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ScienceDirect
European Journal of Operational Research 180 (2007) 1116–1129
EUROPEAN JOURNAL OF OPERATIONAL RESEARCH
www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

Optimal tool replacement for processes with low fraction defective

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Received 22 March 2005; accepted 12 May 2006
Available online 28 August 2006

Abstract

Tool wear is a frequent and natural part in many machining processes and is a systematic assignable cause. The fraction of defectives would rise as the tool deteriorates. When the fraction defective reaches a certain level, the tool must be replaced. To minimize the defective parts and the overall tool costs, the optimal tool replacement time needs to be determined. Process capability indices (PCIs) have been effectively used in the manufacturing industry to measure the fraction of defectives. Conventional methods of capability measurement become inaccurate since the process data is contaminated by the assignable cause variation. In order to determine the optimal tool replacement time to maintain maximum product quality, conventional capability calculation must be modified. Considering process capability changes dynamically, an estimator of C_{pmk} is investigated. We obtain an exact form of the sampling distribution in the presence of a systematic assignable cause. This study provides an effective management policy for optimal tool replacement under low fraction of defectives. To illustrate the application of this procedure, a case study involving the tool wear problem is presented.

Keywords: Quality management; Critical value; Process capability index; Replacement time; Tool wear

1. Introduction

In automated machines, tools occupy a prominent place in producing quality goods. The tool will wear gradually as the manufacturing process proceeds. For instance, the machining operation shapes a production part using, cutting, drilling, or grinding operations, and so on. While such wear is unavoidable, tools must be controlled to maintain product quality and efficient tool utilization. One important issue for tool wear control is the tool replacement policy. The tool should be replaced when product quality becomes worse. Process capability indices have been widely used in the manufacturing industry for measuring process quality, particularly, for processes with low fraction of defectives. In practice, a minimal capability requirement would be preset by the customers/engineers in order to maintain a low fraction of defectives. When the capability index fails to reach the prescribed minimum value, one could conclude that the process is incapable of reaching the

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0377-2217/$ - see front matter
doi:10.1016/j.ejor.2006.05.030

desired production quality and the tool must be reset. In this study, we investigate an effective management policy based on process capability calculation for optimal tool replacement time with low fraction of defectives to meet manufacturing requirement.

In the manufacturing industry, process capability indices have been widely used to provide numerical measures on process reproduction capability, which are convenient and powerful tools for quality assurance and guidance for process improvement. Those indices are easy to understand and straightforward to apply in many industries such as automotive, semiconductor and IC assembly manufacturing industries. Among them, C_p and C_{pk} (see Kane, 1986) are the most extensively-used two in the manufacturing industry. Those indices have been defined explicitly as the following:

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma}, \quad C_{pk} = \min\left\{\frac{\text{USL} - \mu}{3\sigma}, \frac{\mu - \text{LSL}}{3\sigma}\right\}, \quad C_{pm} = \frac{\text{USL} - \text{LSL}}{6\sqrt{\sigma^2 + (\mu - T)^2}},$$

$$C_{pmk} = \min\left\{\frac{\text{USL} - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - \text{LSL}}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\},$$

where T is the target value, μ is the process mean and σ is the standard deviation of the characteristic, USL and LSL are the upper and lower specification limits, respectively. On the topic of PCIs, several authors have presented the use and examined their associated properties with different degrees of completeness. Examples are Kushler and Hurley (1992), Rodriguez (1992), Kotz and Johnson (1993), Vännman and Kotz (1995), Bothe (1997), Spiring (1997), Kotz and Lovelace (1998), Palmer and Tsui (1999), Pearn and Shu (2003), Vännman and Hubele (2003), and references therein. Kotz and Johnson (2002) provided a compact survey for the development of PCIs with interpretations and comments on some 170 publications appeared during 1992–2000. Spiring et al. (2003) consolidated the research findings in the field of process capability analysis for the period 1990–2002.

To understand and correctly interpret process capability indices, the process under investigation must be free from any special or assignable cause (i.e., in-control). Unfortunately, such condition is hardly met in many industrial applications. For example, when the assignable cause is in the form of tool wear, the output values inherently will show a certain increasing or decreasing trend. The causes such as tool wear are responsible for inducing autocorrelation and are not physically removable from the process. As a result, processes with uncontrollable trend are quite common in practice, and process capability analysis becomes a difficult task for practitioners. Quality researchers see this fact, and several approaches have been suggested to deal with problems of assignable cause. Some approaches attempt to remove the variability associated with the systematic assignable cause. For instance, Montgomery (1985) proposed fitting the AR(1) time series model to the auto-correlated data. Yang and Hancock (1990) recommended that in computing the C_p index, the unbiased estimator of σ can be obtained as $\sigma/(1-\rho)^{1/2}$, where ρ is defined as the average correction factor. Time series modeling trend data had been also suggested by Alwan and Roberts (1988), who recommend using residuals in monitoring the process. Other approaches make the general assumption of linear degradation in the tool. For example, Long and De Coste (1988) investigated the procedure to remove the linearity by regressing on the means of the subgroups and then determined the process capability. Quesenberry (1988) also suggested that tool wear can be modeled over an interval of tool life by a regression model and assumes that the tool wear rate is known or a good estimate of it is available, and that the process mean can be adjusted after each batch without an error.

