JOURNAL OF INFORMATION SCIENCE AND ENGINEERING 23, 1849-1863 (2007)

Effective Learning Model and Activate Learning Algorithm for Improving Learning Efficiency*

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In recent years, technology developments are more rapidly. How to learn and obtain desired knowledge efficiently has become an important but complicated problem. We hope that there are methods can give us some suggestions about how to learn knowledge efficiently. In this paper, we introduced some learning behavior of people, and then use our designed Effective Learning Curve Model to imitate this learning phenomenon. Using our learning function model, we can imitate people's learning behavior through pretesting. Every one has different learning behavior functions on learning distinct courses. Different learning sequence of courses will cause different learning efficiency. From this view, we proposed Max Learning Efficiency Slope First Algorithm (MLESFA) by differential learning functions to give people some suggestions about courses learning sequence and obtain desired knowledge efficiently. These algorithms also can help us to understand how much time we have to spend on each course in order to get better learning efficiency under time limitation. Finally, we make some learning example and compare simulation results with other courses learning algorithms. From the simulation results, we can see that our MLESFA algorithm has better learning efficiency than others.

Keywords: e-learning, learning algorithm, learning function, learning model, learning efficiency, learning behavior

1. INTRODUCTION

Technology developments are progressive rapidly in the world. The things people deal with have become more and more complex. Many ten years ago, for the research of building airplane, Wright [1] had used math methods to create learning curve function and develop first thesis about learning curve. From that time, learning methods have been discussed for distinct application plan and different models also had been produced. If we set horizontal axis indicates learning time period on courses, while the vertical axis indicates learning efficiency, we call this figure as learning curve. People's learning curves are different and will be changed because of many reasons such as difficulty of works, learning motivation, knowledge background of learners, and some other reasons. There are some typical learning curves describe as follow, and shown in Fig. 1 (a).

(a) Negative Accelerating Curve

Learning efficiency make a fast progress in the beginning, after that the learning effi-

Received August 31, 2005; revised January 18, 2006; accepted March 9, 2006.
Communicated by Pau-Choo Chung.

* This research was supported in part by the National Science Council (Taiwan), Bestwise International Computing Co., CAISER (National Chiao Tung University, Taiwan) and Ta Hwa Institute of Technology. Shih-Jung Peng and Pi-Feng Liang are the teachers of Ta-Hwa Institute of Technology (THIT).

ciency will slow down gradually. Perhaps the contents are simple at the first and will be harder than before.

(b) Positive Accelerating Curve
Learning efficiency make a slow progress in the beginning, after that the learning efficiency will be speeding gradually. Maybe, learner has been trained at the first time and will get some experience from that. Next time, they will spend less time and get better learning efficiency.

(c) S Accelerating Curve
Learning efficiency curve is the combination behavior of NA curve and PA curve.

(d) Linear Accelerating Curve
The proportion of learning efficiency to spending learning time is linear equation. It can be written as $\eta = a * t$, where η: learning efficiency, a: constant parameter, t: spending time.

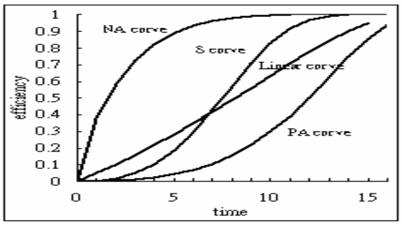


Fig. 1. (a) Typical learning curve.

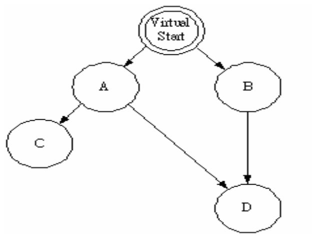


Fig. 1. (b) Courses learning sequence graph.

If there are some knowledge courses we prepare to learn, we can make their relations into a learning graph as shown in Fig. 1 (b). In that, nodes represent learned courses and arrows mean courses learning sequence. For example, people do not allow learning course C till course A has been learned and passed, or the learning efficiency will decrease, and we set this decreasing parameter ∂. Of course, there are some courses learning independent to others, such as course A and course B. That means we can start learning from either course A or course B. For the reason, we use 'virtual start node' as the beginning of learning graph.

