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行政院國家科學委員會補助專題研究計畫成果報告
廣義多使用者多天線技術之自動重傳機制及其性能探究

伍紹勳

摘 要

合作式通訊(cooperative communication)是利用中繼站(relay)當做虛擬天線，以達到類似多重輸入-多重輸出(multiple-input multiple-output; MIMO)系統的效能，可用來增加傳輸的可靠度或是通道容量。此外，配合自動重傳機制(automatic repeat request; ARQ)，亦能讓合作式通訊的功能作進一步的提升。首先我們在一個簡化的數學模型下，利用投機式(opportunistic)的解調傳送(decode-and-forward; DF)中繼技術，並且使用分散式空時編碼(distributed space-time coding; DSTC) 來協助系統之自動重傳協定。分析的結果告訴我們並不一定需要同時佔有所有中繼站的資源才能得到最低的失效機率(outage probability)，在特定的通道環境下，只要投機式的選取中繼站來幫助傳輸即可得到接近最佳的輸出效能。

再者，延續先前的投機式合作中繼技術，依據中繼站的操作複雜度，我們設計出三種合作式自動重傳機制、並且從多樣-多工性取捨(diversity-multiplexing tradeoff; DMT)與吞吐率(throughput)的角度來分析每種機制的使用效率。藉由DMT的分析，我們發現在一般常態的通道環境下，讓所有尚未解調成功的中繼站持續聆聽來自傳送中之中繼站訊號，此舉將使得系統擁有最佳的效能。儘管如此，在特殊的通道環境下，即中繼站距離傳送端或發送端較為靠近時，使用複雜度較低的重傳機制，即使不讓尚未解調成功的中繼站持續聆聽，也能夠提供相同的DMT。

另一方面，若是從吞吐率的角度來分析探討此三種重傳機制的輸出效率，亦能得到類似的結論：在特定的通道環境裡，較簡單的自動重傳機制也能輸出幾乎與最複雜機制相同的最佳吞吐率，此結論可以大大降低為了讓中繼站彼此聆聽訊號所增加的協定設計複雜度。此外，模擬的結果同時顯示出並不需要搭配使用所有的中繼站，只要在所有解調成功的中繼站裡選擇兩個來幫助傳輸亦可得到接近最佳的輸出吞吐率。經由此文的分析，我們便能知道如何使用最少的中繼站數目以及複雜度最低的自動重傳機制，依然可以得到在各種不同的通道環境下所能輸出的最佳效能。

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Abstract

The paper studies the performance of cooperative Automatic-Repeat-reQuest (ARQ) protocols that employ opportunistic distributed space-time coding (ODSTC) for signal relaying. According to the relay selection methods in ARQs, three types of protocols are considered herein which allow us to examine the efficiencies of ODSTC in regards of the number of ARQs, the methods for relay selections and the number of relays for ODSTC. The efficiencies are studied from both the perspective of diversity and multiplexing tradeoff and the viewpoint of the delay and outage limited throughput. Analysis shows that the efficiencies are highly dependent on the link qualities of the relay channels. When the link quality between the source and the relays, or the quality between the relays and the destination are much stronger than the quality between the source and the destination, then simple protocols that use at most two active relays for ODSTC (ODSTC2), with the candidate relays determined only once at the start of relaying with ODSTC, are good enough to provide a near optimal performance. Other than these two extreme channel environments, the performance of ARQ is typically limited by the number of relays available for ODSTC. This relay shortage problem can be resolved by allowing idle relays to overhear the signals sent by active relays, and to participate in ODSTC of the subsequent ARQs once decoding the data. According to our theoretical and simulated analysis, in general, a significant throughput enhancement can be obtained via using an appropriate ARQ protocol that employs ODSTC2 for two times of retransmissions, and no more than five relays are practically necessary for the entire system. These features of ODSTC may have more profound effects on the system throughput from a cross-layer point of view when considering the queuing advantage of ODSTC over a DSTC protocol that may involve more numbers of retransmissions in ARQ.

Keywords

Cooperative relaying, opportunistic DSTC, ARQ, DMT and the delay and outage limited throughput.

I. INTRODUCTION

Cooperative communications have emerged as a new paradigm for wireless communications. Since the work of [1-4], a host of cooperative schemes have been introduced to enhance the system capacity and/or transmission reliability, either through user cooperations or via signal relaying (see [5-12] and the references therein). In view of the cost advantages of relay stations and the performance enhancement that can be brought about with signal relaying, cooperative relaying has been incorporated in IEEE802.16j [13] and other more advanced international standards like IEEE802.16m [14] and 3GPP LTE-Advanced [15], which are under development for the fourth generation cellular networks.

To exploit the spatial diversity offered by distributed relays, distributed versions of space-time coding (DSTC) and beamforming (DBF) schemes have been widely studied and reported in literatures, (see *e.g.* [7,12,16] among others), following the notions of decode-and-forward (DF) or amplify-and-forward (AF) relaying introduced in [2,3]. However, to fully exploit the rich diversities available from multiple relays, the design and implementation of DSTC turn out to be a complicated task when considering the various numbers of relays that can possibly decode the data. In view of the complexities involved in using multiple relays, an opportunistic relaying (OR) scheme that chooses only one out of a set of available relays for forwarding signals is proposed in [10]. By using the relay that possesses the best link quality to the destination, the OR scheme can exploit the full diversity provided by the entire set of relays, and is shown to be optimal when subject to a total power budget for the entire set of relays. Based on a similar idea to OR, an opportunistic relaying scheme that uses the distributed Alamouti code (DAC) is studied in [17]. Assuming an individual power constraint for each relay, the outage probabilities for various types of relay selection methods are discussed therein.

In contrast to the rich diversities offered by cooperative relaying, the multiplexing gain is typically lower for the two-phase half-duplex relaying [3]. To cope with this capacity penalty due to half-duplex relaying (HDR), a dynamic DF (DDF) scheme is proposed for HDR in [5], which is shown to achieve the diversity-multiplexing tradeoff (DMT) of the contrast multiple-input and single-output (MISO) channel [3] when the multiplexing gain is less than 0.5. Moreover, if the relays have the channel state information (CSI) prior to transmissions, [9] shows that the MISO upper bound of DMT can be achieved with the compress-and-forward (CF) HDR [4].

Alternative to the aforementioned physical-layer methods to improve the DMT of the two-phase HDR, cross-layer approaches with Automatic-Repeat-reQuest (ARQ) can be combined with cooperative relaying to alleviate the loss of multiplexing gain either [6,18-20]. Applying DSTC for cooperative ARQ, [18] shows that the effective throughput can be improved either with a simple ARQ or a hybrid type of ARQ (HARQ) with chase combining. Besides, exploiting the extra degrees of freedom offered by cooperative ARQ, [6,19] also demonstrate that both the diversity and the multiplexing gains can be increased via HARQ with incremental redundancy. Motivated by the simplicity of OR, relay selection schemes are proposed in [21] (and the reference therein) for cooperative ARQ protocols to exploit the spatial and temporal diversities with opportunistic AF relaying [10].

Making use of the simplicity of opportunistic relaying and the throughput enhancement via ARQ, we study herein cooperative ARQ protocols that use opportunistic DSTC (ODSTC) for the DF HDR. To characterize the efficiency of ODSTC in cooperative relaying, we first quantify the loss of the signal to noise ratio (SNR) in the outage probability of using an arbitrary number of the available relays for ODSTC than using all of them for DSTC. Furthermore, to examine the effectiveness of the relay reselection function required for ODSTC, we study three types of ARQ protocols which involve different degrees of coordinations among the relays to apply the ODSTC in ARQs. According to the levels of the complexities for coordinations, the three types of protocols employ, respectively, the regular DSTC with fixed relays in the Type-A, relay reselection prior to ODSTC in the Type-B and the combination of overhearing and relay reselection followed by ODSTC in the Type-C protocols, after the first round of ARQ that uses ODSTC. Despite the overhearing function that allows the

Type-C protocol to enlarge the set of available relays during the process of ARQs, the three types of protocols are in fact the same for the first ARQ, which allows us to compare the performance of ARQ that uses ODSTC with the one that uses the regular DSTC, given the same number of relays for space-time coding (STC). Besides, the different functions in the subsequent ARQs allow us to evaluate the effectiveness of the additional procedure for relay reselections and compare the efficiencies of their corresponding ARQ protocols either from the perspective of DMT or from the viewpoint of the delay and outage limited throughput.

According to the system parameters that may affect the outage probabilities of the ARQ protocols, we study the protocols' efficiencies in regards of the number of ARQs, the methods for relay reselections and the number of relays used for ODSTC. Besides, we also examine the effects of the link qualities of the cooperative channels on the performance of the three types of protocols. Analysis shows that either the DMTs or the throughput of these protocols are highly dependent on the ratios of the link qualities among the source, relays and the destination. When the ratio α of the link quality from the source to the relays (S-R) to the link quality from the source to the destination (S-D) is high enough, the relay channel degenerates to a MISO one at high SNR. The Type-B protocol can achieve the DMT of the Type-C protocol in this class of channels. On the contrary, when the ratio β of the link quality from the relays to the destination (R-D) to the link quality of S-D is high enough, then the relay channel resembles a single input and multiple output (SIMO) channel with antenna selection. The destination in this case can decode the data as long as any one of the relays is able to do so. As a result, the three types of protocols perform exactly the same in this class of channels. Other than these two extreme operating environments, in *ordinary relay channels* the diversities of ARQs are typically limited by the cardinality, $\mathcal{D}$, of the set $\mathcal{S}_D$ of relays that successfully decode the data. Only the Type-C protocol can resolve this relay shortage problem and achieve the full diversity of cooperative ARQ by allowing relays $\notin \mathcal{S}_D$ to overhear the signals sent by the active relays $\in \mathcal{S}_D$, and to participate in ODSTC once becoming $\in \mathcal{S}_D$ in the subsequent ARQs.

On the other hand, according to the efficiency analysis for the throughput in *ordinary relay channels*, the S-R to S-D link ratio, α, can be roughly partitioned into three operating regimes as well. When α is large enough such that $\mathcal{D}$ is very close to the total number of relays in the system, then using at most two active relays for ODSTC (ODSTC2) in the Type-B protocol provides a near optimal throughput. However, when α is small such that $\mathcal{D} = 1$ with high probability, then the Type-C protocol with ODSTC2 is capable enough to resolve this relay shortage problem and provide significant throughput enhancement through its repetitive process of overhearing and relay reselection. Other than these two extreme channel conditions, either the Type-A protocol with ODSTC2 or the Type-B protocol with OR provide satisfactory throughput. Based on our theoretical and simulation studies, in general, no more than five relays are practically necessary for the entire system, and using ODSTC2 for two times of ARQs is good enough to offer a significant throughput enhancement with the proposed ARQ protocols, as opposed to the four times or more of ARQs that use randomly selected relays for DAC relaying. This advantage of cooperative ARQ with ODSTC may have a more profound effect on the overall system throughput from a cross-layer point of view [22, 23], when considering the queuing advantage of ODSTC2 over a DSTC that may involve more relays with probably more numbers of retransmissions.

The paper is organized as follows. We introduce in Section II the problem setting for cooperative ARQ based on relaying with ODSTC and review some results on the outage probabilities to be used in the subsequent analysis. Following the approach of outage analysis, the effectiveness and the DMT of relaying with ODSTC are analyzed in Section III. Based on the idea of ODSTC relaying, the retransmission schemes and the outage probabilities of the three types of ARQ protocols are studied in Section IV followed by their DMT analysis in Section V. According to the outage analysis, we study in Section VI the delay and outage limited throughput for the proposed ARQ protocols, and characterize their efficiencies with respect to the number of ARQs and the number of active relays used for ODSTC. The concluding remarks are provided in Section VII.

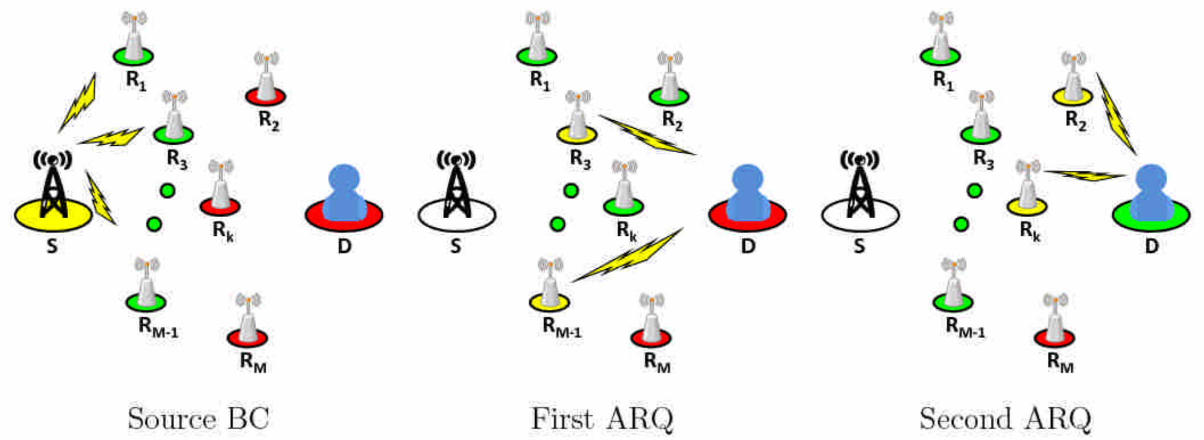


Fig. 1. A cooperative ARQ protocol that uses the Alamouti STC for opportunistic relaying. The green nodes stand for the relays that have decoded the data, while the red nodes stand for the relays or the destination that have not done so. The yellow nodes are th active nodes that are transmitting the data.

II. PRELIMINARY

We consider a relay-assisted communications system as illustrated in Fig. 1. Within the system, there are M relays to help retransmit the signals. In the beginning of a packet transmission, the source first broadcasts (BC) its signal to the relays and the destination. The set of relays that successfully decode the signal is referred to as the decoding set and is denoted by $\mathcal{S}_D$. In case of reception failures at the destination, relays in $\mathcal{S}_D$ are assumed to be able to jointly retransmit the data with DSTC schemes if $\mathcal{S}_D$ is not an empty set, denoted by $\mathcal{S}_D \neq \emptyset$. Otherwise, the source will rebroadcast the signal until either $\mathcal{S}_D \neq \emptyset$ or the destination is able to successfully decode the signal. To simplify our performance investigations, each of the source, destination and relays is assumed to have one antenna.

Inspired by the simplicity of OR and the throughput enhancement via ARQ, we will investigate in the sequel the DMT and the delay and outage limited throughput for cooperative ARQ protocols based on ODSTC relaying. To reduce the complexity in analysis, we perform our investigations from the outage probability point of view. The relay channels are considered flat faded and complex Gaussian distributed with zero mean and unit variance, denoted by $\sim \mathcal{CN}(0,1)$. Furthermore, channel coefficients are assumed unchanged within the duration of a transmission, and change randomly according to $\mathcal{CN}(0,1)$ from one transmission to another. In addition, to simplify our performance investigations, perfect synchronization is assumed achievable for the relays to perform ARQs with ODSTC.

Though somewhat idealized, the above channel assumptions are practical for wireless communications standards such as [13-15] based on the orthogonal frequency division multiple access (OFDMA). Besides, to characterize the influences of the relay channels' link qualities on the DMT and the delay and outage limited throughput, we further assume that the relays receive the same signal power, P_{sr}, from the source, and, similarly, that the destination also receives the same signal power, P_{rd}, from the relays. This type of channel setting can be associated to a relay grouping mode considered in IEEE802.16j [13] where a group of M relays are geometrically close to each other, while are located in a distance from the source and the destination as illustrated in Fig. 2. In addition, when P_{sr} is large enough such that the cardinality of $\mathcal{S}_D$, denoted by $\mathcal{D} \triangleq |\mathcal{S}_D|$, is equal to M or at least with very high probability, the relay channel degenerates to a MISO one. This simplified setting allows us to compare our results with the existing ones for MISO channels.

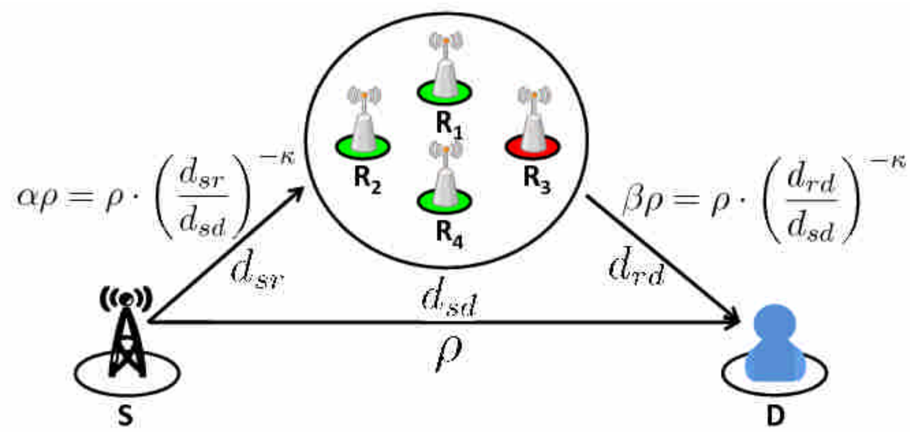


Fig. 2. The geometrical relationship between the source, relays and the destination.

According to the above assumptions, let h_{s,d_s} be the channel coefficients between the source and the destination (S-D), and h_{s,r_m} be the coefficients between the source and the relays (S-R), r_m. The corresponding received signals y_{s,d_s} and y_{s,r_m} are modeled as

$$y_{s,d_s} = \sqrt{P_{sd}} h_{s,d_s} x + n_d \quad (1)$$

$$y_{s,r_m} = \sqrt{P_{sr}} h_{s,r_m} x + n_m, \quad \forall m \in \{1,\ldots,M\} \quad (2)$$

where the noises, n_d and n_m, at the destination and the relays r_m, respectively, are all modeled as $\sim \mathcal{CN}(0, N_0)$. Based on this signal model, the mutual information between the source and the relays, r_m, is given by

$$I_{s,r_m} = \log\left(1 + \frac{P_{sr}|h_{s,r_m}|^2}{N_0}\right), \quad \forall m \in \{1,\ldots,M\}. \quad (3)$$

Thus given the h_{s,r_m}, the decoding set is more specifically defined herein as

$$\mathcal{S}_D \triangleq \{r_m | I_{s,r_m} > R, \quad m = 1, \cdots, M\} \tag{4}$$

where R is the source data rate in bits/sec/channel use, denoted by (b/s/cu).

Similarly, one can obtain the mutual information, I_{s,d_s}, for the S-D channel and that for the channels between the relays and the destination (R-D). Assuming that the signal retransmitted by the relays in $\mathcal{S}_D$ are formed from an orthogonal DSTC, then the mutual information for the R-D channel follows [24]

$$I_{r_m,d_s} = \log\left(1 + \frac{P_{rd}}{N_0} \sum_{r_m \in \mathcal{S}_D} |h_{r_m,d_s}|^2\right) \tag{5}$$

where P_{rd} is the received power at the destination associated with the channel coefficients, h_{r_m,d_s}, between r_m and the destination. The noise at the destination is still $\sim \mathcal{CN}(0,1)$.

Based on the system model described above, we introduce some outage probabilities to be used in the sequel for analysis. For convenience of expression, $\mathfrak{N}$ stands for the natural number and $\mathfrak{R}$ for the real number. The received SNRs for the wireless links of S-D, S-R and R-D are defined as $\rho \triangleq P_{sd}/N_0$, $P_{sr}/N_o \triangleq \alpha\rho$ and $P_{rd}/N_0 \triangleq \beta\rho$, respectively. The parameters α and β characterize the geometrical relationship between the source, relays and the destination, and can be readily obtained given the distances between them and the power loss exponent κ of the wireless relay channels. Besides, following the above assumptions, the probability density function (PDF) of $\rho|h_{s,d_s}|^2$ is $\lambda \exp\{-\lambda |h_{s,d_s}|^2\}$ and is denoted by $\sim$ Exp(λ), with $\lambda \triangleq 1/\rho$. Similarly, we also have $\alpha\rho|h_{s,r_m}|^2 \sim$ Exp(λ_1) and $\beta\rho|h_{r_m,d_s}|^2 \sim$ Exp(λ_2), $\forall m \in [1, M] \cap \mathfrak{N}$, with $\lambda_1 \triangleq 1/(\alpha\rho)$ and $\lambda_2 \triangleq 1/(\beta\rho)$.

A. The Outage Probabilities for DSTC and ODSTC

Define $W \triangleq \rho|h_{s,d_s}|^2 \sim$ Exp(λ). Under the source date rate of R b/s/cu, the outage probability for the direct S-D channel link is given by

$$P_W(\delta_s) \triangleq P\{W < \delta_s\} = 1 - e^{-\lambda\delta_s} \tag{6}$$

where $\delta_s \triangleq (2^R - 1)$. In case of outage events and $\mathcal{S}_D \neq \emptyset$, a cooperative relaying method with DSTC typically uses all relays in $\mathcal{S}_D$ to retransmit the data with STC [18]. Define $X_m \triangleq \beta\rho|h_{r_m,d_s}|^2 \sim \mathrm{Exp}(\lambda_2)$, $r_m \in \mathcal{S}_D$. The outage probability of the relaying with DSTC conditioned on $\mathcal{D}$ is given by [2]

$$P_{\mathcal{Z}|\mathcal{D}}(\delta|d) \triangleq P\left\{\mathcal{Z} \triangleq \sum_{r_m \in \mathcal{S}_D} X_m < \delta \,\middle|\, d \geq 1\right\} = 1 - \sum_{k=0}^{d-1} e^{-\delta\lambda_2} \frac{(\delta\lambda_2)^k}{k!} \tag{7}$$

where $\delta = 2^{\varepsilon R} - 1$ and ε is the rate scaling factor corresponding to the R-D link's SNR. Besides, as $\alpha\rho|h_{s,r_m}|^2$ ~Exp(λ_1), the probability mass function (PMF) of $\mathcal{D}$ follows [2]

$$\mathcal{P}_{\mathcal{D}}(d) = C_d^M (e^{-\delta_s\lambda_1})^d (1 - e^{-\delta_s\lambda_1})^{M-d} \tag{8}$$

where $C_d^M \triangleq \frac{M!}{d!(M-d)!}$.

In contrast to the DSTC relaying that uses all relays in $\mathcal{S}_D$ for retransmission, we consider a relaying method that opportunistically chooses $i_{\mathcal{D}} \triangleq \min\{i, \mathcal{D}\}$ relays, $i \in [1, M] \cap \mathfrak{N}$, out of $\mathcal{S}_D$ to retransmit the data with DSTC (ODSTC). This type of relaying not only saves the resources of relays, but also allows us to evaluate the influences of "i" on the DMT and the throughput of the ARQ protocols that employ ODSTC for relaying

Let $\mathcal{X} \triangleq \{X_m | r_m \in \mathcal{S}_D\}$. Suppose that the ODSTC relaying scheme chooses at most i elements of $\mathcal{X}$ that yield the highest mutual information in (5). Clearly, $i_{\mathcal{D}}$ of the largest elements in $\mathcal{X}$ will be chosen for DSTC. Therefore, sorting the elements of $\mathcal{X}$ in the ascending order into $\mathcal{X}' \triangleq \{X'_1, \ldots, X'_{\mathcal{D}}\}$ such that $X'_k \geq X'_j$ if $k > j$, the outage probability of the ODSTC conditioned on $\mathcal{D}$ is then given by

$$P_{\mathcal{O}_i|\mathcal{D}}(\delta|d) \triangleq P\left\{\mathcal{O}_i \triangleq \sum_{j=\max\{1,d-i+1\}}^{d} X'_j < \delta \middle| d \geq 1\right\}. \tag{9}$$

Given that R is assigned according to the average SNR, ρ, the destination will immediately know whether or not $\mathcal{O}_i \geq \delta$ when the perfect knowledge of $\mathcal{X}'$ is assumed available for the destination to choose the relays for ODSTC. In this case, $P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)$ can be considered as the probability that the destination will wait for another ARQ. On the other hand, in cases where the channel state information (CSI) is imperfect or outdated, or for the Type-C ARQ protocol to be introduced later which requires relays not in $\mathcal{S}_D$ to overhear the signals sent by the relays in $\mathcal{S}_D$ such that the destination will still issue ARQs even if outages may occur, then $P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)$ can be viewed as an idealized lower bound for the packet error rate. In any event, the outage probability (9) can be evaluated with a theorem quoted below from [25].

Theorem 1: [25] Let $\{X'_1 < X'_2 < \cdots < X'_Q\}$ be the order statistics from Q identical and independently distributed ($i.i.d.$) exponential random variables (RVs) with parameter ν. Define $Z_{Q,q} = \sum_{j=Q-q+1}^{Q} X'_j, 1 \leq q \leq Q$. The complementary cumulative distributed function (CCDF) of $Z_{Q,q}$ is given by:

$$\begin{aligned} P\{Z_{Q,q} > z\} &= \sum_{j=1}^{Q-q} a_j e^{\left(\frac{-c_j}{q}\nu z\right)} \times \frac{1}{(q-1)!}\int_0^{z\nu} e^{(b_j t)} t^{(q-1)} dt + \sum_{k=0}^{q-1} e^{(-\nu z)} \frac{(\nu z)^k}{k!} \\ &= \sum_{j=1}^{Q-q} a_j e^{\left(\frac{-c_j}{q}\nu z\right)} \times \frac{1}{(-b_j)^q}\left[1 - e^{b_j \nu z}\sum_{\ell=0}^{q-1}\frac{(-b_j \nu z)^\ell}{\ell!}\right] + \sum_{k=0}^{q-1} e^{(-\nu z)}\frac{(\nu z)^k}{k!} \end{aligned} \tag{10}$$

with $a_j \triangleq \frac{1}{Q-j+1}\frac{Q!}{q!}\frac{(-1)^{Q-q-j}}{(j-1)!(Q-q-j)!}$, $b_j \triangleq \frac{Q-q-j+1}{q}$ and $c_j \triangleq Q - j + 1$.

Applying (10) to (9), the resultant outage probability of $P_{\mathcal{O}_i|\mathcal{D}}$ is given by

$$P_{\mathcal{O}_i|\mathcal{D}}(\delta|d) = 1 - \sum_{k=0}^{i-1} e^{-\delta\lambda_2}\frac{(\delta\lambda_2)^k}{k!} - \sum_{j=1}^{d-i}\tilde{a}_j e^{-\frac{\tilde{c}_j}{i}\delta\lambda_2}\frac{1}{(-\tilde{b}_j)^i}\left[1 - e^{\tilde{b}_j\delta\lambda_2}\sum_{\ell=0}^{i-1}\frac{(-\tilde{b}_j\delta\lambda_2)^\ell}{\ell!}\right] \tag{11}$$

with $\tilde{a}_j \triangleq \frac{1}{d-j+1}\frac{d!}{i!}\frac{(-1)^{d-i-j}}{(j-1)!(d-i-j)!}$, $\tilde{b}_j \triangleq \frac{d-i-j+1}{i}$ and $\tilde{c}_j \triangleq d - j + 1$. Based on the above results, the outage probability for a direct transmission followed by an ARQ with ODSTC using$i_{\mathcal{D}} \triangleq \min\{i, \mathcal{D}\}$, $i \in [1, M]$, relays is given by

$$\mathbb{P}_i = P_W(\delta_s)^2 \mathcal{P}_{\mathcal{D}}(0) + P_W(\delta_s)\sum_{d=1}^{M} P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)\mathcal{P}_{\mathcal{D}}(d). \tag{12}$$

For convenience of expression, and to distinguish from the ordinary DSTC protocol, we use ODSTCi in the sequel to signify the use of i relays at most for ODSTC in ARQs. For ODSTC1, the outage probability is equal to that of the opportunistic relaying (OR) in [10], while for ODSTCM, it is equal to the typical DSTC relaying scheme in [2].

