

The complete, lag and anticipated synchronization of a BLDCM chaotic system

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Abstract

The complete, lag and anticipated synchronizations of two identical autonomous chaotic systems, Brushless DC Motors (BLDCM) systems, are studied in this paper. PC method, linear coupling and active control are used for the achievements of the complete and the lag synchronizations. Linear coupling method and active control is used for the anticipated synchronization. Generalized lag, anticipated and complete synchronization are obtained by active control. Finally, the generalized lag synchronization of BLDCM system and Lorenz system is studied.

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1. Introduction

Chaos synchronization has been applied in many fields such as secure communication [1–3], chemical and biological systems, etc. [4,5]. Many researchers have studied synchronization between two identical chaotic systems [6–21]. Lag synchronization and anticipated synchronization of chaotic systems have been studied widely recently [21–23]. This paper is organized as follows. In Section 2, the complete synchronization of BLDCM systems [24–26] is obtained by PC (Pecora and Corroll) method and linear coupling. In Section 3, the lag synchronization is obtained by PC method and linear coupling. In Section 4, the anticipated synchronization is obtained by linear coupling. In Section 5, generalized lag, anticipated and complete synchronization are obtained by active control. In Section 5, the generalized synchronization of BLDCM system and Lorenz system is also obtained by active control. In Section 6, conclusions are drawn.

2. The complete synchronization of BLDCM systems

2.1. Pecora and Corroll method for complete synchronization

Pecora and Corroll method [27,28] of synchronization for identical systems is used in this subsection. The master and slave systems are described as follows:

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$$\begin{aligned} \dot{x}_1 &= v_q - x_1 - x_2x_3 + \rho x_3, \\ \dot{x}_2 &= v_d - \delta x_2 + x_1x_3, \\ \dot{x}_3 &= \sigma(x_1 - x_3) + \eta x_1x_2 - T_L, \end{aligned} \tag{1}$$

$$\begin{aligned} \dot{y}_1 &= v_q - y_1 - y_2y_3 + \rho y_3, \\ \dot{y}_2 &= v_d - \delta y_2 + y_1y_3, \\ \dot{y}_3 &= \sigma(y_1 - y_3) + \eta y_1y_2 - T_L, \end{aligned} \tag{2}$$

where $v_q = 0.168$, $v_d = 20.66$, $\delta = 0.875$, $\sigma = 4.55$, $\eta = 0.26$, $T_L = 0.53$ are parameters.

Firstly, the variable x_1 , in Eq. (1) is used to replace variable y_1 in Eq. (2), then the new slave system is described as follows:

$$\begin{aligned} \dot{y}_1 &= v_q - x_1 - y_2y_3 + \rho y_3, \\ \dot{y}_2 &= v_d - \delta y_2 + x_1y_3, \\ \dot{y}_3 &= \sigma(x_1 - y_3) + \eta x_1y_2 - T_L. \end{aligned} \tag{3}$$

Take $y_1 - x_1$, $y_2 - x_2$, $y_3 - x_3$ as errors. The complete synchronization can be obtained by simulation which is shown in Fig. 1. Secondly, by the replacement of y_3 by x_3 , the synchronization of these two identical systems can also be obtained, as shown in Fig. 2.

Lastly, it is found that by the replacement of y_1 and y_2 by x_1 and x_2 respectively and by the replacement of y_1 and y_3 by x_1 and x_3 respectively the synchronizations can also be obtained as shown in Figs. 3 and 4 which present that the former case requires less time for the accomplishment of synchronization than the latter case requires.

2.2. Complete synchronization by linear coupling

Take Eq. (1) as master and Eq. (2) with linear coupling as slave

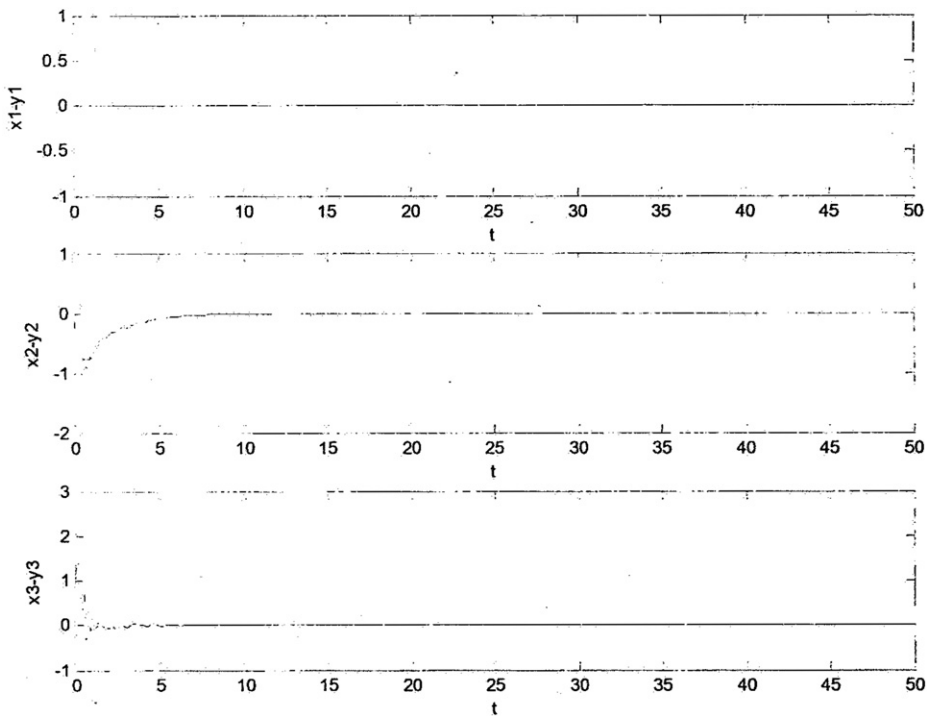


Fig. 1. The complete synchronization of two BLDCM systems by replacement of y_1 by x_1 .

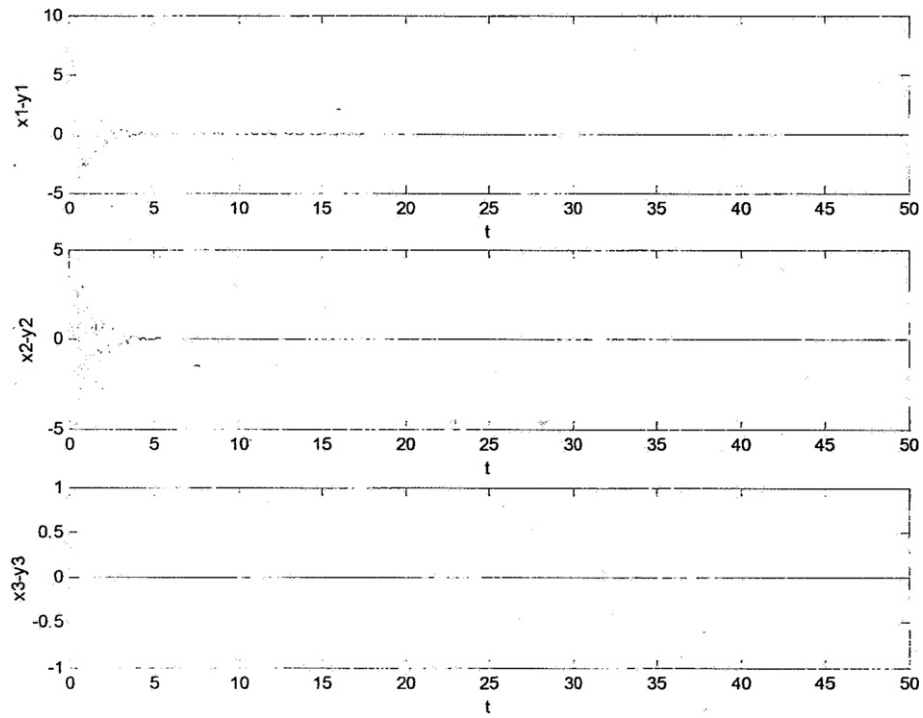


Fig. 2. The complete synchronization of two BLDCM systems by replacement of y_3 by x_3 .

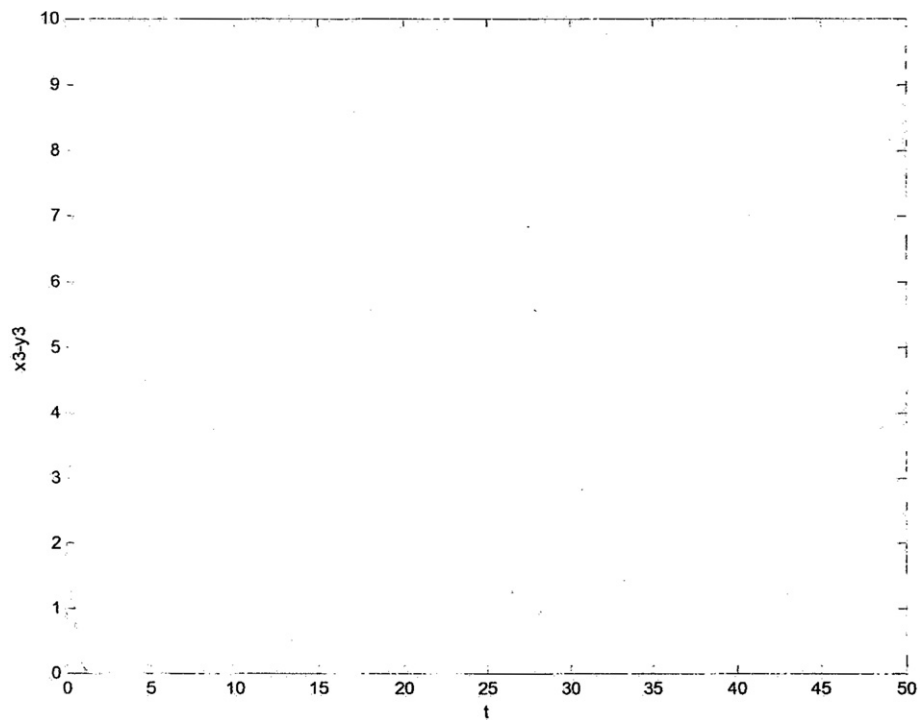


Fig. 3. The complete synchronization of two BLDCM systems by replacement of y_1 and y_2 by x_1 and x_2 respectively.

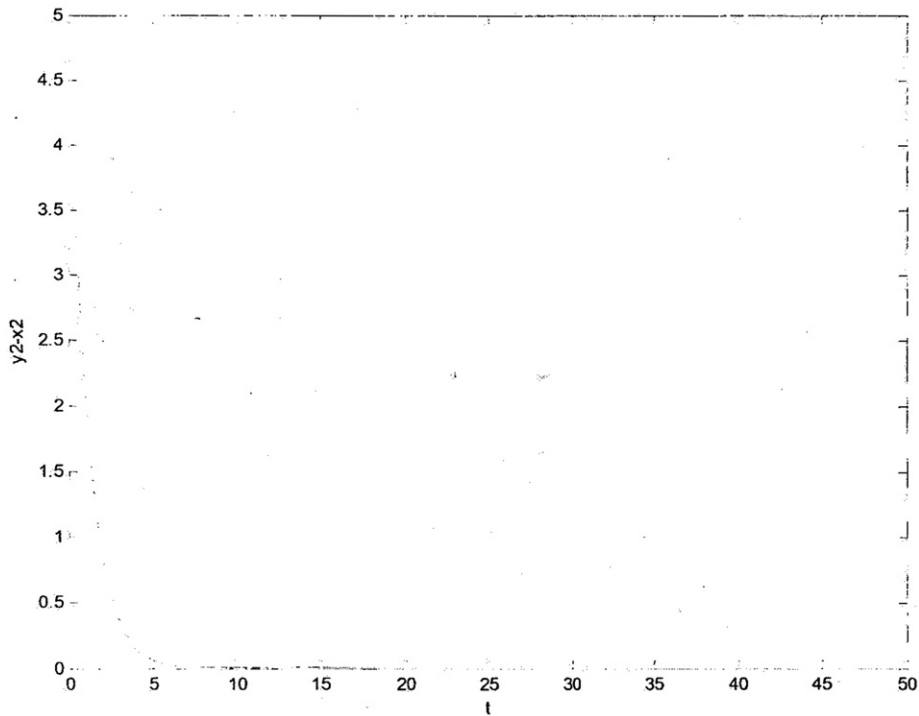


Fig. 4. The complete synchronization of two BLDCM systems by replacement of y_1 and y_3 by x_1 and x_3 respectively.

$$\begin{aligned}
 \dot{y}_1 &= v_q - y_1 - y_2 y_3 + \rho y_3 + K_1(x_1 - y_1), \\
 \dot{y}_2 &= v_d - \delta y_2 + y_1 y_3 + K_2(x_2 - y_2), \\
 \dot{y}_3 &= \sigma(y_1 - y_3) + \eta y_1 y_2 - T_L + K_3(x_3 - y_3).
 \end{aligned}
 \tag{4}$$

where $\mathbf{K} = [K_1 K_2 K_3]^T$ is a coupling strength vector. The synchronization can be obtained with rather small coupling strengths $K_1 = K_2 = K_3 = 1$, as shown in Fig. 5.

