Stochastically Optimal Groundwater Management Considering Land Subsidence

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Abstract: This paper presents a stochastic groundwater management model explicitly considering land subsidence. Through the use of response matrix technique and one-dimensional consolidation equation, a deterministic management model is first developed. By Latin hypercube sampling technique, along with numerical subsurface flow simulation, statistical features of unit response coefficients due to random hydrogeologic parameters, including hydraulic conductivity (K) and Lame constants (μ and λ), are quantified. The first-order-variance-estimation method is adopted to analyze the uncertainties of drawdown and land subsidence based on which the concept of chance-constrained programming is applied to transfer the original deterministic management model into its stochastic form. The stochastic management model enables the determination of optimal total pumpage subject to the constraints that drawdown and land subsidence do not exceed the allowable values with a specified reliability. A hypothetical example is utilized to demonstrate the applicability of the stochastic model to five cases in which various levels of parameter uncertainty are considered. The results indicate that joint consideration of drawdown and land subsidence is essential, and the proposed stochastic management model can be generally applied for regional groundwater resources management in conjunction with controlling land subsidence.

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Introduction

Groundwater is an important water resource, especially for arid or semiarid regions where surface water is highly variable. Due to rapid growth in population and lack of proper management, many groundwater aquifer systems are overdeveloped resulting in serious hazards of land subsidence. The occurrence of land subsidence could have several undesirable consequences including, but not limited to: (1) groundwater quality deterioration and saltwater encroachment; (2) reduction in storage capacity of groundwater systems; and (3) localized flooding due to change in surface drainage features. Therefore, establishment of a proper policy for controlling land subsidence is an important aspect of groundwater management.

Groundwater management has been studied extensively in the past. Only a few studies considered the land subsidence effect. Onta and Gupta (1995) coupled a three-dimensional groundwater flow model and a one-dimensional consolidation model to simulate the piezometric levels and land subsidence in a complex multiaquifer system of lower Central Plain of Thailand. Several different pumping strategies were simulated by the model from which suitable groundwater management policies considering land subsidence control were established.

Unlike simulation approaches, which require trial-and-error in finding the optimal management strategy, the simulation-optimization approaches are widely used in groundwater management, which couple optimization algorithm with groundwater flow simulation to determine the optimal management strategies of aquifer systems. A review of groundwater management models can be found elsewhere (Gorelick 1983; Yeh 1992; Ahlfeld and Heidari 1994; Wagner 1995; Ahlfeld and Mulligan 2000). Recently, Larson et al. (2001) incorporated the effects of land subsidence in an optimal groundwater management model. A linear programming model was developed using the response matrix technique to find the maximum rate of groundwater withdrawal without causing any inelastic compaction in Los Banos-Kettleman City area of San Joaquin Valley, Calif. This is accomplished by setting the preconsolidation head as the lower bound for groundwater levels in the confined aquifer (i.e., drawdown is not allowed to exceed the difference between the initial water level and the preconsolidation head level). Phillips et al. (2003) also considered the land subsidence in a groundwater management model applied in Lancaster, Antelope Valley, Calif. The objective was to maximize the lowest value of head subject to the constraints that the head did not exceed the lower bound and the water demand was satisfied. To prevent the land subsidence caused by delayed drainge from the aquitards, the spring conditions were specified as the lower bound of head initially and decreased along with the management period in the subsidence

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area. Therefore, both models of Larson et al. (2001) and Phillips et al. (2003) do not explicitly consider the magnitude of land subsidence.

The models described earlier are deterministic in that the parameters governing groundwater flow and land subsidence are assumed to be known. In reality, due to inherent heterogeneity and lack of complete information about aquifer parameters, uncertainties exist in specifying parameters values in the management model rendering potential failure to obtain the optimal strategy for the system under consideration. To incorporate these uncertainties into the establishment of optimal groundwater management policy, the development of a stochastic model to account for the presence of uncertainties is essential.

Tung (1986) considered aquifer parameter uncertainties to develop a chance-constrained groundwater management model for confined, homogenous aquifers. The statistical properties of drawdown, calculated from the analytic impulse-response function, were estimated by the first-order variance estimation (FOVE) method. The drawdown was assumed to be normally distributed through the loose use of central limit theorem. The model explicitly considered the uncertainty of transmissivity and storage coefficient in maximizing water supply capability of a groundwater basin. The study showed that the model solution is insensitive to the uncertainty of storage coefficient. Besides, the actual reliability obtained from postoptimality simulation was adopted to estimate the accuracy of normality assumption. The relative differences between stipulated and actual reliability were approximately 11 and 13% when the coefficients of variance for transmissivity were 0.6 and 0.8, respectively.

For stochastic groundwater quality management, Wagner and Gorelick (1987) also used the chance-constrained programming (CCP) to determine minimum pumping rate subject to constraints requiring the compliance reliability of solute concentrations to not exceed the specified standard. The weighted, nonlinear least squares regression analysis was applied to estimate the parameters and their associated uncertainties. The pollutant concentrations, which are function of uncertain parameters, were assumed to be normally distributed. The model was applied to a hypothetical system, and the Monte Carlo simulation (MCS) approach was used to check the normality assumption of concentrations. Morgan et al. (1993) applied the MCS to generate several realizations of the hydraulic conductivity random field, and each realization of hydraulic conductivity was used to calculate the unit response coefficients. The multiple realization method and CCP were combined to develop a management model in the form of mixed-integer CCP. The model considered uncertainty in all constraint coefficients and did not require a priori knowledge of the distribution of drawdown. By using a heuristic algorithm that successively drops realizations when some of the constraints are violated, a trade-off curve for maximum reliability versus minimum pumpage was established.Datta and Dhiman (1996) developed a CCP model to determine optimal groundwater monitoring network that explicitly considered uncertainties associated with transport simulation. Given the upper limit on the number of monitoring wells to be installed, the model determines monitoring locations where the concentrations of the contaminant were expected to be very high. Sawyer and Lin (1998) developed a mixed-integer CCP model for groundwater aquifer remediation in which the uncertainties of management model coefficients due to random hydrogeologic parameters were assumed to be normal and the spatial correlation of hydraulic conductivity was not considered. In addition, the model also considered the uncertainty in unit pumping cost due to variation in energy production cost.

In addition to the CCP formulation, the multiple realization method is an alternative for stochastic groundwater managements. Wagner and Gorelick (1989) consider a similar problem to their earlier work (Wagner and Gorelick 1987) in that the multiple realization method was combined with a stochastic inverse model that incorporates the uncertainty in hydraulic conductivity. Wagner et al. (1992) used the embedding technique along with the multiple realization method to incorporate hydraulic conductivity uncertainty in a stochastic optimization model for groundwater remediation. In addition, a recourse model was incorporated to find the minimum excepted total cost of operating the pumping wells plus the recourse cost incurred when containment of the contaminant plume is not achieved. Chan (1993) used response matrix and multiple realization methods to develop a stochastic management model for groundwater remediation. After the model was developed, many realizations were generated to examine the robustness of multiple realization method. The numerical experiments indicated that the system reliability was insensitive to change in system parameters and structure. This is consistent with the results from the theoretical approach, such as Bayesian analysis or one-dimensional order statistics, which were used to obtain the relationship between reliability and realization size without considering system information. Recently, Feyen and Gorelick (2004) presented a comprehensive analysis for the robustness of the multiple realization method, utilizing approximately 36,000 stochastic-optimization solutions. The results showed that the optimal pumpage decreases with increase in the variance of hydraulic conductivity with the same number of realizations. Chan (1994) further extended the previous stochastic management model (Chan 1993) by using a partial infeasibility method to solve the optimization problem with prespecified system reliability. This overcomes the inability of traditional multiple realization methods to specify system reliability in advance before a postoptimality analysis is performed. The solution technique was accomplished through a heuristic search, and the results showed that the actual system compliance reliability is much closer to the specified value as the numbers of realizations increase. Mylopoulos et al. (1999) also used the multiple realization method for stochastic groundwater remediation in the Kalamaria aquifer, Greece. For a more complete review of stochastic optimization for aquifer remediation, readers are referred to Freeze and Gorelick (1999) which summarizes most research contribution in optimization and decision analysis for aquifer remediation.

