行政院國家科學委員會補助專題研究計畫成果報告

※※※※※※※※※※※※※※※※※※※※※※※※※
※　　　　　　　　　　　　　　　　　　　　　　　※

※　BOT 計畫風險分析之研究　※

※　　　　　　　　　　　　　　　　　　　　　　　※
※※※※※※※※※※※※※※※※※※※※※※※※※

計畫類別：¦ **個別型計畫**　　□**整合型計畫**
計畫編號：NSC 89－2211－E－009－024－
執行期間：　88 **年** 8 **月** 1 **日至** 89 **年** 7 **月** 31 **日**

計畫主持人：馮正民教授
共同主持人：

本成果報告包括以下應繳交之附件：
□**赴國外出差或研習心得報告一份**
□**赴大陸地區出差或研習心得報告一份**
□**出席國際學術會議心得報告及發表之論文各一份**
□**國際合作研究計畫國外研究報告書一份**

執行單位：國立交通大學交通運輸研究所

中　華　民　國 89 **年** 10 **月** 5 **日**

行政院國家科學委員會專題研究計畫成果報告

BOT 計畫風險分析之研究

計畫編號：NSC 89-2211-E-009-024
執行期限：88 年 8 月 1 日至 89 年 7 月 31 日

主 持 人：馮正民　國立交通大學交通運輸研究所
計畫參與人員：康照宗 國立交通大學交通運輸研究所博士候選人

一、中文摘要

本研究目的係以BOT特許公司群體決策觀點，探討 BOT 特許契約之風險特性。在考量群體內部談判者有無討論情形下，對不確定性因素進行風險衡量，以確認風險事件、主要風險事件及次要風險事件。本研究以風險及效用理論為基礎，以數學解析方法構建談判者效用及談判群體效用模式，並研擬求解方法，同時以範例分析方式進行模式之測試。經測試結果顯示，在無討論情形下，若事件之群體總計效用值小於效用期望值時，此事件即屬風險事件；反之，事件為非風險事件。由範例分析結果顯示，特許期限為 BOT 特許公司進行談判時之主要風險事件，匯率為次要風險事件。因此，特許期限應為 BOT 特許公司與政府進行特許契約談判時之首要議題。而在有討論情形下，本研究採動態多目標規劃方式進行事件之風險衡量，就所舉範例之 6 個事件而言，談判者彼此之間經過討論後，特許期限與匯率不再成為風險事件。經由本研究範例分析顯示，本研究所發展之效用衡量模式具有可用性。

關鍵詞：BOT、風險衡量、風險確認、談判群體、討論、效用相依

Abstract

The purpose of the study is to explore the characters of risk of the BOT concession contract from the viewpoint of group decision-making. Considering the discussion behavior among the negotiators, this study develops the utility function of negotiators, group aggregation utility function, dynamic multi-objective programming, and iterative algorithm to evaluate the uncertain factors of BOT concession contract, which based on the utility, risk, and BOT theory. The numerical example shows that the concession period is main risk event, the money exchange rate is secondary risk event, and other events are non-risk event under the independent utility condition among the negotiators. The concession period and money exchange rate could become non-risk events after the discussion among negotiators, which shows that the models of this study developed would be applied to measure the risk of BOT projects in transportation field.

Keywords: BOT, Risk measurement, Risk identification, Negotiation team, Discussion, Dependent utility.

二、緣由與目的

BOT (build, operate and transfer)係交通基礎建設民營化方式之一。由於 BOT 計畫具備特許期限長、不確定性因素多、特許企業聯盟參與廠商多、投入資本龐大以及融資風險高等特色，所以政府採用 BOT 方式之理由，除兼具考量減輕財政困窘，藉用民間技術與管理效率之外，更重要的是在尋找風險分擔對象。對 BOT 特許公司而言，特許契約談判成為風險分擔重要手段之一，藉由契約談判來達到風險移轉，但是欲達特許契約之風險移轉，前題有賴風險衡量與風險確認之分析工作。按 BOT 理論〔7〕，特許契約內所存在之不確定性因素可藉由風險衡量方法，達到尋找風險與非風險因素以及判斷主要風險與次要風險，完成風險移轉與規避管理措施。此風險衡量方法，按文獻可分定性分析與量化分析兩種，BOT 計畫之風險定性分析主要論述 BOT 計畫之風險種類，如政治風險、商業風險、法規變動風險、興建完工風險及營運風險等〔5,7〕，但是此種定性風險

分析並無法有效說明風險水準，風險如何產生以及如何確認主要風險及次要風險。定量風險分析方法如統計分析、財務分析、經濟效用及專家權重方法[4]，這些方法已廣泛運用於交通運輸、管理與投資領域。效用理論[1,3]除可衡量風險與不確定性因素外，亦可反應決策者風險偏好行為，同時該值介於 0 與 1 之間，具備方便判斷之優點。

由文獻可知，風險定性分析欠缺數據佐證，無法得知風險水準；定量分析雖具彌補定性分析的缺點，但仍有許多限制，如財務風險分析[2]無法放寬資金流量假設條件，專家權重法[4]無法知道決策者風險偏好且評估體系不可以一體適用。

由文獻顯示[5,6,7]，BOT 計畫之風險衡量主要是由談判群體內之談判者或專家們，針對不確定性因素進行衡量，此風險衡量過程涉及專家(或決策者)風險偏好態度[3]及不確定性因素之間獨立與否有關，在此兩個因素下可以構成四種情境：1.風險因素及決策偏好皆獨立，2.風險因素不獨立，但決策偏好獨立，3.風險因素獨立但決策偏好不獨立，4.風險因素及決策偏好皆不獨立，第一種情境即所謂確定性風險分析，其餘則為非確定性風險分析。

本研究目的即是考量風險偏好態度與風險因素獨立與否之架構下，以數學解析方法來構建 BOT 計畫之風險衡量模式，藉以求解風險、主要風險事件、次要風險事件，同時兼考慮談判者風險偏好態度。

三、研究內容與成果

(一)、研究內容項目

本研究內容主要共分七個步驟，包含研究問題定義、文獻回顧與評析、風險定義、構建談判者風險衡量模式、構建談判群體風險衡量模式、簡例測試及結論與建議。本研究方法以 BOT 理論為架構，效用及風險理論為分析工具基礎，利用數學解析方式推導談判者效用函數及談判群體效用函數，以及研擬求解方法，並分別利用 SAS 統計軟體與 Turbo Pascal，撰寫程式，進行效用函數與風險衡量之求解。

在簡例分析方面，本研究係假設 BOT 特許公司談判群體有 6 位談判者，並考量談判者彼此之間有討論與無討論兩種情形；同時假設該公司於特許期限內面臨用地取得、銀行融資信款利率、銀行折扣率、特許期限、費率管制及外匯之 6 個事件(或不確定性因素)，由此 6 位談判者對此 6 個不確定性因素進行風險衡量。

(二)、研究成果

經由本研究所發展之談判者效用衡量模式、談判群體效用相依模式及求解法，所得到結果如下：

(1).在假設事件獨立、談判者效用獨立及談判群體追求風險最小化之情境下，推得關鍵風險水準為所有風險事件之效用期望值。另由範例分析得知，特許期限與匯率屬於風險事件；銀行融資信用貸款利率、費率管制、用地徵收與折扣率屬非風險事件。其中，特許期限屬主要風險事件，匯率屬次要風險事件。

(2).在假設事件獨立，談判者效用不獨立情境下，談判群體對此 6 個事件之風險衡量結果，呈現特許期限、匯率、銀行融資信用貸款利率、費率管制、用地徵收與折扣率之事件皆屬非風險事件。

(三)、討論

本研究除獲得上述研究成果，並歸納以下課題可供後續研究。

(1).在效用獨立狀態下，個別談判者之效用風險偏好態度對談判者之風險有影響，但是當談判者彼此之間存有互動關係，個別談判者風險偏好態度對群體效用影響程度是否會展現出來？

(2).影響談判者彼此之間的互動因素，除內部互動外，尚有諸如談判者自我學習能力、資訊充足與否等因素。此外，外部環境因素變動對內部談判者之影響，這些因素改變對模式發展有何改變是值得思考？

(3).迴歸模式雖可構建談判者偏好函數，但只能反應談判者之效用線性函數或多元線性函數。惟在實際狀態下，談判者效用函數可能存在非線性函數形態，此非線性效用函數如何引入談判者彼此之間討論模式中，可為後續發展議題。

(四)、研究成果發表

本研究成果部份已發表或投稿於國內

外相關學術期刊，茲臚列如下。

1. Cheng-Min Feng and Chao-Chung Kang (1999), "Risk Identification and Measurement of BOT Projects," *Journal of the Eastern Asia Society for Transportation Studies,* No. 4, Vol. 4, pp. 331-350.
2. Cheng-Min Feng, Chao-Chung Kang and Gwo-Hshiung Tzeng, "Applications of Group Negotiation to Risk Assessment of BOT Projects," submited to Transportation Planning and Technology, 2000.
3. **馮正民、康照宗，「**BOT**計畫談判群體之風險衡量」，運輸計畫季刊**(29**卷第四期，即將刊登**)**，民國**89**年。**
4. **馮正民、康照宗，「在談判者效用互動下之風險衡量**-**以**BOT**計畫之用地取得事件為例」，運輸計畫季刊**(**已接受**)**，民國**89**年。**
5. Cheng-Min Feng and Chao-Chung Kang, "The risk assessment for the BOT Negotiation team: The dynamic utility model," submited to 9th The World Conference on Transportation Research (2001).

五、參考文獻

[1].Bose, U., Davey, A. M., and Olson, D. L., "Multiattrinute Utility Methods in Group Decision-Making: Past Applications and Potential for Inclusion in GDSS", Omega, *International Journal of Management Science*, Vol. 25, No. 6, 1997, pp.691-706.

[2]Buhlmann, H., *Mathematical Methods in Risk Theory*, Springer-Verlag, 1996.

[3]Keeney, R. L. and Raiffa, H., *Decisions with Multiple Objectives Preferences and Value Tradeoffs*, Cambridge University Press, 1993.

[4]Mustafa, M. A. and Al-Bahar, J. F., "Project Risk Assessment Using the Analytic Hierarchy Process", *IEEE Transactions on Engineering Management*, Vol. 38, No.1, 1991, pp. 46-52.

[5]Philip, N., "Allocation of Risks in BOT Projects", The High Speed Rail BOT Workshop, Taipei, Taiwan, 1995.

[6]Tiong, L. K., "BOT Projects: Risks and Securities", *Construction Management and Economics*, Vol. 8, No. 3, 1990, pp. 315-328.

[7]Walker, C. and Smith, A. J., *Privatized Infrastructure: the Build Operate Transfer*, Thomas Telford Publications, 1996.

Journal of the Eastern Asia Society for Transportation Studies, No. 4. Vol. 1, pp.331-350

RISK IDENTIFICATION AND MEASUREMENT OF BOT PROJECTS

By

Cheng-Min Feng

&

Chao-Chung Kang

Institute of Traffic and Transportation

National Chiao Tung University

114, 4F, Sec.1, Chung Hsiao W. Rd.
Taipei 100, Taiwan
Fax: 886-2-23120082,
E-Mail: cmfeng@cc.nctu.edu.tw

RISK IDENTIFICATION AND MEASUREMENT OF BOT PROJECTS

Cheng-Min Feng
Professor and Chairman
Institute of Traffic and Transportation
National Chiao Tung University
114, 4F, Sec.1, Chung Hsiao W. Rd.
100, Taipei, Taiwan
Fax: 886-2-23120082
E-Mail:cmfeng@sunwk1.tpe.nctu.edu.tw

Chao-Chung Kang
Ph, D. Student
Institute of Traffic and Transportation
National Chiao Tung University
114, 4F, Sec.1, Chung Hsiao W. Rd
100, Taipei, Taiwan
Fax: 886-2-23120082
E-Mail:cckang@iot.gov.tw

Abstract: The purpose of this paper is to analytically measure and rank risk of BOT projects for decision making under an uncertain environment. The individual and group multi-attribute risk utility functions in the risk measurement model are developed based on multiattribute decision making and utility theorems. The preference of the negotiator is considered in the multiattribute risk utility function. The risk event is obtained by the model when the group risk utility value is smaller than the expected risk utility value. Futhermore, the critical risk event is obtained when the group multiattribute risk utility value is not less than the expected utility. In addition the risk measurement model provides an approach to quantify, identify and find critical risk, and to incorporate the preference of the decision-maker in order to share risk under BOT negotiation.

Key Words: BOT; risk identification; critical risk; risk measurement; uncertainty

1. INTRODUCTION

Transportation infrastructure development projects have the following characteristics: large civil works budget, substantial land acquisition over a large area, long construction time frame, large labor force, complex internal government coordination, and multi-national construction/design teams. Few transportation development projects are profitable or self-financing on the basis of user fees only, because of complicating factors such as high capital cost, lengthy construction period with no revenues, and low to moderate fare level favored by government. BOT (Build, Operate and Transfer), one method of privatization, is an approach where the private sector is given a concession to design, construct, finance, manage and operate a project that would normally be built and operated by the government, and transfers ownership of the project back to the government at the end of the concession period. Some important reasons for governments to use the approach are to reduce the government’s financial burden, to use the private sector’s technological know-how, management skills and capital, and to transfer most of the project risks to the private sector.

Since both the government and private sector will take part in a BOT project, the complex contractual negotiation requires considerable cost and time and becomes an important subject of BOT projects. Identification and measurement of risk is fundamental to risk allocation and sharing, which is the basis of contractual negotiation between the government and concession company (Tiong, 1992, 1995, 1996, 1997; Sidney, 1996; Walker and Smith, 1996). Normally, the government wants to transfer most of risk to private sector while the concession company expects to reduce its exposure to risk (Levitt, *et al.* 1980). Philip (1995), Nicole (1995), Tiong (1990, 1995), and Walker and Smith (1996) have discussed different types of risks BOT projects are exposed to, but risk measurement and risk identification are not explored.

Different projects have their own risk profile, although in general there are political risks, commercial risks, legal risks, construction/completion risks, operation risks, etc. How to measure the degree of risk? and how to distinguish between major and minor risk? are the issues this paper will explore. Tiong (1990, 1995) and Tiong and Yeo (1992) have shown that risk analysis is an important issue for BOT projects particularly during the period of bidding, contract negotiation and risk management. Hwang (1995) has employed the notion of property rights to elucidate the essence of BOT projects, and to illustrate their optimal risk level by means of transaction cost and probability distribution. The results show that the optimal risk of the investment is in positive, indeterminable, or negative relationship to the investment rate of return and that the BOT contract is a non-zero-sum game which is completely different from the zero-sum game. They also show the different relationship between risk and investment return but do not show what level of risk is critical. William and Crandall (1982) considered the attributes of risks, suggesting that the risk measurement of infrastructure projects must consider the attributes of risk events. The risk negotiation was affected by risk attributes and the negotiator’s preference (Seo and Sakawa, 1985, 1990). Following the concepts of Seo and Sakawa, this paper will focus on risk measurement and risk identification.

Quantitative and qualitative methods have been used to discuss or measure risk, in past research. Financial risk analysis (Cuthbertson, 1996), utility analysis (Jia and Dyer, 1996; Seo and Sakawa, 1984, 1985, 1990), statistical analysis (Louis, 1990; Jaselskis and Russell, 1992; Ronald, 1990), and expert investigation (Mustafa and Al-Bahar, 1991) were used to analyze risk in the quantitative analysis field. The indices in financial risk analyses, such as the NPV, B/C ratio and IRR, have been widely used for measurement of financial conditions, but they have difficulty estimating future cash flow. Therefore, financial risk analysis is properly applied to evaluate only short-term projects with a certain environment. As for the BOT project with high uncertainty and with a long concession time, it is difficult to accurately estimate cash flow (Sidney, 1996). In addition, the major problem is to determine what level the risk of loss is? Is it 1 million dollars or 1 billion dollars? This problem is hard to answer by NPV. Although the B/C and IRR ratio have an index value from 0 to 1, the index value cannot reflect the different levels of risk for different events.

Buhlman (1996), Ronald (1990), Louis (1990), Jia and Dyer (1996), and Hwang (1995) use the statistical approach to measure risk. The expected value is obtained where the risk probability distribution has a supposed specific distribution. We think the problem lies in what kind of distribution can fit the probability for BOT projects with thirty years concession time, as well as what type of independent or dependent relationships among the risks will lead to measurement error in expected value of loss. As for the utility approach, it is liable to be applied on certainty or uncertainty, and it cannot estimate future cash flow. The approach is especially suited to considering the negotiator’s preference in order to reflect the risk preference during contract negotiation (William and Keith, 1982; Seo and Sakawa, 1984, 1985). Also, the utility approach easily judges with the value between 0 and 1. In addition, Seo and Sakawa have constructed a risk utility function and introduced the fuzzy concept into risk analysis so as to render it more fitting to uncertain negotiation behavior. Seo and Sakawa (1985, 1990) have focused on the preference change for the decision maker’s behavior, but they have not defined the risk by using the utility theorem. Jia and Dyer (1996) have developed risk measurement as $R(X) = -E[u(X - \bar{X})]$, and this study provides the concept of a negative expected utility in preference. The $u(X - \bar{X})$ is a normalization value in mean, where X is a probability distribution and $R(X)$ is a risk measure, but the equations do not consider the stability in measuring risk for factors deriving from different risks, events, attribute samples, etc. In addition, the normalization value in $X - \bar{X}$ results in a positive or negative value, and thus, $R(X)$ value will not hold to

only one value.
Expert investigation is a method to measure risk and the AHP method has been used to evaluate risk in criteria construction (Mustafa and Al-Bahar, 1991). The AHP approach captures the weight value from project experts, engineers or project managers and then, based on the weight of experts and performance value, measures risk value. Nonetheless, the weight and performance value obtained from AHP can hardly demonstrate genuine risk. Moreover, there are different risks for each construction project, the criteria and goals have different importance and hence, the hierarchical structure alone can't indicate the unique conditions of the event.
The purpose of this paper is to analytically identify and measure risk, and to determine the critical risk for the government or BOT concession company. In order to take preference and risk attributes into account, this paper uses multiattribute decision making and utility theorem to construct a multiple attribute utility function to illustrate the measurement of risk, identification of risks and critical risk events
The remainder of this paper is organized as follows: section 2 describes the problem, defines the risk and uncertainty, and develops the multiattribute risk utility function; section 3 develops the group utility function; section 4 analyzes the risk of the BOT contract; and conclusions are made in the final section.

2. MODEL DEVELOPMENT

In this section, we will describe the problem of this study, define the risk and uncertainty, and develop the individual risk utility function in order to establish a group negotiation risk utility function.

2.1 The Problem Description

An Airport-Link Rapid Transit project between CKS Airport and Taipei city will be undertaken by BOT approach in Taiwan. The BOT concession company and government are in contract negotiation for this BOT project as bidding for this transportation infrastructure project recently finished. There are two groups taking part in negotiations, one is the government group and the other is the BOT private group. The government group includes some individual negotiators from other government departments, such as the MOTC (Ministry of Transportation and Communications), EPA (Environment Protection Administration), city government, etc. Also the BOT private group has some individual negotiators including lawyers, financial consultants, participators, participant companies, etc. It must be clarified here, that the two primary negotiators of this contract are groups rather than individuals (see Figure 1). The conditions of contract must be acceptable to both parties, otherwise, the BOT project will be terminated.

In the past, most researches qualitatively identify risks from events, however, risk and uncertainty should be strictly treated different. In addition, what are the critical or important risk items should also be a main issue for negotiators during the period of bidding, negotiation and risk mitigation. This paper aims at developing an approach to decompose the set of event into the set of uncertainty and risk, and furthermore to decompose the set of risk items to critical risks and general risks (see Figure 2).

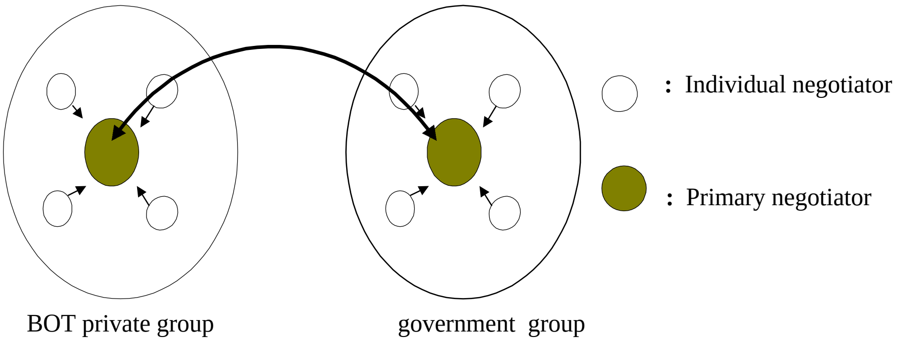


Figure 1. Primary Negotiators and Individuals

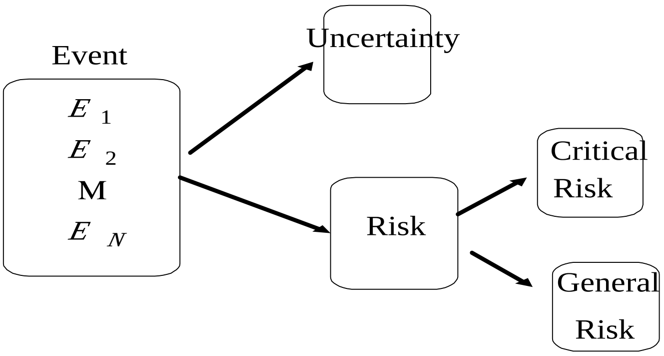


Figure 2. Event, Uncerrtainty, Risk and Critical Risk

2.2 Basic Assumptions

In this paper, some assumptions for model development are as follows:

(1).The asymmetric information does not exist between the negotiation group and individual negotiator.

(2).The decision-making behavior of individual negotiators within the negotiation group is reasonable.

(3).The probability of event occurrence is assumed to be a Bernoulli experiment and the probability of occurrence is the probability of success.

2.3 Model Development

(1). Risk and uncertainty definition

Based on the literature mentioned above, where variance is greater, the risk is higher and where there is a greater difference between actual occurrence and expectation, there will be greater loss. Considering the stability of risk, the risk and uncertainty definition will be expressed as the following:

(a). Risk definition
Since the dislike events will result in lower utility for decision maker, the risk will be defined as a specific event which will result in lower preference for the decision maker. This interprets the occurrence of risk, the loss of preference and the tolerance of choice. Risk R_E is defined as Eq. (1):

$$R_E \equiv u_j(x_j) < \bar{u}(x) \qquad (1)$$

where $u_j(x_j)$ is the utility function of outcome x_j for specific event E, $0 \le u_j(x_j) \le 1$, $\bar{u}(x)$ is the expected utility value for specific event E, $\bar{u}(x) = \sum_j p_j u_j(x_j)$, p_j is the probability of outcome x_j; Eq. (1) implies that the event E has risk when the utility function of outcome x_j for the decision maker is less than expected utility value for event E.

(b). Uncertainty definition

In contrast to risk, uncertainty will be defined as a specific event which will not result in lower preference for the decision maker. The equation can be expressed as Eq. (2).

$$UR_E \equiv (u_j(x_j)) \ge \bar{u}(x) \qquad (2)$$

where the variables $u_j(x_j)$ and $\bar{u}(x)$ are defined as above. The $u_j(x_j)$ value will be $0 \le u_j(x_j) \le 1$.

(2). The risk utility function of the individual decision maker

The risk utility function proposed in this study was based on the multiattribute theory and utility function. The single attribute and multiattribute of a specific event are considered in the risk utility function.

(a). Transformation of the variable

Based on the utility theorem of Keeney and Raiffa (1993), the utility or preference for an event or alternative should be a positive value, that is $0 \le u_j(x_j) \le 1$. This paper proposes the utility transformation variable to satisfy the condition of a utility value between 0 and 1. The utility normalization is defined as Eq. (3).

$$u^*(x_{ij}) = \frac{p_{ij} \times u(x_{ij}) - \min\{p_{ij} \times u(x_{ij})\}}{\max\{p_{ij} \times u(x_{ij})\} - \min\{p_{ij} \times u(x_{ij})\}} \qquad (3)$$

where $u^*(x_{ij})$ is the normalized utility value, p_{ij} is the probability of the utility of outcome i and state j, for all $j = 1,2,...,n$, $i = 1,2,...,m$; $u(x_{ij})$ is the utility value of outcome x_{ij}. Since $0 \le u(x_{ij}) \le 1$ and $0 \le p_{ij} \le 1$, the $u^*(x_{ij})$ value will be located between 0 and 1, $0 \le u^*(x_{ij}) \le 1$. Eq. (3) considers a multiattribute case. When $i = 1$, then Eq. (3) becomes a single attribute case.

(b) Single attribute risk utility

Considering a single attribute of a specific event, let E denote the specific event, S has n states for event E, $S = \{s_1, s_2, ..., s_j, ..., s_n\}$, for $j = 1,2,...,n$; X has outcomes x_n under s_n states for event E, $X = \{x_1, x_2, ..., x_j, ..., x_n\}$; $u(X)$ is the utility value of the individual negotiator of outcome x_n for event E; p is the set of probability corresponding to the utility, $p=\{p_1,p_2,...,p_j,...,p_n\}$, $p_j=prob(x_j,s_j)$, $0\leq p_j\leq 1$. The structure of state, probability and outcome of the attribute for event E are shown in Table 1.

Table 1 The Structure of State, Probability and Attribute

Event E	States S
	s_1 , s_2 ,..., s_j ,..., s_n
Outcome of attribute X	x_1 , x_2 ,..., x_j ,..., x_n
Utility $u(X)$	$u(x_1),u(x_2),...,u(x_j),...,u(x_n)$
Probability P	$p_{1,}$ p_2 ,..., p_j ,..., , p_n

For event E, $p_j=\Pr ob(x_j,s_j)$ is the probability of the utility under the outcome x_j and s_j states, $p_{j-1}=\Pr ob(x_{j-1},s_{j-1})$ is the probability of the outcome under the outcome x_{j-1} and state s_{j-1}. Because there exists a one-to-one relationship among s_j, x_j and p_j, the states s_j and s_{j-1} are mutually independent and the outcome x_j and x_{j-1} are also mutually independent, then $\Pr ob(x_{j-1},s_{j-1})\ \mathrm{I}\ \Pr ob(x_j,s_j)=0$. Based on Table 1, the probability of the utility value is $p_1,p_2,\Lambda\ ,p_j,\Lambda\ ,p_n$, respectively; the utility mean value is obtained from $\bar{u}(x)$, $\bar{u}(x)=\sum_{j=1}^{n}p_j u(x_j)$; the standard deviation of utility value is $\pm(u(x_j))=\sqrt{Var(u(x_j))}$, for all $j=1,2,..,n$. Based on the concept from Eq. (1), the risk utility function of event E for the g individual negotiator can be shown as Eq. (4).

$$u_g(R_E)\equiv((u^*(x_j))<\bar{u}(x)/\pm((u(x)) \qquad (4)$$

where $u_g(R_E)$ is the risk utility function for the g individual negotiator, the risk utility is normalized by $u^*(x_j)$. If $((u^*(x_j))<\bar{u}(x)/\pm((u(x))$ then event E is a risk event, since $0\leq u^*(x_j)\leq 1$, then $0\leq u_g(R_E)\leq 1$. Eq. (4) implies that the event E is a risk event for g individual negotiator when the normalized utility value is smaller than the expected utility value. Otherwise, the event E is an uncertain event.

(c). Multiattribute risk utility

This paper, based on the fundamental single attribute risk utility, will develop the multiattribute risk utility function. Suppose event E has m outcomes and n states, let S be the set of state of event E, $S=\{s_1,s_2,...,s_j,...,s_n\}$; X is the set of outcomes $X=\{x_1,x_2,...,x_i,...,x_m\}$, for $i=1,2,...,m$; the probability P is the set of probability corresponding to the utility, $P=\{p_{1j},p_{2j},...,p_{ij},...,p_{mj}\}$, $p_{ij}=\Pr ob(x_{ij},s_j)$, $0\leq p_{ij}\leq 1$, for $j=1,2,..,n$, $i=1,2,...,m$. The relationship between state, probability and attributes is shown in Table 2.

Table 2 The Structure of State, Probability and Attributes

Event	States		Probability	Weight
E	S	$s_1, s_2, \ldots, s_j, \ldots, s_n$	P	w_j
Outcome Of Attribute X	x_1	$x_{11}, x_{12}, \ldots, x_{1j}, \ldots, x_{1n}$		
	$\vdots$	$\vdots \quad \vdots \quad \vdots \quad \vdots$		
	x_i	$x_{i1}, x_{i2}, \ldots, x_{ij}, \ldots, x_{in}$		
	$\vdots$	$\vdots \quad \vdots \quad \vdots \quad \vdots$		
	x_m	$x_{m1}, x_{m2}, \ldots, x_{mj}, \ldots, x_{mn}$		
Utility $u(x)$	$u(x_1)$	$u(x_{11}), \ldots, u(x_{1j}), \ldots, u(x_{1n})$	p_{1j}	w_1
	$\vdots$	$\vdots \quad \vdots \quad \vdots$	$\vdots$	$\vdots$
	$u(x_i)$	$u(x_{i1}), \ldots, u(x_{ij}), \ldots, u(x_{in})$	p_{ij}	w_i
	$\vdots$	$\vdots \quad \vdots \quad \vdots$	$\vdots$	$\vdots$
	$u(x_m)$	$u(x_{m1}), \ldots, u(x_{mj}), \ldots, u(x_{mn})$	p_{mj}	w_m
Probability	P	$p_{i1} \cdots p_{ij} \cdots p_{in}$		

Let $p_{ij} = \Pr ob(x_{ij}, s_j)$ be probability of utility corresponding to the outcome x_{ij} and state s_j, and $p_{i-1,j} = \Pr ob(x_{i-1j}, s_j)$ is the probability of utility corresponding to the outcome x_{i-1j} and state s_j; because there exists a one-to-one relationship between s_j, x_{ij}, and p_{ij}, thus $\Pr ob(x_{i-1j}, s_j) \cap \Pr ob(x_{ij}, s_j) = 0$. This shows that $p_{i-1,j}$ and $p_{i,j}$ are mutually independent. Let $\bar{u}(x_1) = \sum_{j=1}^{n} p_{1j} u(x_{1j})$ be the expected utility value of outcome x_1, $\bar{u}(x_i) = \sum_{j=1}^{n} p_{ij} u(x_{ij})$ be the expected utility value of outcome x_i, and let $\bar{u}(x) = \sum_{i=1}^{m} w_i \sum_{j=1}^{n} p_{ij} u(x_{ij})$ be the total expected utility value, where w_i is the weight value of utility. $\sigma^2 = Var(u(x)) = E(((u(x_{ij}) - \bar{u}(x))^2)$ is the total variance utility for all $j = 1,2,..,n$, $i = 1,2,...,m$. Then, the multiattribute risk utility function of event E for the g individual negotiator can be shown as Eq. (5).

$$u_g(R_{ME}) \equiv ((u^*(x_{ij})) < \bar{u}(x)/\sigma((u(x)) \tag{5}$$

where $u_g(R_{ME})$ is the multiattribute risk utility function for the g individual negotiator, the multiattribute risk utility is a normalized value, then the multiattribute risk utility will be between 0 and 1, $0 \le u_g(R_{ME}) \le 1$.

3. THE GROUP RISK UTILITY MODEL

In this section, the concession negotiator multiattribute risk utility function will be established for the BOT concession company and government sector respectively, and the function provided will measure risk, identify risk and find out the critical risk event.

The concepts in Eqs. (4) and (5) provided an individual negotiator for risk measurement and results in a constant value, linear, additive, multiplicative or other function form. Although the individual negotiator's preference can be taken into account in risk utility function, the risk measurement is different between the group and individual negotiators. In addition, the notion must be proposed that event E is a risk event by means of individual risk utility if the risk utility function satisfies Eq. (4). This does not ensure event E is a risk event for the concession group. In other words, this implies that the risk event for the group was determined by group risk utility, not by individual negotiators. The concept of group decision making has previously been discussed by Keeney and Raiffa (1993). Their theory infers that the group utility will be adopted instead of an individual utility when group negotiators are making decisions.

3.1 The Concept of Multiattribute Utility Function

Keeney and Raiffa (1993) proposed the multiattribute utility function (MAU). Based on their concept, the additive and multiplicative methods are appropriate for constructing the group decision-maker's multiattribute utility function. The conditions of this function are: (1) total number of attributes is not less than three, (2) the preference structure of decision-maker is preference independent, and (3) the utility of the decision-maker is preference independent. The fundamental mathematical equation for the multiattribute utility function can be expressed as Eq. (6).

$$U(X) = \sum_{i=1}^{n} k_i U_i(x_i) \tag{6}$$

where $U_i(x_i)$ is a single attribute utility function, $0 \le U_i(x_i) \le 1$; U is a multiattribute utility function; k_i is a scaling constant, and $0 \le k_i \le 1$. In Seo's (1990) study, which constructed the fuzzy multiattribute risk function for group decision making based on the concepts of Keneey and Raiffa, the assumptions were event independent, aversion independent and trade-off attribute independent because of his modified MAU theorem. The reason he made the assumptions was to consider the hazard for decision-makers when the event occurs under an uncertain environment.

3.2 The Group Negotiation Risk Utility Function

(1). Multiattribute risk utility function for the government sector

Suppose the government group has g negotiators, risk utility function $u_g(R_E)$ exists for each g individual negotiator, for $g = 1,2,\dots,h$. Assume the negotiators' utility, trade-off attribute, and events are independent. Following the concept of MAU and risk function constructed by Seo (1990), the group risk utility function for government was employed by this study, and is expressed as Eq. (7).

$$\begin{aligned} U_g^E &= U_g^E(u_1(R_1^E), u_2(R_2^E), \dots, u_g(R_g^E)) = \sum_{g=1}^{h} k_g u_g(R_g^E) + k \sum_{g=1}^{h} \sum_{a>g} k_{ag} u_g(R_g^E) u_a(R_a^E) \\ &+ k \sum_{g=1}^{h} \sum_{a>g} \sum_{b>a} k_{gab} u_g(R_g^E) u_a(R_a^E) u_b(R_b^E) + \Lambda + k_{123\Lambda h} u_1(R_1^E) u_2(R_2^E) \Lambda\, u_h(R_h^E) \end{aligned} \tag{7}$$

where U_g^E is a group multiattribute risk utility function for event E, $0 \le U_g^E \le 1$; $u_g(R_g^E)$

is a multiattribute risk utility function for risk event E for g individual negotiator; k_g is a scaling constant, and $0 \le k_g \le 1$. If $U_g^E < \bar{u}_g(R_g^E)$, the event E is a risk event for group decision-makers, otherwise, event E is an uncertain event. That means the risk event E is decided by group decision-makers and $\bar{u}_g(R_g^E)$ is defined as expected value risk utility of event E.

(2). Multiattribute risk utility function of BOT concession company

As for the BOT concession company, the group multiattribute risk utility was established by the same concept described above. Suppose the BOT company group has q negotiators, for $q=1,2,...,l$, the risk utility function of event F is $u_q(R_F)$ for q individual negotiator. Also, we assume for the BOT group risk utility function that the negotiators' utility is independent, trade-off attributes are independent, and events are independent. The group risk utility function for the BOT company was employed and is expressed as Eq. (8).

$$\begin{aligned} U_q^F &= U^F(u_1(R_1^F), u_2(R_2^F),..., u_1(R_1^F)) = \sum_{q=1}^{l} k_q u_q(R_q^F) + k \sum_{q=1}^{l} \sum_{a>q} k_{aq} u_q(R_q^F)\, u_a(R_a^F) \\ &+ k \sum_{q=1}^{l} \sum_{a>g} \sum_{b>a} k_{qab} u_q(R_q^F)\, u_a(R_a^F) u_b(R_b^F) + \Lambda + k_{123\Lambda l} u_1(R_1^F)\, u_2(R_2^F) \Lambda\, u_l(R_l^F) \end{aligned} \tag{8}$$

where U_q^F is a group multiattribute risk utility function for event E; $u_q(R_q^F)$ is a multiattribute risk utility function of g individual negotiator for risk event E; k_q is a scaling constant, and $0 \le k_q \le 1$. Because $0 \le u_q(R_q^F) \le 1$ and $0 \le k_q \le 1$, the U_q^F value will be between 0 and 1. If $U_q^F < \bar{u}_q(R_q^F)$, where $\bar{u}_q(R_q^F)$ is expected value risk utility of event F, then event F is a risk event for q negotiators, $q=1,2,...,l$**;** otherwise, event F is an uncertain event.

4. RISK ANALYSIS

In this section, the risk event and uncertainty event will be defined, and the risks of the contract will be explored.

4.1 Risk Event and Uncertain Event Definition

The concepts in Eqs. (7) and (8) have illustrated one event becoming a risk event for group negotiators instead for an individual negotiator. Because the risks are independent, the risk and uncertain event can be obtained from Eq. (7) or (8). As for the risk to the government, we can get the risk event and uncertain event one-by-one from Eq. (7). The risk event and uncertain event are defined in Eqs. (9) and (10), respectively.

