# New Table-Switching and Data-Multiplexing Schemes of Rate-Compatible Punctured Convolutional Codes for Unequal Error Protection

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***Abstract*— In this letter, rate-compatible punctured convolutional (RCPC) codes are further investigated for the application to unequal error protection (UEP). Besides the rate-compatible restriction, we show that puncturing tables should be switched in a special way called soft-switching to guarantee the designed UEP performance. A new data-multiplexing scheme is also proposed for RCPC codes which can achieve similar UEP performance as the conventional scheme but requires no extra zero-padding for frame termination to improve the system throughput.**

***Index Terms*— Punctured convolutional codes, rate-compatible criterion, unequal error protection.**

## I. INTRODUCTION

PUNCTURED convolutional codes were first introduced in [1] by periodically deleting some coded bits of ordinary convolutional codes. Later in [2], the puncturing process was further regulated by a rate-compatible criterion to guarantee smooth transition between different rates. Owing to flexible choices of rates and ease for decoding all children[1] codes by a single decoder of their parent code, rate-compatible punctured convolutional (RCPC) codes have been extensively employed for the application to unequal error protection (UEP) [2]-[5].

However, we observe that RCPC codes may fail to achieve the designed performance if the puncturing tables are not switched in a proper way. A new way for table-switching, called soft-switching, is presented here which guarantees that distances of the codewords accross the switching boundaries of puncturing tables can be effectively lower-bounded to avoid the unpredictable performance degradation. In addition, RCPC codes are usually equipped with a data multiplexing scheme [2] which can minimize the potential distance loss of RCPC codes but at the expense of extra tail bits for frame termination. In this letter, we demonstrate that for RCPC codes the same distance loss can be obtained no matter the puncturing tables are switched in a forward or backward order by soft-switching. Based on such a result, a new multiplexing scheme is proposed which can achieve similar UEP performance as the conventional one but requires no additional overheads to improve system throughput.

Manuscript received April 22, 2007. The associate editor coordinating the review of this letter and approving it for publication was Dr. Cha'o-Ming Chang. This work was supported by the Aming for the Top University Plan and National Science Council of the Republic of China under Grant NSC 96–2219–E–009–011.

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Digital Object Identifier 10.1109/LCOMM.2007.070627.

[1]For convenience, the original convolutional code to be punctured is called the parent code and the resulting punctured code is called the child code.

## II. SOFT-SWITCHING AND THE NEW MULTIPLEXING SCHEME FOR UEP

Consider $W$ groups of source data $S_l$'s, each of the required bit error rate (BER) $P_{b,l}$; assume $P_{b,1} \geq P_{b,2} \geq \cdots \geq P_{b,W}$ without loss of generality. To provide UEP for $S_l$'s by puncturing, an $(n,\ k)$ parent code $C$ is first chosen together with proper puncturing tables $A(l)$'s of period $p$ satisfying the rate-compatible criterion, i.e.,

$$\begin{aligned}&\text{if } a_{u,v}(i)=1, \text{ then } a_{u,v}(j)=1,\\ &\qquad \forall\ 0\leq u<n,\ 0\leq v<p,\ 1\leq i<j\leq W\end{aligned} \tag{1}$$

where $a_{u,v}(l)$ denotes the $(u,v)$th entry of $A(l)$, to generate a family of children codes $\hat{C}_l$'s, each of free distance $d_f(\hat{C}_l)$ to satisfy $P_{b,l}$. Then $A(l)$ is switched for puncturing as $S_l$ is fed to the encoder of $C$. In this way, $S_l$ can be protected by $\hat{C}_l\ \forall\ l$, thus fulfilling the desire for UEP.

