Thin Solid Films 519 (2011) 2746–2749

Contents lists available at ScienceDirect

Thin Solid Films

journal homepage: www.elsevier.com/locate/tsf

Ultra fast self-corrected polarization modulated ellipsometer

Hsiu-Ming Tsai [a], Leng-Chun Chen [b], Yu-Faye Chao [a],*

[a] Department of Photonics and Institute of Electro-Optical Engineering, National Chiao Tung University, 1001 University Road, Tin ka-ping photonics center, Hsinchu 30010, Taiwan
[b] Department of Biomedical Engineering, University of Michigan, Ann Arbor, 1107 Carl A. Gerstacker Building 2200 Bonisteel, Blvd. Ann Arbor, MI 48109-2099, United States

ARTICLE INFO

Keywords:
Ellipsometry
Polarimetry
Photoelastic modulator

ABSTRACT

A high speed self-corrected algorithm is proposed for the polarization modulated ellipsometry (PME). In this post-flight analysis, we prove that a set of optimized ellipsometric parameters (EPs) can be obtained by using the intensities at 4 specific temporal phases. The correction ability to its initial phase by this technique has been demonstrated through a twisted nematic liquid crystals (TN-LC) cell under the driving of a square wave. Furthermore, the optimal modulation amplitude in obtaining the accurate and precise set of EPs will be discussed in this work.

1. Introduction

Ellipsometry is a powerful and versatile tool in determining physical properties for thin films, such as its refractive indices, thickness, etc. [1]. Recently, the ellipsometry has found increasing applications in industrial [2] and biological [3] for monitoring the dynamic process, a high speed and precise ellipsometric metrology is in strong demand. Generally, there are two photometric configurations in spectroscopic ellipsometry: (i) rotating element [4] ellipsometry which is simple and insensitive to the wavelength; (ii) the polarization modulated ellipsometry (PME), a photoelastic modulator (PEM) has been used to avoid the moving part in the ellipsometry. This PEM operates at 50 kHz in PME versus the rotation frequency of 100 Hz in the rotating element ellipsometry. One can dramatically increase the speed of measurement to obtain the ellipsometric parameters (EPs) in PME.

In this paper, we proposed a self-corrected PME by analyzing the waveform of the modulated signal. Only 4 polarization states are needed to obtain the EPs in one cycle. To verify if this self-corrected method can be employed to operate in the microsecond time scale, we applied a square wave voltage on a twisted nematic liquid crystals (TN-LC) cell. The dynamic process of the switching on and off can be observed by analyzing the recorded waveform, which is considered as the post-flight analysis technique. Around 1995, Ambirajan [5] and Sabatke et al. [6] presented the concept of optimization in polarimetry for square and N × 4 matrices, respectively. These quantitative techniques can be used to evaluate the precision of measurements in the polarimetric metrology. The equally weighted variance (EWV) was named by Sabatke [6], he proved that the square root of EWV is inversely proportional to the signal-to-noise ratio (SNR) in polarimetric measurements. In this paper, we also used this value in analyzing this polarimetric procedure and found the most proper modulation amplitude in our system to be 0.6 λ instead of half-wave retardation.

2. Theoretical background

Ellipsometry/polarimetry measures the changes in the polarization states of the light reflected/transmitted from the sample, then deduce its optical parameters. The EPs are noted as ψ and Δ, and related by

$$\tan\psi e^{i\Delta} = \frac{r_p}{r_s}, \tag{1}$$

where r_p/t_p and r_s/t_s are the complex Fresnel reflection/transmission coefficients for the polarized light parallel and perpendicular to the plane of incidence, respectively. This work, the PEM [7] is used to substitute the compensator in a polarizer–compensator–sample–analyzer (PCSA) ellipsometry. Thus, the polarization state of the reflected/transmitted (S_f) in this configuration can be expressed as