III. EFFECTIVENESS OF COOPERATIVE RELAYING VIA OPPORTUNISTIC DSTC

To motivate the subsequent performance investigations throughout the paper, we would first characterize the relative SNR advantages of DSTC (ODSTCM) over that of an ODSTCi scheme that uses at most i relays in $\mathcal{S}_D$ for retransmission. The purpose is to study the effectiveness of cooperative relaying with ODTSCi, and compare it with the simple OR scheme that uses only one relay out of $\mathcal{S}_D$ and the DSTC scheme that uses all relays in $\mathcal{S}_D$ to forward the signals. This analysis also helps assess the feasibility and provides insight into the practice of DSTC in relay networks, considering the fact that the constantly changing $\mathcal{S}_D$ would make the design and implementation of a universal DSTC relaying scheme particularly difficult.

To alleviate the complexity of analysis, we investigate this problem in the high SNR regime. To this end, we first need to find the asymptotic functions of $P_W(\delta_s)$, $\mathcal{P}_\mathcal{D}(d)$ and $P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)$ of (12) at high SNR. For $P_W(\delta_s)$ and$\mathcal{P}_\mathcal{D}(d)$, it has been shown in [2] that

$$\lim_{\lambda\to 0}\left\{\frac{P_W(\delta_s)}{\lambda}\right\}=\delta_s \tag{13}$$

and

$$\lim_{\lambda\to 0}\left\{\frac{\mathcal{P}_\mathcal{D}(d)}{\lambda^{M-d}}\right\}=\frac{C_d^M\delta_s^{M-d}}{\alpha^{M-d}}. \tag{14}$$

As for $P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)$, we have

Proposition 1: Define $i_d\triangleq\min\{i,d\}$. Given β and δ, we have

$$\lim_{\lambda\to 0}\left\{\frac{P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)}{\lambda^d}\right\}=\frac{\delta^d}{i_d!\,i_d^{d-i_d}\beta^d}. \tag{15}$$

Proof: The proof is provided in Appendix-A

Now, we define an asymptotic equivalence of a function as [26]

$$f(\lambda)\doteq a\lambda^n,\ \text{when}\ \lim_{\lambda\to 0}\left\{\frac{f(\lambda)}{\lambda^n}\right\}=a,\ a\neq 0\ \text{and}\ n\geq 0. \tag{16}$$

Following the definition, it is straightforward to show that

$$a\lambda^n\pm b\lambda^m \doteq a\lambda^n,\ \text{if}\ m>n\geq 0 \tag{17}$$

$$a\lambda^n\cdot b\lambda^m \doteq ab\lambda^{n+m},\ \text{if}\ m+n\geq 0. \tag{18}$$

According to the definition, we may rewrite $P_W(\delta_s)\doteq\delta_s/\rho$ and $\mathcal{P}_\mathcal{D}(d)\doteq C_d^M\left(\delta_s/\alpha\rho\right)^{M-d}$ when $\lambda\to 0$, or equivalently, when $\rho\triangleq\frac{1}{\lambda}\to\infty$ at high SNR. Furthermore, by Proposition 1, we also have

$$P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)\doteq\frac{\delta^d\lambda^d}{i_d!\,i_d^{d-i_d}\beta^d}\equiv\frac{\delta^d}{i_d!\,i_d^{d-i_d}\beta^d\rho^d}. \tag{19}$$

With the results from (13) $\sim$ (19), we next investigate the effectiveness and efficiency of ODSTCi versus i from the perspective of DMT.

A. A DMT Perspective on the Efficiency of ODSTCi

Define the rate adaptation rule for source transmission as $R\triangleq\gamma\log(\rho+1)+R_0$ and the rule for relaying as $\varepsilon R\triangleq\gamma\log(\beta\rho+1)+R_0$, where $\gamma<1$ is referred to as the multiplexing gain [26]. Applying the results of (13) $\sim$ (19) back to (12) immediately gives

$$\mathbb{P}_i \doteq \frac{\delta_0^{M+1}}{\alpha^M \rho^{(1-\gamma)(M+1)}} \left[\frac{\delta_0}{\rho^{1-\gamma}} + \sum_{d=1}^{M} \frac{C_d^M}{i_d! i_d^{d-i_d}} \left(\frac{\alpha}{\beta^{1-\gamma}} \right)^d \right] \tag{20}$$

since $\delta_s \doteq \delta_0 \rho^\gamma$ and $\delta \doteq \delta_0 (\beta\rho)^\gamma$ at high SNR with $\delta_0 \triangleq 2^{R_0}$. Now in cases where $\beta^{1-\gamma} \gg \alpha$, then the summation in the right hand side of (20) will be dominated by the event of $d=1$. Furthermore, if $\beta^{1-\gamma} \gg M\alpha\rho^{1-\gamma}/\delta_0 \geq \alpha$ such that

$$\lim_{\rho\to\infty} \left\{ \frac{\beta}{\rho} \right\} \gg \sqrt[1-\gamma]{\frac{\alpha M}{\delta_0}} \tag{21}$$

then (20) can be approximated at high SNR as

$$\mathbb{P}_i \doteq \frac{\delta_0^{M+1}}{\alpha^M \rho^{(1-\gamma)(M+1)}} \left[\frac{\delta_0}{\rho^{1-\gamma}} + \frac{M\alpha}{\beta^{1-\gamma}} \right] \doteq \frac{\delta_0^{M+2}}{\alpha^M \rho^{(1-\gamma)(M+2)}}. \tag{22}$$

This corresponds to an operating scenario where outage events are mainly attributed to the link failures between the source and the relays. The R-D link strength is so much higher than the S-R link strength such that the destination is able to decode the signal as long as at least one of the relays is able to do so. Except for this extreme operating condition, in normal channel environments of relaying, we usually have

$$\mathbb{P}_i \doteq \frac{\delta_0^{M+1}}{\alpha^M \rho^{(1-\gamma)(M+1)}} \sum_{d=1}^{M} \frac{C_d^M}{i_d! i_d^{d-i_d}} \left(\frac{\alpha}{\beta^{1-\gamma}} \right)^d. \tag{23}$$

Therefore, following the definition of diversity given by [26]

$$\xi \triangleq -\lim_{\rho\to\infty} \left\{ \frac{\log \mathbb{P}_i}{\log \rho} \right\} \tag{24}$$

the diversity and multiplexing tradeoff (DMT) [26] on $\mathbb{P}_i$ can be partitioned into two regimes with respect to ($w.r.t.$) ρ. They are summarized below as

$$\xi \triangleq \left\{ \begin{array}{ll} (1-\gamma)(M+2), & \rho \ll \sqrt[(1-\gamma)]{\frac{\delta_0 \beta^{1-\gamma}}{M\alpha}} \\ (1-\gamma)(M+1), & \text{otherwise} \end{array} \right\}. \tag{25}$$

The threshold SNR $\rho^* \triangleq \sqrt[(1-\gamma)]{\frac{\delta_0 \beta^{1-\gamma}}{M\alpha}}$ not only is determined by the ratio between β and α, but also by the multiplexing gain γ itself.

Given the DMT, the coding gain G_i with ODSTCi relaying can be defined accordingly at high SNR as

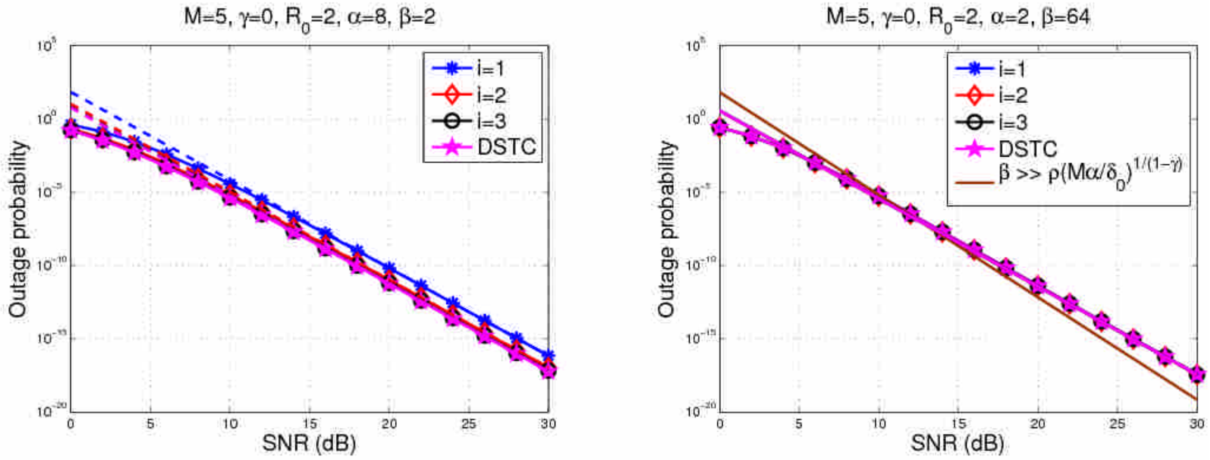
$$G_i \triangleq \lim_{\rho\to\infty} \frac{\mathbb{P}_i}{\rho^{-\xi}} = \lim_{\lambda\to 0} \frac{\mathbb{P}_i}{\lambda^\xi}. \tag{26}$$

As a result, we have

$$G_i \triangleq \left\{ \begin{array}{ll} \frac{\delta_0^{M+2}}{\alpha^M}, & \rho \ll \rho^* \\ \frac{\delta_0^{M+1}}{\alpha^M} \sum_{d=1}^{M} \frac{C_d^M \alpha^d}{i_d! i_d^{d-i_d} \beta^{(1-\gamma)d}}, & \text{otherwise} \end{array} \right\}. \tag{27}$$

Since the first row corresponds to a situation where the destination is not able to decode the signal until at least one of the relays is able to decode the signal successfully, the diversity in this case increases by $(1-\gamma)(M+2)$ for each ARQ and the coding gain is not a function of i any more, making it no need to use for more than one relay in this scenario.

On the other hand when $\rho \gg \rho^*$, every ODSTCi scheme, $i = 1, \ldots, M$, still achieves the same diversity as expected, while with different coding gains. According to (26), the SNR loss of using ODSTCi than DSTC can be quantified as the extra SNR required by the ODSTCi to achieve the same level of the outage probability, $\mathbb{P}'$, achieved with the DSTC at high SNR. Let $\mathbb{P}' = G_i \rho_i^{-\xi} = G_M \rho_M^{-\xi}$, the SNR loss of ODSTCi is thus defined as



(a) (b)

Fig. 3. The outage probabilities of cooperative relaying with ODSTCi for $i=1$, 2 and 3 when $M=5$, $\gamma=0$, $R_0=2$. The subplot (a) corresponds to a channel condition with $\alpha=8$ and $\beta=2$, and the subplot (b) corresponds to a condition wit $\alpha=2$ and $\beta=64$.

$$\mathcal{L}(i) \triangleq \log\{\rho_i\} - \log\{\rho_M\} = \frac{\log\{G_i\} - \log\{G_M\}}{\xi}, \ \forall i \in [1, M] \cap \mathfrak{N}. \qquad (28)$$

It is noted that $\mathcal{L}(i)$ is irrelevant of δ_0.

B. Numerical Studies

The outage probabilities of (12) and their corresponding high-SNR approximations of (23) for various i of ODSTCi are shown in Fig. 3 when $M=5$, $\gamma=0$ and $R_0=2$. Fig. 3(a) presents the results for $\alpha=8$ and $\beta=2$. Clearly, the dashed lines of the approximations match their corresponding exact curves of (12) at high SNR. Besides, the advantage of using more relays loses rapidly when i increases.

In contrast to Fig. 3(a), Fig. 3(b) shows the results for $\alpha=2$ and $\beta=64$. Both the approximations, (22) and (23), are drawn in the figure as opposed to (12) of the exact ones. As expected from (25), the diversity order of (12) is equal to $\xi=(1-\gamma)(M+1)=6$ in this simulation setting when $\rho>\rho^*=14.08$ dB. However, $\xi=(1-\gamma)(M+2)=7$ is only meaningful for $10<\rho<14.08$ dB since the exact curves of (12) start to deviate away from their approximated ones when $\rho<10$ dB. Nevertheless, the results are consistent with (27) which shows no SNR loss is incurred by using ODSTCi, $\forall i \in [1, M]$, for $\rho<\rho^*$.

In addition to the zero SNR loss when $\rho<\rho^*$, the SNR losses for $\rho>\rho^*$ appear to be minor as well either in subplots (a) or (b). This phenomenon is explained with the results in Fig. 4. The analytical SNR losses, $\mathcal{L}(i)$, of (28) versus their simulated counterparts are presented in Fig. 4(a) for various sets of $\{\alpha,\beta\}$ when $\gamma=0.5$ and $R_0=0$. As shown in the figure, when the S-R to S-D link ratio, α, is higher than the R-D to S-D link ratio, β, the SNR losses, $\mathcal{L}(i)$, are more pronounced when the number of active relays, i, for ODSTCi decreases. In fact, if α continues to increase such that $\alpha \gg \beta^{1-\gamma}$, then $\mathbb{P}_i$ of (23) is eventually dominated by the event of $\mathcal{D}=M$, which makes

$$\mathbb{P}_i \doteq \frac{\delta_0^{M+1}}{\beta^{(1-\gamma)M}\rho^{(1-\gamma)(M+1)}} \frac{1}{i! i^{M-i}}. \qquad (29)$$

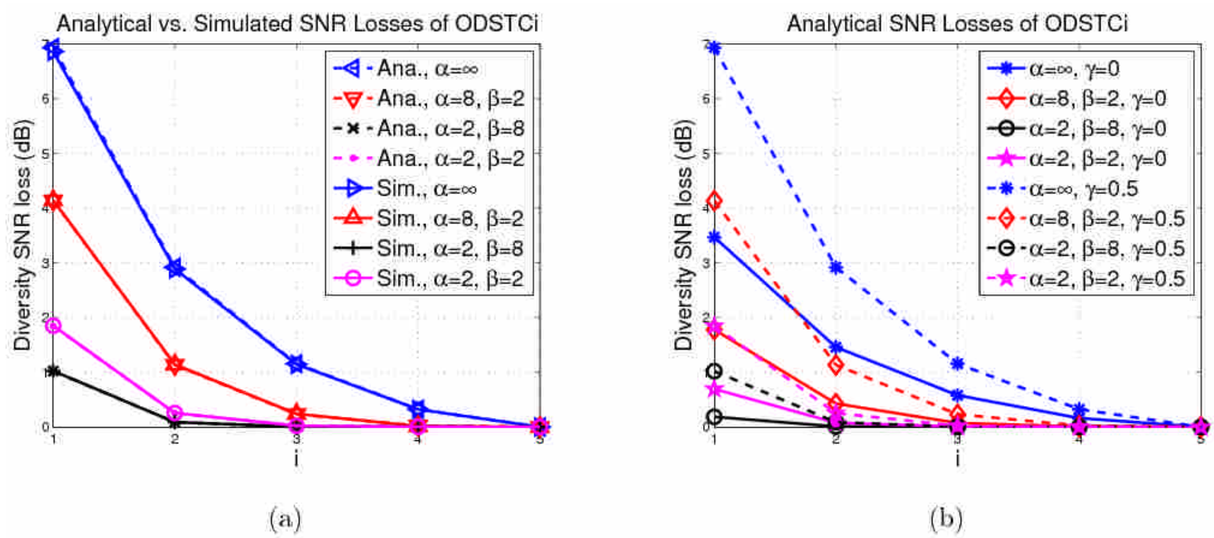


Fig. 4. The SNR losses of ODSTCi at high SNR for $i = 1, \ldots, M$ and $M = 5$. Subplot (a) corresponds to the results for $R_0 = 0$ and $\gamma = 0.5$, while subplot (b) also includes the results for $R_0 = 2$ when $\gamma = 0$.

Then, it follows directly from (25), (26) and (28) that

$$\mathcal{L}_\alpha(i) \triangleq \frac{1}{(1-\gamma)(M+1)} \log\left\{\frac{M!}{i! i^{(M-i)}}\right\} \quad \text{if } \alpha \gg \beta^{1-\gamma}. \tag{30}$$

This shows that when the relay channel resembles a MISO one when $\alpha \gg \beta^{1-\gamma}$, $\mathcal{L}(i)$ become irrelevant of α and β, and decrease exponentially *w.r.t.* i as verified both in subplots (a) and (b) when $\alpha \to \infty$.

In contrast, if $\beta > \alpha$, instead, then $\mathcal{L}(i)$ become very small $\forall i \in [1, M]$. If β continues to increase such that $\beta \gg \sqrt[1-\gamma]{M\alpha\rho^{1-\gamma}/\delta_0}$, then by (27), it follows that $\mathcal{L}(i) = 0$. The destination under this channel condition can decode the signal as long as any one of the relays is able to do so. Thus, the number of i does not really matter in this case. Opportunistic relaying (ODSTC1) appears to be the best choice in this type of channels.

IV. OUTAGE PROBABILITIES OD COOPERATIVE ARQ WITH ODSTC RELAYING

We have studied and characterized the effectiveness of ODSTCi *w.r.t.* the link qualities of a relay channel. To further exploit the opportunistic diversities offered by ODSTCi, we extend this notion to ARQ and study effective protocols with ODSTCi. Three types of cooperative protocols are considered herein to exploit both the temporal and spatial diversities provided by ARQs with ODSTCi. Each one of them requires different levels of coordinations among the source, relays and the destination.

A: Type-A Cooperative ARQ

As introduced earlier, whenever outages occur and $\mathcal{S}_D = \emptyset$, the destination will issue an ARQ to the source. Once $\mathcal{S}_D \neq \emptyset$, by ODSTCi, the most straightforward method is to choose in the beginning the best $i_D = \min\{i, \mathcal{D}\}$ relays out of $\mathcal{S}_D$ to maximize (5) at the destination, and then continue to use these relays in the subsequent ARQs if needed. As such, the Type-A protocol considered herein essentially involves two kinds of relaying methods: the

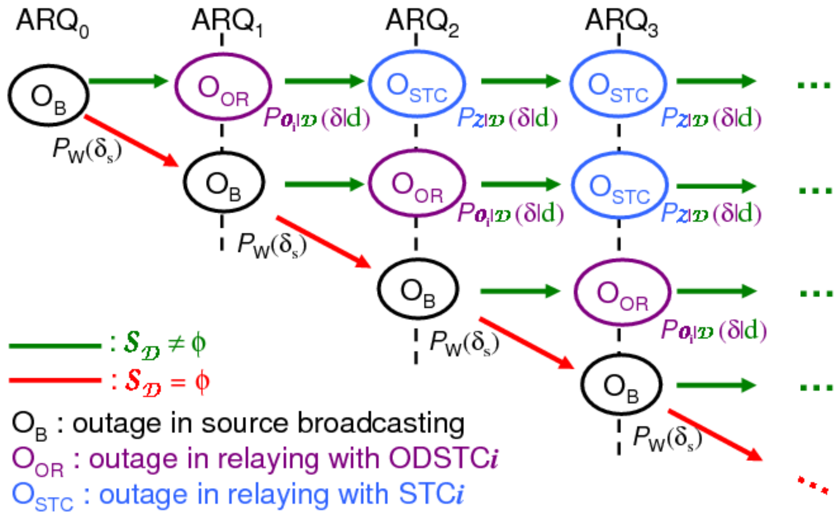


Fig. 5. The outage events in the Type-A ARQ protocol.

ODSTCi at the ARQ when $\mathcal{D}$ turns nonzero, and an ordinary DSTC that uses the same $i_{\mathcal{D}}$ relays in the subsequent ARQs. The reason is that the channel coefficients from relays to the destination change independently in every ARQ, making the opportunistic diversity from channel ordering no longer available in the subsequent ARQs if the same relays are used for retransmission. In the sequel, for clarity, we denote the DSTC by DSTCi to indicate the maximum number of relays that might be used in ARQs.

The outage probability of ODSTCi has been provided in (9), and the outage probability of DSTCi can be obtained from (7) by having $d = i_d$, yielding

$$P_{\mathcal{Z}|\mathcal{D}}(\delta|i_d) = 1 - \sum_{j=0}^{i_d-1} \frac{e^{-\delta\lambda_2}}{j!(\delta\lambda_2)^{-j}}. \quad (31)$$

Based on $P_W(\delta_s)$, $\mathcal{P}_{\mathcal{D}}(d)$, $P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)$ and $P_{\mathcal{Z}|\mathcal{D}}(\delta|i_d)$ of (6), (8), (9) and (31), respectively, we are able to analyze the outage probability of the Type-A protocol for any number of ARQs. For simplicity, we denote the n-th rounds of ARQs by ARQn and also refer to the initial direct transmission from the source to the destination as ARQ0. The outage probability is provided in the following proposition.

Proposition 2: Given R, ε, M and $i \in [1, M]$, the outage probability after n times of the Type-A ARQs is given by

$$\mathbb{P}_{A,i}(n) = P_W(\delta_s)^{n+1}\mathcal{P}_{\mathcal{D}}^n(0) + P_W(\delta_s)\times \sum_{k=1}^{n}[P_W(\delta_s)\mathcal{P}_{\mathcal{D}}(0)]^{n-k}\sum_{d=1}^{M}\mathcal{P}_{\mathcal{D}}(d)P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)P_{\mathcal{Z}|\mathcal{D}}^{k-1}(\delta|i_d) \quad (32)$$

with $\mathbb{P}_{A,i}(0) \triangleq P_W(\delta_s)$ in(6).

Proof: We use a tree diagram in Fig. 5 to illustrate the outage events that might occur in the Type-A protocol. The ellipses marked by O_B stand for the outage events in source broadcasting, whose probability is given by $P_W(\delta_s)$.

Starting from the upper left of the figure, it shows that if an outage occurs after the source broadcasting and $\mathcal{S}_D \neq \emptyset$, then the ARQ that follows immediately after the broadcasting will employ the ODSTCi for retransmission, whose outage event is marked by O_{OR} in the figure with the probability given by $P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)$. In case the retransmission still fails, then the subsequent ARQs will use the same relays for retransmission with DSTCi whose outage events are marked as O_{STC} in the figure, with a probability equal to (31). The time sequence of these outage events is shown as a row of ellipses linked by green arrows from left to right in the figure.

On the other hand, if $\mathcal{S}_D = \emptyset$ after broadcasting, then the source will rebroadcast the signal in case of outages, which will, in turn, invoke another source broadcasting if an outage still happens and $\mathcal{S}_D = \emptyset$ as well. Otherwise, it will initiate another sequence of ARQs the same to what is described above for $\mathcal{S}_D \neq \emptyset$ if outages continue to happen. Since $P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)$ and $P_{\mathcal{Z}|\mathcal{D}}(\delta|i_d)$ condition on $\mathcal{D}$ only, not on the specific relays in $\mathcal{S}_D$, it is clear from the tree diagram of Fig. 5 that the outage probability can be expressed in a recursive form of

$$\mathbb{P}_{A,i}(n) = P_W(\delta_s)\sum_{d=1}^{M}\mathcal{P}_{\mathcal{D}}(d)P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)P_{\mathcal{Z}|\mathcal{D}}^{n-1}(\delta|i_d) + P_W(\delta_s)\mathcal{P}_{\mathcal{D}}(0)\mathbb{P}_{A,i}(n-1). \quad (33)$$

Expanding this recursive form directly gives the result of (32). ■

B: Type-B Cooperative ARQ

Though simple, the diversity order of the Type-A protocol might be very limited after ARQ2 as the opportunistic diversity is no longer available due to the fixed relaying thereafter. Intuitively, a quick modification to resolve this problem is to have the i_D active relays be re-chosen from $\mathcal{S}_D$ according to the channel strength in each round of ARQ. We refer to this type of ARQ as the Type-B protocol. Due to this re-selection mechanism, we know that $P_{\mathcal{Z}|\mathcal{D}}(\delta|i_d)$ in (32) for the Type-A protocol should be replaced by $P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)$ in this case. This gives the outage probability for the Type-B ARQ protocol, summarized in the following corollary.

Corollary 1: Given R, ε, M and $i \in [1, M]$, the outage probability after n times of the Type-B ARQs is given by

$$\mathbb{P}_{B,i}(n) = P_W(\delta_s)^{n+1}\mathcal{P}_{\mathcal{D}}^{n}(0) + P_W(\delta_s)\times\sum_{k=1}^{n}[P_W(\delta_s)\mathcal{P}_{\mathcal{D}}(0)]^{n-k}\sum_{d=1}^{M}P_{\mathcal{O}_i|\mathcal{D}}^{k}(\delta|d)\mathcal{P}_{\mathcal{D}}(d) \quad (34)$$

with $\mathbb{P}_{B,i}(0) \triangleq P_W(\delta_s)$.

Compared with the Type-A protocol, apparently, the Type-B requires all relays in $\mathcal{S}_D$ to hold the decoded data for retransmission until the end of ARQs. However, checking $P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)$ in (34), one may soon find that the diversity order might still be limited by the event of $\mathcal{D}=1$ if $P_{\mathcal{O}_i|\mathcal{D}}(\delta|1)$ dominates the other $P_{\mathcal{O}_i|\mathcal{D}}(\delta|d), \forall d>1$. This situation may happen when the link quality between the source and the relays is poor, and would make the Type-B protocol rather ineffective, taking into account the extra efforts to re-select the relays in each ARQ. A quick remedy to this diversity shortage problem is to allow overhearing on the non-active relays. Nevertheless, a thorough comparative studies on the diversity orders of the proposed protocols will be provided in Section V.

C: Type-C Cooperative ARQ

As pointed out in the previous section, the diversity orders of the Type-A and B protocols may both be limited by the worst occasion of $\mathcal{D}=1$. To prevent from this diversity shortage problem, intuitively, the decoding set should be able to grow with ARQs. To this end, we must have the relays not in $\mathcal{S}_D$ continue to overhear the DSTC signals sent by the active relays in ARQs, and update their status to the destination to allow being picked up in the subsequent ARQs. We refer to this type of ARQ as the Type-C protocol and investigate its outage probability herein. To begin with, we first characterize the outage probability for relays that overhear the DSTC signal sent by the active relays.