(1). Risk event definition
If event E exists and $U_g^E < \bar{u}_g(R_g^E)$, then event E is a risk event, as defined by Eq. (9).

$$R_g^E = \{U_g^E < \bar{u}_g(R_g^E)\}, \text{ for } g = 1,2,...,h \tag{9}$$

(2). Uncertain event definition

Based on Eq. (7), if event E exists and $U_g^E \geq \bar{u}_g(R_g^E)$, then event E is an uncertain event.

4.2. Risk Choice and Ranking Risk

(1.) Risk Choice

For the government agency, suppose the group negotiation has g individual negotiators and G events for $G=1,2,...t$, $g=1,2,...,h$, and suppose these G events are independent. The risk can be measured per event by Eq. (7), then the risk event and uncertain event can be obtained in order to collect risk and uncertainty for the G events.

Let Ω^G denote the set of G events. Based on Eq. (7) and event independence, the events Ω^G can be separated from risk and uncertainty. The set of events Ω^G is defined in Eq. (10). Also, the risk event set and the uncertainty event set can be defined in Eqs. (11) and (12), respectively.

$$\begin{aligned}\Omega^G &= \Omega_R^G + \Omega_{NR}^G \\ &= \{R_g^G \mid U_g^G < \bar{u}(R_g^G), \forall\, G=1,2,..t; g=1,2,...,h\} + \{UR_g^G \mid U_g^G \geq \bar{u}(R_g^G), \forall\, G=1,2,..t; g=1,2,...,h\}\end{aligned} \quad (10)$$

$$\Omega_R^G = \{R_g^G \mid U_g^G < \bar{u}(R_g^G), \forall\, G=1,2,..t; g=1,2,...,h\} \quad (11)$$

$$\Omega_{NR}^G = \{UR_g^G \mid U_g^G \geq \bar{u}(R_g^G), \forall\, G=1,2,..t; g=1,2,...,h\} \quad (12)$$

As for the BOT concession company, let Π^Q denote the set of Q events for $Q=1,2,...r$ and suppose the event is independent. Based on Eq. (8), the risk event and uncertain event can be obtained for the BOT concession company and expressed by Eq. (12). In addition, the risk event set and the uncertain event set can be defined in Eqs. (14) and (15), respectively, for the BOT concession company.

$$\begin{aligned}\Pi^Q &= \Pi_R^Q + \Pi_{NR}^Q \\ &= \{R_q^Q < \bar{u}(R_q^Q),\ \forall\, Q=1,2,...,r; q=1,2,...,l\} + \{R_q^Q \geq \bar{u}(R_q^Q),\ \forall\, Q=1,2,...,r; q=1,2,...,l\}\end{aligned} \quad (13)$$

$$\Pi_R^Q = \{R_q^Q < \bar{u}(R_q^Q),\ \forall\, Q=1,2,...,r; q=1,2,...,l\} \quad (14)$$

$$\Pi_{NR}^Q = \{R_q^Q \geq \bar{u}(R_q^Q),\ \forall\, Q=1,2,...,r; q=1,2,...,l\} \quad (15)$$

(2). Rank risk and critical risk

(a) Rank risk

Based on Eqs. (8) and (9), the multiattribute risk utility function of group negotiation can be used to measure risk, and we can find the risk event set Ω_G^R and Π_Q^R of the government and BOT concession company, respectively. Suppose there are O, P, Q, R, and S risk events $O, P, Q, R, S \in \Omega_G^R$, and the risk utility U_g^O, U_g^P, U_g^Q, U_g^R and U_g^S can be obtained by means of equation, Based on the preference utility theorem, if the degree of risk utility is $U_g^S \,\pi\, U_g^O \,\pi\, U_g^P \,\pi\, U_g^Q \,\pi\, U_g^R$, then the degree of these risks can be ranked by risk utility value, the sequence being $R \phi Q \phi P \phi O \phi S$. Thus, the concept of ranking can provide negotiators knowledge about the risk factor.

(b) Critical risk and the general risk event

Based on the ranking mentioned above, the negotiators can know the maximum and minimum degree risk utility of $O, P, Q, R,$ and S risk events. It can be easily found that risk event R is the most critical and risk event S is least critical in this case. The problem is that it is hard to find the critical risk events when there exist many risk events. We use the expected value to deal with the problem and find the critical risk event.
Suppose for the group negotiation with g negotiators of government $g = 1,2,...,h$, there are $G - N$ risk events and N uncertain events. Let $f_G^g(w)$ denote the weight value of g negotiators. The weight value represents the different influence among the g negotiators. Since the g negotiators' utility is assumed to be dependent, the weight value will be 1, $f_g^G(w) = 1$. Assume the government pursues the maximum utility, that is said, the government party pursues more uncertainty events. The level of optimal risk utility can be obtained by differentiation in U_g^G, the result is expressed as Eq. (16).

$$
\begin{aligned}
&Max(E(U_g^G)) = Max(\sum_{G=1}^{G-N} f_g^G(w_g)U_g^G) \\
&\frac{\partial(E(U_g^G))}{\partial U_g^G} = \frac{\partial(\sum_{G=1}^{G-N} f_g^G(w_g)U_g^G)}{\partial U_g^G} = \frac{\partial(E(U_g^G - \overline{U}_g^G + \overline{U}_g^G)}{\partial U_g^G} \\
&= [\frac{\partial(E(U_g^G - \overline{U}_g^G))}{\partial U_g^G} + \frac{\partial(E(\overline{U}_g^G))}{\partial U_g^G}] \qquad (16) \\
&Let \frac{\partial(E(U_g^G - \overline{U}_g^G)}{\partial U_g^G} = 0, \Rightarrow E(U_g^G - \overline{U}_g^G) = 0 \\
&\therefore E(U_g^G) = \overline{U}_g^G
\end{aligned}
$$

where U_g^G is the total risk utility of the risk events for the government.

The result implies that the optimal risk level is the expected utility value of all risk events, $E(U_g^G) = \overline{U}_g^G$. Then, the critical risk event and general risk event can be found, the risk event is a critical event if $U_g^G > E(U_g^G)$. It is a general risk event if $U_g^G \le E(U_g^G)$, $\exists U_g^G \in \Omega_R^{G-N}$, for all $G = 1,2,...,t$. Therefore, the critical risk event and general risk event can be defined in Eqs. (17) and (18), respectively.

(i). Critical risk event definition
Let $C.R.$ denote the set of critical risk events. For a specific r risk event, $\forall r \in G - N$, if the r risk utility is greater than $\overline{U}_g^{G-N}$, then the r risk event becomes a critical risk event. It can be expressed as Eq. (17).

$$\exists U_g^{G-N} \in \Omega_R^{G-N}, s.t. \; CR \equiv \{U_g^{G-N} > \overline{U}_g^{G-N}, \forall G - N = 1,2,...t\} \qquad (17)$$

(ii). General risk event definition
Based on the concept in Eq. (17), the general risk event can be expressed as Eq. (18).

$$\exists U_g^{G-N} \in \Omega_R^{G-N}, s.t.\ CR \equiv \{U_g^{G-N} \leq \overline{U}_g^{G-N}, \forall G-N=1,2,\ldots t\} \tag{18}$$

The result of the critical risk event can be found by using Eq. (17) and the critical risk events should become critical bargaining chips during negotiation between the BOT concession company and government. Because the utility of general risk is not greater than the utility of critical risk events, these will be secondary in negotiation.

5. CONCLUSION

This paper has constructed a risk measurement model and risk analysis framework based on the multi-attribute decision making and utility theorems The risk utility function has considered a single attribute and multiattribute event, preference of individual negotiators, and the preference of group negotiators. This model can be used to measure risk, rank risk, and to find the critical risk event for the BOT concession company and government agency. Also, we have modified Jia and Dyer’s (1996) definition of risk, which did not consider the factors of stability in risk.

Suppose the negotiator utility and events are both independent, the optimal risk level will then be the expected risk utility value of all events and be able to distinguish the critical risk events and general risk events. Accordingly, preference of utility, the critical risk event and general risk event can be obtained from the risk sets’ optimal risk level, and it can be interpreted that critical risk events are the primary bargaining chips and general risk events are the secondary target of bargaining.

This study was conducted under the assumption that events, event attributes and utility functions of the negotiators are independent. In practical situations, however, events are not entirely independent and other negotiators will affect utility preference and cognition of the negotiators. The results will change when the assumptions are changed and we believe that this can be investigated in the future. Also, the game model of the negotiation contract can be explored.

REFERENCES

Buhlmann, H. (1996) **Mathematical Methods in Risk Theory**. Springer-Verlag.

Cuthbertson, M. (1996) **Quantitative Financial Economics Stock, Bonds and Foreign Exchange.** Published by John Wiley and Sons, Inc.

Hwang, Y. L. (1995) **Project and Policy Analysis of Build-Operate-Transfer Infrastructure Development**, Ph.D. Dissertation, Department of Civil Engineering, University of California at Berkeley.

Jaselskis, E. J. and Russell, J. S. (1992) Risk analysis approach to selection of contractor evaluation method. **Journal of Construction Engineering and Management 118**, 4, 805-812.

Jia, J. and Dyer, J. S. (1996) A standard measure of risk and risk-value models. **Management Science** 42, 12, 1691-1705.

Keeney, R. L. and Raiffa, H. (1993) **Decisions with Multiple Objectives Preferences and Value Tradeoffs.** Cambridge University Press.

Levitt, R. E.; Ashley, D. B. and Logcher, R. D. (1980) Allocating risk and incentive in construction. **Journal of the Construction Division 106**, 297-305.

Louis, A. Cox, Jr. (1990) A probabilistic risk assessment program for analyzing security risks. In Louis, A. Cox Jr. and Paqlo F. R (eds.), **New Risks Issues and Management.** Plenum Press, New York and London.

Mustafa, M. A. and Al-Bahar, J. F. (1991) Project risk assessment using the analytic hierarchy process. **IEEE Transaction on Engineering Management 38**, 1, 46-52.

Nicole, K. (1995) Risks and risk management in project finance. **The High Speed Rail BOT Workshop.** Taipei, Taiwan, December 1995.

Paek, J. H.; Lee Y. W. and Ock, J. H. (1993) Pricing construction risk: fuzzy set application. **Journal of Construction Engineering and Management 119**, 4, 743-755.

Philip, N. (1995) Allocation of risks in BOT projects. **The High Speed Rail BOT Workshop,** Taipei, Taiwan, December 1995.

Ronald, L. I. (1990) Methods used in probabilistic risk assessment for uncertainty and sensitivity analysis. In Louis, A. Cox Jr. and Paqlo, F. R (eds.), **New Risks Issues and Management.** Plenum Press, New York and London.

Seo, F. and Sakawa, M. (1990) A game theoretic approach with risk assessment for international conflict solving. **IEEE Transactions on Systems, Man, and Cybernetics 20**, 1, 141-148.

Seo, F. and Sakawa, M. (1984) An experimental method for diversified evaluation and risk assessment with conflicting objectives. **IEEE Transactions on Systems, Man, and Cybernetics 14**, 2, 213-223.

Seo, F. and Sakawa, M. (1985) Fuzzy multiattribute utility analysis for collective choice. **IEEE Transactions on Systems, Man, and Cybernetics 15**, 1, 45-53.

Seo, F. (1990) On a Construction Fuzzy Multiattribute Risk Function for Group Decision Making Kacprzyk, A. and Fedrizzi, M., **Multiperson Decision Making Models Using Fuzzy Sets and Possibility Theory,** Kluwer Academic Publishers, 98-218.

Sidney, M. L. (1996) **Build, Operate, Transfer Paving the Way for Tomorrow's Infrastructure**, John Wiley and Sons, Inc.

Tiong, L. K. (1990) Comparative Study of BOT Projects. **Journal of Management in Engineering 6**, 1, 107-122.

Tiong, L. K. (1990) BOT Projects: Risks and Securities. **Construction Management and Economics 8**, 3, 315-328.

Tiong, L. K. and Yeo, K. T. (1992) Critical Success Factors in Winning BOT Contracts. **Journal Engineering and Management 118**, 2, 217-228.

Tiong, L. K. (1995) Risks and Guarantees in BOT Tender. **Journal of Construction Engineering and Management, 121,** 2, 183-187.

Tiong, L. K. (1995) Impact of Financial Package versus Technical Solution in a BOT Tender. **Journal of Construction Engineering and Management 121,** 3, 304-311.

Tiong, L. K. and Alum, J. (1997) Final Negotiation in Competitive BOT Tender. **Journal of Construction Engineering and Management 123**, 1, 6-10.

Tiong, L. K. (1996) CSFs in Competitive Tender and Negotiation Model for BOT Projects. **Journal of Construction Engineering and Management 122**, 3, 205-211.

Walker, C. and Smith, A. J. (1996) **Privatized Infrastructure: The Build Operate Transfer.** Thomas Telford Publications.

William, I. C. J. and Crandall, K. C. (1982) Construction Risk: Multiattribute Approach. **Journal of the Construction Division 108**, 2,187-200.

Submitted to *Transportation Planning and Technology* **for review**
(paper Number 695)

Please do not quote without permission
Comments are welcome.

Applications of Group Negotiation to Risk Assessment of BOT Projects

By

Cheng-Min Feng

Chao-Chung Kang

&

Gwo-Hshing Tzeng

Institute of Traffic and Transportation

National Chiao Tung University

114, 4F, Sec.1, Chung Hsiao W. Rd.
Taipei 100, Taiwan
Fax: 886-2-23120082,
E-Mail: cmfeng@cc.nctu.edu.tw

Applications of Group Negotiation to Risk Assessment of BOT Projects

Cheng-Min Feng, Chao-Chung Kang and Gwo-Hshing Tzeng
Institute of Traffic and Transportation, National Chiao Tung University
114, 4F, Sec.1, Chung Hsiao W. Rd., Taipei 100, Taiwan
Fax: 886-2-23120082, E-Mail: cmfeng@cc.nctu.edu.tw

Abstract: This study attempts to identify and assess the potential risks of BOT projects. Based on risk analysis and the multi-attribute utility theory, the negotiator's utility and group utility functions are established via mathematical analysis. The method can evaluate the risk status of event attributes, and can justify whether the event is a risk event or not. This study shows that if the value of group aggregation utility is less than the weight expected utility, the event is regarded as a risk event; otherwise, it is a non-risk event. Numerical examples show that the concession period of the BOT project is the primary risk event; while the foreign exchange ratio is the secondary risk event. The concession period should be the primary consideration in negotiation of the BOT projects. The model developed herein can be applied for use in the contract negotiation of the BOT project. More importantly, this model can find the primary and secondary risk event from among many risk events.

Key Words: BOT; risk assessment; risk identification; uncertainty

1. Introduction

BOT (Build, Operate and Transfer), one type of privatization, is a newly developed approach where the private sector is granted a specific concession to independently plan, design, construct, operate and maintain a project. However, the ownership of the completed project is transferred back to the government at the end of the concession period. Both the public and private sector face great risk in executing a BOT project. The public sector utilizes this approach to alleviate the government's financial burden, learn technological know-how and management skills from the private sector, and share the potential risks with another party. Meanwhile, for the company seeking the BOT-concession, negotiating the concession contract becomes an important job in trying to share and transfer the possible risk to another party. However, the company must analyze and identify risk events in advance before they can transfer the risk. This study examines risk assessment and risk identification regarding concession contracts for BOT projects.

Risk assessment is the key issue for the Concession Company, involving tendering of the BOT Projects and contractual negotiation (Tiong, 1995; Sidney, 1996). Risk assessment is also the key to successful contract negotiation (Tiong, 1990a; Walker and Smith, 1996). Risk sharing is an important incentive for concluding engineering contracts and is important in both investment and contract negotiation (Levitt, et al., 1980; Walker and Smith, 1996).

The analysis of risk assessment can be classified into qualitative and quantitative analysis. Philip (1995), Tiong (1990b, 1995), and Walker and Smith (1996) investigate different types of risks encountered by BOT projects, like political risk, commercial risk, legislative risk, risk of construction completion, operational risk, and so on. However, qualitative analysis can hardly explain some important measures of effectiveness (MOE) such as the level of risk, how risk is produced, how to identify primary and secondary risk and so on. On the other hand, Quantitative analysis, like statistical analysis (Buhlmann, 1996; Jaselskis and Russell, 1992), financial analysis (Cuthbertson, 1996), engineering economic analysisc (Cooper and Chapman, 1987), and the weighted method (Mustafa and Al-Bahar, 1991), have been broadly applied to transportation investment projects. Hwang (1995) investigates the relationship between the level of risk and the rate of return of the investment in the BOT Project using the concept of property rights and transaction cost. He applies the statistical approach to measure the project risk, and the expected value of investment benefit was obtained by assuming risk as a specific probability distribution.

The Financial analysis applied to risk includes Net Present Value (NPV), Benefit-Cost (B/C ratio) and Internal Rate of Return (IRR). These analysis methods must make some assumptions and estimate future cash flow. However, effectively estimating future cash flow is very difficult (Sidney, 1996). As for the utility approach, it can assess risk and also reflect risk preference behavior of the decision-maker (Seo and Sakawa, 1984,1985; Keeney and Raffia, 1993).

Owing to different definitions of risk, differences exist among assessment methods. Risk can be measured via probability, expected value, or variance and so on. Rowe (1977) defines risk as "The potential for unwanted negative consequences of an event or activity". Meanwhile, Rescher (1983) explains that "Risk is the chance of a negative outcome", and Lowrance (1976) defines risk as "A measure of the probability and severity of adverse effects". Ansell and Wharton (1992) define risk as "any unintended or unexpected outcome of a decision or course of action"; and Cooper and Chapman (1987) define risk as "Exposure to the possibility of economic or financial loss or gain, physical damage or injury, or delay, as a consequence of the uncertainty associated with pursuing a particular course of action". Keeney and Raiffa (1993) apply the expected utility value for assessing risk shelter. Meanwhile, Jia and Dyer (1996) develop a risk assessment model based on utility theory, and define risk as negative expected utility in preference, which implies the concept of risk loss. The conception of risk includes two basic elements, one is " the possibility of event ", and another is "the potential consequence".

The above mentioned literature review shows that lack of back-up data is a disadvantage of qualitative analysis, because qualitative analysis cannot determine the level of risk. Though quantitative analysis can overcome these disadvantages, certain limitations exist. Financial analysis cannot loose assumptions regarding fixed discount ratio; while statistical analysis faces the assumption of risk probability distribution. Though the weighting method can reflect the risk assessment of the decision-maker, it cannot be applied to all situations. Risk analysis based on utility approach can not only reflect the risk preference of the decision-maker, but also performs assess the risk of the event, and hence meets the characteristics of contract risk assessment.

This study attempts to establish a risk assessment model for BOT projects by using the mathematical analysis method. The remainder of the paper is structured as follows: Section 2 describes the definition of the BOT problem. Section 3 then makes some assumptions, defines the risk-state and develops the analysis model, which presents the group-utility-function. Subsequently, Section 4 presents the solution algorithm for determining primary and secondary risk. After this, Section 5 presents a numerical example. Finally, some discussion is presented and conclusions are drawn.

2. The description of problem

This section describes the BOT problem in detail, as follows.

This section assumes a transportation infrastructure project exists which will be carried out via BOT, and that the contract negotiation process will be conducted between the Concession Company and government. Generally, the government negotiation team includes the members representing transportation, environmental agencies, and local officials. Meanwhile, the Concession Company negotiation team includes lawyers, financial consultants, the initiator, engineering experts and so on. The principal negotiator from each team will be in charge of the negotiation process. Nevertheless, if the negotiation fails, the concession contract will not be valid. The negotiation aims to discuss possible uncertainties in the contract, define the individual rights and obligations of each party and, finally, write all agreements in a concessional format. Figure 1 presents a conceptual diagram of this process.

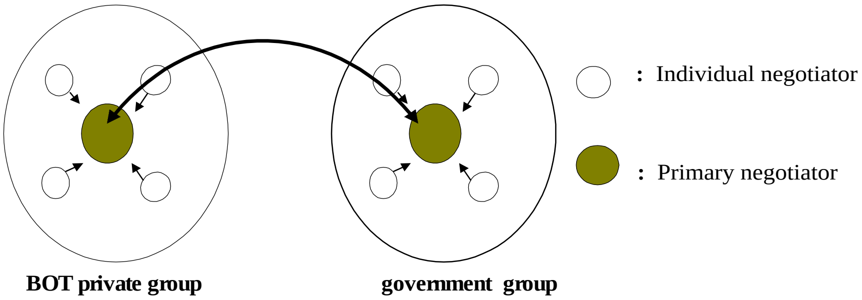
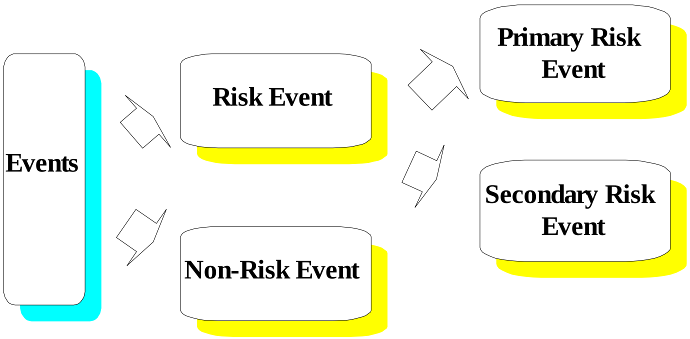


Figure 1. Primary Negotiators and Individuals

Risk analysis is one method widely applied to clarify the uncertainties involved in the BOT contract. Primary and secondary risk events can be identified by assessing the uncertainties, and discussing the primary risk event during negotiation. Restated, both parties will determine the key risk analysis issues requiring discussion during the contract negotiation process. Figure 2 presents a conceptual diagram of risk event, primary event, and secondary risk event.



Fiqure. 2 Coneptual of Event, Risk Event, Primary Risk Event, and Secondary Risk Event

3 The Model

This section presents the assumptions made by the proposed model. Furthermore, the definition of risk state and the individual utility functions, as well as the group aggregation utility function developed in this research are described.

3.1 Assumptions

The assumptions made by the proposed model are as follows:

(1) Agential relationships exist between negotiators and the parties they represent; however, we assume that the agent's costs have noting to do with the negotiators' utility.

(2) The utility function of the negotiator is a continuous function.
(3) The negotiator makes decisions rationally.
(4) The probability distribution of attribute outcome occurrence is assumed to be a Bernoulli experiment, and the probability of occurrence is regarded as the probability of success.

Assumption (1) shows that the negotiator is authorized by a specific organization. However, if the agent cost does not equal zero, adverse-selection behavior might occur. On the contrary, if the agent cost equals zero, the agential relationship cannot influence utility function. Assumption (2) implies that the utility function will not change with the variation of discrete or continuous data. The assumption satisfies the N-M axioms (i.e., Von Neumann-Morgenstern). Meanwhile, Assumption (3) satisfies the principle of maximizing utility while minimizing risk. Finally, Assumption (4) ensures that the negotiator assesses the attribute outcome, state and probability of the event based on previous experience or information.

3.2 Definition of Risk State

This section defines risk state and constructs the risk assessment model used by the negotiator, including single-attribute utility. After this, the assessment model of multi-attribute utility is developed, followed by the group aggregation utility function of negotiation.

(i). Definition of risk state

Assume n states of a specific event exist, $s_1 , s_2 , \Lambda \;, s_n$; state j is denoted as s_j, $j = 1,2,\ldots,n$; the probability of outcome for the state j is p_j; each state corresponds to an outcome of attribute x_j; and each attribute outcome x_j corresponds to a utility value $u_j(x_j)$. $p_j \times u_j(x_j)$ is the utility assessment value of a negotiator regarding each state of the corresponding attribute outcome. Meanwhile, $E(u(x))$ is the expected utility value of all the states. Based on the definition of risk proposed by Ansell & Wharton (1992) and Jia & Dyer (1997), and on the assessment concept of risk preference proposed by Keeney and Raiffa (1993), the "risk state" is defined as : " From the decision-maker’s point of view, an event where the actual value of a specific state is less than the expected utility value of each of all the states". This definition of risk state is shown in Eq. 1.

$$R_j \equiv u_j(x_j) < E(u(x)) \text{ , } \forall\, j \tag{1}$$

Where R_j means state j is a risk state; $u_j(x_j)$ is the utility assessment value of the negotiator regarding state j for a specific event; $E(u(x)) = \sum_j p_j u_j(x_j)$; and $0 \le u_j(x_j) \le 1$; $0 \le p_j \le 1$.

Eq. (1) implies that if a negotiator believes that an event where the utility of its attribute outcome x_j is less than the expected utility value of each of the attribute outcomes, a risk of exists of the event occurring in state s_j. Eq. (1) defines the single attribute outcome of an event. Generally, most events belong to the multi-attributes. For the case of a transportation project, it is assumed here in that ten stations need to be built. Then, some of the stations are categorized as having multi-attributes. Thus, the multi-attribute utility function is developed based on Eq. (1), and is stated as below.

Assume an event has m outcomes for attribute outcomes, $x_1, x_2, \ldots, x_i, \ldots, x_m$, $i = 1,2,\ldots,m$; and n states, $s_1, s_2, \ldots, s_j, \ldots, s_n$, $j = 1,2,\ldots,n$. Denote s_j as the state j, $j = 1,2,\ldots,n$; and $p_{ij} = \Pr ob(x_{ij}, s_j)$ is the probability corresponding to each state and attribute outcome. Let $P_i = \{p_{i1}, p_{i2}, \ldots, p_{ij}, \ldots, p_{in}\}$ be the probability at state s_j corresponding to each attribute outcome and $0 \le p_{ij} \le 1$. Meanwhile, let $u(x_{ij})$ be the utility value of a negotiator regarding attribute outcome x_i at state s_j. Furthermore, $E(u(x_i))$ is the expected utility value of attribute outcome x_i. Finally, $E(u(x))$ is the expected utility value of all attribute outcomes for all states. Table 1 presents the relationship among the outcome of the multi-attribute, state, and probability of an event.

Table 1 Relationship among event state, attribute outcome, and utility

	States of Event		Weight
	S	$s_1, s_2, \ldots, s_j, \ldots, s_n$	
Outcome of Attribute X	x_1	$x_{11}, x_{12}, \cdots, x_{1j}, \cdots, x_{1n}$ $(p_{11}), (p_{12}), \cdots, (p_{1j}), \cdots, (p_{1n})$	
	$\vdots$	$\vdots$ $\vdots$ $\vdots$ $\vdots$	
	x_i	$x_{i1}, x_{i2}, \cdots, x_{ij}, \cdots, x_{in}$ $(p_{i1}), (p_{i2}), \cdots, (p_{ij}), \cdots, (p_{in})$	
	$\vdots$	$\vdots$ $\vdots$ $\vdots$ $\vdots$	
	x_m	$x_{m1}, x_{m2}, \cdots, x_{mj}, \cdots, x_{mn}$ $(p_{m1}), (p_{m2}), \cdots, (p_{mj}), \cdots, (p_{mn})$	
Utility $u(x)$	$u(x_1)$	$u(x_{11}), \ldots, u(x_{1j}), \ldots, u(x_{1n})$	w_1
	$\vdots$	$\vdots$ $\vdots$ $\vdots$	$\vdots$
	$u(x_i)$	$u(x_{i1}), \ldots, u(x_{ij}), \ldots, u(x_{in})$	w_i
	$\vdots$	$\vdots$ $\vdots$ $\vdots$	$\vdots$
	$u(x_m)$	$u(x_{m1}), \ldots, u(x_{mj}), \ldots, u(x_{mn})$	w_m

Note: p_{ij} denote the probability value of attribute outcome x_{ij} at state s_j

Table 1 shows that the relationship among s_j, x_{ij} and p_{ij} is one by one, i.e., $\Pr ob(x_{(i-1)j}, s_j) \cap \Pr ob(x_{ij}, s_j) = 0$. This relationship proves that $x_{(i-1)j}$ and x_{ij} are mutually independent under the state s_j. Consequently, the utility assessment of a negotiator can be defined as Eq. (2).

$$R_{ij} \equiv u_{ij} = p_{ij} \times u(x_{ij}) < E(u(x)') \text{ , } \forall\, i, j \tag{2}$$

Where R_{ij} denotes the risk of attribute outcome i at state j for an event; u_{ij} represents the assessment utility value of a negotiator regarding attribute outcome i at state j of an event; $E(u(x)')$ is the expected utility value for the multi-attribute of an event, $E(u(x)') = \sum_{i=1}^{m} w_i \sum_{j=1}^{n} p_{ij} u(x_{ij})$; and w_i denotes the weight of the outcome of each attribute.

Eq. (2) presents the utility assessment of a negotiator for attribute outcome i at state j of an event. If the utility value is less than the expected utility value of all attributes and states, the negotiator believes this situation represents a risk state for the attribute outcome at the corresponding state. Otherwise, this situation represents a non-risk state. If $i = 1$ in Eq. (2), i.e., $E(u(x)^{'}) = E(u(x))$ **,** then the multi-attribute outcome for the risk state assessment model of Eq. (2) will reduce to a single attribute mode, as illustrated by Eq. (1).

(ii). Normalization of the risk utility

As defined in Eq. (1) and (2), utility value can vary significantly. Although $0 \le u_j(x_j) \le 1$, $0 \le p_j \le 1$, $0 \le u_{ij}(x_{ij}) \le 1$, and $0 \le p_{ij} \le 1$, $E(u(x)^{'})$ may be larger than 1. Consequently, the utility must be normalized. Based on the utility theorem (Keeney and Raiffa, 1993), the utility normalization can be represented as Eq. (3).

$$u_{ij}^{*}(x_{ij}) = \frac{p_{ij} \times u(x_{ij}) - \min_i \{p_{ij} \times u(x_{ij})\}}{\max_i \{p_{ij} \times u(x_{ij})\} - \min_i \{p_{ij} \times u(x_{ij})\}}, \ \max_i \{p_{ij} \times u(x_{ij})\} \ne \min_i \{p_{ij} \times u(x_{ij})\}, \forall\, i, j \qquad (3)$$

Where $u^{*}(x_{ij})$ is the normalized utility value.

Since $0 \le u(x_{ij}) \le 1$, $0 \le p_{ij} \le 1$, so $0 \le u_{ij}^{*}(x_{ij}) \le 1$, if $\max_i \{p_{ij} \times u(x_{ij})\} = \min_i \{p_{ij} \times u(x_{ij})\}$, then $u_{ij}^{*}(x_{ij}) = 0$. The normalized utility value ranges between 0 and 1. The special case for events with the single attribute outcome case is $i = 1$ **.**

3.3 Utility Function of Individual Negotiators

Based on the definition of risk states and utility normalization as proposed above, the utility assessment model of negotiators can now be developed.

From the multi-attribute utility theory (Keeney and Raffia, 1993) and multi-attribute risk utility function (Seo, 1990); the utility assessment model related to multi-attribute outcome and the state expressed as Eq. (4) can be developed.

For negotiator q, $u_q(u_{ij}^{*}(x_{ij})) = u_{ij}^{*}(x_{ij}) < E(u^{*}(x))$ (4)

Where $u_{ij}^{*}(x_{ij})$ is the normalized utility value of Eq. (3); $u_q(u_{ij}^{*}(x_{ij}))$ is the utility value of a negotiator for a given event; and $E(u^{*}(x))$ is the normalized expected utility value.

Because $u_{ij}^{*}(x_{ij})$ is the normalized utility value, the value of $u_q(u_{ij}^{*}(x_{ij}))$ is between 0 and 1. However, if the value of $u_{ij}^{*}(x_{ij})$ is less than $E(u^{*}(x))$, the negotiator believes an event at state s_j and attribute outcome x_{ij} involves some risk. Eq. (4) illustrates the risk assessment for an individual negotiator of a given event.

The above demonstrates that risk assessment and identification of an event is determined by all the negotiators. Additionally, incorporating the utility function of individuals into group aggregation utility function is important. Furthermore, the risk of an event can be assessed based on its utility.

3.4 The Group Aggregation Utility Function

This section presents the utility function of attribute outcome for group negotiators of an event, as well as the group aggregation utility function. The risk assessment of an event is developed based on these.

(i). The Concept of Multi-Attribute Utility (MAU) Function

The multi-attribute utility theory proposed by Keeney and Raiffa (1973, 1993) had been broadly applied in decision-making (Seo and Sakawa, 1985), choice behavior (Tzeng, et al, 1989) and other related research topics (Bosel et al., 1997). The MAU model is based on the utility theory, which can be divided into the additive and multiplicative models. The MAU model makes three assumptions, including (1) the total number of attributes should be no below two, (2) the preference of the decision-maker should be independent, and (3) the utility of the decision-maker is independent of their preference. The multi-attribute utility function can be expressed as Eq. (5).

$$U(x)=\sum_{i=1}^{n}k_iU_i(x_i)+k\sum_{\substack{i=1\\ j>i}}^{n}k_ik_jU_i(x_i)U_j(x_j)+\Lambda+k^{n-1}k_1k_2\Lambda\, k_nU_1(x_1)U_2(x_2)\Lambda\, U_n(x_n) \qquad (5)$$

Where $U_i(x_i)$ is the utility value of the event attribute, and its value is $0\le U_i(x_i)\le 1$, x_i is the event attribute; $U(x)$ is the multi-attribute utility function; k_i is the relative weighting value of attribute i, $0\le k_i\le 1$; and k is the scale constant.

Eq. (5) is the generalized representation of MAU model, when $\sum_i k_i=1$ and the MAU is an additive model. Otherwise, $\sum_i k_i\ne 1$, and the MAU is a multiplicative model.

(ii). The Multi-Attribute Utility Function of Negotiators

As described in Section 2, the process of concession contract negotiation is a kind of group participation; which determines the primary and secondary event through group decision-making. This section, develops the utility function of group negotiators based upon the individual negotiator utility function. To assess the risk or non-risk state, the multi-attribute utility function of negotiators developed here is based on Eq. (5).

We assume there are Q negotiators in the BOT Company's team, $q=1,2,\ldots,Q$ and the utility function of negotiator q is expressed as $u_q(u_{ij}^{*}(x_{ij}))$. Additionally, the utility, i.e., negotiator preference is assumed to be independent. The multi-attribute utility function of negotiators can then be expressed as follows.

$$\begin{aligned}GU_{q,ij}&=U(u_1(u_{ij}^{*}(x_{ij})),u_2(u_{ij}^{*}(x_{ij})),\ldots,u_Q(u_{ij}^{*}(x_{ij})))=\sum_{q=1}^{Q}k_qu_q(u_{ij}^{*}(x_{ij}))+\\ &k\sum_{\substack{q=1\\ a>q}}^{Q}k_qk_au_q(u_{ij}^{*}(x_{ij}))\,u_a(u_{ij}^{*}(x_{ij}))+\Lambda+k^{Q-1}k_1k_2\Lambda\, k_Qu_1(u_{ij}^{*}(x_{ij}))\,u_2(u_{ij}^{*}(x_{ij}))\Lambda\, u_Q(u_{ij}^{*}(x_{ij}))\end{aligned} \qquad (6)$$

Where $GU_{q,ij}$ denotes the utility value of the group negotiators for attribute outcome x_{ij} at state s_j of a given event and , $j=1,2,..,n$, $i=1,2,\ldots,m$; while k_q represents the relative weighting value of a negotiator, where $0\le k_q\le 1$.

Since $0\le u_q(u_{ij}^{*}(x_{ij}))\le 1$ and $0\le k_q\le 1$, $0\le GU_{q,ij}\le 1$. Eq. (6) is the mixed model for group negotiator utility. Let $E(u_q(u_{ij}^{*}(x_{ij})))$ be the expected utility value for all attributes of an event. When $GU_{q,ij}<E(u_q(u_{ij}^{*}(x_{ij})))$, we know that the negotiation group believes a risk exists for an event at state s_j and attribute outcome x_{ij}. Otherwise, no risk exists for an event at state s_j and attribute outcome x_{ij}. When negotiator utility, event attribute and state are independent, Eq. (6) is the additive model of MAU and the utility weighting value k_q of the negotiator can be solved by the weighting method, which was developed by Tzeng et al. (1989) and is shown in Eq. (7).

$$\begin{cases} k_q u_q(\overset{*}{u}_{ij}(x_{ij})^{*}) - k_q u_q(\overset{*}{u}_{ij}(x_{ij})^{**}) = k_{q+1} u_{q+1}(\overset{*}{u}_{ij}(x_{ij})^{*}) - k_{q+1} u_{q+1}(\overset{*}{u}_{ij}(x_{ij})^{**}) \\ \sum\limits_{q}^{Q} k_q = 1 \end{cases} \tag{7}$$

Where $u_q(\overset{*}{u}_{ij}(x_{ij})^{*})$ is the maximum value of the negotiator's (q) utility for a given event at attribute outcome x_{ij} ; $u_q(\overset{*}{u}_{ij}(x_{ij})^{**})$ is the minimum value of the negotiator's (q) utility for a given event at attribute outcome x_{ij} ; $u_{q+1}(\overset{*}{u}_{ij}(x_{ij})^{*})$ is the maximum value of the negotiator's ($q+1$) utility for a given event at attribute outcome x_{ij} ; $u_{q+1}(\overset{*}{u}_{ij}(x_{ij})^{**})$ is the minimum value of the negotiator's ($q+1$) utility for a given event at attribute outcome x_{ij} and k_q is the relative weighting value of the negotiator q.