3. The lag synchronization of BLDCM systems

3.1. Pecora and Corroll method for lag synchronization

The master system and slave systems are described by Eqs. (1) and (2). Firstly, variable $x_1(t - \tau)$ where $\tau = 1s$ in Eq. (1) is chosen to replace variable y_1 in Eq. (2), then new slave system is

$$\begin{aligned}
 \dot{y}_1 &= v_q - x_1(t - \tau) - y_2 y_3 + \rho y_3, \\
 \dot{y}_2 &= v_d - \delta y_2 + x_1(t - \tau) y_3, \\
 \dot{y}_3 &= \sigma(x_1(t - \tau) - y_3) + \eta x_1(t - \tau) y_2 - T_L.
 \end{aligned}
 \tag{5}$$

The lag synchronization can be obtained by simulations which are shown in Figs. 6–8. By simulation results, it is found that the range of time delay τ is unlimited.

Variable $x_3(t - \tau)$ where $\tau = 1s$ in Eq. (1) is used to replace variable y_3 in Eq. (2), the simulation results are in Figs. 9–11. The lag phenomenon is quite clear in Fig. 11. It is found that the range of time delay τ is unlimited also.

Variables $x_1(t - \tau), x_2(t - \tau)$ where $\tau = 1s$ in Eq. (1) are used to replace variables y_1, y_2 respectively in Eq. (2). The simulation results show in Figs. 12 and 13. It is found that range of delay time τ is unlimited also.

Variables $x_1(t - \tau), x_3(t - \tau)$ where $\tau = 1s$ in Eq. (1) are used to replace variables y_1, y_3 respectively in Eq. (2). The simulation results are shown in Figs. 14 and 15.

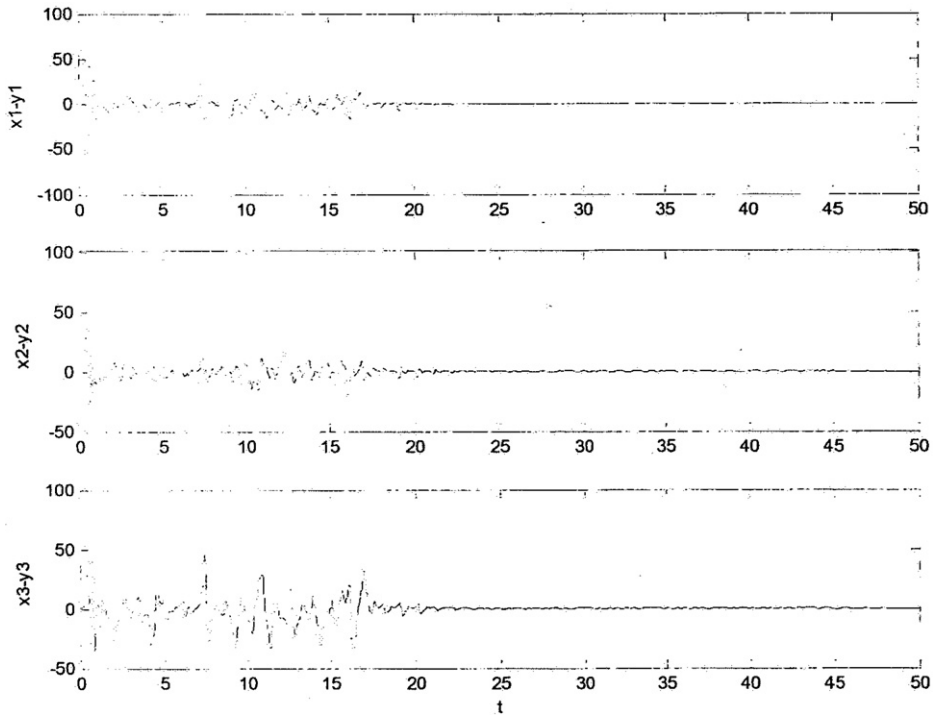


Fig. 5. The complete synchronization of two BLDCM systems by linear coupling.

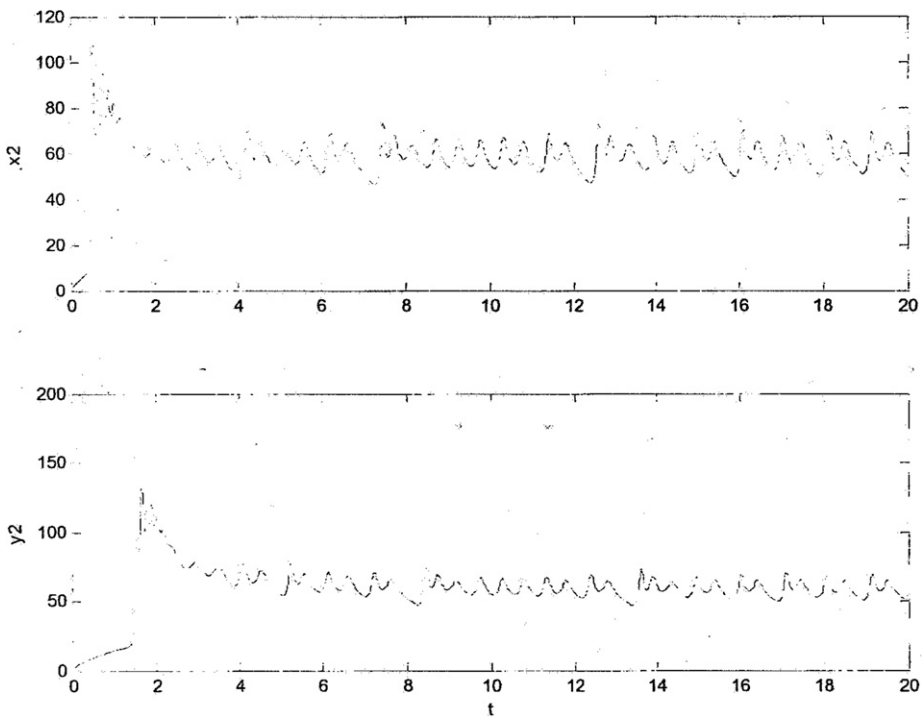


Fig. 6. The time histories of x_2 in Eq. (1) and y_2 in Eq. (5).

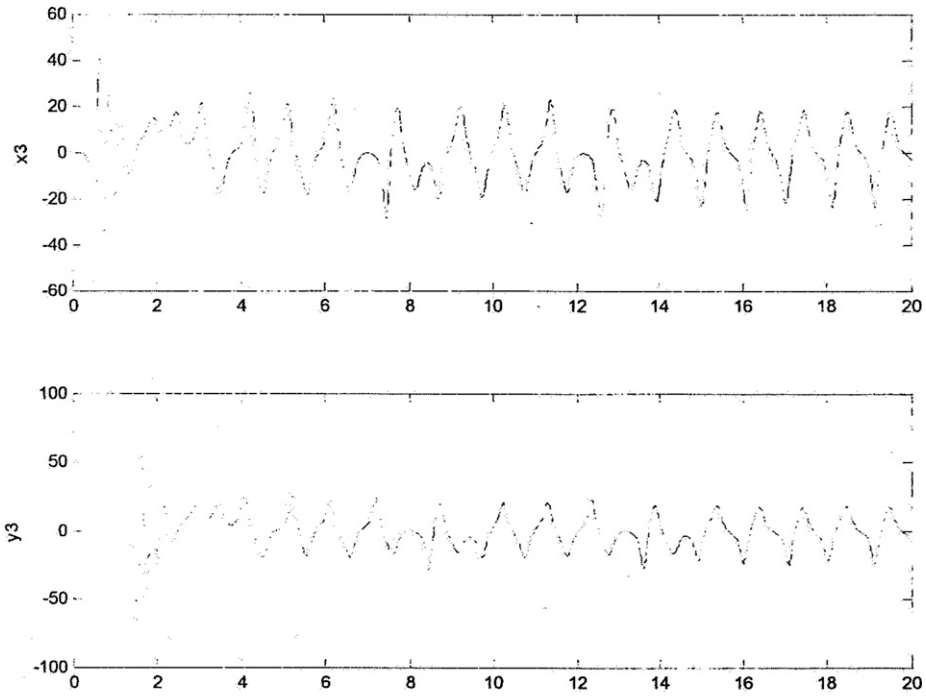


Fig. 7. The time histories of x_3 in Eq. (1) and y_3 in Eq. (5).

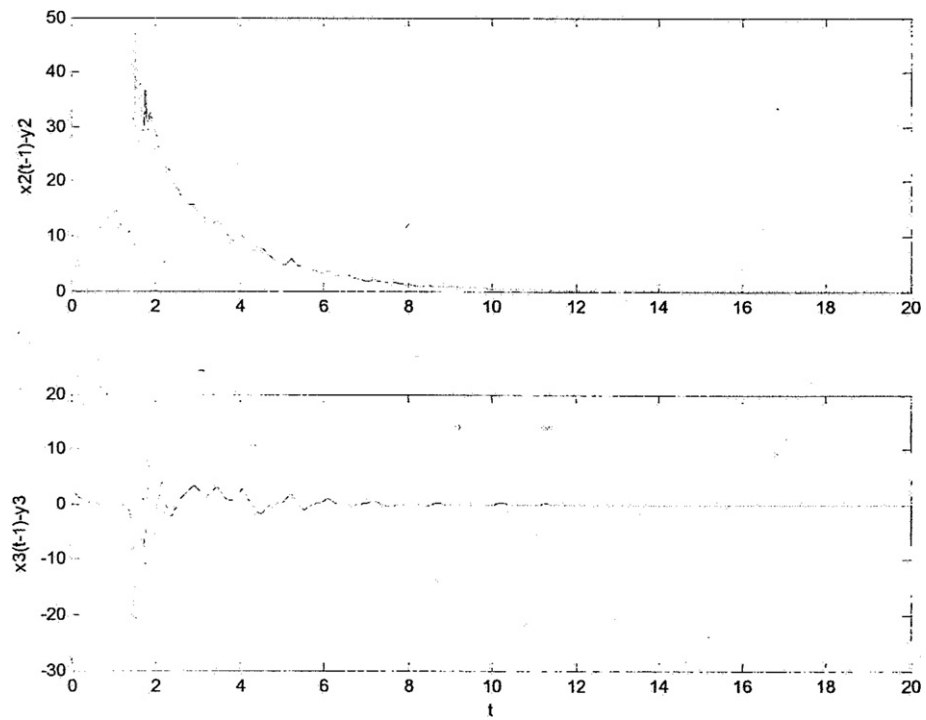


Fig. 8. The lag synchronization of two BLDCM systems of Eq. (1) and of Eq. (5).

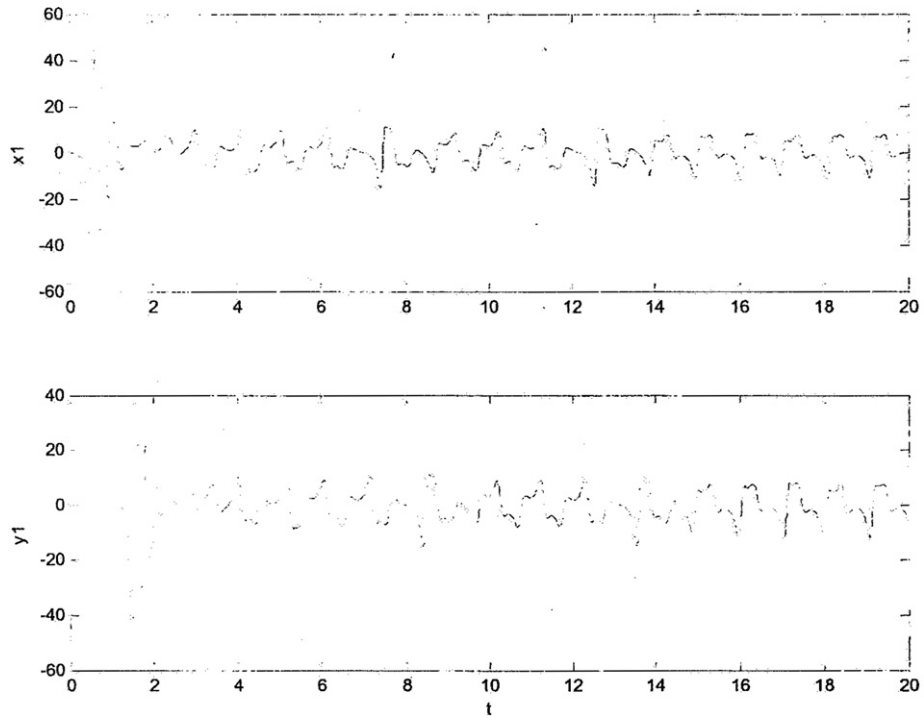


Fig. 9. The time histories of x_1 and y_1 when $y_3(t)$ in Eq. (2) is replaced by $x_3(t - 1)$ in Eq. (1).