Most of the previous works reviewed above, however, did not consider a dynamic process capability over a cycle. By considering the process capability dynamic within a cycle, as well as from cycle to cycle, we could circumvent some of the problems encountered. Spiring (1991) has devised a modification of C_{pm} index for this dynamic process under the influence of systematic assignable causes. Pearn et al. (1992) proposed an index called C_{pmk}, which combines the merits of the three basic indices C_p, C_{pk}, and C_{pm}. In this paper, we consider capability index C_{pmk} for the dynamic process under the influence of systematic assignable cause. This study is divided into six sections beginning with introduction. Section 2 contains the concept of process capability measure when the process involves tool wear problem. In Section 3, a modified estimator of C_{pmk} is proposed and its explicit form of the sampling distribution is derived. Section 4 provides a method for managing a process exhibiting assignable cause. Practitioners can use the proposed approach to determine whether their process meets the preset capability requirement, and make reliable decisions on when to stop the process for tool replacement. In Section 5, an application example involving tool wear is presented. Section 6 concludes the paper by a brief summary and discussion.

2. Measuring process capability with tool wear

Before assessing process capability, it is necessary to ensure that the process is under statistical control and the observations are statistically independent. However, it is not always the case. Porter and Oakland (1991) pointed out that the two specific conditions which make the process capability assessment to be difficult are: (1) ensuring stability of the mean and of the standard deviation; and (2) an absence of any special causes. In practice, processes with uncontrollable but acceptable trend are common. This is also referred to as a constant or consistent process drift, and other examples include accumulation of contaminants and temperature change drift must be quantified and removed before the remaining variability can be analyzed for statistical control (Kotz and Lovelace, 1998). The tool wear problems are responsible for inducing correlation and are not physically removable from the process. The issues of correlation among the samples and its effect on control chart limits have been examined by many authors (see Vasilopoulos and Stamboulis, 1978, Burr, 1979). Although various authors have looked at the issue of correlation from the point of control charts, process capability aspects have seldom been considered.

Fig. 1 illustrates an example of tool wear problem with four cycles, which displays information regarding process specifications (i.e., USL, LSL and T), the starting, stopping, tool replacement times (i.e., t_0, t_1, t_2, t_3, t_4), and the process output. In Fig. 1, the solid line illustrates the general systematic tool wear process with non-linear cycles over time/production. The change times may represent chronological time but are more likely to represent production qualities. The traditional measurement of process capability index C_{pmk} is affected by tool wear slope (see the dashed line in Fig. 1). The causes such as tool wear are responsible for inducing autocorrelation and are not physically removable from the process. Ignoring the unknown trend patterns, the presence of assignable cause variations will make the result of any capability index meaningless.

In order to calculate process capability accurately, the effects such as tool wear with systematic assignable causes must be considered. When systematic assignable causes are present and tolerated, the overall variation of the process (σ^2) is composed of the variation due to random causes (σ_{r}^2) and the variation due to assignable causes (σ_{a}^2). That is, $\sigma^2 = \sigma_{\mathrm{r}}^2 + \sigma_{\mathrm{a}}^2$. The traditional PCI measures fails to acknowledge that portions of the overall variation, (in the presence of tool wear), will be due to assignable causes. Hence any estimates of the process capability will confound the true capability with these two sources. In order to get a true measure of process capability, any variation due to an assignable cause must be removed from the measure of process capability.

Wallgren (1996) has also studied the properties and implications of the index C_{pm} when the consecutive measurements represent observations of dependent variables stemming from a Markov process in discrete

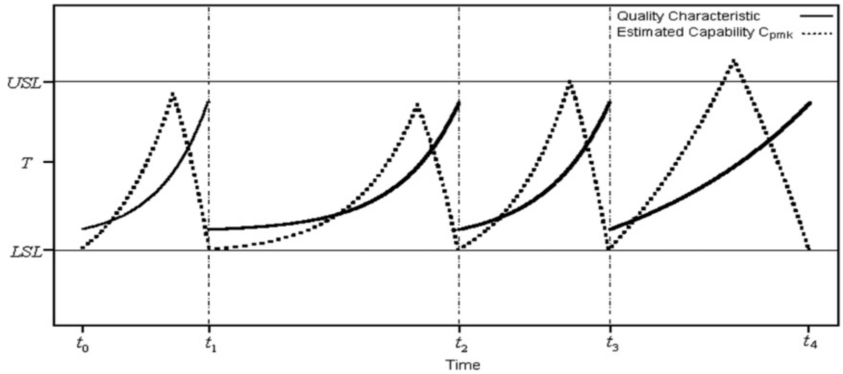

Fig. 1. An example of tool wear problem with four cycles, and the observations (solid) with the corresponding capability $\widehat{C}_{pmk}$ (dashed).

time. This occurs, for example, when consecutive measurements from a process are serially corrected. The author had also developed an augmentation of the index C_{pm}, denoted C_{pmr} based on the first-order autoregressive model (AR(1)). Spring (1989, 1991) viewed this as a dynamic process which is in a constant change as a process, tool, age, etc. In this dynamic model, the capability of the process may vary, possibly in a predictable manner. Spiring has devised a modification of C_{pm} index for this dynamic process under the influence of systematic assignable causes. In this scenario the goal is to maintain some minimum level of capability at all times. To become pro-active in the area of tool wear, steps should be taken to eliminate variation due to an assignable cause.