Under this course learning graph, we hope people can learn more efficiently with spending minimum learning time on each course. For this reason, we design new learning function model to imitate people's learning behavior. We also proposed Max Learning Efficiency Slope First Algorithm (MLESFA) to improve group courses learning efficiency. At the end, we compare simulation results of our MLESFA algorithm with other algorithms. The simulation comparison is shown in Figs. 8 and 9. We can see that our algorithm has better simulation result and can improve learner's group courses learning efficiency.

The remainder of this paper is organized as follows. In section 2, we introduce some related work and compare them with our proposed methods. In section 3, we proposed our learning function model to imitate people's learning behavior. Learning efficiency algorithms are described in section 4. The simulation results and comparisons are presented in section 5. Finally, we provide conclusion in section 6.

2. RELATED WORK

Learning curve can be described in math function for different learning characteristic. Five commonly used learning curves are described in Yelle [2], and are introduced as following:

(a) Log-linear model [1]
$f(x) = a_1x^{-b}$
$f(x)$: time needed for xth production
a_1: time needed for 1st production
b: learning coefficient, $b = -(\ln r/\ln 2)$
r: learning rate
(b) Standford-B model [3]
$f(x) = a_1(x + B)^{-b}$
B: constant, between 0 and 10
(c) De Jong Model [7]
$f(x) = a_1(M + (1 - M)x^{-b})$
M: constant, between 0 and 1
(d) S Curve Model [4]
$f(x) = a_1(M + (1 - M)(x + B)^{-b})$
(e) Time Constant Model [6]
$Y(t) = Y_c + Y_f(1 - e^{-t/\tau})$
Y_t: production numbers at time t
Y_c: production numbers at time $t = 0$
Y_f: adding production numbers through learning
τ : learning time constant

Learning model in Yelle [2] are suitable for special condition, but can not cover all learning behavior we have introduced before, and will be limited in some learning application area. In this paper, we designed Effective Learning Curve Model that can imitate most learning behavior of people by tuning function parameter. After that, we also proposed Max Learning Efficiency Slope First Algorithm (MLESFA) to improve people's learning efficiency under learning group courses, and make some example to prove that our MLESFA algorithm has better learning efficiency.

3. HEURISTIC LEARNING MODEL

In this section, we try to find a learning equation which can imitate most of the learning behavior of people, as in Fig. 1 (a).

At first, we choose e^{-t} as our base function:

Where,

$e^{-xt} = 0$, if $t \to \infty$,

$e^{-xt} = 1$, if $t \to 0$.

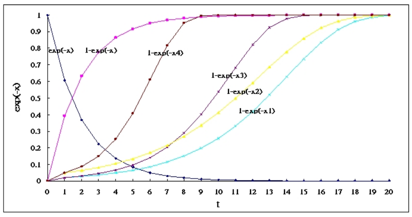


Fig. 2. Exp( ) function under irregularly changing speed of the parameter *x*.

Function $1 - e^{-xt}$ will become that,

$$1 - e^{-xt} = 0, \text{ if } t \to 0,$$
$$1 - e^{-xt} = 1, \text{ if } t \to \infty.$$

We attempt to change parameter '*x*' from 0 to ∞ including with irregular speed. Curve of function $1 - \exp(-x)$ in Fig. 2 is under changing *x* from 0 to 10 stepped 0.5 regularly. In function $1 - \exp(-x1)$, $x1$ changed from 0.05 to 10 under increasing irregular speed 1.3. In the same way, $x2$ with speed 1.35, $x3$ with speed 1.5, $x4$ with speed 1.8 respectively.

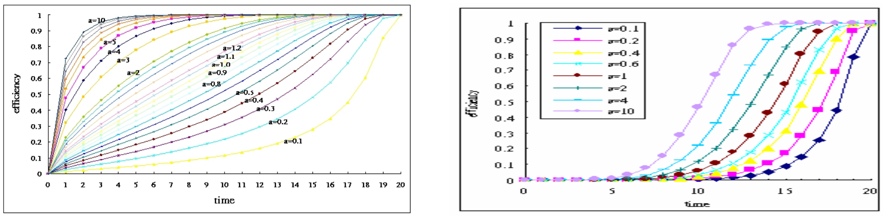
Where,

$$Speed_2 = Speed_1 * 1.3,$$
$$Speed_n = Speed_{n-1} * 1.3,$$
$$Speed_n = Speed_1 * 1.3^{n-1}.$$

From the results of the Fig. 2, we found that proposed learning function model could imitate the learning curves we have introduced in Fig. 1 (a) by tuned the combination of parameters '*a*', '*b*', '*c*'. We proposed $\eta(t) = c(1 - \exp(-a(nt)^b/(1 - nt)))$ as our learning function model under experimentally testing. In that, *t* is time sequence, '*a*' and '*b*' are function parameter in order to imitate learner's learning behavior, *n* is simulation time slot range, used to 0.01, '*c*' is function coefficient indicating learning speed of imitated learning function.