$$S_f = M_A(a)R_S(\psi,\Delta)M_{PEM}(\Delta_P)S_p, \tag{2}$$

where S_p is the initial linear polarized state, M_{PEM} (Δ_P) represents the Mueller matrix of PEM (its modulated phase Δ_P and optical axis is set at zero with respect to the incident plane), R_S (ψ, Δ) represents the Mueller matrix of the sample with ellipsometric parameters ψ and Δ, and M_A (a) is the Mueller matrix of an analyzer with its transmission axis at a. If the azimuth angle of the linear polarized light is set at −45°and the azimuth angle of the analyzer is positioned at 45°, the measured intensity (I_f) can be written as

$$I_f = \frac{I_0}{2}[1-\cos(\Delta-\Delta_P)\sin(2\psi)] \tag{3}$$

---

* Corresponding author. Tel./fax: +886 3 5731914.
E-mail address: yfchao@mail.nctu.edu.tw (Y.-F. Chao).

0040-6090/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.tsf.2010.12.087

where I_0 is the normalized intensity of the system. The phase retardation of the PEM is modulated as $\Delta_P = 2\pi\Delta_o\ \sin(\omega t)$ and the modulation amplitude Δ_o, Eq. (3) can be re-expressed as,

$$I_f(\omega\ t) = \frac{I_0}{2}\{\ 1-\cos[\Delta-2\pi\Delta_o\sin(\omega\ t)]\sin(2\psi)\ \} \tag{4}$$

The digitized waveform acquired by data acquisition (DAQ) system can be saved for post flight analysis. Due to the discrete sampling of DAQ card, it is very important to identify the initial phase in this modulated waveform for obtaining the exact polarization state used in calculating the EPs [8]. Assume there is an x deviation in its initial phase, the intensity reaches its extreme values at the following two positions, i.e.

$$\Delta-2\pi\Delta_o\sin(\omega\ t+x) = 0\ \text{or}\ \pi \tag{5}$$

Thus, the two extreme intensity values are

$$I_{max} = \frac{I_0}{2}[1+\sin(2\psi)], \tag{6}$$

$$I_{\ min} = \frac{I_0}{2}[1-\sin(2\psi)] \tag{7}$$

So, ψ can be obtained by the two extreme intensity values. The algorithm can be easily proven as follows

$$\psi = \frac{1}{2}\sin^{-1}\left(\frac{I_{max}-I_{\ min}}{I_{max}+I_{\ min}}\right) \tag{8}$$

Using this intensity ratio technique, we can avoid the laser intensity fluctuation in the self-corrected PME. Moreover, one can get rid of the influence of the temporal phase error by other two positions, i.e. $\omega t = 90°$ and 270°, which are independent from the sample. By considering the initial phase deviation x, one can re-write the intensity distribution as

$$I(90^\circ+x) = \frac{I_0}{2}\{\ 1-\cos[\Delta-2\pi\Delta_o\sin(90^\circ+x)]\sin(2\psi)\ \}, \tag{9}$$

$$I(270^\circ+x) = \frac{I_0}{2}\{\ 1-\cos[\Delta-2\pi\Delta_o\sin(270^\circ+x)]\sin(2\psi)\ \} \tag{10}$$

Eqs. (9) and (10) can be used to extract Δ by the following relations,

$$A = \Delta-2\pi\Delta_o\sin(90^\circ+x) = \cos^{-1}\left[1-\frac{2\ I(90^\circ+x)}{I_0\sin(2\psi)}\right], \tag{11}$$

$$B = \Delta-2\pi\Delta_o\sin(270^\circ+x) = \cos^{-1}\left[1-\frac{2\ I(270^\circ+x)}{I_0\sin(2\psi)}\right] \tag{12}$$

Obviously; one can obtain Δ without having to worry the effect from the initial temporal phase deviation by $\Delta = \frac{A+B}{2}$. This PME not only can correct its initial phase deviation but also avoid the intensity fluctuation of its light source.