For the convenience of performance investigation, channels between any transmit-and-receive pairs of the relays, (r_m, r_ℓ), for $r_m \in \mathcal{S}_D$ and $r_\ell \notin \mathcal{S}_D$, are assumed to have the same received SNR, $P_{rr}/N_0 \triangleq \eta\rho$, while with individual channel coefficients $\eta\rho|h_{r_m,r_\ell}|^2 \sim \text{Exp}(\lambda_3)$ and $\lambda_3 \triangleq 1/(\eta\rho)$. Since the channels between different pairs of (r_m, r_ℓ) fade independently, the channels from the relays, r_m, which are particularly chosen from $\mathcal{S}_D$ to yield the highest mutual information at the destination do not necessarily provide the overhearing relays, r_ℓ, the same opportunistic diversity via channel ordering. Therefore, the outage probability of overhearing has the ordinary form of MISO channel given in (7). Define $V_{m,\ell} \triangleq \eta\rho|h_{r_m,r_\ell}|^2 \sim \text{Exp}(\lambda_3)$. The outage probability for a relay ℓ overhearing the DSTC signal send by d active relays in $\mathcal{S}_D$ is thus given by

$$P_{\mathcal{V}|\mathcal{D}}(\delta|d) \triangleq P\left\{\mathcal{V} \triangleq \sum_{r_m\in\mathcal{S}_D} V_{m,\ell} < \delta \,\middle|\, d \geq 1\right\} = 1 - \sum_{j=0}^{d-1} e^{-\delta\lambda_3}\frac{(\delta\lambda_3)^j}{j!}. \quad (35)$$

Based on this result, we shall re-formularize the PMF of $\mathcal{S}_D$ as $|\mathcal{S}_D|$ continues to increase with ARQs in the Type-C protocol. To facilitate the analysis, we define some RVs below.

Definition 1: Let $\mathcal{D}_0 \in [0, M]$ be the number of relays that are able to decode the signal sent by the source.

Definition 2: Let $\mathcal{D}_n \in [0, M]$ be the number of increasing relays in the n-th subsequent ARQ after $\mathcal{D}_0 \geq 1$.

Definition 3: Let $\underline{\mathcal{D}}_n = \sum_{\ell=0}^{n}\mathcal{D}_\ell$ and $\underline{\mathcal{D}}_n \in [1, M]$ be the aggregate number of relays that are able to decode the signal up to the n-th subsequent ARQ after $\mathcal{D}_0 \geq 1$.

According to the above definitions, it follows directly from (8) that

$$\mathcal{P}_{\mathcal{D}_0}(d_0) = C_{d_0}^{M} e^{-\lambda_1\delta_s d_0}(1-e^{-\lambda_1\delta_s})^{M-d_0}. \quad (36)$$

Besides, as the source stops sending signal once $\mathcal{D}_0 \geq 1$. By (35), we have the PMF of $\mathcal{D}_n$ conditioned on $\underline{\mathcal{D}}_{n-1}$ given by

$$\mathcal{P}_{\mathcal{D}_n|\underline{\mathcal{D}}_{n-1}}(d_n|\underline{d}_{n-1}) = C_{d_n}^{M-\underline{d}_{n-1}}[1-P_{\mathcal{V}|\mathcal{D}}(\delta|i_n)]^{d_n}[P_{\mathcal{V}|\mathcal{D}}(\delta|i_n)]^{M-\underline{d}_{n-1}-d_n},\ n=1,2,\cdots \quad (37)$$

where $\underline{d}_n \triangleq \sum_{\ell=0}^{n} d_\ell$. In addition, for convenience of notation, we redefine $i_n \triangleq \min\{i, \underline{d}_{n-1}\}$, thus $i_1 = \min\{i, \underline{d}_0\} \equiv \min\{i, d\} \equiv i_d$ defined in Proposition 1.

Given the conditional PMF of $\mathcal{D}_n$, we are in position to analyze the outage probability of the Type-C protocol. However, we note that for the outage probability of ARQ1, there is no difference between the three types of the ARQ protocols. It is exactly equal to $\mathbb{P}_i$ at (12). The difference starts from ARQ2 for which the outage probability of the Type-C protocol can be easily shown

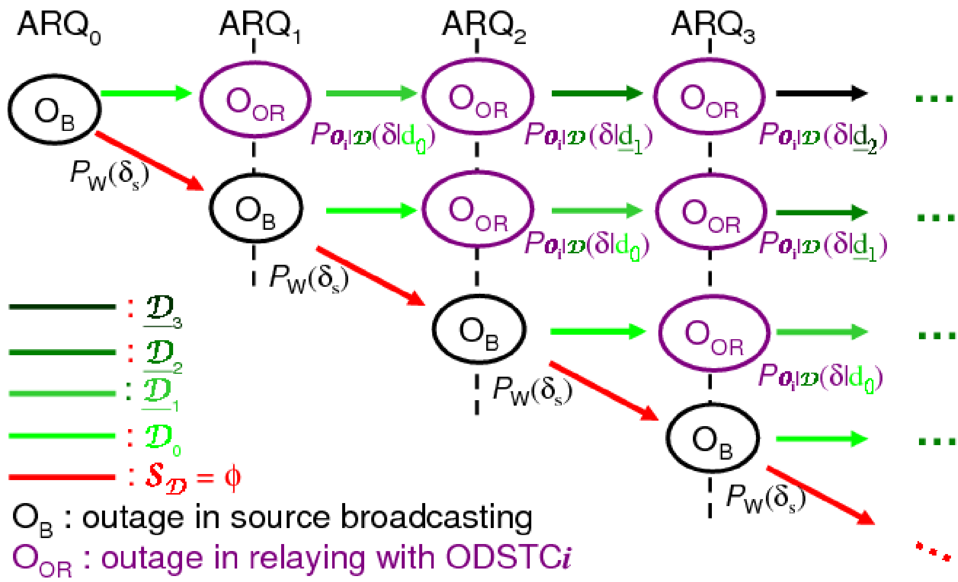


Fig. 6. The outage events of the Type-C ARQ protocol.

to be

$$\mathbb{P}_{C,i}(2) = P_W(\delta_s)\mathcal{P}_{\mathcal{D}_0}(0)\mathbb{P}_{C,i}(1) + P_W(\delta_s)\times \sum_{d_0=1}^{M} P_{\mathcal{O}_i|\mathcal{D}}(\delta|d_0)\mathcal{P}_{\mathcal{D}_0}(d_0) \sum_{d_1=0}^{M-d_0} P_{\mathcal{O}_i|\mathcal{D}}(\delta|\underline{d}_1)\mathcal{P}_{\mathcal{D}_1|\underline{\mathcal{D}}_0}(d_1|\underline{d}_0) \quad (38)$$

with $P_{C,i}(1) \equiv \mathbb{P}_i$. A general formula is provided in the following proposition.

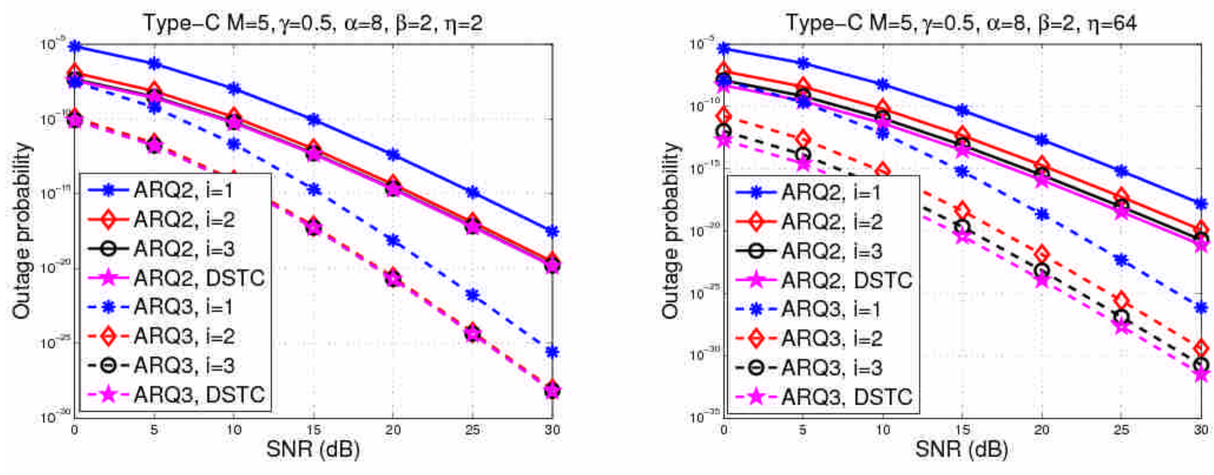
Proposition 3: Given R, ε, M and $i \in [1, M]$, the outage probability after n times of the Type-C ARQs is given by

$$\mathbb{P}_{C,i}(n) = P_W(\delta_s)^{n+1}\mathcal{P}_{\mathcal{D}_0}^{n}(0) + P_W(\delta_s)\sum_{k=1}^{n}[P_W(\delta_s)\mathcal{P}_{\mathcal{D}_0}(0)]^{n-k}\times \sum_{d_0=1}^{M}\sum_{d_1=0}^{M-d_0}\cdots\sum_{d_{k-1}=0}^{M-\underline{d}_{k-2}} P_{\mathcal{O}_i|\mathcal{D}}(\delta|d_0)\mathcal{P}_{\mathcal{D}_0}(d_0)\prod_{\ell=1}^{k-1} P_{\mathcal{O}_i|\mathcal{D}}(\delta|\underline{d}_\ell)\mathcal{P}_{\mathcal{D}_\ell|\underline{\mathcal{D}}_{\ell-1}}(d_\ell|\underline{d}_{\ell-1}) \quad (39)$$

with $\underline{d}_k \triangleq \sum_{q=0}^{k} d_q$ and $\mathbb{P}_{C,i}(0) \triangleq P_W(\delta_s)$.

Proof: The proof follows a similar approach used in proving Proposition 2. Again, the outage events that may occur in the Type-C protocol are illustrated as a tree diagram in Fig. 6. From left to right and top to bottom, the figure shows that the source will rebroadcast the signal until $\mathcal{S}_D \neq \emptyset$, namely $\mathcal{D}_0 \geq 1$, if outages continue to happen. Suppose that $\mathcal{D}_0$ turns nonzero at the $(n-k)$th ARQ. If an outage still happens, then the ODSTCi will start to be used for retransmission by opportunistically choosing $i_1 = \min\{i, \underline{\mathcal{D}}_0\}$ relays out of $\mathcal{S}_D$ that maximize (5) at the end of the $(n-k)$th ARQ. It is noted that the cardinality of $\mathcal{S}_D$ will continue to grow as shown in the figure due to the overhearing mechanism.

If outages still continue to happen, then the ODSTCi will continue to be used for retransmissions as well, while re-choosing $i_{p+1} = \min\{i, \underline{\mathcal{D}}_p\}$ relays from $\mathcal{S}_D$ at the end of the $(n-k+p)$th ARQ, when the retransmission takes place at the $(p+1)$th ARQ after $\mathcal{D}_0 \geq 1$. The time sequence of these outage events is shown as a row of ellipses linked by green arrows from left to right in the figure. The arrows in different darkness represent different numbers of retransmissions after $\mathcal{D}_0 \geq 1$, thus corresponding to different cardinalities, $\underline{\mathcal{D}}_p$, of $\mathcal{S}_D$.



(a) (b)

Fig. 7. Outage probabilities of the Type-C ARQ protocol for $i = 1 \sim 3$ and M of ODSTCi when $M = 5$, $\gamma = 0.5$, $\alpha = 8$ and $\beta = 2$. Subplots (a) and (b) correspond to the cases of $\eta = 2$ and $\eta = 64$, respectively.

Since $P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)$ conditions on $|\mathcal{S}_D|$ only, not on the specific relays in $\mathcal{S}_D$, it is clear from the tree diagram that the outage probability of the Type-C protocol can be expressed in a recursive form given by

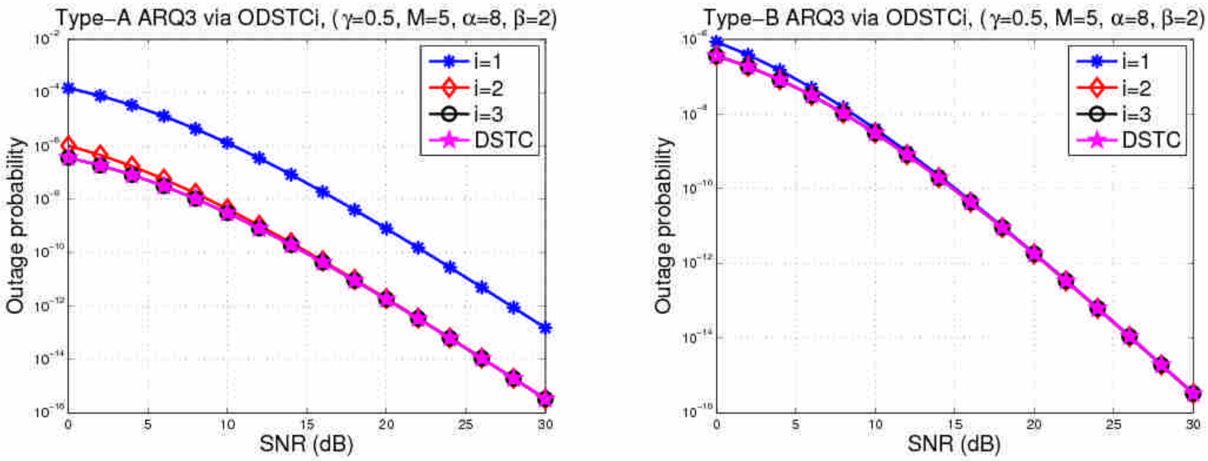
$$\mathbb{P}_{C,i}(n) = P_W(\delta_s)\mathcal{P}_{\mathcal{D}_0}(0)\mathbb{P}_{C,i}(n-1)+$$
$$P_W(\delta_s)\sum_{d_0=1}^{M}\sum_{d_1=0}^{M-\underline{d}_0}\cdots\sum_{d_{n-1}=0}^{M-\underline{d}_{n-2}} P_{\mathcal{O}_i|\mathcal{D}}(\delta|d_0)\mathcal{P}_{\mathcal{D}_0}(d_0)\prod_{\ell=1}^{n-1} P_{\mathcal{O}_i|\mathcal{D}}(\delta|\underline{d}_\ell)\mathcal{P}_{\mathcal{D}_\ell|\underline{\mathcal{D}}_{\ell-1}}(d_\ell|\underline{d}_{\ell-1}). \quad (40)$$

Expanding this recursive form directly gives the result of (39). ■

D. Numerical Studies

We present some simulation results on the outage probabilities and compare their diversity orders for the aforementioned three types of ARQ protocols. Fig. 7 shows the outage probabilities of the Type-C protocol when $M = 5$, $\gamma = 0.5$, $\alpha = 8$ and $\beta = 2$. Subplot (a) presents the results for $\eta = 2$ while subplot (b) corresponds to the case of $\eta = 64$. The outage probabilities for $i = 1, 2$ and 3 of ODSTCi are drawn for ARQ2 and ARQ3, respectively, and contrasted with that of $i = M$, denoted by DSTC in the figure.

Both figures show that the diversities increase significantly with ARQs and that with only two to three relays, the performance can almost achieve what it does using all relays. Even though, for the case of $\eta = 2$, the SNR losses of ODSTCi against DSTC decrease when increasing the number of retransmissions from 2 to 3. For the case of $\eta = 64$, however, the SNR losses increase slightly with the number of retransmissions. The SNR loss of ODSTC1 at ARQ2 is around 4 dB for the case of $\eta = 2$, and it decreases to around 2.6 dB at ARQ3. On the contrary, for $\eta = 64$, the SNR loss of ODSTCi increases by about 1 dB, when the number of retransmissions increases from 2 to 3. This implies that the efficiency of ODSTCi depends on the R-R to S-D link ratio, η, between the relays as well as the maximum allowable retransmission times.



(a) (b)

Fig. 8. Outage probabilities for $i = 1 \sim 3$ and M of ODSTCi after three times of ARQs. Subplot (a) corresponds to $\mathbb{P}_{A,i}(3)$ and subplot (b) corresponds to $\mathbb{P}_{B,i}(3)$ with $M = 5$, $\gamma = 0.5$, $\alpha = 8$ and $\beta = 2$.

On the other hand, the outage probabilities of the Type-A and B protocols are presented in Fig. 8(a) and 8(b), respectively, under the same simulation setting. Different from the Type-C protocol, the outage probabilities of the Type-A protocol merge together at high SNR when $i \geq 2$. However, the results of the Type-B have no difference at all in the high SNR regime for any value of i. This implies that no more than one active relay is needed for the Type-B protocol and that the simplest Type-A protocol can perform equally well with the Type-B protocol at high SNR, using only two active relays. This is because the Type-A and B protocols are actually the same when both use ODSTCM=DSTCM for retransmissions. The characteristics are so different in the outage probabilities of the three types of ARQ protocols, yet they all indicate that the need for using more relays decreases drastically by using ODSTC, which motivates us to investigate the relay efficiency of ODSTCi for each type of the ARQ protocols. We study this problem from both the perspective of DMT and that of the delay and outage limited throughput. The results are presented in Section V and VI, respectively.

V. Diversity and Multiplexing Tradeoffs

To study the relay efficiency, or the efficiency of using more relays, in ODSTCi for different ARQ protocols, we first characterize the DMT of each protocol based on the outage formulas provided in Section IV. We then compare their DMTs under different link qualities parameterized by α, β and η. To alleviate the complexity of analysis, the DMT is investigated in the high SNR regime only. Furthermore, to simplify the expression, the data rate for source transmission is defined as $R \triangleq \gamma \log \rho$ [26] and the code rate for ODSTCi is defined as $\varepsilon R \triangleq \gamma \log(\beta\rho)$ where γ is referred to as the multiplexing gain [26].

A. DMT of the Type-A Protocol

To analyze the DMT from (32) of the Type-A protocol, it is also necessary to obtain the high SNR approximation of $P_{\mathcal{Z}|\mathcal{D}}(\delta|i_{\mathcal{D}})$ in (31). Given the fact that ODSTCi is equivalent to DSTCM when $i=M$, by Proposition 1, it is easy to see that the high SNR approximation is equal to setting $d=i_d$ in (19), which results in

$$P_{\mathcal{Z}|\mathcal{D}}(\delta|i_d) \doteq \frac{\delta^{i_d}}{i_d!\beta^{i_d}\rho^{i_d}}. \quad (41)$$

Together with (13), (14), (19) and the high SNR approximations of $\delta_s \doteq \rho^{\gamma}$ and $\delta \doteq (\beta\rho)^{\gamma}$, we immediately obtain the high SNR approximation of (32), which is given by

$$\mathbb{P}_{A,i}(n) \doteq \frac{\Lambda^n_{\alpha,\rho}}{\rho^{1-\gamma}} + \sum_{k=1}^{n}\sum_{d=1}^{M} \frac{\Lambda^{n-k+1}_{\alpha,\rho}\alpha^d\Omega_{d,k}}{\beta^{(1-\gamma)[d+i_d(k-1)]}\rho^{(1-\gamma)i_d(k-1)}} \quad (42)$$

with $\Lambda_{\alpha,\rho} \triangleq \frac{1}{\alpha^M\rho^{(1-\gamma)(M+1)}}$ and $\Omega_{d,k} \triangleq \frac{C^M_d}{i_d^{(d-i_d)}(i_d!)^k}$.

According to this expression, we will characterize the outage probabilities and the corresponding DMTs of the Type-A protocol under different channel conditions. They are summarized in the following proposition.

Proposition 4: Given the multiplexing gain γ and the channel link qualities parameterized by α and β, for n times of ARQs with the Type-A protocol of ODSTCi relaying, the outage probability can be approximated at high-SNR as:

1. If $2 \le i \le M$, then

$$\mathbb{P}_{A,i}(n) \doteq \left\{ \begin{array}{ll} \frac{g(\alpha,\beta,n)}{\rho^{(1-\gamma)[M+1+i(n-1)]}} & ,\alpha \ge \beta^{(1-\gamma)} \text{ and } \rho < \rho_{A_1} \\ \frac{1}{\alpha^{nM}\rho^{(1-\gamma)[n(M+1)+1]}} & ,\alpha < \beta^{(1-\gamma)} \text{ and } \rho < \rho_{A_2} \\ \frac{M}{\alpha^{M-1}\beta^{(1-\gamma)n}\rho^{(1-\gamma)(M+n)}} & ,\text{ otherwise} \end{array} \right\}. \quad (43)$$

where $g(\alpha,\beta,n) \triangleq \sum_{d=i}^{M}\alpha^{(d-M)}\beta^{-(1-\gamma)[d+i(n-1)]}\Omega_{d,n}$ and

$$\rho_{A_1} = \left[\frac{1}{M}\sum_{d=i}^{M}\frac{\alpha^{d-1}\Omega_{d,n}}{\beta^{(1-\gamma)[n(i-1)+d-i]}}\right]^{\frac{1}{(1-\gamma)(n-1)(i-1)}}. \quad (44)$$

$$\rho_{A_2} = \left[\frac{\beta^{(1-\gamma)n}}{\alpha^{M(n-1)+1}M}\right]^{\frac{1}{(1-\gamma)[M(n-1)+1]}}. \quad (45)$$

2. If $i=1$, then

$$\mathbb{P}_{A,i}(n) \doteq \left\{ \begin{array}{ll} \frac{1}{\alpha^{nM}\rho^{(1-\gamma)[n(M+1)+1]}} & ,\rho < \rho_{A_3} \\ \frac{\sum_{d=1}^{M}C^M_d\alpha^d/\beta^{(1-\gamma)d}}{\alpha^M\beta^{(1-\gamma)(n-1)}\rho^{(1-\gamma)(M+n)}} & ,\rho > \rho_{A_3} \end{array} \right\} \quad (46)$$

with

$$\rho_{A_3} \triangleq \left[\frac{\beta^{(1-\gamma)n}}{\alpha^{M(n-1)+1}\Delta_{\alpha,\beta}}\right]^{\frac{1}{(1-\gamma)[M(n-1)+1]}}. \quad (47)$$

And $\Delta_{\alpha,\beta} \triangleq \sum_{d=1}^{M}C^M_d\alpha^{(d-1)}/\beta^{(1-\gamma)(d-1)}$.

Proof: The proof is provided in Appendix-C.

We note that when $\alpha \ge \beta^{1-\gamma}$, which implies that data packets transmitted from the source can be more successfully decoded at the relays than packets transmitted from the relays and decoded at the destination. Thus, more active relays "i", in this case, give higher diversities. However, when ρ becomes larger, the outage probability contributed by the events of $\mathcal{D} \ge i$ and, hence, $i_{\mathcal{D}} = i$ drops *w.r.t.* ρ far more quickly than those by $i_{\mathcal{D}} = \mathcal{D}$ when $\mathcal{D} < i$. Eventually, when $\rho > \rho_{A_1}$, the outage probability becomes dominated by the event of $\mathcal{D} = 1 = i_{\mathcal{D}}$ which corresponds to the lowest diversity, causing a diversity loss from $(1-\gamma)[M+1+i(n-1)]$ to $(1-\gamma)(M+n)$. This diversity loss phenomenon can be delayed to happen with a higher value of α as suggested in the form of ρ_{A_1} in (44). If we have $\alpha \to \infty$, then the chances for $\mathcal{D} < M$ will become zero, which corresponds to an extreme case of a M by 1 MISO channel. The diversity will remain to be $(1-\gamma)[M+1+i(n-1)]$ in this case when $\rho \to \infty$.

On the other hand, when $\alpha < \beta^{1-\gamma}$, packets transmitted from the relays can be more successfully decoded at the destination than packets transmitted from the source and decoded at the relays. The destination is able to decode the data as long as there is more than one relay being able to do so. Therefore, the source will continue to retransmit the data until the destination successfully decodes it. However, this happens only when the relays fail in decoding all together. Thus, the diversity increases by $M+1$ for every extra ARQ, which also means that the probability for this to happen drops very quickly when ρ gets larger. Eventually when $\rho > \rho_{A_2}$, the probability becomes so small that the outage probability starts to be dominated by the event of $\mathcal{D}=1$ that gives the lowest diversity, causing a diversity loss from $(1-\gamma)[n(M+1)+1]$ to $(1-\gamma)[M+n]$.

Similarly, this diversity loss problem can be delayed to happen with a higher value of β as suggested in the form of ρ_{A_2} in (45). When $\beta \to \infty$, it will guarantee a 100-percent decoding as long as a relay is able to decode the data. This corresponds to another extreme case of a 1 by M SIMO channel. The diversity will remain at $(1-\gamma)[n(M+1)+1]$ in this case when $\rho \to \infty$.

For the case of $i=1$, the arguments for $\alpha \geq \beta^{1-\gamma}$ no longer apply since the diversity will always be equal to $(1-\gamma)(M+n)$. However, similar arguments for $\alpha < \beta^{1-\gamma}$ also apply for $\rho_{A_3} \geq \rho$. Besides, $\rho_{A_2} \geq \rho$ also implies $\beta^{(1-\gamma)} \gg \alpha$ as shown in Appendix-C.3, and thus

$$\rho_{A_2} \triangleq \left[\frac{\beta^{n(1-\gamma)}}{M\alpha^{M(n-1)+1}}\right]^{\frac{1}{(1-\gamma)[M(n-1)+1]}} \gtrsim \left[\frac{\beta^{n(1-\gamma)}}{\alpha^{M(n-1)+1}\Delta_{\alpha,\beta}}\right]^{\frac{1}{(1-\gamma)[M(n-1)+1]}} \triangleq \rho_{A_3} > \rho \quad (48)$$

due to the fact that

$$\Delta_{\alpha,\beta} \triangleq \sum_{d=1}^{M} C_d^M \frac{\alpha^{(d-1)}}{\beta^{(1-\gamma)(d-1)}} = C_1^M + \mathcal{O}\left(\frac{\alpha}{\beta^{(1-\gamma)}}\right) \gtrsim M. \quad (49)$$

Based on the above results, without the loss of generality, by (24), the DMT of the Type-A protocol is summarized as

$$\xi_A \triangleq \left\{ \begin{array}{cc} (1-\gamma)[M+1+i(n-1)], & \alpha \geq \beta^{(1-\gamma)} \text{ and } \rho < \rho_{A_1} \\ (1-\gamma)[n(M+1)+1], & \alpha < \beta^{(1-\gamma)} \text{ and } \rho < \rho_{A_2} \\ (1-\gamma)(M+n), & \text{otherwise} \end{array} \right\}. \quad (50)$$

B: DMT of the Type-B Protocol

For the DMT of the Type-B protocol, given (13), (14) and (19), the high-SNR approximation of its outage probability, (32), can be easily shown to be

$$\mathbb{P}_{B,i}(n) \doteq \frac{\Lambda_{\alpha,\rho}^{n}}{\rho^{1-\gamma}} + \sum_{k=1}^{n}\sum_{d=1}^{M} \frac{\Lambda_{\alpha,\rho}^{n-k+1}\alpha^{d}\Phi_{d,k}}{\beta^{(1-\gamma)dk}\rho^{(1-\gamma)d(k-1)}} \quad (51)$$

where we have defined $\Lambda_{\alpha,\rho} \triangleq \frac{1}{\alpha^{M}\rho^{(1-\gamma)(M+1)}}$ and $\Phi_{d,k} \triangleq \frac{C_d^M}{\left(i_d! i_d^{(d-i_d)}\right)^k}$, and also set $\delta_s \doteq \rho^{\gamma}$ and $\delta \doteq (\beta\rho)^{\gamma}$. Following a similar procedure done in Proposition 4 for the Type-A protocol, (51) can be further simplified in different channel conditions with results given in the following corollary.