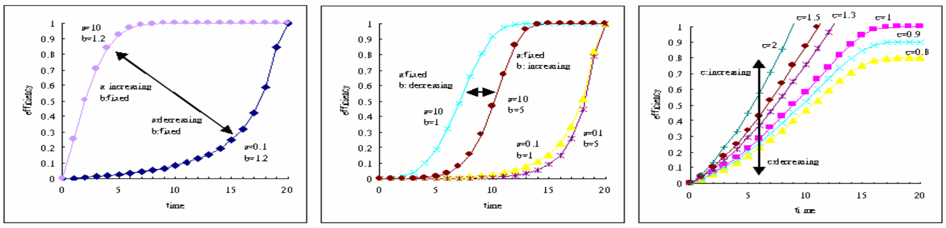
Using function $f = \eta(t)$, and tuning the parameter '*a*', '*b*' and '*c*', we can obtain some different learning curves, including Negative accelerating curves, Positive accelerating curves, S accelerating curves and Linear accelerating curves. In the following, we set parameter '*a*' from 0.1 to 10, parameter '*b*' from 0.7 to 5 and parameter '*c*', the speeding coefficient of learning function, to 1. We can get relative function curves shown in Figs. 3 (a-b), and compare these figures, the effects of parameter '*a*', '*b*' and '*c*' on learning function model are shown in Figs. 4 (a-c).

From Figs. 3 (a-b), if we want to make a imitation of learner's learning curve, we can set the range of parameter '*a*' between 0.1 to 10, parameter '*b*' between 0.7 to 5 and '*c*' = 1. If we want the imitation curves more precisely, we can just tune parameter '*c*' from 0.8 to 2.0. From experiments, effects of parameter '*a*' on learning function are shown in Fig. 4 (a), effects of parameter '*b*' are in Fig. 4 (b), effects of parameter '*c*' are shown in Fig. 4 (c), respectively. Therefore, we can emulate some different learning behavior



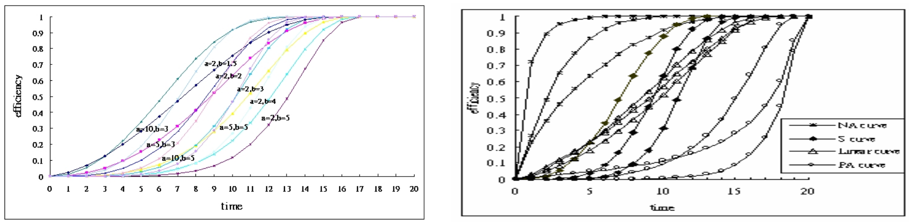
(a) ‘a’ = 0.1 to 10, ‘b’ = 0.7, ‘c’ = 1. (b) ‘a’ = 0.1 to 10, ‘b’ = 5, ‘c’ = 1.

Fig. 3. Imitating learning curve under parameters ‘a’ = 0.1 to 10, ‘b’ = 0.7 to 5, ‘c’ = 1.



(a) Effects of parameter ‘a’. (b) Effects of parameter ‘b’. (c) Effects of parameter ‘c’.

Fig. 4. Effects of parameters ‘a’, ‘b’ and ‘c’ on learning function.



(a) (b)

Fig. 5. Imitate learning function behavior curves under parameters ‘a’: 2~10, ‘b’: 1.5~5, ‘c’ = 1.

Table 1. Learning behavior imitation under combination of parameter ‘a’, ‘b’, ‘c’.

parameter	NA Curve	LA Curve	PA Curve	SA Curve
a	0.1 – 0.5	0.8 – 1.2	2.0 – 10	2.0 – 10
b	0.7 – 5.0	0.8 – 1.2	0.7 – 1.5	1.5 – 5
c	1	1	1	1

under combinations of parameter ‘a’, ‘b’ and ‘c’ between different range, as described in Table 1, and the imitating learning behavior curves are shown in Figs. 5 (a-b) .