Suppose $A(l)$ is switched for puncturing during the interval $[\hat{t}_l, \tilde{t}_l]$. An intuitive way for table-switching, called hard-switching, is to initiate a new puncturing process by $A(l)$ at time $\hat{t}_l$. Denote by $c_{i,t}$ the coded bit of the $i$th output stream of the encoder of $C$ at time $t\ \forall\ 0\leq i<n$. By hard-switching, $c_{i,t}$ is processed as the following, $\forall\ t\in[\hat{t}_l, \tilde{t}_l]$:

$$\begin{cases} c_{i,t} \text{ is allowed for transmission,} & \text{if } a_{i,(t-\hat{t}_l)\bmod p}(l)=1\\ c_{i,t} \text{ is deleted from the encoder outputs,} & \text{if } a_{i,(t-\hat{t}_l)\bmod p}(l)=0.\end{cases} \tag{2}$$

For the case of $p \mid (\tilde{t}_l-\hat{t}_l+1)\ \forall\ l$, which is a common but implicit assumption in the previous researches about RCPC codes for UEP, the potential distance loss due to dynamic switching can be effectively alleviated by the rate-compatible criterion. However, for the other case of $p \nmid (\tilde{t}_l-\hat{t}_l+1)$, we observe that hard-switching may result in unpredictable codeword distance even though the puncturing tables are rate-compatible. For example, consider a (2,1) parent code with the following rate-compatible puncturing tables of $p$ = 5:

$$A(1)=\begin{pmatrix}0&0&0&1&1\\1&0&1&1&1\end{pmatrix} \text{ and } A(2)=\begin{pmatrix}0&0&1&1&1\\1&0&1&1&1\end{pmatrix}. \tag{3}$$

For a codeword $\boldsymbol{c}=(c_{0,0}c_{1,0}, c_{0,1}c_{1,1}, \cdots, c_{0,9}c_{1,9})$ = (10, 00, 10, 11, 11, 10, 00, 10, 11, 11), the resulting sequences after puncturing by either $A(1)$ or $A(2)$ alone are

$$\boldsymbol{c}_{A(1)} = (\times0, \times\times, \times0, 11, 11, \times0, \times\times, \times0, 11, 11)$$

and

$$\boldsymbol{c}_{A(2)} = (\times0, \times\times, 10, 11, 11, \times0, \times\times, 10, 11, 11)$$

with Hamming weights $d(\boldsymbol{c}_{A(1)})=8$ and $d(\boldsymbol{c}_{A(2)})=10$, where bits marked by $\times$ are deleted. Suppose $\boldsymbol{c}$ is punctured by $A(1)$ and $A(2)$ in $[0,4]$ and $[5,9]$ respectively such that $p \mid (\tilde{t}_l - \hat{t}_l+1)$ for $l=1,2$. By (2), the punctured codeword is

$$\boldsymbol{c}_{A(1)|A(2)} = (\times0, \times\times, \times0, 11, 11, \times0, \times\times, 10, 11, 11)$$

with $d(\boldsymbol{c}_{A(1)|A(2)})=9$. Therefore, we can have $d(\boldsymbol{c}_{A(1)}) \leq d(\boldsymbol{c}_{A(1)|A(2)}) \leq d(\boldsymbol{c}_{A(2)})$ as assured by the rate-compatible

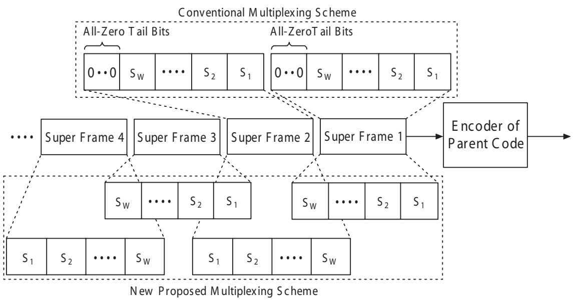


Fig. 1. Multiplexing schemes for UEP.

criterion. However, if $\boldsymbol{c}$ is punctured by $A(1)$ in $[0,2]$ and then switched to $A(2)$ in $[3,9]$ such that $p \nmid (\tilde{t}_l - \hat{t}_l + 1)$ for $l = 1, 2$[2], the codeword after puncturing by (2) turns to be

$$\boldsymbol{c}_{A(1)|A(2)} = (\times 0, \times\times, \times 0, \times 1, \times\times, 10, 00, 10, \times 1, \times\times)$$

with $d(\boldsymbol{c}_{A(1)|A(2)})$=4.In this case, we have $d(\boldsymbol{c}_{A(1)|A(2)}) < d(\boldsymbol{c}_{A(1)})$; the rate-compatibility originally designed to guarantee the UEP performance is now destroyed by hard-switching.