If weighting value k_q is incorporated into Eq. (6), the utility assessment of the negotiation group of a given event at each state can be obtained. This utility value represents mutual assessment result of the negotiation team for a specific state of an event. Restated, the negotiation group can reach a consensus regarding the assessment of states of an event.

(iii). The Aggregation Utility Function of the Negotiation Group

This section develops the risk assessment model for the negotiation group of a given event; while the aggregation utility value of the negotiation group for the risk and non-risk states of an event is integrated by the concept of minimum distance in utility value between the risk and non risk states.

Section 3.4 (ii) provides the utility value of the negotiation group of an event at a given state, which the utility value can be distinguish the risk state or non-risk state. Applying $GU_{q,ij}$ obtained from Eq. (6), which the value of $GU_{q,ij}$ can be ranked from 0 to 1. Let s_j be the variable of the horizontal axial of a given event, and let $GU_{q,ij}$ be the variable of the vertical axial. Based on the risk state defined herein, the utility value of the non-risk state exceeds that of the risk state. Ranking the state by $GU_{q,ij}$ from 0 to 1 distinguishes the risk state and non-risk state of a given event. By multiplying k_q with the utility value of both states, the aggregation utility function of the negotiation group can be obtained. This utility value results from integrating all negotiators' assessments of a given event at various states.

Following the assumptions made in section 3.1, $GU_{q,ij}$ can be obtained through Eqs. (6) and (7). Based on the value of $GU_{q,ij}$, the state of an event can be distinguished into risk state and non-risk state. Let $GUU_{q,ij}^{u}$ be the maximum utility value of the negotiation group of an event in the non-risk state; let $GUU_{q,ij}^{\lambda}$ be the minimum utility value of the negotiation group for an event in the non-risk state; let $GRU_{q,ij}^{u}$ be the maximum utility value of the negotiation group of an event in the risk state; and let $GRU_{q,ij}^{\lambda}$ be the minimum utility value of the negotiation group of an event in the risk state. Finally, GU is the aggregation utility function of the negotiation group, which defined as Eq. (8). And figure 3 presents a conceptual diagram of this process.

$$GU = \sum_{i=1}^{m} \left(\sum_{q=1}^{Q} k_q \times \left(\sum_{j=1}^{n} \left| \frac{(GUU^{u}_{q,ij} - GUU^{\lambda}_{q,ij}) - (GRU^{u}_{q,ij} - GRU^{\lambda}_{q,ij})}{GUU^{u}_{q,ij} - GRU^{\lambda}_{q,ij}} \right| \right) \right) \tag{8}$$

Where k_q is the utility weighting value of the negotiator q.

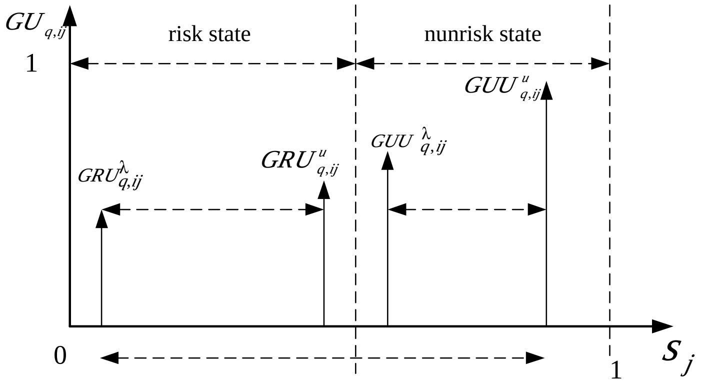


Figure 3. Aggregating utility in risk state and non-risk state

Since $0 \le GUU^{u}_{q,ij} \le 1$, $0 \le GUU^{\lambda}_{q,ij} \le 1$, $0 \le GRU^{u}_{q,ij} \le 1$, and $0 \le GRU^{\lambda}_{q,ij} \le 1$, the GU value is between 0 and 1. The denominator of Eq. (8) is the utility range for the negotiation group of a given event. $(GUU^{u}_{q,ij} - GUU^{\lambda}_{q,ij})$ is the utility difference of the negotiation group of a given event in the non-risk state, while $(GRU^{u}_{q,ij} - GRU^{\lambda}_{q,ij})$ is the utility difference of the negotiation group of a given event in the risk state. However, utility value exists in both the risk and non-risk states of a given event. To integrate the utility value and consider utility weighting value of the negotiator at various states, the distance conception is applied to demonstrate the magnitude of the utility at various states.

The numerator of Eq. (8) is the distance differential between the non-risk and risk states, which shows the magnitude of the difference between two different states and falls between 0 and 1. This process is intended to integrate the assessment results of the negotiator toward utility at various states, and obtain the aggregation utility GU of the negotiation group of a given event. By integrating utility distance and utility weight, a single utility value GU, can be obtained, which represents the consensus of the negotiation group regarding the risk or non-risk state of a given event. When $GU < E(U^*)$, the negotiation group believes a given event is risky. Meanwhile, when $GU \ge E(U^*)$, the event is considered a risky. $E(U^*)$ is the expected utility value of the negotiation group of a given event, $E(U^*) = \sum_{i=1}^{m}((\sum_{q=1}^{Q} \sum_{j=1}^{n} GU_{q,ij}/n)/q)/m$. When an event has a single attribute outcome, which is $i = 1$, then $E(U^*) = \sum_{q=1}^{Q} \sum_{j=1}^{n} GU_{q,ij}/n)/q$.

4. Analysis of Risk Event

This section defines the risk and non-risk events, and classifies primary and secondary risk events.

4.1 Risk and Non-risk Event

Based on the concept addressed in Eq. (8), risk and non-risk events are defined as below.

For the aggregation utility of negotiation group of a given event, if $GU < E(U^*)$, then an event is a risk event defined as Eq. (9). When $GU \geq E(U^*)$, then the given event is a non-risk event defined as Eq. (10).

$$R = \{GU < E(U^*)\} \tag{9}$$

$$UR = \{GU \geq E(U^*)\} \tag{10}$$

Where R denotes a given event is a risk event; UR represents a given event is a non-risk event; GU is the aggregation utility function of negotiation group toward a given event; and $E(U^*)$ denotes the expected utility value of negotiation group toward a given event. As $0 \leq R \leq 1$, $0 \leq UR \leq 1$ and $0 \leq R + UR \leq 1$, a substitutive relationship exists between R and UR. Restated, if a given event is a risk event, it will not be a non-risk event.

According to the concepts presented in Eqs. (9) and (10), if a given event is independent on other events, then the set of risk and non-risk events can be determined. Assuming there are n uncertain events for the BOT Concession Company, $t = 1, 2, \Lambda, T$; U is the union for T events. For the convenience of analysis this study assumes the relationship among the T events is independent. By employing the concept presented in Eq. (8), the aggregation utility value for the T events can be solved. Then, applying the concept presented in Eqs. (9) and (10), the T events can be classified into risk and non-risk events, as shown in Eq. (11).

$$\begin{aligned} U &= R + UR \\ &= \{GU^t < E(U^*(t)),\ \forall\ t \in T\} + \{GU^t \geq E(U^*(t)),\ \forall\ t \in T\} \end{aligned} \tag{11}$$

Where U denotes the union of all the events; R represents the union of risk events; and UR is the union of non-risk events.

4.2 Primary and Secondary risk event

Though the priority of risk events can be obtained by using Eqs. (8) to (10), this approach cannot find the primary and secondary events. Thus, the critical risk level of the risk event union must be further determined, to distinguish the primary and secondary risk events.

(i) Determine the critical risk level

According to the assumptions made in sections 3.1 and 4.2, Q negotiators exist for the Concession Company, $q \in Q$, and N risk events. Let $f^r(k)$ be the probability for event r, and let k be the weighting value for negotiator. For convenience, the relationship among the N events is assumed herein to be independent. As the decision-making behavior of the negotiation group is rational, it can be treated as maximizing the expected utility value. The critical risk level can be solved by first order differentiation of the expected utility value $E(U^N)$, as shown in Eq. (12).

$$Max(E(U^r)) = Max(\sum_{r=1}^{N} f^r(k) GU^r)$$

$$
\begin{aligned}
&\frac{\partial(E(U^r))}{\partial(GU^r)} = \frac{\partial(\sum_{r=1}^{N} f^r(k)GU^r)}{\partial(GU^r)} = \frac{\partial(E(GU^r) - (\overline{GU}^N) + (\overline{GU}^N))}{\partial(GU^r)} \\
&= \frac{\partial(E(GU)^r) - (\overline{GU}^N))}{\partial(GU^r)} + \frac{\partial(E(\overline{GU}^N))}{\partial(GU^r)} \\
&\Theta\ E(\overline{GU}^N)\ is\ const\ , \therefore \frac{\partial(E(\overline{GU}^N))}{\partial(GU^r)} = 0 \\
&\text{when}\quad \frac{\partial(E(U^r))}{\partial(GU^r)} = 0, \frac{\partial(E(GU^r) - (\overline{GU}^N))}{\partial(GU^r)} = 0, \\
&E(GU^r) - (\overline{GU}^N) = 0 \therefore E(GU^r) = (\overline{GU}^N)
\end{aligned}
\tag{12}
$$

Where GU^N is the aggregation utility function for negotiation group toward risk event; $\overline{GU}^N$ is the average utility for risk event.

Eq. (12) demonstrates that the critical risk level is the expected aggregation utility value of the negotiation group toward all the risk events, namely $E(GU^N) = (\overline{GU}^N)$. When $GU^N < E(GU^N)$, which shows that the aggregation value of the negotiation group toward a given risk event is less than the critical risk level, and the given event is a primary risk event. Meanwhile, when $GU^N \geq E(GU^N)$, the given risk event is a secondary risk event.

(ii). Primary and secondary risk event

According to Eq. (12), the expected utility value of the risk event set can be divided by the risk event set into the primary and secondary risk event sets. Based on this result, the primary and secondary risk events are defined as in Eqs. (13) and (14).

$$\exists GU^N \in R, s.t. CRpri \equiv \{Rpri | GU^N < E(GU^N), \forall \quad N \in t\} \tag{13}$$

$$\exists GU^N \in R, s.t. CR_{\sec} \equiv \{R_{\sec} | GU^N \geq E(GU^N), \forall \quad N \in t\} \tag{14}$$

Where $CRpri$ denotes the primary risk event set; $CR_{\sec}$. represents the secondary risk event set; GU^N is the aggregation utility value of negotiation group toward event N; R denotes the risk event set; $E(GU^N)$ represents the critical risk level.

By applying the utility value, the risk events can be divided into the primary and secondary risk event sets. According to the utility preference theory, the lower the utility level, the lower the preference, and thus the higher the risk level. Thus, the main bargaining items for the Concession Company during contract negotiation are the primary risk event, followed by the secondary risk event.

5. Numerical example

This section presents a numerical example describing the risk assessment of a specific project. Events with a single attribute outcome are applied for analytical convenience.

5.1. Description of the Events

Assume rights for contract negotiation regarding a transportation infrastructure project are granted to a BOT Concession Company. This BOT Company will face numerous uncertainties during the concession period, such as: land acquisition, loan credit ratio, discount ratio, concession period, price control, and foreign exchange rates. The example herein assumes the Concession Company's negotiation team contains six negotiators. The research data of these events are supported by Feng and Chung (2000). Assumed utility values and probability for attribute outcome is applied herein. Each event is described in detail below.

a. Land acquisition (L)

The Concession Company must consider that if the government cannot acquired the land for the route and station in time, the Company cannot start construction on schedule, delaying both completion and operation. Assume the delay time is 0,1,2,…, 10 years, corresponding to 10 states (y). Meanwhile, let the increased construction cost (bc) be the outcome of the attribute resulting from a year long delay. Given the utility value $u(L)$ of each negotiator, and the probability value $p(y)$ of event occurrence, $u(L) = u(bc) \times p(y)$, where $u(L)$ denotes the utility value of the negotiator regarding specific attribute and state. Meanwhile, (bc), $u(bc)$, $p(y)$ all correspond to ten states (y), and thus each has ten values.

b. Discount ratio (D)

Let the discount ratio be 7%, 8%, 9%, 10%, 11%, 12%, 13%, 14%, 15%, 16%, 17% and 18%. A total of twelve-states (d) exist, where d is the discount ratio. The cost for paying interest (c) is the outcome of attribute. Meanwhile, the utility value for the negotiator regarding attribute the attribute outcome is $u(c)$, while the probability value is $p(d)$. $u(D) = u(c) \times p(d)$, where $u(D)$ denotes the utility value of the negotiator regarding the outcome of attribute (c) and state. Furthermore, c, $u(c)$ and $p(d)$ are all corresponded to each state d, so they all have twelve values. The outcome of attribute (c) and utility value of the negotiator and the occurrence probability of specific state for each event are given by the assumptions.

c. Loan credit ratio (C)

The Concession Company must pay the loan interest to the financiers within the concession period. If the credit ratio increases, the interest cost will also increase, meaning increased risk. Let the credit ratio be 6.5%, 7%, 7.5%, 8%, 8.5%, 9%, and 10%. A total of seven states (rc) exist where (rc) represents the level of the loan credit ratio. The attribute outcome for this event is interest cost (ic). Meanwhile, $u(ic)$ is the utility value for the negotiator regarding attribute outcome, and the occurrence probability for each state (rc) is $p(rc)$. $u(I) = u(ic) \times p(rc)$, where $u(I)$ denotes the utility value for negotiator regarding attribute and state. Meanwhile, (ic), $u(ic)$ and $p(rc)$ are all corresponded to each state (rc), so each has eight values. The outcome of attribute, negotiator utility value and the probability of a specific state negotiator for each event are given.

d. Price Regulation (P)

Assuming the government regulates the ticket price, and assuming the fare changes every two years, the original unit fare will be set at NTD 180. If no price regulation occurs, the fare will be adjusted according to the fluctuation of the price index (3%), and the future unit fare will be set at $NT 185, $NT 191, $NT 197,… , $NT 298. After implementation of price regulation, the unit fare will be \$NT 180, \$NT 180, \$NT 180, \$NT 191, \$NT 191, … , \$NT 215, \$NT 215, \$NT 215. The state of the price regulation event is the unit fare before and after price regulation, a total of eighteen-states (f) exist, while f is the adjusted unit fare. Meanwhile, the attribute outcome is the revenue loss (R), $u(R)$ is the utility value of the attribute outcome of the negotiator, and the state probability is $p(f)$. $u(P)=u(R)\times p(f)$, where $u(P)$ denotes the utility value of the negotiator regarding attribute and state. R, $u(R)$ and $p(f)$ are corresponded to each of the eighteen-states, so each of them has eighteen values. The outcome of the attribute, negotiator utility value, and the probability of a specific state occurring for each event are given by the assumptions.

e. Concession period (T)

The setting of the concession period is related to the characteristics of the BOT project, and any change in the concession period will significantly impact the Concession Company. Assuming a state of this event is the number of years of the concession period, for concession period from 27 to 35 years, nine states (t) exist. t is the concession year, while attribute outcome is the operational revenue (be). The utility value for negotiators regarding attribute outcome is $u(be)$, while the probability is $p(t)$. $u(T)=u(be)\times p(t)$, where $u(T)$ denotes the utility value for the negotiator regarding the attribute and state. Meanwhile, (be), $u(be)$ and $p(t)$ are all corresponded to each of the nine states, so each has nine values. The utility value of the negotiator and the occurrence probability of specific state for each event are given

f. Foreign exchange rate (E)

The foreign exchange rate is defined in New Taiwan Dollar verses US Dollar, which is $r_e = \$NT/\$US$. Assuming a purchasing plan exists for the concession company during the concession period, and that this plan is priced in US dollars, then this purchasing plan faces a foreign exchange risk. If the magnitude of exchange rate fluctuation is too great, the cost will increase.

Assuming the purchasing plan of the concession company includes buying twenty four vehicles at $r_e = 31.0$ in the second year of the operational period, twenty four vehicles at $r_e = 30.5$ in the seventh year, and twenty four vehicles at $r_e = 31.8$ in the twelfth year. The predicted actual exchange rate is between 29.8 to 32.5, there are 17 states (re). The attribute outcome is the purchasing cost. Meanwhile, the utility of the negotiator regarding attribute is $u(cp)$, while the occurrence probability of the exchange rate state is $p(re)$. Moreover, $u(E)=u(cp)\times p(re)$, where $u(E)$ denotes the utility value $u(cp)$ of the negotiator regarding attribute and state. Meanwhile, $u(cp)$ and $p(re)$ all correspond to the 17 states, and have 17 values. Finally, $u(cp)$ and $p(re)$ are obtained by assumption.

5.2. The utility function for an individual negotiator

(i). Risk assessment of an event

By applying the assumptions made in the previous section, as well as the utility value (obtained from Eq. (2) & (4)) of the negotiator for each state of a given event, the results of the risk analysis are obtained and summarized in Appendix Table 1.

For land acquisition, negotiators #2 and #3 believe that if the delay is under one year, the event is non-risky; otherwise, it is risky. Meanwhile, negotiators #1 and #5 believe if delay equal to zero, the event is non-risky; otherwise, it is risky. Finally, negotiator 6 believes that if the delay is under two years, the event is non-risky; otherwise, it is risky.

Regarding discount ratio, for ratios at 13%, 14% ~15%, 14% ~15%, 14% ~16% and 14% the event is in a non-risk state, and otherwise it is risky. Meanwhile, regarding concession period, negotiators #1, #2, #4, and #6 believe no risk exists when the concession period is setting at exactly 30 years; but otherwise they consider the event risky. Meanwhile, negotiators #3 and #5 believe there no risk exists when the concession period is set at 30 or 31 years, but otherwise a risk exists. Meanwhile, regarding loan credit ratio, negotiators #2, #3 and #5 believe that if loan credit ratio is at 6.5%, the event is in the non-risky state, otherwise, it is in the risky state. However, negotiators #2, #3 and #5 believe that if loan credit ratio is at 6.5%, the event is in the non-risk state, otherwise, it is in the risk state. Furthermore, negotiator #1 believes that if the loan credit ratio at 7.5%, the event is in the non-risk state, otherwise, it is at risk state. Finally, negotiators #4 and #6 believe if the loan credit ratio is at 7% and 7.5%, the event is in the non-risk state, otherwise, it is in the risk state.

In relation to price control, negotiator #1 believes no risk exists for the first-phase fare after implementing price control, but perceives risk for the second-phase fare and thereafter. Meanwhile, negotiators #2 and #6 believe risk exists immediately after implementing price control. Furthermore, regarding exchange rates, negotiator #1 believes no risk exists when the exchange rate is at 30.1, 29.8, 30.4 or 30.7 during purchasing; otherwise, the event is in a risk state. Meanwhile, negotiator #2 believes no risk exists when the exchange rate is at 29.8, 30.4 and 30.7 during purchasing; otherwise, risk exists. Furthermore, negotiator #3 believes no risk exists when the exchange rate is at 29.8, 30.1 and 30.9 during purchasing; otherwise, risk exists. Additionally, negotiator #4 believes no risk exists when the exchange rate is at 30.9, 30.1, 29.8, 30.4 and 30.7; otherwise risk exists. Furthermore negotiator #5 believes no risk exists when the exchange rate is at 30.7; otherwise, risk exists. Finally, negotiator #6 believes no risk exists when the exchange rate is at 30.1, 29.8, 30.4 and 30.7; otherwise risk exists.

Summing up the above results reveals different utilities for negotiators toward various event states, causing the appearance of non-risk and risk states for an event. However, the results match the characteristics of risk-occurring state for the event and risk assessment for the negotiator’s utility.

5.3 The group multi-attribute utility function

As presented in the Appendix Table 1, different assessment results exist for negotiators regarding attribute outcome and event state. Thus, the utility of the negotiation group must be solved by applying Eq. (6), to obtain various views from the negotiation group regarding the state of the event. As the utility of the negotiators is assumed to be independent, while the state of event also is independent, based on the concept in Eq. (6), the utility function of the negotiation group is additive, and can be solved by Eq. (7). For land acquisition, the method of solving Eq. (7) is the simultaneous linear equations illustrated in Eq. (15).

$$\begin{cases} k_1u_1(0)+k_2u_2(10)=k_1u_1(10)+k_2u_2(0) \\ k_1u_1(0)+k_3u_3(9)=k_1u_1(10)+k_3u_3(0) \\ k_1u_1(0)+k_4u_4(10)=k_1u_1(10)+k_4u_4(3) \\ k_1u_1(0)+k_5u_5(10)=k_1u_1(10)+k_5u_5(0) \\ k_1u_1(0)+k_6u_6(10)=k_1u_1(10)+k_6u_6(2) \\ k_1+k_2+k_3+k_4+k_5+k_6=1 \end{cases} \tag{15}$$

Where $u_1(0)$ denotes the minimum utility value of negotiator #1; $u_1(10)$ represents the maximum utility value of negotiator #1; $u_2(0)$ is the minimum utility value of negotiator #2; $u_2(10)$ denotes the maximum utility value of negotiator #2; $u_3(0)$ represents the minimum utility value of negotiator #3; $u_3(9)$ is the maximum utility value of negotiator #3; $u_4(3)$ denotes the minimum utility value of negotiator #4; $u_4(10)$ represents the maximum utility value of negotiator #4; $u_5(0)$ is the minimum utility value of negotiator #5; $u_5(10)$ denotes the maximum utility value of negotiator #5; $u_6(2)$ represents the minimum utility value of negotiator #6; and $u_6(10)$ is the maximum utility value of negotiator #6.

Based on the solution of Eq. (15) and the data of the Appendix Table 2, the utility weight values of the negotiator of an event are $k_1=0.0366$, $k_2=0.0082$, $k_3=0.0125$, $k_4=0.7580$, $k_5=0.1599$ and $k_6=0.0249$, and the group multi-attribute utility function of events can be expressed as in Table 2.

Table 2 the group multiattribute utility function of events

Event	Group multiattribute utility function
Land Acquisition (L)	$GU(y_j)=U(u_1(L),u_2(L),\Lambda\ ,u_6(L))=0.0366\,u_1(L)+0.0082\,u_2(L)$ $+\,0.0125\,u_3(L)+0.7580\,u_4(L)+0.1599\,u_5(L)+0.0249\,u_6(L)$
Discount Ratio (D)	$GU(d_j)=U(u_1(D),u_2(D),\Lambda\ ,u_6(D))=0.2643u_1(D)+0.1386u_2(D)$ $+\,0.1323u_3(D)+0.1584u_4(D)+0.1296u_5(D)+0.1768u_6(D)$
Concession Period (T)	$GU(t_j)=U(u_1(T),u_2(T),\Lambda\ ,u_6(T))=0.1545u_1(T)+0.1548u_2(T)$ $+\,0.1694u_3(T)+0.1931u_4(T)+0.1931u_5(T)+0.1734u_6(T)$
Loan Credit Ratio (I)	$GU(c_j)=U(u_1(C),u_2(C),\Lambda\ ,u_6(C))=0.1478u_1(C)+0.1196u_2(C)$ $+\,0.2494u_3(C)+0.1881u_4(C)+0.1364u_5(C)+0.1573u_6(C)$
Price Regulation (P)	$GU(p_j)=U(u_1(P),u_2(P),\Lambda\ ,u_6(P))=0.1668u_1(P)+0.1651u_2(P)$ $+\,0.1668u_3(P)+0.1651u_4(P)+0.1713u_5(P)+0.1651u_6(P)$
Exchange Rate (E)	$GU(e_j)=U(u_1(E),u_2(E),\Lambda\ ,u_6(E))=0.1848u_1(E)+0.0041u_2(E)$ $+\,0.0038u_3(E)+0.2652u_4(E)+0.2514u_5(E)+0.2906u_6(E)$

Assume all events are independent, then the utility function model of the negotiation group of all events, including: discount ratio, concession period, loan credit ratio, price control and exchange rate, can be obtained via the method shown in Eq. (15) and the data of Appendix Table 2 (continued 1,2). The group multi-attribute utility function toward each event is presented in Table 2, and weight value in the table indicates the relative weight value of a negotiator regarding attribute outcome of the event (refer to Keeney & Raiffa). For land acquisition, the weight value of the negotiator #4 is 0.758, which shows that compared to other negotiators, negotiator #4 has a higher utility regarding the attribute outcome of the event. For exchange rate, the weight value of negotiators #2 and #3 is 0.0041 and 0.0038 respectively, which demonstrates that both individuals have lower utility regarding the attribute outcome of exchange rate. The weight value of the remaining events does not differ significantly among the negotiators, which means no significant recognition difference exists among the six negotiators regarding the attribute outcome of the events.

Employing the group multiattribute utility function of each event, the results of risk state and non-risk state of group negotiators are summarized in Appendix 2.

For land acquisition, all the negotiators believe that if delay is under three years, the event is in the non-risk state; otherwise, it is in the risk state. For the event of discount ratio, from 12% to 16%, the event is in the non-risk state, otherwise, it is in the risk state. For concession period, all the negotiators believe the non-risk state exists when the concession period is set at 30~32 years; otherwise, the event is in the risk state. For loan credit ratio, all the negotiators believe that if loan credit ratio is between 8.5% to 10%, the event is in the risk state, otherwise, it is in the non-risk state. Finally, for price regulation, all the negotiators believe no risk exists for the first-phase fare after implementing price regulation, but risk exists for the second-phase fare and thereafter.

Summarizing the above results reveals that they differ from Appendix Tables 1 and 2. Moreover, the above results cannot identify the risk events or non-risk events. The following section calculates the aggregation utility function of the negotiation team using Eq. (8).

5.4 The risk and non-risk events

This section analyzes the risk and non-risk events, as well as the primary and secondary risk events.

(i). Analysis of risk and non-risk events

By applying the utility function of the negotiation group, the overall assessment results of the negotiation group toward the state of an event can be obtained. However, this approach still cannot determine if the event is a risk event or a non-risk event, and thus Eq. (8) is applied for further analysis. Separating the utility value of the negotiation group toward the state of an event into two categories produces a non-risk event set and a risk event set. Computing Eq. (8) with maximum and minimum utility value of those two categories, respectively, obtains the GU value of the event. The aggregation utility value of each event is 0.5128, 0.6620, 0.4001, 0.4386, 0.2113 and 0.2282, respectively. Based on $E(U^*)$, as defined in Eq. (8), the expected utility value for each event can be calculated, which is 0.0655, 0.2317, 0.2877, 0.2154, 0.2972, 0.2565, respectively. Table 3 shows the results. From Table 3, for the concession period $GU^T = 0.2113$ < $E(U^*(T))$ =0.2672, and for the exchange rate GU^E=0.2282 < $E(U^*(E))$=0.2565; both events are risk events, while the other events are non-risk events. The risk event set is {concession period, exchange rate}, denoted as $R=\{T,E\}$; while the non-risk event union is {land acquisition, discount ratio, credit loan ratio, price control} denoted as $UR=\{L,D,C,P\}$.

(ii). Primary risk event and secondary risk event

Applying the concept addressed in section 4.3, the primary and secondary risk event from the risk events union can be found. In Table 3, concession period and exchange rate both belong to the risk event with aggregation utility value 0.2113 and 0.2282, respectively. Assuming events are independent, then according to the concept presented in Eq. (12), the average of the concession period and exchange ratio can be computed by substituting the above two values into the aggregation expected utility in Eq. (12). This approach obtains the average utility value of the risk event as 0.2198. As the aggregation utility value of the concession period is less than the expected aggregation utility value, the event of concession period (T) is the most risky event in the union of risk events. While the aggregation utility value of the exchange rate exceeds the expected aggregation utility value, the event of exchange rate (E) is the secondary risk event in the union of risk events. Thus, the primary risk event union is {concession period} denoted as $CR_{pri} = \{T\}$, where T is represented as a concession term. Meanwhile, the secondary risk event set is {exchange rate}, denoted as $CR_{\sec} = \{E\}$, with E represented as the exchange rate event. Therefore, during concession contract negotiation, the BOT Company should take concession period as the main negotiation item, followed by the exchange rate.

Table 3 Summary table of risk and non-risk event

Event	Aggregation Utility GU	Utility $E(U^*)$	Risk / Non-risk
Land Acquisition (L)	0.5128	0.0655	Non-risk
Discount Ratio (D)	0.6620	0.2317	Non-risk
Credit Loan Ratio (I)	0.4001	0.2877	Non-risk
Price (fare) Control (P)	0.4386	0.2154	Non-risk
Concession Period (T)	0.2113	0.2972	Risk
Foreign Exchange Rate (E)	0.2282	0.2565	Risk

6. Conclusion and Discussion

This study attempts to identify which uncertainty factors (during group negotiation by the BOT Concession Company with the government) are risk events and which are non-risk events, and then to determine which risk events are primary and which are secondary.

This study is mainly based on the concept of risk assessment, as well as multi-attribute, probability and utility theory. By applying the multi-attribute utility theory of Keeney & Raiffia, this study develops a risk assessment model for the negotiation group, to assess the utility of an uncertain event in relation to the concession contract negotiation of a BOT project. Meanwhile, a case study is presented to demonstrate the feasibility of the risk assessment utility model developed herein. The results of this study show: 1. Assuming the attribute outcome and state of an event are independent, the multi-attribute utility function of an event is a generalized equation of the single attribute utility. 2. By applying the utility function theory of Keeney and Raiffa, utility function of several individual negotiators can be integrated into a single utility function for the negotiation group. Meanwhile, by applying the utility ranking and distance conception, the aggregation utility function can be developed. Furthermore, integrating the utility assessment value of all the negotiators toward various states, the risk level of an event can be assessed. 3. If the negotiation group minimizes risks, they also maximize the expected utility value of the group. The critical risk level of the risk set can then be found, and this critical risk level is the expected utility value of all risk events. After this, the primary and secondary risk event can be obtained from the risk set. 4. Based on the example presented herein, the aggregation utility value is for the six events, exchange rate, loan credit ratio, price control, land acquisition and discount ratio, is 0.5128, 0.6620, 0.4001, 0.4386, 0.2113 and 0.2282, respectively. The concession period and exchange rate are risk events, while others are all non-risk events. Among the union of risk events, the concession period is the primary risk event, while the exchange rate is the secondary risk event.

Additionally, the results herein are achieved under the assumptions that attribute outcome and state of an event are independent, that the six events are independent, and the utility among the negotiators are independent. However, real world events are not entirely independent, and interaction will occur among negotiators. For future studies, the above assumptions can be relaxed, for example: a risk assessment model regarding interactions among negotiators can be developed, and so too can a risk assessment model regarding interaction among events; which will bring the simulation closer to reality. This study considers the perspective of the BOT Concession Company, but in the future the risk assessment model could also be developed from the perspective of government. Furthermore, a real case study based on a BOT project implemented in Taiwan can be presented, to verify the practicability and feasibility of the model developed herein.

Appendix Table 1 Analysis results of a negotiator regarding risk and non-risk state for a given event

Event	Land acquisition		Discount Ratio		Concession Period	
Negotiator	Non-risk state (delay year)	Risk state (delay year)	Non-risk state	Risk state	Non-risk state (Concession term)	Risk state (Concession term)
1st	0	Over one year (including one year)	13%	others	30 years	others
2nd	Less than 1 year	Over one year (including two year)	13%	others	30,31 years	others
3rd	Less than 1 year	Over one year (including two year)	15%	others	30 years	others
4th	Less than 1 year	Over one year (including two year)	14%, 15%	others	30 years	others
5th	0	Over one year (including one year)	14%, 15%, 16%	others	30, 31 years	others
6th	Less than 2 years	Over one year (including three year)	14%	others	30 years	others

Event	Loan Credit Ratio		Price Control		Exchange Rate	
1st	7.5%	others	$NT 180,$NT 191	others	30.1,29.8,30.4,30.9	others
2nd	6.5%	others	$NT 180	others	29.8,30.1,30.9,	others
3rd	6.5%	others	$NT 180	others	29.8,30.4,30.9	others
4th	7.5%	others	$NT 180	others	29.8,30.1,30.4,30.9	others
5th	6.5%	others	$NT 180	others	30.9	others
6th	7.5%	others	$NT 180	others	29.8,30.1,30.4,30.9	others

Appendix Table 2 Analysis results of the risk and non-risk for event

Event	The land acquisition of construction event																		
	State (the year of delay)																		
Utility	0	1	2	3	4	5	6	7	8	9	10	k_q	-	-	-	-	-	-	-
1st	0.2009	0.0241	0.0385	0.0419	0.0318	0.0107	0.0057	0.0004	0.0000	0.0000	0.0000	0.0366	-	-	-	-	-	-	-
2nd	0.8964	0.7704	0.0615	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0082	-	-	-	-	-	-	-
3rd	0.5870	0.5137	0.0615	0.0008	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0125	-	-	-	-	-	-	-
4th	0.0012	0.0048	0.0066	0.0097	0.0034	0.0020	0.0008	0.0001	0.0000	0.0000	0.0000	0.7580	-	-	-	-	-	-	-
5th	0.0460	0.0200	0.0156	0.0319	0.0144	0.0025	0.0000	0.0000	0.0000	0.0000	0.0000	0.1599	-	-	-	-	-	-	-
6th	0.2279	0.2275	0.2958	0.1505	0.0033	0.0099	0.0000	0.0000	0.0000	0.0000	0.0000	0.0249	-	-	-	-	-	-	-
U	0.0360	0.0261	0.0175	0.0177	0.0061	0.0026	0.0008	0.0001	0.0000	0.0000	0.0000	-	-	-	-	-	-	-	-
Outcome	U	U	U	U	R	R	R	R	R	R	R	-	-	-	-	-	-	-	-
Event	Discount ratio event																		
	State (discount ratio)																		
Utility	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	k_q	-	-	-	-	-	-
1st	0.0015	0.0223	0.0376	0.0850	0.2209	0.2866	0.4001	0.3760	0.1769	0.0887	0.0113	0.0091	0.2643	-	-	-	-	-	-
2nd	0.0009	0.0148	0.1093	0.1440	0.2305	0.2907	0.7609	0.3751	0.2043	0.1789	0.0746	0.0110	0.1386	-	-	-	-	-	-
3rd	0.0014	0.0133	0.0599	0.0768	0.1169	0.1472	0.5870	0.6378	0.7973	0.3930	0.2357	0.0598	0.1323	-	-	-	-	-	-
4th	0.0157	0.0369	0.0431	0.0657	0.1301	0.2187	0.3856	0.6647	0.6808	0.5006	0.1070	0.0359	0.1584	-	-	-	-	-	-
5th	0.0000	0.0157	0.0401	0.0797	0.1242	0.3573	0.7569	0.8127	0.7802	0.7880	0.0977	0.0905	0.1296	-	-	-	-	-	-
6th	0.0043	0.0223	0.0440	0.0710	0.1070	0.2552	0.4449	0.6000	0.5584	0.3796	0.1127	0.0183	0.1768	-	-	-	-	-	-
U	0.0039	0.0215	0.0528	0.0859	0.1614	0.2616	0.5267	0.5524	0.4882	0.3487	0.0941	0.0325	-	-	-	-	-	-	-
Outcome	R	R	R	R	R	U	U	U	U	U	R	R	-	-	-	-	-	-	-

Note: "U" denotes the non-risk ; the "R" denotes the risk

Appendix Table 2 Analysis results of the risk and non-risk for event (Continued 1)

Event	The Concession time event																		
	State (concession year)																		
Utility	27	28	29	30	31	32	33	34	35	k_q	-	-	-	-	-	-	-	-	-
1st	0.0000	0.0000	0.0002	1.0000	0.8910	0.6980	0.5095	0.2100	0.1100	0.1545	-	-	-	-	-	-	-	-	-
2nd	0.0000	0.0000	0.0000	0.9980	0.9500	0.8500	0.4500	0.3000	0.1000	0.1548	-	-	-	-	-	-	-	-	-
3rd	0.0000	0.0000	0.0015	0.8910	0.9120	0.4950	0.1000	0.0010	0.0001	0.1694	-	-	-	-	-	-	-	-	-
4th	0.0000	0.0000	0.0200	0.8000	0.4750	0.4950	0.1100	0.0010	0.0001	0.1931	-	-	-	-	-	-	-	-	-
5th	0.0000	0.0000	0.0110	0.9980	0.9000	0.6000	0.3500	0.2500	0.0100	0.1548	-	-	-	-	-	-	-	-	-
6th	0.0000	0.0000	0.0002	0.7984	0.4275	0.2970	0.0385	0.0003	0.0000	0.1734	-	-	-	-	-	-	-	-	-
U	0.0000	0.0000	0.0059	0.9073	0.7444	0.5632	0.2474	0.1180	0.0341	-	-	-	-	-	-	-	-	-	-
Outcome	R	R	R	U	U	U	R	R	R	-	-	-	-	-	-	-	-	-	-
Event	The credit ratio event																		
	State (discount ratio)																		
Utility	6.5%	7.0%	7.5%	8%	8.5%	9%	10%	k_q	-	-	-	-	-	-	-	-	-	-	-
1st	0.1990	0.4701	0.8737	0.4020	0.0613	0.0121	0.0089	0.1478	-	-	-	-	-	-	-	-	-	-	-
2nd	0.9589	0.8693	0.4330	0.2432	0.0992	0.0449	0.0020	0.1196	-	-	-	-	-	-	-	-	-	-	-
3rd	0.4680	0.3306	0.2906	0.0546	0.0660	0.0171	0.0090	0.2494	-	-	-	-	-	-	-	-	-	-	-
4th	0.1157	0.5478	0.6089	0.4990	0.0024	0.0002	0.0090	0.1818	-	-	-	-	-	-	-	-	-	-	-
5th	0.8404	0.6843	0.4235	0.2908	0.0261	0.0018	0.0011	0.1364	-	-	-	-	-	-	-	-	-	-	-
6th	0.2016	0.7225	0.7280	0.3570	0.0990	0.0121	0.0001	0.1573	-	-	-	-	-	-	-	-	-	-	-
U	0.4282	0.5625	0.5364	0.2887	0.0570	0.0136	0.0056	-	-	-	-	-	-	-	-	-	-	-	-
Outcome	U	U	U	U	R	R	R	-	-	-	-	-	-	-	-	-	-	-	-

Note: "U" denotes the non-risk ; the "R" denotes the risk

Acknowledgements

The authors would like to thank the National Science Council of the Republic of China for financially supporting this research under Contract No. NSC89-2211-E-009-024.