But how can we get the parameter ‘a’, ‘b’ and ‘c’ of each user? For this purpose, we could use Item Response Theory (IRT) [12-14] which consider both course material difficulty and learner ability. We also can collect and store much database about the relations of learning efficiency with learner’s personality such as knowledge background, learning attitude, course difficulty, *etc*. Finally, we may use statistical application analysis method to find people’s suitable parameter ‘a’, ‘b’, ‘c’ and ‘n’.

Everyone has his own learning behavior curve in each course, and learning curve of each course would be different. Before start learning, we can make some pre-testing, using Item Response Theory, to predict suitable learning parameter '*a*', '*b*', '*c*' and '*n*' of each course respectively in order to get proper learning function to imitate learner's learning behavior. Under group courses learning, there are some learning sequence relations between courses. So, if we want to get the max learning efficiency in some condition, the model can be formulated as follow.

Object to,

$$\text{Max}\left(\sum_{i=1}^{m}\sum_{j=1}^{n} W_i * \frac{d\eta_i(t_{ij})}{dt}\right)$$

where,

m: courses number
n: time slots of studying
W_i: weights of course i
$\eta_i(t)$: learning function on course i
$d\eta_i(t_{ij})/dt$: learning efficiency of spending Δ time j on course i

Subject to,

$$t_{ij} \geqq 0,\ T \geqq 0,\ \sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij} = T$$

$$W_i \geqq 0,\ \sum_{i=1}^{m} W_i = 1$$

$$\frac{d\eta_i(t_{ij})}{dt} \geq 0,\ \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{d\eta_i(t_{ij})}{dt} \leq 100.$$

In time t_j, using differential method, we can choose max function slop $\text{MAX}(w_1\frac{d\eta_1(t_j)}{dt}$, $w_2\frac{d\eta_2(t_j)}{dt},\ \ldots, w_n\frac{d\eta_n(t_j)}{dt})$ as the first course learning priority.

4. ALGORITHM FOR IMPROVING LEARNING EFFICIENCY

In section 3, we introduced and compared some used learning functions, and proposed a new learning function model through Item Response Theory (IRT) [12-14]. We could use these learning functions to imitate learner's learning behavior, to predict learner's learning efficiency, and give some learning recommendation to the learners. But learner's learning behaviors and learning function are not always fixed. Everyone could have different learning functions under distinct course, for example English and Math, environmental effect, pre-learning *etc*. One has one individual learning function under each course. Someone would have many learning functions, if there are numerous courses to be learned.

Table 2. Learning efficiency of each course under data sampling.

Course	Learning efficiency											
	j = 1	2	3	4	5	6	7	8	9	10	11	12
A	30	25	20	10	5	4	3	2	1	0	0	0
B	40	30	15	10	5	0	0	0	0	0	0	0
C	20	15	15	15	10	5	5	5	5	3	2	0
D	18	17	15	10	10	8	7	5	5	3	2	0
E	25	15	15	15	10	10	5	3	2	0	0	0
F	20	20	15	10	10	10	10	5	0	0	0	0

Table 3. Learning efficiency*normalized weight of each course under data sampling.

Course	Weight	Learning efficiency											
		j = 1	2	3	4	5	6	7	8	9	10	11	12
A	0.1	3.0	2.5	2.0	1.0	0.5	0.4	0.3	0.2	0.1	0	0	0
B	0.1	4.0	3.0	1.5	1.0	0.5	0	0	0	0	0	0	0
C	0.2	4.0	3.0	3.0	3.0	2.0	1.0	1.0	1.0	1.0	0.6	0.4	0
D	0.3	5.4	5.1	4.5	3.0	3.0	2.4	2.1	1.5	1.5	0.9	0.6	0
E	0.2	5.0	3.0	3.0	3.0	2.0	2.0	1.0	0.6	0.4	0	0	0
F	0.1	2.0	2.0	1.5	1.0	1.0	1.0	1.0	0.5	0	0	0	0

Table 4. Sorting courses learning efficiency by descending from Table 3.

