行政院國家科學委員會專題研究計畫 成果報告

策略二元樹：模型與應用
研究成果報告(精簡版)

計 畫 類 別 ：個別型
計 畫 編 號 ：NSC 98-2410-H-009-020-
執 行 期 間 ：98 年 08 月 01 日至 99 年 07 月 31 日
執 行 單 位 ：國立交通大學資訊與財金管理學系

計 畫 主 持 人 ：黃星華

計畫參與人員：博士班研究生-兼任助理人員：莊偉良

公 開 資 訊 ：本計畫可公開查詢

中 華 民 國 99 年 10 月 31 日

Strategic Binomial Tree: Model and Applications

Hsing-Hua Huang*

This Draft: Oct. 31, 2010

Abstract

The project not only provides a discrete-time framework for a strategic binomial tree, but also derives the continuous-time counterpart. We focus on the issue of two-firm strategic investment timing game. When the two firms are symmetric, there are two types of Nash equilibria: pre-emptive and simultaneous equilibria. While the two firms are asymmetric in their investment costs, there are three types of Nash equilibria: pre-emptive, non-preemptive and simultaneous Nash equilibria. The framework can be employed to analyze firms' strategic financing decisions and some financial derivatives, such as game options and convertible bonds.

Keywords: Strategic Binomial Tree, Pre-emptive Nash Equilibrium, Non-preemptive Nash Equilibrium, Simultaneous Nash Equilibrium, Strategic Investment

中文摘要

在本計畫中，我們不僅提出一個離散策略二元樹模型，而且也推導出其相對應之連續時間模型。本計畫藉此模型專門探討二廠商策略投資時間賽局之議題。當二家廠商是對稱的情況下，將存在二種 Nash 均衡：先佔均衡與同時均衡；而當二公司具有不對稱投資成本時，將存在三種 Nash 均衡：先佔均衡、非先佔均衡與同時均衡。此外，本模型也可用以分析企業策略融資決策，以及諸如賽局選擇權與可轉換公司債等金融衍生性商品。

關鍵詞： 策略二元樹、先佔 Nash 均衡、非先佔 Nash 均衡、同時 Nash 均衡、策略投資

* Assistant Professor, Graduate Institute of Finance and Department of Information and Finance Management, National Chiao Tung University. Email: hhhuang@mail.nctu.edu.tw.

I. Introduction

A firm's investment flexibilities are usually not exclusive, and hence the growing importance of strategic interactions between firms leads to a new stream of models situated on the intersection of real options and game theory, known as game-theoretic real options models. In many circumstances, a firm's optimal strategy is not only affected by nature but also by other firms, especially when the firm is in an oligopoly industry.

The game-theoretic binomial model proposed by Smit and Trigeorgis (2004) is considerably intuitive and easy to use, but is only applicable when there exists merely one pure Nash equilibrium in the games at each node of the tree. However, this is not usually the case we deal with. As shown by Nash (1951), a finite non-cooperative game always has at least one equilibrium point. If there is only one pure Nash equilibrium of the game, the choice problem is not encountered, and if there is no pure Nash equilibrium of the game, the mixed Nash equilibrium could be applied. Nevertheless, there may exist multiple pure Nash equilibria in a game.

Weeds (2002) considers irreversible investment in competing R&D projects under a winner-take-all patent system. Depending on the model's parameter values, there are two non-cooperative equilibria. One is the pre-emptive leader-follower equilibrium where firms invest sequentially and option values are reduced by competition, whereas the other is the symmetric equilibrium where firms investment simultaneously. Thijssen et al. (2003) analyze the problem of investment under uncertainty in a duopoly framework, and apply a coordination game to endogenously solve the issue rising when both firms want to invest whereas it is only profitable if just one invests. Miltersen and Schwartz (2004) use a game-theoretic real option framework to investigate patent-protected R&D investment projects when the firm is in an imperfect competition product market. Thijssen et al. (2006) examine the effect of uncertainty and competition on the firm's strategic investment when there exist some imperfect signals of the investment's profitability.