3. Experimental results

For observing the dynamic process, a typical TN-LC cell is employed as the dynamic anisotropic sample and measured by the self-corrected PME, such as shown in Fig.1. Under the proper voltage drive, molecules of TN-LC will align to the employing electric field which will result a change in its anisotropic property. The rubbing direction of the entrance layer of TN-LC cell is arranged parallel to the transmission axis of polarizer, i.e. P = −45°. The strain axis of PEM (Hinds PEM90 operated at an oscillating frequency around 50 kHz), the transmission axis of the polarizer and analyzer (Melles Groit 03FPG015 sheet polarizer with an extinction ratio of 10^{-4}) were all well aligned and calibrated according to references [9,10]. Since there is no mechanical rotating element in this system, each element of the system can be accurately calibrated prior to the measurement.

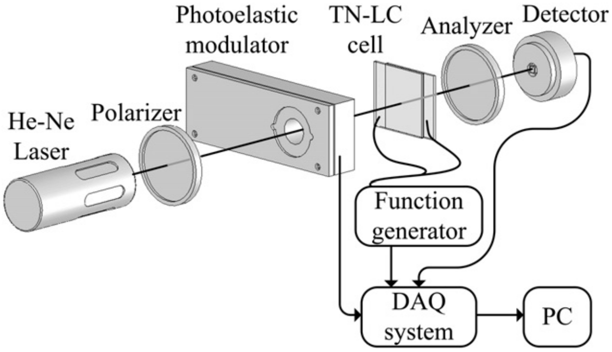

Fig. 1. Experimental setup of the self-corrected polarization modulated ellipsometry.

The measured EPs of the TN-LC under a 10 Hz square wave with applied voltage from 0 to 5 V are shown in Fig 2(a). To prevent the decomposition of TN-LC from charge accumulation, 1 kHz carrier frequency was also applied. We extracted 4 specific points to evaluate the EPs value from the digitized waveform acquired by DAQ system (NI-6115, 12bits, 10 M samples). According to Eqs. (8), (11) and (12), each EPs was obtained in one cycle (i.e. 20 μs). The value of Δ changed from ~180° to ~0° while the voltage was switched on. Spikes around the switching period revealed the resonant effect of RLC circuit in the system; Fig. 2(b) enlarges the grey areas in Fig. 2(a) to zoom in the fine scale in EPs. These results can demonstrate the highly resolving ability of the self-corrected PME.

4. System optimization

The optimization of polarimetry has been widely studied [5,6,11] around 1995. The optimal conditions of rotation angles in polarimetry have been studied by Ambirajan and Look [5] through evaluating the condition number in Stokes polarimetry, which was restricted in the square matrix measurement of four. Based on singular value decomposition, Sabatke et al. [6] introduced the EWV, this value not only quantitatively predicts the system performance, but also evaluates the SNR for any number of steps in the measurement of polarimetry. Since the PEM has been used to substitute the compensator for eliminating the moving part, we utilize four-temporal phases of the photoelastic modulation instead of using conventional four-spatial azimuth angles by rotating a retarder with respect to the analyzer. As the EWV of polarimetry can quantitatively interpret the system performance, we shall present the figure of merit for assessing the noise immunity of the PEM Stokes polarimetry in the temporal domain.

According to a linear model, we can write $\{b\} = [A]\cdot\{s\}$. In this PME, the minimum number of measurement is 4. Thus, $\{b\}$ is a 4-element vector of measurement in irradiance, which is composed of the measured intensity in each temporal phase, [A] is a 4×4 matrix, the measurement matrix [5], and $\{b\}$ is the Stokes vector. Since $\{s\} = [A]^{-1}\cdot\{b\}$, each element represents the response of the unit stimuli of the system. The noise in the measurement of the Stokes vector can be written in a vector form $\{n\}$; thus the error can be expressed as