Corollary 2: Given the multiplexing gain γ and the channel link qualities parameterized by ρ, α and β, for n times of ARQs with the Type-B protocol of ODSTCi relaying, the high-SNR approximations of its outage probability are given by:

$$\mathbb{P}_{B,i}(n) \doteq \left\{ \begin{array}{ll} \frac{\Phi_{M,n}}{\beta^{(1-\gamma)Mn}\rho^{(1-\gamma)(Mn+1)}} & ,\alpha \geq \alpha^* \text{ and } \rho < \rho_{B_1} \\ \frac{1}{\alpha^{nM}\rho^{(1-\gamma)[n(M+1)+1]}} & ,\alpha < \alpha^* \text{ and } \rho < \rho_{A_2} \\ \frac{M}{\alpha^{M-1}\beta^{(1-\gamma)n}\rho^{(1-\gamma)(M+n)}} & ,\text{ otherwise} \end{array} \right\}. \tag{52}$$

where ρ_{A_2} has already defined at (45) and

$$\alpha^* \triangleq \beta^{(1-\gamma)n}\rho^{(1-\gamma)(n-1)} \tag{53}$$

$$\rho_{B_1} = \left[\frac{\alpha^{M-1}}{M\beta^{(1-\gamma)n(M-1)}(i!i^{M-i})^n}\right]^{\frac{1}{(1-\gamma)(n-1)(M-1)}}. \tag{54}$$

Proof: The proof is provided in Appendix-D ■

Based on the above results and the definition of diversity in (24), the DMT of the Type-B protocol is summarized as

$$\xi_B \triangleq \left\{ \begin{array}{ll} (1-\gamma)(nM+1) & ,\alpha \geq \alpha^* \text{ and } \rho < \rho_{B_1} \\ (1-\gamma)[n(M+1)+1] & ,\alpha < \alpha^* \text{ and } \rho < \rho_{A_2} \\ (1-\gamma)(M+n) & ,\text{ otherwise} \end{array} \right\}. \tag{55}$$

Similar to the DMT of the Type-A protocol, the diversity order may change from $(1-\gamma)(nM+1)$ to $(1-\gamma)(M+n)$ when $\alpha \geq \alpha^*$ and $\rho \geq \rho_{B_1}$, or from $(1-\gamma)[n(M+1)+1]$ to $(1-\gamma)(M+n)$ when $\alpha < \alpha^*$ and $\rho \geq \rho_{A_2}$. Nevertheless, in contrast to the Type-A protocol, Type-B can attain the full diversity, $(1-\gamma)(nM+1)$, of relaying when $\rho < \rho_{B_1}$, owing to the reselection mechanism in each subsequent ARQ.

C. DMT of the Type-C Protocol

Even though the form of $\mathbb{P}_{C,i}(n)$ in (39) is a bit more complicated than those of $\mathbb{P}_{A,i}(n)$ and $\mathbb{P}_{B,i}(n)$, its DMT can be analyzed in a way similar to that for the Type-A and B protocols except that it requires the additional high-SNR approximations of $P_{\mathcal{O}_i|\mathcal{D}}(\delta|\underline{d}_\ell)$ and $P_{\mathcal{D}_\ell|\underline{\mathcal{D}}_{\ell-1}}(d_\ell|\underline{d}_{\ell-1})$. By Proposition 1, it follows that

$$P_{\mathcal{O}_i|\mathcal{D}}(\delta|\underline{d}_\ell) \doteq \frac{\delta^{\underline{d}_\ell}}{i_{(\ell+1)}^{(\underline{d}_\ell - i_{\ell+1})} i_{(\ell+1)}!(\beta\rho)^{\underline{d}_\ell}} \tag{56}$$

with $i_\ell \triangleq \min\{i, \underline{d}_{\ell-1}\}$. As for $\mathcal{P}_{\mathcal{D}_\ell|\underline{\mathcal{D}}_{\ell-1}}(d_\ell|\underline{d}_{\ell-1})$ of (37), it requires the high-SNR approximation of $P_{\mathcal{V}|\mathcal{D}}(\delta|d)$ in (35) for relay's overhearing, which is given by

$$P_{\mathcal{V}|\mathcal{D}}(\delta|d) = P\left\{\mathcal{V} \triangleq \sum_{\mu=1}^{d} V_{\mu,z} < \delta \,\middle|\, d \geq 1\right\} \doteq \frac{\delta^d}{d!(\eta\rho)^d}. \tag{57}$$

This expression clearly shows that there is no opportunistic diversity being provided to a relay that decodes the DSTC signal sent from other relays. Besides, the multiplexing gain in general must satisfy

$$\eta\rho \geq \delta \doteq (\beta\rho)^\gamma \Longleftrightarrow \gamma \leq \frac{\log\rho + \log\eta}{\log\rho + \log\beta} \triangleq \gamma^*. \tag{58}$$

Otherwise, we have $P_{\mathcal{V}|\mathcal{D}}(\delta|i_\ell) = 1$ when $d = 1$, which will result in

$\mathcal{P}_{\mathcal{D}_\ell|\underline{\mathcal{D}}_{\ell-1}}(d_\ell|\underline{d}_{\ell-1})=0,\ \forall\ell\in\mathfrak{N}$. Namely, no overhearing relay is able to decode the DSTC signal sent by the active relays. The Type-C protocol will degenerate to the Type-B protocol in this case

Given $\gamma\le\gamma^*$, applying (57) to (37) yields

$$\mathcal{P}_{\mathcal{D}_\ell|\underline{\mathcal{D}}_{\ell-1}}(d_\ell|\underline{d}_{\ell-1})\doteq C_{d_\ell}^{M-\underline{d}_{\ell-1}}\left[\frac{\delta^{i_\ell}}{i_\ell!(\eta\rho)^{i_\ell}}\right]^{M-\underline{d}_\ell}. \tag{59}$$

Now, set $\delta_s\doteq\rho^\gamma$ in (13) and (14), and $\delta\doteq(\beta\rho)^\gamma$ in (56) and (59). Substituting the results back into (39) gives the high-SNR approximation of $\mathbb{P}_{C,i}(n)$:

$$\mathbb{P}_{C,i}(n)\doteq\frac{\Lambda_{\alpha,\rho}^n}{\rho^{1-\gamma}}+\sum_{k=1}^{n}\Lambda_{\alpha,\rho}^{n-k+1}\times$$
$$\sum_{d_0=1}^{M}\sum_{d_1=0}^{M-\underline{d}_0}\cdots\sum_{d_{k-1}=0}^{M-\underline{d}_{k-2}}\Psi_{i,k}\frac{\alpha^{d_0}}{\beta^{(1-\gamma)d_0}}\frac{(\beta^\gamma/\eta)^{\sum_{\ell=1}^{k-1}i_\ell(M-\underline{d}_\ell)}}{\beta^{(1-\gamma)\sum_{\ell=1}^{k-1}\underline{d}_\ell}\rho^{(1-\gamma)\sum_{\ell=1}^{k-1}[\underline{d}_\ell+i_\ell(M-\underline{d}_\ell)]}} \tag{60}$$

where

$$\Psi_{i,k}\triangleq\frac{C_{d_0}^M}{i_k^{(\underline{d}_{k-1}-i_k)}i_k!}\prod_{\ell=1}^{k-1}\frac{C_{d_\ell}^{M-\underline{d}_{\ell-1}}}{i_\ell^{(\underline{d}_{\ell-1}-i_\ell)}(i_\ell!)^{M-\underline{d}_\ell+1}}. \tag{61}$$

Clearly, the second term on the RHS of (60) is dominated at high SNR by the feasible pairs of $(\underline{d}_\ell,i_\ell)$, $\ell=1,\ldots,k-1$, $k\ge2$, that minimize $\sum_{\ell=1}^{k-1}[\underline{d}_\ell+i_\ell(M-\underline{d}_\ell)]$. Since $i_\ell\ge1$ and $\underline{d}_\ell\le M$, we have $\underline{d}_\ell+i_\ell(M-\underline{d}_\ell)\ge M$. The equality always holds if $i=1$. For $i\ge2$, we define the subset of $\mathcal{S}_D$ that satisfies the lower bound in k times of ARQs as

$$\mathcal{S}_{k-1}\triangleq\left\{(d_0,\ldots,d_{k-1})\left|\begin{array}{c}\underline{d}_\ell+i_\ell(M-\underline{d}_\ell)=M\text{ and}\\ d_\ell\in[0,M-\underline{d}_{\ell-1}],\forall\ell\in[1,k-1]\end{array}\right.\right\} \tag{62}$$

for $k\ge2$, $\underline{d}_0\equiv d_0\ge1$ and $i\ge2$. The algorithm to construct $\mathcal{S}_{k-1}$ is provided in the following proposition.

Proposition 5: Assume $\underline{d}_0\equiv d_0\ge1$ and $i\ge2$.

- Let $\mathcal{S}_0\triangleq\{d_0|d_0\in[1,M]\}$
- For $\ell=1$ to $k-1$, $k\ge2$

1. Generate

$$\mathcal{S}_\ell^a\ \triangleq\ \{(d_\ell,\cdots,d_0)|d_\ell\in[0,M-2],d_0=1\text{ and }d_1=\cdots=d_{\ell-1}=0\text{ if }\ell\ge2\} \tag{63}$$
$$\mathcal{S}_\ell^b\ \triangleq\ \{(d_\ell,\cdots,d_0)|d_\ell=M-\underline{d}_{\ell-1},(d_{\ell-1},\cdots,d_0)\in\mathcal{S}_{\ell-1}\} \tag{64}$$

2. Set $\mathcal{S}_\ell\triangleq\mathcal{S}_\ell^a\cup\mathcal{S}_\ell^b$

Proof: The proof is provided in Appendix-E. ■

Given $\mathcal{S}_{k-1}$ and, thus, $i_\ell(M-\underline{d}_\ell)=M-\underline{d}_\ell$, $\forall\ell\in[1,k-1]$, we obtain

$$\mathbb{P}_{C,i}(n)\doteq\frac{\Lambda_{\alpha,\rho}^n}{\rho^{1-\gamma}}+\sum_{k=1}^{n}\frac{q_{i,k}(\alpha,\beta,\eta)}{\alpha^{M(n-k+1)}\beta^{(1-\gamma)(k-1)M}\rho^{(1-\gamma)(nM+n-k+1)}} \tag{65}$$

where

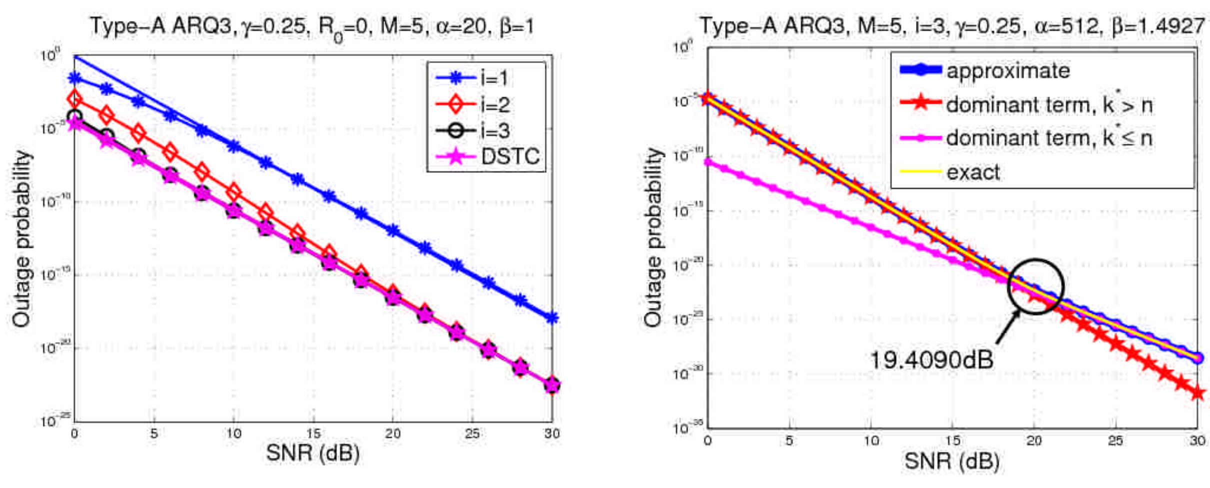
$$q_{i,k}(\alpha,\beta,\eta)\triangleq\sum_{\mathcal{S}_{k-1}}\frac{\alpha^{d_0}\Psi_{i,k}}{\beta^{(1-\gamma)d_0}}\left(\frac{\beta}{\eta}\right)^{\sum_{\ell=1}^{k-1}M-\underline{d}_\ell}. \tag{66}$$

Since the order of ρ in (65) decreases when k increases, keeping only the dominant term corresponding to $k=n$, we obtain at high SNR

$$\mathbb{P}_{C,i}(n)\doteq\frac{\Lambda_{\alpha,\rho}^n}{\rho^{1-\gamma}}+\frac{q_{i,n}(\alpha,\beta,\eta)}{\alpha^M\beta^{(1-\gamma)(n-1)M}\rho^{(1-\gamma)(nM+1)}}. \tag{67}$$

The diversity order of (67) is always equal to $(1-\gamma)(nM+1)$ except for the case of

$$\frac{\Lambda_{\alpha,\rho}^n}{\rho^{(1-\gamma)}}>\frac{q_{i,n}(\alpha,\beta,\eta)}{\alpha^M\beta^{(1-\gamma)(n-1)M}\rho^{(1-\gamma)(nM+1)}} \tag{68}$$



(a) (b)

Fig. 9. The outage probabilities of the Type-A protocol with ODSTC3 when $M=5$, $n=3$ and $\gamma=0.25$. In subplot (a), we have $\alpha=20$ and $\beta=1$, while in subplot (b), we have $\alpha=512$ and $\beta=1.4927$.

which will lead to a lower bound for β, defined as β^*. When the condition of $\beta>\beta^*$ holds, the outage events become mainly attributed to the transmission failures between the source and relays. The Type-C protocol performs exactly the same as the Type-A and B protocols in this case, and the diversity order will turn into $(1-\gamma)[n(M+1)+1]$. In fact, using any relay in $\mathcal{S}_D$ will provide an equal performance under this circumstance.

Based on the above results, the DMT of the Type-C protocol is summarized as below

$$\xi_C \triangleq \left\{ \begin{array}{cc} (1-\gamma)(nM+1), & \beta \le \beta^*, \gamma \le \gamma^* \\ (1-\gamma)[n(M+1)+1], & \beta > \beta^*, \gamma \le \gamma^* \\ \xi_B, & \gamma > \gamma^* \end{array} \right\} \tag{69}$$

with β^* obtained by equating the LHS and RHS of (68).

D: Numerical Studies

We present some simulation results to verify the DMT analysis done in the previous sections. Fig. 9(a) shows $\mathbb{P}_{A,i}(3)$ of (32) and their corresponding high-SNR approximations of (43) when $\alpha=20$, $\beta=1$, $\gamma=0.25$ and $M=5$. The high-SNR approximations are drawn according to the third term in (43). To verify the efficiency of ODSTCi, we consider three values, 1, 2, and 3, for the maximum number of active relays, i, and compare them with the ordinary DSTC that uses all available relays in $\mathcal{S}_D$. Clearly, the approximations match the outage probabilities at high SNR. Besides, the SNR loss of using ODSTCi than DSTC is zero at high SNR if $i \ge 2$. A similar phenomenon has also been observed in Fig. 8 (a). We explain the phenomenon with the results provided in Section V-A.

Recall from (43) and (46) that $\mathbb{P}_{A,i}(n)$ is essentially the same for all $i \in [1, M]$ when $\alpha < \beta^{(1-\gamma)}$ and $\rho < \rho_{A_2}$. While when $\alpha \ge \beta^{(1-\gamma)}$ and $\rho < \rho_{A_1}$, the SNR loss is not an appropriate measure since the diversity changes with i. Thus, when $\alpha < \beta^{(1-\gamma)}$ and $\rho \ge \rho_{A_2} \ge \rho_{A_3}$, by (50), we have $\xi_A = (1-\gamma)(M+n)$ and may define a coding gain of

$$G_{A,i}(n) \triangleq \lim_{\rho\to\infty} \frac{\mathbb{P}_{A,i}(n)}{\rho^{-(1-\gamma)(M+n)}} \doteq \left\{ \begin{array}{cc} \sum_{d=1}^{M} \frac{C_d^M \alpha^d}{\alpha^M \beta^{(1-\gamma)(d+n-1)}} & , i=1 \\ \frac{M}{\alpha^{(M-1)} \beta^{(1-\gamma)n}} & , i \ne 1 \end{array} \right\}. \tag{70}$$

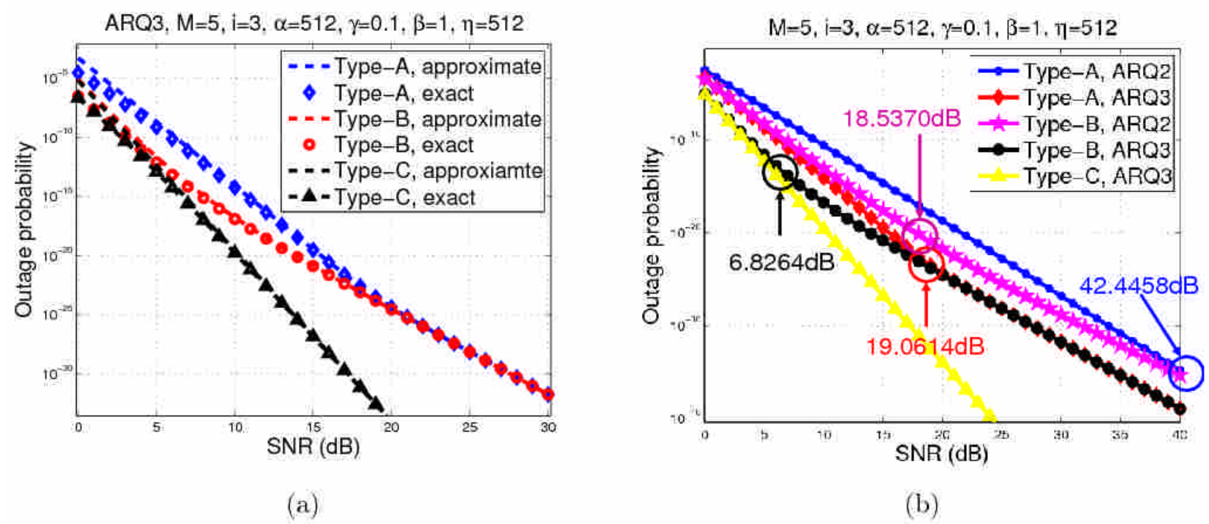


Fig. 10. The outage probabilities of the three types of ARQ protocols with ODSCT3 when $M=5$, $\gamma=0.1$, $\alpha=512$ and $\beta=1$. Subplot (a) shows both the exact and the approximated curves of $\mathbb{P}_{\mathcal{T},3}(3)$ for $\mathcal{T}\in\{A,B,C\}$, and subplot (b) compares the high-SNR approximations of $\mathbb{P}_{\mathcal{T},3}(n)$ for $n=2$ and 3.

This shows that, except for ARQs with OR (ODSTC1), other ODSTCi schemes perform exactly the same as the ODSTCM does at high SNR after twice retransmissions. Therefore, if we follow a definition similar to (28) in Section III to characterize the SNR losses of using ODSTCi than DSTC for the Type-A protocol, then we will obtain

$$\mathcal{L}_A(1,n)=\frac{\log\left\{\frac{G_A(1,n)}{G_A(M,n)}\right\}}{(1-\gamma)(M+n)}=\frac{\log\left\{\sum_{d=1}^{M}\frac{C_d^M}{M}\frac{\alpha^{d-1}}{\beta^{(1-\gamma)(d-1)}}\right\}}{(1-\gamma)(M+n)} \tag{71}$$

and $\mathcal{L}_A(i,n)=0$, for $i\neq 1$. This explains the special advantage of ODSTC2 over the OR observed both in Fig. 8 (a) and 9 (a).

On the other hand, to justify the results of Fig. 8 (b), we recall (52) in Section V-B. Define a similar coding gain for the Type-B protocol as follows

$$G_{B,i}(n)\triangleq\lim_{\rho\to\infty}\frac{\mathbb{P}_{B,i}(n)}{\rho^{-\xi_B}}\doteq\left\{\begin{array}{cc}\frac{1}{\beta^{(1-\gamma)Mn}(i!i^{M-i})^n} & ,\alpha\geq\alpha^* \text{ and } \rho<\rho_{B_1}\\ \frac{1}{\alpha^{nM}} & ,\alpha<\alpha^* \text{ and } \rho<\rho_{A_2}\\ \frac{M}{\alpha^{M-1}\beta^{(1-\gamma)n}} & ,\ \text{otherwise}\end{array}\right\}. \tag{72}$$

Again, following the definition of (28), we obtain

$$\mathcal{L}_B(i,n)=\left\{\begin{array}{cc}\frac{n}{(1-\gamma)(1+nM)}\log\left\{\frac{M!}{i!i^{M-i}}\right\} & ,\alpha\geq\alpha^* \text{ and } \rho<\rho_{B_1}\\ 0 & ,\ \text{otherwise}\end{array}\right\}. \tag{73}$$

This shows that if α is large enough such that $\alpha\geq\alpha^*$, or in other words, when the performance is mainly limited by the S-R link quality, then ODSTCi can introduce effective coding gains by using more relays. Otherwise, there would be no need to us for more than one relay. This justifies the simulation result presented in Fig. 8 (b). In fact, under this channel condition, the Type-A protocol performs exactly the same as the Type-B protocol at high SNR when $i\geq 2$ since $\mathcal{L}_A(i,n)=0$, $\forall i\geq 2$, and $\mathbb{P}_{A,M}(n)=\mathbb{P}_{B,M}(n)$.

In comparison to Fig. 9 (a) where $\alpha=20$ and $\beta=1$, Fig. 9 (b) presents the simulation results for $\alpha=512$ and $\beta=1.4927$. This simulation setting can be associated to a scenario where a group of five relays are located close to each other and situated in the way between the source and the destination. For $\alpha=512$ and $\beta=1.4927$, the relays are 8 times closer to the source than from the destination to the source under a power decay exponent of 3.

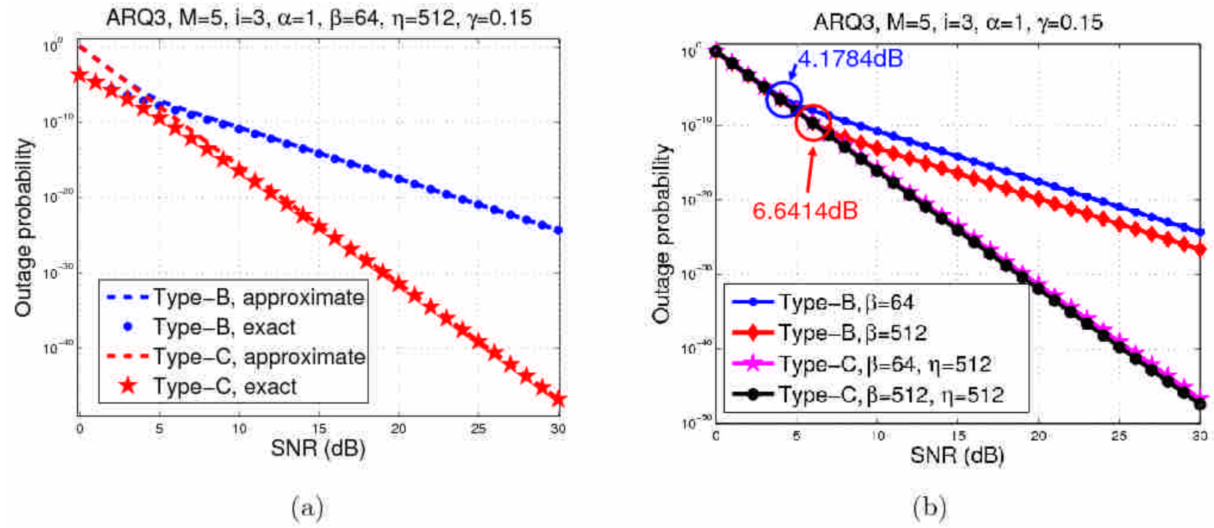


Fig. 11. The outage probabilities of the Type-B and C protocols when $M=5,\ i=3,\ \gamma=0.15$ and $\alpha=1$.

As shown in the figure, the exact expression of (32) can be well approximated by the sum of the first and the third terms of (43) since $\alpha=512>\beta^{(1-\gamma)}=1.7059$. When $\rho<\rho_{A_1}=19.4090$ dB, the diversity order, ξ_A, is $(1-\gamma)[M+1+i(n-1)]=9$, while when $\rho\geq\rho_{A_1}$, ξ_A drops to $(1-\gamma)(M+n)=6$.

In a similar setting to Fig. 9 except for $\beta=1$, the outage probabilities of the three types of ARQ protocols are compared in Fig. 10. As can be seen from the results in Fig. 10 (a) for ARQ3, $\mathbb{P}_{B,3}(3)$ of the Type-B protocol follows the trend of $\mathbb{P}_{C,3}(3)$ of the Type-C when $\rho<5$ dB, while it diverts away from $\mathbb{P}_{C,3}(3)$ when $\rho>5$ dB and merges to $\mathbb{P}_{A,3}(3)$ of the Type-A when $\rho>20$ dB. This diversity changing phenomenon is further examined in subplot (b) with the high-SNR approximations of the three types of ARQ protocols.

For the Type-A protocol, the diversity turning point ρ_{A_1}, [*c.f.* (44)], decreases when the number of ARQs increases. For ARQ2, ρ_{A_1} is 42.4458 dB which is above the normal received SNRs of wireless signal. So, the diversity is $(1-\gamma)[M+1+i(n-1)]=8.1$ over the entire SNR range of interest. In comparison, the diversity becomes $(1-\gamma)(M+n)=7.2$ for ARQ3 when $\rho\geq\rho_{A_1}$ which drops to 19.0614 dB at ARQ3.