Recently, the extensive literature on real options games suggests that, when relatively few firms compete, there does often exist a first-mover advantage (FMA). The simple asymmetric duopoly equilibrium is often employed to analyze a firm's irreversible investment decision. The idea of rent equalization, provided by Funderber and Tirole (1985), is then applied to catching up the threat of pre-emptive investment. For some parameters, Pawlina and Kort (2006) and Mason and Weeds (2009) examine the irreversible investment behavior when there is a competitor who can pre-empt this investment project. They demonstrate that greater FMA will lead to a firm to adopt a pre-emptive investment threshold which is significantly lower than the firm's optimal investment trigger where there is no potential rival. Carlson et al. (2010) focus on the effects of a firm's expansion and contraction options on risk dynamics of the required returns when there is a rival owning the same rights. They generally find that competition will erode the value of wait-and-see options and their Nash equilibria satisfy the requirement of Markov subgame-perfect closed-loop equilibria.

The remainder of this project report is organized as follows. Section 2 introduces the main issue this project concerns, and the methodology is summarized in Section3. Section 5 summarizes results and give some remarks.

II. Main Issue

In this section, we will highlight a simple two-player investment timing game, which can be extended to any multi-player timing games. In a timing game, there are two players, 1 and 2, who have to decide when to make a lump-sum investment at some future time. The player that moves first is called the leader and his/her payoff is equal to $L(t)$, while the other player is called the follower and his/her payoff is $F(t)$. If both players move simultaneously at time t, they both obtain the payoff of $M(t)$.

At any time t before the two firms invest, each firm owns the perpetual right to determine whether to invest or not. This leads to the following game.

	Firm 2		
		Invest	Defer
Firm 1	Invest	$M(t), M(t)$	$L(t), F(t)$
	Defer	$F(t), L(t)$	repeat game

We could easily solve this timing game and there exists a discrete-time mixed-strategy Nash equilibrium. Firstly, we could construct a binomial tree to represent a stochastic environment and then put on the above game into each node on the tree. Next, we could solve individually by randomizing mixed strategies and finally we put backward to the initial and obtain the discrete-time subgame-perfect mixed strategy Nash equilibrium. There is a crucial issue when we extend the discrete-time model to continuous-time model by traditionally formulating the latter as the former's continuous-time limit. As noted by Fudenberg and Tirole (1985), the traditional formulation is not adequate, because it leads to a loss of information in directly passing from discrete-time with short periods to the limit in continuous-time. This loss of information prevents a continuous-time representation for the limits of discrete-time mixed-strategy Nash equilibria.

III. Methodology

We employ the extensive strategy space introduced by Fudenberg and Tirole (1985) to overcome the issue mentioned above. We first define continuous-time strategies, payoffs, and the Nash equilibrium in this strategy space.

Definition 1. A simple strategy for player i, $i=1,2$, in the game beginning at t is a pair of real-valued functions $\left(G_i(s),\alpha_i(s)\right):[t,\infty]\times[t,\infty]\to[0,1]\times[0,1]$ satisfying:

(1) $G_i(s), i=1,2$, is non-decreasing and right continuous.

(2) $\alpha_i(s)>0, i=1,2$, implies $G_i(s)=1, i=1,2$.

(3) $\alpha_i(s), i=1,2$, is right-differentiable.

(4) If $\alpha_i(s)=0, i=1,2$, and $s=\inf\left(u\geq t:\alpha_i(u)>0\right)$, then $\alpha_i(s)$ has positive right derivative.

We need some more notation to define the payoffs leading to a pair of simple strategies. Define

$$\tau_i(t)=\begin{cases}\infty & \text{if } \alpha_i(s)=0\ \forall s\geq t,\\ \inf\left(s\geq t:\alpha_i(s)>0\right) & \text{otherwise.}\end{cases}$$

$\tau_i(t)$ is the time of the first interval of atoms in player i's strategy. Let $\tau(t)=\min\left(\tau_i(t), i=1,2\right)$. Define $G_i^-(s)=\lim_{u\uparrow s}G_i(u)$. The game begins at $t\geq 0$; so let

$G_i^-(t)=0, i=1,2$. Let $a_i(s)=\lim_{\delta\to 0}\left(G_i(s)-G_i(s-\delta)\right)=G_i(s)-G_i^-(s)$ be the size of the jump in G_i at time $s\geq t$.

