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Determination of skin and aquifer parameters for a slug test with wellbore-skin effect

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Summary Slug test is considered to reflect the hydraulic parameters in the vicinity of the test well. The aquifer parameters are usually identified by fitting an appropriate mathematical solution or graphical type curves with slug test data. In this paper, we developed an approach by combining [Moench, A.F., Hsieh, P.A., 1985. Analysis of slug test data in a well with finite-thickness skin. In: Memoirs of the 17th International Congress on the Hydrogeology of Rocks of Low Permeability, U.S.A. Members of the International Association of Hydrologists, Tucson, AZ, vol. 17, pp. 17–29] and simulated annealing (SA) approach to estimate five parameters, i.e., three skin parameters and two aquifer parameters. The three skin parameters are hydraulic conductivity, specific storage, and thickness of the wellbore-skin zone, while the two aquifer parameters are hydraulic conductivity and specific storage of the formation zone. It is worthy to note although the thickness of the wellbore-skin zone is usually taken as an input data for the data-analyzed software, it is actually an unknown parameter that cannot be measured directly. This paper proposes a methodology for estimating the thickness of the wellbore-skin zone with other hydraulic parameters at the same time.

Eight sets of well water-level (WWL) data of aquifers with both positive and negative skins are generated by Moench and Hsieh [Moench and Hsieh, 1985] and four sets of standard normally distributed noise are then added to each set of WWL data. The results indicate that the negative-skin cases generally give a better estimated result than that of the positive-skin cases. Sensitivity analysis is also employed to demonstrate the physical behavior when slug test was performed under positive-skin effect. For the case of an aquifer with a positive-skin, the use of a longer series of WWL data for analysis is strongly recommended for better estimation of aquifer hydraulic conductivity. Analyzing the WWL data of the test and observation wells simultaneously could significantly improve the esti-

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mations on specific storages. Impetuously presuming an arbitrary value for the thickness of the wellbore-skin zone may lead to poor estimation for the other four parameters.
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Introduction

Slug test is a quick and economical way for obtaining the changes in WWL and determining the hydraulic conductivity in the vicinity of the test well. The test is performed by removing (adding) suddenly a small amount of water from (into) the well and measuring the change in WWL simultaneously. Subsequently, the aquifer parameters are identified by fitting an appropriate mathematical solution or graphical type curves with the slug test data. Several slug test mathematical models can be found in the groundwater literature (e.g., Hvorslev, 1951; Cooper et al., 1967; Bouwer and Rice, 1976; Springer and Gelhar, 1991; Hyder et al., 1994; Butler, 1998). In addition, some software, e.g., AQTESOLV (Duffield, 2002), which includes the above-mentioned models has been developed to analyze the slug test data. For a specific well, aquifer configuration, and slug test condition, one can choose an appropriate mathematical model to estimate aquifer parameters.

A wellbore skin is a zone of diverse hydraulic conductivity adjacent to the wellbore face caused by well drilling or completion. A positive skin, usually arisen from the damage of well drilling, is a zone adjacent to the wellbore with a hydraulic conductivity smaller than that of the undisturbed formation. In contrast, a negative skin refers to a zone with a hydraulic conductivity larger than that of the undisturbed formation owing to excessive well completion. The wellbore-skin effect would influence the recovery rate of WWL especially in a small-scale slug test.

Regarding the investigations on wellbore-skin effect, Ramey et al. (1975) presented an analytical solution for a slug test with a skin of infinitesimal thickness. They used the type curves approach, which is developed from a correlating group of dimensionless storage constant and skin effect, to determine the aquifer hydraulic conductivity and the skin effect. However, there may be a large uncertainty in the determination of the correlating group due to the similarity in the shape of the type curves. Therefore, such a curve-matching procedure may result in an inaccurate estimation of aquifer hydraulic conductivity. Faust and Mercer (1984) investigated the effect of a finite-thickness skin on the response of slug tests using a simple analytical solution and a numerical model. They compared the normalized head responses for a slug test performed with positive-skin, negative-skin, and no-skin effects, respectively. Moreover, they concluded that a negative skin has little effect on the shape and does not shift the normalized head response curve horizontally so that it would not obviously affect the estimated result of aquifer hydraulic parameters. On the other hand, a positive skin induces a similar shape but shift the response horizontally when compared with a no-skin response curve and thus leads to an unreliable estimate of aquifer hydraulic parameters. Following the investigation of Faust and Mercer (1984), Moench and Hsieh (1985) presented a semi-analytical solution and used type curves to illustrate the influences of a finite-thickness skin on the open-well and pressurized slug tests. In addition, Yang and Gates (1997) analyzed the effect of wellbore skin on slug test results utilizing the finite-element model and field tests. Further, Yeh and Yang (2006) proposed an analytical solution for a slug test performed in a confined aquifer with finite-thickness skin. Their results showed that the early- and late-time data reflect the ground-water flow within the wellbore-skin and undisturbed formation zones, respectively. The model proposed by Hyder et al. (1994), also known as the KGS model, is useful for determining the hydraulic conductivity and specific storage in a confined or unconfined aquifer for a partially penetrating well with skin effect. The software package AQTESOLV (Duffield, 2002), which includes the KGS model, can analyze the data of slug test under skin effect. However, the AQTESOLV requires the ''outer radius of well skin'' or the ''skin thickness'' as an input data. In fact, the skin thickness is actually unknown and cannot be measured. The identification of skin thickness as well as hydraulic parameters would be more practical and useful for giving hydrologists an insight into the skin-affected slug test.

Simulated Annealing (SA) is a stochastic technique for solving optimization problems. Metropolis et al. (1953) proposed the SA algorithm for evaluating the properties of a material composed of interacting individual molecules. Kirkpatrick et al. (1983) were the first to present an algorithm based on the analogy between the annealing of solids and the problem of solving combinatorial optimization (Pham and Karaboga, 2000). Subsequently, utilization of SA in optimization problems has been widely applied in hydrological engineering. For example, the method has been employed to design the strategies of groundwater remediation (Dougherty and Marryott, 1991; Marryott et al., 1993), identify the best parameter structure for groundwater models (Zheng and Wang, 1996), and determine the optimal network design or water resource management (Pardo-Iguzquiza, 1998; Cunha and Sousa, 1999; Kuo et al., 2001). As presented in these studies, SA is useful in solving optimization problems that consist of many degrees of freedom and several local optima.

The purpose of this paper is to develop an approach that combines the Moench and Hsieh's slug-test-solution (1985) and SA to analyze the slug test data for estimating three skin parameters and two aquifer parameters. The merit of this approach over existing slug test analysis is that skin thickness is treated as an unknown parameter and estimated simultaneously with the other four parameters. Seventeen scenarios and sensitivity analyses for selected scenarios of slug tests were given to demonstrate the capability of the suggested approach in identifying five unknown parameters, i.e. the hydraulic conductivities and specific storages of both the formation and skin zones as well as the skin thickness.