Reference

Ansell and Wharton (1992), Risk : Analysis, Assessment and Management, JOHN WILEY & SONS, Chichester, England.

Bose, U. Davey, A. M., and Olson, D. L. (1997), "Multiattrinute Utility Methods in Group Decision-Making: Past Applications and Potential for Inclusion in GDSS", Omega, International Journal of Management Science, Vol. 25, No. 6, pp.691-706.

Buhlmann, H. (1996), Mathematical Methods in Risk Theory, Springer-Verlag.

Cooper, D. F. and Chapman, C. B. (1987), Risk Analysis for Large Projects Model, Methods and Cases, John Wily & Sons.

Cuthbertson, M. (1996), Quantitative Financial Economics Stock, Bonds and Foreign Exchange, John Wiley and Sons, Inc.

Feng Cheng Min and Chung Chi Chun (2000), " Analyzing Risks Caused by Government to Private Investor in BOT transportation Project", Transportation Planning Journal Quarterly, Vol. 29, No. 1, pp.79-108 (In Chinese)。

Hwang, Y. L. (1995), Project and Policy Analysis of Build-Operate-Transfer Infrastructure Development, Ph.D. Dissertation, Department of Civil Engineering, University of California at Berkeley.

Jaselskis, E. J. and Russell, J. S. (1992), "Risk Analysis Approach to Selection of Contractor Evaluation Method", Journal of Construction Engineering and Management, Vol. 118, No. 4, pp. 805-812.

Jia, J. and Dyer, J. S. (1996), "A Standard Measure of Risk and Risk-Value Models", Management Science, Vol. 42, No. 12, pp. 1691-1705.

Keeney, R. L. and Raiffa, H. (1993), Decisions with Multiple Objectives Preferences and Value Tradeoffs, Cambridge University Press.

Levitt, R. E.; Ashley, D. B. and Logcher, R. D. (1980), "Allocating Risk and Incentive in Construction", Journal of the Construction Division, Vol. 106, No. 3, pp. 297-305.

Lowrance, W. W. (1976), Of Acceptable Risk, William Kaufmann, Los Altos, CA.

Mustafa, M. A. and Al-Bahar, J. F. (1991), "Project Risk Assessment Using the Analytic Hierarchy Process", IEEE Transactions on Engineering Management, Vol. 38, No.1, pp. 46-52.

Philip, N. (1995), "Allocation of Risks in BOT Projects", The High Speed Rail BOT Workshop, Taipei, Taiwan.

Rescher, N. (1983), Risk : A Philosophical Introduction to the Theory of Risk Evaluation and Management, University Press of America.

Rowe, W. D.(1977), An Anatomy of Risk, John Wiley and Sons, NY.

Seo, F. and Sakawa, M. (1984), "An Experimental Method for Diversified Evaluation and Risk Assessment with Conflicting Objectives", IEEE Transactions on Systems, Man, and Cybernetics, Vol. 14, No. 2, pp. 213-223.

Seo, F. and Sakawa, M. (1985), "Fuzzy Multiattribute Utility Analysis for Collective Choice", IEEE Transactions on Systems, Man, and Cybernetics, Vol. 15, No. 1, pp. 45-53.

Seo, F. (1990), "On a Construction Fuzzy Multiattribute Risk Function for Group Decision Making", Multiperson Decision Making Models Using Fuzzy Sets and Possibility Theory, by Kacprzyk, A. and Fedrizzi, M., Kluwer Academic Publishers, pp.198-218.

Sidney, M. L. (1996), Build, Operate, Transfer Paving the Way for Tomorrow's Infrastructure, John Wiley and Sons, Inc.

Tiong, L. K. (1990a), "Comparative Study of BOT Projects", Journal of Management in Engineering, Vol. 6, No. 1, pp. 107-122.

Tiong, L. K. (1990b), "BOT Projects: Risks and Securities", Construction Management and Economics, Vol. 8, No. 3, pp. 315-328.

Tiong, L. K. (1995), "Risks and Guarantees in BOT Tender", Journal of Construction Engineering and Management, Vol. 121, No. 2, pp.183-187.

Tiong, L. K. (1996), "CSFs in Competitive Tender and Negotiation Model for BOT Projects", Journal of Construction Engineering and Management, Vol. 122, No. 3, pp. 205-211.

Tzeng, G, H. Shieh, H. M., and Shiau, T. A. (1989), "Route Choice Behavior in Transportation- An Application of The Multiattribute utility theorem", Transportation Planning and Technology, Vol. 13, No. 3, pp. 289-301.

Walker, C. and Smith, A. J. (1996), Privatized Infrastructure: The Build Operate Transfer, Thomas Telford Publications.

William, I. C. J. and Crandall, K. C. (1982), "Construction Risk: Multiattribute Approach", Journal of the Construction Division, Vol. 108, No. 2, pp.187-200.

運輸計畫季刊(將刊登，第 29 卷第 4 期)

BOT 計畫談判群體之風險評量

By

馮正民, Cheng-Min Feng

康照宗, Chao-Chung Kang

Gwo-Hshing Tzeng

Institute of Traffic and Transportation

National Chiao Tung University

114, 4F, Sec.1, Chung Hsiao W. Rd.
Taipei 100, Taiwan
Fax: 886-2-23120082,
E-Mail: cmfeng@cc.nctu.edu.tw

BOT 計畫談判群體之風險評量[1]

THE RISK ASSESSMENT OF THE NEGOTIATION GROUP FOR BOT PROJECTS

馮正民 Cheng-Min Feng[2]

康照宗 Chao-Chung Kang[3]

(88 年 8 月 17 日收搞，89 年 1 月 21 日第一次修改，89 年 2 月 23 日第二次修改，89 年 9 月 14 日定稿)

摘要

為瞭解 BOT 計畫之風險特性及談判者風險偏好課題，本文以風險及效用理論為基礎，結合多屬性決策理論(multiattribute decision making)，以數學解析方式，構建談判者效用及談判群體效用模式，藉以衡量事件屬性之風險狀態，並進一步發展群體總計效用模式(group aggregation utility function)，衡量事件之風險或非風險。本研究之結果顯示，若事件之群體總計效用值(group aggregation utility value)小於效用期望值(weighted expected utility value)時，即屬風險事件；反之，事件屬非風險事件。本文範例分析顯示，特許期限(the concession period)為主要風險事件，匯率(foreign exchange ratio)屬次要風險事件，特許期限應為 BOT 特許公司進行特許契約談判時之首要事項。本文所發展之群體風險衡量概念與模式可作為衡量 BOT 計畫特許契約風險之用，更重要的是，模式可顯示風險事件中之主要風險事件及次要風險事件，同時可瞭解談判者之風險偏好態度。

關鍵詞：BOT**；風險衡量；風險確認；談判群體**

ABSTRACT

The purpose of this paper is to identify and assess the risk of BOT projects under uncertain condition for decision-makers. The individual and group utility functions in the risk assessment model, which is based on the risk, multiattribute decision making and utility theory are developed. Based on the model, the events with the group aggregation utility value smaller than the weighted expected utility value are considered as risk events. While main risk event is selected with the group aggregation utility value smaller than the expected utility of risk events. The risk assessment model provides an approach to quantify, identify and determine the main risk event under BOT negotiation. Results of the numerical example show that "the concession period" is the main risk event, while the "foreign exchange ratio" is the minor risk event. The main risk event should be treated as the primary bargaining proviso for the BOT parties and government officials when performing BOT contract negotiation. Also, the numerical example shows that the model can be used to identify the risk and determine both main and minor risk events.

Key Words: *BOT; Risk assessment; Risk identification; Negotiation group*

---

1. **本文部份研究承行政院國家科學委員會補助，編號** NSC89-2211-E-009-024**，謹此致謝，作者感謝論文審查委員之寶貴意見。**
2. **國立交通大學交通運輸研究所教授(聯絡地址為** 100 **台北市忠孝西路** 1 **段** 114 **號** 4 **樓交通大學交通運輸研究所)。**
3. **國立交通大學交通運輸研究所博士班研究生，現任交通部運輸研究所專員。**

一、前言

BOT (build, operate and transfer)係交通基礎建設民營化方式之一，其意義係指公共建設經由政府之合法途徑，賦予民間於一定期限內從事該建設之規劃、設計、建造、營運與維護之權利，待特許期限期滿後，將所有權移轉至政府[5,15,21]。由於 BOT 計畫具有高風險特色，無論政府或民間部門來執行，皆會面臨此一特性。一般而言，政府採用 BOT 方式之理由，除考量減輕本身財政困窘，藉用民間技術與管理效率，更重要的是在尋找風險分擔對象。對 BOT 特許公司而言，特許契約談判(concession contract negotiation)為風險分擔重要手段之一[18]，其可藉由特許契約談判達到風險分擔(risk sharing)與風險移轉(risk transfer)效果[15,16,17,18,19]；惟欲達成風險移轉效果，其前題有賴風險衡量與風險確認之分析[15,16,18]，掌握談判事項。本文目的即是進行 BOT 計畫特許契約之風險衡量與風險確認分析研究。

依 Tiong[16,17,18,19]、Sidney[15]及 Walker 與 Smith[21]之研究，風險分析為特許公司參與 BOT 計畫競標(tender)或契約談判之重要課題，亦是促進契約談判成功要素之一。Levitt[9]認為風險分擔為工程契約簽訂重要誘因之一，因此，風險分析無論在投資或契約談判過程中，其所扮演角色皆不可言喻。

有關風險分析之衡量方法可分定性分析(qualitative analysis)與量化分析(quantitative analysis)。Philip[11]、Tiong[16,17]、Sidney[15]及 Walker 與 Smith[21]以風險定性分析方法，論述 BOT 計畫之風險種類，如政治風險、商業風險、法規變動風險、興建完工風險及營運風險等。但是定性風險分析無法說明風險水準，風險如何產生，如何確認主要風險(main risk)及次要風險(minor risk)。定量風險分析如統計分析[2,5]、財務分析[4]、經濟效用[7,12,13]及專家權重之方法[10]已廣泛運用於交通運輸、管理與投資領域，Hwang[5]以財產權(property rights)及交易成本(transaction cost)觀念，詮釋 BOT 計畫機制，探討風險水準與投資報酬率之間的關係，然而 Hwang 並無進一步分析 BOT 計畫之主要風險與次要風險課題。William 與 Crandall[22]運用多屬性方法探討工程契約風險問題，認為風險與工程契約本身特性有關，且工程契約之風險衡量應考慮風險屬性問題。

財務之風險方法有淨現值(net present value, NPV)、本益比(benefit/cost, B/C ratio)及投資報酬率法(investment rate of return, IRR)方式，上述方法須對資金作相關性的假設，進而估計未來資金流量，惟 BOT 計畫具有高度不確定性及特許時間很長之特色，欲有效估計未來資金流量，實屬不易(Sidney[15])。 Buhlmann[2]、Jia 與 Dyer[7]，及 Hwang 利用統計觀念進行計畫之風險衡量，假設風險屬某機率分配，求解風險期望值，然在 BOT 計畫特性之下，如何確認風險屬何種分配，實值得思考。

效用理論除可衡量風險與不確定性因素，亦可反應決策者風險偏好行為(Seo 與 Sakawa[12,13]、Keeney 與 Raffia[8])，且其介於 0 與 1 之間，具方便判斷之優點。Seo 與 Sakawa 利用此理論建立模糊風險效用模式及群體模糊多屬性風險函數(fuzzy multiattribute risk function for group decision making)，分析決策者風險偏好行為。Jia 與 Dyer[7]以效用理論構建風險衡量模式，並將風險定為「偏好為負的期望效用(negative expected utility in preference)」觀念，此觀念具有損失(risk loss)之涵義。

AHP 方法係藉由不同專業背景之專家，以問卷方式評估計畫績效及權重。Mustafa 與 Al-Bahar[10]利用 AHP 方法評估工程計畫風險，惟 AHP 法所構建評估體系係以整個計畫為主。

風險與不確定性至今仍有不同定義，不同定義所採用衡量方法亦有所差異。機率[4]觀點認為風險可以機率來衡量，無法以機率衡量者屬不確定性，如賴士葆[24]將風險定義為「未來變動情形可以用機率分配說明」，「無法以機率分配說明者」屬不確定性；李朝賢[25]將風險定義為「未來事象可藉大量的樣本觀察值來建立其出現的機率」，兩者定義相似；Hammond **將不確定性定義為「決策所可能導致多種結果」；**Cooper **與** Chapman[3]**將風險定義為「經濟、財務上之損益或為實體上之延誤、損害發生的可能性，此發生可能性為某特定行為或決策之不確定性結果」。**Cuthbertson[4]**將風險定義為變異數，決策損益變異越大，風險越高。**Buhlmann[2]**將風險定義為預期結果與實際結果之差。**Keeney **與** Raiffa[8]**並無風險定義，惟風險規避係採效用期望值觀念；**Jia **與** Dyer[7]**將風險定義為「偏好為負的期望效用」觀念，認為風險具有可能發生機率、損益及偏好特性。朱敬一[26]將不確定性定義為「狀態變數有多種可能的不同結果，每一種結果對決策者會產生不同的影響」。**

由文獻可知，風險定性分析欠缺數據佐證缺點，無法得知風險水準；定量分析具彌補定性分析的缺點，惟不同分析方式有不同限制條件。財務風險分析無法放寬資金流量假設條件，統計風險分析則面臨風險機率分配假設問題；專家權重法雖可反應決策者之風險衡量，惟仍面臨無法知道決策者之風險偏好，且評估體系有不可一體適用之限制；效用理論之風險分析除可反應決策者風險偏好，亦可進行事件之狀態風險分析，符合契約風險衡量特性。本文目的即以數學解析方法，構建 **BOT 計畫之風險衡量模式，確認主要、次要風險，兼慮談判者風險偏好態度。**

本文章節結構安排如下：第二節為模式發展，說明本文研究問題及假設條件、並定義風險；第三節為模式發展，構建群體效用模式；第四節進行風險分析，求解主要風險及次要風險；第五節為範例說明；最後為結論與建議。

二、問題分析

本節中，本研究說明研究課題，並定義風險，發展個別談判者效用模式，作為下節構建談判群體效用模式之基礎。

2.1 問題說明

假設一重大交通建設計畫以 **BOT 方式推動，特許公司與政府進行契約談判，談判由政府與特許公司之談判群體共同進行。政府談判群體包含交通、環保及地方政府等之相關代表；特許公司談判群體包含法律、財務顧問、參與廠商、發起人及工程專家之相關人士等。值得注意的是，契約談判是由談判群體之主要談判者進行，非個別談判者。若談判不成，特許契約將無法成立，此談判是雙方就契約之不確定性因素，幾經討論，釐清各自應負權利與義務，行之文字，其概念如圖 **1**。**

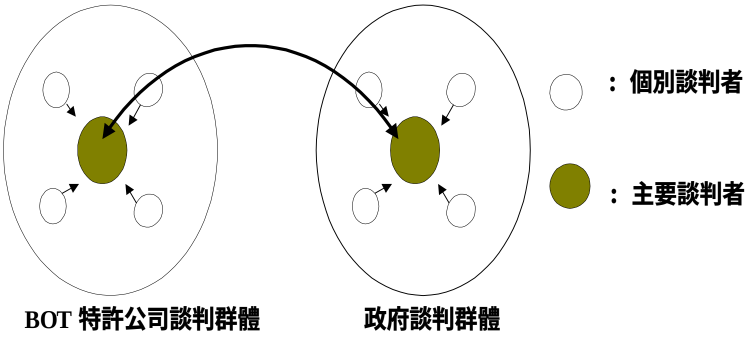


圖 1　談判群體之主要及個別談判者

風險分析為釐清契約不確定性因素手段之一，如能從不確定性因素中進行風險衡量，尋找風險事件，再從風險事件中確認主要風險事件(main risk event)與次要風險事件(minor risk event)，此主要風險事件即為談判時之討論事項。換言之，雙方於契約談判時會選擇關鍵性風險議題進行討論，觀念如圖 2 所示。

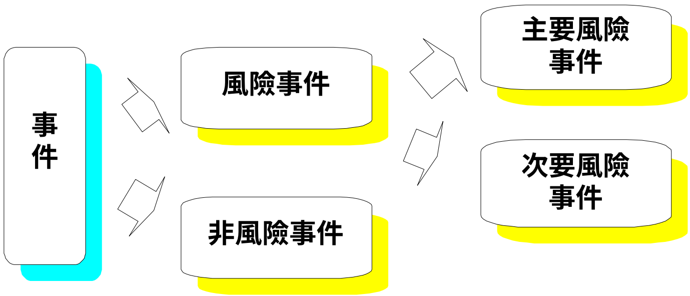


圖 2　事件、風險事件、主要風險事件及次要風險事件之關連性

2.2 假設條件

本文假設條件如下：

1.談判者與所屬機構具有代理關係，但代理成本(agency cost)不對談判者效用產生影響。

2.談判者之效用函數存在且連續。

3.談判者決策屬理性行為(rational)。

4.事件(或屬性產出)之發生機率屬於 Bernoulli 實驗，發生機率為成功之機率。

假設 1 說明，談判者來自於授權關係，因有授權關係故會產生代理關係，若代理關係之代理成本不為零即易生逆選擇行為或道德危機現象，假設代理成本為零，表示代理關係不對效用函數產生影響。假設 2 係談判者存在選擇行為，效用函數不受離散或連續型資料而有所不同，滿足 NM (Von Neumann-Morgenstern)效用公理條件。假設 3 係談判者符合理性之選擇行為，談判者為追求效用最大化及風險最小選擇行為。假設 4 說明談判者根據經驗法則或資訊，對事件之屬性產出、狀態及機率進行評估，機率為事件狀態之可能發生機會，此同於 Bernoulli 分配之成功機率。

2.3 風險定義

本節先定義風險及構建談判者對某事件單屬性之效用風險衡量，而後發展事件多

屬性效用衡量模式，以作為發展談判群體效用模式之基礎。

1. 風險狀態定義

假設某事件有 n 個不確定狀態(states)，s_1, s_2, Λ, s_n，令第 j 種狀態為 s_j，$j = 1,2,...,n$，且該狀態之發生機率為 p_j，每一個狀態皆對應一個屬性產出(outcome of attribute) x_j，而每一個屬性產出 x_j 皆對應一個效用值 $u_j(x_j)$，$p_j \times u_j(x_j)$ 為談判者對每一個狀態及所對應屬性產出之效用衡量值，$E(u(x))$ 為所有狀態之效用期望值。本文根據 Buhlmann[2] 之風險定義，以及 Keeney 與 Raiffa[8] 之風險偏好衡量觀念，將「風險狀態」定義為「對決策者而言，某事件在某狀態下所產生之實際效用小於其所有狀態之期望效用」，如式(1)所示。

$$R_j \equiv u_j = p_j \times u_j(x_j) < E(u(x)), \forall j \qquad (1)$$

其中 R_j 表第 j 種風險狀態；

u_j 為某談判者對某事件第 j 種狀態之效用衡量值；

$E(u(x)) = \sum_j p_j u_j(x_j)$；$0 \le u_j(x_j) \le 1$ 且 $0 \le p_j \le 1$。

式(1)之意義為若某談判者對某事件之屬性產出 x_j 所產生之效用小於所有屬性產出效用期望值，則此某事件於狀態 s_j 下具有風險。事件之狀態、屬性及機率關係結構如表1。

表 1 某事件之狀態、單一屬性及機率結構

項目	狀態(S)
	$s_1, s_2, ..., s_j, ..., s_n$
屬性產出 x	$x_1, x_2, ..., x_j, ..., x_n$
機率 p	$p_1, p_2, ..., p_j, ..., p_n$
效用 u	$u_1, u_2, ..., u_j, ..., u_n$

舉例而言，若某重大交通建設計畫，其「車站用地徵收」為興建時所必須面對之事件，延遲徵收年為事件之狀態，假設為 0,1,…,4 年，屬性產出為延遲徵收土地所造成之成本，假設分別為 100、120、130、140 及 150 萬元；各狀態之發生機率分為 0.9、0.7、0.6、0.2 及 0.01，某談判者對延遲徵收年期之每一狀態所對應屬性產出的效用值分別為 0.9、0.7、0.5、0.1 及 0.02，則每一狀態之效用衡量值為 0.81、0.49、0.3、0.02 及 0.0002，效用期望值為 1.62。由此可知，上述各狀態之效用皆小於效用期望值，故各狀態屬風險狀態。

式(1)係針對某事件所具備單一屬性產出來定義，但一般而言，事件多具有多屬性特性，如某交通建設興建 10 個車站，此數個車站即為事件多屬性特性；因此，本文將式(1)繼續延伸為多屬性之定義。

茲假設某事件有 m 個屬性產出，$x_1, x_2, ..., x_i, ..., x_m$，$i = 1,2,...,m$，且有 n 個不確定狀態，s_1, s_2, Λ, s_n，s_j 為第 j 種狀態，$j = 1,2,...,n$；$p_{ij} = \mathrm{Prob}(x_{ij}, s_j)$ 為每一狀態下及每一屬性產出之所對應發生機率，$P_i = \{p_{1j}, p_{2j}, ..., p_{ij}, ..., p_{mj}\}$ 為 s_j 狀態下之每一屬性產出所對應發

生機率，且$0 \le p_{ij} \le 1$；$u(x_{ij})$為某談判者對屬性產出x_i及狀態s_j下之效用值，$E(u(x_i))$為屬性產出x_i之效用期望值，$E(u(x))$為事件所有屬性產出及狀態之效用期望值。此某事件多屬性產出、狀態、機率之關係結構如表 2 所示。

表 2　某事件多屬性產出、狀態及機率之結構關係

項目	狀態		權重
	S	s_1 , s_2 ,..., s_j ,..., s_n	
屬性產出 x	x_1 M x_i M x_m	x_{11} , x_{12} ,Λ , x_{1j} ,Λ , x_{1n} $(p_{11}), (p_{12}),$Λ $,(p_{1j}),$Λ $,(p_{1n})$ M M M M x_{i1} , x_{i2} ,Λ , x_{ij} , Λ , x_{in} $(p_{i1}), (p_{i2}),$Λ $,(p_{ij}),$Λ , (p_{in}) M M M M x_{m1} , x_{m2} , Λ , x_{mj} , Λ , x_{mn} $(p_{m1}), (p_{m2}),$Λ $,(p_{mj}),$Λ $,(p_{mn})$	
效用 u(x)	$u(x_1)$ M $u(x_i)$ M $u(x_m)$	$u(x_{11})$,..., $u(x_{1j})$,..., $u(x_{1n})$ M M M $u(x_{i1})$,..., $u(x_{ij})$,..., $u(x_{in})$ M M M $u(x_{m1})$,..., $u(x_{mj})$,..., $u(x_{mn})$	w_1 M w_i M w_m

註：p_{ij} 表狀態s_j及屬性產出x_{ij}下之機率值。

由表 2 可知，因s_j、x_{ij}及p_{ij}具一對一關係，故$\mathrm{Prob}(x_{i-1j}, s_j)\ \mathrm{I}\ \mathrm{Prob}(x_{ij}, s_j) = 0$，如此在$s_j$狀態下，屬性$x_{i-1j}$與$x_{ij}$獨立，在獨立關係下，本文可將某談判者對某事件多屬性狀態之效用衡量定義如式(2)。

$$R_{ij} \equiv u_{ij} = p_{ij} \times u(x_{ij}) < E(u(x)') \ \forall\, i, j \tag{2}$$

其中R_{ij}表事件屬性產出i狀態 j之風險狀態；

u_{ij}為某談判者對某事件之屬性產出i狀態 j效用衡量值；

$E(u(x)')$為事件多屬性效用期望值，$E(u(x)') = \sum_{i=1}^{m} w_i \sum_{j=1}^{n} p_{ij} u(x_{ij})$；

w_i為各屬性產出之權重值。

式(2)意義表示某談判者對某事件於屬性產出i狀態 j的效用衡量，若該效用值若小於所有屬性產出及狀態之效用期望值, 則表示某談判者對此屬性產出及所對應狀態下具備風險狀態，反之，則屬非風險狀態。若式(2)中之$i = 1$時，$E(u(x)') = E(u(x))$，則式(2)之多屬性產出會退化成單屬性產出式(1)之風險狀態衡量模式。

(ii). 風險效用之標準化

由式(1)及式(2)定義，效用有大有小，且差距很大，雖然$0 \le u_j(x_j) \le 1$，$0 \le p_j \le 1$，$0 \le u_{ij}(x_{ij}) \le 1$及$0 \le p_{ij} \le 1$，但效用期望值會有大於 1 的現象，為了便於在同一基準下作比較，故效用標準化有其必要。本文利用 Keeney 與 Raiffa 效用觀念，進行效用標準化如下。

考量某事件具備多屬性產出特性，延續式(2)之相關變數定義，式(2)之效用標準化如式(3)所示。

$$u_{ij}^{*}(x_{ij}) = \frac{p_{ij}\times u(x_{ij}) - \min_{i}\{p_{ij}\times u(x_{ij})\}}{\max_{i}\{p_{ij}\times u(x_{ij})\} - \min_{i}\{p_{ij}\times u(x_{ij})\}},\ \max_{i}\{p_{ij}\times u(x_{ij})\} \neq \min_{i}\{p_{ij}\times u(x_{ij})\}, \forall\, i, j \tag{3}$$

其中，$u_{ij}^{*}(x_{ij})$ 為標準化後之效用值。

因 $0 \le u(x_{ij}) \le 1$，$0 \le p_{ij} \le 1$ 且 $0 \le u_{ij}^{*}(x_{ij}) \le 1$，故標準化後之效用值滿足在 0 與 1 之間的條件，若 $\max_{i}\{p_{ij}\times u(x_{ij})\} = \min_{i}\{p_{ij}\times u(x_{ij})\}$ 時，$u_{ij}^{*}(x_{ij}) = 0$。當 $i = 1$ 時，式(3)會退化成單一產出屬性之標準化效用值，故事件之單屬性產出為多屬性產出之特例。

2. 個別談判者效用函數

在完成前述之事件風險狀態定義與效用標準化之構建後，本小節即是利用此定義及效用衡量模式來構建某談判者之效用風險衡量模式。

根據 Keeney 與 Raffia 多屬性效用定理及 Seo (1990)多屬性風險效用函數觀念，某個別談判者 q 對某事件之多屬性產出與狀態效用衡量之模式如式(4)所示：

$$u_{q}(u_{ij}^{*}(x_{ij})) = u_{ij}^{*}(x_{ij}) < E(u^{*}(x)) \tag{4}$$

其中 $u_{ij}^{*}(x_{ij})$ 為式(3)標準化後之效用值；

$u_{q}(u_{ij}^{*}(x_{ij}))$ 為某談判者 q 對某事件之風險效用衡量值；

$E(u^{*}(x))$ 為標準化後之效用期望值。

因 $u_{ij}^{*}(x_{ij})$ 為標準化之效用值，故 $0 \le u_{q}(u_{ij}^{*}(x_{ij})) \le 1$。若式(4)存在小於事件之所有屬性產出及狀態下之 $E(u^{*}(x))$，表示某談判者對某事件於狀態 s_{j} 與屬性產出 x_{ij} 下認為有風險，換言之，該事件之屬性產出 x_{ij} 於狀態 s_{j} 下屬風險狀態。

式(4)乃是某談判者對某事件所進行之風險衡量，但依本研究課題說明顯示，事件之風險衡量與確認係由所有談判者共同決定之，因此須進一步將個別談判者效用予以整合為談判群體效用函數，作為衡量事件風險之依據。

三、 談判群體總計效用模式

本節中，本文將發展談判群體對事件屬性產出效用模式及發展談判群體總計效用函數，以作為事件風險衡量之依據。

3.2 多屬性效用函數(Multiattribute Utility, MAU)概念

Keeney 與 Raiffa (1973，1993)之多屬性效用理論，已廣泛應用於決策行為(Seo 與 Sakawa[12,13,14])、選擇偏好(Tzeng 等人[20])及相關研究課題(Bose1 等人[1])。基本上，該理論係反應群體決策過程，其所建立之 MAU 模式可分加法型及乘法型，模式假設條件有：(1)選擇方案之屬性個數不得小於 2，(2)決策者偏好為獨立，及(3)決策者效用獨立，此多屬性效用函數如式(5)所示。

$$U(x) = \sum_{i=1}^{n} k_{i}U_{i}(x_{i}) + k\sum_{\substack{i=1\\ j>i}}^{n} k_{i}k_{j}U_{i}(x_{i})U_{j}(x_{j}) + \Lambda + k^{n-1}k_{1}k_{2}\Lambda\, k_{n}U_{1}(x_{1})U_{2}(x_{2})\Lambda\, U_{n}(x_{n}) \tag{5}$$

其中 $U_{i}(x_{i})$ 為屬性效用函數，$0 \le U_{i}(x_{i}) \le 1$，x_{i} 為事件屬性；

$U(x)$ 為多屬性效用函數；

k_{i} 為屬性 i 與其他屬性相對權重值，$0 \le k_{i} \le 1$；

k 為尺度常數(scale constant)。

式(5)為 MAU 模式之混合型，當 $\sum_{i} k_i = 1$ 時，MAU 為效用加法型，當 $\sum_{i} k_i \neq 1$ 時，MAU 為效用乘法型，由此可知，加法型為 MAU 之特例。

3.2 談判群體效用函數

如本文研究課題所述，特許契約談判具備群體參與之特性，故主要風險事件或次要風險事件係透過群體來決定。本節中將透過個別談判者效用模式發展談判群體效用模式，藉以衡量事件之風險或非風險狀態；利用式(5)概念所構建之談判群體效用函數如下。

假設 BOT 公司之談判群體有 Q 個談判者，$q = 1,2,...,Q$，第 q 個談判者存在事件效用函數為 $u_q(u_{ij}^{*}(x_{ij}))$；又假設 Q 個談判者效用彼此獨立，偏好結構獨立及事件屬性交換獨立，則談判群體效用函數可構建如式(6)。

$$
\begin{aligned}
GU_{q,ij} &= U(u_1(u_{ij}^{*}(x_{ij})), u_2(u_{ij}^{*}(x_{ij})), ..., u_Q(u_{ij}^{*}(x_{ij}))) = \sum_{q=1}^{Q} k_q u_q(u_{ij}^{*}(x_{ij})) + \\
& k \sum_{\substack{q=1 \\ a>q}}^{Q} k_q k_a u_q(u_{ij}^{*}(x_{ij}))\, u_a(u_{ij}^{*}(x_{ij})) + \Lambda + k^{Q-1} k_1 k_2 \Lambda\, k_Q u_1(u_{ij}^{*}(x_{ij}))\, u_2(u_{ij}^{*}(x_{ij})) \Lambda\, u_Q(u_{ij}^{*}(x_{ij}))
\end{aligned}
\tag{6}
$$

其中 $GU_{q,ij}$ 表談判群體對某事件之 x_{ij} 屬性產出於 s_j 狀態之效用值，$j = 1,2,...,n$

$i = 1,2,...,m$；

k_q 為談判者 q 相對權重值，$0 \le k_q \le 1$。

因 $0 \le u_q(u_{ij}^{*}(x_{ij})) \le 1$ 且 $0 \le k_q \le 1$，故 $0 \le GU_{q,ij} \le 1$；式(6)為談判群體效用之混合型模式，其目的在於整合所有談判者對事件屬性產出之不同的效用衡量，若 $GU_{q,ij} < E(u_q(u_{ij}^{*}(x_{ij})))$ 時，$E(u_q(u_{ij}^{*}(x_{ij})))$ 為事件所有屬性產出 x_{ij} 效用期望值，此表示談判群體認為某事件於屬性產出 x_{ij} 及狀態 s_j 下具有風險，反之，該事件於屬性產出 x_{ij} 及狀態 s_j 下屬非風險狀態。當談判者效用獨立，且事件之屬性狀態獨立時，則式(6)成為 MAU 之加法型效用型態，如此可利用 Tzeng 等人(1989)之權重求解方法，求解談判者效用權重值 k_q，其求解方法如式(7)所示。

$$
\begin{cases}
k_q u_q(u_{ij}^{*}(x_{ij})^{*}) - k_q u_q(u_{ij}^{*}(x_{ij})^{**}) = k_{q+1} u_{q+1}(u_{ij}^{*}(x_{ij})^{*}) - k_{q+1} u_{q+1}(u_{ij}^{*}(x_{ij})^{**}) \\
\sum_{q}^{Q} k_q = 1
\end{cases}
\tag{7}
$$

其中 $u_q(u_{ij}^{*}(x_{ij})^{*})$ 表談判者 q 對某事件於屬性產出 x_{ij} 下之效用上限值；

$u_q(u_{ij}^{*}(x_{ij})^{**})$ 表談判者 q 對某事件於屬性產出 x_{ij} 下之效用下限值；

$u_{q+1}(u_{ij}^{*}(x_{ij})^{*})$ 表談判者 $q+1$ 對某事件於屬性產出 x_{ij} 下之效用上限值；

$u_{q+1}(u_{ij}^{*}(x_{ij})^{**})$ 表談判者 $q+1$ 對某事件於屬性產出 x_{ij} 下之效用下限值；

k_q 為談判者 q 之相對權重值。

將所得到之權重 k_q 值代回式(6)即可得談判群體對某事件之每一狀態效用衡量，此效用值表示談判群體對事件狀態之共同衡量結果，換言之，談判群體對事件狀態取得一致性的衡量結果。

3.3 談判群體總計效用函數(The Aggregation Utility Function of Negotiation Group)

本節重點是在發展談判群體對事件之風險衡量模式，以距離觀念整合事件之風險與非風險狀態之談判群體效用值，其模式發展之觀念如下。

經由 3.2 **節，本文可獲得談判群體對某事件之某狀態下的效用值，此效用可得知事件之風險狀態與非風險狀態。茲利用式**(6)**所得之** $GU_{q,ij}$ **值，該值屬計數**(cardinal)**，且** $0 \le GU_{q,ij} \le 1$ **，故可於第一象限中排序；茲令橫軸變數為事件之狀態** s_j**，縱軸變數為** $GU_{q,ij}$ **，按本文之風險狀態定義可知非風險狀態之效用值會大於風險狀態效用值，故利用** $GU_{q,ij}$ **值在第一象限中由小至大排列，此效用排序後可將某事件之狀態依** $GU_{q,ij}$ **值分為非風險狀態與風險狀態，將此兩種狀態之效用值乘上談判者效用權重，即可得談判群體總計效用函數，該效用值係在整合所有談判者對事件不同狀態之效用衡量，如此可得到談判群體對某事件之風險或非風險的衡量結果，其演算過程如下。**

延續 3.2 **節假設條件，利用式**(6)**與**(7)**求得** $GU_{q,ij}$ **值，依此** $GU_{q,ij}$ **值將事件之狀態分為風險與非風險狀態兩種，令** $GUU^{u}_{q,ij}$ **表談判群體對某事件之非風險狀態效用最大值，** $GUU^{\lambda}_{q,ij}$ **表談判群體對某事件之非風險狀態效用值最小值，並令** $GRU^{u}_{q,ij}$ **為談判群體對某事件之風險狀態效用最大值，** $GRU^{\lambda}_{q,ij}$ **為談判群體對某事件之風險狀態效用最小值，**GU **為談判群體總計效用函數**(the aggregation utility function)**，定義如式**(8)**。**

$$GU = \sum_{i=1}^{m}\left(\sum_{q=1}^{Q} k_q \times \left(\sum_{j=1}^{n}\left|\frac{(GUU^{u}_{q,ij} - GUU^{\lambda}_{q,ij}) - (GRU^{u}_{q,ij} - GRU^{\lambda}_{q,ij})}{GUU^{u}_{q,ij} - GRU^{\lambda}_{q,ij}}\right|\right)\right) \tag{8}$$

其中 k_q **為談判者** q **效用權重值；**

$GUU^{u}_{q,ij} = \max\{GU_{q,ij} \ge E(u_q(u^{*}_{ij}(x_{ij})))\}$ **；**

$GUU^{\lambda}_{q,ij} = \min\{GU_{q,ij} \ge E(u_q(u^{*}_{ij}(x_{ij})))\}$ **；**

$GRU^{u}_{q,ij} = \max\{GU_{q,ij} < E(u_q(u^{*}_{ij}(x_{ij})))\}$ **；**

$GRU^{\lambda}_{q,ij} = \min\{GU_{q,ij} < E(u_q(u^{*}_{ij}(x_{ij})))\}$ **，** $q = 1,2,\ldots,Q$ **。**