Time t_j	j = 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Learning sequence	d1	d2	e1	d3	c1	b1	a1	b2	c2	c3	c4	d4	d5	e2	e3	e4	a2	d6	d7	a3
Each unit score	5.4	5.1	5.0	4.5	4.0	4.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	2.5	2.4	2.1	2.0
Total score	5.4	10.5	15.5	20	24	28	31	34	37	40	43	46	49	52	55	58	60.5	62.9	65	67

Time t_j	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Learning sequence	c5	e5	e6	f1	f2	b3	d8	d9	f3	a4	b4	c6	c7	c8	c9	e7	f4	f5	f6	f7
Each unit score	2.0	2.0	2.0	2.0	2.0	1.5	1.5	1.5	1.5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Total score	69	71	73	75	77	78.5	80	81.5	83	84	85	86	87	88	89	90	91	92	93	94

Time t_j	41	42	43	44	45	46	47	48	49	50	51	52	53
Learning sequence	d10	c10	d11	e8	a5	b5	f8	a6	c11	e9	a7	a8	a9
Each unit score	0.9	0.6	0.6	0.6	0.5	0.5	0.5	0.4	0.4	0.4	0.3	0.2	0.1
Total score	94.9	95.5	96.1	96.7	97.2	97.7	98.2	98.6	99.0	99.4	99.7	99.9	100

In this section, our object is to provide a mathematical analysis in courses learning function and to define some learning behavioral strategies in order to obtain optimal courses learning sequence. And some examples will be briefly discussed. Now, we assumed there are six courses A, B, C, D, E, F and their weights of each course is normalized to be

$$W_i / \sum_{i=1}^{n} W_i.$$

Where, W_1 is signified as the weight of course A before normalizing. Let each course weight equal to $W_A = 1$, $W_B = 1$, $W_C = 2$, $W_D = 3$, $W_E = 2$, $W_F = 1$. After normalizing, the weights of each course are become $W_{nA} = 0.1$, $W_{nB} = 0.1$, $W_{nC} = 0.2$, $W_{nD} = 0.3$, $W_{nE} = 0.2$, $W_{nF} = 0.1$. Using pre-testing, we can obtain user's learning behavior curve of each course and obtain discrete learning efficiency of each spending unit time by differential course learning functions $D\eta_i(t)|_{t=tj}$, as in Table 2. Multiplying value in Table 2 by course weight W_i, we get the results in Table 3. Sorting learning efficiency field by descending in Table 3, we could obtain value result as in Table 4.

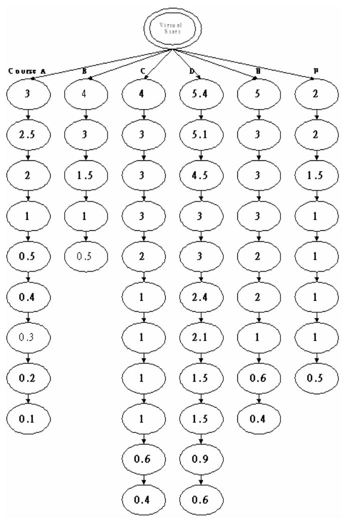


Fig. 6. Sample learning efficiency under courses independence.

In the following discussion, first, we have made the assumption that courses are independent to others, as shown in Fig. 6. In this condition, there are two questions we must face to solve. The first question is how can we know the minimum spending time on each course for the purpose of getting 60 score to pass the courseware, and the second question is how to obtain maximum course learning efficiency in order to get best score under given time limitation. For these reasons, we propose score base Algorithm 1.1 to solve first question, to know the minimum spending time on each course for the purpose of getting 60 score to pass all the courses, and time base Algorithm 1.2 to solve the second question, to get the best score under given time limitation.