Nomenclature

b	aquifer thickness
c_1	$\alpha\gamma pK_0(q\beta) + \beta qK_1(q\beta)$
c_2	$\alpha\gamma pI_0(q\beta) - \beta qI_1(q\beta)$
d_{sk}	skin thickness
e_j	the difference between observed and predicted WWL well water level
$\bar{h}$	well water level of slug test in the Laplace domain
$h_o(t_j)$	observed well water level at time t_j
$h_e(t_j)$	estimated well water level for current solution at time t_j
$h'_e(t_j)$	estimated well water level for trial solution at time t_j
H_0	initial well water level
$I_0(\cdot)$	modified Bessel function of the first kind of order zero
$I_1(\cdot)$	modified Bessel function of the first kind of order one
k_1	hydraulic conductivity in a skin zone
k_2	hydraulic conductivity in a formation zone
$K_0(\cdot)$	modified Bessel function of the second kind of order zero
$K_1(\cdot)$	modified Bessel function of the second kind of order one
N	number of parameters
NS	number of searches for a parameter
NT	number of times for producing NS once at a temperature level
p	Laplace variable
$P(\cdot)$	probability
q	$\sqrt{p}$
r	radial distance from the centerline of well
r_c	radius of casing
r_s	outer radius of wellbore-skin region
r_w	effective radius of test well
RD	random number of a uniform (0, 1) distribution
R_T	cooling rate
S_{s1}	specific storage in a skin zone
S_{s2}	specific storage in a formation zone
t	time since the start of test
T_0	initial temperature
T_e	present temperature
T'_e	new temperature
V	degree of freedom
VM_i	step length vector of the parameter i.
x_i	current solution of parameter i,
x'_i	new trial solution of parameter i
z	objective function value of current solutions
z'	objective function value of trial solutions
α	k_2/k_1
β	$(\alpha S_{s1}/S_{s2})^{1/2}$
γ	$\frac{c}{2\pi r_w^2 S_{s2} b}$
Δ_1	$\alpha I_0(q\beta r_s)K_1(qr_s) + \beta I_1(q\beta r_s)K_0(qr_s)$
Δ_2	$\alpha K_0(q\beta r_s)K_1(qr_s) - \beta K_1(q\beta r_s)K_0(qr_s)$
Δz	difference between trial and current objective functions
SEE	standard error of estimate

Methodology

In past researches, many optimization algorithms were applied for finding optimum values of an objective function with several independent variables. Here, an approach that couples SA with Moench and Hsieh's solution (1985) is developed to determine three skin parameters and two aquifer parameters. This section is thus divided into two parts. We first introduce the concept, process, and parameter requirements in SA, while Moench and Hsieh's solution (1985), which is employed to evaluate the objective function, is reviewed in the second part.

Simulated annealing

SA is known as an optimization algorithm for simulating a material crystallized in the process of annealing. The arrangement of the material molecules is initially disordered at high temperature. The system is gradually cooled; meanwhile, the arrangement becomes more ordered and approaches a thermodynamic equilibrium. The framework of SA is demonstrated in Fig. 1. The parameters required in SA are given in the following.

The algorithm in SA starts with an initial guess at higher temperature and regards the initial guess as the current optimal solution. The five parameters to be estimated are hydraulic conductivities k_1 and k_2, specific storages S_{s1} and S_{s2} in the skin and formation zones, respectively, and the skin thickness d_{sk}. A lower bound (LB) as well as the upper bound (UB) for each trial solution of the target parameter is chosen to make sure that the guessed value is physically reasonable. The hydraulic conductivity for silt sand formation may range from 10^{-7} to 10^{-3} m/s (Freeze and Cherry, 1979). Thus, the LBs and UBs are respectively chosen as 10^{-7} m/s and 10^{-3} m/s for both k_1 and k_2. In addition, the values of storativity generally range from 10^{-5} to 10^{-3} in a confined aquifer (Schwartz and Zhang, 2003). The LBs and UBs for both S_{s1} and S_{s2} are respectively taken to be 10^{-6} m^{-1} and 10^{-4} m^{-1} on the ground that the thickness of a confined aquifer is assumed to be 10 m. The skin thickness d_{sk}, which is equal to $r_s - r_w$ with r_s and r_w representing the outer radius of the wellbore-skin zone and the effective well radius, respectively, is calculated after r_s is estimated in SA. Typically, slug tests result in a variation in radius between 1 and 2 m (Rovey, 1998). Thus, the UB for r_s is set to be 2 m; and the LB for r_s is equal to r_w, i.e., 0.0915 m. Furthermore, the initial guessed values of k_1 and k_2 are chosen as 10^{-3} m/s, S_{s1} and S_{s2} are 10^{-4} m^{-1}, and r_s is 0.0915 m. The parameter estimation process beginning with a higher initial temperature (T_0) may give a more complete exploration of the solution domain but may requires much more computing time than the one with lower temperature. The value of T_0 is thus chosen to be 5 in this study.

The initial guessed values are first employed to generate the WWL data using Moench and Hsieh's slug test solution (1985) and calculate the objective function value. Note that

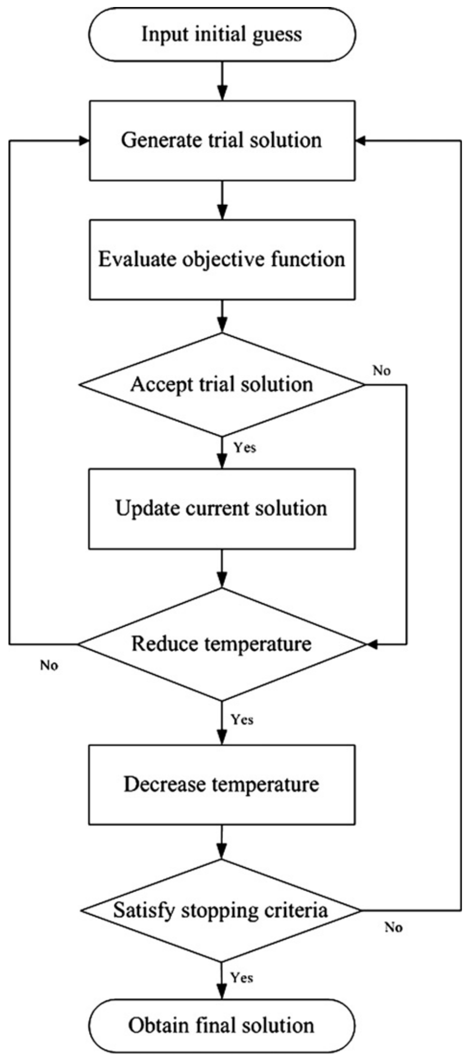

Figure 1 The algorithm framework of SA.

the objective function is analogous to the energy in thermodynamics in SA. Later, SA will randomly generate a neighboring solution (trial solution) and evaluate the objective function. A new trial solution for parameter i namely x'_i is produced randomly from

$$x'_i = x_i + (2 \times \mathrm{RD} - 1) \times \mathrm{VM}_i \tag{1}$$

where x_i is the current solution for parameter i, RD is a random number generated from a uniform (0, 1) distribution, and VM_i is a step length vector of the parameter i. The value of VM_i is first defined as UB − LB. Then VM_i is automatically adjusted according to the objective function value calculated from the trial solution generated at current temperature to make that approximately half of all trial solutions will be accepted at the next temperature. On the other hand, if the trial solution is out of bounds, another procedure for creating a trial solution within bounds is

$$x'_i = \mathrm{LB}_i + \mathrm{RD} \times (\mathrm{UB}_i - \mathrm{LB}_i) \tag{2}$$

For a given temperature, each parameter requires a number of searches (NS) and the search is repeated NT times. Thus, the total number of iterations is $N \times \mathrm{NS} \times \mathrm{NT}$ per temperature level for the target parameters, where N denotes the number of parameters. The total number of iterations at each temperature level is taken as 500 with $N = 5$, NS = 20, and NT = 5. Note that a greater number of searches may give a better result but requires more computing time.

The objective functions for current and trial solutions are respectively defined as

$$z = \sum_{j=1}^{n} (h_o(t_j) - h_e(t_j))^2 \tag{3}$$

and

$$z' = \sum_{j=1}^{n} (h_o(t_j) - h'_e(t_j))^2 \tag{4}$$

where z and z' are the objective functions for current and trial solutions, respectively; $h_o(t_i)$ is the observed WWL; and $h_e(t_j)$ and $h'_e(t_j)$ are the estimated WWL for current solution and trial solution at time t_j, respectively.