因 $0 \le GUU^{u}_{q,ij} \le 1$ **，** $0 \le GUU^{\lambda}_{q,ij} \le 1$ **，** $0 \le GRU^{u}_{q,ij} \le 1$ **及** $0 \le GRU^{\lambda}_{q,ij} \le 1$ **，故** GU **滿足效用介於** 0 **與** 1 **條件。式**(8)**之分母為談判群體對某事件效用之全矩**(range)**，分子** $(GUU^{u}_{q,ij} - GUU^{\lambda}_{q,ij})$ **為群體談判對某事件之非風險狀態效用差，** $(GRU^{u}_{q,ij} - GRU^{\lambda}_{q,ij})$ **為談判群體對某事件之風險狀態效用差，因事件同時具有非風險與風險狀態之效用值，為整合此不同狀態效用值及兼慮談判者對不同狀態之效用權重值，故利用距離觀念來表示不同狀態之效用強度。其中，式**(8)**分子的值為非風險與風險狀態兩者之距離差，其表示兩個不同狀態效用之強度差，且該值必落於全矩之內，取其絕對值是為確保該值介於** 0 1 **之間；再利用此絕對值乘以談判者效用權重值，予以加總，其目的係整合談判者對不同狀態效用之衡量結果，如此即得談判群體對某事件之總計效用值** GU **。此** GU **值表示，談判群體內之談判者對某事件狀態雖有不同效用衡量，衡量強度亦有所差異，但經效用距離與效用權重整合後，可將所有談判者之不同狀態效用值**

予以整合成單一效用值，此值表示談判群體對某事件之風險或非風險的共識看法。若 $GU < E(U^*)$ 時，此表示群體談判認為某事件屬風險事件，反之，當 $GU \geq E(U^*)$ 時，表示某事件不屬風險事件。其中，$E(U^*)$ 為談判群體對某事件效用期望值，$E(U^*) = \sum_{i=1}^{m}((\sum_{q=1}^{Q}\sum_{j=1}^{n} GU_{q,ij}/n)/q)/m$，當事件為單一屬性產出時，即 $i=1$ 時，$E(U^*) = \sum_{q=1}^{Q}\sum_{j=1}^{n} GU_{q,1j}/n)/q$。

四、風險事件分析

本節定義風險事件及非風險事件, 並探討風險優先順序及求解主要風險事件與次要風險事件。

4.1 風險事件及非風險事件

根據式(8)觀念，本文茲定義風險事件及非風險事件如下。

若某事件之談判群體總計效用存在 $GU < E(U^*)$，則某事件為風險事件，定義如式(9)，若 $GU \geq E(U^*)$ 時，則某事件屬非風險事件，定義如式(10)。

$$R = \{GU < E(U^*)\} \tag{9}$$

$$UR = \{GU \geq E(U^*)\} \tag{10}$$

其中，R 表示某事件為風險事件，UR 表示某事件為非風險事件；GU 為某事件之談判群體總計效用函數值，$E(U^*)$ 為談判群體談判對某事件效用期望值，因 $0 \leq R \leq 1$，$0 \leq UR \leq 1$，且 $0 \leq R + UR \leq 1$，此表示二者具有替代關係。換言之，某事件若為風險事件，則將不為非風險事件。

依據式(9)及(10)概念，若事件發生獨立，則可以找出風險事件集合與非風險事件集合。如假設 BOT 特許公司面臨 T 個不確定性事件，$t=1,2,\Lambda,T$；U 為 T 個事件集合，由於不確定性事件存在相關或獨立之關係，為簡化分析，本文假設此 T 個不確定性事件相互獨立。利用式(8)觀念，依序求解此 T 個事件之群體總計效用值，嗣後再利用式(9)及(10)，將 T 個事件分解為風險事件及非風險事件，如式(11)。

$$\begin{aligned} U &= R + UR \\ &= \{GU^t < E(U^*(t)),\ \forall\ t \in T\} + \{GU^t \geq E(U^*(t)),\ \forall\ t \in T\} \end{aligned} \tag{11}$$

其中，U 表所有事件集合，R 表風險事件集合，UR 表非風險事件集合。

4.2 風險排序

根據式(8)可得每個事件之談判群體總計效用值，利用效用偏好定理，可進行事件排序(ranking)，了解非風險事件與風險事件之順序性。

茲假設特許公司之風險集合有 A,B,C,D 及 E 事件，即 $A,B,C,D,E \in R$，各風險事件依序對應談判群體總計效用值 GU^A, GU^B, GU^C, GU^D 及 GU^E。若 A,B,C,D 及 E 事件之談判群體總計效用值依序為 $GU^A \pi GU^B \pi GU^C \pi GU^D \pi GU^E$，根據效用偏好定理，此事件偏好順序為 $A \pi B \pi C \pi D \pi E$，此顯示，談判群體對事件 A 偏好效用最低，對事件 E 偏好效用最高，故談判群體對風險事件集合之風險順序由大至小依序為 $A \pi B \pi C \pi D \pi E$，故得知事件 A

為風險最高者，事件 E 為風險最低者。

4.3 主要風險事件及次要風險事件

利用效用偏好定理雖可得風險事件之排序，惟尚不足以說明風險事件中之主要風險事件與次要風險事件，因此須進一步求風險事件集合中之關鍵風險水準(critical risk level)，以釐清主要與次要風險事件。

(i) 關鍵風險水準之求解

接續 3.2 節及 4.2 節假設條件，假設特許公司談判群體有 Q 個談判者，$q \in Q$，有 N 個風險事件，$r = 1,2,\Lambda,N$；令 $f^r(k)$ 為風險事件 r 之發生機率值，k 為所有談判者權重值。為簡化分析起見，假設此 N 個風險事件相互獨立。因談判群體具理性決策行為，故可視為追求談判群體效用期望最大化，因此，關鍵風險水準可對效用期望值 $E(U^r)$ 一階微分求得，求解如式(12)。

$$\mathrm{Max}(E(U^r)) = \mathrm{Max}\Big(\sum_{r=1}^{N} f^r(k)GU^r\Big)$$

$$\frac{\partial(E(U^r))}{\partial(GU^r)} = \frac{\partial\Big(\sum_{r=1}^{N} f^r(k)GU^r\Big)}{\partial(GU^r)} = \frac{\partial(E(GU^r) - (\overline{GU}^N) + (\overline{GU}^N))}{\partial(GU^r)}$$

$$= \frac{\partial(E(GU)^r) - (\overline{GU}^N))}{\partial(GU^r)} + \frac{\partial(E(\overline{GU}^N))}{\partial(GU^r)} \qquad (12)$$

因 $E(\overline{GU}^N)$ 為常數 故 $\frac{\partial(E(\overline{GU}^N))}{\partial(GU^r)} = 0$

當 $\frac{\partial(E(U^r))}{\partial(GU^r)} = 0$ 時，$\frac{\partial(E(GU^r) - (\overline{GU}^N))}{\partial(GU^r)} = 0,$

$$E(GU^r) - (\overline{GU}^N) = 0 \therefore E(GU^r) = (\overline{GU}^N)$$

其中，GU^r 為第 r 個風險事件之談判群體總計效用函數，$\overline{GU}^N$ 表此 N 個風險事件之效用平均數。式(12)說明，風險事件之關鍵風險水準為此 N 個風險事件之談判群體總計效用的期望值，即 $E(GU^r) = (\overline{GU}^N)$。換言之，若有 5 個風險事件，則其關鍵風險效用水準為此 5 個事件之談判群體總計效用之平均值。因此，當 $GU^r < E(GU^r)$ 時，表第 r 個風險事件之談判群體總計效用值小於關鍵風險水準，此時之第 r 個風險事件屬主要風險事件，若 $GU^r \geq E(GU^r)$ 時，表第 r 個風險事件為次要風險事件。

(ii). 主要風險事件與次要風險事件

依式(12)之觀念，風險事件集合之效用期望值可將風險事件集合一分為二，成為主要風險事件及次要風險事件集合。本文利用此結果將主要風險事件及次要風險事件定義如式(13)及(14)。

$$\exists G\, U^r \in R,\ \text{s.t. } C\, R_{main} \equiv \{R_{main} \,|\, GU^r < E(GU^r), \quad r = 1,2,\Lambda\, N\} \qquad (13)$$

$$\exists G\, U^r \in R,\ \text{s.t. } C\, R_{min} \equiv \{R_{min} \,|\, GU^r \geq E(GU^r), \quad r = 1,2,\Lambda\, N\} \qquad (14)$$

其中，CR_{main}表主要風險事件集合，CR_{min}表次要風險事件集合，GU^r為事件 r之談判群體總計效用值，$E(GU^r)$為關鍵風險水準，此效用值將風險事件分為主要風險事件與次要風險事件兩個事件集合；而依據效用偏好定理觀念，當效用程度愈低，偏好度愈低，相對而言，風險程度愈高，由此可知，主要風險事件應為政府及特許公司在契約談判時主要談判事項，其次為次要風險事件。

五、 範例分析

本節以範例方式說明事件之風險衡量。另為利於分析，事件採單屬性產出情形來說明。

5.1 事件說明

假設某特許公司獲得某一交通建設之特許契約談判權，該公司於特許期限內面臨許多不確定性事件，如用地取得、銀行融資信款利率、銀行折扣率、特許期限、費率管制及外匯等事件，特許公司之談判群體有六個談判者，對此六個事件進行風險衡量，事件之相關資料則採自馮正民、鍾啟椿[23]研究資料，另事件之談判者效用值、屬性產出機率值則採假設性資料，相關事件說明如下︰

(1)用地取得(L)

就特許公司而言，政府若無法於期限內取得路線、場站用地，則無法如期開工，此將導致 BOT 計畫之完工延滯與延遲通車。假設本事件用地取得之延遲年數分別有 0，1，2，...，10 年 之 10 個狀態(y)，屬性產出為延遲年數所增加之建造成本((bc), build cost)，談判者之屬性效用值為 $u(bc)$，事件狀態發生機率值為 $p(y)$，$u(L) = u(bc) \times p(y)$，$u(L)$為談判者對屬性與狀態之效用值，以上(bc)，$u(bc)$，$p(y)$皆對應 10 個狀態y，故皆有 10 個數值。

(2)折扣率 (D)

茲令折扣率水準為 7%、 8%、 9%、 10%、 11%、 12%、 13%、 14%、 15%、 16%、 17% 及 18%，共有 12 個狀態(d)，d為折扣率；屬性產出為支付利率成本(c)，談判者之屬性產出效用值為$u(c)$，機率值為$p(d)$，$u(D) = u(c) \times p(d)$，$u(D)$為談判者對屬性與狀態之效用值，c，$u(c)$及$p(d)$皆對應每一狀態d，故皆有 12 個數值；屬性產出、談判者效用值及事件狀態發生機率分別假設給定。

(3)銀行融資貸款利率 (I)

就特許公司而言，特許公司於特許期限內必須償付銀行融資貸款利息，若信用利率調高，特許公司將增加利息成本支出。對特許公司而言，融資貸款利率變動將產生風險。本事件之信用利率分別有 6.5%、7%、7.5%、8%、8.5%、9%及 10%，有 7 個狀態(rc)，rc為銀行融資貸款利率；事件屬性產出為利息成本(ic , interest cost)，$u(ic)$為談判者之屬性產出效用值，狀態發生機率為$p(rc)$，$u(I) = u(ic) \times p(rc)$，$u(I)$為談判者對屬性與狀態之效用值，(ic)，$u(ic)$及$p(rc)$皆對應每一狀態rc，故皆有 8 個數值；事件屬性產出，談判者效用值及事件狀態發生機率分別假設給定。

(4)費率管制(P)

假設政府對業者實施費率管制措施，費率採每兩年調整一次，最初之基本里程費率為$NT180；若費率無管制下，正常費率價格係隨物價 3%調整，故未來基本里程費率為$NT 185、$NT 191、$NT 197、…及$NT 298，實施費率管制後，基本里程費率分別為$NT 180、$NT 180、$NT 180、$NT 191、$NT 191、…、$NT 215、$NT 215 及$NT 215，事件狀態為管制前與管制後之基本里程費率，計 18 個狀態(f)，f為調整後之基本里程價格，屬性產出為收入損失(R, revenue)，談判者之屬性產出效用值為

$u(R)$，狀態機率值為 $p(f)$， $u(P)=u(R)\times p(f)$， $u(P)$ 為談判者對屬性與狀態之效用值， R， $u(R)$ 及 $p(f)$ 皆對應 18 個狀態，故皆有 18 個數值；屬性產出值，談判者效用值及事件狀態發生機率分別假設給定。

(5)特許期限 (T)

特許期限訂定與 BOT 計畫特性有關，特許期限改變將對業者產生衝擊。假設事件狀態為特許年期，特許期為 27 至 35 年有 9 個狀態(t)，t 為特許年期(year)，屬性產出為營業收入($be, benefit$)，談判者之屬性產出效用值為 $u(be)$，機率為 $p(t)$， $u(T)=u(be)\times p(t)$， $u(T)$ 為談判者對屬性與狀態之效用值， (be)， $u(be)$ 及 $p(t)$ 皆對應 9 個狀態，故皆有 9 個數值；談判者效用值及事件狀態發生機率分別假設給定。

(6)匯率(E)

匯率茲定義每一新台幣與每一美元之比值，即 $r_e=\$NT/\$US$。假設特許公司於特許期間有採購計畫，並以美元報價，此採購計畫面臨匯率變動風險。若匯率變動幅度過大，使業者支付成本大幅增加，此採購計畫將有匯率風險威脅。若特許公司之採購計畫，計畫於興建期間以匯率 $r_e=32.3$ 採購 21 輛車廂，於營運期之第 2 年，以匯率 $r_e=31.0$ 水準購買 24 輛車廂，於營運期第 7 年以匯率 $r_e=30.5$ 水準購買 24 輛，於營運期第 12 年以匯率 $r_e=31.8$ 水準購買 24 輛車廂，假設實質匯率由 29.8 32.5 之間，有 17 個狀態(re)，屬性產出為購車成本(cp, cost of purchase)，談判者之屬性產出效用為 $u(cp)$，匯率狀態發生機率為 $p(re)$， $u(E)=u(cp)\times p(re)$， $u(E)$ 為談判者對屬性與狀態之效用值(cp)；$u(cp)$ 與 $p(re)$ 皆對應於 17 個狀態，故皆有 17 個數值；$u(cp)$ 與 $p(re)$ 分別假設給定。

5.2 個別談判者效用函數之求解

(i) 風險及非風險之衡量

本節利用前節事件之假設條件與資料，並配合式(2)至(4)得事件之每一狀態談判者效用值，其結果整理如附表 1。

就用地取得而言，第 2 及第 3 位談判者認為徵收延遲年限不超過 1 年者，屬非風險狀態，徵收延遲超過 1 年以上者，屬風險狀態；第 1 及第 5 位談判者認為無發生徵收問題者(即徵收延遲為 0 年)，屬非風險狀態，其餘屬風險狀態；第 6 位談判者認為徵收延遲年限在 2 年以內者皆屬非風險狀態，其餘屬風險狀態。

就折扣率而言，非風險狀態之折扣率分別為 13%、14% ~15%、14% ~15%、14% ~16%及 14%，其餘則屬風險狀態。就特許期限而言，第 1、2、4 及第 6 位談判者認為特許年限應為"30 年"，該"30 年"之特許年限屬非風險狀態，其餘特許年限屬風險狀態；第 3 與第 5 位談判者認為"30 年"及"31 年"之特許年限屬非風險狀態，其餘屬風險狀態。就銀行融資信用利率而言，第 2、3 及第 5 位談判者認為貸款利率 6.5%屬非風險狀態，其餘屬風險狀態；第 1 位談判者認為貸款利率為 7.5%者屬非風險狀態，其餘屬風險狀態；第 4 及第 6 位談判者認為貸款利率為 7%與 7.5%者屬非風險狀態，其餘屬風險狀態。

就費率管制而言，第 1 位談判者認為費率管制後之第一期費率水準屬非風險狀態，第二期以後的費率水準屬風險狀態；第 2 及第 6 位認為費率管制後之費率水準皆屬風險狀態。就匯率而言，第 1 位談判者認為採購時之匯率水準屬非風險狀態者有 30.1、29.8、30.4 及 30.7，其餘匯率水準屬風險狀態；第 2 位談判者認為非風險狀態之匯率水準者有 29.8、30.4 及 30.7，其餘匯率水準屬風險狀態；第 3 位談判者認為屬

非風險狀態之匯率水準者有 30.9、30.1 及 29.8，其餘匯率水準屬風險狀態；第 4 位談判者認為屬非風險狀態之匯率水準者有 30.9、30.1、29.8、30.4 及 30.7，其餘匯率水準屬風險狀態，第 5 位談判者認為屬非風險狀態之匯率水準者為 30.7，其餘匯率水準屬風險狀態，第 6 位談判者認為屬非風險狀態之匯率水準者有 30.1、29.8、30.4 及 30.7，其餘屬風險狀態。

綜合上述分析，談判者對事件的每一狀態有其不同效用衡量結果，故會產生某事件會有非風險與風險狀態情形出現，此結果符合事件風險發生狀態及談判者效用風險衡量特性。

(ii) 談判者效用函數之建立

利用談判者效用函數進一步了解談判者風險偏好行為，在談判者效用函數之構建方面則利用談判者效用值及事件狀態資料來進行，相關資料之變數名稱與定義如表 3。

表 3 事件之自變數與應變數

變數 事件		應變數		自變數
用地徵收	(L)	延遲徵收年	(y)	u(L)
折扣率	(D)	折扣率	(d)	u(D)
銀行融資信用貸款	(I)	貸款利率	(rc)	u(I)
費率管制	(P)	管制費率	(p)	u(P)
特許期限	(T)	特許年限	(t)	u(T)
匯率	(E)	匯率	(re)	u(E)

利用表 3 之變數定義與資料，進行談判者效用模式之構建，其結果如附表 2 及續表，茲就效用模式檢定結果說明如下。

就用地徵收而言，所有談判者效用函數模式之係數 t 值及 F 值，於顯著水準 ~ = 0.05 之下呈顯著性，R^2 值介於 0.59 -- 0.87 之間，雖然第 1 及第 6 位談判者效用模式 R^2 不高，惟模式各參數檢定符合假說條件，其餘模式解釋能力亦皆在 0.8 以上。就折扣率而言，談判者效用函數模式之係數 t 值及 F 值，亦於 ~ = 0.05 之下皆呈顯著性，R^2 值介於 0.497 -- 0.89 之間，雖然第 2、4 及 5 位談判者效用模式的 R^2 不高，惟模式之係數 t 值及 F 值檢定亦符合假說條件，故仍可採用。就銀行融資信用貸款而言，第 1 與第 6 位談判者效用函數模式之係數 t 值於 ~ = 0.05 下呈現顯著，其餘談判者效用函數模式之係數 t 值於 ~ = 0.01 呈顯著性，所有模式之 F 值於 ~ = 0.05 呈顯著性，R^2 介於 0.68 -- 0.98 之間。就費率管制而言，談判者效用函數模式之係數 t 值及 F 值於 ~ = 0.001 下呈顯著性，R^2 值介於 0.59 -- 0.94 之間，第 6 位談判者效用函數模式之 R^2 為 0.59，解釋能力較低，惟該模式之係數檢定皆符合檢定條件，故模式仍可採用。就特許期限而言，所有談判者效用函數模式之係數 t 值及 F 值於 ~ = 0.05 下呈顯著性，R^2 值介於 0.62 -- 0.69 之間；就匯率而言，談判者效用函數模式之係數 t 值與 F 值於 ~ = 0.001 下呈顯著性，R^2 值則介於 0.71 -- 0.94。

由模式估計與檢定結果來看，本文所建立效用函數符合模式假設條件，雖然部份

模式之R^2值不高，但R^2值高低主要係反應自變數對應變數之解釋貢獻能力，而本文部份效用模式R^2不高原因主要受限於自變數樣本數太少之故，雖然模式R^2不高，惟函數之檢定仍符合檢定條件，故這些效用函數模式可描述談判者效用偏好行為。

(iii) 談判者風險偏好分析

由附表 2 知，談判者效用函數包含二次式及多項式兩種，依照 Keeney 及 Raffia 之風險規避定理觀念，效用函數若屬二次式，可利用$\theta(u) - \frac{u''}{u'}$進行分析，其中$u$為效用函數；當$(u) < 0$時，談判者偏好態度呈風險規避行為；$(u) > 0$時，談判者屬風險追求行為；$(u) = 0$時，談判者屬風險中立。若效用函數屬多項式型態，則可依 Jia 及 Dyer (1996) 之觀念進行分析，若多項式屬$u(x) = ax - bx^2 + cx^3$型態者，且為$x < b/(3c)$時，此顯示談判者屬風險規避行為，若$x > b/(3c)$時，屬風險追求行為，如$x = b/(3c)$，談判者屬風險中立偏好行為。本文利用 Keeney 與 Raiffa (1993) 及 Jia 與 Dyer (1996)之風險偏好衡量觀念，進行談判者風險偏好行為之分析，結果如表 4。

就用地徵收而言，對於事件之非風險狀態衡量者，此六位談判者屬風險規避行為，對於事件之風險狀態衡量者則呈現風險追求偏好行為。就折扣率而言，談判者效用函數屬多項式，故利用 Jia 及 Dyer 觀念進行分析；結果顯示，第 3 與第 6 位談判者對於事件之風險狀態衡量者屬風險規避行為，其餘談判者對事件之風險狀態衡量者屬風險追求行為。就特許期限而言，第 3 及 4 位談判者效用函數屬多項式型態，其餘屬二次式型態，第 3 及第 4 位談判者之風險規避可按 Jia 及 Dyer 之衡量方式進行分析，結果顯示，該二者對於特許期限之非風險狀態衡量者屬風險規避行為，其餘談判者對於事件之非風險狀態衡量者(可接受特許年限)屬風險規避行為。就銀行融資信用貸款利率而言，第 4 及 6 位談判者對於事件之非風險狀態衡量者屬風險規避行為，其餘談判者對事件之非風險狀態衡量者屬於風險追求行為。就費率管制而言，所有談判者對事件之風險狀態衡量者屬風險規避行為，此說明，政府若採費率管制措施，此政策非談判群體所能接受。就匯率而言，所有談判者對於購車計畫所預訂之匯率水準(即非風險狀態之匯率水準)屬風險追求偏好行為。

綜上討論，就用地徵收而言，談判者對於事件之延遲徵收年限較短者，其效用屬風險規避行為，如談判者可接受較長之延遲徵收年限則屬風險追求行為；就特許期限而言，談判者對於較長時間之特許年限效用衡量屬風險追求行為；就費率管制而言，談判者對於可接受費率管制措施者，其效用屬風險追求行為。

5.3 多屬性效用模式之求解

由附表 1 結果顯示，談判者對事件屬性產出與狀態有不同衡量結果，故須再利用式(6)進一步求談判群體效用，以求得談判群體對事件狀態之共同看法。由於本研究假設談判者間效用獨立，且依表 2 之觀念，事件之狀態屬性具一對一關係，故可依式(6)概念，此談判群體效用模式將成為函數為加法型態，故可利用式(7)進行求解。以用地徵收而言，依式(7)求解方法如式(15)之聯立方程式。

$$\begin{cases} k_1u_1(0)+k_2u_2(10)=k_1u_1(10)+k_2u_2(0)\\ k_1u_1(0)+k_3u_3(9)=k_1u_1(10)+k_3u_3(0)\\ k_1u_1(0)+k_4u_4(10)=k_1u_1(10)+k_4u_4(3)\\ k_1u_1(0)+k_5u_5(10)=k_1u_1(10)+k_5u_5(0)\\ k_1u_1(0)+k_6u_6(10)=k_1u_1(10)+k_6u_6(2)\\ k_1+k_2+k_3+k_4+k_5+k_6=1 \end{cases} \tag{15}$$

其中 $u_1(0)$ 表第 1 位談判者效用之最小值；
$u_1(10)$ 表第 1 位談判者效用之最大值；
$u_2(0)$ 表第 2 位談判者效用之最小值；
$u_2(10)$ 表第 2 位談判者效用之最大值；
$u_3(0)$ 表第 3 位談判者效用之最小值；
$u_3(9)$ 表第 3 位談判者效用之最大值；
$u_4(3)$ 表第 4 位談判者效用之最小值；
$u_4(10)$ 表第 4 位談判者效用之最大值；
$u_5(0)$ 表第 5 位談判者效用之最小值；
$u_5(10)$ 表第 5 位談判者效用之最大值；
$u_6(2)$ 表第 6 位談判者效用之最小值；
$u_6(10)$ 表第 6 位談判者效用之最大值。

根據式(15)之求解得事件談判者效用權重值結果為 $k_1=0.0366$、$k_2=0.0082$、$k_3=0.0125$、$k_4=0.7580$、$k_5=0.1599$，及 $k_6=0.0249$，故事件談判群體效用函數模式結果可如表 5 所示。

假設事件發生獨立，故折扣率、特許期限、銀行融資信用貸款利率、費率管制及匯率之事件皆可依式(15)之求解法，求得事件談判群體效用函數模式，各事件之談判群體效用模式如表 5 所示，表 5 之權重值表談判者對事件之屬性產出的相對權重(參見 Keeney 及 Raiffa)。就用地取得而言，第 4 位談判者對事件之屬性產出權重值為 0.758，此表其對該事件屬性產出相對於其他談判者有較高之效用感受；對匯率而言，第 2 及 3 位談判者之權重為 0.0041 及 0.0038，相較其他談判者為低，二者對匯率之屬性產出效用感受較低；至於其他事件而言，談判者之間的權重值無明顯差距，此 6 位談判者對事件之屬性產出效用感受無明顯認知差距。

表 4　談判者對事件之風險偏好態度分析

事件	用地 徵收		折扣率		特許 期限	
談判者	非風險狀態 (延遲徵收年)	風險狀態 (延遲徵收年)	非風險狀態 (利率水準)	風險狀態 (利率水準)	非風險狀態 (特許年)	風險狀態 (特許年)
第 1 位 偏好態度	0	1 年以上(含)	13%	其他	30	其他
	風險規避	風險追求	風險規避	風險追求	風險規避	風險追求
第 2 位 偏好態度	不超過 1 年	2 年以上(含)	13%	其他	30,31	其他
	風險規避	風險追求	風險規避	風險追求	風險規避	風險追求
第 3 位 偏好態度	不超過 1 年	2 年以上(含)	15%	其他	30	其他
	風險規避	風險追求	風險追求	風險規避	風險規避	風險追求
第 4 位 偏好態度	不超過 1 年	2 年以上(含)	14%, 15%	其他	30	其他
	風險規避	風險追求	風險規避	風險追求	風險規避	風險追求
第 5 位 偏好態度	0	1 年以上(含)	14%, 15%, 16%	其他	30, 31	其他
	風險規避	風險追求	風險規避	風險追求	風險規避	風險追求
第 6 位 偏好態度	不超過 2 年	3 年以上(含)	14%	其他	30	其他
	風險規避	風險追求	風險追求	風險規避	風險規避	風險追求

表 4　談判者對事件之風險偏好態度分析 (續)

事件	銀行融資 信用貸款利率		費率 管制		匯 率	
談判者	非風險狀態 (信用利率)	風險狀態 (信用利率)	非風險狀態 (費率水準)	風險狀態 (費率水準)	非風險狀態 (匯率水準)	風險狀態 (匯率水準)
第 1 位	7.5%	其他	$NT180,$NT191	其他	30.1,29.8,30.4,30.9	其他
偏好態度	風險追求	風險規避	風險追求	風險規避	風險追求	風險規避
第 2 位	6.5%	其他	$NT180	其他	29.8,30.1,30.9,	其他
偏好態度	風險追求	風險規避	風險追求	風險規避	風險追求	風險規避
第 3 位	6.5%	其他	$NT180	其他	29.8,30.4,30.9	其他
偏好態度	風險追求	風險規避	風險追求	風險規避	風險追求	風險規避
第 4 位	7.5%	其他	$NT180	其他	29.8,30.1,30.4,30.9	其他
偏好態度	風險規避	風險追求	風險追求	風險規避	風險追求	風險規避
第 5 位	6.5%	其他	$NT180	其他	30.9	其他
偏好態度	風險追求	風險規避	風險追求	風險規避	風險追求	風險規避
第 6 位	7.5%	其他	$NT180	其他	29.8,30.1,30.4,30.9	其他
偏好態度	風險規避	風險追求	風險追求	風險規避	風險追求	風險規避

表 5 事件之談判群體效用函數

事件種類	談判群體效用模式
用地徵收 (L)	$GU(y_j) = U(u_1(L), u_2(L), \Lambda, u_6(L)) = 0.0366u_1(L) + 0.0082u_2(L)$ $+ 0.0125u_3(L) + 0.7580u_4(L) + 0.1599u_5(L) + 0.0249u_6(L)$
折扣率 (D)	$GU(d_j) = U(u_1(D), u_2(D), \Lambda, u_6(D)) = 0.2643u_1(D) + 0.1386u_2(D)$ $+ 0.1323u_3(D) + 0.1584u_4(D) + 0.1296u_5(D) + 0.1768u_6(D)$
特許期限 (T)	$GU(t_j) = U(u_1(T), u_2(T), \Lambda, u_6(T)) = 0.1545u_1(T) + 0.1548u_2(T)$ $+ 0.1694u_3(T) + 0.1931u_4(T) + 0.1931u_5(T) + 0.1734u_6(T)$
銀行融資信用貸款利率 (I)	$GU(c_j) = U(u_1(C), u_2(C), \Lambda, u_6(C)) = 0.1478u_1(C) + 0.1196u_2(C)$ $+ 0.2494u_3(C) + 0.1881u_4(C) + 0.1364u_5(C) + 0.1573u_6(C)$
費率管制 (P)	$GU(p_j) = U(u_1(P), u_2(P), \Lambda, u_6(P)) = 0.1668u_1(P) + 0.1651u_2(P)$ $+ 0.1668u_3(P) + 0.1651u_4(P) + 0.1713u_5(P) + 0.1651u_6(P)$
匯率 (E)	$GU(e_j) = U(u_1(E), u_2(E), \Lambda, u_6(E)) = 0.1848u_1(E) + 0.0041u_2(E)$ $+ 0.0038u_3(E) + 0.2652u_4(E) + 0.2514u_5(E) + 0.2906u_6(E)$

5.5 風險事件及非風險事件

本小節進行風險事件、非風險事件及主要風險與次要風險事件之分析。

(i) 事件之風險與非風險狀態分析

將利用式(4)所得到之談判者效用值代入表 5 中之談判群體效用模式，得事件各狀態之談判群體效用值，依此效用值進行事件狀態之風險或非風險之分析。

就用地徵收而言，群體談判對於用地徵收延遲超過 4 年者屬風險狀態，該事件於 3 年以內完成徵收者屬非風險狀態，此顯示談判群體可接受用地於 3 年內完成徵收。就折扣率而言，談判群體認為，折扣率介於 7% --11%及 17 % -- 18 % 者屬非風險狀態，其餘折扣率水準屬風險狀態；就銀行融資信用貸款利率而言，貸款利率水準超過 8.5%時，屬風險狀態，其餘利率水準屬非風險狀態；就費率管制而言，談判群體對於實施費率管制後之初期費率水準尚可接受，屬非風險狀態，但對於第二期以後之費率水準則不為談判群體所接受，此屬風險狀態；就特許期限而言，談判群體對於"30"、"31"或"32 年"之特許期限，此屬非風險狀態，其餘屬風險狀態。一般而言，特許公司會要求較長特許期限，以獲更多利益，然從相關 BOT 計畫案例顯示，特許年限訂定與其他事件搭配有關，換言之，若用地徵收發生延遲情形，其會影響特許年限之訂定。

(ii) 風險事件及非風險事件之分析

利用談判群體效用函數雖可得談判群體對事件狀態一致性衡量結果，惟尚無法得知事件是否為風險事件或非風險事件，故須再利用式(8)進行分析。茲利用事件狀態之談判群體效用值分為非風險與風險兩類，將此兩種狀態效用值之最大值與最小值分別代入式(8)中，即得事件 GU值，各事件總計效用值分別為 0.5128、0.6620、0.4001、0.4386、0.2113 及 0.2282，再依式(8)之 $E(U^*)$ 定義，計算每一事件之效用期望值，分

別為 0.0655、0.2317、0.2877、0.2154、0.2972 及 0.2565，結果如表 6。由表 6 知，特許期限之 $GU^T = 0.2113 < E(U^*(T)) = 0.2672$，匯率之 $GU^E = 0.2282 < E(U^*(E)) = 0.2565$，故此二者屬風險事件，其餘屬非風險事件；故得風險事件集合為 {特許期限、匯率}，即 $R = \{T,E\}$，非風險事件集合為 {用地徵收、折扣率、銀行融資信用貸款利匯率、費率管制}，即 $UR = \{L,D,C,P\}$。

表 6 風險事件及非風險事件之一覽表

事件種類		總計效用值 GU	效用值 $E(U^*)$	風險/非風險
用地徵收	(L)	0.5128	0.0655	非風險
折扣率	(D)	0.6620	0.2317	非風險
銀行融資信用貸款	(I)	0.4001	0.2877	非風險
費率管制	(P)	0.4386	0.2154	非風險
特許期限	(T)	0.2113	0.2972	風險
匯率	(E)	0.2282	0.2565	風險

(iii) 事件排序

表 6 之各事件效用值分別為 0.5128、0.6620、0.4001、0.4386、0.2113 及 0.2282，依效用偏好理論，上述事件可依效用值由小至大予以排序，排序結果依序為 $0.2113 \pi 0.2282 \pi 0.4001 \pi 0.4386 \pi 0.5128 \pi 0.6620$，此結果顯示，事件優先順序依序為 $T \pi E \pi C \pi P \pi L \pi D$，即事件之風險程度排序為特許期限<匯率<銀行融資信用貸款利率<費率管制<用地徵收<折扣率，故可知，特許期限之風險程度最高，折扣率事件風險程度最低。

(iv) 主要風險事件(main risk event)與次要風險事件(minor risk event)

本文利用 4.3 節觀念進一步求解風險事件集合中之主要風險事件與次要風險事件。由表 6 可知，特許期限及匯率此二者屬風險事件，總計效用值分別為 0.2113 及 0.2282，假設事件獨立，故利用式(12)觀念，將此二值代入式(12)中之總計效用期望值，即二者之平均值，故得風險事件平均效用值為 0.2198。由於特許期限之總計效用值小於總計效用期望值，故特許期限 (T) 為風險事件集合中之主要風險事件，匯率之總計效用值大於總計效用期望值，故匯率 (E) 為風險事件集合中之次要風險事件，如此，本文可求得主要風險事件集合為 {特許期限}，即 $CR_{main} = \{T\}$，T 表特許期限，次要風險事件集合為 {匯率}，即 $CR_{min} = \{E\}$，E 表匯率事件。故 BOT 特許公司與政府在進行特許契約談判時，特許期限為首要談判事項，其次為匯率事項。

六、結論與建議

本文之主要目的係就 BOT 特許公司與政府進行群體談判時，確認哪些不確定性因素為風險事件或非風險事件，並由風險事件中界定哪些是主要風險或次要風險。

本研究從風險衡量觀念出發，結合多屬性、機率及效用理論，利用 Keeney 及 Raiffa 多屬性效用觀念，構建談判群體之風險衡量模式，衡量 BOT 計畫特許契約之

不確定事件之效用，並以範例分析為例，說明本文所發展之風險效用衡量模式的可用性。本文之研究結果指出：1.**在假設事件屬性產出與狀態獨立之下，事件之多屬性效用函數為單一屬性效用之一般式。**2.**利用** Keeney **及** Raiffa **效用函數觀念，數個別談判者之效用函數可整合成為談判群體效用函數，而利用效用排序與距離觀念，可以構建總計效用函數，整合所有談判者之不同狀態效用衡量值，藉以衡量事件之風險性。**3.**在假設談判群體追求風險最小化，即群體效用期望值最大化條件之下，可得到風險集合之關鍵風險水準，此關鍵風險水準為所有風險事件效用期望值，從而由風險集合中可得到主要風險事件與次要風險事件。**4.**由本研究範例新顯示，就特許期限、匯率、銀行融資信用貸款利率、費率管制、用地徵收與折扣率等六個事件，事件之總計效用值分別為** 0.5128、0.6620、0.4001、0.4386、0.2113 **及** 0.2282**，其風險程度順序性依序為：特許期限**<**匯率**<**銀行融資信用貸款利率**<**費率管制**<**用地徵收**<**折扣率；其中，特許期限與匯率屬於風險事件，其餘屬非風險事件，在特許期限及匯率所屬之風險事件集合中，特許期限屬主要風險事件，匯率屬次要風險事件。**

此外，在本文中係假設事件屬性產出與狀態獨立、事件獨立及談判者效用獨立所進行之研究成果；然而，在實際環境裡，事件並非完全獨立，談判者之間亦會有互動行為。因此，後續可考慮放寬上述假設條件，建立談判者互動風險衡量模式，事件互動風險衡量模式，使模式更符合實際環境特性。另外，本文係就 BOT **特許公司角度，構建談判群體風險衡量模式，未來亦可利用此一觀念，構建政府談判群體之風險衡量模式，求取兩造風險事件，並進行風險談判之相關研究。而未來可就國內之** BOT **計畫推動案例進行實例研究，驗證本文模式之實用性與合理性。**

參考文獻

1. Bose, U., Davey, A. M., and Olson, D. L., "Multiattrinute Utility Methods in Group Decision-Making: Past Applications and Potential for Inclusion in GDSS", Omega, *International Journal of Management Science*, Vol. 25, No. 6, 1997, pp.691-706.
2. Buhlmann, H., *Mathematical Methods in Risk Theory*, Springer-Verlag, 1996.
3. Cooper, D. F. and Chapman, C. B., *Risk Analysis for Large Projects Model, Methods and Cases*, John Wily & Sons, 1987.
4. Cuthbertson, M., *Quantitative Financial Economics Stock, Bonds and Foreign Exchange*, John Wiley and Sons, Inc, 1996.
5. Hwang, Y. L., Project and Policy Analysis of Build-Operate-Transfer Infrastructure Development, Ph.D. Dissertation, Department of Civil Engineering, University of California at Berkeley, 1995.
6. Jaselskis, E. J. and Russell, J. S., "Risk Analysis Approach to Selection of Contractor Evaluation Method", *Journal of Construction Engineering and Management*, Vol. 118, No. 4, 1992, pp. 805-812.
7. Jia, J. and Dyer, J. S., "A Standard Measure of Risk and Risk-Value Models", *Management Science*, Vol. 42, No. 12, 1996, pp. 1691-1705.
8. Keeney, R. L. and Raiffa, H., *Decisions with Multiple Objectives Preferences and Value Tradeoffs*, Cambridge University Press, 1993.
9. Levitt, R. E.; Ashley, D. B. and Logcher, R. D., "Allocating Risk and Incentive in Construction", *Journal of the Construction Division*, Vol. 106, No. 3, 1980, pp. 297-305.
10. Mustafa, M. A. and Al-Bahar, J. F., "Project Risk Assessment Using the Analytic Hierarchy Process", *IEEE Transactions on Engineering Management*, Vol. 38, No.1,

1991, pp. 46-52.

