motivated configurations, there is also a need to investigate simple equations which may capture the essence of chaos in a less involved setting, thereby aiding the understanding of essential characteristics. The original investigation of an extraordinary three-dimensional nonlinear system by the mathematical meteorologist Lorenz [10] who discovered chaos in a simple system of three autonomous ordinary differential equations in order to describe the simplified Rayleigh–Benard problem in 1963 (which is called the contemporary Lorenz system in this paper) is the most popular system for studying.

There are a lot of articles in studying the contemporary Lorenz system [11–15]. Although the contemporary Lorenz system has been analyzed in detail, there are no articles in looking into the history of the Lorenz system. In this paper, we find out that there are rich dynamics in this historical Lorenz system.

Table 3 Range of parameter c of historical Lorenz system

−20.0–(−46.8)	Chaos
−46.8–(−47.7)	Periodic trajectory
−47.7–(−51.3)	Chaos
−51.3–(−52.4)	Periodic trajectory
−52.4–(−59.5)	Chaos
−59.5–(−59.8)	Periodic trajectory
−59.8–(−68.3)	Chaos
−68.3–(−69.6)	Periodic trajectory
−69.6–(−70)	Chaos

Table 4 Range of parameter b of contemporary Lorenz system

0–0.592	Converge to a fixed point
0.592–0.648	Chaos
0.648–0.720	Periodic trajectory
0.720–3.448	Chaos
3.448–4	Converge to a fixed point

Table 5 Range of parameter b of historical Lorenz system

0–(−0.568)	Converge to a fixed point
−0.568–(−0.728)	Chaos
−0.728–(−0.792)	Periodic trajectory
−0.792–(−4.000)	Chaos

Table 6 Range of parameter a of contemporary Lorenz system

5.000–5.760	Converge to a fixed point
5.760–18.368	Chaos
18.368–20.000	Converge to a fixed point

where $\mu \in [-1, 1]$. We choose initial condition $(x_0, y_0, z_0) = (-0.1, 0.2, 0.3)$ and *Yin* parameters $a = -6$, $b = -8/3$ and $c = -28$, the projection of phase portraits, bifurcation diagrams, and Lyapunov exponents with $\mu \in [-1, 1]$ are shown in Figs. 9 and 10. In observation of Figs. 9 and 10, it is clear that there are periodic and chaotic motions in such a family system when μ is varying.

6 Conclusions

In this paper, the *Yin* Lorenz system with "*Yin* parameters" and its one-parameter family are firstly introduced. When the transformation from $(x(t), y(t), z(t), t)$ to $(x(-t), y(-t), z(-t), -t)$ is made, simulation results show that chaos of the *Yin* Lorenz system can be generated via using "*Yin*" parameters $(-a, -c, -b)$. Via numerical simulation, the *Yin* Lorenz system is compared with the *Yang* Lorenz system and we found out there are similarities and differences between them. The approximate symmetry of Lyapunov exponents is most prominent in Figs. 5 and 6. This paper explores the another half battle field for chaos study, and will prove to have epoch-making significance in the future.

Table 7 Range of parameter a of historical Lorenz system

−5.00–(−5.45)	Periodic trajectory (one attractor to two attractors)
−5.45–(−5.60)	Chaos
−5.60–(−6.05)	Periodic trajectory
−6.05–(−6.17)	Chaos
−6.17–(−6.35)	Periodic trajectory
−6.35–(−7.58)	Chaos
−7.58–(−7.76)	Periodic trajectory
−7.76–(−20)	Chaos

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Chaos control of new Ikeda–Lorenz systems by GYC partial region stability theory

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SUMMARY

A new strategy to achieve chaos control by GYC partial region stability theory is proposed. By using the GYC partial region stability theory, the Lyapunov function is a simple linear homogeneous function of error states, the controllers are more simple and have less simulation error because they are in lower degree than that of traditional controllers. Simulation results for a new Ikeda–Lorenz system show the effectiveness of this strategy. Copyright © 2008 John Wiley & Sons, Ltd.

KEY WORDS: chaos control; partial region stability theory; Ikeda–Lorenz system; Genesio system

1. INTRODUCTION

Chaos, as an interesting nonlinear phenomenon, has been intensively investigated. It is well known that chaotic systems have sensitive dependence on initial conditions. A chaotic system is a nonlinear deterministic system that displays complex dynamical behaviors [1].

The theory of chaos control has developed since 1990 [2–4] and today is at the forefront of research in the field of nonlinear dynamics. Techniques have been experimentally implemented in mechanical [5], chemical [6], electronic [7], laser [8], communication [9] and biological [10] systems. Though there are now many different algorithms developed for the control of chaos for specific cases, in general all make use of typical properties of chaotic systems, namely, multiple coexisting solutions, sensitivity and ergodicity.