3. Statistical properties of the estimated C_{pmk}

In this section, we first introduce a modification of C_{pmk} index for the dynamic process under the influence of systematic assignable causes. Accordingly, an explicit form of the cumulative distribution function of the dynamic estimator of C_{pmk} is obtained, which can be expressed in terms of a mixture of the chi-square distribution and the non-central chi-square distribution. We then obtain the rth moment, and the mean and the variance as well as the bias and the mean square error (MSE) of the estimated C_{pmk} for dynamic process.

3.1. Estimation of C_{pmk} with tool wear

Using process capability index can monitor the changing ability of the process. Considering process capability changes dynamically, the goal is to maintain some minimum level of capability. We proposed a modification of C_{pmk} index for dynamic processes at time period t under the influence of systematic assignable cause as

$$C_{pmk} = \min\left\{\frac{\text{USL} - \mu_t}{3\sqrt{\sigma_{\text{r}t}^2 + (\mu_t - T)^2}}, \frac{\mu_t - \text{LSL}}{3\sqrt{\sigma_{\text{r}t}^2 + (\mu_t - T)^2}}\right\}, \tag{1}$$

where μ_t represents the process mean and $\sigma_{\text{r}t}^2$ is the variation (due to random causes only) of the process at time period t. Utilizing the identity $\min\{a,b\} = (a+b)/2 - |a-b|/2$, the index C_{pmk} defined in Eq. (1) can be alternatively rewritten as

$$C_{pmk} = \frac{d - |\mu_t - M|}{3\sqrt{\sigma_{\text{r}t}^2 + (\mu_t - T)^2}}, \tag{2}$$

where $d = (\text{USL} - \text{LSL})/2$ is half of the length of the specification interval, $M = (\text{LSL} + \text{USL})/2$ is the midpoint between the lower and the upper specification limits. Finding the value of C_{pmk} or a suitable estimate at various times t over each cycle in the lifetime of the tool is required for monitoring a process's capability.

In its simplest and most common form, tool wear data tend to have an upward or a downward slope over time. Assuming the effect of the tool deterioration is linear over the sampling window only, then the tool wear data can be modeled by a regression model over the sampling window of tool life. Once control has been activated, the estimates of C_{pmk} are available without involving contribution of the assignable causes. Hence, the proposed estimator of process capability can be obtained by replacing μ_t and $\sigma_{\text{r}t}$ by the estimators $\overline{X}_{t_n}$ and $[(n-2)\text{MSE}_t/(n-1)]^{1/2}$, respectively. Then we have

$$\widehat{C}_{pmk} = \frac{\min\left\{\text{USL} - \overline{X}_{t_n}, \overline{X}_{t_n} - \text{LSL}\right\}}{3\sqrt{\hat{\sigma}_{\text{r}t}^2 + (\overline{X}_{t_n} - T)^2}} = \frac{d - \left|\overline{X}_{t_n} - M\right|}{3\sqrt{\frac{(n-2)\text{MSE}_t}{(n-1)} + (\overline{X}_{t_n} - T)^2}}, \tag{3}$$

where $\overline{X}_{t_n} = \sum_{i=1}^n X_{t_i}/n$, $\text{MSE}_t = \sum_{i=1}^n \left(X_{t_i} - \widehat{X}_{t_i}\right)^2/(n-2)$, n denotes the subgroup sample size, and X_{t_i} represents the ith value of the quality characteristic in the sampling period t. The variation $\sigma_{\text{r}t}^2$ is removed by considering of the sequentially selected points (i.e., $t = 0, 1, 2, \ldots, n$) instead of the sample variance. The MSE_t is the mean square error associated with the regression equation $\widehat{X}_{t_i} = \hat{\alpha}_t + \hat{\beta}_t t_i$, where t_i is the sequence number of the sampling unit and $\hat{\beta}_t$ is the linear change in the tool wear given a unit change in time/production. When

$\hat{\beta}_t$ is large, it indicates that the tool wear trend is significant, and the tool will soon be producing too many defectives. In such situation, the practitioner would choose to replace the tool even if the current process had adequate capability. Alternatively, the practitioner could choose to sample the data more frequently to monitor the tool wear more closely. On the other hand, small value of $\hat{\beta}$ indicates the tool wear situation is not that serious. In general, our method provides a reference for the practitioner to follow for decision making.

Assuming the sampling scheme to be sequential and using ordinary least square (OLS) estimates of $\hat{\alpha}_t$ and $\hat{\beta}_t$, the computational formula for MSE_t can be derived alternatively as follows:

$$\mathrm{MSE}_t = \sum_{i=1}^{n}(X_{t_i} - \widehat{X}_{t_i})^2/(n-2) = \left[\sum_{i=1}^{n} X_{t_i}^2 - \hat{\alpha}_t \sum_{i=1}^{n} X_{t_i} - \hat{\beta}_t \sum_{i=1}^{n} iX_{t_i}\right] \Big/ (n-2),$$

where $\hat{\alpha}_t = \frac{2(2n+1)}{(n-1)}\overline{X}_{t_n} - \frac{6\sum_{i=1}^{n} iX_{t_i}}{n(n-1)}$, and $\hat{\beta}_t = \frac{12\sum_{i=1}^{n} X_{t_i}}{n(n^2-1)} - \frac{6}{(n-1)}\overline{X}_{t_n}$. Then we have that

$$\begin{aligned}\mathrm{MSE}_t &= \left[\sum_{i=1}^{n} X_{t_i}^2 - \hat{\alpha}_t \sum_{i=1}^{n} X_{t_i} - \hat{\beta}_t \sum_{i=1}^{n} iX_{t_i}\right] \Big/ (n-2)\\ &= \left[\frac{\sum_{i=1}^{n} X_{t_i}^2}{(n-1)} - \frac{2n(2n+1)}{(n-1)^2}\overline{X}_{t_n}^2 - \frac{12\left(\sum_{i=1}^{n} iX_{t_i}\right)^2}{n(n^2-1)(n-1)} + \frac{12\overline{X}_{t_n}\sum_{i=1}^{n}(iX_{t_i})}{(n-1)^2}\right] \Big/ (n-2).\end{aligned}$$