In contrast to ρ_{A_1} of (44), ρ_{B_1} of the Type-B protocol, [*c.f.* (54)], is smaller given the same number of ARQs. For ARQ2, $\rho_{B_1}=18.5370$ dB, while for ARQ3, it drops to 6.8264 dB. When $\rho<\rho_{B_1}$, the diversity of ARQ3 is equal to $\xi_B=(1-\gamma)(Mn+1)=14.4$ as $\alpha=512>\beta^{(1-\gamma)n}\rho_{B_1}^{(1-\gamma)(n-1)}=0.0315\geq\alpha^*$, which is the same to the diversity of ARQ3 of the Type-C protocol, [*c.f.* (55) and (69)]. However, ξ_B drops to $(1-\gamma)(M+n)=7.2$ when $\rho\geq\rho_{B_1}$ while the Type-C still enjoys the same full diversity of relaying as $\gamma=0.1<\gamma^*$.

On the contrary, the simulation results for $\alpha\ll\beta$ are presented in Fig. 11 when the quality of the relay channel is mainly determined by the link quality between the source and the relays. Similar to the previous figure, we have $M=5$, $i=3$ and $\eta=512$, while we set $\alpha=1$ and $\gamma=0.15$ in this figure.

As can be expected from (55) and (69), the diversities of $\mathbb{P}_{B,3}(3)$ and $\mathbb{P}_{C,3}(3)$ are equal to $\xi_B=(1-\gamma)(M+n)=6.8$ and $\xi_C=(1-\gamma)(nM+1)=13.6$, respectively, at high SNR. However, the diversity of $(1-\gamma)[n(M+1)+1]$ does

	Ordinary channel	$\alpha > \alpha^*, \rho < \rho_{B_1}$	$\beta > \max\{\alpha^{\frac{1}{1-\gamma}}, \beta^*\}$, $\rho < \rho_{A_2}$
Type-A	$M + n$	$1 + M + i(n-1)$	$n(M+1)+1$, (SIMO + ant. selection)
Type-B	$M + n$	$1 + Mn$, (MISO)	$n(M+1)+1$, (SIMO + ant. selection)
Type-C	$1 + Mn$	$1 + Mn$, (MISO)	$n(M+1)+1$, (SIMO + ant. selection)

TABLE I

The diversity orders of the Type-A, B and C ARQ protocols when $\gamma = 0$ under different link ratios parameterized by α and β.

not seem to be meaningful at low SNR even if $\beta \gg \alpha$ here. For the Type-B protocol, by (55) the diversity of $\mathbb{P}_{B,3}(3)$ should increase to $\xi_B = (1-\gamma)[n(M+1)+1] = 16.15$ when $\rho < \rho_{A_2} = 4.1784$ dB for $\beta = 64$. However, the high-SNR analysis becomes inaccurate at this range of SNR. As can be seen in Fig. 11 (a), $\mathbb{P}_{B,3}(3)$ starts to deviate away from its high-SNR approximation even when $\rho \simeq 5$ dB $> \rho_{A_2}$. A similar problem also occurs to $\mathbb{P}_{C,3}(3)$ whose high-SNR approximation becomes inaccurate when $\rho < 10$ dB.

Despite the accuracy of diversity, the high-SNR analysis does show that all three types of ARQ protocols perform exactly the same when ρ is small and β is large enough such that $\beta > \max\{\alpha^{\frac{1}{1-\gamma}}, \beta^*\}$ and $\rho < \rho_{A_2}$. Besides, as shown in Fig. 11 (b), ρ_{A_2} increases to 6.6414 dB when β is increased to 512, which verifies the result of (45).

Based on the analytical and simulated results presented so far, the relay channel can be roughly classified into three operating regimes. When the S-R to S-D link ratio, α, is high, or more specifically, when $\alpha > \alpha^*$, the relay channel degenerates to a MISO one at high SNR. Suppose that $\rho < \rho_{B_1} \le \rho_{A_1}$, then the Type-A protocol has a diversity order of $\xi_A = (1-\gamma)[M + 1 + i(n-1)]$, and the Type-B has $\xi_B = (1-\gamma)[Mn+1]$ which is the same to ξ_C of the Type-C protocol. The reselection mechanism provides the Type-B and C protocols extra diversities in each subsequent ARQ. On the contrary, when the performance is mainly limited by α, namely when β is large enough, the relay channel degenerates at high SNR to a SIMO channel with antenna selection. If $\beta > \max\{\alpha^{\frac{1}{1-\gamma}}, \beta^*\}$ and $\rho < \rho_{A_2}$, then the destination can successfully decode the data as long as any one of the relays is able to do so. Choosing which relay for ARQ does not matter in this case.

Summarizing the above observations, there seems to be no need to use a highly complex protocol of the Type-C at low SNR either when relays are very close to the source or to the destination. Other than these two extreme operating environments, the cooperative diversities of ARQs are typically limited by the cardinality of $\mathcal{S}_D$ in the non-degenerative case of *ordinary relay channels*. Only the Type-C protocol can resolve this relay shortage problem by allowing overhearing on relays. The diversities of the three types of ARQ protocols are summarized in Table I *w.r.t.* different link ratios parameterized by α and β.

VI. Delay and Outage Limited Throughput

The results in Fig. 8 and Table I show that in *ordinary relay channels* where α and β are neither too large nor too small, ODSTC2 is good enough for the Type-A protocol and seems to be the best choice for the Type-B for the entire range of SNR. Besides, only the Type-C protocol can attain the full diversity, $(1-\gamma)(1+Mn)$, provided by ODSTC relaying. Based on the outage analysis, we continue in this section our analysis from the perspective of the delay and outage limited throughput. This analysis provides more insight on the effectiveness and efficiency of cooperative ARQ with ODSTCi.

We consider transmitting data packets at a rate R. In case of packet losses, the destination will ask for retransmissions from the relays, or from the source if $\mathcal{S}_D=\emptyset$. According to the ARQ protocols introduced in Section IV, define $\mathbf{P}_{\mathcal{T},i}(n_s,n_r)$ to be the probability that the packet will be successfully delivered after n_s chances/attempts of retransmissions with source BC followed by n_r chances/attempts of retransmissions with ODSTCi relaying using the type $\mathcal{T}\in\{A,B,C\}$ protocol. The average throughput in a maximum of n such chances for ARQs can be formulated as

$$\zeta_{\mathcal{T},i}(n) \triangleq \sum_{n_s=0}^{n}\sum_{n_r=0}^{n-n_s}\frac{R\cdot\mathbf{P}_{\mathcal{T},i}(n_s,n_r)}{(1+n_s)\tau_s+n_r\tau_r} \tag{74}$$

where τ_s and $\tau_r\in\mathfrak{R}^+$ are the transmission overhead factors associated with the source BC and the ODSCTi relaying, respectively. The probabilities, $\mathbf{P}_{\mathcal{T},i}(n_s,n_r)$, for the three types of ARQ protocols defined in Section IV are provided in the following proposition.

Proposition 6: Given the rate R and the retransmission pair (n_s,n_r), the probabilities, $\mathbf{P}_{\mathcal{T},i}(n_s,n_r)$, for the ARQ protocols $\mathcal{T}\in\{A,B,C\}$ with ODSTCi are given by:

1. If $n_r=0$, then

$$\mathbf{P}_{\mathcal{T},i}(n_s,0)=[P_W(\delta_s)\mathcal{P}_{\mathcal{D}}(0)]^{n_s}[1-P_W(\delta_s)]. \tag{75}$$

2. If $n_r=1$, then

$$\mathbf{P}_{\mathcal{T},i}(n_s,1)=P_W(\delta_s)[P_W(\delta_s)\mathcal{P}_{\mathcal{D}}(0)]^{n_s}\sum_{d=1}^{M}\mathcal{P}_{\mathcal{D}}(d)[1-P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)]. \tag{76}$$

3. For $n_r\geq 2$, we have

$$\mathbf{P}_{A,i}(n_s,n_r)\triangleq P_W(\delta_s)[P_W(\delta_s)\mathcal{P}_{\mathcal{D}}(0)]^{n_s}\sum_{d=1}^{M}\mathcal{P}_{\mathcal{D}}(d)P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)P_{\mathcal{Z}|\mathcal{D}}^{n_r-2}(\delta|i_d)[1-P_{\mathcal{Z}|\mathcal{D}}(\delta|i_d)]. \tag{77}$$

$$\mathbf{P}_{B,i}(n_s,n_r)\triangleq P_W(\delta_s)[P_W(\delta_s)\mathcal{P}_{\mathcal{D}}(0)]^{n_s}\sum_{d=1}^{M}\mathcal{P}_{\mathcal{D}}(d)P_{\mathcal{O}_i|\mathcal{D}}^{n_r-1}(\delta|d)[1-P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)]. \tag{78}$$

and

$$\begin{aligned}\mathbf{P}_{C,i}(n_s,n_r)\triangleq{}& P_W(\delta_s)[P_W(\delta_s)\mathcal{P}_{\mathcal{D}}(0)]^{n_s}\sum_{d_0=1}^{M}\sum_{d_1=0}^{M-\underline{d}_0}\cdots\sum_{d_{n_r-1}=0}^{M-\underline{d}_{n_r-2}}\\ &\times\mathcal{P}_{\mathcal{D}_0}(d_0)P_{\mathcal{O}_i|\mathcal{D}}(\delta|\underline{d}_0)\left\{\prod_{\ell=1}^{n_r-2}P_{\mathcal{O}_i|\mathcal{D}}(\delta|\underline{d}_\ell)\mathcal{P}_{\mathcal{D}_\ell|\underline{\mathcal{D}}_{\ell-1}}(d_\ell|\underline{d}_{\ell-1})\right\}\\ &\times\mathcal{P}_{\mathcal{D}_{n_r-1}|\underline{\mathcal{D}}_{n_r-2}}(d_{n_r-1}|\underline{d}_{n_r-2})[1-P_{\mathcal{O}_i|\mathcal{D}}(\delta|\underline{d}_{n_r-1})].\end{aligned} \tag{79}$$

Proof: The proof can be readily obtained following the proofs of the outage probabilities of $\mathbb{P}_{\mathcal{T},i}$, $\mathcal{T}\in\{A,B,C\}$, in Section IV. Therefore, it is only sketched below.

If $n_r=0$, then it means the packet delivery succeeds before using relaying. Since this happens only if $\mathcal{S}_D=\emptyset$ before the successful packet delivery, thus (75) is obtained. On the other hand if $n_r=1$, then it implies $\mathcal{S}_D$ turns nonempty right before the successful delivery. As all three types of protocols do relay selection when $\mathcal{S}_D$ first turns nonempty, so $\mathbf{P}_{\mathcal{T},i}(n_s,1)$ are exactly the same and equal to (76). Other than these two cases, $\mathbf{P}_{\mathcal{T},i}(n_s,n_r)$ are different for the three types of ARQ protocols and are discussed below separately.

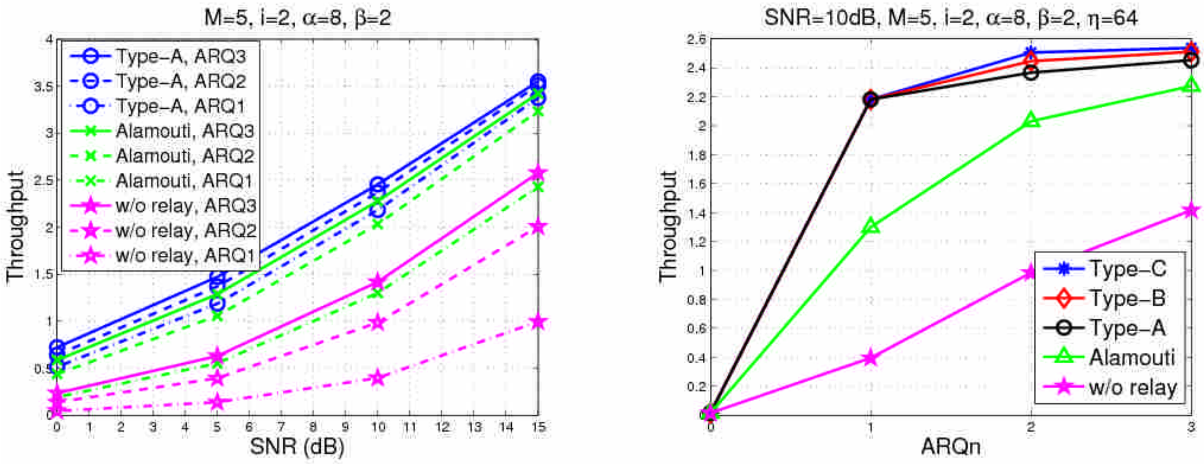
- Type-A: Recall from Fig. 5 that for $n_r \geq 2$, $\mathbf{P}_{\mathrm{A},i}(n_s,n_r)$ is the probability of n_s+1 times of the outage event $\mathrm{O_B}$'s followed by another outage event $\mathrm{O_{OR}}$ and then by n_r-2 times of the outage event $\mathrm{O_{STC}}$'s before the successful delivery, $\overline{\mathrm{O}}_{\mathrm{STC}}$. Since the active relays are fixed after the first relay forwarding, the probability of $\overline{\mathrm{O}}_{\mathrm{STC}}$ is given by $1-P_{\mathcal{Z}|\mathcal{D}}(\delta|i_d)$. Therefore, $\mathbf{P}_{\mathrm{A},i}(n_s,n_r)$ is expressed in the form of (77).
- Type-B: The only difference between $\mathbf{P}_{B,i}(n_s,n_r)$ and $\mathbf{P}_{A,i}(n_s,n_r)$ is that $\mathbf{P}_{B,i}(n_s,n_r)$ is the probability of n_s+1 times $\mathrm{O_B}$'s followed by n_r-1 times $\mathrm{O_{OR}}$'s before the successful delivery, $\overline{\mathrm{O}}_{\mathrm{OR}}$, owing to the reselection mechanism in each ARQ. As a result, $\mathbf{P}_{B,i}(n_s,n_r)$ is expressed in the form of (78) when $n_r \geq 2$.
- Type-C: The difference between the Type-C and the Type-B protocols lies in the fact that the decoding set $\mathcal{S}_D$ in the Type-C will continue to grow due to the overhearing capability once $|\mathcal{S}_D|>0$. Making use of the tree diagram of Fig. 6 to account for the increment of $|\mathcal{S}_D|$ in each ARQ then $\mathbf{P}_{C,i}(n_s,n_r)$ can be readily obtained by replacing the ending state $\mathrm{O_{OR}}$ of the corresponding branch in the figure with $\overline{\mathrm{O}}_{\mathrm{OR}}$, which leads to (79).

■

Based on the results of (74) $\sim$ (79), we are able to study the maximum delay-limited throughput for the three types of ARQ protocols subject to $(s.t.)$ a constraint, $\overline{P}_e$, on the outage probability. Given the average S-D and R-D link qualities, ρ and $\beta\rho$, the transmission rates for the source and the relays are defined as $R \triangleq \gamma\log(1+\rho)$ and $\varepsilon R \triangleq \gamma\log(1+\beta\rho)$, respectively. Thus, the throughput maximization problem is formulated as

$$\begin{aligned} &\max_{\gamma} \zeta_{\mathcal{T},i}(n) \\ \text{s.t.}\quad &\mathbb{P}_{\mathcal{T},i}(n) \leq \overline{P}_e,\ \mathcal{T} \in \{A,B,C\}. \end{aligned} \tag{80}$$

This maximization problem can be solved with the typical steepest descent algorithm. We next study the optimal throughput, $\hat{\zeta}_{\mathcal{T},i}(n)$, for the three types of ARQ protocols in practical ranges of received SNRs. According to the diversity analysis presented in Section V-D, the relay channel degenerates to a MISO one when $\alpha>\alpha^*$, or to a SIMO channel with antenna selection when $\beta>\max\{\alpha^{\frac{1}{1-\gamma}},\beta^*\}$. Other than these two degenerative cases, in the non-degenerative cases of *ordinary relay channels*, the cooperative diversities of ARQs are typically limited by the cardinality of $\mathcal{S}_D$, which is in turn closely related to value of α. Therefore, we study in particular the effects of α on the optimal throughput in the non-degenerative relay channels. The system parameters for simulations are set to be $M=5$, $i=2$, $\alpha=8$, $\beta=2$ and $\eta=64$ if not specified particularly. Besides, counting the data transmission time only, we have $\tau_s=1$ and $\tau_r=\log(1+\rho)/\log(1+\beta\rho)$ in (74).



(a) (b)

Fig. 12. The optimal throughput $\hat{\zeta}_{\mathcal{T},2}(n)$ subject to $\mathbb{P}_{\mathcal{T},2}(n) \leq 10^{-3}$. Subplot (a) compares $\hat{\zeta}_{A,2}(n)$ with $\hat{\zeta}_{RA}(n)$ of the random Alamouti (RA) and $\hat{\zeta}_{DR}(n)$ of the direct retransmissions (DR) for $n = 1, \ldots, 3$. Subplot (b) shows the throughput of $\hat{\zeta}_{DR}(n)$, $\hat{\zeta}_{RA}(n)$ and $\hat{\zeta}_{\mathcal{T},2}(n)$, $\mathcal{T} \in \{A, B, C\}$, versus n at $\rho = 10$dB.

A: ARQ Efficiency with ODSTCi

We first exam the effects on throughput enhancement via cooperative ARQ with ODSTCi. Fig. 12 presents the optimal throughput $\hat{\zeta}_{\mathcal{T},2}(n)$ subject to $\mathbb{P}_{\mathcal{T},2}(n) \leq \overline{P}_e = 10^{-3}$. Subplot (a) compares $\hat{\zeta}_{A,2}(n)$ with the throughput of randomly choosing two relays out of $\mathcal{S}_D$ for forwarding with the Alamouti scheme (RA), and the ones using direct retransmissions (DR) from the source for $n = 1, \ldots, 3$. The results show that $\hat{\zeta}_{A,2}(n)$ converge $w.r.t.$ n much faster than the throughput of RA and that of DR, denoted by $\hat{\zeta}_{RA}(n)$ and $\hat{\zeta}_{DR}(n)$, respectively. The $\hat{\zeta}_{A,2}(1)$ of ARQ1 is even higher than the $\hat{\zeta}_{RA}(2)$ of ARQ2 in this case. Besides, $\hat{\zeta}_{A,2}(2)$ is very close to $\hat{\zeta}_{A,2}(3)$ and has roughly a 2dB gain in SNR against $\hat{\zeta}_{RA}(2)$.

The throughput of $\hat{\zeta}_{DR}(n)$, $\hat{\zeta}_{RA}(n)$ and $\hat{\zeta}_{\mathcal{T},2}(n)$, for $\mathcal{T} \in \{A, B, C\}$, at $\rho = 10$ dB are drawn in subplot (b) versus the number of ARQs, n. We note that $\hat{\zeta}_{\mathcal{T},i}(1)$ is the same $\forall \mathcal{T} \in \{A, B, C\}$. The opportunistic selection of relays provides 70\% improvement over the $\hat{\zeta}_{RA}(1)$ that uses random selection and can still offer about 20\% enhancement over $\hat{\zeta}_{RA}(2)$. This ARQ efficiency by ODSTCi is elaborated in the following simulation studies.

Observe $\zeta_{\mathcal{T},i}(n)$ of (74) that the contributions from the terms with $n_s \neq 0$ are in fact marginal because $\mathbf{P}_{\mathcal{T},i}(n_s \neq 0, n_r) \ll \mathbf{P}_{\mathcal{T},i}(0, n_r)$ for $[P_W \mathcal{P}_D(0)]^{n_s} \ll 1$ in (75) ~ (79). Besides, the denominators of the summands in (74) make the influence of $\mathbf{P}_{\mathcal{T},i}(n_s, n_r)$ to $\zeta_{\mathcal{T},i}(n)$, for $n_s \neq 0$, even smaller. Therefore, the throughput can be approximated by

$$\zeta_{\mathcal{T},i}(n) \quad \simeq \quad \sum_{n_r=0}^{n} \frac{R \cdot \mathbf{P}_{\mathcal{T},i}(0, n_r)}{\tau_s + n_r \tau_r}. \tag{81}$$

Fig. 13 (a) shows the $\hat{\zeta}_{\mathcal{T},2}(2)$, $\mathcal{T} \in \{A, B, C\}$, obtained with this approximation when $\alpha = 2$ and compares them with the $\hat{\zeta}_{\mathcal{T},2}(2)$ obtained according to (74). As shown in the figure, for the Type-A and B protocols, $\hat{\zeta}_{\mathcal{T},2}(2)$ obtained with two summation terms corresponding to $n_r = \{0, 1\}$ in (81) exhibit no difference than the ones obtained with (74). Besides, the two-term

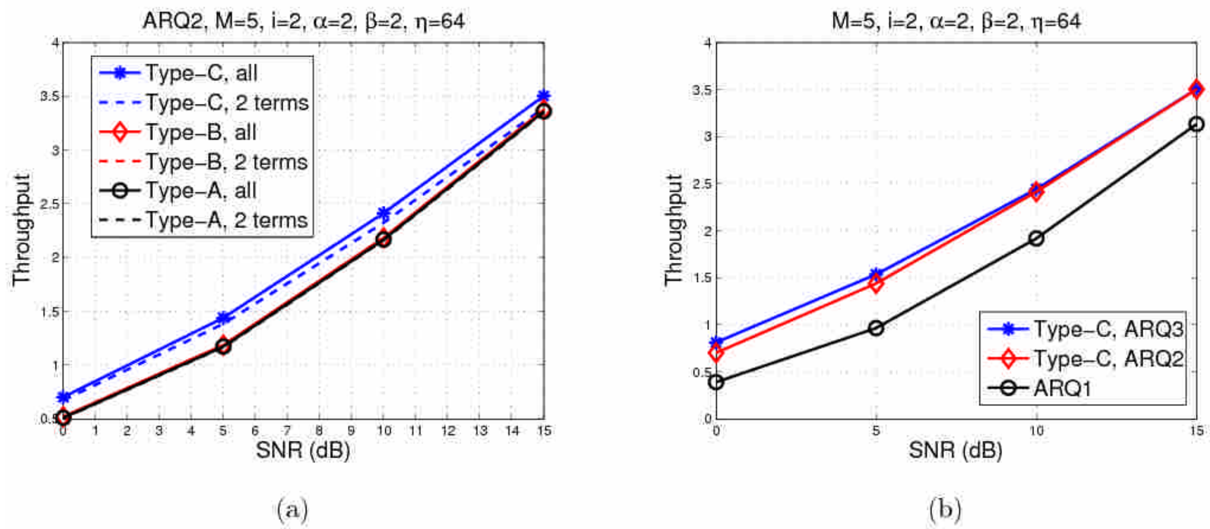


Fig. 13. The ARQ efficiency of the proposed protocols. Subplot (a) compares the $\hat{\zeta}_{\mathcal{T},2}(2)$ obtained according to (74) with the ones obtained by (81). Subplot (b) shows the convergence of $\hat{\zeta}_{C,2}(n)$ $w.r.t.$ n.

approximation of the Type-C protocol is almost the same to the one using three terms with $n_r = \{0, 1, 2\}$ in (81). This explains what are observed in Fig. 12 (b) and Fig. 13 (b) that the asymptotic throughput of the ARQ protocols with ODSTCi can be attained with two or three times of retransmissions.

B: Protocol Efficiency with ODSTCi

In addition to the ARQ efficiency with ODSTCi, the protocols' efficiencies in regards of overhearing, relay reselection and ODSTCi are demonstrated in Fig. 14. Subplot (a) presents $\hat{\zeta}_{\mathcal{T},i}(2)$, $\forall \mathcal{T} \in \{A, B, C\}$, for $i = 1$ and 2 when $\alpha = 2$ and $\overline{P}_e = 10^{-3}$, and compares them with the optimal throughput corresponding to randomly choosing at most i relays out of $\mathcal{S}_D$ for ARQ with DSTC. The throughput of this type is denoted by $\hat{\zeta}_{RSTC,i}(2)$ with $\hat{\zeta}_{RSTC,2}(2) \equiv \hat{\zeta}_{RA}(2)$. The results show that $\hat{\zeta}_{A,1}(2) \simeq \hat{\zeta}_{RSTC,2}(2)$ and $\hat{\zeta}_{A,2}(2) \simeq \hat{\zeta}_{RSTC,M}(2) \simeq \hat{\zeta}_{B,2}(2) \simeq \hat{\zeta}_{B,1}(2)$. The overhearing and reselection function of the Type-C protocol can provide $\hat{\zeta}_{C,2}(2)$ roughly a 0.4b/s/cu gain over $\hat{\zeta}_{B,2}(2) \simeq \hat{\zeta}_{RSTC,M}(2)$. While, the relay reselection function alone does not offer $\hat{\zeta}_{B,2}(2)$ a meaningful gain over $\hat{\zeta}_{B,1}(2)$ or $\hat{\zeta}_{A,2}(2)$ of the Type-A protocol when $\alpha = 2$. The small advantage in $\hat{\zeta}_{C,2}(2)$ makes the Type-B with ODSTC1 an attractive ARQ scheme for $\alpha = 2$ as there is no synchronization and coordination required among the participant relays to perform DSTC.

On the other hand, when α increases from 2 to 8 as shown in subplot (b), both $\hat{\zeta}_{A,2}(2)$ and $\hat{\zeta}_{B,2}(2)$ get improved, with $\hat{\zeta}_{B,2}(2)$ gaining some extra but small advantages from the relay reselection at ARQ2. On the contrary, $\hat{\zeta}_{C,2}(2)$ only gets some minor improvement from the functions of overhearing and relay reselection. The Type-A protocol with ODSTC2 turns out to be an effective yet efficient scheme as relay selection takes place only once when $\mathcal{S}_D$ turns non-empty. This result together with those for $\alpha = 2$ in subplot (b) are reminiscences of the results in Fig. 8 for outage probabilities, which also show that ODSTC2 is necessary for the Type-A protocol, while ODSTC1 works well with the Type-B protocol at high SNR.

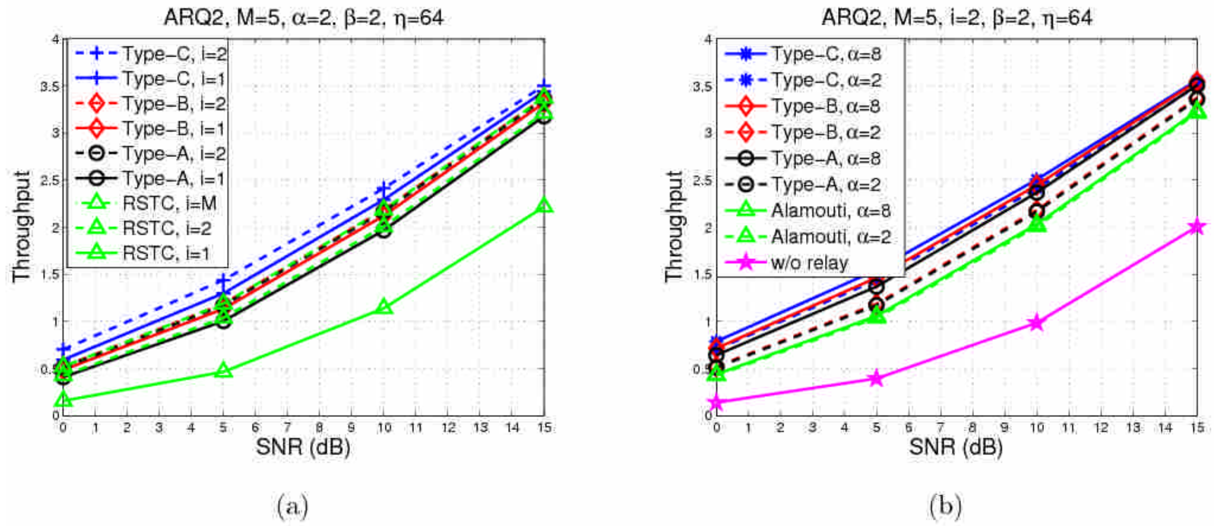


Fig. 14. The protocols' efficiencies in regards of overhearing, relay reselection and ODSTCi for ARQs. Subplot (a) compares the throughput of various protocols when $\alpha = 2$, $n = 2$ and $\overline{P}_e = 10^{-3}$ for $i = 1, 2$ and M. Subplot (b) demonstrates the effects of α on the throughput.

