變動訂購期間定期盤存模式之研究

國立交通大學管理科學系姜齊

I. Introduction

Although the use of computer systems has made continuous review models more attractive, periodic review models are still applied in many situations (e.g., Prasad et al., 2005; Silver et al., 1998), especially for inventory systems in which the coordination of ordering and transportation for different items is important (which is especially true if these items are purchased from the same supplier). Also, as Porteus (1985) observes, continuous review systems that keep inventory records current, but order periodically are equivalent to periodic review systems. Often, periodic systems have the review periods that are possibly longer than the supply lead-times.

One fundamental assumption about periodic systems is that the review periods are of a fixed length (i.e., the successive time between orders is constant). In practice, however, the review periods may be of a variable length. Such periodic systems result mainly from supply uncertainties. For example, many supermarkets have suppliers come to visit regularly and replenish the inventory of various items (and even sell) for them. However, the supplier does not always come in constant (say, 10-day) intervals. Depending on her visit plans or work schedules and loads, she often arrives at a particular supermarket one or few hours (or days) early or late. The elapsed time between two consecutive visits varies basically. Ertogral and Rahim (2005) also observed institutional settings or constraints that are internal to the supply chain, in which the supplier is strategically dominant, in a relatively more powerful position, and/or the retailers are located in a geographically disadvantageous remote location, so that the supplier decides when to visit and replenish the retailers' inventories. In general, for such situations, the replenishment epochs are not under retailers' control; rather, they are under the supplier's control. Hence, if the supplier arrives at a particular retailer in irregular intervals, the replenishment cycle length for that retailer is random in nature.

To our knowledge, the possibility of stochastic review periods or replenishment intervals has not been investigated in the inventory literature, though there are some works on inventory models with supply uncertainties (e.g., Lee et al., 1997; Mohebbi, 2004; Ozekici and Parlar, 1999). It was studied only recently by Ertogral and Rahim (2005) who derived the expected profit per replenishment cycle by assuming independently and identically distributed (i.i.d.) replenishment intervals, constant demand and zero replenishment lead-time.

In this research, we use dynamic programming to model the supply chain situations where the supplier's visit intervals (i.e., replenishment intervals) are random. We will assume that the supplier's visit intervals are i.i.d., as in Ertogral and Rahim (2005). However, unlike Ertogral and Rahim, we will assume stochastic demand that is usually true in the real world; also, we will allow the replenishment lead-time to be positive (i.e., it may take a positive time to replenish inventories after the supplier arrives at a retailer and reviews his inventories). We will develop both the backlogged and lost-sales periodic review inventory models. With a suitable transformation, the backlogged model derived becomes a standard discrete-time model. Thus, an order-up-to policy is optimal for the infinite horizon problem. This is also true of the lost-sales problem with zero lead-time (due to a result from the inventory literature). For the lost-sales problem with positive lead-time, we suggest a simple heuristic policy in Hadley and Whitin (1963).

The computation shows that ignoring the variability of the supplier's visit intervals can incur unnecessary large costs, especially if shortage is costly, the replenishment lead-time is short, and/or demand variability is not high. It is thus important for a retailer to incorporate this variability into inventory models when the supplier does not visit in constant intervals. It would be better if the retailer can have the supplier to visit in more regular intervals (i.e., the visit interval has a smaller variability) so that his cost can be reduced, as shown in the computation. This may not be an easy task, since the institutional constraints are perhaps difficult to change in the short run (for example, the supplier is in a relatively more powerful position as described above). For such institutional contexts, we suggest that the retailer should somehow persuade the supplier to visit more regularly. The retailer should at least communicate with the supplier often so that she understands the consequence of the irregular visit intervals and hopefully, she will continue to improve on her visit schedule in terms of the stability/reliability in the future.

Of course, it is possible that the supplier completely fixes the visit interval after the retailer's persuasion. The supplier and retailer may even cooperate closely in the supply chain, or form a strategic alliance in the long run. Then the supplier will also visit the retailer and replenish his inventories more often (not only more regularly) so as to further decrease his costs, and in return, the retailer could negotiate a long-term supply contract or purchase other products from the supplier, for example. All of these are certainly a significant change of status-quo, i.e., a break-through of the supply chain. Note that we are not saying that it is not good to have the replenishment epochs under the supplier's control; it may be one of the most efficient ways of operating the supply chain (in terms of replenishing the downstream retailers' inventories), especially for the institutional settings described above. We simply say that cooperation between the supplier and retailer could result in a win-win situation. If indeed the supplier no longer visits the retailer in irregular intervals, then the ordinary periodic review models found in textbooks can be used, i.e., the periodic review models derived in this research need not be used.

II. Backlogged Periodic Review Inventory Models

We first assume that all demand not filled immediately is backlogged. Let c denote the unit item cost and L the (deterministic) lead-time. Demand is stochastic with mean rate μ per unit time, and is assumed to be non-negative and independently distributed in disjoint time intervals. Let T be the period length, i.e., the supplier's visit interval. Successive T's are assumed to be i.i.d. random variables. Let $\phi(\cdot)$ be the probability density function (pdf) of T and D the demand during T. Also let $g(\cdot|\tau)$ be the conditional pdf of demand during a time interval of length τ. Thus, $g(\cdot|T)$ is the conditional pdf of D.