In this paper, a new chaos control strategy by GYC (Ge-Yao-Chen) partial region stability theory is proposed [11–13]. By using the GYC partial region stability theory, the Lyapunov function is a simple linear homogeneous function of error states and the controllers are more simple and have

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which is, obviously, in contradiction with Equation (A17). The contradiction thus obtained shows that the function $V(t, x_1(t), \ldots, x_n(t))$ approaches zero as t increases without limit. Consequently, the same will be true for the function $W(x_1(t), \ldots, x_n(t))$ as well, from which it follows directly that

$$\lim_{t\to\infty} x_s(t) = 0 \quad (s = 1, \ldots, n)$$

which proves the theorem. □

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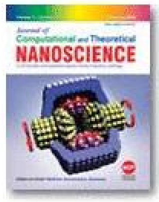

Fuzzy Modeling and Synchronization of Chaotic Quantum Cellular Neural Networks Nano System via a Novel Fuzzy Model and Its Implementation on Electronic Circuits

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< previous article | view table of contents | next article >

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

■ 達成目標

□ 未達成目標（請說明，以 100 字為限）

□ 實驗失敗

□ 因故實驗中斷

□ 其他原因

第一年：Mathieu-van der Pol 系統與 Ikeda- Mackey-Glass 系統的渾沌行為與新式非耦合渾沌同步新方法之研究，並探討對此二系統之應用。

1. 完成採用諸多相圖、分歧圖、功率譜圖、參數圖及李亞普諾夫指數及碎形維度等研究自治的 Mathieu-van der Pol 系統與 Ikeda- Mackey-Glass 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。(二個月)
2. 完成採用諸多相圖、分歧圖、功率譜圖、參數圖及李亞普諾夫指數及碎形維度等研究非自治的 Mathieu-van der Pol 系統與 Ikeda- Mackey-Glass 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。(二個月)
3. 完成研究新式非耦合渾沌同步新方法之理論。(三個月)
4. 完成研究新式非耦合渾沌同步新方法對自治的 Mathieu-van der Pol 系統與 Ikeda- Mackey-Glass 系統之應用。(二個月)
5. 完成研究新式非耦合渾沌同步新方法對非自治的 Mathieu-van der Pol 系統與 Ikeda- Mackey-Glass 系統之應用。(二個月)
6. 完成撰寫年度報告書。(一個月)

第二年：Duffing-van der Pol 系統與 Mathieu-Duffing 系統之渾沌行為與實用渾沌適應控制反控制新方法之研究，及對此二系統的應用 1. 完成採用諸多相圖、分歧圖、功率譜圖、參數圖、李亞普諾夫指數及碎形維度等研究 Duffing-van der Pol 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。(二個月) 2. 完成採用諸多相圖、分歧圖、功率譜圖、參數圖、李亞普諾夫指數及碎形維度等研究 Mathieu-Duffing 系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。(二個月) 3. 完成研究實用渾沌適應控制反控制新方法之理論。(三個月) 4. 完成研究實用渾沌適應控制反控制新方法對雙 Duffing-van der Pol 系統之應用。(二個月) 5. 完成研究實用渾沌適應控制反控制新方法對雙 Mathieu-Duffing 系統之應用。(二個月) 6. 完成撰寫年度報告書。(一個月)
第三年：兩種慣性測速器新系統的渾沌行為與指數渾沌同步新方法之研究。 1. 完成採用諸多相圖、分歧圖、功率譜圖、參數圖研究第一種種慣性測速器新系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。（二個月） 2. 完成採用諸多相圖、分歧圖、功率譜圖、參數圖研究第二種慣性測速器新系統之週期運動、準週期運動、渾沌運動及超渾沌運動各種行為。（二個月） 3. 完成研究指數渾沌同步新方法之理論。（一個月） 3. 完成研究在各種不同起始條件下，第一種慣性測速器新系統之延遲或預期同步。（二個月） 4. 完成研究在各種不同起始條件下，第二種慣性測速器新系統之延遲或預期同步。（二個月） 5. 完成研究在各種不同起始條件下，第一種慣性測速器新系統之延遲或預期反同步。（一個月） 6. 完成研究在各種不同起始條件下，第二種慣性測速器新系統之延遲或預期反同步。（一個月） 8. 完成撰寫年度報告書。（一個月）