Despite the functions of overhearing and relay reselection that provide huge diversities to the Type-C protocol, the throughput $\hat{\zeta}_{C,2}(n)$ does not show a proportional gain over that of the other types of protocols in the above cases. This motivates us to study the interplay between the outage and the delay constraints, and their effects on the throughput. We investigate this problem by studying the system parameters that might affect the outage probabilities of ARQs, which include the S-D to S-R link ratio (α), relay reselection and overhearing, and examining the *relay* efficiency in terms of M and i of ODSTCi.

B.1 Effects of the Outage Constraint and the S-D to S-R Link Ratio

Recall from (75) to (79) that the forms of $\mathbf{P}_{\mathcal{T},i}(0, n_r)$ are in fact the same $\forall \mathcal{T} \in \{A, B, C\}$ when $n_r = 0$ or 1. Based on the observations from Fig. 13, the differences among $\hat{\zeta}_{\mathcal{T},2}(2)$ mainly result from the terms corresponding to $n_r \le 2$ in (81) and the individual constraint of $\mathbb{P}_{\mathcal{T},2}(2) \le \overline{P}_e$ for each type of the protocol. Since the two-term approximations with $n_r = \{0, 1\}$ are almost the same for the Type-A and B protocols, the fact that $\hat{\zeta}_{A,2}(2) \simeq \hat{\zeta}_{B,2}(2)$ disproves the advantage of relay reselection of the Type-B protocol when $i = 2$. In contrast, that $\hat{\zeta}_{C,2}(2) > \hat{\zeta}_{B,2}(2)$ justifies the throughput advantage of overhearing of the Type-C protocol when the S-R to S-D link ratio is relatively low at $\alpha = 2$. Nevertheless, the results in Fig. 13 (b) show that this overhearing advantage is marginal when $n \ge 2$.

The simulation studies presented thus far all show that $\hat{\zeta}_{\mathcal{T},i}(n)$, $\mathcal{T} \in \{A, B, C\}$, appear to be greatly affected by α and the constraint on $\mathbb{P}_{\mathcal{T},i}(n) \le \overline{P}_e$. To study the effects of α and $\overline{P}_e$ on $\zeta_{\mathcal{T},i}(n)$ while to circumvent the complexity in the analysis for the optimal $\hat{\zeta}_{\mathcal{T},i}(n)$, we investigate this problem from the viewpoint of the maximum multiplexing gain, $\gamma = 1$. We assign $R = \log(1 + \sigma\rho)$ and $\varepsilon R = \log(1 + \sigma\beta\rho)$ for the source and relay transmissions, respectively, and use a SNR scaling factor σ to control the outage probability. Since this rate assignment method sets the highest possible multiplexing gain,

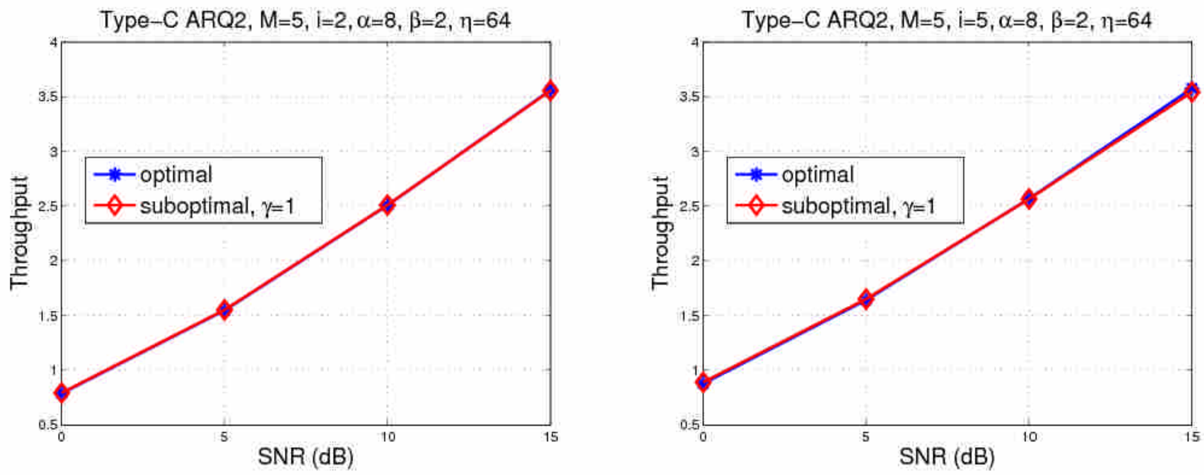


Fig. 15. The comparison of the delay and outage limited throughput between the optimal approach and a suboptimal one with $R = \log(1+\sigma\rho)$ and $\epsilon R = \log(1+\sigma\beta\rho)$ when $M = 5$ and $\overline{P}_e = 10^{-3}$ for ARQ2. The left plot compares the results for the Type-C protocol with ODSTC2, while the right plot for ODSTCM.

$\gamma = 1$, for transmissions, it will cause the highest outage probability as well for all $\gamma < 1$ subject to the same outage constraint. Though not optimal, this approximated analysis is mathematically tractable and provides valuable insight into the interplay between $\overline{P}_e$ and $\hat{\zeta}_{\mathcal{T},i}(n)$. Besides, simulation results in Fig. 15 also show that the resultant throughput with $\gamma = 1$ is very close to the optimal one in the practical operating range of SNR even if $\gamma = 1$ may lead to a lower throughput at high SNR due to its higher chances of retransmissions.

Given $R = \log(1+\sigma\rho)$ and $\varepsilon R = \log(1+\sigma\beta\rho)$, we have $\delta_s = \sigma\rho$ and $\delta = \sigma\beta\rho$, respectively. The outage probabilities from (6) to (9) become invariant to the SNR under this setting as $\delta_s\lambda = \delta\lambda_2 = \sigma$ and $\delta_s\lambda_1 = \frac{\sigma}{\alpha}$. More specifically, $P_W(\delta_s)$, $P_{\mathcal{Z}|\mathcal{D}}(\delta|d)$ and $P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)$ become functions of $\sigma_{\mathcal{T},i}$s only, and

$$\mathcal{P}_{\mathcal{D}}(d) = C_d^M \left[e^{\frac{-\sigma_{\mathcal{T},i}}{\alpha}}\right]^d \left[1 - e^{\frac{-\sigma_{\mathcal{T},i}}{\alpha}}\right]^{M-d} \tag{82}$$

whose value depends on the ratio of $\frac{\sigma_{\mathcal{T},i}}{\alpha}$. Here we use $\sigma_{\mathcal{T},i}$ to stand for the different values of σ for the different protocols to achieve $\overline{P}_e$ with ODSTCi relaying. Following this rule for notationing, the probability, $P_{\mathcal{V}|\mathcal{D}}(\delta|d)$, in (37) for the Type-C protocol becomes

$$P_{\mathcal{V}|\mathcal{D}}(\delta|d) = 1 - \sum_{y=0}^{d-1} e^{-\delta\lambda_3} \frac{(\delta\lambda_3)^y}{y!} = 1 - \sum_{y=0}^{d-1} e^{-\frac{\beta\sigma_{C,i}}{\eta}} \frac{\left(\frac{\beta\sigma_{C,i}}{\eta}\right)^y}{y!}. \tag{83}$$

Denote the probabilities, $P_W(\delta_s)$, $\mathcal{P}_{\mathcal{D}}(d)$, $P_{\mathcal{Z}|\mathcal{D}}(\delta|d)$, $P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)$ and $P_{\mathcal{V}|\mathcal{D}}(\delta|d)$, under this setting by $\overline{P}_W$, $\overline{\mathcal{P}}_{\mathcal{D}}(d)$, $\overline{P}_{\mathcal{Z}|\mathcal{D}}(d)$, $\overline{P}_{\mathcal{O}_i|\mathcal{D}}(d)$ and $\overline{P}_{\mathcal{V}|\mathcal{D}}(\delta|d)$, respectively. Substituting them back into (32), (34) and (39), the outage probabilities, $\mathbb{P}_{\mathcal{T},i}(n)$, become solely determined by $\sigma_{\mathcal{T},i}$, given the values of the link ratios, α, β and η. In particular, $\mathbb{P}_{A,i}(n)$ and $\mathbb{P}_{B,i}(n)$ are functions of $\sigma_{\mathcal{T},i}$ and α only. As a result, given a predetermined constraint, $\overline{P}_{out}$, on $\mathbb{P}_{\mathcal{T},i}(n)$, one can obtain the corresponding $\sigma_{\mathcal{T},i}$ for each $\mathcal{T} \in \{A, B, C\}$ and $i = 1, \ldots, M$. With $\sigma_{\mathcal{T},i}$ and, hence, $R = \log(1+\rho\sigma_{\mathcal{T},i})$, the throughput, $\zeta_{\mathcal{T},i}(n)$, can be determined accordingly. More specifically, for the $\zeta_{\mathcal{T},i}(n)$ of each type, the associated $\mathbf{P}_{\mathcal{T},i}$ in (75) to (79) are evaluated by setting $\sigma = \sigma_{\mathcal{T},i}$ in $\overline{P}_W$, $\overline{\mathcal{P}}_{\mathcal{D}}(d)$, $\overline{P}_{\mathcal{Z}|\mathcal{D}}(d)$, $\overline{P}_{\mathcal{O}_i|\mathcal{D}}(d)$ or $\overline{P}_{\mathcal{V}|\mathcal{D}}(\delta|d)$.

	$\alpha=2$	$\alpha=8$	$\alpha=27$
$\sigma_{A,2}$	0.4998	0.7216	0.7981
$\sigma_{A,2}$	0.5174	0.8993	1.1164
$\sigma_{A,2}$	0.8879	1.1184	1.1985

TABLE II

The values of $\sigma_{\mathcal{T},i}$ to achieve $\overline{P}_{out}=10^{-3}$ at ARQ2 for different α whe $M=5$ and $i=2$.

As pointed out earlier, for $n_r=\{0,1\}$, the approximated $\zeta_{\mathcal{T},i}(n)$ in (81) are in fact the same $\forall\mathcal{T}\in\{A,B,C\}$. Therefore, the difference between $\zeta_{A,i}(n)$ and $\zeta_{B,i}(n)$ lies mainly in the distinct $\sigma_{A,i}$ and $\sigma_{B,i}$ for each type of the protocol to achieve the same outage constraint $\overline{P}_e$. Table II shows the values of $\sigma_{\mathcal{T},i}$ to achieve $\overline{P}_e=10^{-3}$ at ARQ2 for different values of α when $M=5$, $i=2$, $\beta=2$ and $\eta=64$. As shown in the table, when $\alpha=2$, $\sigma_{A,2}\simeq\sigma_{B,2}$, thus, $\zeta_{A,2}(n)$ and $\zeta_{B,2}(n)$ are almost indistinguishable. On the other hand, when $\alpha=8$, $\sigma_{\mathcal{T},2}$ of different types are already close to each other. Besides, for the Type-C protocol, $P_{\mathcal{D}}(M)+P_{\mathcal{D}}(M-1)=0.87$, which is large enough such that the overhearing function of the Type-C protocol does not make much difference in enlarging $\mathcal{D}$. Therefore, the differences among $\zeta_{\mathcal{T},2}(n)$ of different protocols are small as shown in Fig. 14 when $\alpha=8$.

C. Relay Efficiency with ODSTCi

From the analytic and simulated results presented in the previous sections and from Fig. 12 to 14, one may soon notice that the huge diversities contributed by multiple retransmissions and/or multiple relays in ODSCTi do not necessarily lead to a proportional enhancement in throughput. In fact, the throughput comes saturated quickly either when i or M for ODSTCi increase as shown in Fig. 14 (a) and Fig. 16, respectively. The throughput enhancement $w.r.t.$ M are presented in Fig. 16 when $\rho=10$dB, $i=2$ and $\overline{P}_e=10^{-3}$ at ARQ2. As shown in the figure, $\hat{\zeta}_{RA}(2)$ is lower than $\hat{\zeta}_{C,2}(2)$ by about 0.5 b/s/cu and becomes saturated when the probability of $\mathcal{D}\geq 2\approx 1$. In addition, the improvement of $\hat{\zeta}_{C,2}(2)$ has dropped down to around 2% when M increases from 5 to 6. This mainly results from the fact that $\zeta_{\mathcal{T},i}(n)$ of (74) is linearly proportional to $\mathbf{P}_{\mathcal{T},i}$. As such, the number of relays, M, does not play as an important role to the throughput as it does to the diversity.

On the other hand, the results in Fig. 14 show that the enhancement is almost indistinguishable for $i\geq 2$ when $M=5$. Moreover, $\hat{\zeta}_{\mathcal{T},i}(2)$ of different protocols increase $w.r.t.$ ρ almost at the same rate when the SNR is in the practical range of 5 dB $\leq\rho\leq 15$ dB. This allows us to exam the SNR loss of using ODSTCi than using ODSTCM from the viewpoint of throughput as opposed to the diversity point of view in (28) of Section III.

Recall from (81) and Fig. 13 and 15 that $\hat{\zeta}_{\mathcal{T},i}(n)$ can be approximated as

$$\hat{\zeta}_{\mathcal{T},i}(n)\simeq\sum_{n_r=0}^{n}\frac{\log(1+\sigma_{\mathcal{T},i}\rho)\cdot\overline{\mathbf{P}}_{\mathcal{T},i}(0,n_r)}{\tau_s+n_r\tau_r}\simeq(\log\sigma_{\mathcal{T},i}+\log\rho)\sum_{n_r=0}^{2}\frac{\overline{\mathbf{P}}_{\mathcal{T},i}(0,n_r)}{\tau_s+n_r\tau_r}\qquad(84)$$

in the practical SNR range of 0 dB $\leq\rho\leq 15$ dB. Since $\overline{\mathbf{P}}_{\mathcal{T},i}(0,n_r)$ are solely

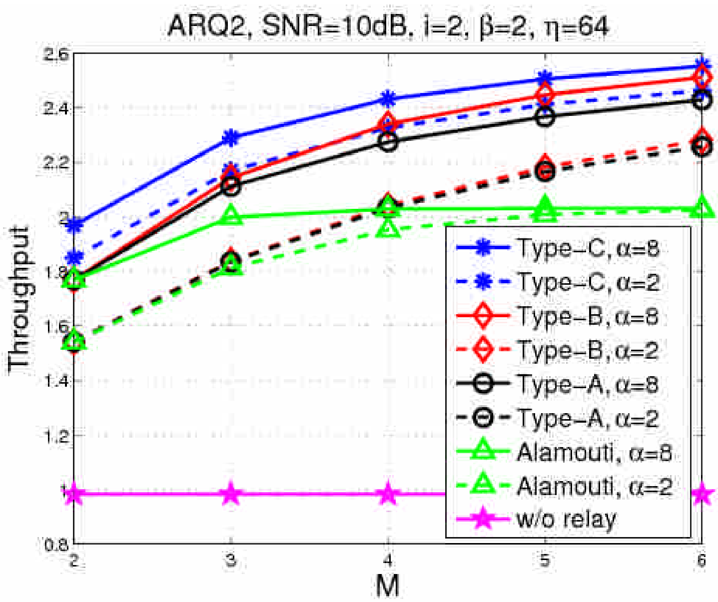


Fig. 16. The throughput versus M of ODSTC2 when $\overline{P}_e = 10^{-3}$ at ARQ2. The results of $\hat{\zeta}_{\mathcal{T},2}(2)$ are evaluated at $\rho = 10$ dB when $i = 2$. For comparison, $\hat{\zeta}_{RA}(2)$ and $\hat{\zeta}_{DR}(2)$ are also shown in the figure.

determined by α and $\sigma_{\mathcal{T},i}$, for $n_r = 0, 1$, which are constant given a fixed $\overline{P}_e = 10^{-3}$, thus $\hat{\zeta}_{\mathcal{T},i}(n)$ increase approximately linearly *w.r.t.* $\log \rho$. As a result, we may define an alternative SNR loss, $\overline{\mathcal{L}}_{\mathcal{T}}(i, n)$, in dB as the additional SNR required for ODSTCi to achieve the same throughput, ζ_{level}, achieved with ODSTCM at ARQn. Different from the SNR loss defined in (28) from the perspective of outage probability, the SNR loss, $\overline{\mathcal{L}}_{\mathcal{T}}(i, n)$, vary slightly with SNR as shown in Fig. 14 (a). Nevertheless, it still effectively characterizes the relay efficiency in terms of i for ODSTCi.

The values of $\overline{\mathcal{L}}_{\mathcal{T}}(i, 2)$, $\mathcal{T} \in \{A, B, C\}$, at $\zeta_{\text{level}} = 2$ for different α and $i = 1, \ldots, M$ are drawn in Fig. 17 when $M = 5$ and $\overline{P}_e = 10^{-3}$ at ARQ2. As shown in the figure that the SNR losses diminish rapidly when i increase. This characterizes what are observed in Fig. 14 (a) that the efficiency of using a large i for ODSTCi decreases rapidly. This is particularly true for a small value of α. Compared with the Type-A protocol, the reselection function of the Type-B makes it more efficient in using the relays when α becomes larger. Having $i = 2$ only incurs about a 0.4 dB loss in $\overline{\mathcal{L}}_B(2, 2)$ for the Type-B protocol even if $\alpha = 27$.

On the other hand, the results in subplot (b) present the effects of the R-R to S-D link ratio, η, on $\overline{\mathcal{L}}_C(i, 2)$. The SNR losses, $\overline{\mathcal{L}}_C(i, 2)$, increase with η in particular when α is small. This is because the cardinality of $\mathcal{S}_D$ can be effectively increased in this case through the overhearing function of the Type-C protocol. Even though the advantage of a larger i increases slightly when α gets larger, we still have $\overline{\mathcal{L}}_C(2, 2) \simeq 0.55$ dB even if $\alpha = 27$. This once again makes the ODSTC2 an appealing choice for ARQ with ODSTCi.

Based on the results presented in Fig. 12, 14, 16 and 17 for the analysis of efficiencies in regards of the number of ARQs, the functions of the protocols and the number of relays for ODSTCi, we may reach a rule of thumb for choosing and setting a cooperative ARQ protocol that is based on ODSTCi. In general, $M = 5$ and $i = 2$ accompanied by two times of ARQs turns out to be a good tradeoff

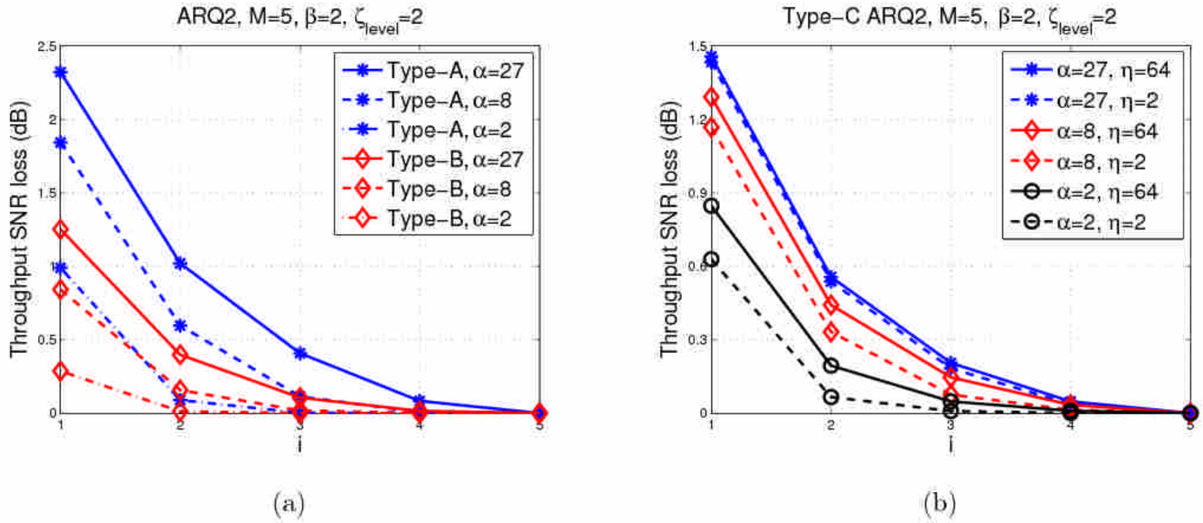


Fig. 17. The SNR losses versus i of ODSTCi when $\zeta_{\rm level} = 2$ and $\overline{P}_e = 10^{-3}$ at ARQ2. Subplot (a) shows the results for $\overline{\mathcal{L}}_A(i,2)$ and $\overline{\mathcal{L}}_B(i,2)$. Subplot (b) compares $\overline{\mathcal{L}}_C(i,2)$ for $\eta = 2$ and 64, respectively.

between the protocol complexity and the system performance. When α is large enough, *e.g.* $\alpha = 27$, such that $P_{\mathcal{D}}(M) \approx 1$, the Type-B protocol with ODSTC2 provides an excellent performance. On the other hand, when α is small such as $\alpha = 2$, the Type-C protocol with ODSTC2 has obvious advantages provided by its functions of overhearing and relay reselection. Other than these two extreme channel conditions, *e.g.* $\alpha = 8$, either the Type-A protocol with ODSTC2 or the Type-B protocol with ODSTC1 provide satisfactory performance. Overall, when the protocol's complexity is of an important concern, then the Type-B protocol with ODSTC2 serves as a good compromise between the complexity and performance.

VII. CONCLUSIONS

We proposed three types of ARQ protocols which involve three different levels of complexities in using cooperative relaying with ODSTCi. According to the degrees of coordinations required for the relays to jointly perform ODSTCi, the three types of protocols employ the functions of fixed relaying in Type-A, opportunistic relaying in Type-B and the overhearing and opportunistic relaying in Type-C, respectively, which allow us to evaluate the effectiveness versus the complexities of the three types of protocols from both the perspectives of DMT and the delay and outage limited throughput.

According to the outage analysis, the DMTs of these protocols are highly related to the S-R to S-D and the R-D to S-D link ratios, α and β, respectively. When α is high enough, the relay channel degenerates to a MISO one at high SNR. As such, the Type-B protocol can achieve the DMT of the Type-C protocol. On the contrary, when β is high enough, then the relay channel degenerates at high SNR to a SIMO channel with antenna selection. The destination in this channel can decode the data as long as any one of the relays is able to do so. Thus, the three types of protocols perform exactly the same in this case. Other than these two extreme operating environments, the cooperative diversities of ARQs are typically limited by the cardinality, $\mathcal{D}$, of $\mathcal{S}_D$ in the non-degenerative case of *ordinary relay channels*. Only the Type-C protocol can resolve this relay shortage problem and achieve the full diversity by enabling overhearing on relays $\notin \mathcal{S}_D$.

On the other hand, based on the efficiency analysis for the throughput in *ordinary relay channels*, the S-R to S-D link ratio, α, can be roughly partitioned into three operating regimes as well. When α is large enough such that $P_{\mathcal{D}}(M) \approx 1$, the Type-B protocol with ODSTC2 provides a near optimal throughput. However, when α is small such that ODSTCi suffers from severe relay shortages as $P_{\mathcal{D}}(1)$ becomes dominant, then the Type-C protocol can serve as the rescuer with its function of overhearing, and offer a significant throughput enhancement even with ODSTC2 in two times of ARQs, making use of its repetitive process of overhearing and relay reselection. Other than these two extreme channel conditions, either the Type-A protocol with ODSTC2 or the Type-B protocol with ODSTC1 provide satisfactory throughput.

Overall, from the practice point of view, this research study shows that a significant throughput enhancement can be obtained by using an appropriate ARQ protocol for two times of retransmissions that employ at most two active relays out of a total of $M = 5$ relays for ODSTC. This advantage of ODSTC may have a more profound effect on the overall system throughput from a cross-layer point of view. Consider the sum of the queue lengths of all relays as a system resource. An ARQ with DSTC will duplicate the packets to be retransmitted on all relays in $\mathcal{S}_D$, while for the Type-A protocol with ODSTC2, each of the packets will be duplicated on two available relays in $\mathcal{S}_D$ at most. In addition, the total duration for packets to reside in relays' queues will become shorter in average due to the higher efficiency in retransmissions with ODSTC. These features will make the queues of relays less likely to overflow with retransmission packets, and hence, more capable to sustain a higher system throughput due to a lower probability in dropping packets. A more thorough study on this is worth pursuing, while is beyond the scope of this paper.

APPENDICES

A. Proof of Proposition 1

To find the closed form expression of

$$\lim_{\lambda \to 0} \frac{1}{\lambda^d} P_{\mathcal{O}_i|\mathcal{D}}(\delta|d) \tag{85}$$

we would need the asymptotic result of a CDF given below.

Assume $\mathcal{D} \geq i$. Let $\mathcal{O}_i \triangleq \sum_{j=\mathcal{D}-i+1}^{\mathcal{D}} X'_j$ stand for the summation of the largest i out of $\mathcal{D}$ *i.i.d.* exponential RVs, $X_j \sim$Exp(λ/β), with $X'_{\mathcal{D}} \geq X'_{\mathcal{D}-1} \geq, \ldots, X'_1 \geq 0$. For the CDF of $\mathcal{O}_i < \delta$, we have

$$F_{\mathcal{O}_i}(\delta) = \int_0^{\frac{\delta}{i}} \int_{x'_{\mathcal{D}-i+1}}^{\frac{\delta - x'_{\mathcal{D}-i+1}}{i-1}} \cdots \int_{x'_{\mathcal{D}-1}}^{\delta - x'_{\mathcal{D}-i+1} - \ldots - x'_{\mathcal{D}-1}} f_{X'_{\mathcal{D}}, \cdots, X'_{\mathcal{D}-i+1}}(x'_{\mathcal{D}}, \cdots, x'_{\mathcal{D}-i+1}) dx'_{\mathcal{D}} \cdots dx'_{\mathcal{D}-i+1} \tag{86}$$

where the joint PDF of the ordered RVs $X'_{\mathcal{D}}, \cdots, X'_{\mathcal{D}-i+1}$ is given by [25]

$$f_{X'_{\mathcal{D}}, \cdots, X'_{\mathcal{D}-i+1}}(x'_{\mathcal{D}}, \cdots, x'_{\mathcal{D}-i+1}) = \frac{\mathcal{D}!}{(\mathcal{D}-i)!} \frac{\lambda^i}{\beta^i} e^{-\frac{\lambda}{\beta} \sum_{\ell=0}^{i-1} x'_{\mathcal{D}-\ell}} \left(1 - e^{-\frac{\lambda}{\beta} x'_{\mathcal{D}-i+1}}\right)^{\mathcal{D}-i}. \tag{87}$$

We want to derive the expression of

$$\lim_{\lambda \to 0} \frac{1}{\lambda^{\mathcal{D}}} F_{\mathcal{O}_i}(\delta). \tag{88}$$

To this end, we compute its upper and lower bound respectively.

