Optimal condition for full-field heterodyne interferometry

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Abstract. An area scan digital camera is used to record the full-field heterodyne interference signals, and the processes to derive the associated phases from the data of the recorded frames under a convenient condition are described. By calculating the possible errors under several different cases, an optimal condition to get better results is proposed.
 [DOI: 10.1117/1.2802095]

Subject terms: full-field heterodyne interferometry; digital camera; IEEE 1241 sine wave fitting algorithm.

Paper 070112R received Feb. 8, 2007; revised manuscript received May 29, 2007; accepted for publication May 30, 2007; published online Nov. 14, 2007.

1 Introduction

Heterodyne interferometry has many merits, such as rapid measurement, high resolution, and easy application to other interferometers.[1–4] As it is used for two-dimensional measurements,[1,2] it needs electromechanical equipment to scan the test surface. As a result, it becomes tedious and its resolution is also decreased. Consequently, full-field heterodyne interferometry[5–7] was proposed to avoid these drawbacks. An area scan digital camera is used to record the full-field heterodyne interference signal. The measurement resolution depends on the frequency and the visibility of the heterodyne interference signal, the camera recording time, the frame period, the frame exposure time, and the number of gray levels. For convenience, the camera recording time and the frame exposure time are always set to be equivalent to one period of the heterodyne interference signal and a half-frame period, respectively. Under convenient conditions, the processes to derive the associated phases from the data of a series of recorded frames are described. After calculating the possible errors under several different cases, an optimal condition to get better results is proposed.

2 Principle

In a general heterodyne interferometry, two interfered waves can be written as

$$E_1(t) = A_1 \exp(i2\pi f_1 t), \tag{1a}$$

and

$$E_2(t) = A_2 \exp(i2\pi f_2 t + i\phi), \tag{1b}$$

where $A_{1,2}$, $f_{1,2}$, and ϕ are their amplitudes, frequencies, and the phase difference between them, respectively. Its interference intensity is

$$\begin{aligned} I(t) &= |E_1 + E_2|^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos(2\pi(f_2 - f_1)t + \phi) \\ &= I_0[1 + r\cos(2\pi f t + \phi)], \end{aligned} \tag{2}$$

where $f = |f_2 - f_1|$ is the frequency of the heterodyne interference signal, and I_0 and r are the average intensity and the visibility, respectively. ϕ is often measured by comparing $I(t)$ with an optical/electronic reference signal. It can also be derived theoretically with the Fourier cosine and sine transforms[8] under $I(t)$ is specified. Thus, we have

$$\begin{aligned} A &= \int_{-\infty}^{\infty} I(t)\sin(2\pi f t)dt, \\ B &= \int_{-\infty}^{\infty} I(t)\cos(2\pi f t)dt, \end{aligned} \tag{3}$$

and

$$\phi = \tan^{-1}\left(-\frac{A}{B}\right). \tag{4}$$

When the heterodyne interference signal is sampled with a comb function as shown in Fig. 1, the intensity of the k'th point can be written as

$$I_k = I_0\left\{1 + r\cos\left(\frac{2\pi(k-1)}{m} + \phi\right)\right\}. \tag{5}$$

In full-field heterodyne interferometry, an area scan digital camera with frame frequency f_s and frame exposure time Δt is used to sample the interference signals. For convenience, we let the camera recording time T is equivalent to one period $(1/f)$ of the heterodyne interference signals and m frames are taken, as shown in Fig. 2. Here, f_s/f equals $m+n$, where m and n are an integer and a decimal, respectively. Consequently, the interference intensity measured at anyone pixel on the k'th frame becomes

$$\begin{aligned} I_{sk} &= \frac{1}{\Delta t}\int_{(k-1)/f_s}^{(k-1)/f_s+\Delta t} I_0(1 + r\cos(2\pi f t + \phi))dt \\ &= I_0\left\{1 + r'\cos\left(\frac{2\pi f(k-1)}{f_s} + \pi f\Delta t + \phi\right)\right\}, \end{aligned} \tag{6}$$