	Firm 2		
		$\alpha_2(s)$	$1-\alpha_2(s)$
		Invest	Defer
Firm 1 $\alpha_1(s)$	Invest	$M(t), M(t)$	$L(t), F(t)$
Firm 1 $1-\alpha_1(s)$	Defer	$F(t), L(t)$	repeat game

Definition 2. The payoff of player $i, i=1,2$, $V^i\left(t,\left(G_1,\alpha_1\right),\left(G_2,\alpha_2\right)\right)$, in the subgame starting at time t if the player $j, j=1,2, j\neq i$ adopts the simple strategy $\left(G_j,\alpha_j\right)$ is given by

$$V^i\left(t,\left(G_1,\alpha_1\right),\left(G_2,\alpha_2\right)\right)$$
$$=\int_{s=t}^{\tau(t)^-}\left(L(s)\left(1-G_j(s)\right)dG_i(s)+F(s)\left(1-G_i(s)\right)dG_j(s)\right)+\sum_{s<\tau(t)}a_i(s)a_j(s)M(s)$$
$$+\left(1-G_i^-(\tau(t))\right)\left(1-G_j^-(\tau(t))\right)W^i\left(\tau(t),\left(G_1,\alpha_1\right),\left(G_2,\alpha_2\right)\right),$$

where

$$W^i\left(\tau(t),\left(G_1,\alpha_1\right),\left(G_2,\alpha_2\right)\right)$$
$$=\begin{cases}\left(\dfrac{a_j(\tau(t))}{1-G_j^-(\tau(t))}\right)\left(\left(1-\alpha_i(\tau(t))\right)F(\tau(t))+\alpha_i(\tau(t))M(\tau(t))\right)+\left(\dfrac{1-G_j(\tau(t))}{1-G_j^-(\tau(t))}\right)L(\tau(t)), \text{ if } \tau_j(t)>\tau_i(t)\\ \left(\dfrac{a_i(\tau(t))}{1-G_i^-(\tau(t))}\right)\left(\left(1-\alpha_j(\tau(t))\right)L(\tau(t))+\alpha_j(\tau(t))M(\tau(t))\right)+\left(\dfrac{1-G_i(\tau(t))}{1-G_i^-(\tau(t))}\right)F(\tau(t)), \text{ if } \tau_i(t)>\tau_j(t)\end{cases}$$

if $\tau_i(t)=\tau_j(t)$,

$$W^i\left(\tau(t),\left(G_1,\alpha_1\right),\left(G_2,\alpha_2\right)\right)$$
$$=\begin{cases}M(\tau(t)), & \text{if } \alpha_i(\tau(t))=\alpha_j(\tau(t))=1,\\ \left(\dfrac{\alpha_i(\tau(t))\left(1-\alpha_j(\tau(t))\right)L(\tau(t))+\alpha_j(\tau(t))\left(1-\alpha_i(\tau(t))\right)F(\tau(t))+\alpha_i(\tau(t))\alpha_j(\tau(t))M(\tau(t))}{\alpha_i(\tau(t))+\alpha_j(\tau(t))+\alpha_i(\tau(t))\alpha_j(\tau(t))}\right), & \\ & \text{if } 0<\alpha_i(\tau(t))+\alpha_j(\tau(t))<2,\\ \dfrac{\alpha_i'(\tau(t))L(\tau(t))+\alpha_j'(\tau(t))F(\tau(t))}{\alpha_i'(\tau(t))+\alpha_j'(\tau(t))}, & \text{if } \alpha_i(\tau(t))=\alpha_j(\tau(t))=0,\end{cases}$$

where $\alpha_i'=\dfrac{\partial\alpha_i(\tau(t))}{\partial\tau(t)}, i=1,2.$

Using the payoff functions we can now define the Nash equilibrium of a game starting at time t.