For the lower bound, exploiting Fatou's lemma [27], we obtain

$$\liminf_{\lambda \to 0} \frac{F_{\mathcal{O}_i}(\delta)}{\lambda^{\mathcal{D}}} \geq \int_0^{\frac{\delta}{i}} \int_{x'_{\mathcal{D}-i+1}}^{\frac{\delta - x'_{\mathcal{D}-i+1}}{i-1}} \cdots \int_{x'_{\mathcal{D}-1}}^{\delta - x'_{\mathcal{D}-i+1} - \ldots - x'_{\mathcal{D}-1}}$$

$$\liminf_{\lambda \to 0} \frac{1}{\lambda^{\mathcal{D}}} f_{X'_{\mathcal{D}}, \cdots, X'_{\mathcal{D}-i+1}}(x'_{\mathcal{D}}, \cdots, x'_{\mathcal{D}-i+1}) dx'_{\mathcal{D}} \cdots dx'_{\mathcal{D}-i+1} \tag{89}$$

where

$$\begin{aligned} &\liminf_{\lambda \to 0} \frac{1}{\lambda^{\mathcal{D}}} f_{X'_{\mathcal{D}}, \cdots, X'_{\mathcal{D}-i+1}}(x'_{\mathcal{D}}, \cdots, x'_{\mathcal{D}-i+1}) \\ &= \liminf_{\lambda \to 0} \frac{1}{\lambda^{\mathcal{D}}} \frac{\mathcal{D}!}{(\mathcal{D}-i)!} \frac{\lambda^i}{\beta^i} \left(1 - e^{-\frac{\lambda}{\beta} x'_{\mathcal{D}-i+1}}\right)^{\mathcal{D}-i} \prod_{\ell=0}^{i-1} e^{-\frac{\lambda}{\beta} x'_{\mathcal{D}-\ell}} \\ &= \frac{\mathcal{D}!}{(\mathcal{D}-i)!} \frac{(x'_{\mathcal{D}-i+1})^{\mathcal{D}-i}}{\beta^{\mathcal{D}}} \end{aligned} \tag{90}$$

due to the fact that $\lim_{\lambda \to 0} \inf e^{-\frac{\lambda}{\beta} x'_{\mathcal{D}-\ell}} = 1$ and

$$\liminf_{\lambda \to 0} \frac{1}{\lambda^{\mathcal{D}-i}} \left(1 - e^{-\frac{\lambda}{\beta} x'_{\mathcal{D}-i+1}}\right)^{\mathcal{D}-i} = \left(\frac{x'_{\mathcal{D}-i+1}}{\beta}\right)^{\mathcal{D}-i} \tag{91}$$

by L'hôpital's rule. As a result, we have

$$\liminf_{\lambda \to 0} \frac{F_{\mathcal{O}_i}(\delta)}{\lambda^{\mathcal{D}}} \geq \frac{\mathcal{D}!}{(\mathcal{D}-i)! \beta^{\mathcal{D}}} q(\delta, \mathcal{D}, i) \tag{92}$$

where we define

$$q(\delta, \mathcal{D}, i) \triangleq \int_0^{\frac{\delta}{i}} \int_{x'_{\mathcal{D}-i+1}}^{\frac{\delta - x'_{\mathcal{D}-i+1}}{i-1}} \cdots \int_{x'_{\mathcal{D}-1}}^{\delta - x'_{\mathcal{D}-i+1} - \ldots - x'_{\mathcal{D}-1}} (x'_{\mathcal{D}-i+1})^{\mathcal{D}-i} dx'_{\mathcal{D}} \cdots dx'_{\mathcal{D}-i+1}. \tag{93}$$

On the other hand, we have $e^{-\frac{\lambda}{\beta} \sum_{\ell=0}^{i-1} x'_{\mathcal{D}-\ell}} \leq 1$ and $1 - e^{-x'_{\mathcal{D}-i+1}/\beta\rho} \leq x'_{\mathcal{D}-i+1}/\beta\rho$, $\forall\, x'_{\mathcal{D}-\ell} \geq 0,\ \ell = 0, \ldots, i-1$. Substituting these upper bounds back into (87), we obtain

$$\limsup_{\lambda \to 0} \frac{F_{\mathcal{O}_i}(\delta)}{\lambda^{\mathcal{D}}} \leq \frac{\mathcal{D}!}{(\mathcal{D}-i)! \beta^{\mathcal{D}}} q(\delta, \mathcal{D}, i). \tag{94}$$

Since $\inf(\cdot) \leq \sup(\cdot)$. By (92) and (94), we have

$$\lim_{\lambda \to 0} \frac{F_{\mathcal{O}_i}(\delta)}{\lambda^{\mathcal{D}}} = \frac{\mathcal{D}!}{(\mathcal{D}-i)! \beta^{\mathcal{D}}} q(\delta, \mathcal{D}, i). \tag{95}$$

To further process $q(\delta, \mathcal{D}, i)$, we define a change of variable:

$$X'_j \triangleq \sum_{p=\mathcal{D}-j+1}^{i} T_p,\ j = (\mathcal{D}-i+1), \ldots, \mathcal{D}. \tag{96}$$

By the change of variable, we have

$$0 \leq T_i = X'_{\mathcal{D}-i+1} \leq \frac{1}{i} \sum_{j=\mathcal{D}-i+1}^{\mathcal{D}} X'_j = \frac{\mathcal{O}_i}{i} < \frac{\delta}{i}. \tag{97}$$

and,

$$0 \leq T_{i-k} < \frac{\delta - \sum_{j=0}^{k-1} (i-j) T_{i-j}}{i-k},\ k = 1, \ldots, i-1 \tag{98}$$

or, equivalently,

$$0 \leq T_k < \frac{\delta - \sum_{j=k+1}^{i} j T_j}{k}, k = 1, \ldots, i-1. \tag{99}$$

As a result, it can be easily shown that

$$q(\delta,\mathcal{D},i)=\int_0^{\frac{\delta}{i}}\int_0^{\frac{\delta-it_i}{i-1}}\cdots\int_0^{\delta-\sum_{j=2}^{i}j\cdot t_j}t_i^{\mathcal{D}-i}dt_1\cdots dt_i. \tag{100}$$

This multiple integrals can be solved with Lemma 1 provided in AppendixA-B which results in

$$q(\delta,\mathcal{D},i)=\frac{\delta^{\mathcal{D}}(\mathcal{D}-i)!}{i!i^{\mathcal{D}-i}\mathcal{D}!}. \tag{101}$$

Substituting this expression back into (95), we obtain

$$\lim_{\lambda\to 0}\frac{F_{\mathcal{O}_i}(\delta)}{\lambda^{\mathcal{D}}}=\frac{\delta^{\mathcal{D}}}{i!i^{\mathcal{D}-i}\beta^{\mathcal{D}}}. \tag{102}$$

Now, applying this formula to (85) gives

$$\lim_{\lambda\to 0}\frac{1}{\lambda^d}P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)=\frac{\delta^d}{i!i^{d-i}\beta^d},\ d\geq i. \tag{103}$$

In cases where $d<i$, then all the relays in $\mathcal{S}_D$ will be used to forward the signal. The corresponding expression of (85) is equal to setting $i=d$ in (103). Thus, define $i_d\triangleq\min\{i,d\}$, we conclude that

$$\lim_{\lambda\to 0}\frac{1}{\lambda^d}P_{\mathcal{O}_i|\mathcal{D}}(\delta|d)=\frac{\delta^d}{i_d!i_d^{d-i_d}\beta^d},\quad i_d\triangleq\min\{i,d\}. \tag{104}$$

B. Proof of Lemma 1

Lemma 1:

$$\int_0^{\frac{T}{n}}\int_0^{\frac{T-nt_n}{n-1}}\cdots\int_0^{\frac{T-nt_n-\cdots-2t_2}{1}}t_n^{L-n}dt_1dt_2\cdots dt_n$$
$$=\frac{T^L(L-n)!}{n!n^{L-n}L!}\triangleq F_n(T,L)\ ,\ L\geq n,\ L,n\in\mathfrak{N}. \tag{105}$$

Proof:

1. Let $n=2$, we have

$$\int_0^{\frac{T}{2}}\int_0^{T-2t_2}t_2^{L-2}dt_1dt_2=\int_0^{\frac{T}{2}}(T-2t_2)t_2^{L-2}dt_2. \tag{106}$$

From the table of integrals [28], it follows

$$\int_0^{u}(u-x)^{p-1}x^{v-1}dx=u^{p+v-1}\frac{(p-1)!(v-1)!}{(p+v-1)!}. \tag{107}$$

Therefore, (106) is equal to

$$2\int_0^{\frac{T}{2}}\left(\frac{T}{2}-t_2\right)t_2^{L-2}dt_2=\frac{T^L(L-2)!}{2\cdot 2^{L-2}\cdot L!}=F_2(T,L). \tag{108}$$

2. For $n=3$, the general form of the formula becomes

$$\int_0^{\frac{T}{3}}\int_0^{\frac{T-3t_3}{2}}\int_0^{\frac{T-3t_3-2t_2}{1}}t_3^{L-3}dt_1dt_2dt_3=\int_0^{\frac{T}{3}}\int_0^{\frac{T-3t_3}{2}}(T-3t_3-2t_2)t_3^{L-3}dt_2dt_3. \tag{109}$$

Set $S_3=T-3t_3$, we have

$$\int_0^{\frac{S_3}{2}}(S_3-2t_2)dt_2=2\int_0^{\frac{S_3}{2}}\left(\frac{S_3}{2}-t_2\right)dt_2=\frac{S_3^2\cdot 0!}{2!\cdot 2^0\cdot 2!}=F_2(S_3,2).$$

Substituting this result back into (109), we obtain

$$\int_0^{\frac{T}{3}} \frac{(T-3t_3)^2}{2!\cdot 2!} t_3^{L-3} dt_3 = \frac{3^2}{2!\cdot 2!} \int_0^{\frac{T}{3}} \left(\frac{T}{3} - t_3\right)^2 t_3^{L-3} dt_3$$
$$= \frac{3^2}{2!\cdot 2!} \frac{(T/3)^L 2!(L-3)!}{L!} = \frac{T^L (L-3)!}{3!\cdot 3^{L-3}\cdot L!} = F_3(T,L). \quad (110)$$

3. Now, we assume the following equality holds for $n = k$

$$\int_0^{\frac{T}{k}} \int_0^{\frac{T-kt_k}{k-1}} \cdots \int_0^{\frac{T-kt_k-\cdots-2t_2}{1}} t_k^{L-k} dt_1 dt_2 \cdots dt_k = \frac{T^L (L-k)!}{n!\cdot n^{L-n}\cdot L!} = F_k(T,L). \quad (111)$$

For $n = k+1$, we have

$$\int_0^{\frac{T}{k+1}} \cdots \int_0^{\frac{T-(k+1)t_{k+1}-\cdots-2t_2}{1}} t_{k+1}^{L-(k+1)} dt_1 dt_2 \cdots dt_{k+1}$$
$$= \int_0^{\frac{T}{k+1}} \cdots \int_0^{\frac{T-(k+1)t_{k+1}-\cdots-2t_2}{1}} dt_1 \cdots dt_k t_{k+1}^{L-(k+1)} dt_{k+1}. \quad (112)$$

Set $S_{k+1} = T-(k+1)t_{k+1}$. By (111), the integral becomes

$$\int_0^{\frac{T}{k+1}} \frac{\mathcal{S}_{k+1}^k}{k!\cdot k!} t_{k+1}^{L-(k+1)} dt_{k+1} = \frac{(k+1)^k}{k!\cdot k!} \int_0^{\frac{T}{k+1}} [T/(k+1) - t_{k+1}]^k t_{k+1}^{L-(k+1)} dt_{k+1}$$
$$= \frac{(k+1)^k}{k!\cdot k!} \times \frac{[T/(k+1)]^L k!(L-k-1)!}{L!}$$
$$= \frac{T^L (L-k-1)!}{(k+1)!(k+1)^{L-(k+1)} L!} = F_{k+1}(T,L). \quad (113)$$

Thus, by induction, the proof is completed. ■

C. Proof of Proposition 4

Continued from (42), $\mathbb{P}_{A,i}(n)$ is approximated at high SNR as

$$\mathbb{P}_{A,i}(n) \doteq \frac{\Lambda_{\alpha,\rho}^n}{\rho^{1-\gamma}} + \sum_{k=1}^{n} \Upsilon_k(\alpha,\beta,\rho) \quad (114)$$

where

$$\Upsilon_k(\alpha,\beta,\rho) \triangleq \Lambda_{\alpha,\rho}^{n-k+1} \underbrace{\sum_{d=1}^{M} \frac{\alpha^d \Omega_{d,k}}{\beta^{(1-\gamma)[d+i_d(k-1)]} \rho^{(1-\gamma)i_d(k-1)}}}_{(a)} \quad (115)$$

with $i_d \triangleq \min\{i,d\}$, $\Lambda_{\alpha,\rho} \triangleq \frac{1}{\alpha^M \rho^{(1-\gamma)(M+1)}}$ and $\Omega_{d,k} \triangleq \frac{C_d^M}{i_d^{(d-i_d)} (i_d!)^k}$. Now, suppose that $i > 1$. The expansion of (a) on the right hand side (RHS) of (115) is given by

$$(a) = \sum_{d=1}^{i-1} \frac{\alpha^d \Omega_{d,k}}{\beta^{(1-\gamma)dk} \rho^{(1-\gamma)d(k-1)}} + \sum_{d=i}^{M} \frac{\alpha^d \Omega_{d,k}}{\beta^{(1-\gamma)[d+i(k-1)]} \rho^{(1-\gamma)i(k-1)}}. \quad (116)$$

With this expression, we discuss below the diversity of (115) in different channel conditions.

C.1 When $\alpha \ll \beta^{(1-\gamma)k} \rho^{(1-\gamma)(k-1)}$:

Given α and β, at the kth ARQ, this condition also means that $\rho \gg \sqrt[(1-\gamma)(k-1)]{\alpha/\beta^{(1-\gamma)k}}$, which is common to many of the operating scenarios and is valid only if

$$k \geq \left\lfloor \frac{\log \alpha + (1-\gamma)\log \rho}{(1-\gamma)(\log \beta + \log \rho)} \right\rfloor + 1 \triangleq k' + 1, \ k \in \mathfrak{N}. \tag{117}$$

- If $\alpha < \beta^{(1-\gamma)}$ as well, then $k' \leq 0$. As a result, we have $k \geq 1$. Besides,

$$\frac{\alpha^d}{\beta^{(1-\gamma)[d+i_d(k-1)]}\rho^{(1-\gamma)i_d(k-1)}} = \left\{ \begin{array}{ll} \left[\frac{\alpha}{\beta^{(1-\gamma)k}\rho^{(1-\gamma)(k-1)}}\right]^d & , i_d = d \\ \frac{1}{(\beta\rho)^{(1-\gamma)i(k-1)}}\left[\frac{\alpha}{\beta^{(1-\gamma)}}\right]^d & , i_d = i \end{array} \right\} \tag{118}$$

decreases when d increases under this channel condition. Therefore, (a) of (116) is dominated at high SNR by the lowest order of $d = 1$, which gives

$$(a) \doteq \frac{\alpha M}{\beta^{(1-\gamma)k}\rho^{(1-\gamma)(k-1)}} \tag{119}$$

since $\Omega_{1,k} = M$. Substituting the result back into (115), we further have

$$\begin{aligned} \sum_{k=1}^{n} \Upsilon_k(\alpha, \beta, \rho) &\doteq \sum_{k=1}^{n} \frac{\alpha M}{\alpha^{M(n-k+1)}\beta^{(1-\gamma)k}\rho^{(1-\gamma)[M(n-k+1)+n]}} \\ &\doteq \frac{\alpha M}{\alpha^M \beta^{(1-\gamma)n}\rho^{(1-\gamma)(M+n)}}. \end{aligned} \tag{120}$$

- If $\beta^{(1-\gamma)} \leq \alpha \ll \beta^{(1-\gamma)k}\rho^{(1-\gamma)(k-1)}$, then at high SNR we still have

$$\sum_{d=1}^{i-1} \frac{\alpha^d \Omega_{d,k}}{\beta^{(1-\gamma)dk}\rho^{(1-\gamma)d(k-1)}} \doteq \frac{\alpha \Omega_{1,k}}{\beta^{(1-\gamma)k}\rho^{(1-\gamma)(k-1)}}. \tag{121}$$

Consequently, (116) can be approximated at high SNR as

$$(a) \doteq \underbrace{\frac{\alpha \Omega_{1,k}}{\beta^{(1-\gamma)k}\rho^{(1-\gamma)(k-1)}}}_{(b)} + \underbrace{\sum_{d=i}^{M} \frac{\alpha^d \Omega_{d,k}}{\beta^{(1-\gamma)[d+i(k-1)]}\rho^{(1-\gamma)i(k-1)}}}_{(c)}. \tag{122}$$

— If (b) ≫ (c) such that

$$\frac{\alpha \Omega_{1,k}}{\beta^{(1-\gamma)k}\rho^{(1-\gamma)(k-1)}} > \overline{\Omega}_k \sum_{d=i}^{M} \frac{\alpha^d}{\beta^{(1-\gamma)[d+i(k-1)]}\rho^{(1-\gamma)i(k-1)}} \geq \sum_{d=i}^{M} \frac{\alpha^d \Omega_{d,k}}{\beta^{(1-\gamma)[d+i(k-1)]}\rho^{(1-\gamma)i(k-1)}} \tag{123}$$

where $\overline{\Omega}_k \triangleq \max\{\Omega_{d,k}, \ \forall d \in [i, M]\}$, then it follows that

$$\rho > \left[\frac{\overline{\Omega}_k}{M} \sum_{d=i}^{M} \frac{\alpha^{d-1}}{\beta^{(1-\gamma)[d-i+(i-1)k]}}\right]^{\frac{1}{(1-\gamma)(i-1)(k-1)}} \tag{124}$$

which is valid only if

$$k \geq \left\lceil \frac{\log\left\{\frac{\overline{\Omega}_k}{M}\sum_{d=i}^{M}(\alpha/\beta^{1-\gamma})^{d-1}\right\}}{(1-\gamma)(i-1)(\log \beta + \log \rho)} \right\rceil + 1 \triangleq k" + 1. \tag{125}$$

Let $k^* = \max\{k', k"\}$. Then, if $k^* + 1 \leq n$, we have

$$\begin{aligned} \sum_{k=k^*+1}^{n} \Upsilon_k(\alpha, \beta, \rho) &\doteq \sum_{k=k^*+1}^{n} \Lambda_{\alpha,\rho}^{n-k+1} \frac{\alpha \Omega_{1,k}}{\beta^{(1-\gamma)k}\rho^{(1-\gamma)(k-1)}} & (126) \\ &= \sum_{k=k^*+1}^{n} \frac{\alpha M}{\alpha^{M(n-k+1)}\beta^{(1-\gamma)k}\rho^{(1-\gamma)[M(n-k+1)+n]}} & (127) \\ &\doteq \frac{\alpha M}{\alpha^M \beta^{(1-\gamma)n}\rho^{(1-\gamma)(M+n)}}. & (128) \end{aligned}$$

— If $k' + 1 \leq k < k"$ instead, such that (b) ≪ (c), then we will have

$$\sum_{k=k'+1}^{k^*} \Upsilon_k(\alpha,\beta,\rho) \doteq \sum_{k=k'+1}^{k^*}\sum_{d=i}^{M} \frac{\Lambda_{\alpha,\rho}^{n-k+1}\alpha^d\Omega_{d,k}}{\beta^{(1-\gamma)[d+i(k-1)]}\rho^{(1-\gamma)i(k-1)}} \tag{129}$$

$$= \sum_{k=k'+1}^{k^*} \frac{\sum_{d=i}^{M}\Omega_{d,k}\alpha^d/\beta^{(1-\gamma)d}}{\alpha^{M(n-k+1)}\beta^{(1-\gamma)i(k-1)}\rho^{(1-\gamma)[(M+1)n+(k-1)(i-M-1)]}} \tag{130}$$

$$\doteq \frac{\sum_{d=i}^{M}\Omega_{d,k^*}\alpha^d/\beta^{(1-\gamma)d}}{\alpha^{M(n-k^*+1)}\beta^{(1-\gamma)i(k^*-1)}\rho^{(1-\gamma)[(M+1)n+(k^*-1)(i-M-1)]}}. \tag{131}$$

C.2 When $\rho > 1$ and $\alpha \gg \beta^{(1-\gamma)k}\rho^{(1-\gamma)(k-1)} > \beta^{(1-\gamma)}$:

Under this channel condition, we have

$$\frac{\alpha^d}{\beta^{(1-\gamma)[d+i(k-1)]}\rho^{(1-\gamma)i(k-1)}} = \frac{\alpha^{d-i}}{\beta^{(1-\gamma)(d-i)}}\left[\frac{\alpha}{\beta^{(1-\gamma)k}\rho^{(1-\gamma)(k-1)}}\right]^i \gg 1 \tag{132}$$

which is increasing with d and holds only if $1 \le k \le k'$. Therefore, when α is large enough, the summation in (a) of (116) can be approximated at high SNR by its dominant terms associated with $d \ge i$, namely

$$(a) \doteq \sum_{d=i}^{M} \frac{\Omega_{d,k}\alpha^{d-i}}{\beta^{(1-\gamma)(d-i)}}\left[\frac{\alpha}{\beta^{(1-\gamma)k}\rho^{(1-\gamma)(k-1)}}\right]^i. \tag{133}$$

As a result, we have

$$\sum_{k=1}^{k'} \Upsilon_k(\alpha,\beta,\rho) \doteq \sum_{k=1}^{k'}\sum_{d=i}^{M} \frac{\Lambda_{\alpha,\rho}^{n-k+1}\alpha^d\Omega_{d,k}}{\beta^{(1-\gamma)[d+i(k-1)]}\rho^{(1-\gamma)i(k-1)}}$$

$$= \sum_{k=1}^{k'} \frac{1}{\alpha^{Mn}}\left[\frac{\alpha^M}{\beta^{(1-\gamma)i}}\right]^{k-1} \frac{\sum_{d=i}^{M}\Omega_{d,k}\alpha^d/\beta^{(1-\gamma)d}}{\rho^{(1-\gamma)[(M+1)n+(k-1)(i-M-1)]}}. \tag{134}$$

Obviously, the summands increase with k since $\alpha \gg \beta^{(1-\gamma)}$ and $M \ge i$. Thus, we further have

$$\sum_{k=1}^{k'} \Upsilon_k(\alpha,\beta,\rho) \doteq \frac{\sum_{d=i}^{M}\Omega_{d,k'}\alpha^d/\beta^{(1-\gamma)d}}{\alpha^{M(n-k'+1)}\beta^{(1-\gamma)i(k'-1)}\rho^{(1-\gamma)[(M+1)n+(k'-1)(i-M-1)]}}. \tag{135}$$

From the results of (120), (128), (131) and (135), we can see that except for the case of $\alpha < \beta^{(1-\gamma)}$, $\sum_{k=1}^{n}\Upsilon_k(\alpha,\beta,\rho)$ can be dominated by different terms in different range of $k \in [1,n]$. If $k^* = \max\{k', k"\} < n$, it is equal to (120) or the summation of (128), (131) and (135), both of which satisfy

$$\lim_{\lambda\to 0} \frac{\sum_{k=1}^{n}\Upsilon_k(\alpha,\beta,\rho)}{\lambda^{(1-\gamma)(M+n)}} = \frac{\alpha M}{\alpha^M\beta^{(1-\gamma)n}} \tag{136}$$

where $\lambda \triangleq 1/\rho$. Otherwise, if $\alpha \ge \beta^{(1-\gamma)}$ and $k' \ge n$ or $k" \ge n$, then $\sum_{k=1}^{n}\Upsilon_k(\alpha,\beta,\rho)$ is equal to (135) or plus (131), thus satisfying

$$\lim_{\lambda\to 0} \frac{\sum_{k=1}^{n}\Upsilon_k(\alpha,\beta,\rho)}{\lambda^{(1-\gamma)[M+1+i(n-1)]}} = g(\alpha,\beta,n) \triangleq \sum_{d=i}^{M} \frac{\alpha^{(d-M)}\Omega_{d,n}}{\beta^{(1-\gamma)[d+i(n-1)]}}. \tag{137}$$

Summarizing the above two results, we obtain at high SNR

$$\sum_{k=1}^{n}\Upsilon_k(\alpha,\beta,\rho) \doteq \left\{\begin{array}{ll} \frac{g(\alpha,\beta,n)}{\rho^{(1-\gamma)[M+1+i(n-1)]}} & ,\alpha \ge \beta^{(1-\gamma)} \text{ and } k^* \not< n \\ \frac{\alpha^{1-M}\beta^{-(1-\gamma)n}M}{\rho^{(1-\gamma)(M+n)}} & ,\text{otherwise} \end{array}\right\}. \tag{138}$$

Given α, β and $\alpha \ge \beta^{(1-\gamma)}$, this show that the logarithm of $\sum_{k=1}^{n}\Upsilon_k(\alpha,\beta,\rho)$ can be approximated with two straight lines in different ranges of SNR with slopes equal to $-(1-\gamma)(M+N)$ and $-(1-\gamma)[M+1+i(n-1)]$, respectively. The intersection point of the two lines is given by equating

$$\frac{M}{\alpha^{M-1}\beta^{(1-\gamma)n}\rho^{(1-\gamma)(M+n)}} = \frac{g(\alpha,\beta,n)}{\rho^{(1-\gamma)[M+1+i(n-1)]}} \tag{139}$$

which results in

$$\rho_{A_1}=\left[\frac{\alpha^{M-1}\beta^{(1-\gamma)n}g(\alpha,\beta,n)}{M}\right]^{\frac{1}{(1-\gamma)(n-1)(i-1)}}. \tag{140}$$

When $\rho>\rho_{A_1}$ the diversity is $(1-\gamma)(M+N)$, otherwise it is $(1-\gamma)[M+1+i(n-1)]$.