To avoid such an unexpected distance loss due to hard-switching, we propose a new switching scheme called soft-switching which processes $c_{i,t}$'s by

$$\begin{cases} c_{i,t} \text{ is allowed for transmission,} & \text{if } a_{i,t \bmod p}(l)=1 \\ c_{i,t} \text{ is deleted from the encoder outputs,} & \text{if } a_{i,t \bmod p}(l)=0 \end{cases} \quad (4)$$

as $A(l)$ is used for puncturing in $[\hat{t}_l, \tilde{t}_l]$. By soft-switching, as presented in Appendix, we can show that all codewords across the switching boundary between $A(i)$ and $A(j)$ $\forall\ 1 \le i, j \le W$ will have a distance $\min(d_f(\hat{C}_i),\ d_f(\hat{C}_j))$ at least, no matter whether $p \mid (\tilde{t}_l - \hat{t}_l + 1)$ or $p \nmid (\tilde{t}_l - \hat{t}_l + 1)$; the UEP performance assured by the rate-compatibility can hence been achieved successfully. Revisit the above example. If $\boldsymbol{c}$ is now punctured by $A(1)$ and $A(2)$ in $[0,2]$ and $[3,9]$ respectively by (4), we have

$$\boldsymbol{c}_{A(1)|A(2)} = (\times 0, \times\times, \times 0, 11, 11, \times 0, \times\times, 10, 11, 11)$$

and $d(\boldsymbol{c}_{A(1)|A(2)}) = 9$ which implies $d(\boldsymbol{c}_{A(1)}) \le d(\boldsymbol{c}_{A(1)|A(2)}) \le d(\boldsymbol{c}_{A(2)})$. Such an observation also indicates that around the boundary of rate change the bits in the $A(1)$-phase will receive more protection than those encoded by the pure $\hat{C}_1$ while the bits in the $A(2)$-phase will suffer a little BER loss compared with those protected by the pure $\hat{C}_2$. In general, at the expense of some BER compromise between the bits near the transition boundary, soft-switching can successfully guarantee the designed performance of RCPC codes for UEP.

In addition, as RCPC codes are used for UEP, $S_l$'s are recommanded to be grouped into super frames before encoding as the conventional multiplexing scheme in Fig. 1 [2]. In this scheme, $S_l$ is followed by $S_{l+1}$ $\forall\ 1 \le l < W$ such that for the adjacent children codes the minimum loss of free distance guaranteed by the rate-compatible criterion, i.e., $d_f(\hat{C}_{l+1}) - d_f(\hat{C}_l)$ $\forall\ 1 \le l < W$, can be achieved, but extra zero bits are inserted at the end of every super frame to avoid the abrupt switching from $A(W)$ to $A(1)$. However, as revealed in Appendix, we also show that the distance between any two codewords will still be lower bounded by $\min(d_f(\hat{C}_i),\ d_f(\hat{C}_j))$ no matter the puncturing tables are switched either from $A(i)$ to $A(j)$ or from $A(j)$ to $A(i)$ $\forall\ 1 \le i, j \le W$. Suppose $S_l$'s are multiplexed by the new proposed scheme in Fig. 1, where no extra bits are required for frame termination but $S_l$'s are multiplexed in a reverse order for alternate super frames. Accordingly, the puncturing tables are restricted to switch either from $A(l)$ to $A(l+1)$ or from $A(l+1)$ to $A(l)$ $\forall\ 1 \le l < W$; the same amount of distance loss as the conventional scheme can thus be obtained during the transition phase between the adjacent tables. In general, for a parent code of constraint length $K$ and a super frame of $L$ bits, the new multiplexing scheme can improve the system throughput by a factor of $\frac{L+K-1}{L}$, since $K$-1 zero bits are required in the conventional scheme for frame termination [2].

