High-energy String Scattering Amplitudes and Signless Stirling Number Identity

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Abstract

We give a complete proof of a set of identities Eq.(14) proposed recently from calculation of high-energy string scatterings. These identities allow one to extract ratios among high-energy string sacttering amplitudes in the fixed angle regime from high-energy amplitudes in the Regge regime. The proof is based on a signless Stirling number identity in combinatorial theory. The results are valid for arbitrary real values L rather than only for $L = 0, 1$ proved previously. The identities for non-integer real value L were recently shown to be realized in high-energy compactified string scatterings [31].

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Recently high-energy fixed angle string scattering amplitudes were intensively investigated [1–11] for string states at arbitrary mass levels. One of the motivation of this calculation has been to uncover the fundamental hidden stringy spacetime symmetry conjectured more than twenty years ago in [12–14]. An infinite number of linear relations among high energy scattering amplitudes of different string states were derived and the complete ratios among the amplitudes at each fixed mass level can be determined. An important new ingredient of this string amplitude calculation was based on an old conjecture of [15–17] on the decoupling of zero-norm states (ZNS) in the spectrum, in particular, the identification of inter-particle symmetries induced by the inter-particle ZNS [15] in the spectrum.

Another fundamental regime of high-energy string scattering amplitudes is the Regge regime (RR) [18–23]. See also [24–26]. Since the decoupling of ZNS applies to all kinematic regimes, one expects some implication of this decoupling in the RR. Moreover, it is conceivable that there exists some link between the patterns of the high energy scattering amplitudes in the fixed angle regime, or Gross Regime (GR), and RR. It was found that the number of high-energy scattering amplitudes for each fixed mass level in the RR is much more numerous than that of GR calculated previously. In contrast to the case of scatterings in the GR, there is no linear relation among scatterings in the RR. Moreover, it was discovered that the leading order amplitudes at each fixed mass level in the RR can be expressed in terms of the Kummer function of the second kind. More surprisingly, for those leading order high energy amplitudes $A^{(N,2m,q)}$ in the RR with the same type of $(N, 2m, q)$ as those of GR, one can extract from them the ratios $T^{(N,2m,q)}/T^{(N,0,0)}$ in the GR by using this Kummer function. The calculation was based on a set of identities which depend on a parameter $L(M_i^2) = 1 - N$ where $M_i^2 = 2(N-1), i = 1, ..4$, are the mass square of the string scattering states. The proof of these identities for $L = 0, 1$ was previously given in [27–29] based on a set of signed Stirling number identities developed in 2007 [30].

In this letter, we are going to prove these identities for arbitrary real values L by using a signless Stirling number identity. It is remarkable to see that the identities suggested by string theory calculation can be rigorously proved by a totally different mathematical method in combinatorial theory. It is also very interesting to see that, physically, the identities for arbitrary real values L in Eq.(16) can only be realized in high-energy *compactified* string scatterings considered very recently [31]. This is mainly due to the relation $M^2 = (K^{25})^2 + \hat{M}^2$ where K^{25} is the winding momentum corresponding to the compactified string coordinate

[31]. All other high-energy string scattering amplitudes calculated previously [27–29] correspond to integer value of L only. A recent work on string D-particle scatterings [32] also gave integer values L.

We begin with a brief review of high energy string scatterings in the fixed angle regime,

$$s, -t \rightarrow \infty, t/s \approx -\sin^2 \frac{\phi}{2} = \text{fixed (but } \phi \neq 0) \tag{1}$$

where s, t and u are the Mandelstam variables and ϕ is the CM scattering angle. It was shown [4, 5] that for the 26D open bosonic string the only states that will survive the high-energy limit at mass level $M_2^2 = 2(N-1)$ are of the form

$$|N, 2m, q\rangle \equiv (\alpha_{-1}^T)^{N-2m-2q}(\alpha_{-1}^L)^{2m}(\alpha_{-2}^L)^q|0, k\rangle, \tag{2}$$