Therefore

$$\widehat{C}_{pmk} = \frac{d - \left|\overline{X}_{t_n} - M\right|}{3\left[\frac{\sum_{i=1}^{n} X_{t_i}^2}{(n-1)} - \frac{2n(2n+1)}{(n-1)^2}\overline{X}_{t_n}^2 - \frac{12\left(\sum_{i=1}^{n} iX_{t_i}\right)^2}{n(n^2-1)(n-1)} + \frac{12\overline{X}_{t_n}\sum_{i=1}^{n}(iX_{t_i})}{(n-1)^2} + \frac{n(\overline{X}_{t_n}-T)^2}{(n-1)}\right]^{1/2}}, \tag{4}$$

where $M = T$.

3.2. Sampling distribution of the estimated C_{pmk}

From Eq. (3), estimator $\widehat{C}_{pmk}$ for the dynamic process can be rewritten as follows:

$$\widehat{C}_{pmk} = \frac{d - \left|\overline{X}_{t_n} - M\right|}{3\sqrt{\frac{(n-2)}{(n-1)}\mathrm{MSE}_t + (\overline{X}_{t_n} - T)^2}} \sim \frac{b\sqrt{n} - H}{3\sqrt{\frac{n}{n-1}K + H^2}}, \tag{5}$$

where $b = d/\sigma_{\mathrm{r}t}$, $K = (n-2)\mathrm{MSE}_t/\sigma_{\mathrm{r}t}^2$, $H = \left|\sqrt{n}(\overline{X}_{t_n} - M)/\sigma_{\mathrm{r}t}\right|$, and $\overline{X}_{t_n} = \sum_{i=1}^{n} X_{t_i}/n$. Under the assumption of normality, K is distributed as χ_{n-2}^2, a chi-square distribution with $n-2$ degrees of freedom, H^2 is distributed as $\chi_{1,\lambda}'^2$ a non-central chi-square distribution with one degree of freedom and non-centrality parameter $\lambda = n(\mu_t - T)^2/\sigma_{\mathrm{r}t}^2$. And H is distributed as a folded-normal distribution, $N(\xi\sqrt{n}, 1)$ with probability density function $f_H(h) = \phi(h + \xi\sqrt{n}) + \phi(h - \xi\sqrt{n})$ for $h \geqslant 0$, where $\phi(\cdot)$ is the probability density function of the standard normal distribution and $\xi = (\mu_t - T)/\sigma_{\mathrm{r}t}$.

For $x > 0$, the cumulative distribution function of $\widehat{C}_{pmk}$ can be derived as

$$\begin{aligned}F_{\widehat{C}_{pmk}}(x) &= P\left(\widehat{C}_{pmk} \leqslant x\right) = P\left(\frac{(b\sqrt{n} - H)}{3\sqrt{\frac{n}{n-1}K + H^2}} \leqslant x\right) = 1 - P\left(\sqrt{\frac{n}{n-1}K + H^2} < \frac{(b\sqrt{n} - H)}{3x}\right)\\ &= 1 - \int_0^\infty P\left(\sqrt{\frac{n}{n-1}K + H^2} < \frac{(b\sqrt{n} - H)}{3x}\middle| H = h\right) f_H(h)\mathrm{d}h\\ &= 1 - \int_0^\infty P\left(\sqrt{\frac{n}{n-1}K + h^2} < \frac{(b\sqrt{n} - h)}{3x}\right) f_H(h)\mathrm{d}h\\ &= 1 - \int_0^\infty P\left(K < \frac{(n-1)(b\sqrt{n} - h)^2}{9nx^2} - \frac{(n-1)h^2}{n}\right) f_H(h)\mathrm{d}h.\end{aligned}$$

Since K is distributed as χ^2_{n-2}, we have

$$P\left(K < \frac{(n-1)(b\sqrt{n}-h)^2}{9nx^2} - \frac{(n-1)h^2}{n}\right) = 0 \quad \text{for } h > b\sqrt{n}/(1+3x).$$

Therefore,

$$\begin{aligned} F_{\widehat{C}_{pmk}}(x) &= 1 - \int_0^{b\sqrt{n}/(1+3x)} P\left(K < \frac{(n-1)(b\sqrt{n}-h)^2}{9nx^2} - \frac{(n-1)h^2}{n}\right) f_H(h)\mathrm{d}h \\ &= 1 - \int_0^{b\sqrt{n}/(1+3x)} G\left(\frac{(n-1)(b\sqrt{n}-h)^2}{9nx^2} - \frac{(n-1)h^2}{n}\right) f_H(h)\mathrm{d}h \quad \text{for } x > 0, \end{aligned} \tag{6}$$

where $G(\cdot)$ is the cumulative distribution function of χ^2_{n-2}. Substituting $f_H(h)$ into Eq. (6) leads to the result:

$$F_{\widehat{C}_{pmk}}(x) = 1 - \int_0^{b\sqrt{n}/(1+3x)} G\left(\frac{(n-1)(b\sqrt{n}-t)^2}{9nx^2} - \frac{(n-1)t^2}{n}\right)\left[\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})\right]\mathrm{d}t \quad \text{for } x > 0. \tag{7}$$