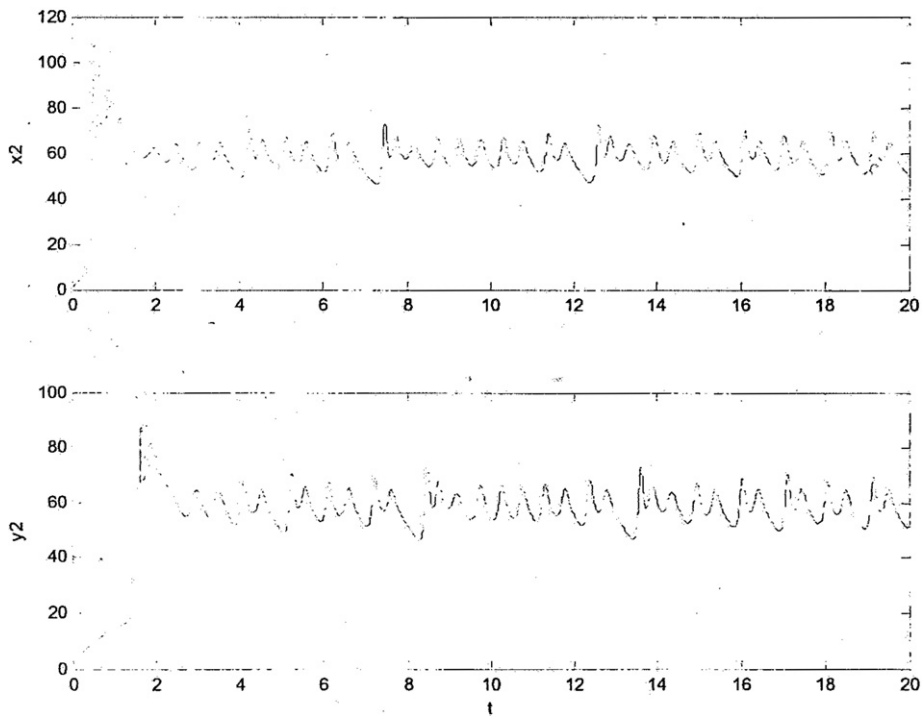


Fig. 10. The time histories of x_2 and y_2 when $y_3(t)$ in Eq. (2) is replaced by $x_3(t - 1)$ in Eq. (1).

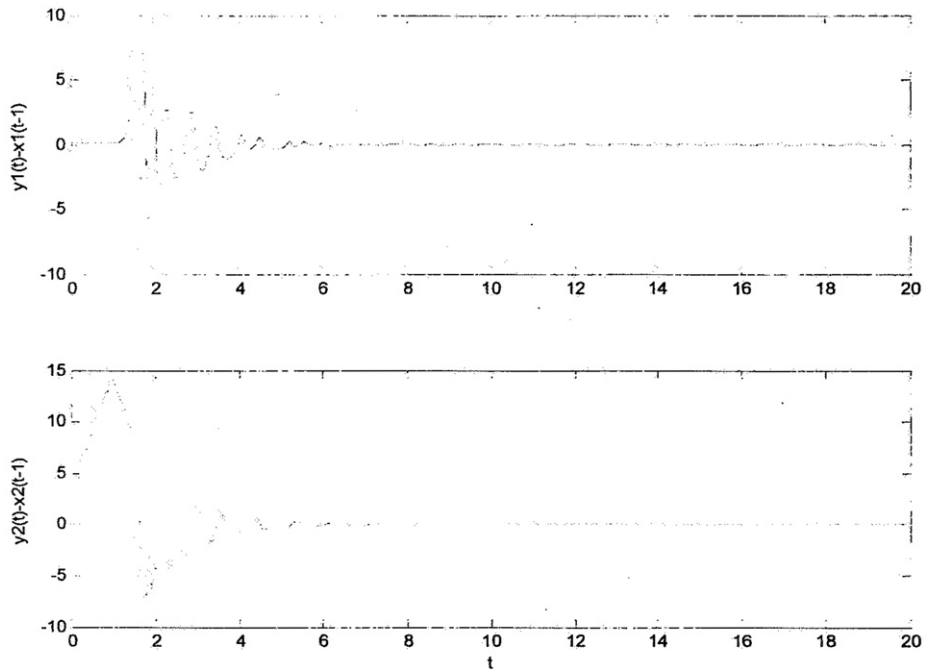


Fig. 11. The lag synchronization of two BLDCM systems when $y_3(t)$ in Eq. (2) is replaced by $x_3(t - 1)$ in Eq. (1).

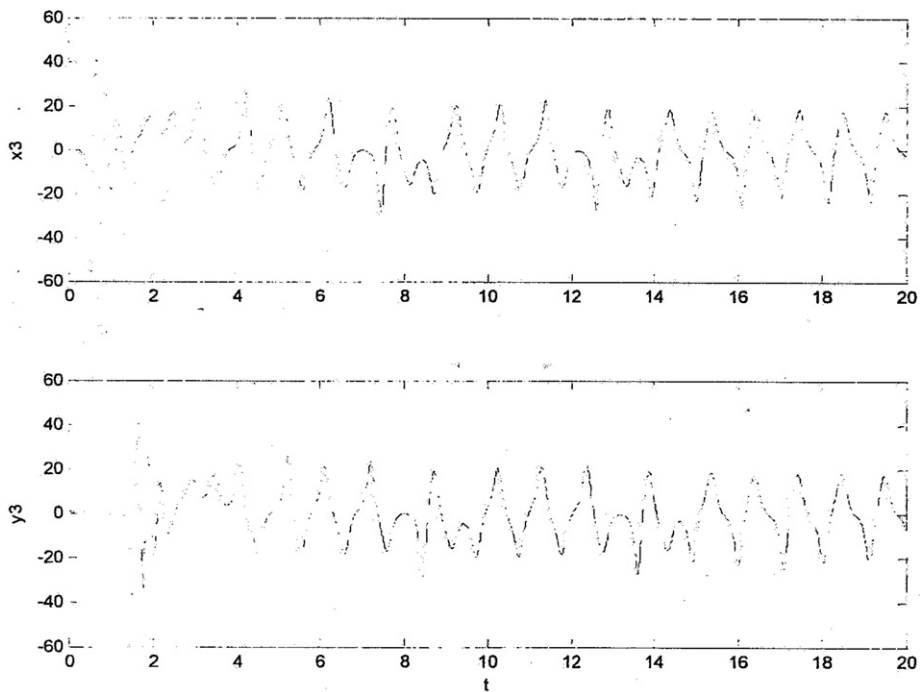


Fig. 12. The time histories of x_3 and y_3 when $y_1(t)$, $y_2(t)$ in Eq. (2) are replaced by $x_1(t - 1)$, $x_2(t - 1)$ in Eq. (1) respectively.

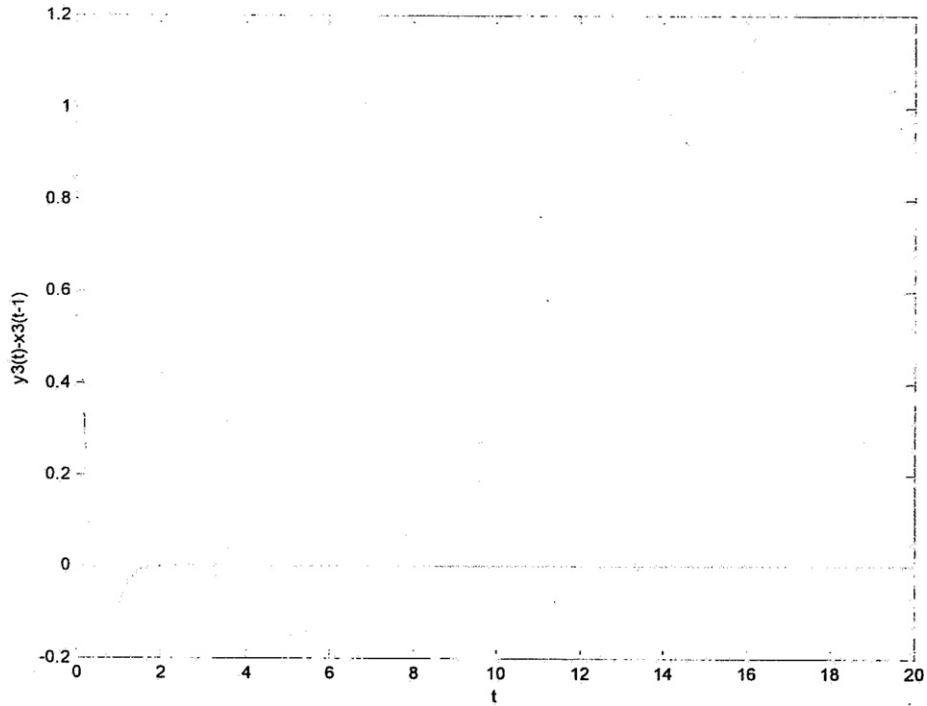


Fig. 13. The lag synchronization of two BLDCM systems when $y_1(t)$, $y_2(t)$ in Eq. (2) are replaced by $x_1(t-1)$, $x_2(t-1)$ in Eq. (1) respectively.

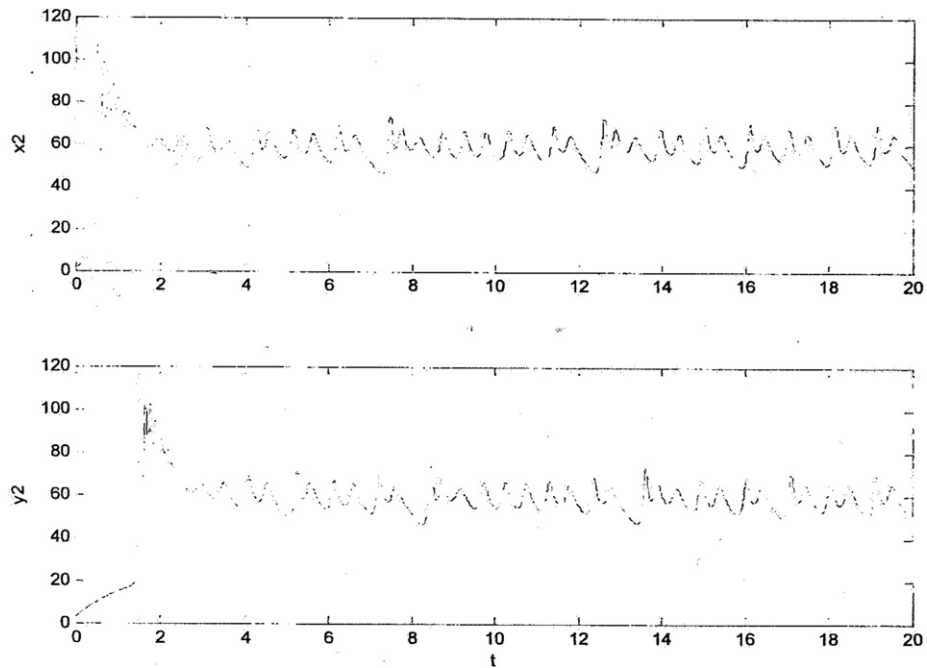


Fig. 14. The time histories of x_2 and y_2 when $y_1(t)$, $y_3(t)$ in Eq. (2) are replaced by $x_1(t-1)$, $x_3(t-1)$ in Eq. (1) respectively.

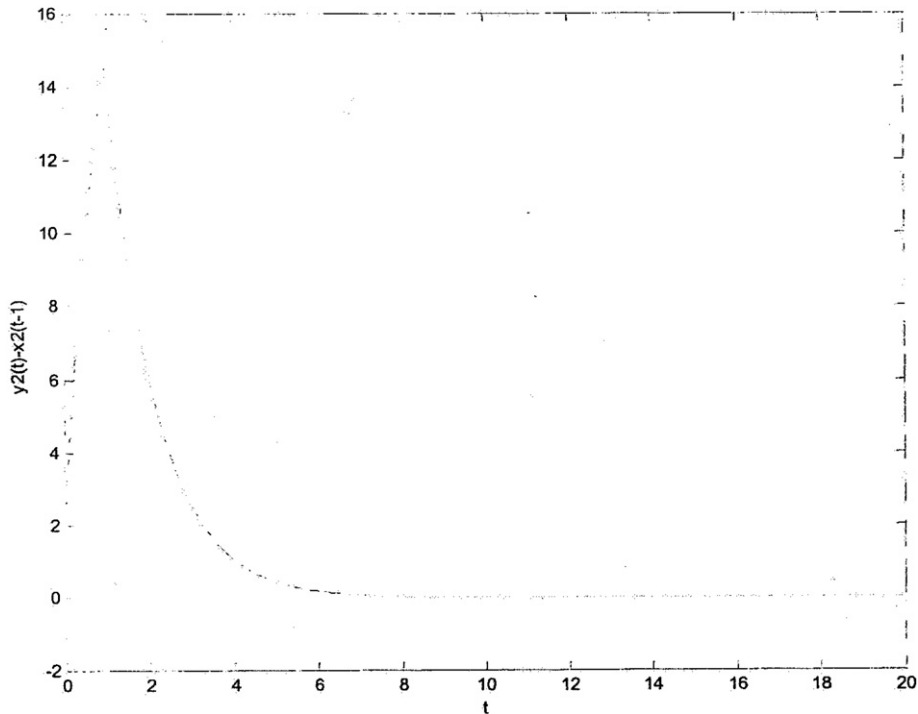


Fig. 15. The lag synchronization of two BLDCM systems when $y_1(t)$, $y_3(t)$ in Eq. (2) are replaced by $x_1(t-1)$, $x_3(t-1)$ in Eq. (1) respectively.

Lastly, it is found that the time required for synchronizations by the replacement of two variables are less than that by the replacement of one variables. And the x_1 , x_2 case is faster than the x_1 , x_3 case.