where

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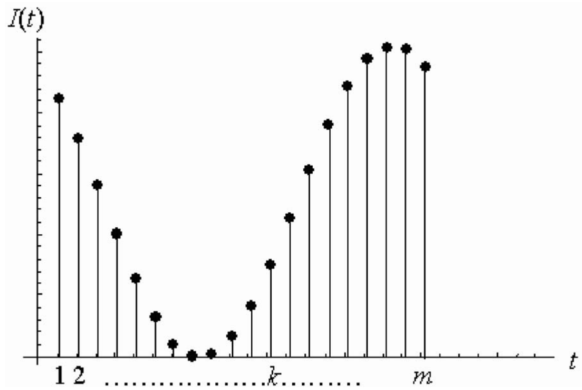

Fig. 1 The interference signal is sampled by a comb function.

$$r' = \frac{r \sin(\pi f \Delta t)}{\pi f \Delta t} = r\,\mathrm{sinc}(f\Delta t). \tag{7}$$

Next, I_{sk} will be quantized in gray-level units and it can be expressed as

$$I_{gk} = Round\left[\frac{I_0\left\{1 + r' \cos\left(\frac{2\pi f(k-1)}{f_s} + \pi f\Delta t + \phi\right)\right\}}{2I_0} \cdot g\right], \tag{8}$$

where g is the number of gray levels and $Round[x]$ is a mathematical operator to round the number x to an integer. A series of data I_{gk} are calculated with the IEEE 1241 sine wave fitting algorithm,[9] the optimal fitted cosine curve of the heterodyne interference signal $I'(t)$ can be obtained. Substituting $I'(t)$ into following equations

$$A' = 2f\int_0^{1/f} I'(t)\sin(2\pi f t)dt,$$
$$B' = 2f\int_0^{1/f} I'(t)\cos(2\pi f t)dt, \tag{9}$$

and

$$\phi' = \tan^{-1}\left(-\frac{A'}{B'}\right), \tag{10}$$

can be calculated. According to Eq. (6), it can be see that ϕ' includes a phase drift term $\pi f\Delta t$. That term can be obtained under the experimental conditions in which f and Δt are specified. Consequently the measured phase and its sampling error are

$$\phi_s = \phi' - \pi f\Delta t, \tag{11}$$

and

$$\Delta\phi = |\phi_s - \phi|, \tag{12}$$

respectively. If we apply the measurement processes repeatedly to every pixel, then its associated phase and sampling error can be obtained.

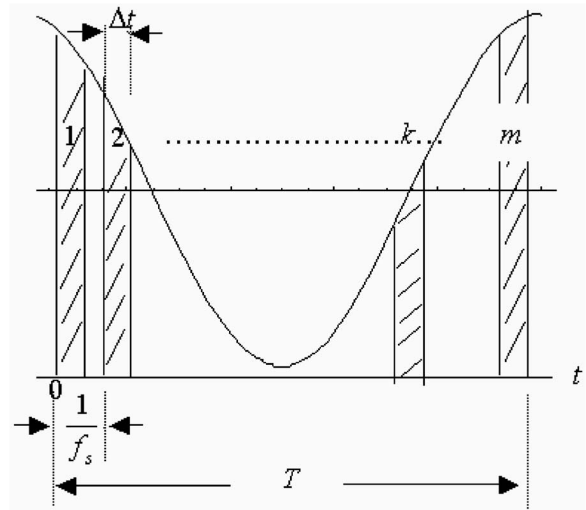

Fig. 2 The interference signal is sampled by a digital camera.