Most of the previous studies on groundwater management, deterministic or stochastic, did not consider land subsidence effect. Although Onta and Gupta (1995) and Larson et al. (2001) had considered land subsidence in the optimal groundwater management, the former is by simulation involving trial-and-error, and the latter did not explicitly incorporate land subsidence in model constraints. Both works were deterministic, which do not account for the uncertainty in land subsidence due to random hydrogeologic parameters.

In this paper, an optimal stochastic groundwater management model explicitly considering land subsidence is developed. By incorporating a one-dimensional consolidation equation model (Tsai 2001) with the response matrix technique, a deterministic management model is developed to maximize total pumpage subject to drawdown and land subsidence constraints. To further account for the uncertainty of land subsidence due to the random hydrogeologic parameters (i.e., hydraulic conductivity and Lame constants), CCP is applied to transfer the deterministic management model into the stochastic form. The resulting stochastic management model enables the determination of optimal total pumpage subject to the constraints that drawdown and land subsidence do not exceed the maximum allowable values under specified reliabilities.

Methodology

The development of proposed stochastic groundwater management model consists of following four major steps: (1) using the response matrix technique in conjunction with one-dimensional consolidation equation to formulate a deterministic groundwater management model with land subsidence constraints; (2) analyzing statistical features of unit response coefficients; (3) assessing statistical features of drawdown and land subsidence by the FOVE method; and (4) converting deterministic constraints into CCP format using the results from Step (3). The theoretical basis for each step is described in detail in the following.

Simulation Model for Land Subsidence

Land subsidence is a very complex phenomenon which could occur in many ways (Whittaker and Reddish 1989). In this study, groundwater overpumping is considered as the only factor causing land subsidence. As the total stress of soil is a constant, dewatering of aquifers due to pumpage results in decreased pore water pressure and increased effective stress, which causes consolidation and subsequently land subsidence. The general three-dimensional governing equation of groundwater flow for a saturated aquifer can be stated as

$$\frac{\partial}{\partial x}\left(K_x\frac{\partial \Delta h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial \Delta h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial \Delta h}{\partial z}\right)=-S_s\frac{\partial \Delta h}{\partial t}-S \quad (1)$$

where K_x, K_y, and K_z=components of hydraulic conductivity $[\mathrm{L\,T^{-1}}]$ in x, y, and z directions of Cartesian coordinate system; Δh=drawdown with positive value denoting a decrease in hydraulic head [L]; t=time [T]; S_S=specific storage $[\mathrm{L^{-1}}]$; and S=sink or source term.

In this study, an uncoupled numerical model (Tsai 2001) consisting of depth-averaged two-dimensional groundwater simulation and one-dimensional consolidation equation is used to simulate land subsidence due to groundwater extraction. Integrating Eq. (1) vertically over the thickness of one layer, one obtains

$$\int_b\left[\frac{\partial}{\partial x}\left(K_x\frac{\partial \Delta h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial \Delta h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial \Delta h}{\partial z}\right)\right]dz$$
$$=\int_b\left[-S_s\frac{\partial \Delta h}{\partial t}-S\right]dz \quad (2)$$

where b=boundary for the layer considered.

From Leibnitz rule and chain rule, assuming that the soil parameters are constant along vertical direction, Eq. (2) can further be written as

$$KB\frac{\partial^2\overline{\Delta h}}{\partial x^2}+KB\frac{\partial^2\overline{\Delta h}}{\partial y^2}+\left[\frac{\partial}{\partial x}(KB)\right]\frac{\partial\overline{\Delta h}}{\partial x}+\left[\frac{\partial}{\partial y}(KB)\right]\frac{\partial\overline{\Delta h}}{\partial y}$$
$$=-S_sB\frac{\partial\overline{\Delta h}}{\partial t}+K\left(\left.\frac{\partial \Delta h}{\partial z}\right|_{b1}-\left.\frac{\partial \Delta h}{\partial z}\right|_{b2}\right)-\bar{S} \quad (3)$$

where $\overline{\Delta h}$ and $\bar{S}$=vertical averaged drawdown and sink/source term, respectively. Eq. (3) is a depth-averaged two-dimensional governing equation adopted in this study for ground water flow simulation.

According to Bear and Verruijt (1987), by assuming that (1) soil matrix is isotropic; (2) soil stress–strain relationship relating average effective stress and the average displacement follows Hooke's law of linear elasticity; and (3) displacements occur only in the vertical direction, the relationship between pore water pressure change and soil vertical strain can be stated as

$$\frac{\partial w_z}{\partial z}=\frac{p^e}{2\mu+\lambda} \quad (4)$$

where w_z=vertical displacement of soil [L]; p^e=incremental pore water pressure $[\mathrm{M\,L^{-1}\,T^{-2}}]$; μ and λ=Lame constant at a point $[\mathrm{M\,L^{-1}\,T^{-2}}]$. The two Lame constants μ and λ are statistically independent and they represent the elastic coefficients, which are determined experimentally for a given porous matrix. More detail descriptions about Lame constants can be found elsewhere (Reismann and Pawlik 1980; Bear and Verruijt 1987). Integrating Eq. (4) along the z axis, and neglecting the soil swell due to the increase of pore water pressure, one can obtain the one-dimensional consolidation equation as

$$\Delta s(k,t)=\begin{cases}\dfrac{\rho_w g B_k[\Delta h(k,t)-\Delta h(k,t-1)]}{2\mu_k+\lambda_k} & \text{if } \Delta h(k,t)>\Delta h(k,t-1)\\ 0 & \text{if } \Delta h(k,t)\leqslant\Delta h(k,t-1)\end{cases} \quad (5)$$

where $\Delta s(k,t)$=land subsidence at point k during the tth time period [L]; $\Delta h(k,t)$, $\Delta h(k,t-1)$=drawdowns of point k at the end of the tth and $(t-1)$th time periods, respectively; ρ_w=density of water $[\mathrm{M\,L^{-3}}]$; g=gravitational acceleration $[\mathrm{L\,T^{-2}}]$; and B_k=layer thickness at point k[L].

Tsai (2001) performed an order-of-magnitude analysis on the general three-dimensional governing equations involving groundwater flow and soil displacement. Assume that the displacement of soil in vertical direction is much larger than that in horizontal, the order-of-magnitude analysis indicates that one-dimensional simplification is adequate when groundwater flow pattern is approximately horizontal or vertical. This approximation is plausible for large-scale regional multilayer aquifer systems as groundwater flow is commonly assumed to be horizontal in aquifer and vertical in aquitard (Anderson and Woessner 1991).

Notice that the consolidation equation is derived through elastic body theorem, thus it cannot simulate the time delay effect of soil compaction. According to Biot (1941), Helm (1987), and Gutierrez and Lewis (2002), the one-dimensional approximation of consolidation can provide satisfactory estimation for general application. However, one should realize that the time delay effect will be more significant when the soil is very soft or the layer is thick.

Numerically, finite analytic method is applied to solve the depth-averaged two-dimensional groundwater flow governing equation, Eq. (3), and then Eq. (5) to compute land subsidence for each time period and layer. Detailed descriptions of this model can be found in Tsai (2001).