Algorithm 1.1 Score base under courses independence

/* How much time we spend at least in order to obtain the score we wanted */

1: make a pretest about learning courses in order to obtain learning behavior function
2: differential learning function $d\eta_i(t_j)/dt$, we can obtain the Δscore of spending time on course learning, as in Table 2
3: multiplying obtained score in Table 2 by each course weight
4: sort the learning efficiency in Table 2 by descending, and shown the result in Table 3
5: input courses number *cn* and wanted score ws, where $60 \leqq ws \leqq 100$
6: initialize variable to zero
7: total score $ts = 0$
8: total learning time $tlt = 0$
9: course chapter learning sequence LS = ‘’
10: for $i = 1$ to *cn*
11: time spending on each course chapter $n(i) = 0$
12: each course score obtained $cs(i) = 0$
13: next *i*
14: do while ($ts < ws$)
15: for $i = 1$ to *cn*
16: finding better course learning efficiency score bcs from $\eta_i'|_{t=n(i)+1}$
17: next *cn*
/* from step 16, we could see that course *j* had better course learning efficiency and obtain score *bc*s, where *j* between 1 to *cn* */
18: $n(j) = n(j) + 1$
19: $cs(j) = cs(j) + bcs$
20: $ts = ts + bcs$
21: record chapter learning sequence to *LS*
22: end do
23: print the result *ts*, *tlt*, LS
24: for $i = 1$ to *cn*
25: print $tsc(i)$, $cs(i)$
26: next *i*
27: end

Algorithm 1.2 Time base under courses independence

/* How many score we could obtain under learning time limited */

1: to do the same steps 1 to 13 as in Algorithm 1.1
2: input time limited of learning courses *tl*
3: do while ($tlt < tl$)
4: for $i = 1$ to *cn*
5: finding better course learning efficiency score *bcs* from $\eta_i'|_{t=n(i)+1}$
6: next *cn*
/* from step 5, we could see that course *j* had better course learning efficiency and obtain score *bcs*, where *j* between 1 to *cn* */
7: $n(j) = n(j) + 1$
8: $cs(j) = cs(j) + bcs$
9: $ts = ts + bcs$

```
    record chapter learning sequence to LS
end do
print the result ts, tlt, LS
for i = 1 to cn
    print tsc(i), cs(i)
next i
end
```

From the examples of courses independent learning graph as in Fig. 6, using Algorithm 1.1, if we want 60 score at least in order to pass the courseware, we must spend 17 unit times on courses. The time spending on each course is TA = 2, TB = 2, TC = 4, TD = 5, TE = 4, and TF = 0, and the final score we may obtain equal to '60.5'. Course learning sequence suggestion is stored in variable *LS* = 'd1 d2 e1 d3 c1 b1 a1 b2 c2 c3 c4 d4 d5 e2 e3 e4 a2', as in Table 4. Under another time limitation condition, if we have 25 unit time, what score we can get max? Using time base Algorithm 1.2, we can get max final score equal to 77 and the time spending on each course is TA = 3, TB = 2, TC = 5, TD = 7, TE = 6, and TF = 2. Course learning sequence suggestion is *LS* = 'd1 d2 e1 d3 c1 b1 a1 b2 c2 c3 c4 d4 d5 e2 e3 e4 a2 d6 d7 a3 c5 e5 e6 f1 f2', as in Table 4.

Finally, we assume that some courses are dependent to others, as shown in Fig. 7. In Fig. 7, we also want to know the minimum spending time on each course for the purpose of getting 60 score to pass the courseware, and the maximum course learning efficiency in order to obtain best score under given time limitation. But some courses are dependent to each other, if course $n - 1$ has not passed, the learning efficiency of courses n will be influent. We raise parameter ∂ as this effect parameter, and learning efficiency will be changed to learning efficiency*(∂^m), in that $0 \leqq \partial \leqq 1$, m is the fail courses number before learned course n. As to these questions, we propose score base Algorithm 2.1 to solve first question, to know the minimum spending time on each course for the purpose of getting 60 score to pass all the courses, and time base Algorithm 2.2 to solve the second question, to get the best score under given time limitation. At first, we made a course learning choosing sequence principle, such that,

(a) high level learned course first,
(b) high course learning weight first with the same level,
(c) effecting more learning courses first,
(d) learning from left to right.

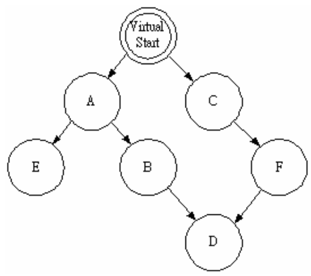


Fig. 7. Course learning sequence under courses dependence.