The proposed sampling scheme is similar to the schemes used in monitoring a process for control charting procedures. The general format is to gather k subgroups of size n from each cycle (e.g., the period form t_0 to t_1 in Fig. 1) over the lifetime of the tool. The value of k will be unique to each process and, in fact, may change from cycle to cycle within a process. On the other hand, sample size of less than five (i.e., $n < 5$) are cautioned against, while larger samples (e.g., $n > 30$) may also pose a problem. The optimal sample size for assessing process capability in the presence of systematic assignable cause will vary for each process considered (Spiring, 1991).

3.3. The rth moment of the estimated C_{pmk}

Under the assumption of normality and for the general case with $T = M$, Pearn et al. (1992) derived the rth moment of $\widehat{C}_{pmk}$. Using the similar technique to derive the rth moment of the modified $\widehat{C}_{pmk}$, we obtained as follows:

$$E(\widehat{C}^r_{pmk}) = \frac{e^{-\lambda/2}}{3^r}\sum_{i=0}^{r}(-1)^i\binom{r}{i}\left(\frac{d}{\sigma}\sqrt{\frac{n}{2}}\right)^{r-i} \times \sum_{j=0}^{\infty}\frac{(\frac{\lambda}{2})^j}{j!}\left[\frac{\Gamma(\frac{i+1}{2}+j)}{\Gamma(\frac{1}{2}+j)}\frac{\Gamma(\frac{n-1-r+i}{2}+j)}{\Gamma(\frac{n+i}{2}+j)}\right]. \tag{8}$$

Taking $r = 1$ and 2, the expected value and variance of $\widehat{C}_{pmk}$ can be expressed as the following:

$$E(\widehat{C}_{pmk}) = \frac{e^{-\lambda/2}}{3}\sum_{j=0}^{\infty}\frac{(\frac{\lambda}{2})^j}{j!}\left[\frac{d}{\sigma}\sqrt{\frac{n}{2}}\cdot\frac{\Gamma(\frac{n}{2}-1+j)}{\Gamma(\frac{n}{2}+j)} - \frac{j!\Gamma(\frac{n-1}{2}+j)}{\Gamma(\frac{1}{2}+j)\Gamma(\frac{n+1}{2}+j)}\right],$$

and

$$\mathrm{Var}(\widehat{C}_{pmk}) = E(\widehat{C}^2_{pmk}) - [E(\widehat{C}_{pmk})]^2,$$

where

$$E(\widehat{C}^2_{pmk}) = \frac{e^{-\lambda/2}}{9}\sum_{j=0}^{\infty}\frac{(\frac{\lambda}{2})^j}{j!}\left[\left(\frac{d\sqrt{n}}{\sigma}\right)^2\cdot\frac{1}{n+2j-3} - \frac{d\sqrt{2n}}{\sigma}\cdot\frac{j!}{\Gamma(\frac{1}{2}+j)}\cdot\frac{2}{n+2j-2} + \frac{1+2j}{n+2j-1}\right],$$

where $\Gamma(u) = \int_0^\infty t^{u-1}e^{-t}\mathrm{d}t$ is a gamma function. Therefore, the bias and the MSE of $\widehat{C}_{pmk}$ are: $\mathrm{Bias}(\widehat{C}_{pmk}) = E(\widehat{C}_{pmk}) - C_{pmk}$, $\mathrm{MSE}(\widehat{C}_{pmk}) = \mathrm{Var}(\widehat{C}_{pmk}) + [\mathrm{Bias}(\widehat{C}_{pmk})]^2$.

4. Testing procedure for process capability

Under the assumption of normality, the cumulative distribution function of $\widehat{C}_{pmk}$ for dynamic process can be expressed in terms of a mixture of the chi-square distribution and the non-central chi-square distribution. Therefore, to test whether a given process is capable, we can consider the following statistical testing hypotheses:

$$H_0 : C_{pmk} \leqslant C \quad \text{(process is not capable)},$$
$$H_1 : C_{pmk} > C \quad \text{(process is capable)}.$$

Using the index $\widehat{C}_{pmk}$, the engineers can access the process performance and monitor the manufacturing processes on routine basis. A testing procedure similar to those used in monitoring a process with control chart can be used to monitor the process and determine whether the process should stop and reset the tool to avoid producing non-conforming products. Defining the decision making rule $\phi^*(x)$ as the following:

$$\phi^*(x) = \begin{cases} 1 & \text{if } \widehat{C}_{pmk} > c_\alpha, \\ 0 & \text{otherwise}, \end{cases}$$

where $\alpha(c_\alpha) = \alpha$ is the type I error, the chance of incorrectly concluding an incapable process ($C_{pmk} \leqslant C$) as capable ($C_{pmk} > C$), thus, the test $\phi^*(x)$ rejects the null hypothesis $H_0(C_{pmk} \leqslant C)$ if $\widehat{C}_{pmk} > c_\alpha$. Based on the cumulative distribution function of $\widehat{C}_{pmk}$ expressed in Eq. (7), given values of capability requirement C, the α-risk, the sample size n and the parameter ξ, hence the critical value c_α can be obtained by solving the equation $P(\widehat{C}_{pmk} \geqslant c_\alpha | C_{pmk} = C) = \alpha$ using available numerical integration methods. That is,

$$\int_0^{b\sqrt{n}/(1+3C)} G\left(\frac{(n-1)(b\sqrt{n}-t)^2}{9nC^2} - \frac{(n-1)t^2}{n}\right)\left[\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})\right]\mathrm{d}t = \alpha. \tag{9}$$