Deterministic Management Model

For a groundwater hydraulic management problem involving pumpage maximization subject to drawdown and land subsidence constraints, the deterministic management model can be formulated as:

Maximize

$$\sum_{j=1}^{NP}\sum_{t=1}^{NT} Q(j,t) \tag{6}$$

Subject to

$$\Delta h(k,t) \leqslant \Delta h^*(k,t), \quad t=1,\ldots,NT; \quad k=1,\ldots,NC \tag{7}$$

$$\Delta s(k,t) \leqslant \Delta s^*(k,t) \quad t=1,\ldots,NT; \quad k=1,\ldots,NC \tag{8}$$

$$0 \leqslant Q(j,t) \leqslant Q^*(j,t), \quad t=1,\ldots,NT; \quad j=1,\ldots,NP \tag{9}$$

where NP = number of pumping wells; NT = number of time periods; NC = number of control points; $Q(j,t)$ = pumpage at the jth pumping well during the tth time period $[L^3\,T^{-1}]$; $Q^*(j,t)$ = allowable pumpage at the jth pumping well during the tth time period $[L^3\,T^{-1}]$; $\Delta h^*(k,t)$ = allowable drawdown of the kth control point at the end of the tth time period [L]; $\Delta s^*(k,t)$ = allowable land subsidence at the kth control point during the tth time period [L]. Without considering constraints (8), there is a potential to overestimate total pumpage and, subsequently, undesirable land subsidence would occur.

To calculate the drawdown by the response matrix technique Eq. (7) can be rewritten as

$$\Delta h(k,t) = \sum_{j=1}^{NP}\sum_{i=1}^{t} \beta(k,j,t-i+1)Q(j,i) \leqslant \Delta h^*(k,t),$$
$$t=1,\ldots,NT; \quad k=1,\ldots,NC \tag{10}$$

where β = unit response coefficient representing the drawdown at the kth control point at the end of the tth time period due to unit pumpage at the jth pumping well during the ith time period.

Note that in the aquitard, the response matrix method would not be appropriate due to the storage effect results in the nonlinear relation between head and pumpage. However, according to Bredehoeft and Pinder (1970), if the nondimensional time factor $t^* = Kt/S_sB^2$ is larger than 0.5, the storage effect can be neglected. For general aquitards, the values of K and S_s are in the order of 10^{-9} m/s and 10^{-5} m^{-1}, respectively (Bear and Verruijt 1987). Assume that the aquitard thickness (B) is in the order of 10^1 m, as long as the management time period (t) is longer than 10^7 s (i.e., approximately 4 months), the value of t^*, through an order-of-magnitude analysis, would be larger than 0.5 which justifies the applicability of the response matrix method.

Using Eq. (5), constraint Eq. (8) can be rewritten as

$$\Delta s(k,t) = \frac{\rho_w g B_k G(k,t)}{2\mu_k + \lambda_k} \leqslant \Delta s^*(k,t),$$
$$t=1,\ldots,NT; \quad k=1,\ldots,NC \tag{11}$$

where $G(k,t) = \max(0, \Delta h(k,t) - \Delta h(k,t-1))$, indicating that the land subsidence occurs only if the drawdown is increased during the tth time period.

The deterministic management model is composed of objective function (6) and constraints (9)–(11). Because G is a nondifferentiable function at the origin, the management model is a nonsmooth optimization problem which can be solved by several algorithms. However, the convergence and global optimality of the solution could not be guaranteed, especially when the problem size (in terms of the number of constraints and decision variables) is large (Uryas'ev and Dong 1991; Yang, 2001). To circumvent such a situation, the nonsmooth optimization problem is transformed into mixed integer linear programming (MILP) by introducing additional binary variables, $m(k,t)$, and new constraints as

$$\Delta h(k,t) - \Delta h(k,t-1) + LO \times m(k,t) \geqslant LO \quad \forall\, t; \forall\, k \tag{12}$$

$$\Delta h(k,t) - \Delta h(k,t-1) - UP \times m(k,t) \leqslant 0 \quad \forall\, t; \forall\, k \tag{13}$$

$$\Delta h(k,t) - \Delta h(k,t-1) - G(k,t) \leqslant 0 \quad \forall\, t; \forall\, k \tag{14}$$

$$\Delta h(k,t) - \Delta h(k,t-1) - G(k,t) - UP \times m(k,t) \geqslant -UP \quad \forall\, t; \forall\, k \tag{15}$$

$$G(k,t) - UP \times m(k,t) \leqslant 0 \quad \forall\, t; \forall\, k \tag{16}$$

$$G(k,t) \geqslant 0 \quad \forall\, t; \forall\, k \tag{17}$$

where LO, UP = negative and positive coefficients with large value, respectively, and $m(k,t) = 0$ or 1 only. If $m(k,t) = 0$, the drawdown is decreasing at control point k during the tth time period [i.e., Eq. (13)], and the land subsidence would not occur. On the other hand, if $m(k,t) = 1$, the drawdown is increasing at control point k during the tth time period [i.e., Eq. (12)] and constraint Eqs. (14) and (15) impose that the value of G equals the drawdown during the tth time period.

In this paper, the previous MILP is solved by the branch-and-bound (B&B) method (Floudas 1995).

Analysis of Statistical Features of Unit Response Coefficients

By the response matrix technique the assessment of statistical features of unit response coefficient is essential for quantifying the uncertainty associated with the resulting drawdown. A unit response coefficient represents the drawdown at one control point due to a unit pumpage at a production well which is a function of random hydrogeologic parameters and boundary conditions, etc. As the consequence of geologic process through which groundwater systems evolve, hydrogeologic parameters of an aquifer vary through space. In practical groundwater system modeling, one normally would not have sufficient data to completely describe the heterogeneity of an aquifer. Therefore, unit response coefficients are subject to uncertainty. In a transient groundwater flow model, hydraulic conductivity and specific storage are the two major parameters with random spatial variability. According to Tung (1986), the variation of specific storage does not significantly affect groundwater flow prediction; hence only the uncertainty of hydraulic conductivity is considered to assess the uncertainty of unit response coefficients. A typical assumption made for most hydraulic conductivity random field is that it is second-order stationary (Wagner and Gorelick 1989; Mylopoulos et al. 1999).

Assume that distribution of hydraulic conductivity is lognormal (Gelhar 1993) and the log-hydraulic conductivity random field, $Y = \ln(K)$, is isotropic with exponential covariance structure, the mean and covariance function for the random log-hydraulic conductivity field are

$$E[Y_k] = \mu_Y \tag{18}$$

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$$\mathrm{Cov}[Y_1,Y_2]=\sigma_Y^2\exp\left\{-\frac{\|\Delta\mathbf{x}_{12}\|}{a_Y}\right\} \tag{19}$$

where $E[\ ]$ denotes expectation; $\mathrm{Cov}[\ ]$ denotes covariance; $Y_k=\ln(K)$=natural, logarithm of hydraulic conductivity at point $\mathbf{x}_k$; μ_Y=mean of log-hydraulic conductivity; σ_Y=standard deviation (S.D.) of log-hydraulic conductivity; $|\Delta\mathbf{x}_{12}|$=distance separating the two points in space; and a_Y=correlation scale of log-hydraulic conductivity. Thus, the random log-hydraulic conductivity field considered herein is statistically characterized by μ_Y, σ_Y, and a_Y.

Once the three statistical parameters of the second-order stationary and correlated two-dimensional random field, $\mathbf{Y}$, are known, the covariance matrix $\mathbf{C}(\mathbf{Y})$ and correlation matrix $\mathbf{R}(\mathbf{Y})$ can be determined from which stastically plausible realizations of $\mathbf{Y}$ can be generated by

$$\mathbf{Y}=\boldsymbol{\mu}_Y+\mathbf{D}^{1/2}\mathbf{V}\boldsymbol{\Lambda}^{1/2}\mathbf{w} \tag{20}$$

where $\mathbf{D}^{1/2}$=diagonal matrix of standard deviations, $\boldsymbol{\sigma}_Y$; $\mathbf{V}$=eigenvector matrix consisting of normalized eigenvectors of the correlation matrix, $\mathbf{R}(\mathbf{Y})$; $\boldsymbol{\Lambda}$=diagonal matrix of eigenvalues of $\mathbf{R}(\mathbf{Y})$; and $\mathbf{w}$=vector of independent standard normal random variables. A large number of realizations of $\mathrm{In}(K)$ can be generated and used in the groundwater simulation model to produce samples of unit response coefficients for assessing their statistical properties.