2. 研究成果在學術期刊發表或申請專利等情形：

論文：■已發表 □未發表之文稿 □撰寫中 □無

專利：□已獲得 □申請中 □無

技轉：□已技轉 □洽談中 □無

其他：（以 100 字為限）

研究內容寫成之期刊論文已有 31 篇，其中已有 7 篇被接受：

1. Zheng-Ming Ge and Shih-Yu Li, 2009, "Chaos Control of New Mathieu-Van der Pol Systems with New Mathieu -Duffing Systems as Functional System by GYC Partial Region Stability Theory", Nonlinear Analysis: Theory, Methods, and Applications, Vol. 71, pp. 4047-4059. (SCI, Impact Factor: 1.295)
2. Shih-Yu Li and Zheng-Ming Ge, 2009, "A Novel Study of Parity and Attractor in the Time Reversed Lorentz System", Physics Letter A, Vol. 373, pp, 4053-4059. (SCI, Impact Factor: 2.174).
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8. Zheng-Ming Ge and Shih-Yu Li, 2009, "Pragmatical Adaptive Synchronization of Different Orders Chaotic Systems with All Uncertain Parameters via Nonlinear Control" submitted to Nonlinear Dynamics. (SCI, Impact Factor: 1.295)
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31. Zheng-Ming Ge and Chun-Yu Chiang, 2009, “Pragmatical Hybrid Projective Hyperchaotic Symplectic Synchronization of Hyperchaotic Tachometer Systems with Different Order System by Adaptive Backstepping Control”, submitted to Philosophical Transactions of the Royal Society A: Mathematical, Physical & Engineering Sciences. (SCI, Impact Factor: 2.282)

本計畫經費贊助之已出版及接受之期刊論文7篇：

1. Zheng-Ming Ge and Shih-Yu Li, 2009, "Chaos Control of New Mathieu-Van der Pol Systems with New Mathieu-Duffing Systems by GYC Partial Region Stability Theory", Nonlinear Analysis: Theory, Methods, and Applications, Vol. 71, pp. 4047-4059. (SCI, Impact factor: 1.295).
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5. Zheng-Ming Ge and Shih-Yu Li, 2010, "Fuzzy Modeling and Synchronization of Chaotic Quantum Cellular Neural Networks Nano System via A Novel Fuzzy Model and Its Implementation on Electronic Circuits" Journal of Computational and Theoretical Nanoscience, Vol. 7, pp. 1-10. (SCI, Impact Factor: 1.256)
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11. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以500字為限）

Mathieu系統、van der Pol系統、Ikeda系統及Mackey-Glass系統皆為經典之系統。經巧妙之取代與結合既可獲得自治新Mathieu−van der Pol系統，自治新Ikeda-Mackey Glass系統亦可得非自治之新Mathieu−van der Pol系統與非自治新Ikeda-Mackey Glass系統大大擴大了經典渾沌系統的範圍及研究領域。而新的非耦合渾沌同步方法具有很大的機動性及發展潛力。取代之參數可以變化多端，採用之取代函數更是取之不盡。對用於秘密通訊而言，機密性大為增加。Duffing系統、van der Pol系統及Mathieu系統皆為經典之系統。經巧妙之取代與結合既可獲得自治新Duffing−van der Pol系統，自治新Mathieu-Duffing系統亦可得非自治之新Duffing−van der Pol系統與非自治新Mathieu-Duffing系統大大擴大了經典渾沌系統的範圍及研究領域。而不同系統實用渾沌適應控制新方法具有很大的發展潛力，涉及的各系統參數可以都是未知參數。大大地擴展了渾沌控制反控制的能力。取代之參數可以變化多端，採用之取代函數更是取之不盡。對用於秘密通訊而言，機密性大為增加。

本計畫之研究不僅對渾沌動力學學科中最重要最典型的四種渾沌系統的研究的拓廣與深化，更重要的是它們本身顯然具有更複雜的，未經發現的複雜渾沌行為，本研究對渾沌動力學學科具重大意義。其應用於機械、電機、物理、化學、生科、奈米之耦合系統，具有重要的實用價值。

渾沌同步除本身之重要理論價值外，其研究在秘密通訊、神經網路、自我組織等方面有日益廣泛之應用。廣義渾沌同步則為渾沌同步之進一步發展，其應用亦方興未艾。本計畫提出三種新的渾沌同步。實用適應廣義同步法糾正了目前國際文獻中未經証明即認為估值參數趨於為之參數之錯誤，首次在渾沌同步中引入概率概念，具重大理論及實用意義。由於$\dot{V}$之要求降低，實際應用亦較易實現。純誤差穩定的廣義同步，則彌補了國際文獻中需用數值計算結果為條件之理論，即有缺陷之理論。在理論與實用上有重要意義。不同起始條件的延遲同步等多種渾沌同步則為新發現的渾沌運動之現象，特別是Ikeda系統的永遠性延遲同步或反同步，不同於傳統理論，尤具重大意義。