3 Numerical Simulations

We define $u = f_s\Delta t$, which means the ratio of the frame exposure time to the frame period. A general commerical digital camera for the heterodyne interferometry has $u = \frac{1}{2}$ and $g = 256$ gray levels (that is, 8 bits). We then make some numerical simulations for several cases with these specifications to find out the optimal condition for better results. Figure 3 shows the relation curves of $\Delta\phi$ versus f_s/f under $\phi = 0°$ as $r = 1$, 0.75, and 0.5, respectively. According to the sampling theory,[10] the condition $f_s/f \geqslant 2$ must be chosen to avoid sampling errors. However, it is obvious that the condition $f_s/f \geqslant 3$ should be chosen in the full-field heterodyne interferometry from the curves shown in Fig. 3. Next, because of the calculation of operator $Round[x]$ in Eq. (8) and the following derivations for estimating the value of $\Delta\phi$ in Eqs. (9)–(12), it is obvious that $\Delta\phi$ is dependent on ϕ. Figure 4 shows the relation curve of $\Delta\phi$ versus ϕ under the condition $f_s/f = 3$ as $r = 1$. It shows that the maximum sampling error equals 0.21 deg. Consequently, some relation curves of $\Delta\phi$ versus f_s/f are depicted as shown in Fig. 5, under the conditions (a) $r = 1$ and (b) $r = 0.5$ at $\phi = 0°$, 20°, 40°, and 60°, respectively. From those curves, it can seen

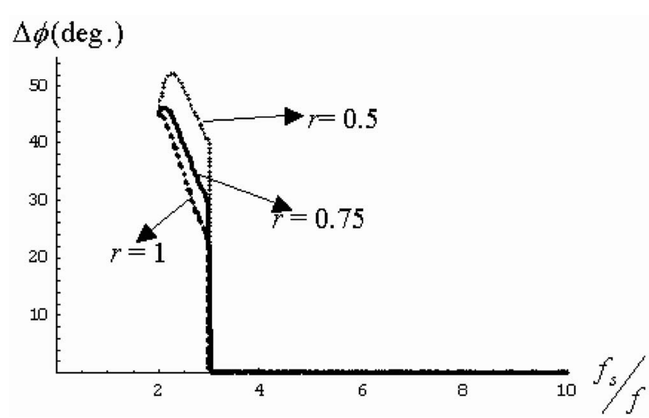

Fig. 3 The relation curves of $\Delta\phi$ versus f_s/f under the condition $\phi = 0°$ at $r = 1$, 0.75, and 0.5.

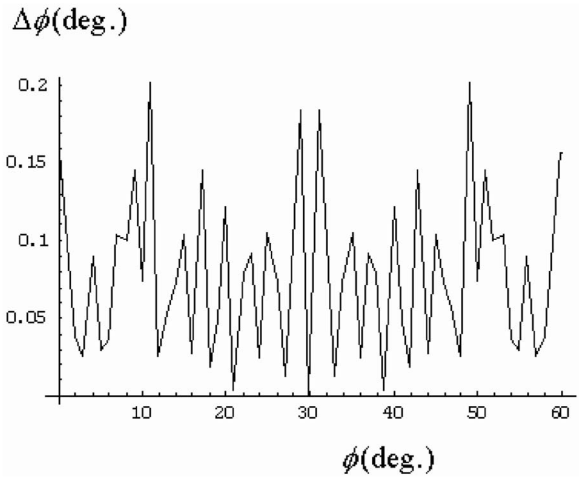

Fig. 4 The relation curve of $\Delta\phi$ versus ϕ under the condition $f_s/f=3$ at $r=1$.

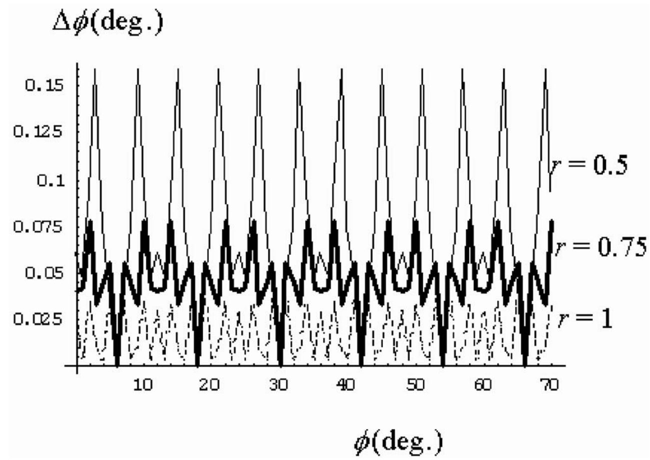

Fig. 6 The relation curves of $\Delta\phi$ versus ϕ under the condition $f_s/f=15$ at $r=1$, 0.75, and 0.5.