where the polarizations of the 2nd particle with momentum k_2 on the scattering plane were defined to be $e^P = \frac{1}{M_2}(E_2, \mathrm{k}_2, 0) = \frac{k_2}{M_2}$ as the momentum polarization, $e^L = \frac{1}{M_2}(\mathrm{k}_2, E_2, 0)$ the longitudinal polarization and $e^T = (0, 0, 1)$ the transverse polarization. In Eq.(2), N, m and q are non-negative integers and $N \geq 2m + 2q$. It can be shown that the high-energy vertex in Eq.(2) are conformal invariants up to a subleading term in the high-energy expansion. Note that e^P approaches to e^L in the GR. For simplicity, we choose k_1, k_3 and k_4 to be tachyons. It turned out that the high-energy fixed angle scattering amplitudes can be calculated by using the saddle-point method. An infinite number of linear relations among high-energy scattering amplitudes of different string states were derived and the complete ratios among the amplitudes at each fixed mass level can be calculated to be [4, 5]

$$\frac{T^{(N,2m,q)}}{T^{(N,0,0)}} = \left(-\frac{1}{M_2}\right)^{2m+q} \left(\frac{1}{2}\right)^{m+q} (2m-1)!!. \tag{3}$$

Alternatively, the ratios can be calculated by the method of decoupling of two types of ZNS in the old covariant first quantized string spectrum. Similarly, the ratios for closed string [9], superstring [8] and D-brane scatterings [10] can be obtained.

Another high-energy regime of string scattering amplitudes, which contains complementary information of the theory, is the fixed momentum transfer t or RR. That is in the kinematic regime

$$s \rightarrow \infty, \sqrt{-t} = \text{fixed (but } \sqrt{-t} \neq \infty). \tag{4}$$

It was found [27] that the number of high energy scattering amplitudes for each fixed mass level in this regime is much more numerous than that of fixed angle regime calculated previously. On the other hand, it seems that both the saddle-point method and the method of decoupling of zero-norm states adopted in the calculation of fixed angle regime do not apply to the case of Regge regime. However the calculation is still manageable, and the general formula for the high energy (s,t) channel open string scattering amplitudes at each fixed mass level can be written down explicitly.

It was shown that a class of high-energy open string states in the Regge regime at each fixed mass level $N=\sum_{n,m} lk_n + mq_m$ are [27, 29]

$$|p_l, q_m\rangle = \prod_{l>0}(\alpha_{-l}^T)^{p_l}\prod_{m>0}(\alpha_{-m}^L)^{q_m}|0,k\rangle. \tag{5}$$

For our purpose here, however, we will only calculate scattering amplitudes corresponding to the vertex in Eq.(2). The relevant kinematics are

$$e^P\cdot k_1 \simeq -\frac{s}{2M_2},\quad e^P\cdot k_3 \simeq -\frac{\tilde{t}}{2M_2} = -\frac{t-M_2^2-M_3^2}{2M_2}; \tag{6}$$

$$e^L\cdot k_1 \simeq -\frac{s}{2M_2},\quad e^L\cdot k_3 \simeq -\frac{\tilde{t}'}{2M_2} = -\frac{t+M_2^2-M_3^2}{2M_2}; \tag{7}$$

and

$$e^T\cdot k_1 = 0,\quad e^T\cdot k_3 \simeq -\sqrt{-t}. \tag{8}$$

Note that e^P does not approach to e^L in the RR. The Regge scattering amplitude for the (s,t) channel was calculated to be [27] (We choose to calculate e^L amplitudes. The e^P amplitudes can be similarly discussed.)

$$\begin{aligned} A^{(N,2m,q)}(s,t) &= B\left(-1-\frac{s}{2},-1-\frac{t}{2}\right)\sqrt{-t}^{N-2m-2q}\left(\frac{1}{2M_2}\right)^{2m+q} \\ &\cdot 2^{2m}(\tilde{t}')^q U\left(-2m\,,\,\frac{t}{2}+2-2m\,,\,\frac{\tilde{t}'}{2}\right). \end{aligned} \tag{9}$$