Note that, for fixed values of C, n and α, Eq. (9) is an even function of ξ. Thus, we obtain the same critical value c_α for both $\xi = \xi_0$ and $\xi = -\xi_0$. Since the process parameters μ and σ are unknown, the distribution characteristic parameter $\xi = (\mu - T)/\sigma$ is also unknown, which has to be estimated in real applications, naturally by substituting μ and σ by the sample mean and the sample variance. To eliminate the need for estimating the parameter ξ, we examine the behavior of the critical values c_α against the parameter $0 \leqslant \xi \leqslant 3$. Further, we perform extensive calculations to obtain the critical values c_α for $0 \leqslant \xi \leqslant 3$, $n = 5, 10, 20, 30$ and C_{pmk} form 0 to 2 with risk $\alpha = 0.05$. Note that the parameter values we investigated, $0 \leqslant \xi \leqslant 3$, cover a sufficiently wide range of applications with process capability analysis. Figs. 2(a)–2(d) display the surface plots of the critical value c_α versus the parameter $0 \leqslant \xi \leqslant 3$, $0 \leqslant C_{pmk} \leqslant 2$ with type I error $\alpha = 0.05$ for sample size $n = 5, 10, 20, 30$, respectively. The results indicate that (i) the critical value c_α is increasing in ξ, and is decreasing in n, (ii) the critical value c_α obtains its maximum at $\xi = 0.5$ in all cases with accuracy up to 10^{-3}. Hence, for practical purpose we may solve Eq. (9) with $\xi = 0.5$ to obtain the required critical values c_α for given C_{pmk}, n, and α, without having to further estimate the parameter ξ. Thus, the risk α can be ensured, and the decisions made based on such approach are indeed more reliable.

Therefore, for users' convenience in applying our proposed procedure, we tabulate the critical values of C_{pmk} for various values of $\alpha = 0.01$, 0.025 and 0.05 with $n = 5(1)30$ in Table 1 for commonly recommended minimum capability requirement $C = 1.00$, 1.33, 1.50, 1.67 and 2.00. For example, if $C = 1.00$ is the minimum capability requirement, then for $\alpha = 0.05$, with sample size $n = 15$ we can find $c_\alpha = 1.60$ from Table 1. That is, as the estimated process capability drops below the critical value of C_{pmk}, the practitioner should stop the process and reset the tool because there is an evidence to consider that the process is nearing the end of its ability to produce agreeable product. Otherwise, if the values of C_{pmk} greater than the critical value, then the process is considered capable and is allowed to continue. In the following, we calculate the power of the test as

$$\begin{aligned}\pi(C_{pmk}) &= 1 - \beta = P(\widehat{C}_{pmk} > C | C_{pmk}) \\ &= \int_0^{b\sqrt{n}/(1+3C)} G\left(\frac{(n-1)(b\sqrt{n}-t)^2}{9nC^2} - \frac{(n-1)t^2}{n}\right)\left[\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})\right]\mathrm{d}t.\end{aligned} \tag{10}$$

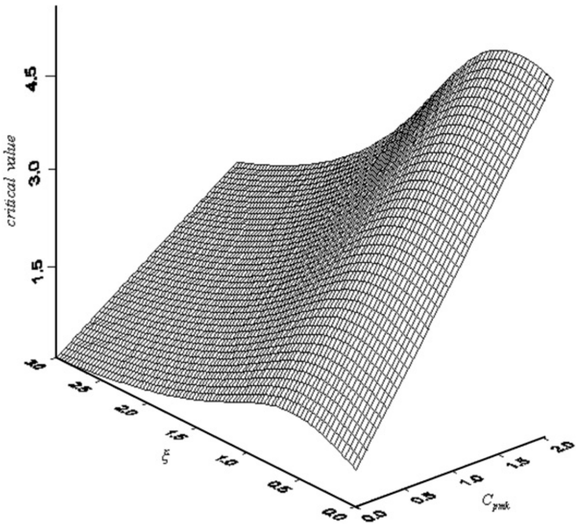

Fig. 2a. Surface plot of c_α with $0 \leqslant \xi \leqslant 3$ and $0 \leqslant C_{pmk} \leqslant 2$ for $n = 5$ and $\alpha = 0.05$.