3.2. Lag synchronization by linear feedback

The coupling scheme for the dynamics of the master and the slave is written as

$$\begin{aligned}\dot{x}(t) &= f(x(t)), \\ \dot{y}(t) &= f(y(t)) + K[x(t-\tau) - y(t)],\end{aligned}\quad (6)$$

where x , y are state vectors, $f(x(t))$ is an arbitrary vector function, K is a coupling strength matrix and τ is the time delay. Using the above scheme, simulations are given. Results are shown in Figs. 16–19. From simulation results, when the range of time delay τ is between 1 and 10, good performances are obtained.

4. The anticipated synchronization of BLDCM systems

The synchronization of chaotic systems in a unidirectional coupling configuration has attracted great interest due to its potential applications to secure communication systems. Particular attention has been paid to the so-called *anticipating synchronization* regime, where two identical chaotic systems can be synchronized by unidirectional delayed coupling in such a manner that the “slave” (the system with coupling) anticipates the “master” (the one without coupling). More specifically, the coupling scheme proposed in [29–33] for the dynamics of the master $x(t)$ and slave $y(t)$ is

$$\begin{aligned}\dot{x}(t) &= f(x(t)), \\ \dot{y}(t) &= f(y(t)) + K[x(t) - y(t-\tau)],\end{aligned}\quad (7)$$

where x and y are state vectors, f is a vector function, τ is a delay time, K is a coupling strength matrix.

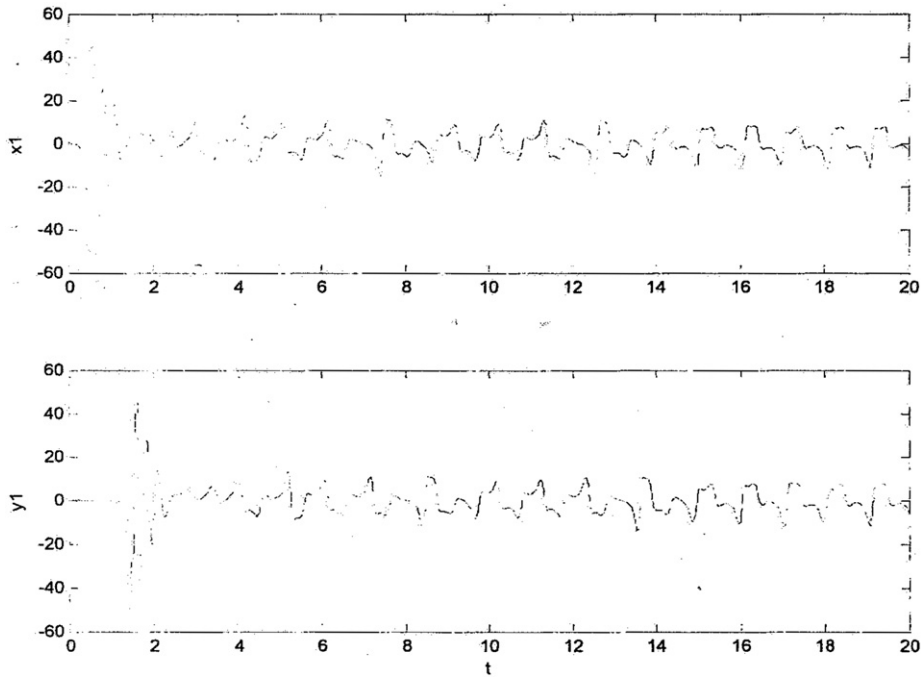


Fig. 16. The time histories of x_1 and y_1 in Eq. (6), $\tau = 1$ s.

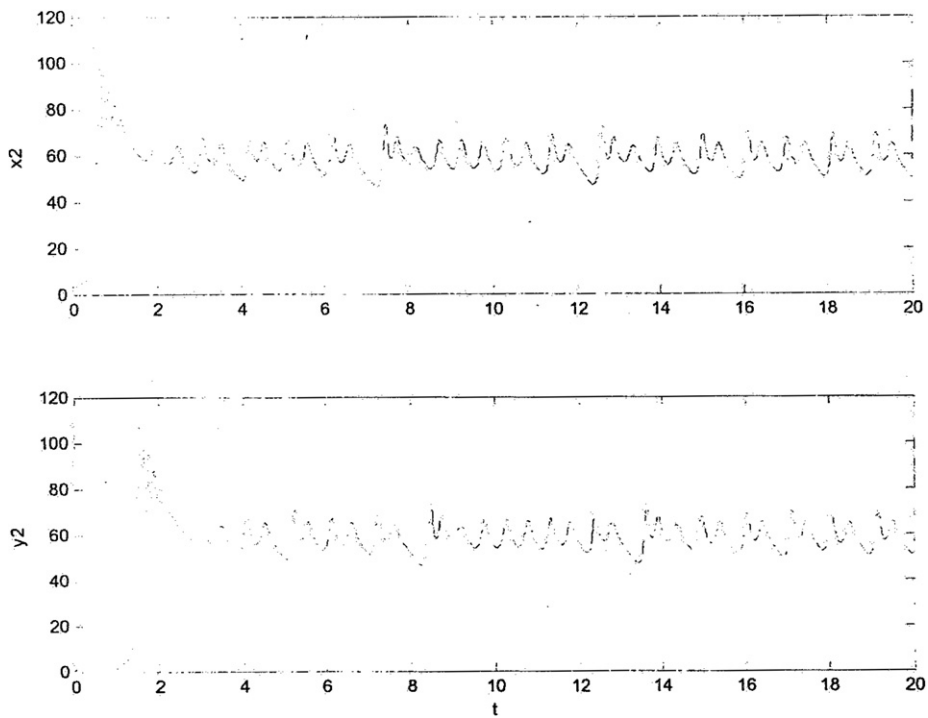


Fig. 17. The time histories of x_2 and y_2 in Eq. (6), $\tau = 1$ s.

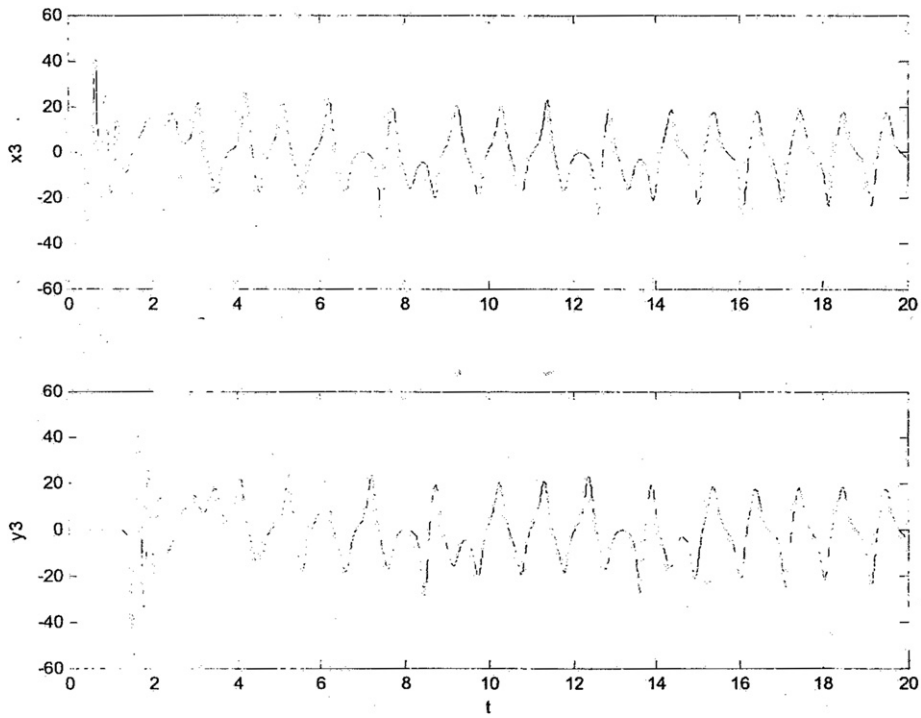


Fig. 18. The time histories of x_3 and y_3 in Eq. (6), $\tau = 1s$.

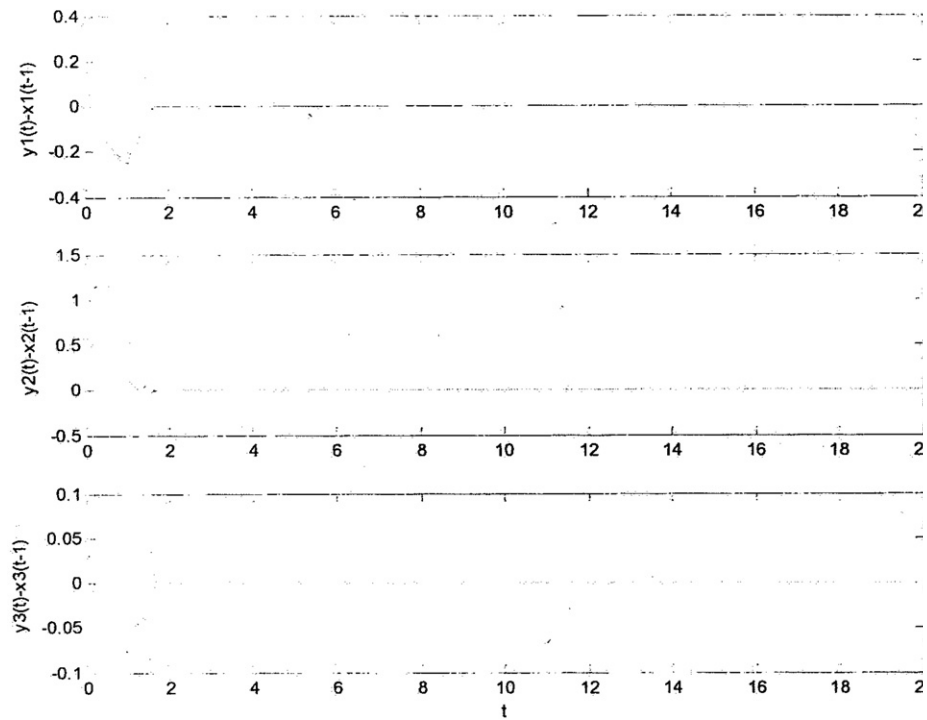


Fig. 19. The lag synchronization of two BLDCM systems in Eq. (6), $\tau = 1s$.

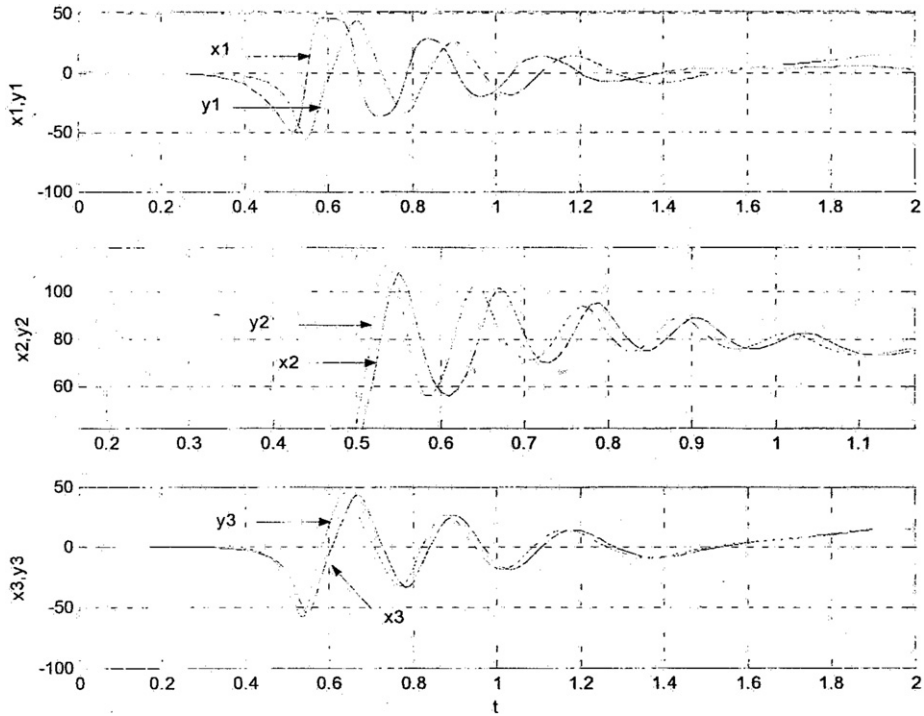


Fig. 20. The time histories of all states, $K_1 = K_2 = K_3 = 145$, and $\tau = 0.02$.