[2]In this case, the maximum distance loss due to hard-switching is presented. For other choices of $\hat{t}_1$ and $\hat{t}_2$, a better distance can be obtained. However, soft-switching can still result in a larger distance than hard-switching.

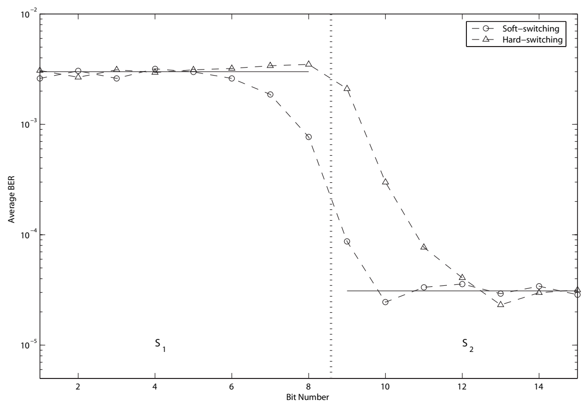


Fig. 2. Average BER of source bits in different positions within a super frame at signal-to-noise ratio 5 dB. (The solid lines denote the designed BERs of the children codes.)

## III. Simulation Results

In this section, simulation results are presented to verify performance of soft-switching and the new proposed multiplexing scheme. Coded bits are assumed with binary phase-shift keying modulation for transmission over additive white Gaussian noise channels. Consider the best rate-1/2 parent code with generator matrix $[D^4+D+1,\ D^4+D^3+D^2+1]$ of constraint length 5 searched in [3] which is punctured by the tables in (3) to provide 2-level UEP. Suppose $S_1$ and $S_2$ are of 8 and 7 bits, respectively, per super frame such that $p \nmid 8$ and $p \nmid 7$. The effect of BER compromise mentioned in Section II can be observed from the BER curves in Fig. 2. As expected, soft-switching can provide smooth transition between rates. By hard-switching, the first two bits in $A(2)$-phase however suffer an unacceptable loss of BER compared with the designed performance.

Furthermore, based on the discussion in Section II, the higher the switching rate is, the more gain of soft-switching over hard-switching is expected. Consider super frames of various lengths containing $S_1$: $8 * l$ bits and $S_2$: $7 * l$ bits for $l$ = 1,2,3,4,6,7,8,9,11,22,44,111 (with $p \nmid 8 * l$ and $p \nmid 7 * l$ $\forall\ l$), respectively; the shorter the super frame is, the higher the switching rate is. By employing the above RCPC codes, the average BERs of $S_1$ and $S_2$ for different lengths, i.e., different switching rates, are depicted in Fig. 3. From the simulation results, soft-switching is observed to fulfil the

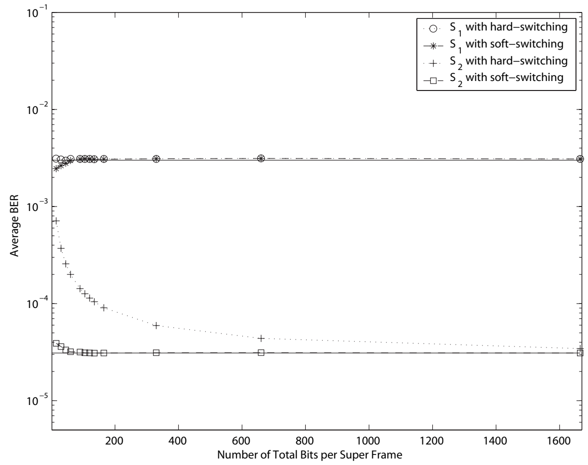


Fig. 3. Average BER of $S_1$ and $S_2$ for super frames of various lengths at signal-to-noise ratio 5 dB. (The upper and lower solid lines denote the designed BERs of $S_1$ and $S_2$, respectively.)