C.3 When $\beta \gg M^{\frac{1}{n(1-\gamma)}}[\alpha\rho^{(1-\gamma)}]^{\frac{M(n-1)+1}{n(1-\gamma)}} \triangleq \beta^*$:

It has been known from (114) and (120) that when $\beta^{(1-\gamma)}>\alpha$, we have

$$\mathbb{P}_{A,i}(n)\doteq\frac{1}{\alpha^{nM}\rho^{(1-\gamma)[n(M+1)+1]}}+\frac{M}{\alpha^{M-1}\beta^{(1-\gamma)n}\rho^{(1-\gamma)(M+n)}}. \tag{141}$$

If we further have $\beta^{(1-\gamma)}\gg(\beta^*)^{(1-\gamma)}\geq[\alpha\rho^{(1-\gamma)}]^{\frac{M(n-1)+1}{n}}\geq\alpha\rho^{(1-\gamma)}\gg\alpha$, then the first term on the RHS of (141) will become dominant. Therefore, if

$$1\leq\rho<\rho_{A_2}\triangleq\alpha^{\frac{1}{(1-\gamma)}}\left[\frac{\beta^{(1-\gamma)n}}{M}\right]^{\frac{1}{(1-\gamma)[M(n-1)+1]}} \tag{142}$$

then, the outage probability can be approximated at high SNR as

$$\mathbb{P}_{A,i}(n)\doteq\frac{1}{\alpha^{nM}\rho^{(1-\gamma)[n(M+1)+1]}}. \tag{143}$$

Otherwise, we have

$$\mathbb{P}_{A,i}(n)\doteq\left\{\begin{array}{ll}\frac{g(\alpha,\beta,n)}{\rho^{(1-\gamma)[M+1+i(n-1)]}} & ,\alpha\geq\beta^{(1-\gamma)}\text{ and }\rho<\rho_{A_1}\\ \frac{\alpha^{1-M}\beta^{-(1-\gamma)n}M}{\rho^{(1-\gamma)(M+n)}} & ,\text{otherwise}\end{array}\right\}. \tag{144}$$

We note that it is impractical to have $\alpha\geq\beta^{(1-\gamma)}$ and

$$\frac{1}{\alpha^{nM}\rho^{(1-\gamma)[n(M+1)+1]}}>\frac{g(\alpha,\beta,n)}{\rho^{(1-\gamma)[M+1+i(n-1)]}} \tag{145}$$

since this would lead to

$$\rho^{(1-\gamma)[(n-1)(M+1-i)+1]}<\frac{1}{\alpha^{nM}g(\alpha,\beta,n)}\ll 1 \tag{146}$$

due to the fact that, in general,

$$\alpha^{nM}g(\alpha,\beta,n)=\sum_{d=i}^{M}\frac{\alpha^{[M(n-1)+d]}\Omega_{d,n}}{\beta^{(1-\gamma)[i(n-1)+d]}}\geq\sum_{d=i}^{M}\Omega_{d,n}\beta^{(1-\gamma)(M-i)(n-1)}\gg 1. \tag{147}$$

C.4 For ARQs with ODSTC1 ($i=1$):

We note that the above derivations are conducted under the assumption that $i>1$. For $i=1$, we always have

$$\sum_{k=1}^{k}\Upsilon_k(\alpha,\beta,\rho) = \sum_{k=1}^{k}\frac{\sum_{d=1}^{M}\Omega_{d,k}\alpha^d/\beta^{(1-\gamma)d}}{\alpha^{M(n-k+1)}\beta^{(1-\gamma)(k-1)}\rho^{(1-\gamma)[(M+1)n-(k-1)M]}} \tag{148}$$

$$\doteq \frac{\sum_{d=1}^{M}C_d^M\alpha^d/\beta^{(1-\gamma)d}}{\alpha^{M}\beta^{(1-\gamma)(n-1)}\rho^{(1-\gamma)(M+n)}}. \tag{149}$$

As a result, the outage probability can be approximated at high SNR as (143) only if

$$\lim_{\rho\to\infty}\frac{\beta^{n(1-\gamma)}}{[\alpha\rho^{(1-\gamma)}]^{M(n-1)+1}\Delta_{\alpha,\beta}}>1 \tag{150}$$

where $\Delta_{\alpha,\beta}\triangleq\sum_{d=1}^{M}C_d^M\alpha^{(d-1)}/\beta^{(1-\gamma)(d-1)}$. Therefore, define

$$\rho_{A_3}\triangleq\left[\frac{\beta^{n(1-\gamma)}}{\alpha^{M(n-1)+1}\Delta_{\alpha,\beta}}\right]^{\frac{1}{(1-\gamma)[M(n-1)+1]}}. \tag{151}$$

For $i=1$, we have

$$\mathbb{P}_{A,i}(n) \doteq \left\{ \begin{array}{ll} \frac{1}{\alpha^{nM}\rho^{(1-\gamma)[n(M+1)+1]}} & ,\rho < \rho_{A_3} \\ \frac{\sum_{d=1}^{M} C_d^M \alpha^d/\beta^{(1-\gamma)d}}{\alpha^M \beta^{(1-\gamma)(n-1)}\rho^{(1-\gamma)(M+n)}} & ,\rho > \rho_{A_3} \end{array} \right\}. \tag{152}$$

D. Proof of Corollary 2

Continued from (51), $\mathbb{P}_{B,i}(n)$ is approximated at high SNR as

$$\mathbb{P}_{B,i}(n) \doteq \frac{\Lambda_{\alpha,\rho}^n}{\rho^{1-\gamma}} + \sum_{k=1}^{n} \Gamma_k(\alpha,\beta,\rho) \tag{153}$$

where

$$\Gamma_k(\alpha,\beta,\rho) \triangleq \Lambda_{\alpha,\rho}^{n-k+1} \sum_{d=1}^{M} \frac{\alpha^d \Phi_{d,k}}{\beta^{(1-\gamma)dk}\rho^{(1-\gamma)d(k-1)}} \tag{154}$$

with $\Lambda_{\alpha,\rho} \triangleq \frac{1}{\alpha^M \rho^{(1-\gamma)(M+1)}}$ and $\Phi_{d,k} \triangleq \frac{C_d^M}{\left(i_d! i_d^{(d-i_d)}\right)^k}$.

1. If $\alpha \ll \beta^{(1-\gamma)k}\rho^{(1-\gamma)(k-1)}$, hence, $k \geq k'+1$, [*c.f.* (117)], then the term associated with $d=1$ will dominate the summation of $\Gamma_k(\alpha,\beta,\rho)$ at high SNR, resulting in

$$\Gamma_k(\alpha,\beta,\rho) \doteq \frac{\alpha M}{\alpha^{M(n-k+1)}\beta^{(1-\gamma)k}\rho^{(1-\gamma)[M(n-k+1)+n]}}. \tag{155}$$

Therefore, if $k' \leq n-1$, we have

$$\sum_{k=k'+1}^{n} \Gamma_k(\alpha,\beta,\rho) \doteq \frac{\alpha M}{\alpha^M \beta^{(1-\gamma)n}\rho^{(1-\gamma)(M+n)}}. \tag{156}$$

2. On the contrary, if $\alpha \gg \beta^{(1-\gamma)k}\rho^{(1-\gamma)(k-1)} > \beta^{(1-\gamma)}$, when $\rho > 1$. Then $1 \leq k < k'+1$. The term associated with $d=M$ will dominate the summation of $\Gamma_k(\alpha,\beta,\rho)$, resulting in

$$\Gamma_k(\alpha,\beta,\rho) \doteq \frac{\Phi_{M,k}}{\alpha^{Mn}} \left[\frac{\alpha}{\beta^{(1-\gamma)}}\right]^{Mk} \frac{1}{\rho^{(1-\gamma)[(M+1)n-k+1]}}. \tag{157}$$

and, hence,

$$\sum_{k=1}^{k'} \Gamma_k(\alpha,\beta,\rho) \doteq \frac{\Phi_{M,k'}}{\alpha^{M(n-k')}\beta^{(1-\gamma)Mk'}\rho^{(1-\gamma)[(M+1)n-k'+1]}}. \tag{158}$$

It is clear that $[(M+1)n-k'+1] \geq Mn+1 \geq M+n$. Therefore, combing the above two results, we obtain

$$\sum_{k=1}^{n} \Gamma_k(\alpha,\beta,\rho) \doteq \left\{ \begin{array}{ll} \frac{\Phi_{M,n}}{\beta^{(1-\gamma)Mn}\rho^{(1-\gamma)(Mn+1)}} & ,k' \geq n \\ \frac{M}{\alpha^{M-1}\beta^{(1-\gamma)n}\rho^{(1-\gamma)(M+n)}} & ,k' \leq n-1 \end{array} \right\}. \tag{159}$$

Since k' decreases when ρ increases, this shows that the logarithm of $\sum_{k=1}^{n}\Gamma_k(\alpha,\beta,\rho)$ can be approximated with two straight lines in different ranges of SNR with slopes equal to $-(1-\gamma)(Mn+1)$ and $-(1-\gamma)(M+N)$, respectively. When $k' \geq n$, namely, $\alpha \geq \beta^{(1-\gamma)n}\rho^{(1-\gamma)(n-1)} \triangleq \alpha^*$, the diversity is $(1-\gamma)(nM+1)$. However, as ρ keeps increasing, the diversity may change from $(1-\gamma)(Mn+1)$ to $(1-\gamma)(M+N)$ when ρ is higher than a threshold SNR, ρ_{B_1}. The intersection point of the two lines is obtained by solving

$$\frac{\alpha M}{\alpha^M \beta^{(1-\gamma)n}\rho^{(1-\gamma)(M+n)}} = \frac{1}{(i! i^{M-i})^n \beta^{(1-\gamma)nM}\rho^{(1-\gamma)(nM+1)}} \tag{160}$$

which yields

$$\rho_{B_1} = \left[\frac{\alpha^{M-1}}{M\beta^{(1-\gamma)n(M-1)}(i!i^{M-i})^n}\right]^{\frac{1}{(1-\gamma)(n-1)(M-1)}}. \tag{161}$$

On the other hand, when $k' < n$, which also means $\alpha < \beta^{(1-\gamma)n}\rho^{(1-\gamma)(n-1)} \triangleq \alpha^*$, the $\mathbb{P}_{B,i}(n)$ can be approximated as

$$\mathbb{P}_{B,i}(n) \doteq \frac{1}{\alpha^{nM}\rho^{(1-\gamma)[n(M+1)+1]}} + \frac{M}{\alpha^{M-1}\beta^{(1-\gamma)n}\rho^{(1-\gamma)(M+n)}}. \tag{162}$$

If we further have $\beta^{(1-\gamma)} \gg (\beta^*)^{(1-\gamma)} \geq [\alpha\rho^{(1-\gamma)}]^{\frac{M(n-1)+1}{n}} \geq [\alpha\rho^{(1-\gamma)(1-n)}]^{\frac{1}{n}}$, where $\beta^* \triangleq M^{\frac{1}{n(1-\gamma)}}[\alpha\rho^{(1-\gamma)}]^{\frac{M(n-1)+1}{n(1-\gamma)}}$, then the first term on the RHS will become dominant. The outage probability can thus be approximated at high SNR as

$$\mathbb{P}_{B,i}(n) \doteq \frac{1}{\alpha^{nM}\rho^{(1-\gamma)[n(M+1)+1]}}. \tag{163}$$

The threshold SNR, ρ_{B_2}, is thus given by

$$\rho_{B_2} = \left[\frac{\beta^{(1-\gamma)n}}{M\alpha^{M(n-1)+1}}\right]^{\frac{1}{(1-\gamma)[M(n-1)+1]}} = \rho_{A_2}. \tag{164}$$

We note that it is unlikely to see at high SNR that

$$\frac{1}{\alpha^{nM}\rho^{(1-\gamma)[n(M+1)+1]}} = \frac{1}{(i!i^{M-i})^n\beta^{(1-\gamma)nM}\rho^{(1-\gamma)(nM+1)}}. \tag{165}$$

Since, in this case, we would have $\rho = \sqrt[1-\gamma]{i!i^{M-i}\beta^{M(1-\gamma)}/\alpha^M} \ll 1$ when requiring at least $\alpha \gg \beta^{1-\gamma}$ for (158) to be valid. Summarizing the above results, we have

$$\mathbb{P}_{B,i}(n) \doteq \left\{\begin{array}{ll} \frac{\Phi_{M,n}}{\beta^{(1-\gamma)Mn}\rho^{(1-\gamma)(Mn+1)}} & ,\alpha \geq \alpha^* \text{ and } \rho < \rho_{B_1} \\ \frac{1}{\alpha^{nM}\rho^{(1-\gamma)[n(M+1)+1]}} & ,\alpha < \alpha^* \text{ and } \rho < \rho_{A_2} \\ \frac{M}{\alpha^{M-1}\beta^{(1-\gamma)n}\rho^{(1-\gamma)(M+n)}} & ,\text{ otherwise} \end{array}\right\}. \tag{166}$$

This completes the proof.

E. Proof of Proposition 5

Since $\{\underline{d}_\ell + i_\ell(M-\underline{d}_\ell)\} \geq M$, with the equality holding either when $i_\ell \triangleq \min\{i, \underline{d}_{\ell-1}\} = 1$ or $\underline{d}_\ell = M$. Given $i > 1$ and $d_0 \geq 1$, for $k \geq 2$, by inspection, we have:

1. Define $\mathcal{S}_0 \triangleq \{d_0 | d_0 \in [1, M]\}$.
2. ARQ2:

To satisfy $\underline{d}_1 + i_1(M-\underline{d}_1) = M$:

- For ($i_1 = 1$ and $\underline{d}_1 < M$), then $\underline{d}_0 = 1$, we have
$$\mathcal{S}_1^a \triangleq \{(d_1, d_0) | d_0 = 1, d_1 \in [0, M-2]\}.$$
- For $\underline{d}_1 = M$. Since $\underline{d}_0 \geq 1$, we have
$$\mathcal{S}_1^b \triangleq \{(d_1, d_0) | d_1 = M - \underline{d}_0, d_0 \in \mathcal{S}_0\}.$$
- Thus, $\mathcal{S}_1 \triangleq \mathcal{S}_1^a \cup \mathcal{S}_1^b$ satisfies $\underline{d}_1 + i_1(M-\underline{d}_1) = M$.

3. ARQ3:

- For ($i_2 = 1$ and $\underline{d}_2 < M$), then $\underline{d}_1 = 1$. Since $d_0 \geq 1$, $\underline{d}_\ell + i_\ell(M-\underline{d}_\ell) = M$, $\ell \in [1, 2]$ is satisfied when
$$\mathcal{S}_2^a \triangleq \{(d_2, d_1, d_0) | d_0 = 1, d_1 = 0, d_2 \in [0, M-2]\}.$$
- For $\underline{d}_2 = M$, to also satisfy $\underline{d}_1 + i_1(M-\underline{d}_1) = M$, we have $(d_1, d_0) \in \mathcal{S}_1$. Thus,
$$\mathcal{S}_2^b \triangleq \{(d_2, d_1, d_0) | d_2 = M - \underline{d}_1, (d_1, d_0) \in \mathcal{S}_1\}.$$
- $\mathcal{S}_2 \triangleq \mathcal{S}_2^a \cup \mathcal{S}_2^b$ satisfies $\underline{d}_\ell + i_\ell(M-\underline{d}_\ell) = M$, $\ell \in [1, 2]$.

4. For ARQk-1:
Assume $\mathcal{S}_{k-2} \triangleq \mathcal{S}_{k-2}^a \cup \mathcal{S}_{k-2}^b$ satisfies $\underline{d}_\ell + i_\ell(M - \underline{d}_\ell) = M,\ \forall \ell \in [1, k-2]$, with

$$\mathcal{S}_{k-2}^a \triangleq \{(d_{k-2}, \cdots, d_0) | d_0 = 1, d_1 = \cdots = d_{k-3} = 0, d_{k-2} \in [0, M-2]\}.$$
$$\mathcal{S}_{k-2}^b \triangleq \{(d_{k-2}, \cdots, d_0) | d_{k-2} = M - \underline{d}_{k-3}, (d_{k-3}, \cdots, d_0) \in \mathcal{S}_{k-3}\}.$$

5. For ARQk:
- For ($i_{k-1} = 1$ and $\underline{d}_{k-1} < M$), then $\underline{d}_{k-2} = 1$. Since $d_0 \geq 1$, $\underline{d}_\ell + i_\ell(M - \underline{d}_\ell) = M$, $\forall \ell \in [1, k-1]$ is satisfied when

$$\mathcal{S}_{k-1}^a \triangleq \{(d_{k-1}, \cdots, d_0) | d_0 = 1, d_1 = \cdots = d_{k-2} = 0, d_{k-1} \in [0, M-2]\}. \quad (167)$$

- For $\underline{d}_{k-1} = M$, to also satisfy $\underline{d}_\ell + i_\ell(M - \underline{d}_\ell) = M, \forall \ell \in [1, k-2]$, we have $(d_{k-2}, \cdots, d_0) \in \mathcal{S}_{k-2}$. Thus,

$$\mathcal{S}_{k-1}^b \triangleq \{(d_{k-1}, \cdots, d_0) | d_{k-1} = M - \underline{d}_{k-2}, (d_{k-2}, \cdots, d_0) \in \mathcal{S}_{k-2}\}. \quad (168)$$

- Having $\mathcal{S}_{k-1} \triangleq \mathcal{S}_{k-1}^a \cup \mathcal{S}_{k-1}^b$ satisfies $\underline{d}_\ell + i_\ell(M - \underline{d}_\ell) = M, \forall \ell \in [1, k-1]$.

By induction, the proof is completed.

REFERENCE

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-part I: System description," IEEE Trans. on Communications, vol. 51, no. 11, pp. 1927–1938, 2003.

[2] J. N. Laneman and G. W. Wornell, "Distributed space-time coded protocols for exploiting cooperative diversity in wireless networks," IEEE Trans. on Information Theory, vol. 49, no. 10, pp. 2415–2525, 2003.

[3] J. N. Laneman, D. N.C. Tse, and G. W. Wornell, "Coopeative diversity in wireless networks: Efficient protocols and outage behavior," IEEE Trans. on Information Theory, vol. 50, pp. 3062–3080, 2004.

[4] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," IEEE Trans. on Information Theory, vol. 51, no. 9, pp. 3037–3063, 2005.

[5] K. Azarian, H. E. Gamal, and P. Schniter, "On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels," IEEE Trans. on Information Theory, vol. 51, no. 12, pp. 4152–4172, Dec. 2005.

[6] K. Azarian, H. E. Gamal, and P. Schniter, "On the optimality of the ARQ-DDF protocol," IEEE Trans. on Information Theory, vol. 54, no. 4, pp. 1718–1724, Apr. 2008.

[7] Y. Jing and B. Hassibi, "Distributed space-time coding in wireless relay networks," IEEE Trans. on Wireless Communications, vol. 5, no. 12, pp. 3524–3536, 2006.

[8] K. G. Seddik, A. K. Sadek, and K.J. Ray Liu, "Outage analysis and optimum power allocation for multinode relay networks," Signal Processing Letters, vol. 14, no. 6, pp. 1126–1131, 2007.

[9] M. Yuksel and E. Erkip, "Multi-antenna cooperative wireless systems: a diversity-multiplexing tradeoff perspective," IEEE Trans. on Information Theory, vol. 53, no. 10, pp. 3371–3393, 2007.

[10]A. Bletssa, H. Shin, and M. Win, "Cooperative communicaions with outage-optimal opportunistic relaying," IEEE Trans. on Wireless Communications, vol. 6, no. 9, pp. 3450–3460, 2007.
[11]D. Chen, K. Azarian, and J. N. Laneman, "A case for amplify-and-forward relaying in the block-fading multi-access channel," IEEE Trans. on Information Theory, vol. 54, no. 8, pp. 3728–3733, 2008.
[12]R. Mudumbai, D. R. Brown III, U. Madhow, and H. V. Poor, "Distributed transmit beamforming: Challenges and recent progress," IEEE Communications Magazine, vol. 47, no. 2, pp. 102–110, Feb. 2009.
[13]IEEE standard for local and metropolitan area networks part 16: Air interface for fixed and mobile broadband wireless access systems: Multihop relay specification, P802.16j/D9, May 2009, available at http://www.ieee802.org/16.
[14]IEEE standard for local and metropolitan area networks part 16: Task Group m (TGm), Apr. 2009, available at http://www.ieee802.org/16/tgm.
[15]Third-Generation partnership project's (3GPP) long term evolution (LTE): Release 10 & beyond (LTE-Advanced), May 2009, available at http://www.3gpp.org/LTE-Advanced.
[16]Y. Zhao, R. Adve, and T. J. Lim, "Improving amplify-and-forward relay networks: optimal power allocation versus selection," IEEE Trans. on Wireless Communications, vol. 6, no. 8, pp. 3114–3123, 2007.
[17]P. Zhang, F. Wang, Z. Xu, S. Diouba, and L. Tu, "Opportunistic distributed space-time coding with semi-distributed relay selection method," Research Journal of Information Technology, vol. I, pp. 41–50, 2009.
[18]I. Stanojev, O. Simeone, and Y. Bar-Ness, "Performance of multi-relay collaborative hybrid-ARQ protocols over fading channels," IEEE Communications Letters, vol. 10, no. 7, pp. 522–524, 2006.
[19]T. Tabet, S. Dusad, and R. Knopp, "Diversity-Multiplexing-Delay tradeoff in half-duplex ARQ relay channels," IEEE Trans. on Information Theory, vol. 53, no. 10, pp. 3797–3805, Oct. 2007.
[20]R. Narasimhan, "Throughput-Delay performance of half-duplex Hybrid-ARQ relay channels," in Proc. IEEE ICC. Beijing, China, May 2008.
[21]C.-K. Tseng and S.-H. Wu, "Simple cooperative ARQ protocols with selective amplify-and-forward relaying," in Proc. IEEE PIMRC. Tokyo, Japan, Sep. 2009.
[22]I. Cerutti, A. Fumagalli, and P. Gupta, "Delay models of single-source single-relay cooperative ARQ protocols in slotted radio networks with Poisson frame arrivals," IEEE/ACM Trans. on Networking, vol. 16, no. 2, pp. 371–382, 2008.
[23]J. Cai, A. S. Alfa, P. Ren, X. (Sherman) Shen, and J. W. Mark, "Packet level performance analysis in wireless user-relaying networks," IEEE Trans. on Wireless Communications, vol. 7, no. 12, pp. 5336–5345, 2008.
[24]E. Telatar, "Capacity of multi-antenna gaussian channels," European Trans. on Telecommunications, vol. 10, no. 6, pp. 585–595, 1999.
[25]H. A. David and H. N. Nagarja, Order Statistics, Wiley, 3rd edition, 2003.
[26]L. Zhang and D. N.C. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels," IEEE Trans. on Information Theory, vol. 49, no. 5, pp. 1073–1069, May 2003.

[27]M. Adams and V. Guillemin, Measure Theory and Probability, Boston, MA:Birkh¨auser, 1996.
[28]I.S. Gradshteyn and I.M. Ryzhik, Table of Integrals, Series, and Products, Elsevier, 7th edition, 2007.

行政院國家科學委員會補助國內專家學者出席國際學術會議報告

99 年 6 月 24 日

報告人姓名	伍紹勳	服務機構及職稱	交通大學電信工程所
會議時間 地點	16-22 April 2010 Sydney, Australia	本會核定 補助文號	
會議 名稱	（中文） （英文）IEEE WCNC 2010		
發表 論文 題目	（中文） （英文）Time Synchronization Protocol for Small Scale Wireless Sensor Networks		

報告內容應包括下列各項：

一、參加會議經過

1. 至會議報告所發表之論文:

Time Synchronization Protocol for Small Scale Wireless Sensor Networks

2. 此外並參與聆聽會議其他論文之發表，以及大會之Key Note speeches:

   i. The Future of the Wireless Communications.

      Dr. Adam T. Drobot (CTO, Telcordia Technologies)

   ii. The Future of the Mobile Internet.

      Dr. Hugh S. Bradlow (CTO, Telstra Corporation).

   iii. Towards a New World of Cognitive and Agile Networks.

      Professor Khaled Ben Letaief (Hong Kong University of Science and Technology)

3. 另外也參加了會議前周日的 Tutorial 課程

   Biologically-inspired and Nano-scale Communication and Networking

   並蒐集了以下課程的投影片

   1. Biologically-inspired and Nano-scale Communication and Networking.
   2. Aspects of Multiuser MIMO - Principles and Standardization in LTE-Advanced.
   3. Coalitional Game Theory in Wireless Networks.
   4. Towards 4G - Technical Overview of LTE and WiMAX.

二、與會心得

參與此次通訊會議之目的除發表研究成果外，更注重觀察合作式通訊與感知無線電之發展方向。

1. 合作式通訊:

   在此方面，由於802.16m以及 LTE-Advanced 均將合作式中繼納入標準，此研究益發受到重視。可藉由此中繼技術解決寬頻峰巢式通訊之頻率相互干擾問題，以及拓展覆蓋範圍及增加通訊容量。

2. 感知無線電

   此技術之概念在於能讓系統自動搜尋並利用尚未被佔用的頻譜，因此頻譜的安排變的更有彈性。其中 Caltech Tao Cui et al 之 “Blind spectrum sensing in cognitive radio” 提出在沒有預先得知訊號統計特性之條件做頻譜感測市值得注意的技術。

三、考察參觀活動(無是項活動者省略)

四、回資料名稱及內容

Conference papers

表 Y04

無研發成果推廣資料

98 年度專題研究計畫研究成果彙整表

計畫主持人：伍紹勳	計畫編號：98-2221-E-009-055-

計畫名稱：廣義多使用者多天線技術之自動重傳機制及其性能探究

成果項目			量化			單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）
			實際已達成數（被接受或已發表）	預期總達成數(含實際已達成數)	本計畫實際貢獻百分比		
國內	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%		
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（本國籍）	碩士生	2	2	100%	人次	
		博士生	3	3	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		
國外	論文著作	期刊論文	1	1	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	3	3	100%		
		專書	0	0	100%	章/本	
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（外國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		

其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)	無

	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

■達成目標

□未達成目標（請說明，以 100 字為限）

　□實驗失敗

　□因故實驗中斷

　□其他原因

說明：

2. 研究成果在學術期刊發表或申請專利等情形：

論文：■已發表 □未發表之文稿 □撰寫中 □無

專利：□已獲得 □申請中 ■無

技轉：□已技轉 □洽談中 ■無

其他：（以 100 字為限）

已發表三篇會議論文於 IEEE ITW, 2010, IEEE VTC 2010-Spring, IEEE PIMRC 2010.
及投稿一篇期刊論文至 IEEE Trans. on Information Theory

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限）

我們在一個簡化的數學模型下，利用投機式(opportunistic)的解調傳送(decode-and-forward; DF)中繼技術,並且使用分散式空時編碼(distributed space-time coding; DSTC) 來協助系統之自動重傳協定。分析的結果告訴我們並不一定需要同時佔有所有中繼站的資源才能得到最低的失效機率(outage probability)，在特定的通道環境下，只要投機式的選取中繼站來幫助傳輸即可得到接近最佳的輸出效能。此外，模擬的結果同時顯示出並不需要搭配使用所有的中繼站，只要在所有解調成功的中繼站裡選擇兩個來幫助傳輸亦可得到接近最佳的輸出吞吐率。經由此文的分析，我們便能知道如何使用最少的中繼站數目以及複雜度最低的自動重傳機制，依然可以得到在各種不同的通道環境下所能輸出的最佳效能。