In this study, the Latin hypercube sampling (LHS), (McKay 1988) is adopted for data generation. McKay et al. (2000) showed that the LHS is a good method for generating model inputs and had been applied to various problems in groundwater (Gwo et al. 1996; Christiaens and Feyen 2001). It is a variance-reduction technique and requires fewer computer runs to achieve the degree of precision comparable to that obtained from a simple random sampling scheme.

Statistical Properties of Drawdown and Land Subsidence

From Eqs. (10) and (11), the drawdown and land subsidence are functions of uncertain parameters including unit response coefficients and Lame constants. Hence, the estimated drawdown and land subsidence by the model are subject to uncertainty. As the drawdown is a linear combination of random unit response coefficients, its distribution can be approximated by a normal distribution based on the central limit theorem. For simplicity, the distribution of land subsidence also is assumed to be normal. The validity of normality assumption will be discussed later through an example application. With the distributions of drawdown and land subsidence assumed normal, their statistical information can be defined by the respective expected values and variances. Several methods can be applied to estimate the expected value and variance of a function involving multiple stochastic parameters. In this study, the FOVE method is applied to estimate the first two moments of unit response coefficients and its applications to groundwater problems can be found elsewhere (Nguyen and Chowdhury 1985; Ricardo et al. 1999; Ricardo and Keith 2000). In theory, random unit response coefficients are correlated due to spatial correlation of hydraulic conductivity. Although the covariance among unit response coefficients can be estimated by the LHS technique, in conjunction with numerical groundwater flow simulation, the independence assumption of unit response coefficients is adopted herein for the sake of ignoring nonlinear terms in the CCP. From Eq. (10), the expected value and variance of drawdown can be obtained as

$$E[\Delta h(k,t)]=\sum_{j=1}^{\mathrm{NP}}\sum_{i=1}^{t}E[\beta(k,j,t-i+1)]Q(j,i)\quad \forall\, t, \forall\, k \tag{21}$$

$$\mathrm{Var}[\Delta h(k,t)]=\sum_{j=1}^{\mathrm{NP}}\sum_{i=1}^{t}\mathrm{Var}[\beta(k,j,t-i+1)]Q^2(j,i)\quad \forall\, t, \forall\, k \tag{22}$$

where $\mathrm{Var}[\ ]$ denotes the variance operator.

Referring to Eq. (5), B_k is assumed to be constant as land subsidence is typically several orders of magnitude smaller than the thickness of an aquifer. Thus from the FOVE method, the expectation and variance of land subsidence can be estimated as

$$E[\Delta s(k,t)]=\frac{\rho_w g B_k\overline{G(k,t)}}{2\overline{\mu}_k+\overline{\lambda}_k}\quad \forall\, t, \forall\, k \tag{23}$$

$$\mathrm{Var}[\Delta s(k,t)]=\left(\frac{\rho_w g B_k}{2\overline{\mu}_k+\overline{\lambda}_k}\right)^2\mathrm{Var}[G(k,t)]+\{4\,\mathrm{Var}[\mu_k]+\mathrm{Var}[\lambda_k]\}\times\left[\frac{\rho_w g B\overline{G(k,t)}]}{(2\overline{\mu}_k+\overline{\lambda}_k)^2}\right]^2\quad \forall\, t, \forall\, k \tag{24}$$

In Eq. (24), random variables μ, λ, and G are assumed to be independent of each other.

In Eqs. (21)–(24), the $Q(i,j)$'s are the decision variables. The expectation and variance of β can be found by the LHS technique along with model simulation described in the previous section whereas the expectation and variance of parameters μ and λ can be determined from field investigation. The remaining problem is how to quantify the statistical properties of G. As $G(k,t)=\max[0,\Delta h(k,t)-\Delta h(k,t-1)]$, G is a mixed variable having a continuous probability density function for $G>0$ and a probability mass function at $G=0$. The actual expected value and variance of G are

$$E[G(k,t)]=E[\Omega]-\int_{-\infty}^{0}\omega f(\omega)d\omega\quad \forall\, t, \forall\, k \tag{25}$$

$$\mathrm{Var}[G(k,t)]=E[G^2(k,t)]-E[G(k,t)]^2\quad \forall\, t, \forall\, k \tag{26}$$

where $\Omega=\Delta h(k,t)-\Delta h(k,t-1)$; $f(\omega)$ denotes the probability density function of random Ω which can be assumed normal because it is the linear combination of two normally distributed random drawdowns. The graphical representation of Eq. (25) is shown in Fig. 1.

The explicit form of Eq. (25) can be obtained, but it would greatly increase the computing complexity due to the presence of complementary error function arising from the integration operation. As the objective function is to maximize pumping rate, the probability density function of Ω moves to the right of x axis which makes the integration part of Eq. (25) smaller. Thus, the expected value and variance of G can be approximated as

$$\overline{G(k,t)}=\max[0,E[\Delta h(k,t)-\Delta h(k,t-1)]]\quad \forall\, t, \forall\, k \tag{27}$$

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$$\mathrm{Var}[G(k,t)] = \begin{cases} \mathrm{Var}[\Delta h(k,t) - \Delta h(k,t-1)] & \text{if } E[\Delta h(k,t) - \Delta h(k,t-1)] \geqslant 0 \\ 0 & \text{if } E[\Delta h(k,t) - \Delta h(k,t-1)] < 0 \end{cases} \quad \forall\, t, \forall\, k \tag{28}$$

where

$$E[\Delta h(k,t) - \Delta h(k,t-1)] = \sum_{j=1}^{\mathrm{NP}} \sum_{i=1}^{t} E[\beta(k,j,t-i+1)]Q(j,i) - \sum_{j=1}^{\mathrm{NP}} \sum_{i=1}^{t-1} E[\beta(k,j,t-i)]Q(j,i) \quad \forall\, t, \forall\, k \tag{29}$$

Stochastic Management Model

The CCP model specifies a required reliability for the operational constraints that the optimal pumping pattern would not fail due to parameter uncertainty. The deterministic constraints (7) and (8) can be transformed to probabilistic statements in the form of chance constraints as

$$\Pr[\Delta h(k,t) \leqslant \Delta h^*(k,t)] \geqslant \alpha_h(k,t), \quad t=1,\ldots,\mathrm{NT}; \quad k=1,\ldots,\mathrm{NC} \tag{30}$$

$$\Pr[\Delta s(k,t) \leqslant \Delta s^*(k,t)] \geqslant \alpha_s(k,t), \quad t=1,\ldots,\mathrm{NT}; \quad k=1,\ldots,\mathrm{NC} \tag{31}$$

where Pr[ ] is the probability operator; and $\alpha_h(k,t)$ and $\alpha_s(k,t)$ are, respectively, required reliabilities that the actual drawdown and land subsidence would not exceed the specified maximum allowable values at the kth control point at the end of the tth time period. The value of required reliability is stipulated by the decision-maker which could vary with locations and times.