In Eq.(9) U is the Kummer function of the second kind and is defined to be

$$U(a,c,x) = \frac{\pi}{\sin\pi c}\left[\frac{M(a,c,x)}{(a-c)!(c-1)!} - \frac{x^{1-c}M(a+1-c,2-c,x)}{(a-1)!(1-c)!}\right] \quad (c\neq 2,3,4...) \tag{10}$$

where $M(a,c,x) = \sum_{j=0}^{\infty}\frac{(a)_j}{(c)_j}\frac{x^j}{j!}$ is the Kummer function of the first kind. Note that the second argument of Kummer function $c=\frac{t}{2}+2-2m$, and is not a constant as in the usual case.

It can be seen from Eq.(9) that the Regge scattering amplitudes at each fixed mass level are no longer proportional to each other. The ratios are t dependent functions and can be calculated to be [27, 28]

$$\frac{A^{(N,2m,q)}(s,t)}{A^{(N,0,0)}(s,t)} = (-1)^m \left(-\frac{1}{2M_2}\right)^{2m+q} (\tilde{t}' - 2N)^{-m-q} (\tilde{t}')^{2m+q} \cdot \sum_{j=0}^{2m} (-2m)_j \left(-1 + N - \frac{\tilde{t}'}{2}\right)_j \frac{(-2/\tilde{t}')^j}{j!} + O\left\{\left(\frac{1}{\tilde{t}'}\right)^{m+1}\right\} \tag{11}$$

where $(x)_j = x(x+1)(x+2)...(x+j-1)$ is the Pochhammer symbol which can be expressed in terms of the signed Stirling number of the first kind $s\,(n,k)$ as following

$$(x)_n = \sum_{k=0}^{n} (-)^{n-k} s\,(n,k)\, x^k. \tag{12}$$

It was proposed in [27] that the coefficients of the leading power of $\tilde{t}'$ in Eq.(11) can be identified with the ratios in Eqs.(3). To ensure this identification

$$\lim_{\tilde{t}' \to \infty} \frac{A^{(N,2m,q)}}{A^{(N,0,0,)}} = \frac{T^{(N,2m,q)}}{T^{(N,0,0)}} = \left(-\frac{1}{M_2}\right)^{2m+q} \left(\frac{1}{2}\right)^{m+q} (2m-1)!!, \tag{13}$$

one needs the following identity

$$\sum_{j=0}^{2m} (-2m)_j \left(-L - \frac{\tilde{t}'}{2}\right)_j \frac{(-2/\tilde{t}')^j}{j!} = 0(-\tilde{t}')^0 + 0(-\tilde{t}')^{-1} + ... + 0(-\tilde{t}')^{-m+1} + \frac{(2m)!}{m!}(-\tilde{t}')^{-m} + O\left\{\left(\frac{1}{\tilde{t}'}\right)^{m+1}\right\} \tag{14}$$

where $L = 1 - N$ and is an integer. For all four classes [8] of high-energy superstring scattering amplitudes, L is an integer too [29]. A recent work on string D-particle scatterings [32] also gives an integer value of L. Note that L effects only the sub-leading terms in $O\left\{\left(\frac{1}{\tilde{t}'}\right)^{m+1}\right\}$. Here we give a simple example for $m = 3$ [28, 29]

$$\sum_{j=0}^{6} (-2m)_j \left(-L - \frac{\tilde{t}'}{2}\right)_j \frac{(-2/\tilde{t}')^j}{j!} = \frac{120}{(-\tilde{t}')^3} + \frac{720L^2 - 2640L + 2080}{(-\tilde{t}')^4} + \frac{480L^4 - 4160L^3 + 12000L^2 - 12928L + 3840}{(-\tilde{t}')^5} + \frac{64L^6 - 960L^5 + 5440L^4 - 14400L^3 + 17536L^2 - 7680L}{(-\tilde{t}')^6}. \tag{15}$$