Algorithm 2.1 Score base under courses dependence

1: make a pretest about learning courses in order to obtain learning behavior function
2: differential learning function, $d\eta_i(t_j)/dt$, we can obtain the Δscore of spending time on course learning, as in Table 2
3: multiplying obtained score in Table 2 by each course weight
4: sort the learning efficiency in Table 2 by descending, and shown the result in Table 3
5: input courses number *cn* and wanted score *ws*, where $60 \leqq ws \leqq 100$
6: initialize variable to zero
7: total score $ts = 0$
8: total learning time $tlt = 0$
9: course chapter learning sequence LS = ‘’
10: for $i = 1$ to cn
11: time spending on each course chapter $n(i) = 0$
12: each course score obtained $cs(i) = 0$
13: using choosing principle for course learning, store course reading sequence into $crs(i)$ and course weight into $cw(i)$
14: next i
15: for $i = 1$ to cn
16: do while $(cs(crs(i)) < ws * cw(crs(i)))$
17: finding better course learning efficiency score *bcs* from $\eta_i'|_{t=n(i)+1}$, as in Table 2
/* from step 17, we could see that chapter j of course $crs(i)$ had better course learning efficiency and could obtain score *bcs* */
18: $n(crs(i)) = n(crs(i)) + 1$
19: $cs(crs(i)) = cs(crs(i)) + bcs$
20: $ts = ts + bcs$
21: record chapter learning sequence to LS
22: end do
23: next i
24: print the result ts, tlt, LS
25: for $i = 1$ to cn
26: print $tsc(i)$, $cs(i)$, $crs(i)$, $cw(i)$
27: next i
28: end

Algorithm 2.2 Time base under courses dependence

1: to do the same steps 1 to 14 as in Algorithm 2.1
2: input time limited of learning courses tl
3: for $i = 1$ to cn
4: do while $(cs(crs(i)) < 60 * cw(crs(i))$ and $tlt < tl)$
5: finding better course learning efficiency score *bcs* from $\eta_i'|_{t=n(i)+1}$, as in Table 2
/* from step 5, we could see that chapter j of course $crs(i)$ had better course learning efficiency and could obtain score *bcs* */
6: $n(crs(i)) = n(crs(i)) + 1$
7: $cs(crs(i)) = cs(crs(i)) + bcs$
8: $ts = ts + bcs$

```
      record chapter learning sequence to LS
end do
next i
to do the same steps 3 to 11 as in Algorithm 1.2
print the result ts, tlt, LS
for i = 1 to cn
      print tsc(i), cs(i)
next i
end
```

From the course learning graph as in Fig. 7, using Algorithms 2.1 and 2.2, we can get course learning sequence such as course C, A, E, B, F, and D. If we want to pass all courses, the time we must spend on each course is TA = 3, TB = 2, TC = 4, TD = 4, TE = 4, TF = 4 and the score of each course we will get is as follow, SA = 75, score of course A, SB = 70, SC = 65, SD = 60, SE = 70, SF = 65, respectively. Therefore if we want to pass all the courses, we must spend at least to 3 + 2 + 4 + 4 + 4 + 4 = 21 unit times, and will obtain final score equal to (75 * 1 + 70 * 1 + 65 * 2 + 60 * 3 + 70 * 2 + 65 * 1)/10 = 66 with multiplying each course score and course weighting.

Another question, if we have unit time > 21, for example 30, what score we can get max? First, we use Algorithm 2.1 to make all courses pass, and then use Algorithm 1.2 to sort and get all the other course contains that we will spend learning times on it. Finally, following these steps, we can obtain the time we spend on each course, TA = 3, TB = 3, TC = 5, TD = 9, TE = 6, TF = 4 and get related scores as follow, SA = 75, SB = 85, SC = 75, SD = 95, SE = 90, SF = 65. At last, we obtain final score equal to (75 * 1 + 85 * 1 + 75 * 2 + 95 * 3 + 90 * 2 + 65 * 1)/10 = 84. And the course learning sequence suggestion is stored in variable *LS*.