As the objective function is minimized, any downhill step is accepted or an uphill step may be accepted according to the Metropolis criterion. The Metropolis's criterion is expressed as (Pham and Karaboga, 2000)

$$P(\text{accept trial solution}) = \begin{cases} 1 & \text{for } \Delta z \leqslant 0 \\ \exp\left(\frac{-\Delta z}{T_e}\right) & \text{for } \Delta z > 0 \end{cases} \tag{5}$$

where P is the acceptance probability of the trial solution, $\Delta z = z' - z$ is the difference between the trial and current objective functions, and T_e is the present temperature. The trial solution supplants the current optimal solution as the starting point of the next state on condition that $\Delta z \leqslant 0$. On the other hand, if $\Delta z > 0$, the current optimal solution also has a probability to be replaced by the trial solution even though it is inferior to the current one. The replacement happens when P is larger than a random number generated from a uniform (0, 1) distribution. In addition, according to the Metropolis's criterion, the probability of accepting inferior trial solutions decreases with the cool-down process. The major advantage of SA over other optimization algorithms is its ability to accept inferior trial solution and to escape from local optimum.

After $N \times \mathrm{NS} \times \mathrm{NT}$ iterations at a temperature level, the system decreases the temperature by a constant cooling rate (R_T). Therefore, the new temperature becomes

$$T'_e = R_T \times T_e \tag{6}$$

The cooling rate which ranges from zero to one is chosen as 0.85 as suggested in Corana et al. (1987). The cool-down process continues until the system satisfies the termination criteria.

Two termination criteria are used in this study. The first criterion is that the difference of the objective functions between two consecutive temperatures is less than 10^{-6} for four times sequentially. The second and minor one is that the maximum number of total iterations allowed is set to be 2×10^7. Once the total number of iterations reaches this number, the identification process will be terminated even the solution is not optimal.

Moench and Hsieh's slug test solution

Moench and Hsieh's solution (1985) is chosen to couple with SA for determining the skin and aquifer parameters. Their solution assumes that a fully penetrating well is installed in a confined, homogeneous, isotropic, and infinite lateral extent aquifer of constant thickness with a finite-thickness skin. The dimensionless form of WWL solution in the Laplace domain is expressed as

$$\bar{h} = \frac{\alpha\gamma[\Delta_1 K_0(q\beta) - \Delta_2 I_0(q\beta)]}{c_1\Delta_1 - c_2\Delta_2} \quad (7)$$

with

$$\Delta_1 = \alpha I_0(q\beta r_s)K_1(qr_s) + \beta I_1(q\beta r_s)K_0(qr_s) \quad (8)$$

$$\Delta_2 = \alpha K_0(q\beta r_s)K_1(qr_s) - \beta K_1(q\beta r_s)K_0(qr_s) \quad (9)$$

$$c_1 = \alpha\gamma p K_0(q\beta) + \beta q K_1(q\beta) \quad (10)$$

$$c_2 = \alpha\gamma p I_0(q\beta) - \beta q I_1(q\beta) \quad (11)$$

$$\alpha = k_2/k_1 \quad (12)$$

$$\beta = (\alpha S_{s1}/S_{s2})^{1/2} \quad (13)$$

$$\gamma = \frac{\pi r_c^2}{2\pi r_w^2 S_{s2} b} \quad (14)$$

and

$$q = p^{1/2} \quad (15)$$

The notation r_w is the effective radius of test well; r_c is the radius of well casing; b is the aquifer thickness; and p is the Laplace variable. Moreover, $I_0(\cdot)$ and $I_1(\cdot)$ denote the modified Bessel functions of the first kind of order zero and one, respectively; and $K_0(\cdot)$ and $K_1(\cdot)$ are the modified Bessel functions of the second kind of order zero and one, respectively. Eq. (7) is first employed to generate several sets of WWL data, i.e., the observed WWLs $h_o(t_j)$. Then, the estimated WWL data $h_e(t_j)$ or $h'_e(t_j)$ are calculated according to Eq. (7) to estimate the objective function, that is, either Eq. (3) or (4).

Table 1a The target values and estimated results by analyzing the WWL data within 15 s for aquifers with a positive skin

Case	Estimated results					
	k_1 (m/s)	k_2 (m/s)	Ss_1 (1/m)	S_{s2} (1/m)	d_{sk} (m)	SEE
Target value	1.00×10^{-5}	1.00×10^{-4}	1.00×10^{-4}	1.00×10^{-4}	0.9085	–
1a	1.00×10^{-5}	1.14×10^{-4}	9.94×10^{-5}	4.63×10^{-5}	0.895	3.43×10^{-4}
1b	1.03×10^{-5}	2.80×10^{-4}	8.98×10^{-5}	2.03×10^{-5}	1.051	5.97×10^{-4}
1c	1.07×10^{-5}	5.02×10^{-4}	8.17×10^{-5}	7.82×10^{-5}	1.267	8.65×10^{-4}
1d	1.04×10^{-5}	1.20×10^{-4}	8.76×10^{-5}	8.07×10^{-5}	1.024	7.41×10^{-4}
1e	1.02×10^{-5}	1.45×10^{-4}	9.51×10^{-5}	5.32×10^{-5}	0.973	5.76×10^{-4}
Mean	1.03×10^{-5}	2.32×10^{-4}	9.07×10^{-5}	5.57×10^{-5}	1.042	–
RE	3.20%	132.20%	−9.28%	−44.26%	14.69%	–
Target value	1.00×10^{-5}	1.00×10^{-4}	1.00×10^{-4}	1.00×10^{-4}	0.3085	–
2a	1.53×10^{-5}	8.50×10^{-5}	3.18×10^{-5}	9.10×10^{-5}	0.856	2.82×10^{-4}
2b	1.82×10^{-5}	1.08×10^{-4}	1.68×10^{-5}	3.48×10^{-5}	1.486	6.81×10^{-4}
2c	1.55×10^{-5}	8.09×10^{-5}	3.15×10^{-5}	9.97×10^{-5}	0.907	8.56×10^{-4}
2d	1.51×10^{-5}	1.07×10^{-4}	3.17×10^{-5}	5.36×10^{-6}	0.683	7.35×10^{-4}
2e	1.58×10^{-5}	1.08×10^{-4}	2.92×10^{-5}	1.78×10^{-5}	0.869	7.78×10^{-4}
Mean	1.60×10^{-5}	9.78×10^{-5}	2.82×10^{-5}	4.97×10^{-5}	0.960	–
RE	59.80%	−2.22%	−71.80%	−50.27%	211.25%	–
Target value	1.00×10^{-5}	1.00×10^{-3}	1.00×10^{-4}	1.00×10^{-4}	0.9085	–
3a	9.98×10^{-6}	7.99×10^{-4}	1.00×10^{-4}	7.93×10^{-5}	0.895	3.26×10^{-4}
3b	9.98×10^{-6}	9.32×10^{-4}	9.93×10^{-5}	1.33×10^{-5}	0.891	6.01×10^{-4}
3c	1.03×10^{-5}	9.65×10^{-4}	9.20×10^{-5}	8.15×10^{-5}	1.007	8.73×10^{-4}
3d	1.05×10^{-5}	9.96×10^{-4}	8.41×10^{-5}	5.52×10^{-5}	1.039	7.45×10^{-4}
3e	1.00×10^{-5}	9.83×10^{-4}	9.98×10^{-5}	5.50×10^{-5}	0.902	7.96×10^{-4}
Mean	1.02×10^{-5}	9.35×10^{-4}	9.50×10^{-5}	5.69×10^{-5}	0.947	–
RE	1.52%	−6.50%	−4.96%	−43.14%	4.22%	–
Target value	1.00×10^{-5}	1.00×10^{-3}	1.00×10^{-4}	1.00×10^{-4}	0.3085	–
4a	1.07×10^{-5}	9.85×10^{-4}	8.57×10^{-5}	7.43×10^{-6}	0.346	3.15×10^{-4}
4b	1.90×10^{-5}	9.72×10^{-4}	1.53×10^{-5}	9.70×10^{-5}	1.449	7.83×10^{-4}
4c	1.14×10^{-5}	8.04×10^{-4}	7.78×10^{-5}	5.94×10^{-5}	0.400	8.17×10^{-4}
4d	1.58×10^{-5}	9.99×10^{-4}	2.82×10^{-5}	2.94×10^{-5}	0.856	7.39×10^{-4}
4e	1.22×10^{-5}	9.94×10^{-4}	6.53×10^{-5}	3.80×10^{-5}	0.462	7.91×10^{-4}
Mean	1.38×10^{-5}	9.51×10^{-4}	5.45×10^{-5}	4.62×10^{-5}	0.703	–
RE	38.20%	−4.92%	−45.54%	−53.75%	127.75%	–