Definition 3. A pair of simple strategies $\left\{\left(G_i(\cdot),\alpha_i(\cdot),i=1,2\right)\right\}$ is an open-loop Nash equilibrium of the game starting at time t, when neither player has not invested yet, if each player i's strategy attempts to maximize the payoff V^i holding the other player's strategy fixed.

Definition 4. A closed-loop strategy for players is a collection of simple strategies $\left\{\left(G_i^t(\cdot),\alpha_i^t(\cdot),i=1,2,t\geq 0\right)\right\}$ satisfying the intertemporal consistency conditions:

(1) $G_i^t(v)=G_i^t(u)+\left(1-G_i^t(u)\right)G_i^u(v)$ for $t\leq u\leq v\leq 1$.

(2) $\alpha_i^t(v)=\alpha_i^u(v)=\alpha_i(v)$ for $t\leq u\leq v\leq 1$.

Definition 5. A pair of closed-loop strategies $\left\{\left(G_i^t(\cdot),\alpha_i^t(\cdot),i=1,2,t\geq 0\right)\right\}$ is a subgame-perfect Nash equilibrium if for every t, the pair of simple strategies $\left\{\left(G_i^t(\cdot),\alpha_i^t(\cdot),i=1,2\right)\right\}$ is Nash equilibrium.

We further make the following assumptions on the value functions to focus on analyzing a particular class of symmetric pre-emption game.

Assumption 1. $L(t)$, $F(t)$ and $M(t)$ are continuous.

Assumption 2. $\exists T_F$ such that $L(t)=F(t)=M(t)\ \forall t\geq T_F$ and $F(t)>M(t)\ \forall t<T_F$.

Assumption 3. $F(t)$ is strictly increasing on $t\leq T_F$.

Assumption 4. $L(t)-F(t)$ is quasi concave.

Let $T_P=\inf\left(t\geq 0: L(t)\geq F(t)\right)$, $T_L=\arg\max_{t\leq T_F} L(t)$, $T_M=\arg\max_{0\leq t} M(t)$ and $T_S=\inf\left(t\geq T_F: M(t)=L(T_L)\right)$.

Next, several lemmas and propositions are provided. For the formal proofs, we refer to Fudenberg and Tirole (1985).

Lemma 1. $T_P\leq T_L$.

Lemma 2. $T_M\geq T_F$.

Proposition 1. If $L(T_L)>M(T_M)$, then

$$G^t(s)=\begin{cases}0, & s<T_P\\ 1, & s\geq T_P\end{cases} \quad\text{and}\quad \alpha(s)=\begin{cases}0, & s<T_P\\ \dfrac{L(s)-F(s)}{L(s)-M(s)}, & T_P<s<T_C\\ 1, & s\geq T_C\end{cases}$$

is the unique subgame-perfect mixed-strategy Nash equilibrium for the pre-emption game satisfying Assumptions 1-4.

Proposition 2. If $L(T_L)\leq M(T_M)$, then there are two types of subgame-perfect mixed-strategy Nash equilibria. The first type is the pre-emptive equilibrium as defined in Proposition 1, and there are an infinite number equilibria of the second type which are characterized by its investment date u, where $u\in[T_S,T_M]$, given by

$$G^t(s)=\begin{cases}0, & t\leq s<u\\ 1, & u\leq s\end{cases} \quad\text{and}\quad \alpha(s)=\begin{cases}0, & t\leq s<u\\ 1, & s\geq u\end{cases}.$$

Fudenberg and Tirole (1985) argue that the second type, if exists, would Pareto dominates the first type and is therefore the most reasonable outcome of the preemption satisfying Assumptions 1-4.

IV. Results and Remarks

Based on the methodology developed above, the two-firm symmetric investment timing game under uncertainty can be solved. Overall, there are three scenarios. In the first scenario when the first mover's advantages are large, a pre-emptive equilibrium occurs where the two firms' investment timings are dispersed. In the second, the two firms simultaneously invest when the uncertainty is large and thus the wait-and-see option value is high. In the last scenario, it turns out that the pre-emption is applied when there is lower uncertainty, while the simultaneous equilibrium appears at the moment that the uncertainty is larger. Compared to the first-best monopolistic investment timing, the investment timing of the leader in the pre-emptive equilibrium is earlier. In order to preempt its rival, the firm is satisfied with lower profits when it invests. On the other hand, the investment timing in the simultaneous equilibrium is late relative to that of the monopolistic case. This is because the two firms share the wait-and-see option value in the simultaneous equilibrium.