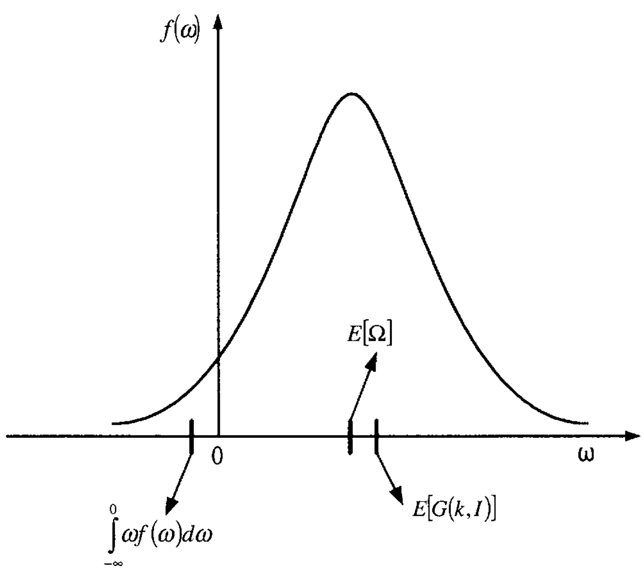

Fig. 1. Schematic diagram of Eq. (24)

To solve chance-constrained equations (30) and (31), conversion to their respective deterministic equivalents is required. As the distribution function of total drawdown is assumed to be normal, Eq. (30) can be expressed as

$$\Pr\left\{Z \leqslant \frac{\Delta h^*(k,t) - E[\Delta h(k,t)]}{\sqrt{\mathrm{Var}[\Delta h(k,t)]}}\right\} \geqslant \alpha_h(k,t) \quad \forall\, t, \forall\, k \tag{32}$$

where Z=standard normal random variable. The deterministic equivalent of Eq. (32) can be written as

$$\sqrt{\mathrm{Var}[\Delta h(k,t)]} \times \Phi^{-1}[\alpha_h(k,t)] + E[\Delta h(k,t)] \leqslant \Delta h^*(k,t) \quad \forall\, t, \forall\, k \tag{33}$$

where $\Phi^{-1}[\alpha_h(k,t)]$=standard normal quantile corresponding to the compliance probability of $\alpha_h(k,t)$. Similarly, the deterministic equivalent of Eq. (31) can be expressed as

$$\sqrt{\mathrm{Var}[\Delta s(k,t)]} \times \Phi^{-1}[\alpha_s(k,t)] + E[\Delta s(k,t)] \leqslant \Delta s^*(k,t) \quad \forall\, t; \forall\, k \tag{34}$$

In Eqs. (33) and (34), the expectation and variance of drawdown and land subsidence can be determined by Eqs. (21)–(24). Again, to avoid solving nonsmooth constraints [i.e., Eqs. (27) and (28)], binary variables are introduced. Thus, in addition to constraints (9), (33), and (34), the following new constraints are involved:

$$E[\Delta h(k,t) - \Delta h(k,t-1)] + \mathrm{LO} \times m(k,t) \geqslant \mathrm{LO} \quad \forall\, t, \forall\, k \tag{35}$$

$$E[\Delta h(k,t) - \Delta h(k,t-1)] - \mathrm{UP} \times m(k,t) \leqslant 0 \quad \forall\, t, \forall\, k \tag{36}$$

$$E[\Delta h(k,t) - \Delta h(k,t-1)] - E[G(k,t)] \leqslant 0 \quad \forall\, t, \forall\, k \tag{37}$$

$$E[\Delta h(k,t) - \Delta h(k,t-1)] - E[G(k,t)] - \mathrm{UP} \times m(k,t) \geqslant -\mathrm{UP} \quad \forall\, t, \forall\, k \tag{38}$$

$$E[G(k,t)] - \mathrm{UP} \times m(k,t) \leqslant 0 \quad \forall\, t, \forall\, k \tag{39}$$

$$E[G(k,t)] \geqslant 0 \quad \forall\, t, \forall\, k \tag{40}$$

$$\mathrm{Var}[G(k,t)] - m(k,t)\mathrm{Var}[\Delta h(k,t) - \Delta h(k,t-1)] = 0 \quad \forall\, t, \forall\, k \tag{41}$$

Except for Eq. (41), the newly introduced constraints [Eqs. (35)–(40)] are similar to those in the deterministic MILP management model. As the value of $m(k,t)$ indicates whether the hydraulic head at control point k during the time period t is increasing or decreasing, Eq. (41) can be easily explained by Eq. (28). The resulting stochastic management model becomes a mixed integer nonlinear programming (MINLP). In this study, the B&B method is applied to solve the stochastic management model.

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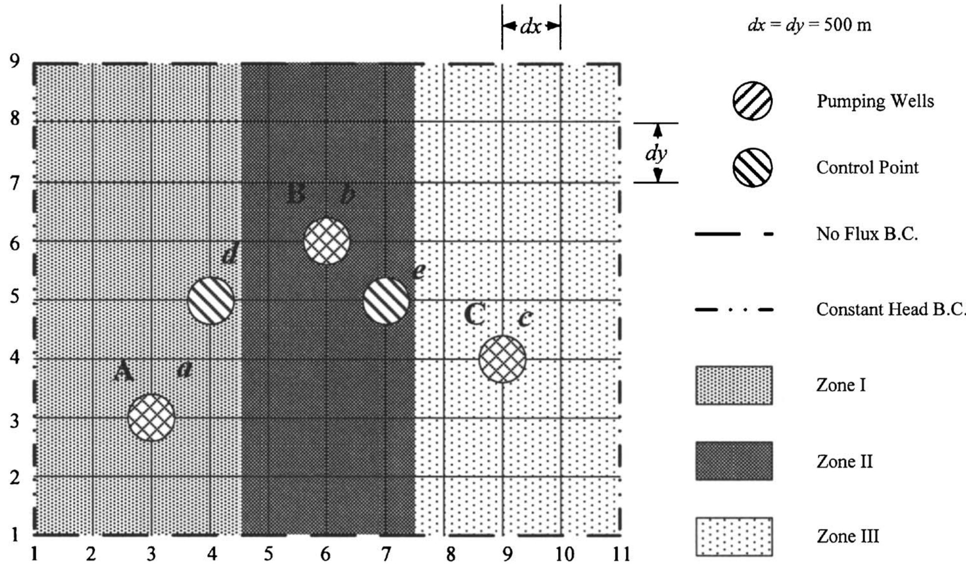

Fig. 2. Hypothetical groundwater system used in example

Application

Consider a hypothetical confined aquifer basin with three pumping wells (A, B, and C) and five control points (*a*, *b*, *c*, *d*, and *e*) as shown in Fig. 2. The control points (*a*, *b*, and *c*) coincide with the Production Wells A, B, and C, respectively. The aquifer basin is divided into three zones (I, II, and III) based on their different, but isotropic, hydrogeologic parameters in each zone. The aquifer domain is discretized into 99 (11×9) nodes with equal grid space of $500 \text{ m} \times 500 \text{ m}$. The boundary conditions are no flux at the top and bottom while a constant head on the sides. Initially, the piezometric heads are uniformly distributed and no groundwater flow occurred. For alluvial sand aquifer, the standard deviation of log-hydraulic conductivity ($\ln K$) varies from 0.4 to 1.2 (Gelhar 1993) and the Lame constant varies up to five times of the typical range (Das 1983). Both hydraulic conductivity and Lame constants are assumed to follow log-normal distribution. Five cases involving different uncertainty levels of parameters are considered (see Table 1) in that parameters uncertainties increase from Case 1 to Case 5, whereas the mean values of the parameters are maintained constant. The layer thickness is 80 m and the correlation scale of log-hydraulic conductivity is assumed to be 1,000 m for all three zones. The specific storage values are 5.3×10^{-6}, 7.0×10^{-6}, and $1.4 \times 10^{-5}\ \text{m}^{-1}$, respectively, in Zones I, II, and III.