$$\{\varepsilon\} = [A]^{-1}\cdot\{n\} \tag{13}$$

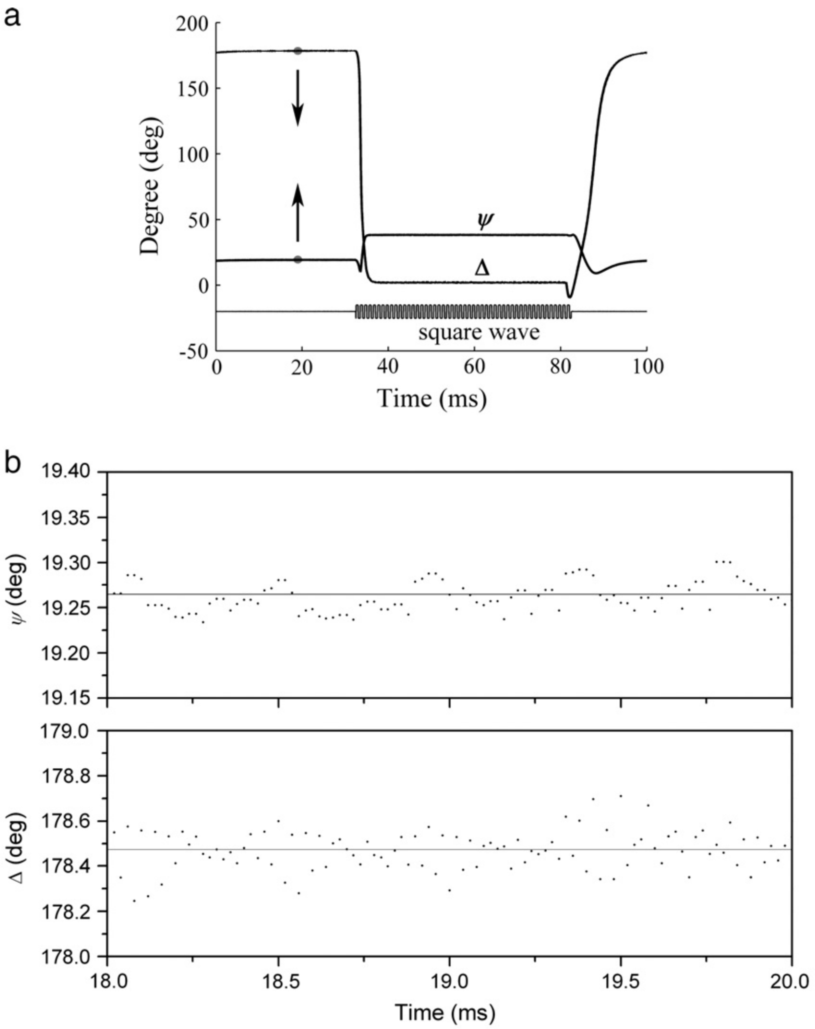

Fig. 2. (a) The EPs measured by the self-corrected PME for the dynamic process of a TN-LC cell. A square wave of 10 Hz with strength 5 V was applied, and its carrier frequency was 1 kHz. (b) Enlarged the grey circle areas of (a), which is from 18 ms to 20 ms. The standard deviation of ψ is 0.02°. The solid line indicates the mean value which is 19.27°; the standard deviation of Δ is 0.11°. The solid line indicates the mean value which is 178.47°.

Since all components in the Stokes vector are equally weighted in noise production, we can therefore use EWV to be the value for the figure of merit of the system,

$$EWV = \sum_{i=0}^{3}\sum_{j=0}^{3}\left([A]^{-1}\right)^{2}_{i,j} = Tr\left[[A]^{-1}\left([A]^{-1}\right)^{T}\right] \tag{14}$$

This value provides us the measurement errors by summing all entries in the measurement matrix. The jth row of the matrix $[A]^{-1}$ of the PME can be expressed as $[1\ 0\ \cos(\Delta_{Pj})\ \sin(\Delta_{Pj})]^{\mathrm{T}}$, where $\Delta_{Pj}=2\pi\Delta_o \sin(\omega t_j)$, $j=0\sim3$. Those $\varphi_j=\omega t_j$ have been fixed in four specific temporal phases instead of using the conventional rotating elements ellipsometry. In Eq. (5), the positions of two extreme intensity values are related to Δ, which plays an important role on the value of EWV, such as shown in Fig. 3. EWV of this PME was calculated for the evaluation of the measurement of any Δ under two different modulation amplitudes. It is worth noting that there is no singularity by using the modulation amplitude at 0.6 λ in the self-corrected PME, while when the modulation amplitude is at 0.5 λ (can be used for the convenient of calculation) there are singular points around $\Delta=180°$ or 0°. In this PME, one can minimize the EWV by adjusting the value of modulation amplitude of PEM to measure dielectric materials even when Δ is 180° or 0°, which are the inferiorities of the rotating Polarizer–Sample–analyzer polarimetric system. In addition, the EWV figure of merit will vary less than 1% when the modulation deviated by 0.002 λ. In this self-corrected PME, we chose the modulation amplitude to be 0.6 λ instead of 0.5 λ for its optimal operation condition.