11. Philip, N., "Allocation of Risks in BOT Projects", The High Speed Rail BOT Workshop, Taipei, Taiwan, 1995.
12. Seo, F. and Sakawa, M., "An Experimental Method for Diversified Evaluation and Risk Assessment with Conflicting Objectives", *IEEE Transactions on Systems, Man, and Cybernetics*, Vol. 14, No. 2, 1984, pp. 213-223.
13. Seo, F. and Sakawa, M., "Fuzzy Multiattribute Utility Analysis for Collective Choice", *IEEE Transactions on Systems, Man, and Cybernetics*, Vol. 15, No. 1, 1985, pp. 45-53.
14. Seo, F., "On a Construction Fuzzy Multiattribute Risk Function for Group Decision Making", *Multiperson Decision Making Models Using Fuzzy Sets and Possibility Theory*, by Kacprzyk, A. and Fedrizzi, M., Kluwer Academic Publishers, 1980, pp.198-218.
15. Sidney, M. L., *Build, Operate, Transfer Paving the Way for Tomorrow's Infrastructure*, John Wiley and Sons, Inc, 1996.
16. Tiong, L. K., "Comparative Study of BOT Projects", *Journal of Management in Engineering*, Vol. 6, No. 1, 1990, pp. 107-122.
17. Tiong, L. K., "BOT Projects: Risks and Securities", *Construction Management and Economics*, Vol. 8, No. 3, 1990, pp. 315-328.
18. Tiong, L. K., "Risks and Guarantees in BOT Tender", *Journal of Construction Engineering and Management*, Vol. 121, No. 2, 1995, pp.183-187.
19. Tiong, L. K., "CSFs in Competitive Tender and Negotiation Model for BOT Projects", *Journal of Construction Engineering and Management*, Vol. 122, No. 3, 1996, pp. 205-211.
20. Tzeng, G, H. Shieh, H. M., and Shiau, T. A., "Route Choice Behavior in Transportation- An Application of The Multiattribute utility theorem", *Transportation Planning and Technology*, Vol. 13, No. 3, 1989, pp. 289-301.
21. Walker, C. and Smith, A. J., *Privatized Infrastructure: the Build Operate Transfer*, Thomas Telford Publications, 1996.
22. William, I. C. J. and Crandall, K. C., "Construction Risk: Multiattribute Approach", *Journal of the Construction Division*, Vol. 108, No. 2, 1982, pp.187-200.
23. **馮正民、鐘啟椿，交通建設BOT案政府對民間造成之風險分析，運輸計畫季刊，第29卷，第1期，頁79-108，民國八十九年。**
24. **賴士葆，工程經濟，華泰書局，民國七十六年。**
25. **李朝賢，區域發展規劃，華泰書局，民國八十二年。**
26. **朱敬一，個體經濟分析，新陸書局，民國七十九年。**

附錄

附表 1　談判者對事件之風險及非風險狀態分析結果

用地 徵收		折扣率		特許 期限	
非風險狀態 (延遲徵收年)	風險狀態 (延遲徵收年)	非風險狀態 (折扣率水準)	風險狀態 (折扣率水準)	非風險狀態 (特許年限)	風險狀 (特許年
0	1 年以上(含 1 年)	13%	其他	30 年	其他
1 年以內	2 年以上(含 2 年)	13%	其他	30,31 年	其他
1 年以內	2 年以上(含 2 年)	15%	其他	30 年	其他
1 年以內	2 年以上(含 2 年)	14%, 15%	其他	30 年	其他
0	1 年以上(含 1 年)	14%, 15%, 16%	其他	30, 31 年	其他
2 年以內	3 年以上(含 1 年)	14%	其他	30 年	其他
銀行融資信用 貸款利率		費率 管制		匯 率	
7.5%	其他利率水準	$NT 180,$NT 191	其他費率水準	30.1,29.8,30.4,30.9	其他匯率
6.5%	其他利率水準	$NT 180	其他費率水準	29.8,30.1,30.9,	其他匯率
6.5%	其他利率水準	$NT 180	其他費率水準	29.8,30.4,30.9	其他匯率
7.5%	其他利率水準	$NT 180	其他費率水準	29.8,30.1,30.4,30.9	其他匯率
6.5%	其他利率水準	$NT 180	其他費率水準	30.9	其他匯率
7.5%	其他利率水準	$NT 180	其他費率水準	29.8,30.1,30.4,30.9	其他匯率

附表 2 個別談判者效用函數估計情形

用地徵收(L)									
談判者	1		2		3		4		
變數	**係數**	t **值**	**係數**	t **值**	**係數**	t **值**	**係數**	t **值**	**係**
截距項	0.1345	4.74***	0.824	6.75***	0.548	7.06***	-	-	0.
y^3	-	-	-	-	-	-	0.00009	4.79***	-
y^2	0.0028	2.22*	0.021	3.84***	0.0138	3.97**	0.0014	-5.56***	
y	-0.04	- 3.05**	-0.28	-4.93***	- 0.185	-5.12***	0.0056	6.59***	
R^2	0.67		0.82		0.83		0.87		
F **值**	8.19**		17.99***		19.62***		18.11***		
折扣率 (D)									
截距項	-1.512	-4.21**	-1.962	-3.13*	5.93	2.72*	-1.95	-2.41*	
d^3	-	-	-	-	-4755.11	-3.80***	-	-	-
d^2	-110.52	-4.61***	-142.64	-3.41**	1652.29	3.51**	-124.64	-2.31*	
d	28.11	4.65***	36.48	3.46**	-176.37	- 3.11*	34.27	2.52*	
R^2	0.71		0.57		0.83		0.497		
F **值**	10.82**		6.01*		12.62***		4.44*		
銀行融資信用貸款(I)									
截距項	-	-	0.982	5.53***	4.67	4.39**	-98.24	-4.61**	
$(rc)^3$	-3303.03	-2.41	-	-	-	-	174841.0	4.49*	-
$(rc)^2$	320.51	2.68*	1056.26	3.99**	509.72	3.21*	-44010.0	-4.56**	8
(rc)	-	-	-203.86	-4.67***	-97.64	-3.73**	3635.2	4.60*	
R^2	0.68		0.968		0.95		0.92		
F **值**	5.47*		61.7***		36.5***		12.05*		

註："***" **表於** ~ = 0.001 **水準下顯著**; "**" **於表於** ~ = 0.01 **水準顯著**; "*"**於** ~ = 0.05 **水準下顯著。**

附表 2 談判者效用函數估計情形(續)

						費率管制 (P)			
談判者	1		2		3		4		
變數	**係數**	t **值**	**係數**	t **值**	**係數**	t **值**	**係數**	t **值**	**係**
截距項	9.58	5.42***	7.44	4.79***	8.75	9.19***	10.003	11.09***	8.61
f^3	-	-	-	-	-	-	-	-	-
f^2	0.00015	4.58***	0.00011	3.77***	0.00012	7.16***	0.00014	8.88***	
f	-0.075	-4.96***	-0.056	-4.22***	-0.066	- 8.06***	-0.076	-9.86***	- 0.06
R^2	0.76		0.80		0.94		0.95		
F **值**	27.52***		30.27***		117.28***		150.38***		
						特許期限 (T)			
截距項	-48.27	-3.15**	-51.38	-3.13*	-	-	-	-	
t^3	-	-	-	-	-0.001	-2.52*	-0.001	-2.53*	-
t^2	-0.05	-3.12*	-0.05	-3.41**	0.091	2.50*	0.069	2.51*	
t	3.13	3.15**	3.33	3.46**	10387.0	-2.46*	-1.06	-2.47*	
R^2	0.63		0.65		0.68		0.69		
F **值**	5.21*		5.59*		4.3*		4.5*		
						匯率 (E)			
截距項	-	-	-	-	-	-	-	-	-
$(re)^3$	-	-	-	-	-	-	-	-	-
$(re)^2$	- 0.0084	-6.13***	-0.0087	-5.92***	-0.0075	-4.64***	-0.0053	-4.810**	
(re)	0.273	6.34***	0.285	6.15***	0.246	4.82***	0.173	5.00***	
R^2	0.83		0.84		0.76		0.92		
F **值**	39.35***		40.56***		25.20***		12.05*		

註："***" **表於** ~＝0.001 **水準下顯著**；"**" **於表於** ~＝0.01**水準顯著**；"*"**於** ~＝0.05**水準下顯著。**

運輸計畫季刊(編號 634，已接受)

在談判者效用互動下之風險衡量
--以 BOT 計畫用地取得事件為例[1]

By

馮正民,　Cheng-Min Feng

康照宗,　Chao-Chung Kang

Gwo-Hshing Tzeng

Institute of Traffic and Transportation

National Chiao Tung University

114, 4F, Sec.1, Chung Hsiao W. Rd.
Taipei 100, Taiwan
Fax: 886-2-23120082,
E-Mail: cmfeng@cc.nctu.edu.tw

在談判者效用互動下之風險衡量
--以 BOT 計畫用地取得事件為例[1]

The Risk Measurement of the BOT Projects Under Interactive Utility among the Negotiators

馮正民 Cheng-Min Feng[2]　**康照宗** Chao-Chung Kang[3]

摘要

本文目的在探討談判者或決策者彼此之間有相互討論行為時，談判者如何對特許契約所存在之不確定性因素進行風險衡量。本文從效用線性相依及偏好可分性(weakly separable preference)**觀念出發，以動態規劃方法構建談判群體效用衡量模式，同時研擬疊代求解法**(iterative algorithm)**以及以範例分析方式，說明談判群體效用衡量模式之可用性。經由本文所發展之模式、求解方法與範例分析顯示，影響談判者之間效用獨立與否取決於效用交互影響值**(interactive utility value, IUV)**之和是否為** 0**；當效用交互影響值之和趨近於** 0 **時，談判者彼此之間的效用相依程度愈低，獨立越強；當效用交互影響值之和趨近於** 1 **時，談判者彼此之間的效用相依程度越高。此外，當談判者彼此之間的效用差異越小，模式求解易於收斂；若談判者彼此之間的效用差異越大，討論次數會增加；而當討論次數增加時，談判者之間的效用交互影響值會遞減。當第** 1 **次討論即獲得模式求解時，此時談判群體效用值可由談判者之最初效用予以加總，此結果與談判者在獨立狀態下所進行效用衡量結果相同。**

關鍵詞：BOT**、討論、風險、風險衡量、效用相依**

ABSTRACT

The purpose of this paper is to measure the risk of BOT projects when there are interactive relationship among the negotiators. Based on the utility theorem and the weak separability theorem, a dynamic programming of the risk measurement model is developed to simulate the utility dependent behavior among the negotiators. The results of numerical example show that the interactive relationship of negotiators increase when the sum of interactive utility value near to 1. Otherwise, the interactive relationship of negotiators becomes independent when the sum of interactive utility value is 0. The algorithm of the model can be converged when the difference value between negotiator's utility is small, and the discussion frequency increases when the difference value between negotiator's utility becomes large. It shows that the group utility model and iterative algorithm in this paper can be applied to analyze the interactive behavior and risk measurement in BOT projects.

Key Words: *BOT; Discussion; Risk; Risk measurement; Utility dependent*

---

4. **本文部份研究承行政院國家科學委員會補助，編號** NSC89-2211-E-009-024**，謹此致謝，作者感謝論文審查委員之寶貴意見。**
5. **國立交通大學交通運輸研究所教授(聯絡地址：台北市忠孝西路一段** 114 **號四樓，連絡電話：**02-23146515)**。**
6. **國立交通大學交通運輸研究所博士班研究生，現任交通部運輸研究所專員。**

一、前言

BOT 計畫風險分擔係政府與 BOT 特許公司透過談判手段達成特許契約風險移轉[1,31,32,35]，此特許契約談判過程反應群體參與及反覆討論特色[33]。而無論公私部門之談判群體，其在進行特許契約談判之前，談判群體之談判者(或專家)藉由經驗法則及資訊蒐集，針對特許契約所存在之不確定性因素進行風險評估(risk assessment)工作，來確認特許契約內之風險事項，以期有效掌握風險事件中之主要風險(primary risks)及次要風險(secondary risks)事件，俾作為特許契約談判工作之用。

然而就實際決策環境而言，談判群體之談判者或專家，個自完成對不確定性因素風險衡量後，通常談判群體內部會有「討論」行為，此內部討論行為主要目的是透過內部之溝通協調來決定特許契約內所存在之風險事件。當談判群體內部有「討論」行為時，此即反映談判者彼此之間有了互動現象，此互動因素會影響談判者在原先獨立狀態下之風險衡量。

由於談判群體內部有討論行為，此隱約可見，某談判者的衡量改變會影響其他談判者的評估，而此時的影響因素即涉及人與人之間的互動，此種互動現象在賽局理論(game theory)中有兩種狀況要討論，一種是彼此獨立做決策，但是所作決策彼此影響，二是大家一起來商量，在決策階段時相互協調。後者即是本文所要探討主題。

雖然國內外已有相當多文獻針對 BOT 計畫之風險課題有所探討，然甚少以群體決策觀點對 BOT 計畫進行風險衡量分析，縱以馮正民與康照宗[2,17]應用 MAU 效用理論對 BOT 特許契約之不確定性因素進行風險衡量分析，反應談判者之風險偏好，求解特許契約之主要風險與次要風險課題，惟尚未就談判者彼此之間討論行為課題，深入分析。因此，本文目的在於探討談判群體內部之談判者有討論時，如何就不確定性因素進行風險衡量。

本文內容結構共分七大部份，第二部份為文獻回顧，第三則說明本文研究問題；第四部份說明假設條件及構建談判者與談判群體效用相依模式；第五部份則研擬模式求解法；第六部份則為範例分析，最後為結論與建議。

二、文獻回顧

本節主要回顧相關文獻之風險定義，效用相依及相關風險衡量方法。

2.1 風險概念與定義

風險是個複雜且抽象概念，文獻對於風險(Risk)定義，至今仍有眾多不同觀點。Rowe[30]將風險詮釋為「某負面事件(unwanted negative)潛在損失結果(negative consequence)」。Lowrance[26]則是將風險定義為「負面影響的嚴重程度(severity of adverse effects)及事件發生機率之衡量效果(probability)」；Rescher[29]對風險詮釋為「負面效果之可能性(chancing of a negative outcome)」；Ansell & Wharton[7]則是將風險詮釋為「決策所產生之非期望效果(unexpected outcome)」，此非期望效果所指的是「事件發生次數及事件規模的組合乘積」；

Gratte[19]對風險之定義係指「事件發生機率及事件發生後的結果」。而 Haupymanns & Werner[21]則是將風險定義為「危險或損失可以被衡量者」。Buhlmann[12]將風險詮釋為「預期結果與實際結果之差」。Cooper 與 Chapman[13]則是將定義風險為「某特定行為或決策，在經濟、財務上之損益或實體上之延誤、損害發生的可能性，此發生可能性為不確定性之結果」。蔡明志[4]則是將風險詮釋為「風險係事件的發生及發生後事件的後果之組合乘積」，二者缺一則無法完整表達風險內容。

由上述風險定義概念，衡量風險有期望值與機率測度兩種，但是就財務風險而言，其主要是將風險詮釋為主要決策變數之變異幅度。在此概念之下，變異數或者期望值與變異數混合使用成為衡量風險方法，如 Cuthbertson[14]將風險定義為「決策變數之變異數，決策變異越大，風險越高」。效用觀念亦是衡量風險的另一理論方法，如 Jia 與 Dyer[21]以效用觀點將風險定義為「偏好為負之效用期望(negative expected utility)」。

從 Ansell & Wharton[6]、Gratte[18]、Lowrance[25]、Rescher[28]、Cooper & Chapman[13]及蔡明志[4]之風險定義概念來看，風險期望值是目前公認可接受之定義(蔡明志,2000)，由此一定義顯示，事件之發生機率與發生結果，二者為構成風險缺一不可的要素。

此外，由文獻顯示，衡量風險除可採貨幣方法外，亦可採非貨幣化方式進行如效用衡量方法(朱敬一[5]；Keeney & Raiffa[22]；Jia 與 Dyer[21])。在效用、風險與報酬之間的關係方面，依 Bell[9]研究證明，效用期望理論可顯示決策者之風險偏好及可作為風險衡量之基礎，其亦證明效用期望、報酬與風險三者之間存有正向關係特性。Keeney & Raiffa (1993)之多屬性效用理論(Multiattribute Utility Theory, MAU)亦以效用觀念來衡量決策者之決策風險，該風險規避指標建立於效用期望理論基礎上，決策者之決策風險會隨效用期望值增減而有所不同。一般而言，效用期望值愈高(遞增狀態)，決策者會頃向選擇(享受較多報酬)，故風險性較高。而 Keeney & Raiffa 之 MAU 理論亦已廣泛應用於決策行為的研究(Bose, *et al*,[10])及工程計畫風險衡量領域(William & Crandall[35])，雖然此理論模式有效用乘法型(multiplicative utility)，惟仍屬於靜態分析，無法有效詮釋決策者互動行為課題。

2.2 效用互動之相關研究

過去有關效用互動方面之研究，有蒙地卡羅(Monte Carlo)、效用相依數學解析及團隊理論(Team Theory; Kim & Roush[24])觀念。就模擬而言，Carbone[12]以蒙地卡羅方法模擬效用期望排序(complete rank dependent expected utility theory)、成對選擇效用(pairwise choice utility)、效用期望理論(expected utility theory)、顯示價值理論(prospective reference theory)及加權效用理論(weight utility theory)模式之五個模式在決策分析上之差異性。由於蒙地卡羅在模擬時須預設效用值、模擬參數值及機率值，但是，當決策者之效用與其他決策者有互為因果關係時，便無法預先決定輸入哪些效用值與機率值。

在效用相依數學解析方面，此方面觀念係在探討事件效用與機率之間的關係(Belichrosdt & Quiggin[9]；Daniels & Keller[16])。Belichrosdt & Quiggin (1997)之 Rank dependent utility theory 模式即是將機率設為效用內生變數，探討效用與

聯合機率或邊際機率之關係。當效用機率為聯合機率時，事件之效用不能由效用期望值求得。Quiggin[28]則認為效用與產出具有某種關係，並非完全獨立，在放寬效用與產出獨立條件下，利用機率權重、線性轉換觀念及以介於 0~1 之間的尺度常數，將不同產出效用予以結合，再以期望值求總效用，進行偏好排序，構建效用期望排序模式(RDEU, rank-dependent expected utility theory)，模式中之機率權重與線性轉換即屬機率線性組合觀念。Kelsey[24]之效用相依模式(state-dependent utility function, SDUF)則是將狀態機率設為效用函數之外生變數，將狀態權重納入機率函數內，當狀態機率不為聯合機率時，則模式成為機率與效用之加權模式；換言之，此模式與效用期望模式無異。

由上述文獻知，機率可為效用函數之外生或內生變數。效用相依模式是否為效用期望模式取決於聯合機率或條件機率，若效用之聯合機率或條件機率成為邊際機率時，此效用相依模式會成為效用期望模式。

Kim & Roush (1987)之團隊理論(team theory)觀念係另一種處理效用互動方法，該理論著眼於決策者與外在環境因素之互動關係。其以協調觀念分析決策者與外在環境因素之互動問題，以並聯(parallel team)、串聯(chain team)、協調(coordination team)及蒐尋(search team)之四種團隊型態觀念，分析決策者與其他影響因素(motivation)之互動行為。Kim & Roush 所提之協調觀念重點於分析外在環境因素之改變對團隊效用之衝擊，但此理論並未就團隊內部決策者彼此之間的互動進行相關課題之論述；同時並聯或串聯團隊型態須預設外在因素與團隊之間的關係架構之限制條件。

在處理決策者與外在環境互動課題中，Tzeng 等人[34]以習慣領域(Habit Domain, HD)觀念，分析決策者與外在環境因素互動問題，此研究著眼於決策者之自我學習與能力拓展因素，並以動態求解方法，求解決策者學習能力與能力集合問題；惟求解方法甚為繁複，且 Tzeng 尚忽略決策者與決策者之間之互動問題。

2.3 其他風險衡量概念

按相關文獻對風險定義概念，衡量風險除機率、期望值與變異數方法之外，數學規劃亦為衡量風險之另一種方法(Haimes[20]; Orlovski, 1990)。Haimes 提出多目標規劃與動態規劃模式觀念，以數學規劃模式來分析決策者在資源有限之下的決策風險水準。動態規劃觀念之風險衡量則可以衡量不同階段性之風險水準；此外，Orlovski(1990)提出模糊二階規劃觀念，用以分析二人賽局(two-person game)問題。

三、問題說明

BOT 特許契約談判是由政府部門與特許公司兩個談判群體共同進行，政府談判群體成員包含交通、都計、環保、地政、法規顧問及地方政府單位。特許公司談判群體成員包含法律、財務顧問、工程專家及參與廠商。此特許契約談

判係由雙方主要談判者進行，假若談判群體內成員間同質性高，則特許契約談判即如一對一談判，概念如圖 1 所示。

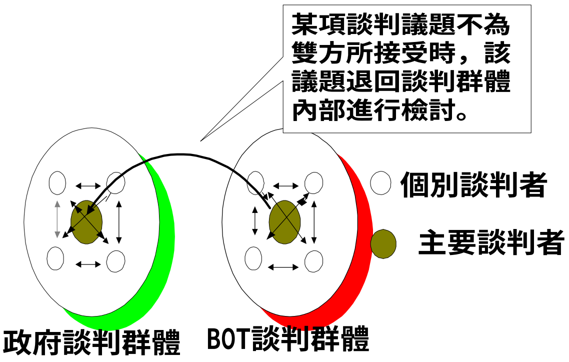


圖 1　談判群體之談判者互動關係概念

當政府與 BOT 特許公司在進行特許契約簽訂之前，雙方就契約內之風險事項進行風險分擔談判。在談判時，若某些風險事項無法為雙方所接受，則此風險事項將為特許契約談判後續談判議題，談判群體各自就此具爭議之風險事項進行內部討論，重新評估其風險性，其概念如圖 1。或者，當某談判群體欲進行特許契約之前，該談判群體在決定談判事項之前，談判群體內部就所衡量之風險事件進行討論，以確認該事件是否屬於特許契約談判對象，觀念如圖 2。

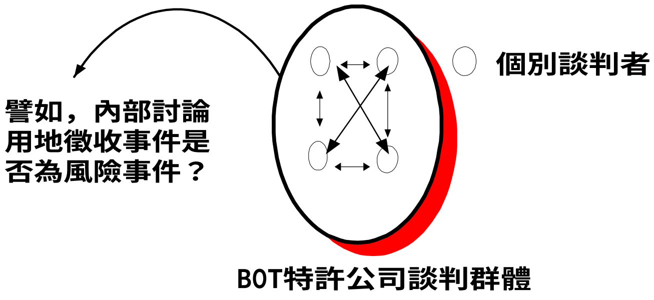


圖 2　BOT 談判群體對風險事件內部討論概念

由於風險衡量涉及談判者本身之效用及事件兩個因素，若考量單一事件情況下，談判者之間若無效用互動干擾，則呈效用獨立(utility independent)現象；若有效用互動(interactive utility)，如談判者彼此之間有相互討論情形，則屬效用相依(utility dependent)。一般而言，只要談判者之間有討論行為，即產生效用互動情形，此互動具有雙向性與迴饋性。所謂雙向性係指任何談判者之間，自己效用改變時會透過討論過程去影響他人效用，其他談判者效用改變時亦透過討論過程影響本身效用。所謂迴饋性係指談判者彼此之間在某次討論，透過效用互動之雙向性影響某談判者在下一次討論時之效用。當談判者彼此之間的互動影響趨於相同或穩定時，此時即可達到所有談判者對事件之風險衡量，即談判群體內部對事件之風險衡量獲得「共識」，此即本文所探討議題範圍。

四、模式發展

本節說明模式假設條件，並進行談判者及談判群體效用相依模式之構建。

4.1 模式假設條件

在構建談判群體效用相依模式前，茲將模式發展相關假設條件說明如下。

1.個別談判與所屬機構具有代理關係，但代理成本(agency cost)不對個別談判者效用產生影響。

2.談判者存在效用函數，且效用函數不受離散型或連續型資料而有所不同。

3.個別談判者之決策屬於理性行為(rational)。

4.事件(或屬性產出)之發生機率屬於 Bernoulli 實驗，發生機率為成功之機率。

假設 1 說明，談判者來自於授權關係，此授權關係即是所謂代理關係。一般而言，有代理關係即有代理成本，若代理成本不為零即易生逆選擇行為或道德危機現象，假設代理成本為零，即代理關係不對效用函數產生影響。假設 2 說明，每一談判者存在選擇行為，效用函數不受離散型或連續型資料而有所不同，且滿足 NM (Von Neumann-Morgenstern)效用公理條件。假設 3 說明，談判者符合理性條件進行選擇，在理性選擇行為下，談判者係追求效用最大化，即追求風險最小。假設 4 說明，談判者依其經驗法則或所蒐集資訊進行事件、屬性產出或狀態發生機率等之計算。機率採 Bernoulli 實驗乃說明 BOT 計畫具有眾多不確定性因素，計畫特許期很長，並無法以任何一種分配來計算，以狀態發生與否之機會作為發生機率，恰同於 Bernoulli 分配之成功機率。

4.2 風險狀態定義

本節定義風險狀態及構建談判者效用風險衡量模式，作為發展談判群體效用模式之基礎。

本文根據 Buhlmann[12] 之風險概念，參酌 Keeney & Raiffa 之風險偏好觀念，茲將「風險狀態」定義為「對決策者而言，某事件在某狀態下所產生之實際效用小於其所有狀態之期望效用」，其衡量概念說明如下。

假設某事件有 n 個不確定狀態(uncertainties)，s_j 表第 j 種狀態，$j=1,2,\ldots,n$。令 p_j 為狀態之發生機率，每一個狀態皆對應一個屬性產出(outcome of attribute) x_j，而每個屬性產出 x_j 皆對應一個效用值 $u_j(x_j)$，$p_j \times u_j(x_j)$ 為談判者對每一個狀態及所對應屬性產出之效用衡量值，$E(u(x))$ 為所有狀態之效用期望值，模式定義如式(1)。

$$R_j \equiv u_j = p_j \times u_j(x_j) < E(u(x)), \forall\ j \qquad (1)$$

其中 R_j 表第 j 種風險狀態；

u_j 為某談判者對某事件第 j 種狀態之效用衡量值；

$$E(u(x))=\sum_{j} p_j \times u_j(x_j)$$ ；$0 \le u_j(x_j) \le 1$ 且 $0 \le p_j \le 1$。

式(1)**說明，若某談判者對某事件之屬性產出** x_j **所產生效用小於所有屬性產出之效用期望值，則此事件於狀態** s_j **下具有風險。有關事件之狀態、屬性及機率關係結構如表** 1**。**

表 1　事件之狀態、屬性及機率結構

	狀態 S
	s_1 , s_2 ,..., s_j ,..., s_n
屬性產出 x	x_1 , x_2 ,..., x_j ,..., x_n
機率 p	p_1 , p_2 ,..., p_j ,..., p_n
效用 u	u_1 , u_2 ,..., u_j ,..., u_n

雖然 $0 \le u_j(x_j) \le 1$**，** $0 \le p_j \le 1$**，但若不同談判者之效用衡量差距很大時，式**(1)**會產生大於** 1 **現象，如此便無法有效進行風險衡量。為了便於在同一基準下作比較，本文利用** Keeney & Raiffa **效用轉換觀念，將式**(1)**予以正規化**(normalization)**，如式**(2)**。**

$$u_j^*(x_j)=\frac{p_j \times u(x_j)-\min\limits_{i}\{p_j \times u(x_j)\}}{\max\limits_{j}\{p_j \times u(x_j)\}-\min\limits_{j}\{p_j \times u(x_j)\}},\ \max_{j}\{p_j \times u(x_j)\} \ne \min_{j}\{p_j \times u(x_j)\}, \forall\ j \qquad (2)$$

其中， $u_j^*(x_j)$ **為正規化後效用值。**

由於 $0 \le u_j(x_j) \le 1$ **與** $0 \le p_j \le 1$**，故正規化後之** $u_j^*(x_j)$ **滿足** 0~1 **之間的條件。另外，若式**(2)**之分母** $\max\limits_{j}\{p_j \times u(x_j)\} = \min\limits_{j}\{p_j \times u(x_j)\}$ **時，則** $u_j^*(x_j)=0$ **。若** $u_j^*(x_j) < E(u^*(x))$**，此表示某談判者認為某事件在狀態** s_j **與屬性產出** x_j **下具有風險，其中** $E(u^*(x))$ **為正規化後效用期望值，。**

4.3 談判群體效用相依效用模式

本節構建談判者效用相依模式，作為群體效用相依模式發展之基礎。

4.3.1.群體效用相依概念

假設某談判群體內部有 2 **位談判者，茲定義** $\sim_{2,1}(t)$ **為效用交互影響值**(interactive utility value, IUV)**，其意義表示在第** t **次討論時，第** 2 **位談判者效用影響第** 1 **位談判者效用；同理，** $\sim_{1,2}(t)$ **表第** t **次討論時，第** 1 **位談判者效用影響第** 2 **位談判者效用。另定義** f **為** BOT **特許契約內之不確定性因素或特許公司所欲進行之決策，或政府部門所辦理事項但對** BOT **特許公司會產生潛在影響，此事件** f **在某情況下會成為風險事件，某狀況下會成為非風險事件。另假設每個談判者對事件** f **存在效用函數，此效用函數滿足** NM **定理。另按** Fishburn[18] **與** Quiggin[28] **效用線性觀念，不同效用函數滿足連續性**(continue)**及遞移性**(transitive)**定理且具弱獨立性**(weak independent**，參見朱敬一**[5]**)，則不同效用函**

數可以線性組合，即利用一個尺度常數á ， $á \in [0,1]$**且** $u_1^f(t), u_2^f(t) \in U^f$ **，使得** $U^f = \sim u_1^f(t) + (1-\sim)u_2^f(t)$ **，線性轉換後之** U^f **仍符合效用雙元偏好** "ϕ" (binary relation preference)**特性。**

按效用線性轉換觀念，將第 1 **位與第** 2 **位談判者效用予以線性組合，如式**(3)**及**(4)**，其效用互動概念如圖** 3**。**

$$u_1^f(t+1) = \sim_{2,1}(t)u_2^f(t) + (1-\sim_{2,1}(t))u_1^f(t) \tag{3}$$

$$u_2^f(t+1) = \sim_{1,2}(t)u_1^f(t) + (1-\sim_{1,2}(t))u_2^f(t) \tag{4}$$

其中 $u_1^f(t)$ **表第** 1 **位談判者於第** t **次討論之效用；**

$u_1^f(t+1)$ **表第** 1 **位談判者於第** $t+1$ **次討論之效用；**

$u_2^f(t)$ **表第** 2 **位談判者於第** t **次討論之效用；**

$u_2^f(t+1)$ **表第** 2 **位談判者於第** $t+1$ **次討論之效用；**

t **為討論次數，** $t = 0,1,2,\Lambda\ ,T$ **。**

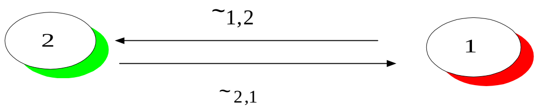


圖 3　**談判群體之** 2 **位談判者效用互動關係概念**

式(3)**中, 若** $\sim_{2,1}(t) = 0$ **時, 表第** t **次討論, 第** 2 **位談判者效用不影響第** 1 **位談判者，故得** $u_1^f(t+1) = u_1^f(t)$ **；若** $\sim_{1,2}(t) = \sim_{2,1}(t) = 0$ **時，表第** t **次討論第** 1 **位談判者與第** 2 **位談判者之間無效用互動影響現象，故屬效用獨立狀態；若** $\sim_{1,2}(t) \neq 0$ **，但** $\sim_{2,1}(t) \neq 0$ **時, 表第** t **次討論時, 第** 1 **位與第** 2 **位談判者之間有效用相互影響現象，此屬效用相依狀態。**

當談判群體有 3 **位談判者時，分別令** $\sim_{1,3}(t)$ **表第** t **次討論第** 1 **位談判者效用影響第** 3 **位談判者效用，** $\sim_{3,1}(t)$ **表第** t **次討論第** 3 **位談判者影響第** 1 **位談判者效用，其餘** $\sim_{1,2}(t)$ **、** $\sim_{2,1}(t)$ **、** $\sim_{2,3}(t)$ **及** $\sim_{3,2}(t)$ **意義依此類推。利用前述效用線性轉換觀念，將此** 3 **位談判者效用與其他談判者效用進行線性轉換，轉換後效用模式如式**(5)**至**(7)**；而此** 3 **位談判者效用互動關係如圖** 4**。**

$$u_1^f(t+1) = (1-\sim_{2,1}(t)-\sim_{3,1}(t))u_1^f(t) + \sim_{2,1}(t)u_2^f(t) + \sim_{3,1}(t)u_3^f(t) \tag{5}$$

$$u_2^f(t+1) = (1-\sim_{1,2}(t)-\sim_{3,2}(t))u_2^f(t) + \sim_{1,2}(t)u_1^f(t) + \sim_{3,2}(t)u_3^f(t) \tag{6}$$

$$u_3^f(t+1) = (1-\sim_{1,3}(t)-\sim_{2,3}(t))u_3^f(t) + \sim_{1,3}(t)u_1^f(t) + \sim_{2,3}(t)u_2^f(t) \tag{7}$$

其中 $u_1^f(t)$ **表第** 1 **位談判者於第** t **次之效用；**

$u_1^f(t+1)$ 表第 1 位談判者於第 $t+1$ 次之效用；

$u_2^f(t)$ 表第 2 位談判者於第 t 次之效用；

$u_2^f(t+1)$ 表第 2 位談判者於第 $t+1$ 次之效用；

$u_3^f(t)$ 表第 3 位談判者於第 t 次之效用；

$u_3^f(t+1)$ 表第 3 位談判者於第 $t+1$ 次之效用。

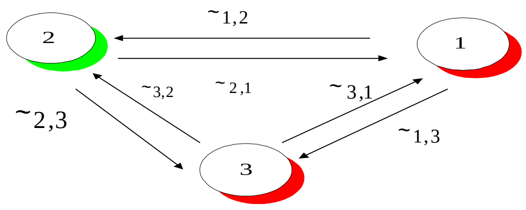


圖 4 談判群體之 3 位談判者效用互動關係概念

由式(5)至式(7)顯示，在 3 位談判者效用互動關係架構中，某談判者在第 t 次討論時會透過 $\sim(t)$ 去影響其他談判者之第 t 次討論的效用，而其他談判者亦藉由第 t 次討論時之效用，透過 $\sim(t)$ 去影響某談判者之第 $t+1$ 次效用，此種即是效用迴饋現象。通常談判群體內部有討論行為時，即存有此迴饋現象。

4.3.2. 群體效用相依模式發展

本文利用 3 位談判者效用轉換觀念，繼續發展談判群體有 q 位談判者之效用相依模式。

假設 BOT 談判群體有 q 位談判者，$q=1,2,\Lambda,Q$，談判者之間的效用關係如圖 5。由圖 5 觀念顯示，談判者之間的 $\sim_{k,q}(t)$ 或 $\sim_{q,k}(t)$ 屬於效用函數之內生變數，談判者藉由 $\sim_{k,q}(t)$ 與 $\sim_{q,k}(t)$ 去影響其他談判者效用, 進而達到效用互動及討論過程之影響。