Results and discussion

Assume that the slug test is performed in a confined, homogeneous, isotropic, and infinite lateral extent aquifer system. The test well is of full penetration and the radius of effective well and well casing are 0.0915 m and 0.0508 m, respectively. The initial WWL is assumed to be 1 m, while the aquifer thickness is assumed to be 10 m. The WWL data are produced according to Moench and Hsieh's solution (1985) and analyzed by the present approach for determining the four hydraulic parameters and skin thickness for a slug test with a positive or negative skin.

Table 1 presents the ''true'' aquifer conditions, i.e., the target values of the four hydraulic parameters and skin thickness, and the results estimated by the present approach for eight scenarios. In Case 1a of Scenario 1, the WWL data were directly generated using Moench and Hsieh's solution (1985) and listed in Table 2. The WWL data for the other four cases in Scenario 1, i.e., Cases 1b–1e, were obtained by adding four different sets of noise to the WWL data of Case 1a. These four sets of noise are generated by the routine RNNOF of IMSL (1997), which can generate normally distributed random numbers with zero mean and unit variance. They are then divided by 1000 to represent the measurement errors of the water-level meter with the magnitude on the order of millimeter. Note that the standard error of the estimate (SEE) listed in the last column of Table 1 is defined as $\left(\sum_{j=1}^{n} e_j^2/v\right)^{1/2}$, where e_j represents the difference between the observed WWL and the predicted WWL according to the estimated parameters and v, the degree of freedom, equals the number of observed data point minus the number of unknowns (Yeh, 1987). In addition, the relative error (RE) is defined as the difference between the

Table 1b The target values and estimated results by analyzing the WWL data within 15 s for aquifers with a negative skin

Case	Estimated results					
	k_1 (m/s)	k_2 (m/s)	S_{s1} (1/m)	S_{s2} (1/m)	d_{sk} (m)	SEE
Target value	1.00×10^{-4}	1.00×10^{-5}	1.00×10^{-4}	1.00×10^{-4}	0.9085	–
5a	1.15×10^{-4}	9.31×10^{-6}	6.03×10^{-5}	7.35×10^{-5}	1.150	3.40×10^{-4}
5b	1.30×10^{-4}	9.94×10^{-6}	3.50×10^{-5}	3.66×10^{-5}	1.547	6.97×10^{-4}
5c	1.13×10^{-4}	9.04×10^{-6}	6.39×10^{-5}	8.06×10^{-5}	1.120	1.01×10^{-3}
5d	1.12×10^{-4}	9.74×10^{-6}	6.43×10^{-5}	6.94×10^{-5}	1.134	6.95×10^{-4}
5e	1.11×10^{-4}	1.06×10^{-5}	6.60×10^{-5}	5.88×10^{-5}	1.141	6.99×10^{-4}
Mean	1.16×10^{-4}	9.73×10^{-6}	5.79×10^{-5}	6.38×10^{-5}	1.218	–
RE	16.20%	−2.74%	−42.10%	−36.22%	34.11%	–
Target value	1.00×10^{-4}	1.00×10^{-5}	1.00×10^{-4}	1.00×10^{-4}	0.3085	–
6a	1.69×10^{-4}	9.85×10^{-6}	2.69×10^{-5}	3.07×10^{-5}	0.628	3.27×10^{-4}
6b	1.39×10^{-4}	9.90×10^{-6}	7.94×10^{-5}	8.95×10^{-5}	0.322	6.69×10^{-4}
6c	1.67×10^{-4}	9.42×10^{-6}	5.48×10^{-5}	7.67×10^{-5}	0.381	9.52×10^{-4}
6d	8.15×10^{-5}	1.06×10^{-5}	8.98×10^{-5}	6.77×10^{-5}	0.381	7.90×10^{-4}
6e	1.39×10^{-4}	9.94×10^{-6}	5.58×10^{-5}	6.12×10^{-5}	0.414	7.95×10^{-4}
Mean	1.39×10^{-4}	9.94×10^{-6}	6.13×10^{-5}	6.52×10^{-5}	0.425	–
RE	39.10%	−0.58%	−38.66%	−34.84%	37.83%	–
Target value	1.00×10^{-3}	1.00×10^{-5}	1.00×10^{-4}	1.00×10^{-4}	0.9085	–
7a	9.98×10^{-4}	1.00×10^{-5}	8.93×10^{-5}	8.88×10^{-5}	0.968	3.21×10^{-4}
7b	1.00×10^{-3}	1.09×10^{-5}	3.01×10^{-5}	2.36×10^{-5}	1.785	8.17×10^{-4}
7c	9.99×10^{-4}	1.03×10^{-5}	4.16×10^{-5}	3.73×10^{-5}	1.490	1.03×10^{-3}
7d	9.63×10^{-4}	1.03×10^{-5}	6.93×10^{-5}	6.46×10^{-5}	1.121	7.08×10^{-4}
7e	8.52×10^{-4}	1.06×10^{-5}	7.60×10^{-5}	6.41×10^{-5}	1.084	6.89×10^{-4}
Mean	9.62×10^{-4}	1.04×10^{-5}	6.13×10^{-5}	5.57×10^{-5}	1.290	–
RE	−3.76%	4.20%	−38.74%	−44.32%	41.95%	–
Target value	1.00×10^{-3}	1.00×10^{-5}	1.00×10^{-4}	1.00×10^{-4}	0.3085	–
8a	5.75×10^{-4}	1.00×10^{-5}	7.78×10^{-5}	7.62×10^{-5}	0.370	3.07×10^{-4}
8b	9.86×10^{-4}	1.02×10^{-5}	7.11×10^{-5}	6.61×10^{-5}	0.388	7.50×10^{-4}
8c	9.83×10^{-4}	9.64×10^{-6}	5.58×10^{-5}	6.43×10^{-5}	0.432	9.44×10^{-4}
8d	2.26×10^{-4}	1.04×10^{-5}	6.38×10^{-5}	4.96×10^{-5}	0.476	7.05×10^{-4}
8e	2.18×10^{-4}	1.07×10^{-5}	6.86×10^{-5}	4.69×10^{-5}	0.470	7.57×10^{-4}
Mean	5.98×10^{-4}	1.02×10^{-5}	6.74×10^{-5}	6.06×10^{-5}	0.427	–
RE	−40.24%	1.88%	−32.58%	−39.38%	38.48%	–