Mathematically, Eq.(14) was exactly proved [27–29] for $L = 0, 1$ by a calculation based on a set of signed Stirling number identities developed very recently in combinatorial theory in

[30]. For general integer L cases, only the identity corresponging to the nontrivial leading term $\frac{(2m)!}{m!}(-\tilde{t}')^{-m}$ was rigoursly proved [29], but not for other "0 identities". A numerical proof of Eq.(14) was given in [29] for arbitrary real values L and for non-negative integer m up to $m = 10$. It was then conjectured that [29] Eq.(14) is valid for any *real* number L and any non-negative integer m. Physically, it is important to discover recently [31] that Eq.(14) for any non-negative integer m and arbitrary real values L can be realized in high-energy *compactified* string scatterings. This is due to the dependence of the value L on winding momenta K_i^{25} [31]

$$L = 1 - N - (K_2^{25})^2 + K_2^{25} K_3^{25}. \tag{16}$$

All other high-energy string scatterings calculated previously [27–29, 32] correspond to integer value of L only. It is thus of importance to rigorously prove the validity of Eq.(14) for arbitrary real values L.

We now proceed to prove Eq.(14). We first rewrite the left-hand side of Eq.(14) in the following form

$$\begin{aligned} &\sum_{j=0}^{2m}(-2m)_j \left(-L - \frac{\tilde{t}'}{2}\right)_j \frac{(-2/\tilde{t}')^j}{j!} \\ =&\sum_{j=0}^{2m} (-1)^j \binom{2m}{j} \sum_{l=0}^{j} \binom{j}{l} (-L)_{j-l} \sum_{s=0}^{l} c\left(l,s\right) \left(-\frac{2}{\tilde{t}'}\right)^{j-s} \end{aligned} \tag{17}$$

where we have used the signless Stirling number of the first kind $c\left(l,s\right)$ to expand the Pochhammer symbol

$$\left(x\right)_n = \sum_{k=0}^{n} c\left(n,k\right) x^k. \tag{18}$$

The coefficient of $(-2/\tilde{t}')^i$ in Eq.(17), which will be defined as $G\left(m,i\right)$, can be read off from the equation as

$$G\left(m,i\right) = \sum_{j=0}^{2m} \sum_{l=0}^{j} (-1)^{j+i} \binom{2m}{j} \binom{j}{l} (-L)_{j-l} c\left(l, j-i\right). \tag{19}$$

One needs to prove that

$$1. G\left(m,m\right) = (2m-1)!!, \text{ for all } L \in \mathbb{R}; \tag{20}$$

$$2. G\left(m,i\right) = 0, \text{ for all } L \in \mathbb{R} \text{ and } 0 \le i < m. \tag{21}$$

From the definition of $c\,(n,k)$ in (18), we note that $c\,(n,k)\neq 0$ only if $0\leq k\leq n$. Thus $c\,(l,j-i)\neq 0$ only if $j\geq i$ and $l\geq j-i$. We can rewrite $G\,(m,i)$ as