The problem is to determine the maximum total amount of

Table 1. Statistical Properties of Aquifer Parameters of Each Zone and Case

		Zone-I		Zone-II		Zone-III	
Parameter	Case	Mean	S.D.	Mean	S.D.	Mean	S.D.
K (m/s) ($\ln K$)	1	5.0×10^{-5} (−9.98)	2.0×10^{-5} (0.39)	2.0×10^{-4} (−8.60)	8.3×10^{-5} (0.40)	5.0×10^{-4} (−7.68)	2.0×10^{-4} (0.39)
	2	5.0×10^{-5} (−10.08)	3.3×10^{-5} (0.60)	2.0×10^{-4} (−8.70)	1.3×10^{-4} (0.59)	5.0×10^{-4} (−7.78)	3.3×10^{-4} (0.60)
	3	5.0×10^{-5} (−10.22)	4.8×10^{-5} (0.81)	2.0×10^{-4} (−8.84)	1.9×10^{-4} (0.80)	5.0×10^{-4} (−7.92)	4.7×10^{-4} (0.80)
	4	5.0×10^{-5} (−10.40)	6.6×10^{-5} (1.00)	2.0×10^{-4} (−9.02)	2.6×10^{-4} (1.00)	5.0×10^{-4} (−8.10)	6.6×10^{-4} (1.00)
	5	5.0×10^{-5} (−10.62)	9.0×10^{-5} (1.20)	2.0×10^{-4} (−9.24)	3.5×10^{-4} (1.18)	5.0×10^{-4} (−8.32)	9.0×10^{-4} (1.20)
μ (Nt/m^2)	1	5×10^{8}	5×10^{7}	5×10^{8}	5×10^{7}	1×10^{8}	5×10^{7}
	2		1×10^{8}		1×10^{8}		1×10^{8}
	3		2×10^{8}		2×10^{8}		2×10^{8}
	4		3×10^{8}		3×10^{8}		3×10^{8}
	5		5×10^{8}		5×10^{8}		5×10^{8}
λ (Nt/m^2)	1	1×10^{9}	5×10^{7}	5×10^{8}	5×10^{7}	5×10^{8}	5×10^{7}
	2		1×10^{8}		1×10^{8}		1×10^{8}
	3		2×10^{8}		2×10^{8}		2×10^{8}
	4		3×10^{8}		3×10^{8}		3×10^{8}
	5		5×10^{8}		5×10^{8}		5×10^{8}

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Table 2. Pumping Rate, Drawdown, and Land Subsidence under Optimality for Deterministic Application

Time period	Pumping rate at pumping well (m^3/s)			Drawdown at control point (m)			Subsidence at control point (cm)		
	A	B	C	*a*	*b*	*c*	*a*	*b*	*c*
1	0.00	0.28	0.95	3.52	15.00	15.00	0.14	0.79	1.68
2	0.00	0.57	1.77	3.41	15.00	13.38	0.13	0.79	1.50
3	0.00	0.89	2.23	3.07	15.00	8.92	0.12	0.78	1.00

pumpage from the three production wells over three time periods of one year each, such that the resulting drawdown and land subsidence at all control points will not exceed specified allowable values. The allowable drawdown values at each control points over each time period are 15 m. To control land subsidence, the allowable subsidence at each control points are 3, 1.5, and 1.0 cm for Time Periods 1, 2, and 3, respectively. The decision variables, i.e., pumping rate at each production well over the three time periods, are non-negative and a uniform reliability is specified for all constraints.

Deterministic Application

Mean values of the parameters are used in the deterministic management model. The optimal pumping rate for each production well is summarized in Table 2 from which one can find that Well A does not produce groundwater, whereas Well C is the most productive for groundwater extraction. This is because Zone I has the smallest hydraulic conductivity (5×10^{-5} m/s). For drawdown, the active constraints (i.e., constraints with equality sign under optimality) occur at Control Point *b* in all three time periods and *c* in the first time period. For land subsidence, the active constraints occur at Control Point *c* in the last two time periods. This consequence implies that, without land subsidence constraints, the optimal total pumpage will be higher. To further increase pumpage will result in subsidence exceeding the maximum allowable values of 1.5 and 1.0 cm at Point *c* in the second and third time periods, respectively. Thus, the optimal total pumpage may be overestimated if one only considers drawdown constraints.

Stochastic Application

Based on 5,000 realizations of random hydraulic conductivity field and Lame constants generated by the LHS, statistical mean and variance of unit response coefficients are assessed. Fig. 3 shows the trade-off curves between reliability and optimal total pumpage for the five cases listed in Table 1. Table 1 shows that the optimal total pumpage decreases with increase in both required compliance reliability and the degree of uncertainty of hydrogeological parameters. Referring to Eqs. (33) and (34), a 50% compliance reliability corresponds to risk-neutral management in which the variances of drawdown and land subsidence have no effect on the stochastic constraints as the corresponding value of standard normal quantile is zero. Thus, the optimal pumping scheme obtained from the deterministic model and stochastic model with 50% compliance reliability should be identical. However, Fig. 3 shows that the optimal pumpage decreases with increase in parameter uncertainty under the condition of 50% compliance reliability. This phenomenon can be explained due to the fact that the unit impulse response coefficients for drawdown are not linearly related to the hydrogeological parameter, *K*. Under the condition of constant mean values for the hydrogeological parameter, an increase in its corresponding uncertainty would enhance the likelihood of realizing both smaller and larger values of hydrogeologic parameter. In addition, the sensitivity of unit impulse response coefficients with respect to hydrogeological parameter is higher when the parameter values are smaller. The combined effects of nonlinearity and uneven sensitivity result in an increase in the mean value of the unit impulse response coefficients as the uncertainty of hydrogeological parameter increases. Fig. 4 shows the histograms of the unit response coefficient for Cases 1, 3, and 5. Hence, even the effect of variances can be ignored under the 50% compliance reliability, the mean values of unit impulse response coefficients on the left-hand side of Eqs. (33) and (34) increase with hydrogeological parameter uncertainty. As a result, to meet the drawdown constraints the optimal total pumpage would have to decrease with increase in hydrogeological parameter uncertainty.

Fig. 3 also shows that, for Cases 1–3 with relatively small uncertainty, the rate of decrease in optimal total pumpage increases as the required compliance reliability reaches 90% or higher. This is because the value of standard normal quantile, $\Phi^{-1}(\alpha)$ in Eqs. (33) and (34), increases in a faster rate than the reliability value, α. The implication is that when the aquifer system is to be operated at very high reliability level, the optimal total pumpage will becomes increasingly sensitive to the stipulated compliance reliability and the decision-maker would have to pay more attention to the trade-off between the total pumpage and desired target reliability.

Verification of Stochastic Management Model

To verify the developed stochastic groundwater management model, a postoptimality analysis involving the LHS technique is

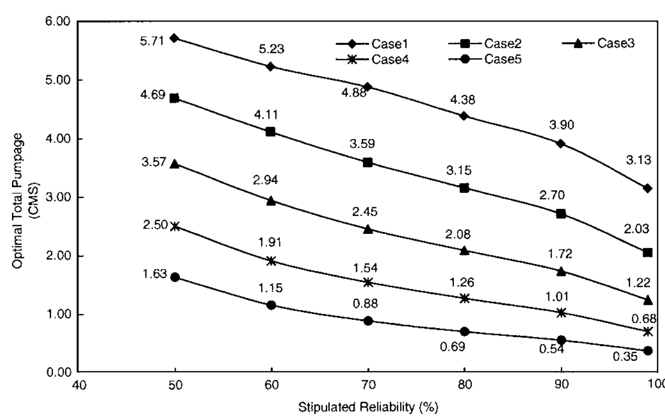

Fig. 3. Trade-off curves between stipulated reliability and optimal total pumpage