Table 2 The WWL responses simulated by Moench and Hsieh's solution (1985) for the hypothetical Scenarios 1–8

Time (s)	WWL (m)							
	1a	2a	3a	4a	5a	6a	7a	8a
0.5	0.967	0.967	0.967	0.967	0.838	0.895	0.644	0.868
1.0	0.947	0.944	0.947	0.942	0.754	0.856	0.599	0.830
1.5	0.930	0.921	0.930	0.918	0.698	0.826	0.567	0.800
2.0	0.914	0.900	0.914	0.895	0.657	0.801	0.542	0.775
2.5	0.899	0.879	0.899	0.872	0.624	0.778	0.521	0.752
3.0	0.884	0.859	0.884	0.850	0.596	0.758	0.502	0.732
3.5	0.870	0.840	0.870	0.829	0.573	0.739	0.485	0.713
4.0	0.857	0.821	0.856	0.808	0.552	0.721	0.470	0.695
4.5	0.844	0.802	0.843	0.788	0.534	0.705	0.457	0.679
5.0	0.831	0.784	0.830	0.768	0.518	0.689	0.444	0.663
6.0	0.806	0.750	0.804	0.730	0.488	0.661	0.421	0.635
7.0	0.783	0.717	0.780	0.694	0.463	0.635	0.401	0.609
8.0	0.760	0.686	0.756	0.659	0.441	0.611	0.384	0.586
9.0	0.738	0.656	0.732	0.627	0.422	0.589	0.368	0.564
10.0	0.716	0.628	0.710	0.596	0.404	0.568	0.353	0.544
11.0	0.695	0.601	0.688	0.566	0.388	0.549	0.340	0.525
12.0	0.675	0.576	0.667	0.538	0.373	0.531	0.328	0.508
13.0	0.656	0.551	0.647	0.512	0.360	0.515	0.316	0.492
14.0	0.637	0.528	0.627	0.486	0.347	0.499	0.306	0.476
15.0	0.619	0.506	0.608	0.462	0.335	0.484	0.296	0.461

mean and target value divided by the target value to reflect the magnitude of parameter uncertainty on estimation.

Positive-skin scenarios

Table 1a demonstrates the estimated results in four scenarios for the aquifer with a positive skin. Both Scenarios 1 and 2 had the same target values of $k_1 = 10^{-5}$ m/s and $k_2 = 10^{-4}$ m/s but different target values of d_{sk}, i.e., 0.9085 m and 0.3085 m, respectively. In Cases 1a–1e of Scenario 1, the estimated k_1, S_{s1} and d_{sk} with fairly good mean values of 1.03×10^{-5} m/s, 9.07×10^{-5} m^{-1} and 1.042 m, respectively, and RE of 3.20%, −9.28% and 14.69%. However, relatively inaccurate results were obtained for k_2 and S_{s2} with a mean value of 2.32×10^{-4} m/s and 5.57×10^{-5} m^{-1} and *RE* of 132.20% and −44.26%, respectively. Scenario 2 shows fairly good results on k_2 for a mean value of 9.78×10^{-5} m/s and RE of −2.22%. However, in this scenario, the estimated values of d_{sk}, ranging from 0.683 to 1.486 m with a mean value of 0.960 m and RE of 211.25%, are exaggerated while the target d_{sk} is only 0.3085 m. In addition, slightly inaccurate results were obtained for k_1, S_{s1} and S_{s2} with a mean value of 1.60×10^{-5} m/s, 2.82×10^{-5} m^{-1} and 4.97×10^{-5} m^{-1}, and RE of 59.80%, −71.80% and −50.27%, respectively.

Scenarios 3 and 4 use the same target values of parameters as Scenarios 1 and 2 except that $k_2 = 10^{-3}$ m/s, which is one order of magnitude higher than that of Scenarios 1 and 2. The estimated results of Scenario 3 are fairly good for skin parameters k_1, S_{s1} and d_{sk} with the averages of 1.02×10^{-5} m/s, 9.50×10^{-5} m^{-1} and 0.947 m and RE of 1.52%, −4.96% and 4.22%, respectively. The mean value and RE are respectively 9.35×10^{-4} m/s and −6.50% for k_2 and 5.69×10^{-5} m^{-1} and −43.14% for S_{s2}, which are relatively inaccurate but acceptable. Overall, the estimated results of Scenario 3 are similar to those of Scenario 1 in that aquifer parameters, i.e., k_2 and S_{s2} were poorly estimated. In Scenario 4, the unfavorable results of d_{sk} vary from 0.346 to 1.449 m with a mean value of 0.703 m and a RE of 127.75% while the target d_{sk} equals 0.3085 m. In addition, relatively inaccurate estimations were also obtained for k_1, S_{s1}, and S_{s2} with a mean value of 1.38×10^{-5} m/s, 5.45×10^{-5} m^{-1} and 4.62×10^{-5} m^{-1} and RE of 38.20%, −45.54% and −53.75%, respectively. Both Scenarios 2 and 4 represent the positive-skin scenarios that have a thinner skin zone than that simulated in Scenarios 1 and 3. Moreover, relatively unfavorable estimations on k_1, S_{s1}, S_{s2}, and d_{sk} are seen in these scenarios.

Negative-skin scenarios

In contrast to Tables 1a and 1b lists the estimated results for four negative-skin scenarios. Scenarios 5 and 6 have the same target values of $k_1 = 10^{-4}$ m/s and $k_2 = 10^{-5}$ m/s, yet different target values of d_{sk}, i.e., 0.9085 m and 0.3085 m, respectively; while Scenarios 7 and 8 differ from Scenarios 5 and 6 in that one-order higher magnitude of target values of $k_1 = 10^{-3}$ m/s. Note that the estimated k_2 for the four negative-skin scenarios are accurate with each having a RE less than 5%. In addition, the estimated results of the other four parameters are acceptable. In short, the negative-skin scenarios as a whole give a better estimation than that of the positive-skin scenarios. Overall, the values of the five parameters estimated using SA approach shown in Table

1 can produce a set of WWL data coinciding with those created by the target values of parameters with the SEE values being all less than 10^{-3}.

Sensitivity analysis

In this section, sensitivity analysis is employed to investigate the problem of unfavorable estimated results in Scenarios 1 and 2. Fig. 2 displays the normalized sensitivities, representing the temporal change in WWL with respect to a relative change in one of the five parameters (Kabala, 2001).

Analyzing a longer set of WWL data

Fig. 2a illustrates the normalized sensitivities of the five parameters over a 1000-s interval for Scenario 1. As can be seen, all parameters have their own influence on the WWL. A positive change in the value of r_s poses positive influence on the WWL through the 1000-s interval, while a positive change in the values of k_1, k_2 and S_{s2} has negative influence. In addition, a positive change in the value of S_{s1} has negative influence at the early 50-s interval, but exerts positive influence over the 50- to 1000-s interval. The WWL is almost insensitive to the change in S_{s1} and S_{s2}. Thus, these two parameters allow higher degree of uncertainty in estimations than the other parameters. The influence of k_2 on the WWL is less sensitive than that of S_{s1} at the prior 20-s interval; afterwards, it has more sensitive characteristic than that of S_{s1} and S_{s2}. The unfavorable estimate on k_2 in Scenario 1 may be attributed to the short WWL period within 15 s and 20 data points used in identifying the hydraulic parameters.