Moreover, we also extend the methodology by introducing asymmetric investment costs between the two firms. The results show that the potential of rival's pre-emption also precipitates investment in the asymmetric setting, but in a milder intensity. More precisely, there are two possibilities. When the investment costs of the rival (say firm 2) are very high (i.e., the two firms are extremely asymmetric), firm 1 will simply invest at its first-best monopolistic investment timing. This result appears in both the cases of negative and positive externalities. On the other hand, when the rival's investment costs are low enough, the preemptive effect prevails. Both in the cases of negative and positive externalities, this effect turns out to precipitate investment, but for totally different reasons. When there are negative externalities, the threat of rival's pre-emption leads firm 1 to invest earlier; when there are positive externalities, the two firms simultaneously invest early in anticipation that the other firm will also invest early.

The methodology developed in this project could be further employed to analyze game options and convertible bonds since the two financial derivatives are both involved closely in strategic interactions between issuers and buyers of derivatives. Firms' strategic financing strategies can also be investigated in the present framework. The two topics are research in progress.

References

1. Aguerrevere, F. L., 2003, Investment Strategies and Output Price Behavior: A Real Options Approach, *Review of Financial Studies* 16(4), 1239-1272.
2. Aguerrevere, F.L., 2009, Real Options, Product Market Competition, and Asset Returns, *Journal of Finance* 64, 957-983.
3. Back, K., and D., Paulsen, Open-Loop Equilibria and Perfect Competition in Option Exercise Games, *Review of Financial Studies* 22, 4531-4552.
4. Aumann, R. J., 1974, Subjectivity and Correlation in Randomized Strategies, *Journal of Mathematical Economics* 1, 67-96.
5. Aumann, R. J., 1990, Nash Equilibria Are Not Enforceable, in *Economic Decision Making* (J. J. Gabszewitz et al., Eds.), 201-206, Amsterdam, Elsevier.
6. Carlson, M., E.J. Dockner, A. Fisher, and R. Giammarino, 2010, Leaders,