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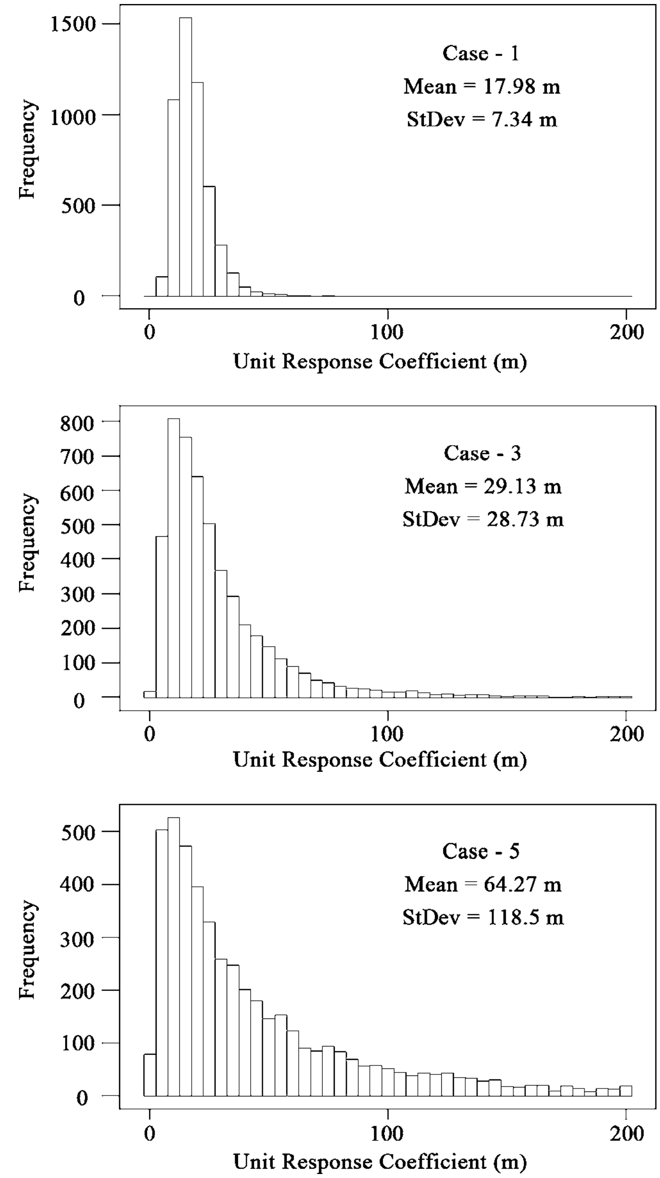

Fig. 4. Histogram of 5,000 unit response coefficient data at control point *a* for the first time period due to the unit pumpage of Well A

performed (see Fig. 5). In the analysis, the LHS technique generates 5,000 realizations of random hydrogeological parameter (i.e., K, μ, and λ) field following Eqs. (18)–(20). The generated realizations of parameter field, along with the optimal pumping rates under various compliance reliabilities of 50, 60, 70, 80, 90, and 99%, are used to compute the corresponding drawdown and land subsidence from the groundwater flow simulation model at each control point. To assess the accuracy of the developed stochastic management model, following analyses are performed: (1) compare the means and standard deviations of drawdown and land subsidence between the FOVE method and LHS simulation; (2) compare the actual and specified reliabilities at the control points where the corresponding constraint are active; and (3) check the normality assumption of land subsidence and drawdown. The details of the analyses are described in the following.

Although several compliance reliabilities have been considered in this study, all the verification results for the proposed stochastic management model show a similar tendency. Therefore, only the results under the condition of 90% compliance reliability are presented and discussed herein. Table 3 lists the mean and standard deviation of drawdown and land subsidence, as well as the actual compliance reliability at the control points corresponding to active constraints and system reliability under the optimal pumping pattern. The system reliability is defined as the condition under which drawdown and land subsidence constraints at all control points and time periods do not violate the allowable limits. In Table 3, "LHS" denotes that the mean and standard deviation are calculated on the basis of 5,000 realizations and "FOVE" denotes that the mean and standard deviation are calculated by Eqs. (21)–(24). The actual reliability (α^*) is calculated from LHS simulation.

From Table 3, one observes that even if the actual compliance reliability of each individual constraint is close to 90%, the actual system reliability ranges from 73 to 78%. Assuming that all constraints are statistically independent and the reliability associated with an inactive constraint is 100%, the expected system reliability is the multiple of reliabilities for each active constraint, which would be much lower than actual system reliability. This implies that the assumption of statistical independence among the constraints would not be reasonable; drawdown and subsidence constraints must be correlated.

From Table 3, the differences in drawdown statistics between the FOVE and LHS methods are minimal, even when the parameter uncertainty is large. This is expected because Eq. (21) yields exact mean drawdown as it is a linear function of random unit response coefficients whose mean values are obtained by the LHS technique. The standard deviation of drawdown between the FOVE and LHS has slight differences primarily due to the assumption of statistical independence between the unit response coefficients by Eq. (22). The results imply that independence assumption between the unit response coefficients in this example application is reasonable.

On the other hand, the differences in statistics of land subsidence between the FOVE and LHS methods get larger as parameter uncertainty increases. The largest relative difference, occurring in Case 5, reaches 50% for the mean and 45% for the standard deviation at control Point *c* during the second time period. The difference arises mainly from the nonlinearity of the consolidation equation. With the parameter uncertainty getting larger, the contribution of higher-order terms to the total variability of land subsidence, ignored by the FOVE method, would become significant. Although the accuracy of land subsidence uncertainty obtained by the FOVE method decreases with an increase in parameter uncertainty, the actual compliance reliabilities are quite close to the stipulated values. In Table 3, the largest discrepancy between the actual and stipulated compliance reliabilities of 90% is about 4% for Case 5, which has a rather large discrepancy in mean and standard deviation between the LHS and FOVE methods.

Based on the 5,000 realizations of land subsidence generated in the postoptimality analysis, Fig. 6 shows the histograms of land subsidence for Cases 1 and 5 at Control Point *c* in the third time period with 90% compliance reliability. From Fig. 6, it is observed that the distribution of land subsidence at Control Point *c* (same for other control points) becomes more positively skewed as parameter uncertainty increases. This is mainly owing to the combined effects of: (1) increased likelihood of realizing small Lame constants; and (2) increased model sensitivity with relatively small parameter values (K, μ, and λ), which results in a higher likelihood of getting large value of, and increased variability of, land subsidence.

From the previous discussions, the FOVE method tends to

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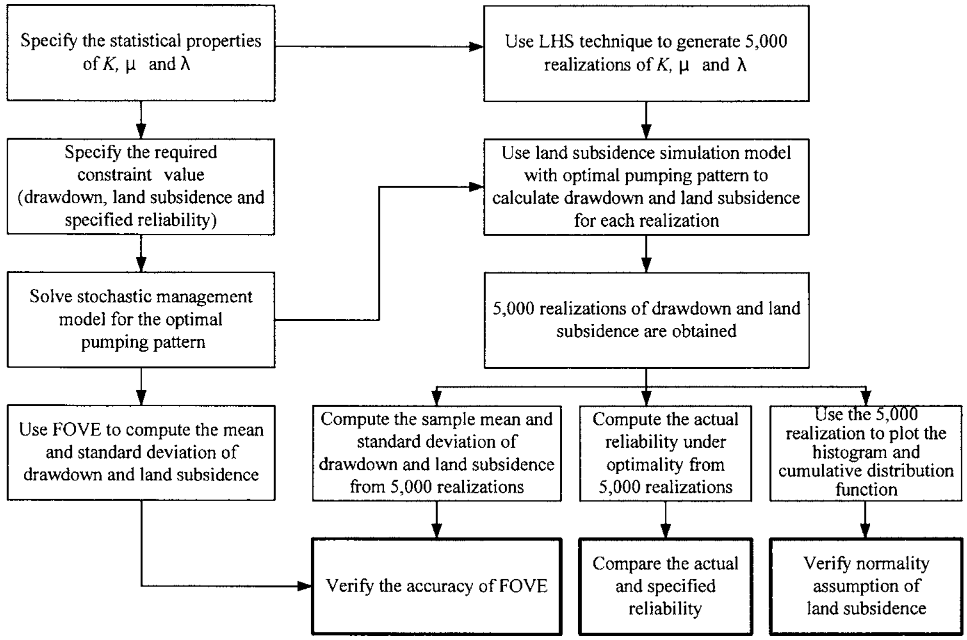

Fig. 5. Flow chart of postoptimality analysis

underestimate the mean and standard deviation of land subsidence as parameter uncertainty increases. If the land subsidence was to remain normally distributed when parameter uncertainty increases, the underestimated mean and standard deviation would result in the actual compliance reliability being much smaller than 90%. However, the land subsidence distribution becomes more positively skewed with an increase in parameter uncertainty. This results in a higher likelihood of occurring smaller and large land subsidence values. This phenomenon compensates the effect of underestimated mean and standard deviation, along with the normality assumption, resulting in relatively little difference between actual and stipulated compliance reliabilities.