that the associated $\Delta\phi$ change very differently, but they have smaller values together as $f_s/f=15$. The relation curves of $\Delta\phi$ versus ϕ under the condition $f_s/f=15$ for different r are also depicted, as shown in Fig. 6. From those curves, we can see that the maximum sampling errors for $r=1$ and 0.5 are 0.036° and 0.16°, respectively. Such low sampling errors are suitable for practical measurements. Therefore, we propose an optimal condition

$$f_s/f = 15, \tag{13}$$

which is suitable for full-field heterodyne interferometry by using a commercial area scan digital camera under the conditions $T=1/f$, $u=1/2$, and $r\geqslant 0.5$.

4 Discussion

All the parameters f, f_s, u, T, and g can affect $\Delta\phi$. Theoretically, $\Delta\phi$ becomes smaller as T, g, and f_s/f increase, and the data processing also becomes time-consuming. If Tf is an integer that is larger than 1, the same frame data will be recorded Tf times. Consequently, $\Delta\phi$ hardly becomes smaller. Compromising those parameters, only the conditions $Tf=1$ and $u=\frac{1}{2}$ are discussed in this paper. In addition, because each digital camera has its own responsibility, the phase drift term $\pi f\Delta t$ in Eq. (6) should be measured in advance by using a reference signal for practical measurements.

5 Conclusion

A commercial area scan digital camera is always used to record the full-field heterodyne interference signals under the conditions $T=1/f$ and $u=1/2$. The processes to derive the associated phases from a series of recorded frames are described, based on IEEE 1241 sine wave fitting algorithm and Fourier cosine and sine transforms. After calculating the possible errors under several different cases, an optimal condition shown in Eq. (13) is proposed for better measurement results.

Acknowledgments

This study was supported in part by the National Science Council, Taiwan, under Contract No. NSC 95-2221-E-009-236-MY3.

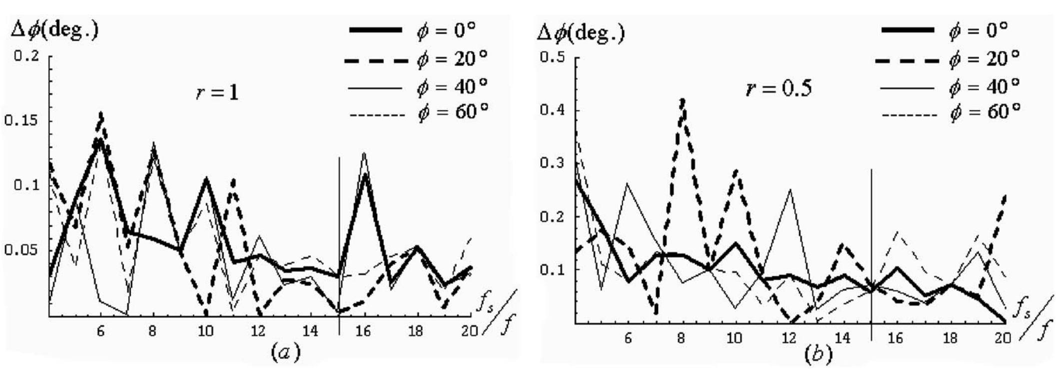

Fig. 5 The relation curves of $\Delta\phi$ versus f_s/f under the conditions (a) $r=1$ and (b) $r=0.5$ as $\phi=0°$, 20°, 40°, and 60°, respectively.

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