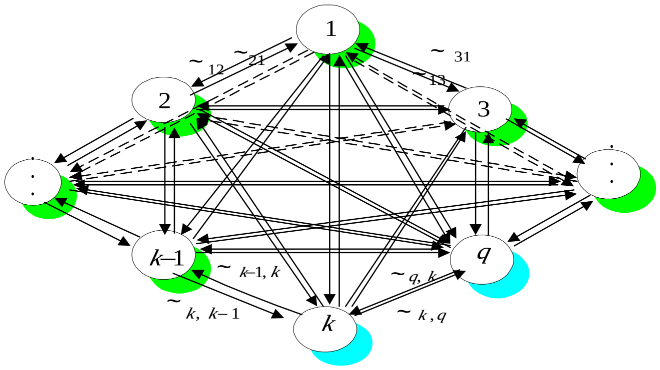


圖 5 談判群體 q 位談判者效用互動關係概念

茲假設此 q 位談判者於第 t 次與第 $t+1$ 次討論時，存在事件 f 之效用且 $0 \le u_q^f(t+1), u_q^f(t) \le 1$。依 Fishburn (1990)與 Quiggin (1991)之效用轉換觀念，茲將第 1 位談判者效用與其他談判者效用函數予以線性組合，轉換後效用函數如式(8)。

$$\begin{aligned} u_1^f(t+1) &= (1 - \sim_{2,1}(t) - \ldots - \sim_{k,1}(t) - \sim_{k+1,1}(t) - \ldots - \sim_{Q,1}(t))u_1^f(t) + \sim_{2,1}(t)u_2^f(t) + \Lambda \\ &\quad + \sim_{k,1}(t)u_k^f(t) + \sim_{k+1,1}(t)u_{k+1}^f(t) + \ldots + \sim_{Q,1}(t)u_Q^f(t) \\ &= (1 - \sum_{q=2}^{Q} \sim_{q,1}(t))u_1^f(t) + \sum_{q=2}^{Q} \sim_{q,1}(t)u_q^f(t) \end{aligned} \tag{8}$$

其中 $\sim_{q,1}(t)$ 表第 t 次討論時，第 q 位談判者效用影響第 1 位談判者效用；

$\sim_{1,q}(t)$ 表第 t 次討論時，第 1 位談判者效用影響第 q 位談判者效用；

$u_q^f(t)$ 表第 t 次討論時之第 q 位談判者效用，$\forall q = 2,\ldots,Q$。

同理，其他談判者效用函數可依式(8)觀念分別予以構建，就第 k 位談判者效用函數而言，此第 k 位談判者與其他談判者效用關係可如式(9)之一般式來表示。

$$u_k^f(t+1) = (1 - \sum_{q=1}^{Q} \sim_{q,k}(t))u_k^f(t) + \sum_{q=1}^{Q} \sim_{q,k}(t)u_q^f(t) \tag{9}$$

依式(8)而言，式(8)左邊之 $u_1^f(t+1)$ 會受 $\sum_{q=2}^{Q} \sim_{q,1}(t)$， $u_1^f(t)$ 及 $\sum_{q=2}^{Q} \sim_{q,1(t)} u_q^f(t)$ 值之影響而變動，此表示第 1 位談判者於第 t 次討論後，除受本身效用影響外亦受其他談判者之第 t 次討論時的效用，以及談判者之間的效用交互影響值(IUV)之和，而有所改變。依式(8)而言，若 $\sum_{q=2}^{Q} \sim_{q,1}(t) = 0$，則 $u_1^f(t+1) = u_1^f(t)$，此表第 1 位談判者於第 $t+1$ 次與第 t 次效用相等，此時第 1 位談判者效用不受其他談判者效用影響有所改變，故屬效用獨立現象。當 $\sum_{q=2}^{Q} \sim_{q,1}(t) \ne 0$ 時，表其他談判者與第 1 位談判者具相關性，若 $\sum_{q=2}^{Q} \sim_{q,1}(t) = 1$，表第 1 位談判者效用完全受其他談判者影響放棄其原來對事件 f 之效用衡量；當 $0 < \sum_{q=2}^{Q} \sim_{q,1}(t) < 1$ 時，式(8)即為第 1 位談判者與其他談判者效用相依模式。換言之，第 1 位談判者在與其他談判者效用互為影響下進行效用風險衡量。由此顯示，當 $\sum_{q=2}^{Q} \sim_{q,1}(t) \to 1$ 時，表談判者彼此之間效用相依程度越高；反之，當 $\sum_{q=2}^{Q} \sim_{q,1}(t) \to 0$，效用相依程度愈低，獨立越強。故談判者彼此之間的效用是否獨立取決於變數 $\sum_{q=1}^{Q} \sim_{q,k}(t)$ 及 $\sum_{q=1}^{Q} \sim_{q,k}(t)u_q^f(t)$ 是否為 0。

由圖 5 及式(8)觀念知，第 k 位談判者於第 $t+1$ 次討論時之效用與其他談判者之第 t 次討論時的效用存有線性關係, 此第 k 位談判者透過 IUV 值去影響其他談判者效用, 故此 IUV 值為第 k 位談判者於 $t+1$ 次討論時為該效用函數之內生變

數。又此談判群體有Q位談判者，故對談判群體之$GU^f(t+1)$而言，此$GU^f(t+1)$函數受此Q位談判者效用變動，即$GU^f(t+1)=U(u_1^f(t),u_2^f(t),\Lambda,u_Q^f(t))$。因此將式(8)及(9)觀念納入談判群體效用函數$GU^f(t+1)$中，因 IUV 值為個別談判者效用函數之變數，故此 IUV 值將為$GU^f(t+1)$函數之內生變數，即$GU^f(t+1)=U(u_1^f(\sum\sim_{q,1}(t)),\ u_2^f(\sum\sim_{q,2}(t)),\Lambda,u_k^f(\sum\sim_{q,k}(t)),\ \Lambda,u_Q^f(\sum\sim_{Q-1,Q}(t)))$。

由於此Q位談判者效用函數$u_k^f(t)\ \forall k=1,2,...,Q$滿足 NM 定理，故線性轉後之$GU^f(t+1)$仍滿足效用雙元偏好"$\phi$"特性(Fishburn, 1990)。另依 Bleichorodt 與 Quiggin (1997)之效用偏好分解定理定理(preference decomposition)觀念，其證明轉換後之談判群體效用值$GU^f(t+1)$可以用效用期望值來表示。故利用此二定理觀念，將個別談判者效用函數予以加總，即可得談判群體對事件f於第$t+1$次討論時之效用值$GU^f(t+1)$，如式(10)。

$$
\begin{aligned}
&GU^f(t+1)=U(u_1^f(t),u_2^f(t),\Lambda,u_Q^f(t))=\sum_{q=1}^{Q}u_q^f(t)\\
&=[(1-\sum_{q=1}^{Q}\sim_{q,1}(t))u_1^f(t)+\sum_{q=1}^{Q}\sim_{q,1}(t)u_q^f(t)]+[(1-\sum_{q=1}^{Q}\sim_{q,2}(t))u_2^f(t)+\sum_{q=1}^{Q}\sim_{q,2}(t)u_q^f(t)]+\Lambda+\\
&[(1-\sum_{q=1}^{Q}\sim_{q,\lambda}(t))u_\lambda^f(t)+\sum_{q=1}^{Q}\sim_{q,\lambda}(t)u_q^f(t)]+\Lambda+[(1-\sum_{q=1}^{Q}\sim_{q,Q}(t))u_Q^f(t)+\sum_{q=1}^{Q}\sim_{q,Q}(t)u_q^f(t)]\\
&=\sum_{q=1}^{Q}[(1-\sum_{k=1}^{Q}\sim_{q,k}(t))u_k^f(t)]+\sum_{q=1}^{Q}(\sum_{k=1}^{Q}\sim_{q,k}(t))u_q^f(t)\\
&\forall\ k=1,2,\Lambda,\lambda,\Lambda,Q;\ q=1,2,\Lambda,\lambda,\Lambda,Q.
\end{aligned}
\tag{10}
$$

因$u_k^f(t)$與$u_q^f(t)$為$GU^f(t+1)$函之變數且$GU^f(t+1)$與$u_k^f(t,t+1,t+2,\Lambda,t+n-1,\Lambda,T)$及$u_q^f(t,t+1,t+2,\Lambda,t+n-1,\Lambda,T)$存有遞迴關係，故屬動態數學規劃問題。在動態數學規劃問題中，此問題可採向前(Forward)或向後(Backward)方式來處理。本文採 Backward 方式處理此一遞迴關係，其方式乃是分別令$t=1,2,\Lambda\ T$，$k=1,2,\Lambda,q,\Lambda\ Q$分別代入式(7)中，如此可得此$Q$位談判者之第$t,t+1,t+2,\Lambda,t+n-1,\Lambda,T$次討論效用式子，並將此$Q$位談判者討論後效用式子代入式(10)中，再利用每個談判者效用式子反覆相互疊代，進行運算，由於計算繁複，茲整理如式(11)所示。

$$
\begin{aligned}
&GU^f(t+1)=\sum_{q=1}^{Q}[((1-\sum_{k=1}^{Q}\sim_{q,k}(t))^t u_k^f(1))+((1-\sum_{k=1}^{Q}\sim_{q,k}(t))^{t-1}\sum_{k=1}^{Q}\sim_{q,k}(t)u_q^f(1))\\
&+((1-\sum_{k=1}^{Q}\sim_{q,k}(t))^{t-1}\sum_{k=1}^{Q}\sim_{q,k}(t)u_k^f(1))+((1-\sum_{k=1}^{Q}\sim_{q,k}(t))^{t-3}\sum_{k=1}^{Q}\sim_{q,k}(t)u_q^f(1))\\
&+\sum_{k=1}^{Q}\sim_{q,k}(t)((1-\sum_{k=1}^{Q}\sim_{q,k}(t))^{t-3}u_k^f(1))+((1-\sum_{k=1}^{Q}\sim_{q,k}(t))^{t-3}(\sum_{k=1}^{Q}\sim_{q,k}(t))^{t-2}u_q^f(1))\\
&+((1-\sum_{k=1}^{Q}\sim_{q,k}(t))^{t-3}(\sum_{k=1}^{Q}\sim_{q,k}(t))^{t-2}u_k^f(1))+(\sum_{k=1}^{Q}\sim_{q,k}(t))^t u_q^f(1)]\\
&\forall\ k=1,2,\Lambda,\lambda,\Lambda,Q;\ q=1,2,\Lambda,\lambda,\Lambda,Q;
\end{aligned}
\tag{11}
$$

式(11)係經 Backward 方式運算後得談判群體於第$t+1$次之效用函數。若將$t=1$代入式(11)右邊之第 1 與 2 項，得$GU^f(2)=\sum_{k=1}^{Q}(u_k^f(1)+\sum_{q=1}^{Q}\sim_{q,k}(1)u_q^f(1))$；若$t=1$且$\sum_{k=1}^{Q}\sim_{q,k}(1)=0$時，式(11)成為$GU^f(2)=\sum_{q=1}^{Q}u_q^f(1)$，此時談判群體效用為個別談判者效用

之和；換言之，當談判者彼此之間處於效用獨立時，無論談判者彼此之間討論幾次，談判群體效用值可由談判者之最初效用值予以加總。此研究結果與 Bleichorodt 與 Quiggin (1997)之效用加總及 Luce 與 Fishburn[27] 之效用加法模式 (utility additive independence)觀念相同。而當 $t=1$ 且 $\sum_{q=1}^{Q}\sim_{q,k}(1)=1$ 時，式(11)成為 $GU^{f}(2)=\sum_{q=1}^{Q}\sum_{k=1}^{Q}\sim_{q,k}(1)u_{q}^{f}(1)$，此表示談判群體對事件 f 之效用係由談判者之間的 IUV 值與個別談判者效用之乘積加總，此屬於效用互動加權觀念。

由上述分析結果顯示，當談判者於討論 1 次即達共識時，談判者之間並無遞迴關係，此時會回到個別談判者效用之加權。因此，此談判群體效用值 $GU^{f}(t+1)$ 可由此 Q 位談判者於第 1 次討論加權效用而得。由此可知，若於內部討論於第 1 次討論時即達到收斂，則群體內部討論不會有擴散與遞迴現象。但是，當談判者彼此之間的討論達 $t\geq 3$ 且 $0<\sum_{k=1}^{Q}\sim_{q,k}(1)<1$ 時，談判群體對事件 f 之效用衡量會受個別談判者效用、談判者之間的 IUV 值及討論次數多寡而改變。因 $0<\sum_{k=1}^{Q}\sim_{q,k}(t)<1$，$0<u_{k}^{f}(t)<1$ 及 $0<u_{q}^{f}(t)<1$，故談判群體效用會隨著討論次數增加而遞減，談判者之間的 IUV 值亦會趨於穩定(詳附錄 B 推導)。當 $t=1$ 時且 $\sum_{k=1}^{Q}\sim_{q,k}(t)=1$ 及 $\sum_{k=1}^{Q}\sim_{q,k}(t)=0$ 時，此結果恰為馮正民與康照宗[3]之研究特例，因此，式(11)為其研究之擴充模式。

4.4 基本模式之建立

就式(3)與(4)而言，若能求得 $\sim_{2,1}(t)$ 及 $\sim_{1,2}(t)$ 值即可得第 1 位與第 2 位談判者之第 $t+1$ 次效用值，該 $\sim_{2,1}(t)$ 及 $\sim_{1,2}(t)$ 可由式(1)及(2)聯立求解獲得且此解屬 Cournot 的 Nash 均衡解(詳朱敬一,1990)。但當談判群體有 3 位談判者以上時，$\sim_{q,k}(t)$ 或 $\sim_{k,q}(t)$ 值無法由聯立求解方式獲得，須另研擬求解方法。由於談判者之間的關係是由 $\sim_{q,k}(t)$ 或 $\sim_{k,q}(t)$ 決定之，符合動態規劃問題，故本文以動態規劃觀念將問題模化如式(12)至(20)所示。

基本模式

$$\begin{aligned}
&Max\quad GU^{f}(t+1)=\sum_{q=1}^{Q}[((1-\sum_{k=1}^{Q}\sim_{q,k}(t))^{t}u_{k}^{f}(1))+((1-\sum_{k=1}^{Q}\sim_{q,k}(t))^{t-1}\sum_{k=1}^{Q}\sim_{q,k}(t)u_{q}^{f}(1))\\
&+((1-\sum_{k=1}^{Q}\sim_{q,k}(t))^{t-1}\sum_{k=1}^{Q}\sim_{q,k}(t)u_{k}^{f}(1))+((1-\sum_{k=1}^{Q}\sim_{q,k}(t))^{t-3}\sum_{k=1}^{Q}\sim_{q,k}(t)u_{q}^{f}(1))\\
&+\sum_{k=1}^{Q}\sim_{q,k}(t)((1-\sum_{k=1}^{Q}\sim_{q,k}(t))^{t-3}u_{k}^{f}(1))+((1-\sum_{k=1}^{Q}\sim_{q,k}(t))^{t-3}(\sum_{k=1}^{Q}\sim_{q,k}(t))^{t-2}u_{q}^{f}(1))\\
&+((1-\sum_{k=1}^{Q}\sim_{q,k}(t))^{t-3}(\sum_{k=1}^{Q}\sim_{q,k}(t))^{t-2}u_{k}^{f}(1))+(\sum_{k=1}^{Q}\sim_{q,k}(t))^{t}u_{q}^{f}(1)]
\end{aligned}\tag{12}$$

$$s.t.\quad u_{q}^{f}(t+1)=(1-\sum_{q=1}^{Q}\sim_{q,k}(t))u_{k}^{f}(t)+\sum_{q=1}^{Q}\sim_{q,k}(t)u_{q}^{f}(t)\qquad\forall q,k\in Q,q\neq k;t\in T\tag{13}$$

$$u_{k}^{f}(t+1)=(1-\sum_{q=1}^{Q}\sim_{q,k}(t))u_{k}^{f}(t)+\sum_{q=1}^{Q}\sim_{q,k}(t)u_{q}^{f}(t)\tag{14}$$

$$0\le u_{q}^{f}(t),u_{q}^{f}(t+1)\le 1,\forall q\in Q,t\in T;\tag{15}$$

$$0\le u_{k}^{f}(t),u_{k}^{f}(t+1)\le 1,\forall k\in Q,k\neq q;\ t\in T;\tag{16}$$

$$0\le\sum_{q=1}^{Q}\sim_{q,k}(t)\le 1,\ 0\le\sum_{k=1}^{Q}\sim_{q,k}(t)\le 1,\forall q,k\in Q,t\in T;\tag{17}$$

$$0\le\sum_{q=1}^{Q}\sum_{k=1,q\neq k}^{Q}\sim_{q,k}(t)\le 1,\forall q,k\in Q,\ t\in T;\tag{18}$$

$0\le\sim_{q,k}(t)\le 1,0\le\sim_{k,q}(t)\le 1,\forall\ q,k\in Q;q\neq k,t\in T$ **，決策變數；** (19)

$0\le u_{q}^{f}(0),0\le GU^{f}(t+1)\le 1\le 1$**，個別談判者之效用函數；已知。** (20)

其中， $GU^{f}(t+1)$ **：談判群體在第** $t+1$ **次討論後之事件** f **效用；**

$u_{q}^{f}(t+1)$ **：談判者** q **於第** $t+1$ **次討論之事件** f **效用；**

$u_{q}^{f}(t)$ **：談判者** q **於第** t **次討論之事件** f **效用；**

$u_{q}^{f}(t)$ **：談判者** q **於第** t **次討論之事件** f **效用；**

$u_{k}^{f}(t)$ **：談判者** k **於第** t **次討論之事件** f **效用；**

$u_{k}^{f}(t+1)$ **：談判者** k **於第** $t+1$ **次討論之事件** f **效用；**

$u_{q}^{f}(1)$ **：談判者** q **於第** 1 **次討論之事件** f **效用；**

$u_{k}^{f}(1)$ **：談判者** k **於第** 1 **次討論之事件** f **效用；**

$\sum_{q=1}^{Q}\sim_{q,k}(t)$ **：談判者** q **影響其他談判者於第** t **次討論之效用影響和，且介於** 0~1 **之間的實數，** $\forall q,k\in Q;\ k\neq q$ **；**

$\sum_{k=1}^{Q}\sim_{q,k}(t)$ **：談判者** k **受到其他談判者於第** t **次討論之效用影響和，且介於** 0~1 **之間的實數，** $\forall q,k\in Q;\ k\neq q$ **；**

$\sim_{k,q}(t)$：談判者 k 於第 t 次討論影響談判者 q 之效用變化，其值介於 0~1 之間的實數，$\forall\, q,k \in Q;\ k \neq q$；

$\sim_{q,k}(t)$：談判者 q 於第 t 次討論影響談判者 k 之效用變化，其值介於 0~1 之間的實數，$\forall\, q,k \in Q;\ k \neq q$；

t：談判群體討論事件 f 之次數，$t \in \{0,1,2,\Lambda\ ,T\}$。

按本文風險及式(1)定義觀念，談判者對事件 f 之風險衡量可由效用來衡量，當效用期望高時，談判者效用滿足水準隨之增加，風險降低。另依經濟理論，假設談判者(或決策者)是在理性狀況下對事件 f 進行風險衡量，故屬追求風險最小行為，即談判者在追求效用最大化。由於談判群體係由數個談判者所組成，故談判群體亦在追求效用最大化，風險最小行為，如此談判群體對事件 f 在第 $t+1$ 次之風險衡量可由群體效用水準來衡量，即以式(12)之目標式來表示。

由於基本問題之目標式多寡會受到其他事件發生與否有關，若事件 f 與其他事件發生不獨立，則目標式會產生多目標形態或數個單目標規劃模式形態出現；此外，目標式亦會受到公與私部門特許契約談判之影響，該目標式亦有可能以兩階段方式出現，形成二階規劃問題(Bi-level programming)，若將特許契約談判與多個事件同時考慮在內，則即有可能產生二階多目標規劃問題。由於本文研究課題係著重於 BOT 特許公司談判群體內部之討論行為，由談判者內部討論如何決定風險事件，其目的在於構建討論模式及研擬求解法，故為簡化分析起見，乃先考量單一事件下，談判者彼此之間的討論行為，茲假設事件 f 與其他事件具獨立關係，故基本模式之目標式採單一目標式形態。

在限制式中，式(13)與(14)分別表示第 q、k 位談判者於第 $t+1$ 次討論時之效用函數，此效用函數與其他談判者於第 t 次討論時之效用變化有關，且第 q、k 位談判者於第 $t+1$ 次討論之效用會影響下一次討論其他談判者效用，其意義在表現討論時效用之雙向性，而且目標式(12)之談判群體效用亦會受式(13)與(14)之影響。另限制式之(15)與(16)表示所有談判者之效用須滿足 0~1 條件，此在使談判者效用不為負；限制式(17)與(18)在使談判者之間的 IUV 值之總和須滿足 0~1 條件，此在確保談判者效用值不為負且不超過 1；此外，式(17)與(18)滿足 0~1 條件，使得式(20)之談判群體效用值 $GU^{f}(t+1)$ 符合 0　1 條件；而式(19)之 $\sim_{k,q}(t)$ 及 $\sim_{q,k}(t)$ 表談判者之間的效用交互影響值，其值界於 0 與 1 之間，為決策變數。

五、模式求解

由式(12)至(20)而言，決策變數為 $\sim_{k,q}(t)$，$\sim_{q,k}(t)$。在求解方面，若能得 $\sim_{k,q}(t)$ 與 $\sim_{q,k}(t)$，則 $GU^{f}(t+1)$ 與討論變數 t 皆可得求得。惟考量基本模式之遞迴與雙向性，本文研擬疊代求解法(*iterative algorithm*)，求解步驟說明如下。

步驟 1：設定效用起始值

令 $u_{q}^{f}(0)$ 與 $u_{k}^{f}(0)$ 分別為談判者 q 與 k 之討論前的效用，$u_{q}^{f}(1)$ 與 $u_{k}^{f}(1)$ 為談判者 q 與 k 之第 1 次討論效用，$q = 1,2,\Lambda\ ,k,k+1,\Lambda\ Q$。

步驟 2：求取效用交互影響起始值

假設談判群體在進行討論時，某主要談判者先行發言，如此可得以主要談

判者為中心之效用影響起始值，再計算其他談判者之間的效用影響起始值。換言之，以某人先發言為中心去求任兩個談判者之效用交互影響；計算過程如下。

利用步驟 1 之 $u_q^f(0)$ 與 $u_k^f(0)$ 及 $u_q^f(1)$ 與 $u_k^f(1)$ 值，代入附錄 A 之式(A-4)及(A-5)中，求談判者之間的效用交互影響起始值，如式(21)。為使 $\sim_{q,k}(t+1)$ 與 $\sim_{k,q}(t+1)$ 符合非負條件，將式(21)取絕對值。

$$\sim_{q,k}(t+1)=\left|\frac{u_k^f(t+1)-u_k^f(t)}{(u_q^f(t+1)-u_k^f(t))}\right|\text{，}\sim_{k,q}(t+1)=(1-\sim_{q,k}(t+1))\left|\frac{u_{k+1}^f(t)-u_q^f(t+1)}{u_k^f(t+1)-u_q^f(t)}\right| \tag{21}$$

若式(21)之 $u_k^f(t+1)=u_k^f(t)$ 或 $u_q^f(t)=u_k^f(t+1)$ 時，則令 $u_k^f(t+1)-u_k^f(t)=\pounds$ 或 $u_q^f(t)-u_k^f(t+1)=\pounds$，$\pounds=1\times 10E-06$。

步驟 3：效用交互影響值之正規化(normalization)

步驟 3 主要是考量談判者於討論過程中，產生具有強烈影響者(稱之堅持己見者)與完全受他人意見影響者(稱之無己見者)之情形，如此將使模式不易收斂。因此，將步驟 2 之 $\sim_{k,q}(t+1)$ 與 $\sim_{q,k}(t)$ 予以正規化(normalization)，正規化步驟如式(22)。

$$\sim_{k,q}^{adj}(t+1)=\frac{\sim_{k,q}(t+1)-\sim_{\min}(t+1)}{\sim_{\max}(t+1)-\sim_{\min}(t+1)}\text{，}\sim_{q,k}^{adj}(t+1)=\frac{\sim_{q,k}(t+1)-\sim_{\min}(t+1)}{\sim_{\max}(t+1)-\sim_{\min}(t+1)} \tag{22}$$

其中，$\sim_{\max}(t+1)=\max\{\sim_{k,q}(t+1),\sim_{q,k}(t+1)\}$；

$\sim_{\min}(t+1)=\min\{\sim_{k,q}(t+1),\sim_{q,k}(t+1)\}$。

若 $\sim_{\max}(t+1)=\sim_{\min}(t+1)$ 則 $\sim_{k,q}^{adj}(t+1)=0$，$\sim_{q,k}^{adj}(t+1)=0$

步驟 4：求解討論後之效用交互影響值

將步驟 3 正規化後之 $\sim_{k,q}^{adj}(t+1)$，$\sim_{q,k}^{adj}(t+1)$ 代入基本模式中，得討論後之談判者效用值，$u_q^{f*}(t+1), \forall q\in Q$。由於 $\sim_{k,q}^{adj}(t+1)$ 與 $\sim_{q,k}^{adj}(t+1)$ 為正規後之值，非經過討論後之效用交互影響值，故再依步驟 2 求討論後之 $\sim_{k,q}^{*}(t+1)$ 與 $\sim_{q,k}^{*}(t+1)$。

步驟 5：判定效用交互影響值是否收斂

由步驟 1 至步驟 4 所求之 $u_q^{f*}(t+1)$、$\sim_{k,q}^{*}(t+1)$ 與 $\sim_{q,k}^{*}(t+1)$ 須檢測是否收斂，此收斂條件可如附錄 B 之計算。依附錄 B 顯示，當 $\sim_{k,q}^{*}(t+1)=\sim_{q,k}^{*}(t+1)$ 或 $\sum_{q,k\neq q}\sim_{q,k}^{*}(t+1)=\sum_{k,k\neq q}\sim_{k,q}^{*}(t+1)$ 時，其滿足模式收斂條件。其意義表示談判者彼此之間看法趨於一致。因此，$\sim_{k,q}^{*}(t+1)=\sim_{q,k}^{*}(t+1)=0$ 或 $\sim_{k,q}^{*}(t+1)=\sim_{q,k}^{*}(t+1)=1$ 皆為模式收斂條件情形之一。當模式滿足收斂條件，則跳至步驟 9 求談判群體效用值；若無法收斂，則進行步驟 6，修正效用交互影響值，並開始進行下次討論。

步驟 6：修正效用交互影響值

當模式無法收斂時，則修正談判者之間的效用交互影響值，其步驟如下。當 $|\sim_{k,q}^{*}(t+1)-\sim_{q,k}^{*}(t+1)|\rhd E$ 時，令 $\sim_{k,q}^{*}(t+1)=\sim_{k,q}^{*}(t)-E$；$|\sim_{q,k}^{*}(t+1)-\sim_{k,q}^{*}(t+1)|\rhd E$ 時，令 $\sim_{q,k}^{*}(t+1)=\sim_{q,k}^{*}(t)-E$，$E$ 為所設定之容忍誤差值。

步驟 7：修正個別效用值

將步驟 6 所得之修正後效用交互影響值 $\sim_{k,q}^{*}(t+1)$ 與 $\sim_{q,k}^{*}(t+1)$ 代入基本模式。

步驟 8：重複執行步驟 4 至步驟 7，直到模式收斂或討論上限次數完畢為止。

步驟 9：計算個別談判者與談判群體之效用值

當模式符合收斂條件，則可得 $\sim^{*}_{q,k}(t)$ 與 $\sim^{*}_{k,q}(t)$ 值及第 t 次討論後之談判者 $u_k^f(t)$ 與談判群體 $GU^f(t)$ 值。

六、範例分析

本節以範例方式說明談判者彼此之間對某事件進行討論，範例中之相關資料採自馮正民與康照宗[2]及馮正民與鍾啟椿[6]之研究。

6.1　用地取得事件背景說明

假設某 BOT 特許公司獲得某一交通建設之特許契約談判權，特許公司之談判群體有六個談判者或專家，由此六位談判者針對用地取得事件進行風險衡量，事件之相關資料除引用相關研究資料外，事件之談判者效用值、屬性產出機率值則採假設性資料，至於用地取得事件特性說明如下。

就特許公司而言，政府若無法於期限內取得交通建設之路線及場站用地，則無法如期開工竣工，如此將導致 BOT 計畫完工延滯與延遲通車，產生完工風險及延遲通車風險。假設用地取得事件之延遲徵收年數分別為 0，1，2，...，10 年計 10 個狀態(y)，屬性產出為延遲年數所增加之建造成本((bc), build cost)，談判者之屬性效用值為 $u(bc)$，事件狀態發生機率值為 $p(y)$， $u(L)=u(bc)\times p(y)$， $u(L)$ 為談判者對事件於屬性與狀態下之效用值，以上 (bc)， $u(bc)$， $p(y)$ 皆對應 10 個狀態 y，故皆有 10 個數值。

6.2　範例之求解

在未討論前，談判者各自對用地取得事件進行效用衡量，此效用衡量可將所給定之 (bc)、 $u(bc)$ 及 $p(y)$ 值代入式(1)中，並利用式(2)進行效用正規化，即得此 6 位談判者之 $u(L)$ 值，結果如表 2。現在此 6 位談判者就用地取得事件進行討論，故利用基本模式與求解法，配合表 2 資料，並以 Turbo Pascal 7.0 軟體撰寫模擬程式，求討論後談判者彼此之間的效用交互影響值(IUV)，個別談判者效用值，其演算步驟如下。

表 2　未討論前，談判者對用地取得事件之效用衡量值

事件	狀態 (延遲徵收年)										
效用	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
第 1 位	0.2009	0.0241	0.0385	0.0419	0.0318	0.0107	0.0057	0.0004	0.0000	0.0000	0.0000
第 2 位	0.8964	0.7704	0.0615	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
第 3 位	0.5870	0.5137	0.0615	0.0008	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
第 4 位	0.0012	0.0048	0.0066	0.0097	0.0034	0.0020	0.0008	0.0001	0.0000	0.0000	0.0000

第 5 位	0.0460	0.0200	0.0156	0.0319	0.0144	0.0025	0.0000	0.0000	0.0000	0.0000	0.0000
第 6 位	0.2279	0.2275	0.2958	0.1505	0.0033	0.0099	0.0000	0.0000	0.0000	0.0000	0.0000

資料來源：馮正民、康照宗[2]

步驟 1：藉由表 2 之延遲徵收第 6 年效用值，各談判者效用依序為 0.0057、0、0、0.0008、0 **及** 0，**將此 6 位談判者效用值加上微小值** e，$e = 0.00001$ ，**得此 6 位談判者效用起始值，分別為** 0.00571、0.00001、0.00001、0.00081、0.00001 **及** 0.00001。

步驟 2：將步驟 1 之效用起始值代入式(21)**中，得第 1 次討論之效用交互影響起始值** $\sim_{q,k}(1)$，**如表 3 所示。就表 3 之** $\sim_{1,2}(1) = 0.00175$ **而言，其意義表示，在第 1 次討論過程中，第 1 位談判者效用影響第 2 位談判者效用為** 0.00175，**其餘表 3 中之效用交互影響值意義依此類推。**

表 3　延遲徵收 6 年之效用交互影響起始值

談判者	第 1 位	第 2 位	第 3 位	第 4 位	第 5 位	第 6 位
第 1 位	-	0.00175	0.00175	0.00204	0.00175	0.00175
第 2 位	0.00175	-	0.90000	0.09875	0.01111	0.90000
第 3 位	0.00175	0.10000	-	0.90000	0.90000	0.90000
第 4 位	0.00204	0.01250	0.01250	-	0.00000	0.00000
第 5 位	0.00175	0.10000	0.10000	0.01250	-	0.90000
第 6 位	0.00175	0.10000	0.10000	0.01250	0.10000	-

步驟 3：由表 3 資料顯示，部份談判者之間的 IUV **值差距不大，或呈現相等現象，如** $\sim_{1,2}(1)$ **與** $\sim_{2,1}(1)$ **相等，但亦有** IUV **值差距甚大者，如** $\sim_{2,3}(1) = 0.9$ **與** $\sim_{3,2}(1) = 0.1$，**此** IUV **值差距甚大反應談判者彼此之間效用互動影響甚為明顯，故在內部討論過程中會產生不同談判者對自己效用衡量有堅持己見與看法迴異現象。因此，本文再利用表 3 之** IUV **值利用式**(22)**予以正規化。**

步驟 4：將步驟 3 正規後之效用交互影響值，代入基本模式中進行模式求解，得第 1 次討論後 6 位談判者效用值，分別為 $u_1^*(6,1) = 0.00571$，$u_2^*(6,1) = 0.000062$，$u_3^*(6,1) = 0.000062$，$u_4^*(6,1) = 0$，$u_5^*(6,1) = 0.0000621$，**及** $u_6^*(6,1) = 0.0000621$。**此 6 位談判者效用值係第 1 次討論後之效用值，與步驟 1 之效用值微調不同。因此，再將此 6 位談判者效用值代入式**(21)**中，得第 1 次討論後之** $\sim_{q,k}^*(1)$，**結果如表** 4。

表 4　第 1 次討論後之效用交互影響值

談判者	第 1 位	第 2 位	第 3 位	第 4 位	第 5 位	第 6 位	小計
第 1 位	-	0.00000	0.00001	0.00000	0.00000	0.00002	0.00004
第 2 位	0.00000	-	0.00000	0.00000	0.00001	0.00000	0.00001
第 3 位	0.00000	0.00001	-	0.00000	0.00010	0.00000	0.00011
第 4 位	0.00000	0.00000	0.00000	-	0.00000	0.00000	0.00000
第 5 位	0.00000	0.00000	0.00000	0.00000	-	0.00022	0.00022

第 6 位	0.00004	0.00000	0.00010	0.00000	0.00010	-	0.00024
小計	0.00004	0.00001	0.00011	0.00000	0.00021	0.00024	0.00062

步驟 5**：此步驟則是利用步驟** 4 **所得到之** $u_1^*(6,1), u_2^*(6,1), \Lambda, u_6^*(6,1)$ **與** $\sim^*_{q,k}(1)$ **值，進一步求** $\sum_{q,k\neq q}\sim^*_{q,k}(1)$ **與** $\sum_{k,k\neq q}\sim^*_{k,q}(1)$ **，意義是在檢核模式求解是否符合收斂條件。換言之，藉由此步驟來檢測談判者之間是否達到討論之「共識」。**

此步驟本文利用附錄 B **所推導數學式來檢測模式收斂條件，此一分析結果如表** 4 **資料所示。依表** 4 **中之** IUV **值可計算此** 6 **為談判者之間** IUV **值之和，分別為** 0.00004、0.00001、0.00011、0.00000、0.00021 **及** 0.00024 **，亦即** $\sum_{q,1}\sim^*_{q,1}(1) = \sum_{1,q}\sim^*_{1,q}(1) = 0.00004$ **，** $\sum_{q,2}\sim^*_{q,2}(1) = \sum_{2,q}\sim^*_{2,q}(1) = 0.00001$ **，** $\sum_{q,3}\sim^*_{q,3}(1) = \sum_{3,q}\sim^*_{3,q}(1) = 0.00011$ **，** $\sum_{q,4}\sim^*_{q,4}(1) = \sum_{4,q}\sim^*_{4,q}(1) = 0.00000$ **，** $\sum_{q,5}\sim^*_{q,5}(1) = \sum_{5,q}\sim^*_{5,q}(1) = 0.00021$ **，以及** $\sum_{q,6}\sim^*_{q,6}(1) = \sum_{6,q}\sim^*_{6,q}(1) = 0.00024$ **；由** IUV **值之和呈現由某位談判者影響其他談判者之效用，與由其他談判者去影響某位談判者效用呈現相等現象，此結果符合** $\sum_{q,k\neq q}\sim^*_{q,k}(1) = \sum_{k,k\neq q}\sim^*_{k,q}(1)$ **模式收斂條件。其意義說明此** 6 **位談判者在經過第** 1 **次討論後，談判者彼此之間效用影響處於相等狀態。此外，除表** 4 **中之** IUV **值介於** 0~1 **外，**$\sum_{q,1}\sim^*_{q,1}(1)$**、**$\sum_{1,q}\sim^*_{1,q}(1)$**、**$\sum_{q,2}\sim^*_{q,2}(1)$**、**$\sum_{2,q}\sim^*_{2,q}(1)$**、**$\sum_{q,3}\sim^*_{q,3}(1)$**、**$\sum_{3,q}\sim^*_{3,q}(1)$**、**$\sum_{q,4}\sim^*_{q,4}(1)$**、**$\sum_{4,q}\sim^*_{4,q}(1)$**、**$\sum_{q,5}\sim^*_{q,5}(1)$**、**$\sum_{5,q}\sim^*_{5,q}(1)$**、**$\sum_{q,6}\sim^*_{q,6}(1)$ **及** $\sum_{6,q}\sim^*_{6,q}(1)$ **值介於** 0~1**，符合收斂條件。另由步驟** 4 **所得之** $u_1^*(6,1)$ **，** $u_2^*(6,1)$ **，** Λ **，** $u_6^*(6,1)$ **亦符合** 0~1 **條件。**