Followers, and Risk Dynamics in Industry Equilibrium, *Working Paper*, Sauder School of Business, University of British Columbia.
7. Cox, J. C., S. A. Ross and M. Rubinstein, 1979, Option Pricing: A Simplified Approach, *Journal of Financial Economics* 7, 229-263.
8. Dixit, A. K. and R. S. Pindyck, 1994, *Investment under Uncertainty*, Princeton, New Jersey, Princeton University Press.
9. Fudenberg, D. and J. Tirole, 1985, Preemption and Rent-Equalization in the Adoption of New Technology, *Review of Economic Studies* 52, 383-401.
10. Grendadier, S. R., 2002, Option Exercise Games: An Application to the Equilibrium Investment Strategies of Firms, *Review of Financial Studies* 15(3), 691-721.
11. Gryglewicz, S., K. J. M. Huisman and P. M. Kort, 2008, Finite Project Life and Uncertainty Effects on Investment, *Journal of Economic Dynamics and Control* 32, 2191-2213.
12. Harsanyi, J. C. 1975, The Tracing Procedure, *International Journal of Game Theory* 4, 61-94.
13. Harsanyi, J. C. and R. Selten, 1988, *A General Theory of Equilibrium Selection in Games*, Cambridge, MA, MIT Press.
14. Harsanyi, J. C., 1995, A New Theory of Equilibrium Selection for Games with Complete Information, *Games and Economic Behavior* 8, 91-122.
15. Koln, W. G., 1985, A Remark on the Harsanyi-Selten Theory of Equilibrium Selection, *International Journal of Game Theory* 14, 31-39.
16. Lambrecht, B.M., and W. Perraudin, 2003, Real Options and Preemption under Incomplete Information, *Journal of Economic Dynamics and Control* 27, 619-643.
17. Mason, R., and H. Weeds, 2009, Investment, Uncertainty and Pre-emption, *Working Paper*, University of Essex and CEPR.
18. Miltersen, K. R. and E. S. Schwartz, 2004, R&D Investments with Competitive Interactions, *Review of Finance* 8, 355-401.
19. Nash, J. F., 1950, The Bargaining Problem, *Econometrica* 18, 128-140.
20. Nash, J. F., 1951, Non-Cooperate Games, *Annals of Mathematics* 54(2), 286-295.
21. Nash, J. F., 1953, Two-Person Cooperative Games, *Econometrica* 21, 128-140.
22. Pawlina, G., and P.M. Kort, 2006, Real Options in an Asymmetric Duopoly: Who Benefits from Your Competitive Disadvantage?, Journal of Economics and Management 15, 1-35.
23. Samuelson, 1997, *Evolutionary Games and Equilibrium Selection*, Cambridge, MA, MIT Press.
24. Smit, H. T. J. and L. Trigeorgis, 2004, *Strategic Investment: Real Options and Games*, Princeton, New Jersey, Princeton University Press.
25. Smit, H. T. J. and L. Trigeorgis, 2006, Real Options and Games: Competition, Alliances and Other Applications of Valuation and Strategy, *Review of Financial Economics* 15, 95-112.
26. Thijssen, J. J. J., K. J. M. Huisman and P. M. Kort, 2003, Symmetric Equilibria in Game Theoretic Real Option Models, *Working Paper*, Department of Econometrics & Operations Research and CentER, Tilbrug University, Tilbrug, Netherlands.
27. Thijssen, J. J. J., K. J. M. Huisman and P. M. Kort, 2006, The Effects of Information on Strategic Investment and Welfare, *Economic Theory* 28, 399-424
28. Weeds, H., 2002, Strategic Delay in a Real Options Model of R&D Competition, *Review of Economic Studies* 69(3), 729-747.

無衍生研發成果推廣資料

98 年度專題研究計畫研究成果彙整表

計畫主持人：黃星華			計畫編號：98-2410-H-009-020-				
計畫名稱：策略二元樹：模型與應用							
成果項目			量化			單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）
			實際已達成數（被接受或已發表）	預期總達成數(含實際已達成數)	本計畫實際貢獻百分比		
國內	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%		
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（本國籍）	碩士生	0	0	100%	人次	
		博士生	1	1	5%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		
國外	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%	章/本	
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（外國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		

其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)	無

	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

■達成目標

□未達成目標（請說明，以 100 字為限）

□實驗失敗

□因故實驗中斷

□其他原因

說明：

2. 研究成果在學術期刊發表或申請專利等情形：

論文：□已發表 □未發表之文稿 ■撰寫中 □無

專利：□已獲得 □申請中 ■無

技轉：□已技轉 □洽談中 ■無

其他：（以 100 字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限）

本研究計畫結果之最主要成果在於，成功將直覺的策略二元樹離散時間模型，擴展至連續時間隨機模型。因此，我們可以將此連續時間隨機賽局模型應用於分析：企業策略投資，略策融資與衍生性商品訂價。以下將簡述此三項應用及其學術與應用價值。

(1) 企業策略投資

學術價值：可結合實質選擇權與寡佔市場競爭模型。

應用價值：可用於分析企業於高度不確定性與高度競爭性產業之投資分析。

(2) 企業策略融資

學術價值：可結合或有求償權分析與寡佔市場競爭模型。

應用價值：可用於分析企業於高度不確定性與高度競爭性產業之最適資本結構或債權人與債務人之互動分析。

(3) 衍生性商品訂價

學術價值：可結合原本之衍生性商品訂價理論與隨機賽局模型

應用價值：可用於評價並分析可轉換公司債或賽局選擇權等具有賽局性質之衍生性商品。

上述三項應用之完成進度如下：

第一項策略投資初稿已接近完成，約 90%。

第二項已初步有相關架構，約 10%。

第三項賽局選擇權與可轉換公司債已開始進行，約 50%。

註：本計畫申請時本為二年共計畫，故總進度約完成 50%。