$$\begin{aligned}
G\,(m,i) &= \sum_{j=i}^{2m}\sum_{l=j-i}^{j}(-1)^{j}\binom{2m}{j}\binom{j}{l}(-L)_{j-l}c\,(l,j-i) \\
&= \sum_{k=0}^{2m-i}\sum_{l=k}^{k+i}(-1)^{k+i}\binom{2m}{i+k}\binom{i+k}{l}(-L)_{k+i-l}c\,(l,k) \\
&= \sum_{k=0}^{2m-i}\sum_{p=0}^{i}(-1)^{k+i}\binom{2m}{i+k}\binom{i+k}{p+k}(-L)_{i-p}c\,(k+p,k) \\
&= \sum_{p=0}^{i}(-L)_{i-p}\sum_{k=0}^{2m-i}(-1)^{k+i}\binom{2m}{i+k}\binom{i+k}{p+k}c\,(k+p,k) \\
&= (-1)^{i}\sum_{p=0}^{i}(-L)_{i-p}\binom{2m}{i-p}\sum_{k=0}^{2m-i}(-1)^{k}\binom{2m-i+p}{k+p}c\,(k+p,k) \\
&\equiv (-1)^{i}\sum_{p=0}^{i}(-L)_{i-p}\binom{2m}{i-p}S_{2m-i}\,(p)
\end{aligned} \tag{22}$$

where we have defined

$$S_N\,(p)=\sum_{k=0}^{N}(-1)^{k}\binom{N+p}{k+p}c\,(k+p,k)\,. \tag{23}$$

It is easy to see that for fixed m and $0\leq i<m$, $G\,(m,i)$ is a polynomial of L of degree i, expanded with the basis 1, $(-L)_1$, $(-L)_2$,.... Note that $p\leq i<m$, so $2m-i\geq p+1$. For Eq.(21), we want to show that $S_N\,(p)=0$ for $N\geq p+1$. For this purpose, we define the functions

$$C_n\,(x)=\sum_{k\geq 0}c\,(k+n,k)\,x^{k+n}. \tag{24}$$

The recurrence of the signless Stirling number identity

$$c\,(k+n,k)=(n+k-1)\,c\,(n+k-1,k)+c\,(n+k-1,k-1) \tag{25}$$

leads to the equation

$$C_n\,(x)=\frac{x^2}{1-x}\frac{d}{dx}C_{n-1}\,(x)\,, \tag{26}$$

with the intial value

$$C_0\,(x)=\frac{1}{1-x}. \tag{27}$$

The first couple of $C_n(x)$ can be calculated to be

$$C_1(x) = \frac{x^2}{(1-x)^3},\ \ C_2(x) = \frac{x^4+2x^3}{(1-x)^5},\ \ C_3(x) = \frac{x^6+8x^5+6x^4}{(1-x)^7}. \tag{28}$$

Now by induction, it is easy to show that

$$C_n(x) = \frac{f_n(x)}{(1-x)^{2n+1}},\ \ \text{where } f_n(x) = x^{2n} + \mathcal{O}\left(x^{2n-1}\right), \tag{29}$$

and

$$f_n(1) = (2n-1)!!. \tag{30}$$

In order to prove Eq.(21), we note that $(-1)^N S_N(p)$ is the coefficient of x^{N+p} in the function

$$(1-x)^{N+p} C_p(x) = f_p(x)(1-x)^{N-p-1} = x^{N+p-1} + \mathcal{O}(\cdots), \tag{31}$$

which is obviously zero for $N \geq p+1$. This proves $S_N(p) = 0$ for $N \geq p+1$ and thus Eq.(21).

In order to prove the first identity in Eq.(20), we first note that the above argument remains true for $i = m$ and $0 \leq p < i$. So Eq.(20) corresponds to the case $p = i = m$. By using Eq.(22), we can evaluate

$$G(m,m) = \sum_{k=0}^{m} (-1)^{k+m} \binom{2m}{k+m} \binom{k+m}{k+m} c(k+p,k) = \sum_{k=0}^{m} (-1)^{k+m} \binom{2m}{k+m} c(k+p,k). \tag{32}$$

Equation.(32) corresponds to the coefficient of x^{2m} in the function

$$(1-x)^{2m} C_m(x) = \frac{f_m(x)}{1-x} = f_m(x)(1+x+x^2+....). \tag{33}$$

By Eq.(30), this coeffieient is

$$f_m(1) = (2m-1)!!. \tag{34}$$

This proves Eq.(20). We thus have completed the proof of Eq.(14) for any non-negative integer m and any real value L.

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