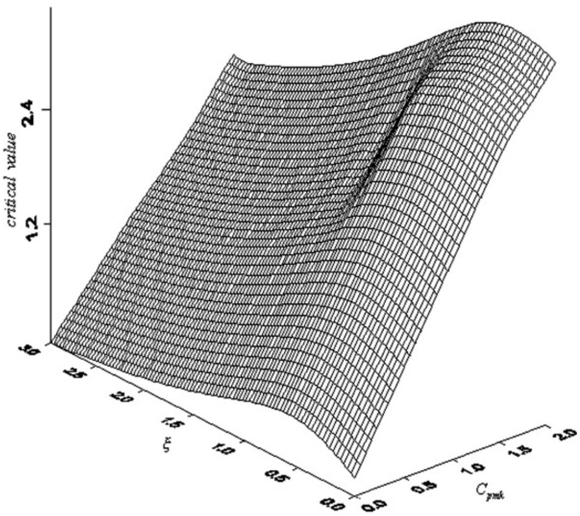

Fig. 2b. Surface plot of c_α with $0 \leqslant \xi \leqslant 3$ and $0 \leqslant C_{pmk} \leqslant 2$ for $n = 10$ and $\alpha = 0.05$.

In Figs. 3(a)–3(d), we plotted the power curves, $\pi(C_{pmk})$ versus c_α value, for the quality conditions with $C = 1.00$, 1.33, α-risk = 0.01, 0.05 and $n = 10(5)30$. It can be seen from Figs. 3(a)–3(d), the power is quite good.

5. Capability testing with applications

5.1. Capability requirement

In the general case, a manufacturing process is said to be inadequate if $C_{pmk} < 1.00$; it indicates that the process is not adequate with respective to the manufacturing tolerances, the process variation needs to be reduced (often using design of experiments). The fraction of defectives for such process exceeds 2700 ppm (parts per million). A manufacturing process is said to be marginally capable if $1.00 \leqslant C_{pmk} < 1.33$; it indicates that caution needs to be taken regarding the process consistency and some process control is required (usually

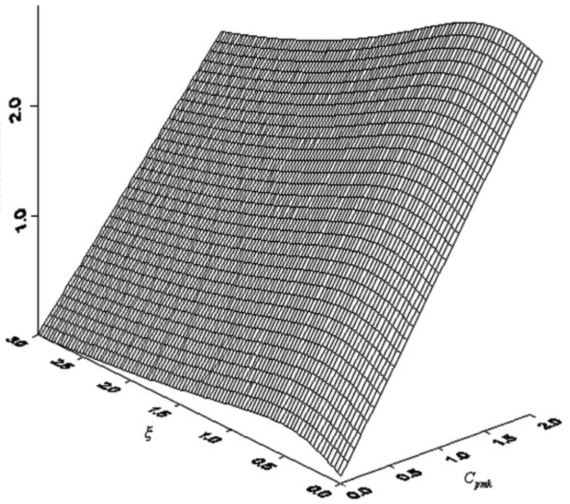

Fig. 2c. Surface plot of c_α with $0 \leqslant \xi \leqslant 3$ and $0 \leqslant C_{pmk} \leqslant 2$ for $n = 20$ and $\alpha = 0.05$.

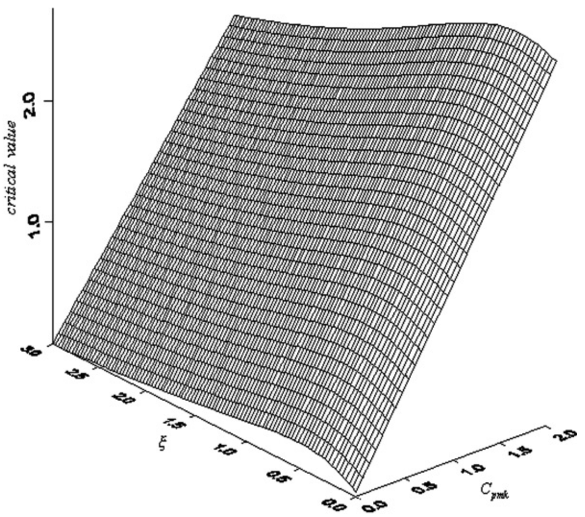

Fig. 2d. Surface plot of c_α with $0 \leqslant \xi \leqslant 3$ and $0 \leqslant C_{pmk} \leqslant 2$ for $n = 30$ and $\alpha = 0.05$.

using R or S control charts). The fraction of defectives for such process is within 66–2700 ppm. A manufacturing process is said to be satisfactory if $1.33 \leqslant C_{pmk} < 1.67$; it indicates that process consistency is satisfactory, material substitution may be allowed, and no stringent precision control is required. The fraction of defectives for such process is within 0.54–66 ppm. A manufacturing process is said to be excellent if $1.67 \leqslant C_{pmk} < 2.00$; it indicates that process precision exceeds satisfactory. The fraction of defectives for such process is within 0.002–0.54 ppm. Finally, a manufacturing process is said to be super if $C_{pmk} \geqslant 2.00$. The fraction of defectives for such process is less than 0.002 ppm.

5.2. An example

To illustrate the practicality of our proposed approach to actual data, we consider the following real case taken from a metal crown company engaged mainly in making aluminum lids components, which are produced on a press. Each press contains 22 dies and the differences exist die-to-die and press-to-press. Slight fluctuations are observed with lot-to-lot changes in steel. Since the press of the interfaces may affect wear rates of

Table 1
Critical values c_α for $C_{pmk} = 1.00$, 1.33, 1.50, 1.67, 2.00, $n = 5(1)30$, and $\alpha = 0.01$, 0.025, 0.05