5. EFFICIENCY COMPARISON

To prove our learning behavior function and learning algorithms having better learning efficiency, we use learning graph, Figs. 6 and 7 as our comparison example. Table 2 is learner's learning efficiency of each course by differential course learning function and data sampling. At the first, we take Fig. 6 into consideration that courses are independent, and we compare MLESFA1 simulation result with course Depth First Algorithm (DFA), course Bread First Algorithm (BFA) and course Random Algorithm (RA). DFA learns all chapters of course A by sequence, after learned completely, next to learn course B, C, D, E and F. BFA learns chapter 1of course A at the first, next to chapter 1 of course B, C, D, E, and F, after that, learns chapter 2 of course A, B, C, D, E, F and will not stop till all chapters have been learned completely. RA means random choosing course chapter to learn. Because of course chapter having its learning sequence, random choosing course chapter to learn will affect chapter learning efficiency, we assume effect parameter η = '0.8'. The comparison learning efficiency curves are shown in Fig. 8 under course learning graph Fig. 6.

Next, we think Fig. 7 that courses are dependent as our simulation graph. The courses

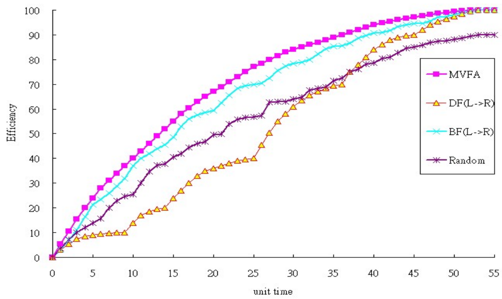


Fig. 8. Learning efficiency curves comparison under courses independence.

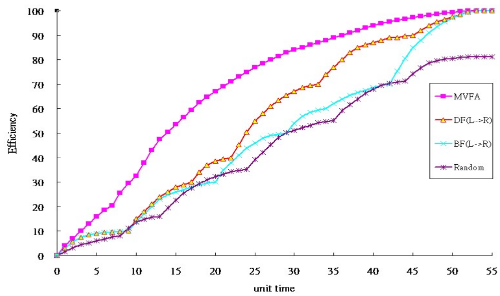


Fig. 9. Learning efficiency curves comparison under courses dependence.

learning sequence of our algorithm MLESFA by choosing course learning sequence principle are course A, C, B, E, F, D, courses learning sequence by algorithm DFA are course A, E, B, D, C, F, courses learning sequence by algorithm BFA are course A, C, E, B, F, D respectively. As to algorithm RA, we get randomly courses learning sequence, at here we supposed its course learning sequence are course F, B, D, C, E, A. Because courses are dependent, if parent courses haven't passed yet, and we insist on learning following courses, the course learning efficiency will be influenced. This effect will bigger than courses dependent and we assumed this effect parameter $\eta = 0.8$. If learning course have m parent courses not passed, the influence will be changed to η^m. The comparison learning efficiency curves are shown in Fig. 9 under condition Fig. 7.

From above discussions, we can see that our algorithms has better learning efficiency result both in Fig. 8 under Fig. 6 courses independence, and in Fig. 9 under Fig. 7 courses dependence.

6. CONCLUSION

In this paper, we have made some contributions, (1) we proposed new effective heuristic learning function $\eta(t) = c(1 - \exp(-a(nt)^b/(1 - nt)))$ to imitate people's learning behavior. From tuning parameter 'a', 'b', 'c' and 'n', we can imitate most people's learning behavior including Negative accelerating learning curve, Positive accelerating learning curve, S accelerating learning curve and 'linear' accelerating learning curve on learning courses or works. (2) Under time limitation, every one wants to understand how to learn will get the best result and how much time they have to spend on each course. Using effective heuristic learning function, we raised two different course learning algorithm, score-based algorithm and time-based algorithm under the conditions of courses dependent and independent separately.

For getting better learning efficiency, we follow the steps:

Step 1: Obtain the learner's learning function under pre-testing and (IRT).
Step 2: Sample learning efficiency of each course.
Step 3: Multiply learning efficiency by normalized weight of each course.
Step 4: Sort courses learning efficiency by descending.

Step 5: Choose a suitable Algorithms 1.1, 1.2, 2.1 or 2.2 in section 4.

Through proposed algorithm simulation results, we could get suggestions about the course learning sequence, and the times we spend on each course in order to get better learning efficiency under time limitation. From the result comparison in Figs. 8 and 9, we could see that our proposed algorithm has better learning efficiency than others.

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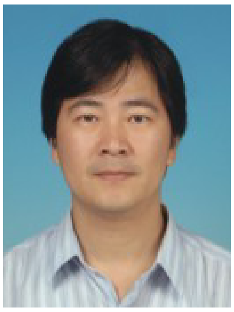

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