Let α be the discount rate, y the inventory position (i.e., inventory on hand minus backorder plus inventory on order) after an order is placed at a review epoch (i.e., upon the supplier's visit), and H the expected one-period inventory holding and shortage cost (H is a function of y). Given time 0 at a review epoch, H is charged for the time interval $[L, T + L)$. Denote $V_n(x)$ as the expected discounted cost with n periods remaining until the end of the planning horizon when the starting inventory position is x and an optimal ordering policy is used at every review epoch. $V_n(x)$ satisfies the functional equation

$$V_n(x) = \min_{x \le y}\left\{e^{-\alpha L}[cy + H(y)] + E_T\left[e^{-\alpha T}E_{D|T}[V_{n-1}(y - D)]\right]\right\} - e^{-\alpha L}cx,$$

where the procurement cost $c(y - x)$ is paid upon delivery. The above dynamic program is an inventory problem with discrete but random epochs. See Chiang (2008) for how to transform the above program into a standard discrete-time model and how to derive the optimal ordering policy.

III. Lost-sales Periodic Review Inventory Models

Suppose that demand not satisfied at once is lost. Assume that L is less than the minimum T (i.e., there is at most one order outstanding). Let D_1 be the demand during the lead-time L and D_2 the demand during the time interval $[L, T)$ (thus $D = D_1 + D_2$). Also, let z be the order quantity placed at a review epoch and redefine x as the starting on-hand inventory. Let $(\cdot)^+ \equiv \max\{\cdot, 0\}$. Then, $V_n(x)$ satisfies the recursive equation

$$V_n(x) = \min_{z \geq 0}\{e^{-\alpha L}(cz + E_{D1}[H((x - D_1)^+ + z)]) + E_T[e^{-\alpha T}E_{D1,\, D2|T}[V_{n-1}(((x - D_1)^+ + z - D_2)^+)]]\}.$$

If L is positive, the above dynamic program is difficult to solve. We will suggest a heuristic policy in this research, which is similar to that in Hadley and Whitin (1963). See Chiang (2008) for details.

IV. Computational Results

We investigate the effect of a variable T on the expected cost, if a retailer fails to incorporate it when developing inventory policies. We assume that demand is normal with mean $\mu\tau$ and variance $\sigma^2\tau$ for a time interval of length τ. The common data used are μ = 10/day (unit time is one day), h = \$0.1, α = 0, and $E[T]$ = 10 days. Also, T is either triangularly or uniformly (discretely) distributed. In the former case, $Pr(T = 8) = Pr(T = 12) = 1/9$, $Pr(T = 9) = Pr(T = 11) = 2/9$, and $Pr(T = 10) = 1/3$; in the latter case, $Pr(T = 8) = Pr(T = 9) = Pr(T = 10) = Pr(T = 11) = Pr(T = 12) = 1/5$. Thus, T has a larger variability if it is uniformly distributed.

The computation (see Chiang, 2008 for details) showed that ignoring the variability of the supplier's visit intervals could incur large losses if shortage was costly. It also showed that a retailer was more vulnerable to this variability if the replenishment lead-time was short and/or demand variability was small. This was because the introduction of T's variability into an inventory model increased the overall variability of demand during T plus L more significantly when lead-time was shorter and/or demand variability was smaller. In the extreme case where demand was deterministic, ignoring T's variability could increase the retailer's cost by more than three hundred percent!

Furthermore, the computation showed that a retailer could avoid some losses by reducing the variability of the supplier's visit intervals (e.g., from a uniform to triangular distribution). This might not be an easy task, because the replenishment epochs were under the supplier's control and such institutional constraints were perhaps difficult to change in the short run. However, the retailer could discuss this issue and explain its effect on his cost with the suppler. The retailer should at least communicate with the supplier often so that she would come to visit and replenish inventories more punctually (or even in constant intervals) in the future.

V. Conclusion

In this research, we considered periodic inventory models with stochastic supplier's visit intervals. We assumed that the supplier's visit intervals were independently and identically distributed. With a suitable transformation, the backlogged dynamic programming model derived became a standard discrete-time model. In addition, we suggested a simple order-up-to policy for the lost-sales periodic problem with positive lead-time. The periodic review policies developed in this research were thus easy to implement.

References

Chiang, C., 2008. Periodic review inventory models with stochastic supplier's visit intervals. International Journal of Production Economics 115 (2), 433-438.

Ertogral, K., Rahim, M.A., 2005. Replenish-up-to inventory control policy with random replenishment intervals. International Journal of Production Economics 93-94, 399-405.

Hadley, G., Whitin, T.M., 1963. Analysis of Inventory Systems. Prentice-Hall, Englewood Cliffs, NJ.

Lee, H.L., Padmanabhan, V., Whang, S., 1997. Information distortion in a supply chain: the bullwhip effect. Management Science 43, 546-558.

Mohebbi, E., 2004. A replenishment model for the supplier-uncertainty problem. International Journal of Production Economics 87, 25-37.

Ozekici, S., Parlar, M., 1999. Inventory models with unreliable suppliers in a random environment. Annals of Operations Research 91, 123-136.

Porteus, E., 1985. Numerical comparisons of inventory policies for periodic review systems. Operations Research 33, 134-152.

Prasad, S., Tata, J., Madan, M., 2005. Build to order supply chains in developed and developing countries. Journal of Operations Management 23, 551-568.

Silver, E.A., Pyke, D.F., Peterson, R., 1998. Inventory Management and Production Planning and Scheduling. Wiley, New York.

計畫成果自評

In this research, we consider periodic inventory models with stochastic supplier's visit intervals. We assume that the supplier's visit intervals were independently and identically distributed. We derive the backlogged dynamic programming model and transform it to a standard discrete-time model. We also show that the lost-sales zero-lead-time dynamic programming model could be handled in the same way as the literature suggested. These results make a significant contribution to the periodic-review inventory literature. In addition, we develop a simple order-up-to policy for the lost-sales periodic problem with positive lead-time. The periodic review policies developed in this research can be easily implemented in practice.