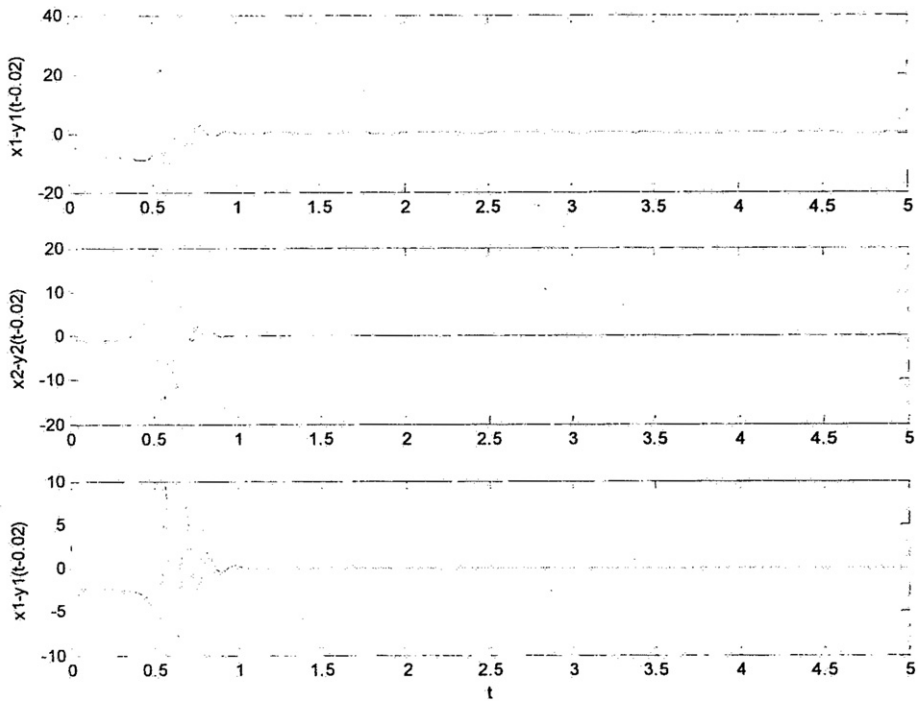


Fig. 21. Anticipated synchronization of two BLDCM systems, $K_1 = K_2 = K_3 = 145$, and $\tau = 0.02$.

For BLDCM system, the master is described in Eq. (1) and slave is

$$\begin{aligned} \dot{y}_1 &= v_q - y_1 - y_2 y_3 + \rho y_3 + K_1(x_1 - y_1(t - \tau)), \\ \dot{y}_2 &= v_d - \delta y_2 + y_1 y_3 + K_2(x_2 - y_2(t - \tau)), \\ \dot{y}_3 &= \sigma(y_1 - y_3) + \eta y_1 y_2 - T_L + K_3(x_3 - y_3(t - \tau)). \end{aligned} \tag{8}$$

For appropriate values of the delay time τ and coupling strength K , the basic results can be obtained such that $y(t) = x(t + \tau)$, i.e., the slave “anticipates” by an amount τ the output of the master while the value of τ is limited. The simulation results are shown in Figs. 20 and 21 where $K_1 = K_2 = K_3 = 145$, $\tau = 0.02$.

5. Generalized lag, anticipated, and complete synchronizations of BLDCM chaos system by active control

In this section, active control [27] is used to the generalized lag, anticipated, and complete synchronizations. When generalized synchronization is accomplished, the response state vector y is a given function of the drive state vector x . We use a type of generalized (lag, anticipated, and complete) synchronization which is defined as the presence of certain relationship between the states of the drive and response systems, i.e., there exists a smooth vector function H such that $y(t) = H(x(t - \tau))$ with $\tau \in R$, which includes the generalized lag synchronization (GLS, $y(t) = H(x(t - \tau))$ with $\tau \in R^+$), the generalized anticipated synchronization (GAS, $y(t) = H(x(t - \tau))$ with $\tau \in R^-$), and generalized complete synchronization GS($y(t) = H(x(t))$ with $\tau = 0$).

5.1. Linear vector function H

The drive and response system is as following:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + F(x), \quad A = \begin{bmatrix} -1 & 0 & \rho \\ 0 & -\delta & 0 \\ \sigma & 0 & -\sigma \end{bmatrix}, \quad F(x) = \begin{bmatrix} -x_2 x_3 + v_q \\ x_1 x_3 + v_d \\ \eta x_1 x_2 - T_L \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix} = B \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} + G(x) + U(x, y),$$

$$B = \begin{bmatrix} -1 & 0 & \rho \\ 0 & -\delta & 0 \\ \sigma & 0 & -\sigma \end{bmatrix}, \quad G(x) = \begin{bmatrix} -y_2 y_3 + v_q \\ y_1 y_3 + v_d \\ \eta y_1 y_2 - T_L \end{bmatrix}, \quad U(x, y) = \begin{bmatrix} U_1(x, y) \\ U_2(x, y) \\ U_3(x, y) \end{bmatrix}, \tag{10}$$

Let the error state vector $e(t) = y(t) - H(x(t - \tau))$, where $\tau \in R$ and $H(x(t - \tau)) = [H_1(x(t - \tau)), H_2(x(t - \tau)), \dots, H_n(x(t - \tau))]^T$ is a smooth vector function. We can obtain the error dynamic system and choose controller $U(x, y)$ as follows:

$$\begin{aligned} \dot{e}(t) &= Ae(t) + BH(x(t - \tau)) + G(y(t - \tau)), \\ &+ DH(x(t - \tau))[Ax(t - \tau) + F(x(t - \tau)) + U(x, y)], \end{aligned} \tag{11}$$

$$\begin{aligned} U &= \Delta e - BH(x(t - \tau)) - G(y(t - \tau)), \\ &- DH(x(t - \tau))[Ax(t - \tau) + F(x(t - \tau))], \end{aligned} \tag{12}$$

where Δ is a constant matrix and $DH(x(t - \tau))$ is the Jacobian matrix of $H(x(t - \tau))$. A linear vector function $H(x(t - \tau))$ is chosen as follows:

$$H(x_1, x_2, x_3) = \begin{pmatrix} h_{11} & 0 & 0 \\ 0 & h_{22} & 0 \\ 0 & 0 & h_{33} \end{pmatrix} \begin{pmatrix} x_1(t - \tau) \\ x_2(t - \tau) \\ x_3(t - \tau) \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}, \tag{13}$$

where $h_{11}, h_{22}, h_{33}, c_1, c_2, c_3$ are constants. Then the error dynamic system (11) becomes

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = (B + \Delta) \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \Delta_{11} - 1 & \Delta_{12} & \Delta_{13} + \rho \\ \Delta_{21} & \Delta_{22} - \delta & \Delta_{23} \\ \Delta_{31} - \sigma & \Delta_{32} & \Delta_{33} + \sigma \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}. \tag{14}$$

Proper Δ_{ij} can be obtained such that all eigenvalues of system (14) have negative real parts, i.e., the null solution of the system (14) is globally asymptotically stable. The parameters in system (9) and system (10) are chosen as $v_q = 0.168$, $v_d = 20.66$, $T_L = 0.53$, $\sigma = 4.55$, $\rho = 60$, $\eta = 0.26$. Let $\Delta_{11} = -3$, $\Delta_{12} = -2$, $\Delta_{13} = 3$, $\Delta_{21} = 0$, $\Delta_{22} = -30$, $\Delta_{23} = 2$, $\Delta_{31} = -4.55$, $\Delta_{32} = 0$, $\Delta_{33} = -30$. The initial values of the states of system (9) and system (10) are $x_1 = 0.01$, $x_2 = 0.01$, $x_3 = 0.01$, $y_1 = 0.1$, $y_2 = -5$, $y_3 = -10$. The time constants $\tau = 1$ and $\tau = -1.5$ are used in the lag synchronization and the anticipated synchronization respectively. Finally, the dynamics of generalized lag, anticipated and complete synchronization errors for the drive system (9) and the response system (10) are shown in Figs. 22–25.

5.2. Nonlinear vector function

The drive system and response system are also Eqs. (9) and (10). In this section, we choose nonlinear vector function $H(x(t - \tau))$

$$H(x_1, x_2, x_3) = \begin{pmatrix} 2x_1(t - \tau) & 0 & 0 \\ 0 & 2x_2(t - \tau) & 0 \\ 0 & 0 & 2x_3(t - \tau) \end{pmatrix} \begin{pmatrix} x_1(t - \tau) \\ x_2(t - \tau) \\ x_3(t - \tau) \end{pmatrix}. \tag{15}$$

We can also obtain the error dynamic system (14) from (9) and (10). Choose proper Δ_{ij} such that all eigenvalues of system (14) have negative real parts, i.e., the null solution of system (14) is globally asymptotically stable. Take the parameters in system (9) and (10) as $v_q = 0.168$, $v_d = 20.66$, $T_L = 0.53$, $\sigma = 4.55$, $\rho = 60$, $\eta = 0.26$. Let $\Delta_{11} = -30$, $\Delta_{12} = 2$, $\Delta_{13} = 3$, $\Delta_{21} = 0$, $\Delta_{22} = -30$, $\Delta_{23} = 2$, $\Delta_{31} = -4.55$, $\Delta_{32} = 0$, $\Delta_{33} = -33$ and the initial values of the states of system (9) and (10) as $x_1 = 10$, $x_2 = 5$, $x_3 = 7$, $y_1 = 21$, $y_2 = 30$, $y_3 = 15$. The time constant $\tau = 0.5$ and $\tau = -1$ are used in lag and anticipated synchronization respectively. Finally the dynamics of the generalized complete lag and anticipated synchronization errors for the drive system (9) and the response system (10) are shown in Figs. 26–31.

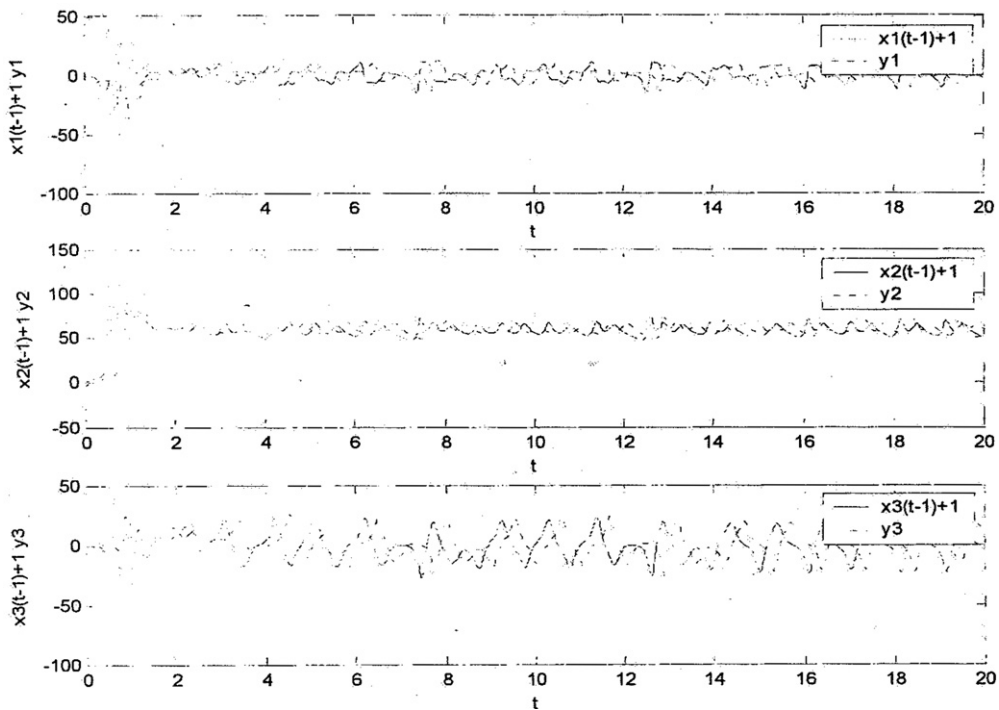


Fig. 22. The time histories of y_1, y_2, y_3 and $x_1(\tau - 1) + 1, x_2(\tau - 1) + 1, x_3(\tau - 1) + 1$ for linear vector function H .

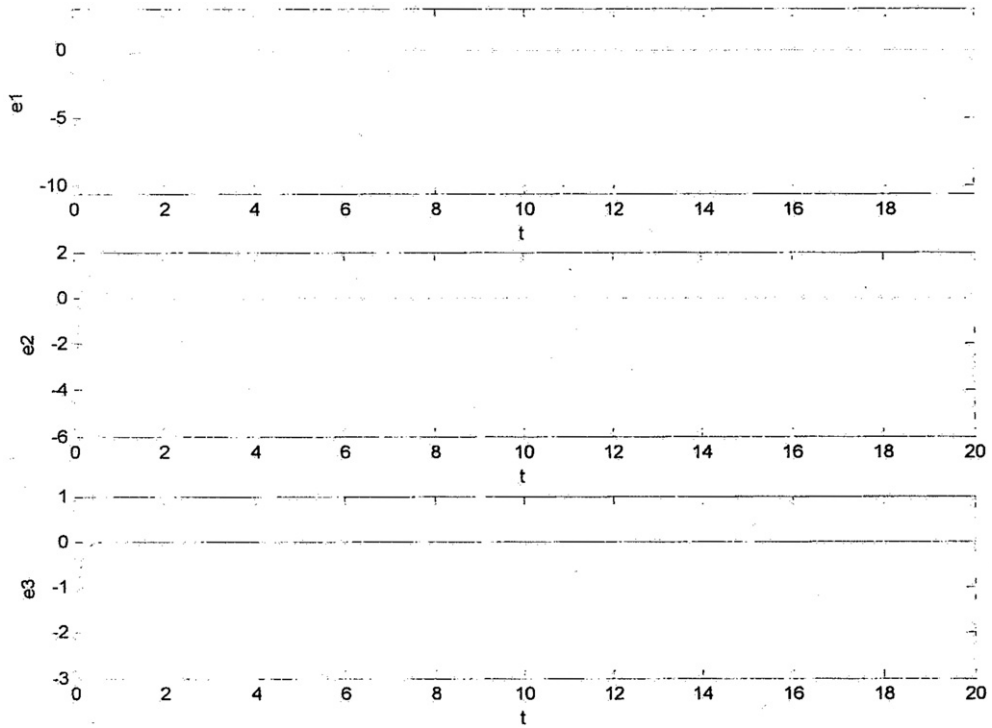


Fig. 23. The time histories of the generalized lag synchronization error $e = y(t) - (x(t - 1) + 1)$ for linear vector function H .