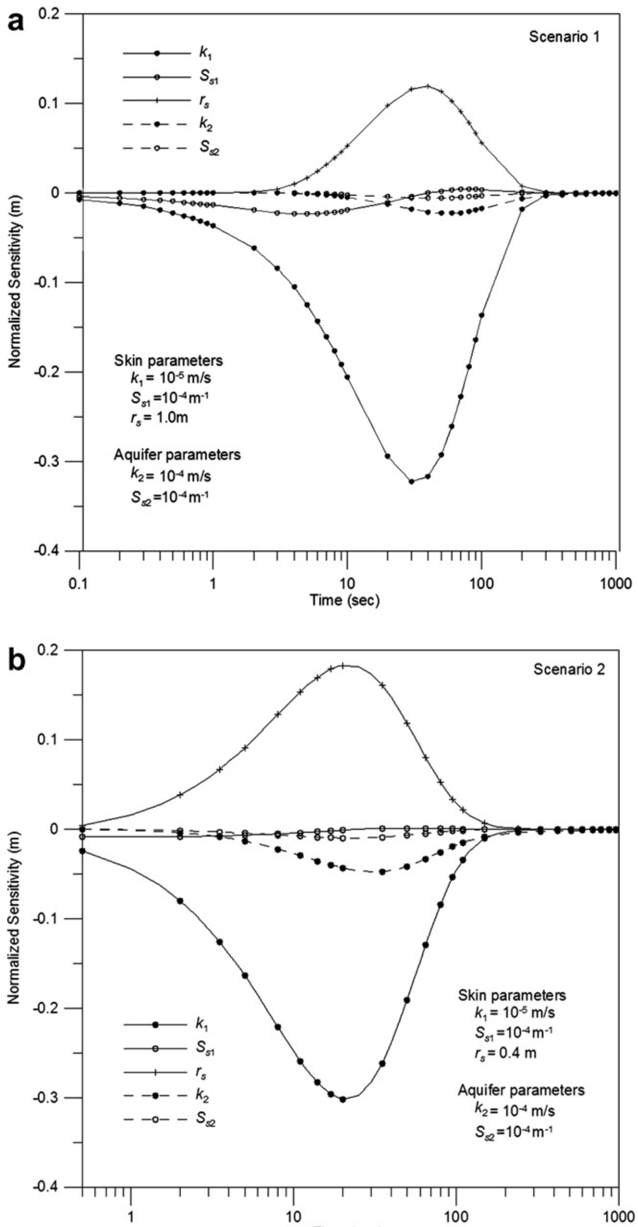

Figure 2 (a) The normalized sensitivities of the five parameters over a period of 1000 s for (a) Scenario 1 and (b) Scenario 2.

Scenario 9, listed in Table 3, gives the estimated results of analyzing the 180-s WWL data with the same target parameters as Scenario 1. An obvious improvement can be observed from the estimated results of k_2 and S_{s2}. It is noteworthy that the averages of k_2 and S_{s2} become 1.02×10^{-4} m/s and 9.44×10^{-5} m^{-1}, which are very close to the target values of 1.00×10^{-4} m/s and 1.00×10^{-4} m^{-1}, respectively. The parameter estimation took about 7.6 hours for analyzing a set of 47 WWL data of Case 9a and about 3.6 hours when analyzing a set of 20 WWL data in Case 1a when using a personal computer with 3.6 G *Pentium IV* CPU and 1 GB RAM. Though the use of a longer set of WWL data consumes more computing time, it gives a more accurate estimation of k_2 in a positive-skin case.

Fig. 2b shows the normalized sensitivities of the five parameters for Scenario 2. The influences of S_{s1} and S_{s2} on the WWL are also insensitive over the 1000-s interval. In addition, the sensitivity of k_2 is apparently larger than that of S_{s1} and S_{s2} after 4 s and the estimated result of k_2 in Scenario 2 is fairly good. The 180-s WWL data with the same target parameters as Scenario 2 are also employed to estimate the five parameters. The results represented as Scenario 10 and listed in Table 3 reveal a more accurate estimation on k_2 with a mean value of 9.97×10^{-5} m/s; nevertheless, there are only minor improvements in k_1, S_{s1}, and d_{sk} and even slight deterioration in S_{s2}.

Highly correlated parameters: k_1 and r_s

The normalized sensitivities of k_1 and r_s, shown in Fig. 2, are relatively high when compared with those of the other parameters. Nevertheless, the sensitivity curves of k_1 and r_s are symmetrical in shape on the horizontal axis but have different magnitudes, implying that the parameters k_1 and r_s are highly correlated. This may be the reason why SA may sometimes misidentify the values of k_1 and d_{sk}.

Table 4 shows the estimated results for Scenario 11 when the unknown parameters are the same as those in Scenario 2 except that d_{sk} is known. In this scenario, k_1 is precisely identified with a mean value of 1.06×10^{-5} m/s and RE of 6.20%; additionally, k_2 and S_{s1} are accurately estimated when compared with those presented in Scenario 2. Similarly, in Scenario 12, k_1 is regarded as a known parameter and other parameters are the same as those given in Scenario 2. The results listed in Table 4 show a fairly good estimation on d_{sk} with a mean value of 0.286 m and RE of -7.42%. In addition, more accurate estimations on k_2 and S_{s1} are presented when compared with those shown in Scenario 2. Scenarios 11

Table 3 The target values and estimated results for Scenarios 1 and 2 while the analyzed WWL data is extended to 180 s

Case	Estimated results					
	k_1 (m/s)	k_2 (m/s)	S_{s1} (1/m)	S_{s2} (1/m)	d_{sk} (m)	SEE
Target value	1.00×10^{-5}	1.00×10^{-4}	1.00×10^{-4}	1.00×10^{-4}	0.9085	–
9a	1.01×10^{-5}	1.01×10^{-4}	9.67×10^{-5}	8.67×10^{-5}	0.817	3.26×10^{-4}
9b	1.10×10^{-5}	9.91×10^{-5}	7.13×10^{-5}	9.83×10^{-5}	1.206	8.99×10^{-4}
9c	1.00×10^{-5}	9.95×10^{-5}	9.95×10^{-5}	9.57×10^{-5}	0.915	7.88×10^{-4}
9d	1.00×10^{-5}	1.10×10^{-4}	9.96×10^{-5}	9.12×10^{-5}	0.936	8.48×10^{-4}
9e	1.04×10^{-5}	1.02×10^{-4}	8.68×10^{-5}	9.99×10^{-5}	1.029	1.05×10^{-3}
Mean	1.03×10^{-5}	1.02×10^{-4}	9.08×10^{-5}	9.44×10^{-5}	0.981	–
RE	3.00%	2.32%	−9.22%	−5.64%	7.94%	–
Target value	1.00×10^{-5}	1.00×10^{-4}	1.00×10^{-4}	1.00×10^{-4}	0.3085	–
10a	1.51×10^{-5}	1.01×10^{-4}	3.22×10^{-5}	1.95×10^{-5}	0.756	3.08×10^{-4}
10b	1.33×10^{-5}	1.00×10^{-4}	4.69×10^{-5}	6.72×10^{-5}	0.588	9.90×10^{-4}
10c	9.99×10^{-6}	9.93×10^{-5}	9.57×10^{-5}	4.30×10^{-6}	0.243	7.92×10^{-4}
10d	1.63×10^{-5}	1.05×10^{-4}	2.64×10^{-5}	1.50×10^{-5}	0.940	8.47×10^{-4}
10e	1.63×10^{-5}	9.62×10^{-5}	2.70×10^{-5}	9.86×10^{-5}	1.104	9.87×10^{-4}
Mean	1.42×10^{-5}	1.00×10^{-4}	4.56×10^{-5}	4.09×10^{-5}	0.726	–
RE	41.98%	0.30%	−54.36%	−59.08%	135.40%	–