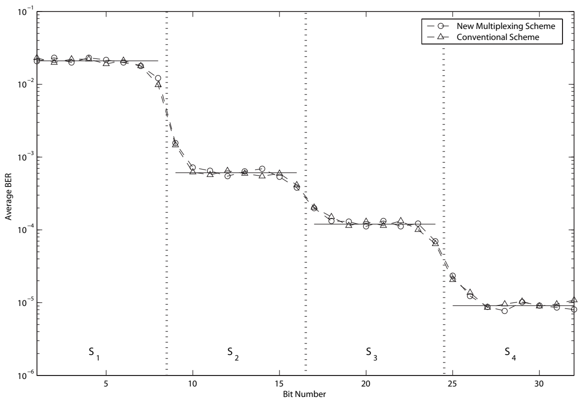


Fig. 4. Average BER of source bits in different positions within a super frame for different multiplexing schemes at signal-to-noise ratio 2 dB; for the new proposed scheme, the plotted BER of the $i$th source bit is averaged with respect the $i$th bit and the (33-$i$)-th bit in alternate super frames $\forall\ 1 \leq i \leq 32$. (The solid lines denote the designed BERs of the children codes.)

designed BERs for both of $S_1$ and $S_2$, while hard-switching results in an unacceptable loss of BER for $S_2$ for the case of high switching rate.

In addition, the same parent code with the following rate-compatible tables [3]:

$$\begin{pmatrix} 1&1&1&1 \\ 0&1&0&0 \end{pmatrix}, \begin{pmatrix} 1&1&1&1 \\ 0&1&0&1 \end{pmatrix}, \begin{pmatrix} 1&1&1&1 \\ 1&1&0&1 \end{pmatrix}, \begin{pmatrix} 1&1&1&1 \\ 1&1&1&1 \end{pmatrix}$$

are simulated together with both of the multiplexing schemes for comparison. Suppose there are four groups of data $S_1$, $S_2$, $S_3$, $S_4$, each containing 8 bits per super frame. In this case, four extra all-zero bits are appended at the end of every frame for the conventional scheme. Observed from Fig. 4, the new scheme can provide almost the same UEP performance as the conventional one, which is consistent with the analysis in Section II.

## Appendix

Consider an $(n, k)$ parent code $C$ which is punctured by $W$ rate-compatible tables $A(l)$'s of period $p$ to generate a family of children codes $\hat{C}_l$'s, each of free distance $d_f(\hat{C}_l)$, with $d_f(\hat{C}_l) \leq d_f(\hat{C}_{l+1})\ \forall\ 1 \leq l < W$. Suppose $\phi$ arbitrary tables $A(l_1)$, $A(l_2)$, $\cdots$, $A(l_\phi)$ with $1 \leq l_i \leq W$ are successively switched for puncturing during $[\hat{t}_i, \tilde{t}_i]\ \forall\ 1 \leq i \leq \phi$, respectively. By soft-switching, we would like to show that all codewords across the switching boundaries between $A(l_1)$, $A(l_2)$, $\cdots$, $A(l_\phi)$ will have a distance $\min_{1\leq i\leq\phi} d_f(\hat{C}_{l_i})$ at least, hence achieving the UEP performance assured by the rate-compatibility no matter whether $p \mid (\tilde{t}_i - \hat{t}_i + 1)$ or $p \nmid (\tilde{t}_i - \hat{t}_i + 1)\ \forall\ i$.