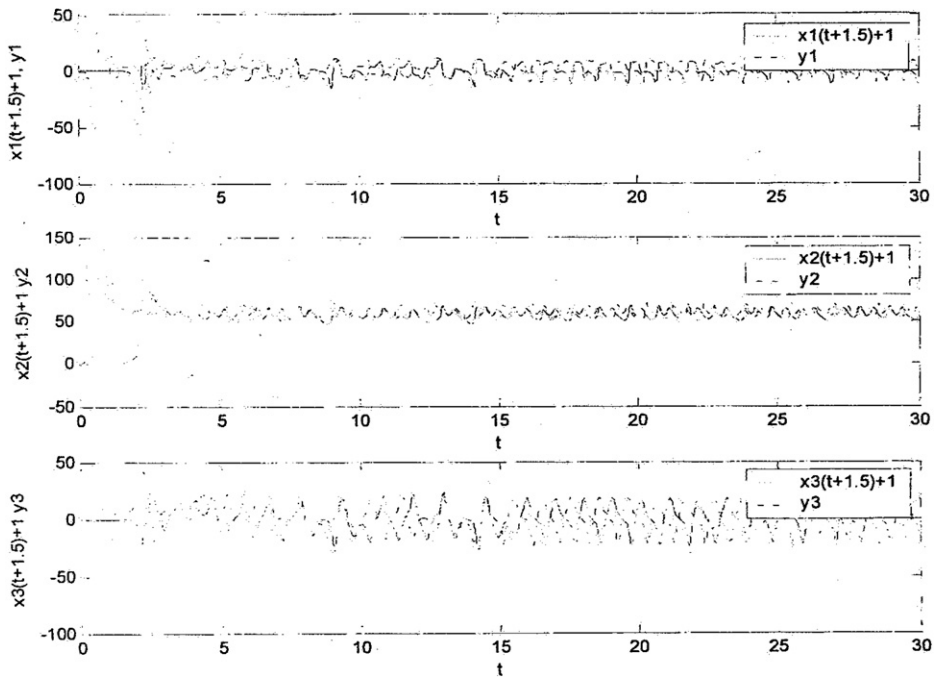


Fig. 24. The time histories of y_1, y_2, y_3 and $x_1(\tau + 1.5) + 1, x_2(\tau + 1.5) + 1, x_3(\tau + 1.5) + 1$ for linear vector function H .

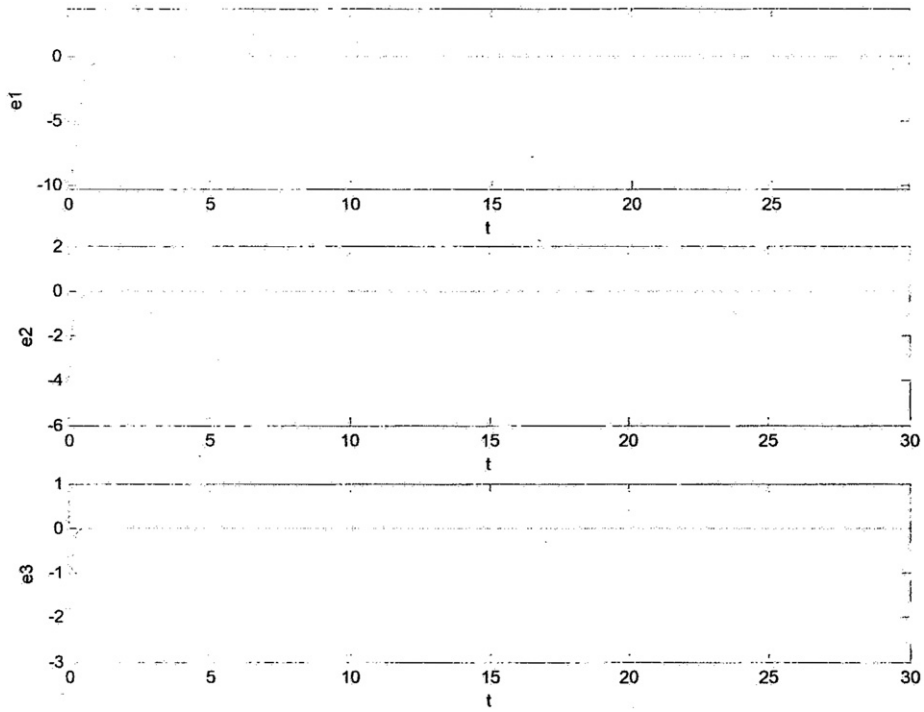


Fig. 25. The time histories of the generalized anticipated synchronization error $e = y(t) - (x(t + 1.5) + 1)$ for linear vector function H .

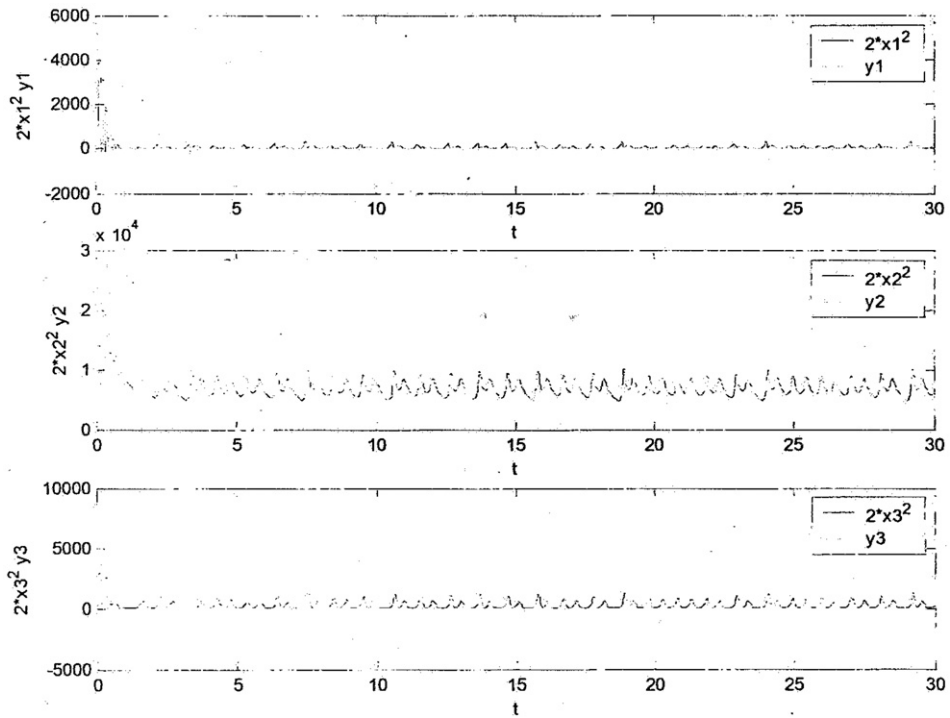


Fig. 26. The time histories of y_1, y_2, y_3 and $2x_1^2, 2x_2^2, 2x_3^2$ for nonlinear vector function H .

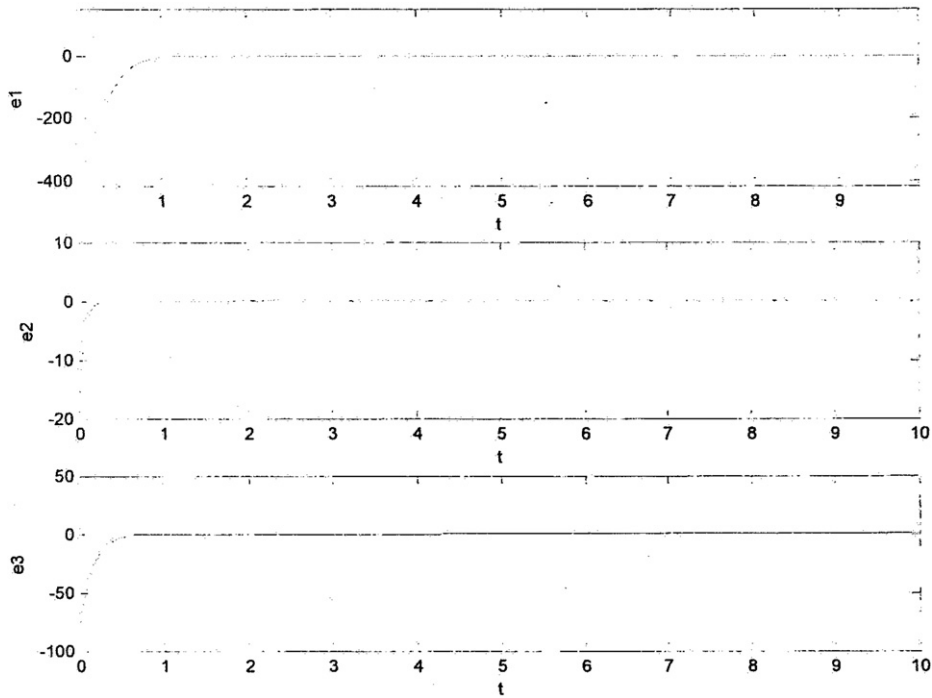


Fig. 27. The generalized complete synchronization error $e = y(t) - (2x^2)$ for nonlinear vector function H .

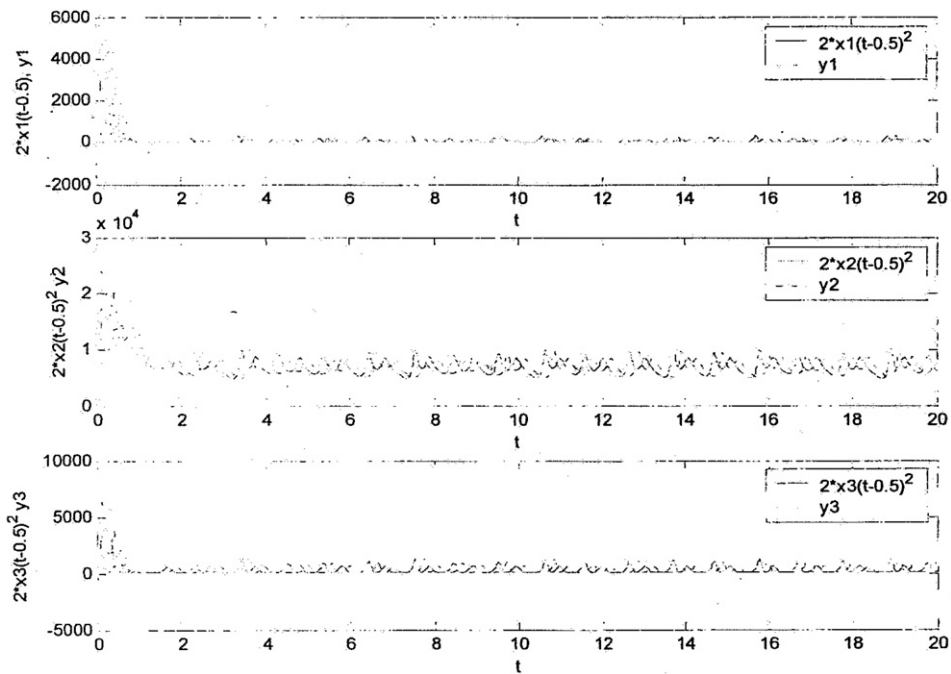


Fig. 28. The time histories of y_1, y_2, y_3 and $2x_1^2(t - 0.5), 2x_2^2(t - 0.5), 2x_3^2(t - 0.5)$ for nonlinear vector function H .

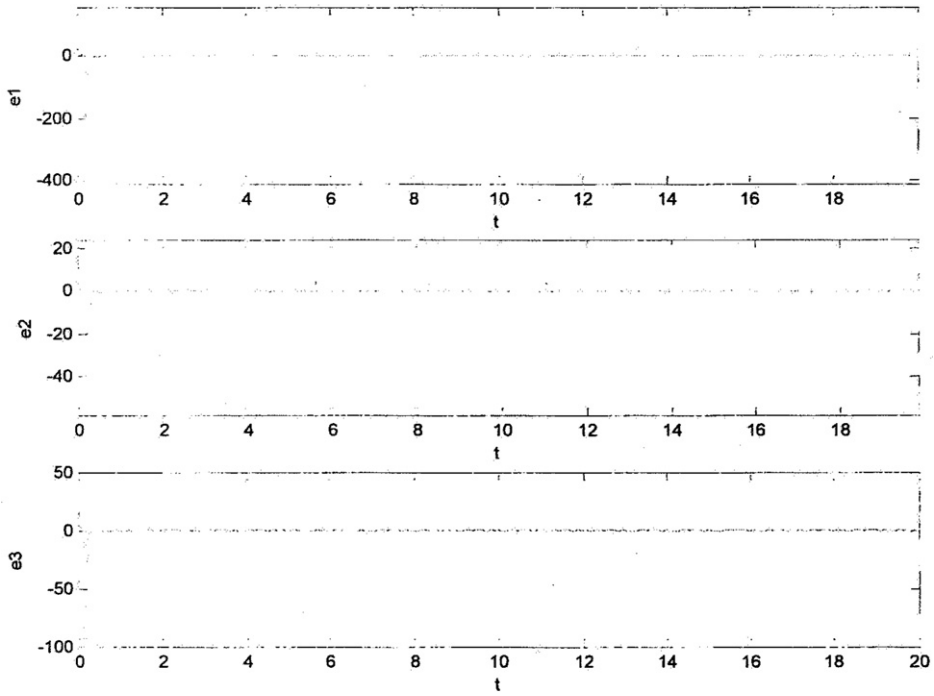


Fig. 29. The generalized lag synchronization error $e = y(t) - (2x^2(t - 0.5))$ for nonlinear vector function H .