Table 4 The target values and estimated results of Scenario 2 on condition that d_{sk} and k_1 is regarded as a known parameter, respectively, in Scenarios 11 and 12

Case	Estimated results					
	k_1 (m/s)	k_2 (m/s)	S_{s1} (1/m)	S_{s2} (1/m)	d_{sk}(m)	SEE
Target value	1.00×10^{-5}	1.00×10^{-4}	1.00×10^{-4}	1.00×10^{-4}	0.3085	–
11a	1.05×10^{-5}	9.64×10^{-5}	8.44×10^{-5}	2.75×10^{-5}	–	2.85×10^{-4}
11b	1.01×10^{-5}	1.10×10^{-4}	9.50×10^{-5}	4.27×10^{-5}	–	6.70×10^{-4}
11c	1.06×10^{-5}	9.58×10^{-5}	8.82×10^{-5}	2.41×10^{-5}	–	8.41×10^{-4}
11d	1.13×10^{-5}	1.01×10^{-4}	7.14×10^{-5}	3.17×10^{-6}	–	7.51×10^{-4}
11e	1.06×10^{-5}	1.03×10^{-4}	8.73×10^{-5}	1.74×10^{-5}	–	8.12×10^{-4}
Mean	1.06×10^{-5}	1.01×10^{-4}	8.53×10^{-5}	2.30×10^{-5}	–	–
RE	6.20%	1.24%	−14.74%	−77.03%	–	–
Target value	1.00×10^{-5}	1.00×10^{-4}	1.00×10^{-4}	1.00×10^{-4}	0.3085	–
12a	–	9.55×10^{-5}	9.42×10^{-5}	3.89×10^{-5}	0.284	2.80×10^{-4}
12b	–	1.09×10^{-4}	9.84×10^{-5}	7.84×10^{-5}	0.310	6.72×10^{-4}
12c	–	9.08×10^{-5}	9.99×10^{-5}	8.00×10^{-5}	0.296	8.38×10^{-4}
12d	–	1.00×10^{-4}	9.35×10^{-5}	1.74×10^{-5}	0.272	7.55×10^{-4}
12e	–	1.01×10^{-4}	9.80×10^{-5}	1.30×10^{-5}	0.266	8.14×10^{-4}
Mean	–	9.93×10^{-5}	9.68×10^{-5}	4.55×10^{-5}	0.286	–
RE	–	−0.74%	−3.20%	−54.46%	−7.42%	–

The analyzed WWL data is within 15 s.

and 12 show a highly correlated relationship between the unknown parameters k_1 and d_{sk} in a positive-skin scenario, which may lead to misidentification of these two parameters. Other positive-skin scenarios also have a normalized sensitivity plot similar to that shown in Fig. 2.

Analyzing WWL data in an observation well

Assume that an observation well is located at 3 m away from the test well. In Scenario 13, we analyzed the 15-s WWL data simulated in the test and observation wells simultaneously. The estimated results are presented in Table 5.

Fig. 3 shows the normalized sensitivities of the five parameters using the WWL data measured in the observation well of Scenario 13. The parameter k_1 appears to have positive influence on the WWL over the 1000-s interval, while r_s has negative influence at early 100-s interval, which becomes positive afterwards. In addition, the normalized sensitivity curves of k_1 and r_s are not highly correlated. Table 5 shows obvious improvements in the estimations of k_1 and d_{sk} with

Table 5 The target values and estimated results for Scenario 2 while the hypothetical WWL data is analyzed in the test well as well as in an observation well which is located at 3 m away from the test well

Case	Estimated results					
	k_1 (m/s)	k_2 (m/s)	S_{s1} (1/m)	S_{s2} (1/m)	d_{sk} (m)	SEE
Target value	1.00×10^{-5}	1.00×10^{-4}	1.00×10^{-4}	1.00×10^{-4}	0.3085	–
13a	1.01×10^{-5}	1.02×10^{-4}	9.98×10^{-5}	1.00×10^{-4}	0.314	3.00×10^{-4}
13b	1.02×10^{-5}	1.10×10^{-4}	9.83×10^{-5}	9.99×10^{-5}	0.329	6.89×10^{-4}
13c	1.06×10^{-5}	1.06×10^{-4}	9.95×10^{-5}	1.00×10^{-4}	0.354	7.33×10^{-4}
13d	9.70×10^{-6}	9.26×10^{-5}	1.00×10^{-4}	9.98×10^{-5}	0.282	7.91×10^{-4}
13e	1.06×10^{-5}	1.12×10^{-4}	9.71×10^{-5}	9.99×10^{-5}	0.356	7.83×10^{-4}
Mean	1.02×10^{-5}	1.05×10^{-4}	9.89×10^{-5}	9.99×10^{-5}	0.327	–
RE	2.40%	4.52%	−1.06%	−0.08%	6.00%	–

Both the analyzed WWL data in test and observation wells are within 15 s.

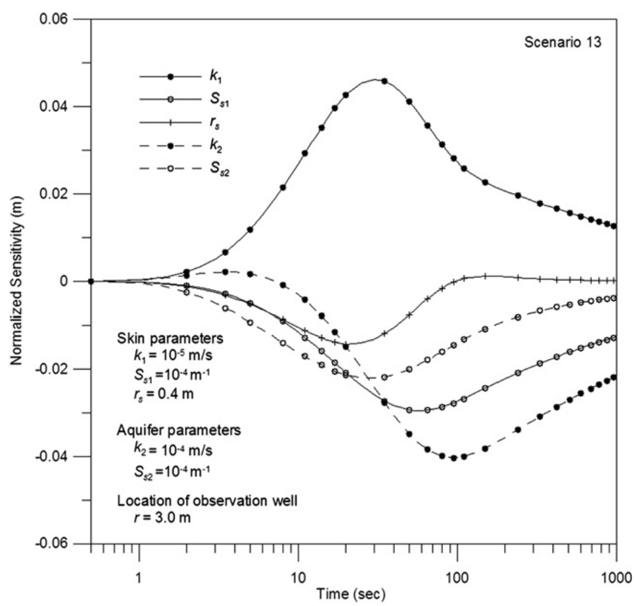

Figure 3 The normalized sensitivities of the five parameters over a period of 1000 s for Scenario 13.

mean values of 1.02×10^{-5} m and 0.327 m, respectively, and RE of 2.40% and 6.00%. Moreover, the WWL data simulated in the observation well are more sensitive to the change in S_{s1} and S_{s2} than that in r_s. Particularly, the normalized sensitivities of S_{s1} and S_{s2} depicted in Fig. 3 become more sensitive than those displayed in Fig. 2b. The results also show significant improvements in estimations of S_{s1} and S_{s2} with mean values of 9.89×10^{-5} m^{-1} and 9.99×10^{-5} m^{-1}, and RE of −1.06% and −0.08%, respectively. In this case, analyzing the composite WWL data in the test and observation wells gives better results for the five parameters.