Consider any non-zero punctured sequence $\boldsymbol{c}_{A(l_1)|A(l_2)|\cdots|A(l_\phi)}$ across the boundaries between $A(l_i)$'s, which is obtained by puncturing a codeword $\boldsymbol{c} = (c_{u,t}\ \forall\ 0 \leq u < n, t)$ of $C$. Denote by $d(\boldsymbol{c}_{A(l_1)|A(l_2)|\cdots|A(l_\phi)})$ the Hamming weight of $\boldsymbol{c}_{A(l_1)|A(l_2)|\cdots|A(l_\phi)}$. By soft-switching, we have

$$d(\boldsymbol{c}_{A(l_1)|A(l_2)|\cdots|A(l_\phi)}) = \sum_{i=1}^{\phi} \sum_{t=\hat{t}_i}^{\tilde{t}_i} \sum_{u=0}^{n-1} a_{u,t \bmod p}(l_i) \cdot 1(c_{u,t}) \quad \text{(A-1)}$$

where $a_{u,v}(l)$ denotes the $(u,v)$th entry of $A(l)\ \forall\ u, v, l$, and $1(x)$ is defined as the function with $1(x) = 1$ if $x \neq 0$ and $1(x) = 0$ if $x = 0$. Let $m = \arg\min_i d_f(\hat{C}_{l_i})$, i.e., $l_m = \min_{1\leq i\leq\phi} l_i$. Since $C$ is successively punctured by $A(l_1)$, $A(l_2)$, $\cdots$, $A(l_\phi)$, the whole time interval can be divided into the subsequent non-overlapping subintervals $[\hat{t}_i, \tilde{t}_i]\ \forall\ 1 \leq i \leq \phi$. Suppose $\boldsymbol{c}$ is now punctured by $A(l_m)$ alone; by soft-switching, the corresponding weight after puncturing can be expressed as

$$\begin{aligned} d(\boldsymbol{c}_{A(l_m)}) &= \sum_{t} \sum_{u=0}^{n-1} a_{u,t \bmod p}(l_m) \cdot 1(c_{u,t}) \\ &= \sum_{i=1}^{\phi} \sum_{t=\hat{t}_i}^{\tilde{t}_i} \sum_{u=0}^{n-1} a_{u,t \bmod p}(l_m) \cdot 1(c_{u,t}). \end{aligned} \quad \text{(A-2)}$$

By (A-1), (A-2), and the rate-compatible restriction: $a_{u,v}(l_i) - a_{u,v}(l_m) \geq 0\ \forall\ u, v, i$, we have

$$\begin{aligned} &d(\boldsymbol{c}_{A(l_1)|A(l_2)|\cdots|A(l_\phi)}) - d(\boldsymbol{c}_{A(l_m)}) \\ &= \textstyle\sum_{i=1}^{\phi} \sum_{t=\hat{t}_i}^{\tilde{t}_i} \sum_{u=0}^{n-1} (a_{u,t \bmod p}(l_i) - a_{u,t \bmod p}(l_m)) \cdot 1(c_{u,t}) \geq 0. \end{aligned} \quad \text{(A-3)}$$

Moreover, by definition, we also have

$$d_f(\hat{C}_{l_m}) = \min_{\forall \boldsymbol{c}_{A(l_m)} \neq \mathbf{0}} d(\boldsymbol{c}_{A(l_m)}) \quad \text{(A-4)}$$

where $\mathbf{0}$ denotes the all-zero codeword. By (A-3) and (A-4), it implies

$$\begin{aligned} d(\boldsymbol{c}_{A(l_1)|A(l_2)|\cdots|A(l_\phi)}) &\geq d(\boldsymbol{c}_{A(l_m)}) \geq \min\nolimits_{\forall \boldsymbol{c}_{A(l_m)} \neq \mathbf{0}} d(\boldsymbol{c}_{A(l_m)}) \\ &= d_f(\hat{C}_{l_m}) = \min\nolimits_{1\leq i\leq\phi} d_f(\hat{C}_{l_i}). \end{aligned}$$

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