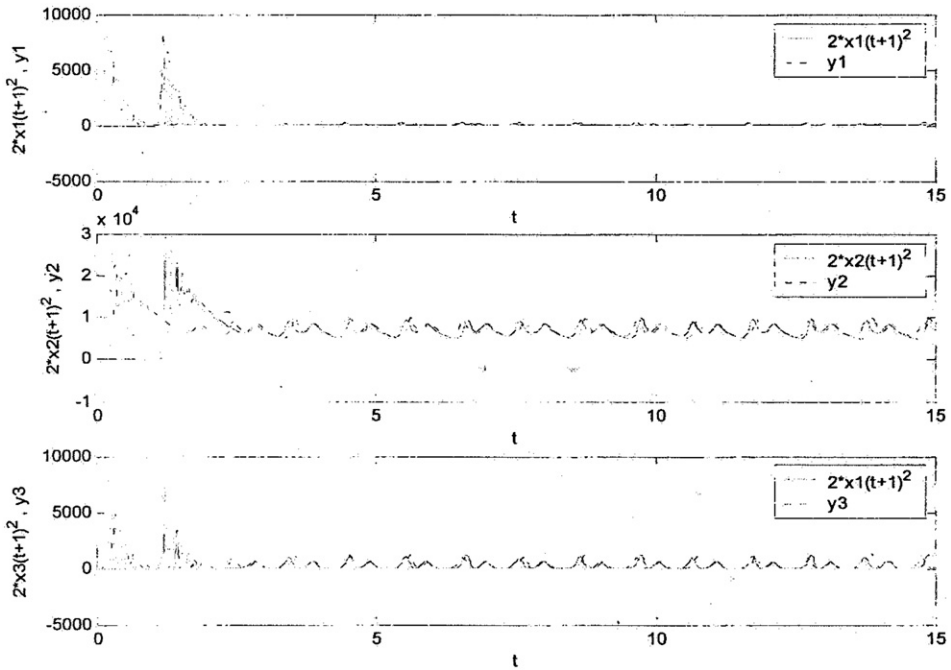


Fig. 30. The time histories of y_1, y_2, y_3 and $2x_1^2(t + 1), 2x_2^2(t + 1), 2x_3^2(t + 1)$ for nonlinear vector function H .

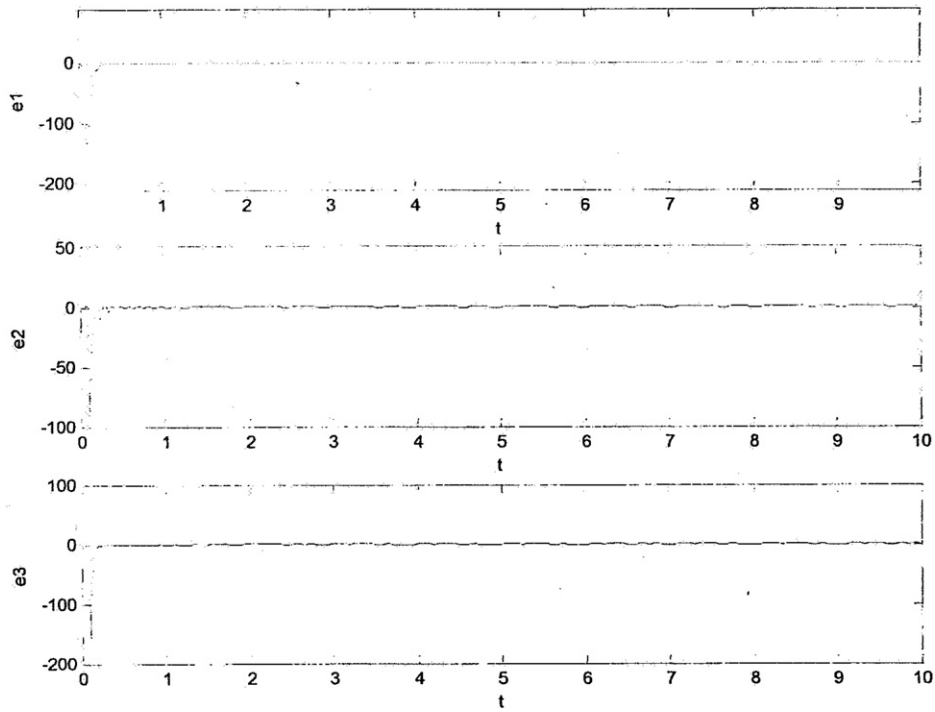


Fig. 31. The generalized anticipated synchronization error $e = y(t) - (2x^2(t + 1))$ for nonlinear vector function H .

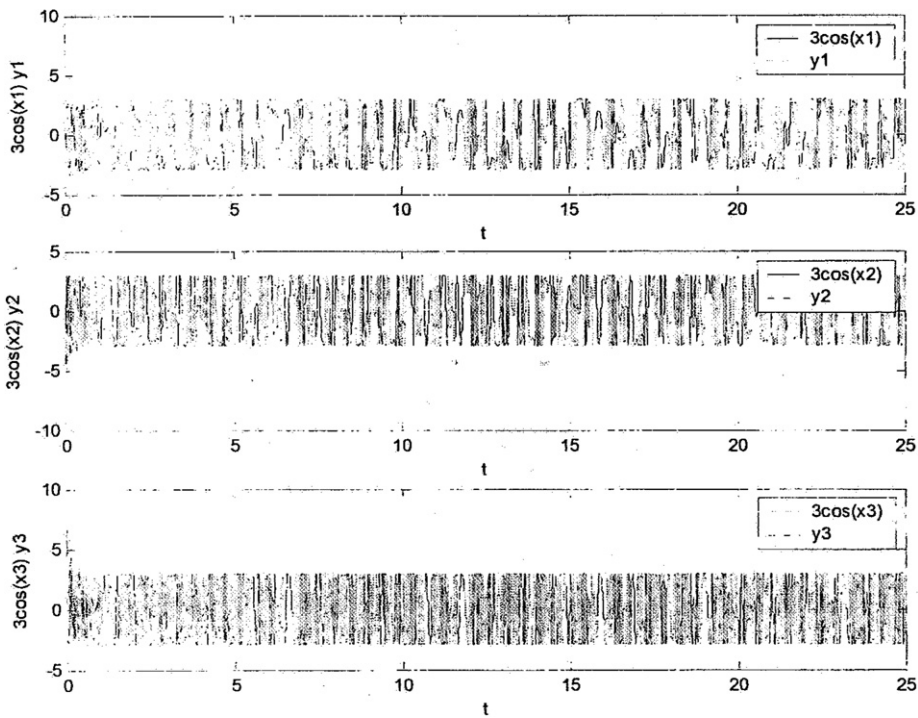


Fig. 32. The time histories of y_1, y_2, y_3 for Lorenz system and of $3 \cos(x_1), 3 \cos(x_2), 3 \cos(x_3)$ for BLDCM system, with nonlinear vector function H .

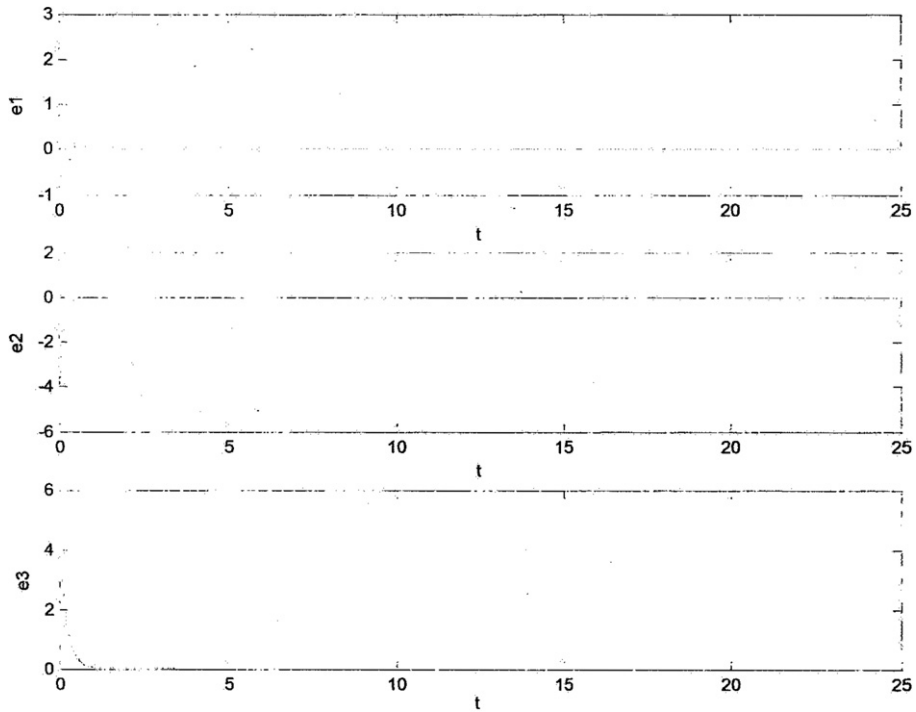


Fig. 33. The generalized complete synchronization error $e = y(t) - 3 \cos(x)$ for different systems for nonlinear vector function H .

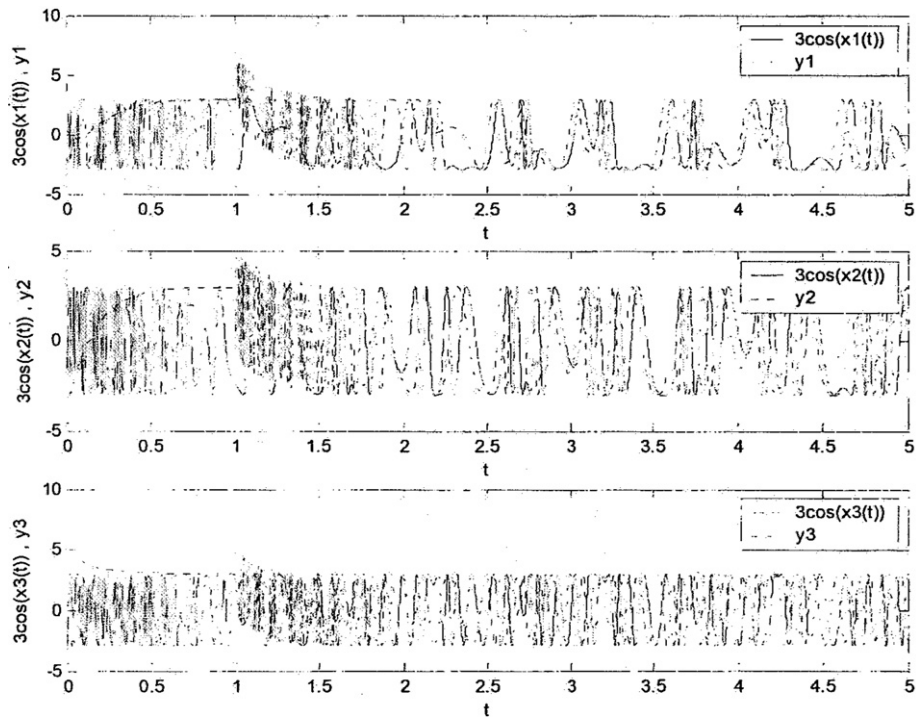


Fig. 34. The time histories of y_1, y_2, y_3 for Lorenz system and of $3 \cos(x_1(t-1)), 3 \cos(x_2(t-1)), 3 \cos(x_3(t-1))$ for BLDCM system, with nonlinear vector function H .

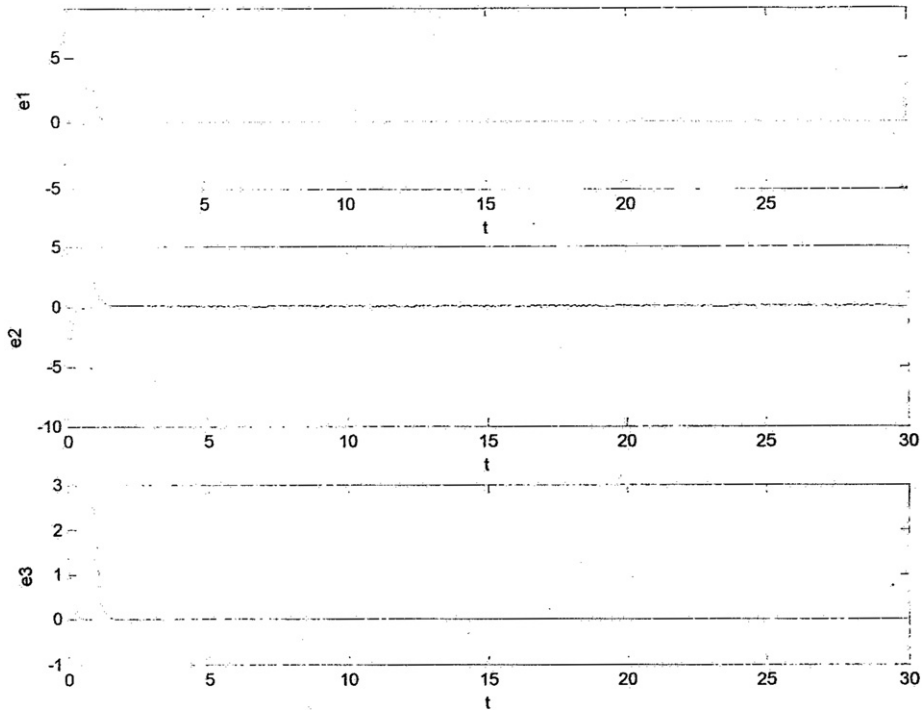


Fig. 35. The generalized lag synchronization error $e = y(t) - 3 \cos(x(t - 1))$ for different systems with nonlinear vector function H .