Comparison of different positive-skin scenarios

Table 6 displays the estimated results of the five parameters for four positive-skin scenarios by analyzing their 15-s WWL data. Scenario 14, whose results are shown in Table 6a, represents an aquifer with a positive skin having target values of parameters same as those in Scenario 2, except that k_1 equals 10^{-6}. Comparing Scenario 14 with Scenario 2 shows that Scenario 14 gives a much better estimation on skin parameters k_1, S_{s1}, and d_{sk}, but a worse estimation on k_2. Such unfavorable estimation on k_2 can be improved by using a longer set of WWL data that explicitly reflects the aquifer behavior. This phenomenon also appears in Scenario 4 in contrast to Scenario 2. In other words, it is much easier to identify skin parameters accurately from the case having a distinct difference between k_1 and k_2. Table 6b demonstrates three positive-skin scenarios having different values of d_{sk} and shows that the estimations on skin parameters k_1,

Table 6a The target values and estimated results for a positive-skin aquifer with a distinct difference between the hydraulic conductivities of skin and formation zones

Case	Estimated results					
	k_1 (m/s)	k_2 (m/s)	S_{s1} (1/m)	S_{s2} (1/m)	d_{sk} (m)	SEE
Target value	1.00×10^{-6}	1.00×10^{-4}	1.00×10^{-4}	1.00×10^{-4}	0.3085	–
14a	1.00×10^{-6}	8.51×10^{-4}	9.70×10^{-5}	8.51×10^{-6}	0.315	3.26×10^{-4}
14b	1.00×10^{-6}	6.51×10^{-4}	9.41×10^{-5}	8.97×10^{-5}	0.315	7.38×10^{-4}
14c	1.03×10^{-6}	6.59×10^{-4}	9.94×10^{-5}	5.50×10^{-5}	0.349	8.21×10^{-4}
14d	1.38×10^{-6}	8.33×10^{-5}	4.87×10^{-5}	7.10×10^{-5}	0.623	7.49×10^{-4}
14e	1.02×10^{-6}	7.66×10^{-4}	9.97×10^{-5}	9.11×10^{-5}	0.324	6.98×10^{-4}
Mean	1.09×10^{-6}	6.02×10^{-4}	8.78×10^{-5}	6.31×10^{-5}	0.385	–
RE	8.60%	502.06%	−12.22%	−36.94%	24.86%	–

Table 6b The target values and estimated results for positive-skin scenarios with d_{sk} range from 0.1085 to 1.3085

Case	Estimated results					
	k_1 (m/s)	k_2 (m/s)	S_{s1} (1/m)	S_{s2} (1/m)	d_{sk} (m)	SEE
Target value	1.00×10^{-5}	1.00×10^{-4}	1.00×10^{-4}	1.00×10^{-4}	0.1085	–
15a	2.31×10^{-5}	9.97×10^{-5}	1.71×10^{-5}	1.83×10^{-5}	0.493	3.16×10^{-4}
15b	3.16×10^{-5}	1.05×10^{-4}	7.25×10^{-6}	8.91×10^{-6}	1.324	7.57×10^{-4}
15c	2.47×10^{-5}	9.34×10^{-5}	1.55×10^{-5}	2.82×10^{-5}	0.629	6.99×10^{-4}
15d	2.99×10^{-5}	9.97×10^{-5}	6.71×10^{-6}	9.38×10^{-6}	1.015	7.75×10^{-4}
15e	1.92×10^{-5}	9.66×10^{-5}	2.93×10^{-5}	5.02×10^{-5}	0.346	7.07×10^{-4}
Mean	2.57×10^{-5}	9.89×10^{-5}	1.52×10^{-5}	2.30×10^{-5}	0.761	–
RE	157.00%	−1.12%	−84.83%	−77.00%	601.75%	–
Target value	1.00×10^{-5}	1.00×10^{-4}	1.00×10^{-4}	1.00×10^{-4}	0.6085	–
16a	1.04×10^{-5}	1.11×10^{-4}	9.10×10^{-5}	8.79×10^{-5}	0.684	2.67×10^{-4}
16b	1.10×10^{-5}	2.23×10^{-4}	7.70×10^{-5}	1.31×10^{-5}	0.824	7.84×10^{-4}
16c	1.11×10^{-5}	1.04×10^{-4}	7.43×10^{-5}	3.72×10^{-5}	0.768	8.64×10^{-4}
16d	1.17×10^{-5}	1.16×10^{-4}	6.25×10^{-5}	4.69×10^{-5}	0.914	8.02×10^{-4}
16e	1.13×10^{-5}	1.13×10^{-4}	7.25×10^{-5}	8.08×10^{-5}	0.851	8.07×10^{-4}
Mean	1.11×10^{-5}	1.33×10^{-4}	7.55×10^{-5}	5.32×10^{-5}	0.808	–
RE	11.00%	33.40%	−24.54%	−46.82%	32.82%	–
Target value	1.00×10^{-5}	1.00×10^{-4}	1.00×10^{-4}	1.00×10^{-4}	1.3085	–
17a	1.01×10^{-5}	2.40×10^{-4}	9.69×10^{-5}	3.19×10^{-5}	1.402	2.98×10^{-4}
17b	9.95×10^{-6}	1.08×10^{-4}	9.99×10^{-5}	4.30×10^{-5}	1.241	6.63×10^{-4}
17c	1.00×10^{-5}	9.02×10^{-4}	9.99×10^{-5}	2.33×10^{-5}	1.463	8.76×10^{-4}
17d	1.02×10^{-5}	7.88×10^{-5}	9.37×10^{-5}	8.36×10^{-5}	1.348	8.84×10^{-4}
17e	1.00×10^{-5}	5.12×10^{-4}	9.97×10^{-5}	8.75×10^{-5}	1.401	7.42×10^{-4}
Mean	1.01×10^{-5}	3.68×10^{-4}	9.80×10^{-5}	5.39×10^{-5}	1.371	–
RE	0.50%	268.16%	−1.98%	−46.14%	4.78%	–

S_{s1}, and d_{sk} with a larger target d_{sk} are better than those with a smaller one. Similarly, the unfavorable estimations on the two aquifer parameters k_2 and S_{s2} can be improved by using a longer set of WWL data. In short, it is difficult to identify the skin parameters correctly for a positive-skin aquifer having an obscure skin characteristic, e.g., a similar k_1 and k_2 and/or a small d_{sk}; nevertheless, the aquifer parameters could be identified accurately by analyzing a longer set of WWL data.

Conclusions

In this paper, we propose a methodology that couples Moench and Hsieh's solution (1985) with SA algorithm to identify three skin parameters (k_1, S_{s1}, and d_{sk}) and two aquifer parameters (k_2 and S_{s2}) simultaneously for a slug test performed in a skin-affected confined aquifer system. The WWL data analyzed in this paper were generated by Moench and Hsieh's solution (1985) and with a set of standard normally distributed noise added for both positive-skin and negative-skin scenarios.

From the hypothetical Scenarios 1 to 8, the negative-skin scenarios as a whole give better estimated results than those of the positive-skin scenarios. The sensitivity analysis is performed to demonstrate the WWL response to the relative change in aquifer parameters when the well of slug test has a positive skin. In the sensitivity plots, the influence of k_2 on the WWL is insensitive at the initial period of the test. Accordingly, analyzing a longer set of WWL data, though consumes much computing time, would result in a better estimation on k_2 for aquifers with a positive skin. In addition, the parameters k_1 and r_s in the case of a positive-skin aquifer are highly correlated. Hence, one may misidentify the values of k_1 and d_{sk} by estimating them using an optimization technique, especially for positive-skin aquifers. Note that d_{sk} (or r_s) is usually taken as an input data in some data-analyzed software but it is actually an invisible and unknown parameter. Impetuously presuming an arbitrary value of d_{sk} and only estimating the other four parameters may confound the parameter estimation for the other four parameters. On the other hand, the results of sensitivity analysis reveal that whichever optimization technique is applied, the insensitivities of S_{s1} and S_{s2} usually lead to inaccurately estimated results when determining the skin and aquifer parameters simultaneously. Nevertheless, analyzing the composite WWL data, which were obtained from the test and observation wells, could significantly improve the estimations of S_{s1} and S_{s2}.

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