From the results of numerical application, it is shown that the accuracy of proposed management model is dependent on the uncertainty level of parameters. Among the five test cases considered, according to previous studies (Das 1983; Gelhar 1993), the

Table 3. Comparison of Actual Reliability and Statistical Properties of Drawdown and Land Subsidence between Different Methods under Desired Compliance Reliability of 90%

Response	Control point	Time period (year)	Method	Case 1		Case 2		Case 3		Case 4		Case 5	
				Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
Δh (m)	b	1	FOVE	10.57	3.47	9.46	4.32	8.13	5.36	7.05	6.20	6.08	6.96
			LHS	10.55	3.47	9.46	4.29	8.12	5.32	7.04	6.20	6.03	6.87
			α^*	90.1%		90.2%		91.8%		92.4%		92.9%	
		2	FOVE	10.23	3.72	9.23	4.50	7.84	5.59	6.90	6.32	5.75	7.22
			LHS	10.23	3.72	9.23	4.50	7.83	5.60	6.90	6.40	5.75	7.35
			α^*	90.1%		90.2%		91.9%		92.5%		92.9%	
		3	FOVE	10.12	3.81	8.91	4.75	7.50	5.86	6.70	6.47	5.47	7.44
			LHS	10.12	3.82	8.91	4.76	7.49	5.88	6.70	6.55	5.47	7.58
			α^*	90.0%		90.4%		92.0%		92.6%		93.0%	
	c	1	FOVE	10.37	3.61	8.99	4.69	7.75	5.66	6.55	6.59	5.53	7.39
			LHS	10.37	3.61	8.99	4.66	7.74	5.63	6.54	6.60	5.50	7.32
			α^*	89.6%		90.6%		92.0%		93.1%		94.3%	
Δs (cm)	c	2	FOVE	1.00	0.39	0.85	0.51	0.68	0.64	0.56	0.74	0.42	0.84
			LHS	1.03	0.40	0.92	0.54	0.83	0.70	0.79	0.94	0.84	1.53
			α^*	88.5%		89.0%		88.0%		88.2%		87.4%	
		3	FOVE	0.69	0.24	0.58	0.33	0.46	0.42	0.39	0.48	0.29	0.57
			LHS	0.70	0.25	0.63	0.34	0.57	0.45	0.55	0.60	0.57	1.00
			α*	88.5%		89.5%		87.7%		87.2%		86.0%	
Expected system reliability				51.27%		53.08%		55.11%		56.67%		56.89%	
Actural system reliability				78.04%		72.64%		74.0%		74.02%		73.6%	

Note: a^*=actual reliability obtained from LHS simulation.

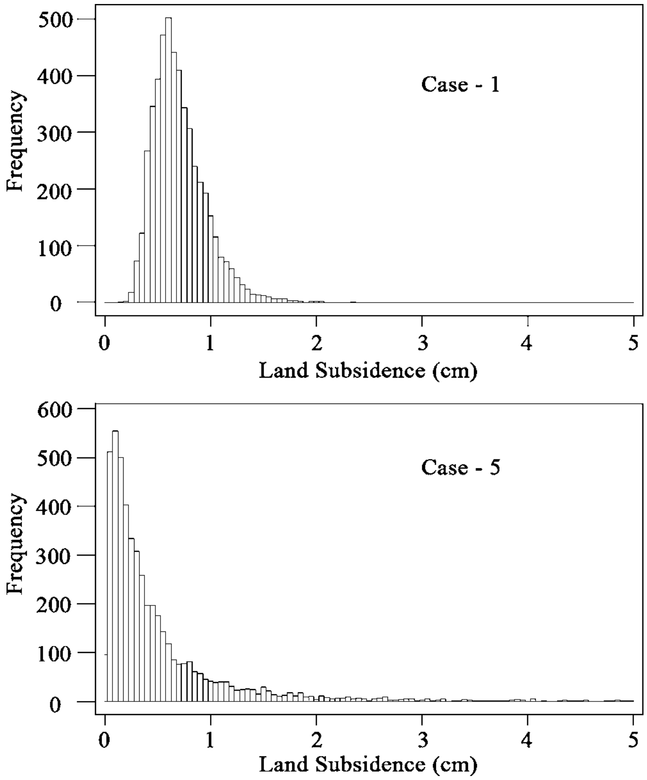

Fig. 6. Histogram of land subsidence data at control point *c* in the third time period under 90% compliance reliability

uncertainty level of hydraulic conductivity and Lame constants are chosen with a sufficiently wide range of variation to cover the hydrogeological parameter values for aquifers. Thus, the results of the application study substantiate the accuracy and applicability of the proposed stochastic management model.

Conclusions

In this paper, a stochastic groundwater management model explicitly considering land subsidence control is developed using the response matrix technique within the chance-constraint framework. A one-dimensional consolidation equation is used in the management model to explicitly consider land subsidence. The stochastic groundwater management model also explicitly considers the spatial randomness of hydraulic conductivity and Lame constants in the consolidation equation.

The developed management model was applied to a hypothetical example in that the deterministic management model was found to overestimate the optimal total pumpage that would cause undesirable land subsidence if only drawdown constraints were considered. Hence, joint consideration of drawdown and land subsidence is necessary if the latter is an important factor to consider in the groundwater management.

Five cases of varying degrees of parameter uncertainty are considered in the application of stochastic management model. The results of postoptimality indicated that the use of LHS to estimate the statistical features of the unit impulse response coefficients can provide a quite satisfactory estimation of uncertainty features of total drawdown even under high level of parameter uncertainty. On the other hand, the accuracy of estimated uncertainty features of land subsidence by the FOVE method deteriorates with increases in parameters uncertainty. This is primarily due to nonlinearity of the consolidation equation and higher sensitivity under relatively small parameter values. However, the actual compliance reliability interestingly matches closely to the stipulated value due to the compensating effect of positively skewed subsidence.

For complex real-world problems, the implementation for the proposed management model has two major concerns. The first one is the stability of the MINLP solver which is mostly dependent on the number of nonlinear constraints. Thus, removal of the nonnecessary control points can improve the efficiency of the MINLP solver. A review of the implementation for large scale MINLP can be found in Floquet et al. (1993). The second concern is the determination of parameter values. Based on the available field-experiment data and Wagner and Gorelick (1989), the parameter values can be generated. An issue arises as the experimental values of Lame constants are rare but available in literature. Fortunately, Reismann and Pawlik (1980) showed that the relationship between Lame constants and Young's modulus is linear. Once Young's modulus for an interesting aquifer is obtained, the Lame constants can then be decided.

In summary, in the region where the groundwater is the major resource for water supply and one wishes to control overpumpage to mitigate land subsidence hazards, the proposed stochastic model could be useful for optimal groundwater management such that the land subsidence could be controlled to achieve desired management objectives.

Acknowledgments

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Notation

The following symbols are used in this paper:

a_Y = correlation scale of log-hydraulic conductivity;
B = layer thickness of soil;
G = max[0 drawdown during one time period];
g = gravitation acceleration;
i = index of time period;
j = index of pumping well;
K = hydraulic conductivity;
k = index of control point;
LO = large negative coefficient;
m = binary variable which only equals 0 or 1;
Q = pumping rate;
S = sink or source term;
S_s = specific storage;
t = time;
UP = large positive coefficient;
Var[ ] = variance operator;
$\mathbf{W}$ = vector of independent standard normal random variables;
Y = natural logarithm of hydraulic conductivity;

α_h, α_s	=	specified reliability for drawdown and subsidence constraints, respectively;
β	=	unit response coefficient;
Δh	=	magnitude of drawdown;
Δs	=	magnitude of land subsidence;
λ	=	Lame constant;
μ	=	Lame constant;
μ_Y	=	expectation of log-hydraulic conductivity;
ρ_w	=	density of water;
σ_Y	=	standard deviation of log-hydraulic conductivity;
$\Phi^{-1}[\alpha]$	=	standard normal quantile corresponding to the probability α;
Ω	=	random variable of drawdown during one time period; and
ω	=	the magnitude of drawdown during one time period.

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