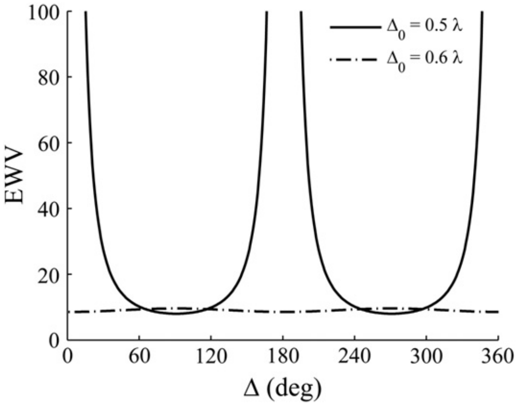

Fig. 3. EWV under various Δ of sample: (solid line) $\Delta_o=0.5\ \lambda$, singularities are at $\Delta=1°$, 179°, 181°, and 359°, respectively. (dashed line) $\Delta_o=0.6\lambda$ no singularities for any sample.

5. Conclusion

In general, the waveform recorded in the DAQ system is discrete; it may not record the exact initial temporal phase to resolve the physical parameters of the sample. In most cases, an interpolation can be employed to improve the precision. However, we proposed a four-point post-flight analysis for self-corrected PEM based ellipsometry which not only can operate in wide dynamic range, but also can avoid the drawback of misposition of temporal phase in the discrete waveform. This extreme values and zero crossing of the modulation waveform allow us to measure EPs without having to synchronize the pulses. Since one can obtain all measurements within one cycle, i.e. 20 μs, this technique has already achieved the time limit of the PEM. Due to the common path and intensity–ratio measurement technique in the self-corrected PME, the fluctuation in light source can be effectively reduced. While the EWV value can be used to quantitatively evaluate the noise immunity of a polarimetry, the optimal phase retardation and the four temporal phases set give us with great confidence in developing for probing fast dynamical processes in biological applications.

References

[1] R.M.A. Azzam, N.M. Bashara, Ellipsometry and Polarized Light, North-Holland, Amsterdam, 1988.
[2] K. Riedling, Ellipsometry for Industrial Applications, Springer-Verlag, New York, 1988.
[3] H. Arwin, Thin Solid Films 313–314 (1998) 764.
[4] D.E. Aspnes, A.A. Studna, Appl. Opt. 14 (1975) 220.
[5] A. Ambirajan, D.C. Look Jr., Opt. Eng. 34 (1995) 1651.
[6] D.S. Sabatke, M.R. Descour, E.L. Dereniak, W.C. Sweatt, S.A. Kemme, G.S. Phipps, Opt. Lett. 25 (2000) 802.
[7] J.C. Kemp, J. Opt. Soc. Am. 59 (1969) 950.
[8] D.J. Diner, A. Davis, B. Hancock, G. Gutt, R.A. Chipman, B. Cairns, Appl. Opt. 46 (2007) 8428.
[9] M.W. Wang, Y.F. Chao, K.C. Leou, F.H. Tsai, T.L. Lin, S.S. Chen, Y.W. Liu, Jpn. J. Appl. Phys. 43 (2004) 827.
[10] M.W. Wang, F.H. Tsai, Y.F. Chao, Thin Solid Films 455–456 (2004) 78.
[11] G.H. Golub, C.F. Van Loan, Matrix Computations, 3rd ed, Johns Hopkins U, Press, Baltimore, Md, 1996.