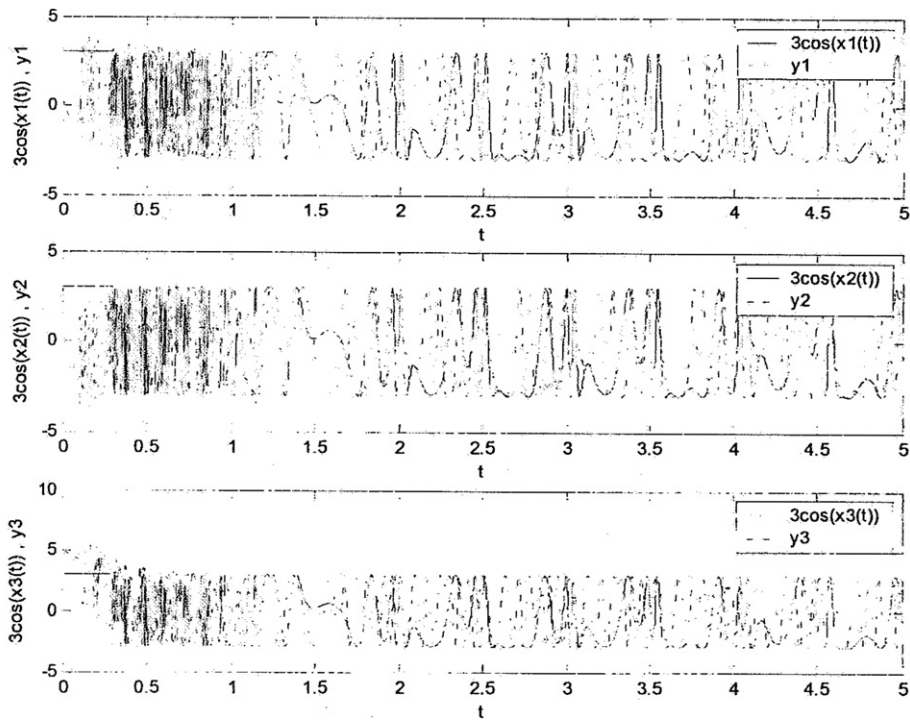


Fig. 36. The time histories of y_1, y_2, y_3 for Lorenz system and of $3 \cos(x_1(t + 0.2)), 3 \cos(x_2(t + 0.2)), 3 \cos(x_3(t + 0.2))$ for BLDCM system, with nonlinear vector function H .

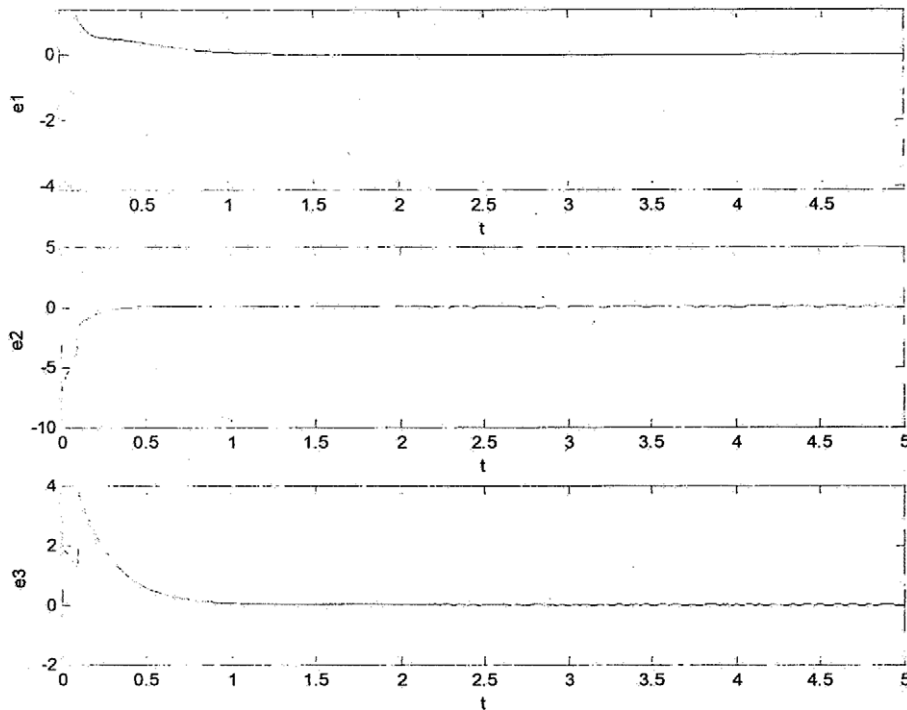


Fig. 37. The generalized anticipated synchronization error $e = y(t) - 3 \cos(x(t+0.2))$ for different systems with nonlinear vector function H .

5.3. The generalized synchronization of BLDCM and Lorenz chaotic system for nonlinear vector function

In this section, we use nonlinear vector function $H(x(t-\tau))$ and different chaotic systems for generalized synchronization. The drive system is Eq. (9) and the response system is Lorenz system :

$$\begin{cases} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{cases} = B \begin{cases} y_1 \\ y_2 \\ y_3 \end{cases} + G(y) + U, \quad B = \begin{bmatrix} -a & a & 0 \\ c & 0 & -1 \\ 0 & 0 & -b \end{bmatrix}, \quad G(y) = \begin{bmatrix} 0 \\ -y_1 y_3 \\ y_1 y_2 \end{bmatrix}, \quad (16)$$

$$H(x_1, x_2, x_3) = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} \cos(x_1(t-\tau)) \\ \cos(x_2(t-\tau)) \\ \cos(x_3(t-\tau)) \end{pmatrix}. \quad (17)$$

We can also obtain the error dynamic system (14) from (9) and (16). Choose proper Δ_{ij} such that all eigenvalues of system (14) have negative real parts, i.e., the null solution of system (14) is globally asymptotically stable. Take the parameters in system (9) and (16) as $v_q = 0.168$, $v_d = 20.66$, $T_L = 0.53$, $\sigma = 4.55$, $\rho = 60$, $\eta = 0.26$, $a = 10$, $b = 8/3$, and $c = 28$. Let $\Delta_{11} = -5$, $\Delta_{12} = 2$, $\Delta_{13} = 5$, $\Delta_{21} = -28$, $\Delta_{22} = -5$, $\Delta_{23} = 1$, $\Delta_{31} = 0$, $\Delta_{32} = 0$, $\Delta_{33} = -5$, and the initial values of the states of system (9) and (16) as $x_1 = -15$, $x_2 = 5$, $x_3 = 30$, $y_1 = -5$, $y_2 = -4$, $y_3 = 5$. The time constant $\tau = 1$ and $\tau = -0.2$ are used in lag and anticipated synchronization respectively. Finally, the dynamics of generalized lag, anticipated and complete synchronization errors for the drive system (9) and the response system (16) are shown in Figs. 32–37.

6. Conclusions

By Pecora and Corroll method, complete synchronization and lag synchronization are accomplished successfully. The larger the number of states of the response system replaced by states of the drive system is, the quicker the chaos synchronization can be accomplished. Linear coupling for complete, lag and anticipated synchronization is achieved, and time delay τ can be arbitrarily chosen for lag synchronization, and the range of negative τ has limitation for antic-

ipated synchronization. The generalized complete, lag and anticipated synchronization are achieved by active control. Finally generalized lag synchronization are studied for different systems, BLDCM system and Lorenz system, by active control.

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References

- [1] Chien T-I, Liao T-L. Design of secure digital communication systems using chaotic modulation, cryptography and chaotic synchronization. *Chaos, Solitons & Fractals* 2005;24:241–55.
- [2] Li Z, Xu D. A secure communication scheme using projective chaos synchronization. *Chaos, Solitons & Fractals* 2004;22:477–81.
- [3] Terry John R, VanWiggeren GD. Chaotic communication using generalized synchronization. *Chaos, Solitons & Fractals* 2001;12:145–52.
- [4] Xu Wei Guo, Li Qian Shu. Chemical chaotic schemes derived from NSG system. *Chaos, Solitons & Fractals* 2003;15:663–71.
- [5] Petrovskii Sergei, Li B-L, Horst Malchow. Quantification of the spatial aspect of chaotic dynamics in biological and chemical systems. *Bull Math Biol* 2003;65:425–46.
- [6] Yang SS, Duan CK. Generalized synchronization in chaotic systems. *Chaos, Solitons & Fractals* 1998;9:1703–7.
- [7] Ge Z-M, Lee C-I. Control, anticontrol and synchronization of chaos for an autonomous rotational machine system with time-delay. *Chaos, Solitons & Fractals* 2005;18:55–64.
- [8] Ge Z-M, Chen Y-S. Adaptive synchronization of unidirectional and mutual coupled chaotic systems. *Chaos, Solitons & Fractals* 2005;26:881–8.
- [9] Jiang G-P, Wallace K-S, Chen G. A simple global synchronization criterion for coupled chaotic systems. *Chaos, Solitons & Fractals* 2003;15:925–35.
- [10] Z.-M. Ge, C.-H. Yang. Generalized synchronization of quantum-CNN chaotic oscillator with different order systems, in press.
- [11] Ge Z-M, Cheng J-W. Chaos synchronization and parameter identification of three time scales brushless DC motor system. *Chaos, Solitons & Fractals* 2005;24:597–616.
- [12] Ge Z-M, Chang CM. Chaos synchronization and parameter identification of single time scale brushless DC motors. *Chaos, Solitons & Fractals* 2004;20:883–903.
- [13] Lü J, Zhou T, Zhang S. Chaos synchronization between linearly coupled chaotic systems. *Chaos, Solitons & Fractals* 2002;14:529–41.
- [14] Ge Z-M, Chen Y-S. Synchronization of unidirectional coupled chaotic systems via partial stability. *Chaos, Solitons & Fractals* 2004;21:101–11.
- [15] Ge Z-M, Leu W-Y. Anti-control of chaos of two-degrees-of-freedom loudspeaker system and chaos synchronization of different order systems. *Chaos, Solitons & Fractals* 2004;20:503–21.
- [16] Ge Z-M, Leu W-Y. Chaos synchronization and parameter identification for loudspeaker system. *Chaos, Solitons & Fractals* 2004;21:1231–47.
- [17] Ge Z-M, Chen C-C. Phase synchronization of coupled chaotic multiple time scales systems. *Chaos, Solitons & Fractals* 2004;20:639–47.
- [18] Ge Z-M, Yu T-C, Chen Y-S. Chaos synchronization of a horizontal platform system. *J Sound Vibr* 2003;268:731–49.
- [19] Ge Z-M, Cheng J-W, Chen Y-S. Anticipation in the synchronization of chaotic time-delay systems. *Chaos, Solitons & Fractals* 2004;22:1165–82.
- [20] Ge Z-M, Lee C-I. Control, anticontrol and synchronization of chaos for an autonomous rotational machine system with time-delay. *Chaos, Solitons & Fractals* 2005;23:1855–64.
- [21] Voss HU. Anticipating chaotic synchronization. *Phys Rev E* 2000;61:5115.
- [22] Masoller C. Anticipation in the synchronization of chaotic time-delay systems. *Physica A* 2001;295:301.
- [23] Calvo O, Chialvo DR, Eguiluz VM, Mirasso C, Toral R. Anticipated synchronization : a metaphorical liner view. *Chaos* 2004;14:7–13.
- [24] Krause PC. Analysis of electric machinery. McGraw-Hill; 1986.
- [25] Hemati N, Leu MC. A complete model characterization of brushless DC motors. *IEEE Trans Ind Appl* 1992;28:172–80.
- [26] Hemati N. Strange attractors in brushless DC motors. *IEEE Trans Circ Syst* 1994;41:40–5.
- [27] Pecora LM, Carroll ThL. Synchronization in chaotic system. *Phys Rev Lett* 1990;64:821–4.
- [28] Pecora L, Carroll T, Johnson G. Fundamentals of synchronization in chaotic systems, concepts and applications. *Chaos* 1997;7:520.
- [29] Li Z, Xu D. A secure communication scheme using projective chaos synchronization. *Chaos, Solitons & Fractals* 2004;22:477–81.

- [30] Agiza HN, Yassen MT. Synchronization of rossler and chen chaotic dynamical systems using active control. *Phys Lett A* 2001;278:191.
- [31] Yan Z. A new scheme to generalized (lag, anticipated and complete) synchronization in chaotic and hyperchaotic systems. *Chaos* 2005;15:013101.
- [32] Chen G, Liu S. On generalized synchronization of spatial chaos. *Chaos, Solitons & Fractals* 2003;15:311–8.
- [33] Jiang G, Zheng W, Chen G. Global chaos synchronization with channel time-delay. *Chaos, Solitons & Fractals* 2004;20:267–75.