由步驟 5 **檢測結果顯示，用地取得事件之延遲徵收第六年，在經過此** 6 **位談判者第** 1 **次討論後，得** $u_1^*(6,1), u_2^*(6,1), \Lambda, u_6^*(6,1)$ **、** $\sim^*_{q,k}(1)$ **、** $\sum_{q,k\neq q}\sim^*_{q,k}(1)$ **、** $\sum_{k,k\neq q}\sim^*_{k,q}(1)$ **皆符合模式求解條件，故得談判群體效用值** $GU(6,1)$ **，** $GU(6,1) =$ 0.006018**，此值亦介於** 0~1**，亦符合模式收斂條件。**

由於模式達收斂條件，表示此 6 **位談判者在經過第** 1 **次討論後，彼此之間的效用影響趨於相等或者** IUV **值之和相等，談判者彼此之間的效用交互影響值達到收斂狀態。此意義表示，此** 6 **位談判者於第** 1 **次討論後即對延遲徵收第** 6 **年之狀態風險衡量取得討論之「共識」，因此，此** 6 **位談判者無須再就此狀態進行討論。而此** $GU(6,1)$ **值即可作為該談判群體對事件** f **對延遲徵收第** 6 **年狀態下之效用衡量，此** $GU(6,1)$ **值表示，談判群體內部在經過** 1 **次討論後，即對延遲收第** 6 **年狀態取得最後之風險衡量。**

另外本文另以該事件之延遲徵收第 10 **年為例，繼續說明本文模式與求解法之操作性。**

由表 3 **之延遲徵收第** 10 **年資料，此** 6 **位談判者之最初效用值皆為** 0**，故利用依步驟** 1 **與步驟** 2 **計算，得效用交互影響起始值**(IUV **值**)**皆為** 0**，如表** 5 **所示；**

而此 IUV 起始值皆為 0，表示此 6 位談判者在討論之前，談判者彼此之間並無效用互動現象。而後，將表 5 中之值依步驟 3 予以正規化，然由於 IUV 起始值皆為 0，故正規化後之 IUV 值亦為 0。換言之，在討論時，此 6 位談判者並無出現強烈影響者與無己見者之情形。因此，將此正規化後之 IUV 值與表 2 中之效用值代入基本模式中進行求解，得討論後之 IUV 值，如表 6 所示。

表 5　延遲徵收第 10 年之效用交互影響起始值

談判者	第 1 位	第 2 位	第 3 位	第 4 位	第 5 位	第 6 位
第 1 位	-	0.00000	0.00000	0.00000	0.00000	0.00000
第 2 位	0.00000	-	0.00000	0.00000	0.00000	0.00000
第 3 位	0.00000	0.00000	-	0.00000	0.00000	0.00000
第 4 位	0.00000	0.00000	0.00000	-	0.00000	0.00000
第 5 位	0.00000	0.00000	0.00000	0.00000	-	0.00000
第 6 位	0.00000	0.00000	0.00000	0.00000	0.00000	-

表 6　延遲徵收第 10 年收斂後之效用交互影響值

談判者	第 1 位	第 2 位	第 3 位	第 4 位	第 5 位	第 6 位	小計
第 1 位	-	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
第 2 位	0.00000	-	0.00000	0.00000	0.00000	0.00000	0.00000
第 3 位	0.00000	0.00000	-	0.00000	0.00000	0.00000	0.00000
第 4 位	0.00000	0.00000	0.00000	-	0.00000	0.00000	0.00000
第 5 位	0.00000	0.00000	0.00000	0.00000	-	0.00000	0.00000
第 6 位	0.00000	0.00000	0.00000	0.00000	0.00000	-	0.00000
小計	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

由表 6 之討論後 IUV 值顯示其值皆為 0，其意義說明，就延遲徵收第 10 年狀態而言，此 6 位談判者在經過第 1 次討論後，談判者彼此之間的效用並無有所改變，亦無產生效用互動或交互影響現象。換言之，此 6 位談判者在內部第 1 次討論時，彼此之間對此狀態之效用衡量已經取得效用「共識」之衡量結果。此外，由於 IUV 值皆為 0，故符合介於 0~1 條件且該 IUV 值之和亦為 0，亦符合模式收斂條件；由於表 6 中之行列 IUV 之和皆為 0 且相等，故模式初步已達收斂狀態，因此，由基本模式可得討論後之談判者效用值，分別為 $u_1^*(10,1)=0$，$u_2^*(10,1)=0$，$u_3^*(10,1)=0$，$u_4^*(10,1)=0$，$u_5^*(10,1)=0$ 及 $u_6^*(10,1)=0$，此討論後效用皆為 0，與討論前之效用值沒有增減。而此 6 位談判者之 $u_q^*(10,1)$ 亦符合 0~1 條件，因此可計算 $GU(10,1)$ 值，得 $GU(10,1)=0$，此 $GU(10,1)$ 亦介於 0~1 之間，符合模式收斂條件，故此 6 位談判者於第 1 次討論後，對此狀態之效用衡量即達到共識衡量解果。換言之，談判群體在內部經過 1 討論後達到對延遲徵收第 10 年狀態之效用衡量，其結果為 0，此值與討論前此 6 位談判者之效用衡量沒有改變。

而由表 6 討論後 IUV 值顯示，討論後之 IUV 值皆為 0，此顯示此 6 位談判者的效用，彼此之間不受其他談判者效用影響而有所改變。因此，可得一個特

性，當談判者之最初效用為0且效用改變亦相當微小時，談判者彼此之間的討論行為，其效用交互影響與獨立狀態相同，如此，談判群體效用值可由此6位談判者之最初效用值予以加總，此種特性與效用加總模式相同，而此一結果驗證本文基本模式之推論。

經由前述範例分析顯示，本文基本模式與求解方法可以詮釋談判者彼此之間的討論行為。因此，本文利用此一模式與求解法繼續分析該事件之其他狀態下的效用衡量。為利於分析，本文延續基本模式假設條件，假設事件狀態發生獨立及談判者討論次數50次為上限，換言之，求解模擬次數為50次。本文利用表2資料之效用值進行談判者之間討論模擬。茲將表2之資料代入基本模式與求解法，求事件之每一狀態下的討論後IUV值，結果如表6。

就延遲徵收之第0年與1年狀態而言，談判群體內部雖經50次討論，惟討論後之IUV值之和大於1，此IUV值之和大於1會使得討論後之個別談判者效用值出現負的現象，但由於效用須不為負，故此IUV值不符合模式收斂條件，故無討論後之IUV值；而此一結果涵意說明，在此6位談判者在討論50次後，談判者在討論過程中無法取得「共識」效用值，其原因說明，某位談判者對其他談判者之效用影響過鉅，亦即出現IUV值之和大於1現象，而此現象說明，在討論過程中出現有堅持己見者與完全無己見者的現象，如果有此種現象者，則在有限討論次數下，並無法產生最後取得大家妥協之效用衡量值。

其次，就延遲徵收第4、8、9及第10年而言，由於此6位談判者之最初效用值皆為0，故討論後之IUV值亦為0，此顯示此6位談判者對這些狀態進行討論時，並未產生效用互動現象，故屬效用獨立狀態。而就延遲徵收第2、3、5及第6年而言，IUV值介於0~0.00054，此IUV值部份為0，部份不為0的IUV值亦趨向微小之特性。此討論後之IUV值顯示此6位談判者在討論過程中，效用互動影響並不明顯。而在延遲徵收第2年狀態時，則以第2、3、5及第6位之間的效用互動最為明顯，其中又以第2、3及第6位三者之間的效用互動影響較其他談判者更為明顯，而第6位談判者影響第2位談判者效用之0.00054最高。至於延遲徵收第3、4及第5年狀態，談判者效用影響值介於0~0.00005之間，彼此效用互動影響程度不大。

表7　收斂後之談判者效用交互影響值

狀態	延遲徵 收第2年						
談判者	第1位	第2位	第3位	第4位	第5位	第6位	小計
第1位	-	0.00000	0.00000	0.00004	0.00003	0.00004	0.00011
第2位	0.00000	-	0.00002	0.00002	0.00002	0.00050	0.00056
第3位	0.00000	0.00002	-	0.00000	0.00002	0.00049	0.00052
第4位	0.00000	0.00000	0.00000	-	0.00010	0.00010	0.00020
第5位	0.00007	0.00000	0.00000	0.00014	-	0.00032	0.00053
第6位	0.00004	0.00054	0.00050	0.00001	0.00036	-	0.00145
小計	0.00011	0.00056	0.00052	0.00020	0.00053	0.00145	0.00336
	延遲徵 收第3年						
第1位	-	0.00001	0.00000	0.00000	0.00000	0.00001	0.00002
第2位	0.00001	-	0.00000	0.00002	0.00000	0.00000	0.00003

第 3 位	0.00000	0.00000	-	0.00000	0.00000	0.00000	0.00000
第 4 位	0.00000	0.00000	0.00000	-	0.00003	0.00000	0.00003
第 5 位	0.00000	0.00002	0.00000	0.00001	-	0.00000	0.00003
第 6 位	0.00001	0.00000	0.00000	0.00000	0.00000	-	0.00001
小計	0.00002	0.00003	0.00000	0.00003	0.00003	0.00001	0.00012
	延遲徵 收第 4 年						
第 1 位	-	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
第 2 位	0.00000	-	0.00000	0.00000	0.00000	0.00000	0.00000
第 3 位	0.00000	0.00000	-	0.00000	0.00000	0.00000	0.00000
第 4 位	0.00000	0.00000	0.00000	-	0.00000	0.00000	0.00000
第 5 位	0.00000	0.00000	0.00000	0.00000	-	0.00000	0.00000
第 6 位	0.00000	0.00000	0.00000	0.00000	0.00000	-	0.00000
小計	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	延遲徵 收第 5 年						
第 1 位	-	0.00000	0.00000	0.00000	0.00000	0.00002	0.00002
第 2 位	0.00000	-	0.00000	0.00000	0.00001	0.00003	0.00004
第 3 位	0.00000	0.00001	-	0.00000	0.00000	0.00003	0.00004
第 4 位	0.00000	0.00000	0.00004	-	0.00001	0.00000	0.00005
第 5 位	0.00000	0.00000	0.00000	0.00003	-	0.00000	0.00003
第 6 位	0.00002	0.00003	0.00000	0.00002	0.00001	-	0.00008
小計	0.00002	0.00004	0.00004	0.00005	0.00003	0.00008	0.00026
	延遲徵 收第 6 年						
第 1 位	-	0.00000	0.00001	0.00000	0.00000	0.00002	0.00004
第 2 位	0.00000	-	0.00000	0.00000	0.00001	0.00000	0.00001
第 3 位	0.00000	0.00001	-	0.00000	0.00010	0.00000	0.00011
第 4 位	0.00000	0.00000	0.00000	-	0	0	0.00000
第 5 位	0.00000	0.00000	0.00000	0.00000	-	0.00022	0.00022
第 6 位	0.00004	0.00000	0.00010	0.00000	0.00010	-	0.00024
小計	0.00004	0.00001	0.00011	0.00000	0.00021	0.00024	0.00062
	延遲徵 收第 7 年						
第 1 位	-	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
第 2 位	0.00000	-	0.00000	0.00000	0.00000	0.00000	0.00000
第 3 位	0.00000	0.00000	-	0.00000	0.00000	0.00000	0.00000
第 4 位	0.00000	0.00000	0.00000	-	0.00000	0.00000	0.00000
第 5 位	0.00000	0.00000	0.00000	0.00000	-	0.00000	0.00000
第 6 位	0.00000	0.00000	0.00000	0.00000	0.00000	-	0.00000
小計	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	-
	延遲徵 收第 8 年						
第 1 位	-	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
第 2 位	0.00000	-	0.00000	0.00000	0.00000	0.00000	0.00000

第 3 位	0.00000	0.00000	-	0.00000	0.00000	0.00000	0.00000
第 4 位	0.00000	0.00000	0.00000	-	0.00000	0.00000	0.00000
第 5 位	0.00000	0.00000	0.00000	0.00000	-	0.00000	0.00000
第 6 位	0.00000	0.00000	0.00000	0.00000	0.00000	-	0.00000
小計	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	-
狀態	延遲徵 收第 9 年						
第 1 位	-	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
第 2 位	0.00000	-	0.00000	0.00000	0.00000	0.00000	0.00000
第 3 位	0.00000	0.00000	-	0.00000	0.00000	0.00000	0.00000
第 4 位	0.00000	0.00000	0.00000	-	0.00000	0.00000	0.00000
第 5 位	0.00000	0.00000	0.00000	0.00000	-	0.00000	0.00000
第 6 位	0.00000	0.00000	0.00000	0.00000	0.00000	-	0.00000
小計	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	-
狀態	延遲徵收 第 10 年						
第 1 位	-	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
第 2 位	0.00000	-	0.00000	0.00000	0.00000	0.00000	0.00000
第 3 位	0.00000	0.00000	-	0.00000	0.00000	0.00000	0.00000
第 4 位	0.00000	0.00000	0.00000	-	0.00000	0.00000	0.00000
第 5 位	0.00000	0.00000	0.00000	0.00000	-	0.00000	0.00000
第 6 位	0.00000	0.00000	0.00000	0.00000	0.00000	-	0.00000
小計	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	-

將事件之所有狀態經過模擬 50 次(即討論 50 次)後, 各狀態之討論次數分別為 50、50、11、5、5、6、1、1、1、1 及 1，結果如表 8 所示。以延遲徵收第 6 年至第 10 年而言，其經過 1 次討論後即獲得收斂解，得到 IUV 值；此結果說明，此 6 位談判者在第 1 次討論後即對該狀態求得談判群體效用值。此外求得事件之各狀態的談判群體效用值，分別為 1.9427、1.3332、0.6639、0.2262、0.0359、0.018836、0.006018、0.000432、0、0 及 0；由於談判群體效用值須介於 0~1, 故 1.9427 與 1.3332 不符模式限制條件, 其餘談判群體效用值皆小於 1。延遲徵收第 0 年與第 1 年，其雖經模擬 50 次仍無法收斂，無法得到效用交互影響值之收斂解，此種現象可說明，此 6 位談判者對於延遲徵收第 0 年與第 1 年仍存有相當大之效用衡量差距。就個別談判者效用變化而言，第 1、第 2 與第 3 位談判者之效用值，其討論前後效用值變化不大，但第 4、5 與第 6 位之效用值變化較大；此一結果顯示，在討論過程中，第 4、5 與第 6 位談判者效用受其他談判者效用影響很大，改變其原來效用衡量。

在談判群體效用變化方面, 討論前之 GU 值係利用 Keeney 與 Raiffa 之 MAU 模式加法型求得(馮正民與康照宗[2])，討論後之 GU 值係依本文所研擬之動態規劃模式求得。比較討論前後 GU 值變化情形，除延遲徵收第 8、9 與第 10 年之 GU 值並無增減外，其餘狀態之討論前後 GU 皆有明顯變化，尤其討論後之 GU 值明顯較討論前 GU 大幅度增加，其增加原因乃是因為討論後所得到之 $\sim^{*}_{q,k}(t)$ 十分微小，故造成討論後，第 1、2 與第 3 位談判者效用值無明顯遞減，而第 4、5 與第 6 位談判者討論後效用值有明顯增加的現象，因此，討論後之談判群體效

用值相對於討論前來得高。而此一結果顯示，在討論過程中，部份談判者的效用明顯受到其他談判者效用的影響，改變原來效用衡量結果，部份談判者明顯不受其他談判者效用之影響而有所改變。

6.3 討論

經由本文之風險定義、模式構建、模式求解法及範例分析之結果，本文歸納以下特性：(1)就用地取得事件而言, 用地延遲徵收年增加時, 無論討論前後，談判群體效用趨近於 0，風險性越高。故依本文風險意義，當談判群體效用越低時，談判群體內之 6 位談判者對此狀態下之效用滿足感愈低，此顯示，當政府在執行用地取得時，若產生土地徵收延遲過久，經內部討論後，所展現出來的是談判群體對用地徵收延遲年遞增，風險性很高。(2)當談判者彼此之間的效用差異趨小，模式求解較易收斂；反之，當談判者之間的效用差異甚大，模式求解則不易收斂。此特性說明，當談判者在最初之效用衡量，大家效用衡量差距很小，彼此之間的效用影響就會縮小，在討論過程中，比較容易取得效用衡量之「共識」結果，因此討論次數就會減少。(3)當討論次數越多，談判者之間的效用交互影響值會遞減, 如延遲徵收第 3 年、4 年及第 5 年, 討論次數分別為 5、5 及 6 次，談判者之間的效用交互影響值呈現遞減。此特性說明，談判者彼此之間在討論過程中，雖有效用交互影響因素存在，但是只要在討論過程中不出現有堅持己見者、完全無意見者，或效用衡量出現差距過大現象，只要有討論行為，談判者彼此間的效用交互影響值差距會隨之縮減。此說明，在討論過程中，談判者彼此之間效用差距會因討論產生縮小，使得彼此之間的效用趨於「共識」之衡量結果。(4)就事件之延遲徵收第 6 年致第 10 年，此 6 位談判者在經 1 次討論後即達共識時，因此，談判群體效用值會由此 6 位談判者之最初效用予以加總獲得，故此 6 位談判者討論時並不會有效用擴散與遞迴現象，此結果與談判者在獨立狀態下所進行效用衡量結果相同。由此推之，當此 6 位談判者之最初效用相同時，其討論後結果應與原來談判者在獨立狀態所進行之效用衡量果相同。

表 8 討論前後之談判者效用值變化情形

事件		**狀態 (延遲徵收年)**										
談判者	**效用**	0	1	2	3	4	5	6	7	8	9	10
第 1 位	**討論前**	0.2009	0.0241	0.0385	0.0419	0.0318	0.0107	0.005700	0.0004	0.0000	0.0000	0.0000
	討論後	0.2009	0.0241	0.0385	0.0419	0.0318	0.0107	0.005710	4.00E-04	0.0000	0.0000	0.0000
	增減	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.000010	0.0000	0.0000	0.0000	0.0000
第 2 位	**討論前**	0.8964	0.7704	0.0615	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	討論後	0.8963	0.7702	0.0614	0.0008	0.0001	1.80E-05	0.000062	6.30E-06	0.0000	0.0000	0.0000
	增減	-1E-04	-0.0002	-1E-04	0.0000	0.0000	0.000018	0.000062	6.3E-06	0.0000	0.0000	0.0000
第 3 位	**討論前**	0.587	0.5137	0.0615	0.0008	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	討論後	0.5871	0.5141	0.0614	0.0008	0.0000	1.80E-05	0.000062	6.30E-06	0.0000	0.0000	0.0000
	增減	1E-04	0.0004	-1E-04	0.0000	0.0000	0.000018	0.000062	6.3E-06	0.0000	0.0000	0.0000
第 4 位	**討論前**	0.0012	0.0048	0.0066	0.0097	0.0034	0.002	0.000800	0.0001	0.0000	0.0000	0.0000
	討論後	0.0023	0.0011	0.0614	0.0096	0.0026	0.0019	0.000062	6.30E-06	0.0000	0.0000	0.0000
	增減	0.0011	-0.0037	0.0548	-0.0001	-0.0008	-0.0001	-0.000738	-9.4E-05	0.0000	0.0000	0.0000
第 5 位	**討論前**	0.046	0.02	0.0156	0.0319	0.0144	0.0025	0.000010	0.0000	0.0000	0.0000	0.0000
	討論後	0.0453	0.0161	0.2206	0.0316	0.0023	0.0023	0.000062	5.90E-06	0.0000	0.0000	0.0000
	增減	-0.0007	-0.0039	0.205	-0.0003	-0.0121	-0.0002	0.000052	5.90E-06	0.0000	0.0000	0.0000
第 6 位	**討論前**	0.2279	0.2275	0.2958	0.1505	0.0033	0.0099	0.0000	0.0000	0.0000	0.0000	0.0000
	討論後	0.2108	0.0076	0.2206	0.1415	0.0021	0.0039	0.000062	5.90E-06	0.0000	0.0000	0.0000
	增減	-0.0171	-0.2199	-0.0752	-0.009	-0.0012	-0.006	0.000062	5.90E-06	0.0000	0.0000	0.0000
收斂情形		×	×	Í	Í	Í	Í	Í	Í	Í	Í	Í
討論次數		50	50	11	5	5	6	1	1	1	1	1
討論前	*GU*	0.0360	0.0261	0.0175	0.0177	0.0061	0.0026	0.0008	0.0001	0.0000	0.0000	0.0000
討論後	*GU*	1.9427	1.3332	0.6639	0.2262	0.0389	0.018836	0.006018	0.000432	0.0000	0.0000	0.0000

備註："×"：不收斂；"Í"：收斂

七、結論與建議

本文目的主要是以 BOT 特許公司談判群體觀點，探討談判群體內部討論行為以及分析談判者彼此之間討論下，如何就某事件或不確定性因素進行風險衡量。本文從效用線性相依及效用偏好分解觀念出發，以動態規劃方法，構建談判群體效用相依模式，並研擬求解方法，以範例分析方式，說明本文基本模式之可用性，以期詮釋談判者彼此之間的效用互動及事件之風險衡量。

由基本模式之推導與範例分析顯示, 影響談判者彼此之間效用獨立與否取決於效用交互影響值之和是否為 0；當效用交互影響值之和趨近於 0 時，談判者彼此之間的效用相依程度愈低，獨立越強。當效用交互影響值之和趨近於 1 時，談判者彼此之間的效用相依程度越高。此外，當談判者彼此之間的效用值差異甚小時，模式求解易於收斂。當談判者彼此之間的效用差異甚大時，談判者彼此之間的討論次數會遞增；當討論次數越多時，談判者彼此之間的效用交互影響值會遞減，並會趨近於 0。若談判者之討論前後效用值差異十分微小，則談判者效用交互影響值會趨近於 0，此與效用獨立狀態衡量結果相同。當談判群體於第 1 次討論後即獲得結果時，談判群體效用值可由談判者之最初效用予以加總，此與效用獨立狀態下所進行之效用衡量結果相同。

經由範例分析顯示，談判群體對用地取得事件之每一個狀態進行內部討論，除延遲徵收之第 0 與第 1 年經過討論後並無產生共識外，其餘狀態皆獲得內部討論後之效用衡量結果。此外，談判者彼此之間的效用影響值介於 0~0.00054 間，談判者彼此之間效用影響不明顯。由範例分析顯示，本文所構建之談判群體效用相依模式，可以顯現談判者彼此之間的效用互動特性及談判群體對事件之效用風險衡量，足以反應談判群體之實際決策環境，而此一結果是傳統 MAU 模式加法型所無法達到之效果。

由於本文基本模式係將效用影響值視為談判者效用函數之內生變數，惟影響效用函數因素除談判者彼此之間的效用互動因素外，尚包含談判者學習能力與資訊不完全之因素，後續研究可將此因素納入模式中，使之更符合談判群體實際討論環境。另外，考量特許契約所存在之許多不確定性因素，這些因素中可能具有相關特性，建議後續研究可放寬事件獨立假設條，構建事件相關之風險衡量模式。而有關討論前後談判者之風險偏好態度改變之課題，亦值得後續研究。另外，亦可就本文模式應用於國內 BOT 計畫進行實例研究，驗證本文模式之實用性。

附錄 A

假設第 k 位與第 $k+1$ 位談判者之效用關係分別如式(A-1)與(A-2), 利用聯立方程式求解 $\sim_{k,k+1}(t)$ 與 $\sim_{k+1,k}(t)$，求解過程如下。

$$\begin{cases} u_k^f(t+1) = (1-\sim_{k+1,k}(t))u_k^f(t) + \sim_{k+1,k}(t)u_{k+1}^f(t) & \text{(A−1)} \\ u_{k+1}^f(t+1) = (1-\sim_{k,k+1}(t))u_{k+1}^f(t) + \sim_{k,k+1}(t)u_k^f(t) & \text{(A−2)} \end{cases}$$

$\forall k \in Q,\ t = 0,1,2,\Lambda\ T.$， t 為討論次數。將式(A-1)整理後，得(A-3)式。

$$u_k^f(t) = \frac{1}{1-\sim_{k+1,k}(t)}[u_k^f(t+1) - \sim_{k+1,k}(t)u_{k+1}^f(t)] \qquad \text{(A-3)}$$

將式(A-3)代入式(A-2)中之 $u_k^f(t)$，整理後得 $\sim_{k+1,k}(t)$ 值，如式(A-4)；再將式(A-4)之 $\sim_{k+1,k}(t)$ 值代入式(A-3)中，得 $\sim_{k,k+1}(t)$ 值如式(A-5)。

$$\sim_{k+1,k}(t) = \left(\frac{u_k^f(t+1) - u_k^f(t)}{u_{k+1}^f(t) - u_k^f(t)}\right) \qquad \text{(A-4)}$$

$$\sim_{k,k+1}(t)=(1-\sim_{k+1,k}(t))(\frac{u_{k+1}^{f}(t)-u_{k+1}^{f}(t+1)}{u_{k}^{f}(t+1)-u_{k+1}^{f}(t)}) \quad ,\forall\, k,k+1\in Q;\, k\neq k+1 \qquad \text{(A-5)}$$

附錄 B

效用交互影響值之收斂結果，說明如下：

利用式(A-1)與式(A-2)關係式，將式(A-1)改為式(B-1)，式(B-1)左邊表第k位與第$k+1$位談判者效用增量差，式(B-1)右邊增量差表第$k+1$位影響第k位談判者之效用，兩者透過效用交互影響值達到恆等，因此，式(B-1)可以式(B-2)表示。

$$u_{k}^{f}(t+1)-u_{k}^{f}(t)=\sim_{k+1,k}(t)(u_{k+1}^{f}(t)-u_{k}^{f}(t)) \qquad \text{(B-1)}$$

$$\Delta u_{k}^{f}(t+1)/\Delta u_{k+1}^{f}(t)=\lambda nu_{k,k+1}^{f}=\sim_{k+1,k}(t) \qquad \text{(B-2)}$$

其中式(B-2)之$\Delta u_{k}^{f}(t+1)=u_{k}^{f}(t+1)-u_{k}^{f}(t)$，$\Delta u_{k+1}^{f}(t)=u_{k+1}^{f}(t)-u_{k}^{f}(t)$；同理，利用式(A-2)改為式(B-3)，此式(B-3)表第$k+1$位與第k位談判者效用增量差，故式(B-3)可由效用增量方式表示之，如式(B-4)。

$$u_{k+1}^{f}(t+1)-u_{k+1}^{f}(t)=\sim_{k,k+1}(t)(u_{k}^{f}(t)-u_{k+1}^{f}(t)) \qquad \text{(B-3)}$$

$$\Delta u_{k+1}^{f}(t+1)/\Delta u_{k}^{f}(t)=\lambda nu_{k+1,k}^{f}=\sim_{k,k+1}(t) \qquad \text{(B-4)}$$

其中式(B-4)之$\Delta u_{k+1}^{f}(t+1)=u_{k+1}^{f}(t+1)-u_{k+1}^{f}(t)$，$\Delta u_{k}^{f}(t)=u_{k}^{f}(t)-u_{k+1}^{f}(t)$。式(B-4)之意義表示談判者效用增量比，若效用變化趨於穩定，則式(B-2)與式(B-4)之效用增量比會趨於相等，即$\Delta u_{k+1}^{f}(t+1)/\Delta u_{k}^{f}(t)=\Delta u_{k}^{f}(t+1)/\Delta u_{k+1}^{f}(t)$，此隱含第$k$位與第$k+1$位談判者效用交互影響值會趨於相等，即$\sim_{k+1,k}(t)=\sim_{k,k+1}(t)$，其推導過程如下。

因效用增量趨於穩定，表示第k位與第$k+1$位談判者之效用增量比的差值愈小愈好，即$\min[(\Delta u_{k+1}^{f}(t+1)/\Delta u_{k}^{f}(t))-(\Delta u_{k}^{f}(t+1)/\Delta u_{k+1}^{f}(t))]$。

$$\begin{aligned}&\min[(\Delta u_{k+1}^{f}(t+1)/\Delta u_{k}^{f}(t))-(\Delta u_{k}^{f}(t+1)/\Delta u_{k+1}^{f}(t))]=\min(\lambda nu_{k,k+1}^{f}-\lambda nu_{k+1,k}^{f})\\&=\min(\sim_{k+1,k}(t)-\sim_{k,k+1}(t))\end{aligned} \qquad \text{(B-5)}$$

為確保$0\leq\sim_{k+1,k}(t)-\sim_{k,k+1}(t)\leq 1$，式(B-5)進一步改為式(B-6)

$$\min\cdot_{k+1,k}=\min|\sim_{k+1,k}(t)-\sim_{k,k+1}(t)| \qquad \text{(B-6)}$$

將式(B-6)改為式(B-7)。

$$\min\cdot_{k+1,k}=\min(\sim_{k+1,k}(t)-\sim_{k,k+1}(t))^{2}=\min(\lambda nu_{k+1,k}^{f}-\lambda nu_{k,k+1}^{f})^{2} \qquad \text{(B-7)}$$

對式(B-7)之$\sim_{k+1,k}(t)$或$\sim_{k,k+1}(t)$微分，結果如式(B-8)。

$$\begin{aligned}&\min\cdot_{k+1,k}=\min(\lambda nu_{k+1,k}^{f}-\lambda nu_{k,k+1}^{f})^{2}\\&\frac{\partial\cdot_{k+1,k}}{\partial\lambda nu_{k+1,k}}=\frac{\partial(\lambda nu_{k+1,k}^{f}-\lambda nu_{k,k+1}^{f})^{2}}{\partial\lambda nu_{k+1,k}^{f}}=2\lambda nu_{k+1,k}^{f}-2\lambda nu_{k,k+1}^{f}\\&Let\,\frac{\partial\cdot_{k+1,k}}{\partial\lambda nu_{k+1,k}}=0,\quad \therefore 2\lambda nu_{k+1,k}^{f}-2\lambda nu_{k,k+1}^{f}=0 \quad \Rightarrow \lambda nu_{k+1,k}^{f}=\lambda nu_{k,k+1}^{f}\end{aligned} \qquad \text{(B-8)}$$

即$\Delta u_{k+1}^{f}(t+1)/\Delta u_{k}^{f}(t)=\Delta u_{k}^{f}(t+1)/\Delta u_{k+1}^{f}(t)\Rightarrow\sim_{k+1,k}(t)=\sim_{k,k+1}(t)$。

由式(B-8)推導顯示，第k位與第$k+1$位談判者之效用趨於穩定時，彼此之間的效用交互影響會趨於相等；換言之，當$\sim_{k+1,k}(t)=\sim_{k,k+1}(t)$時，模式有收斂解。如此當$\sim_{k+1,k}(t)=\sim_{k,k+1}(t)=0$，$\sim_{k+1,k}(t)=\sim_{k,k+1}(t)=1$或$\sum_{k}\sim_{k+1,k}(t)=\sum_{k}\sim_{k,k+1}(t)$時，滿足模式求解收斂條件。該收斂解意義表示，談判者之間的討論行為，若談判者之效用不再有改變時，談判者之間已取得看法趨於一致的結果。

參考文獻

1. **江前良**(1996)**，國際** BOT **方式理論與實務，中國對外經濟貿易出版社。**
2. **馮正民，康照宗**(2000)**，「**BOT **計畫談判群體之風險評量」，運輸計畫季刊(將刊登)。**
3. **馮正民，康照宗**(1999a)**，「**BOT **計畫群體決策在效用相依下之風險衡量模式」，中華民國第十四屆運輸學會年會論文集，台北科技大學。**
4. **蔡明志** (2000)**，風險管理在大眾運輸安全管理管制課題之發展應用，運輸計畫季刊，**29 **卷，第** 1 **期，頁** 181-211**。**
5. **朱敬一**(1990)**，個體經濟分析，新陸書局。**
6. **馮正民、鐘啟椿** (2000)**，交通建設** BOT **案政府對民間造成之風險分析，運輸計畫季刊，**29 **卷，第** 1 **期，頁** 79-108**。**
7. Ansell, A. and Wharton, F. (1992), Risk: Analysis, Assessment and Management, JOHN WILEY & SONS, Chichester, England.
8. Becker, J. L., and Sarin, R. K. (1987), "Lottery Dependent Utility," Management Science, Vol. 33, No. 11, pp.1367-1382.
9. Belichrosdt, H. and Quiggin, J. (1997), "Characterizing QALYs under a General Rank Dependent Utility Model," Journal of Risk and Uncertainty, Vol.15, No. 2, pp. 151-165.
10. Bell, D. E. (1995), "Risk, Return, and Utility," Management Science, Vol. 41, No. 1, pp23-30.
11. Bose, U., Davey, A. M., and Olson, D. L. (1997), "Multiattrinute Utility Methods in Group Decision-Making: Past Applications and potential for Inclusion in GDSS," Omega, International Journal of Management Science, Vol. 25, No. 6, pp.691-706.
12. Buhlmann, H. (1996), Mathematical Methods in Risk Theory, Springer-Verlag .
13. Carbone, E. (1997), "Discriminating Between Preference Functionals: A Monte Carlo Study," Journal of Risk and Uncertainty, Vol. 15, No. 1, pp. 29-54.
14. Cooper, D. F., and Chapman, C. B. (1987), Risk Analysis for Large Projects Models, Method and Cases, John Wiley & Sons.
15. Cuthbertson, M. (1996), Quantitative Financial Economics Stock, Bonds and Foreign Exchange, Published by John Wiley & Son Ltd.
16. Daniels, R. L. and Keller, L. R. (1990), "An Experimental Evaluation of the Descriptive Validity of Lottery-Dependent Utility Theory," Journal of Risk and Uncertainty, Vol. 3, No. 2, pp.115-134.
17. Feng, C.M. and Kang, C.C. (1999b), "Risk Identification and Measurement of BOT Projects," Journal of the Eastern Asia Society for Transportation Studies, No. 4, Vol. 4, pp. 331-350.
18. Fishburn, P. C. (1990), "Representation of Preferences," The New Palgrave Utility and Probability by Eatwell, J. Milgate, M. and Newman, P., W.W. Norton & Company, Inc.
19. Gratte, L.B. (1987), Risk Analysis or Risk Assessment: a Proposal for Consistent Definitions, Plenum Press, NY, USA.
20. Haimes, Y. Y. (1998), Risk Modeling, Assessment, and Management, A Wiley-Interscience Publication, JOHN WILEY & SONS. INC.
21. Haupymanns, U. and Werner, W. (1991), Engineering Risks Evaluation and Valuation, Springer-Verlag.
22. Jia, J. and Dyer, J. S. (1996), "A Standard Measure of Risk and Risk-Value Models," Management Science, Vol. 42, No. 12, pp. 1691-1705.
23. Keeney, R. L. and Raiffa, H. (1993), Decisions with Multiple Objectives Preferences and Value Tradeoffs, Cambridge University Press.
24. Kelsey, D. (1992), "Risk and Risk Aversion State-Dependent Utility," Theory and Decision, Vol. 33, pp.71-82.
25. Kim, K. H. and Roush, F. W. (1987), Team Theory, ELLIS HORWOOD LIMITED Publishers, Chichester.
26. Lowrance, W.W., (1976) Acceptable Risk, William Kaufmannm Los Altos, CA, USA.

27. Luce, R. D. and Fishburn, P. (1995), " A Note on Deriving Rank-Dependent Utility Using Additive Joint Receipts," Journal of Risk and Uncertainty, Vol. 11, No. 1, pp. 5-16.
28. Quiggin, J. (1991), "Comparative Static for Rank-Dependent Expected Utility Theory," Journal of Risk and Uncertainty, Vol. 4, No. 4, pp.339-350.
29. Rescher, N., (1983) Risk: A Philosophical Introduction to the Theory of Risk Evaluation and Managemnet, University Press of America, 1983.
30. Rowe, W. D.(1977), An Anatomy of Risk, John Wiely and Sons, NY.
31. `Tiong, L. K. (1995), "Risks and Guarantees in BOT Tender," Journal of Construction Engineering and Management, Vol. 121, No. 2, pp.183-187.
32. Tiong, L. K. (1996), "CSFs in Competitive Tender and Negotiation Model for BOT Projects," Journal of Construction Engineering and Management, Vol. 122, No. 3, pp. 205-211.
33. Tiong, L. K. (1997), "Final Negotiation in Competitive BOT Tender," Journal of Construction Engineering and Management, Vol. 123, No. 1, pp.6-10.
34. Tzeng, G. H. Chen, T. Y. and Wang, J. C. (1998), "A Weight-Assessing method with Habitual Domains," European Journal of Operational Research, Vol. 110, No. 4, pp. 342-367.
35. Walker, C. and Smith, A. J. (1996), Privatized Infrastructure: The Build Operate Transfer, Thomas Telford Publications.
36. William, I. C. J., and Crandall, K. C. (1982), "Construction Risk : Multiattribute Approach," Journal of the Construction Division, Vol. 108, No. 2, pp.187-200.