C_{pmk}	1.00			1.33			1.50			1.67			2.00		
n	α														
	0.01	0.025	0.05	0.01	0.025	0.05	0.01	0.025	0.05	0.01	0.025	0.05	0.01	0.025	0.05
5	4.02	3.12	2.55	5.21	4.06	3.33	5.83	4.54	3.73	6.44	5.02	4.13	7.64	4.02	3.12
6	3.27	2.65	2.24	4.24	3.45	2.92	4.74	3.86	3.27	5.25	4.27	3.62	6.22	3.27	2.65
7	2.85	2.37	2.05	3.70	3.09	2.67	4.13	3.46	2.99	4.57	3.82	3.31	5.42	2.85	2.37
8	2.58	2.19	1.92	3.35	2.85	2.50	3.74	3.19	2.80	4.14	3.53	3.11	4.91	2.58	2.19
9	2.39	2.06	1.83	3.10	2.68	2.38	3.47	3.00	2.67	3.84	3.32	2.96	4.55	2.39	2.06
10	2.24	1.96	1.75	2.92	2.55	2.29	3.26	2.86	2.56	3.61	3.16	2.84	4.29	2.24	1.96
11	2.14	1.88	1.70	2.78	2.45	2.21	3.11	2.75	2.48	3.44	3.04	2.75	4.08	2.14	1.88
12	2.05	1.82	1.65	2.66	2.37	2.15	2.98	2.66	2.41	3.30	2.94	2.67	3.92	2.05	1.82
13	1.98	1.77	1.61	2.57	2.30	2.10	2.88	2.58	2.36	3.19	2.86	2.61	3.78	1.98	1.77
14	1.92	1.72	1.58	2.50	2.25	2.06	2.79	2.52	2.31	3.09	2.79	2.56	3.67	1.92	1.72
15	1.87	1.69	1.55	2.43	2.20	2.02	2.72	2.46	2.27	3.01	2.73	2.52	3.58	1.87	1.69
16	1.82	1.65	1.52	2.37	2.16	1.99	2.66	2.42	2.23	2.94	2.68	2.48	3.49	1.82	1.65
17	1.78	1.62	1.50	2.32	2.12	1.96	2.60	2.38	2.20	2.88	2.63	2.44	3.42	1.78	1.62
18	1.75	1.60	1.48	2.28	2.09	1.94	2.55	2.34	2.17	2.83	2.59	2.41	3.36	1.75	1.60
19	1.72	1.58	1.46	2.24	2.06	1.92	2.51	2.31	2.15	2.78	2.55	2.38	3.30	1.72	1.58
20	1.69	1.56	1.45	2.21	2.03	1.90	2.47	2.28	2.13	2.74	2.52	2.36	3.25	1.69	1.56
21	1.67	1.54	1.43	2.17	2.01	1.88	2.44	2.25	2.11	2.70	2.49	2.33	3.20	1.67	1.54
22	1.65	1.52	1.42	2.15	1.99	1.86	2.40	2.23	2.09	2.66	2.47	2.31	3.16	1.65	1.52
23	1.63	1.50	1.41	2.12	1.97	1.84	2.37	2.20	2.07	2.63	2.44	2.29	3.13	1.63	1.50
24	1.61	1.49	1.40	2.10	1.95	1.83	2.35	2.18	2.05	2.60	2.42	2.28	3.09	1.61	1.49
25	1.59	1.48	1.39	2.07	1.93	1.82	2.32	2.16	2.04	2.57	2.40	2.26	3.06	1.59	1.48
26	1.57	1.46	1.38	2.05	1.91	1.80	2.30	2.15	2.02	2.55	2.38	2.24	3.03	1.57	1.46
27	1.56	1.45	1.37	2.03	1.90	1.79	2.28	2.13	2.01	2.52	2.36	2.23	3.00	1.56	1.45
28	1.54	1.44	1.36	2.02	1.88	1.78	2.26	2.11	2.00	2.50	2.34	2.22	2.98	1.54	1.44
29	1.53	1.43	1.35	2.00	1.87	1.77	2.24	2.10	1.99	2.48	2.33	2.20	2.95	1.53	1.43
30	1.52	1.42	1.34	1.98	1.86	1.76	2.22	2.09	1.98	2.46	2.31	2.19	2.93	1.52	1.42

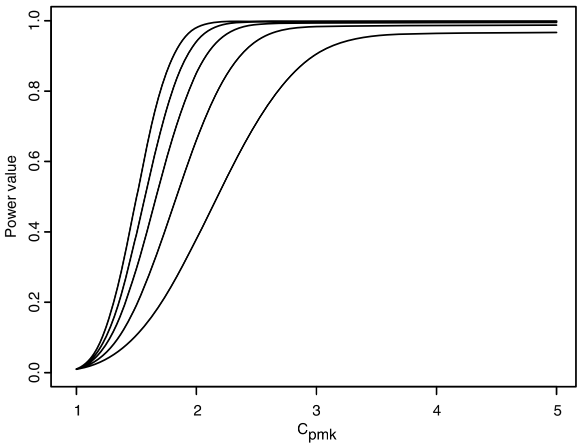

Fig. 3a. Power curves for $C = 1.00$ and $n = 10(5)30$ (from bottom to top in the plot) with $\alpha = 0.01$.

tool, the process exhibits tool wear. Each day, lids from each station are sampled and lid height is measured. Data collected over one-week period. Each sample contains a single lid from each station. We investigated a particular type of the lid product with the upper and lower specification limits of the key characteristic, lid height, are set to USL $= 68.4$ mm, LSL $= 64.65$ mm, respectively and the target value is set to $T = 66.525$ mm. The lid height is measured and recorded when the product comes out of the process. The collected

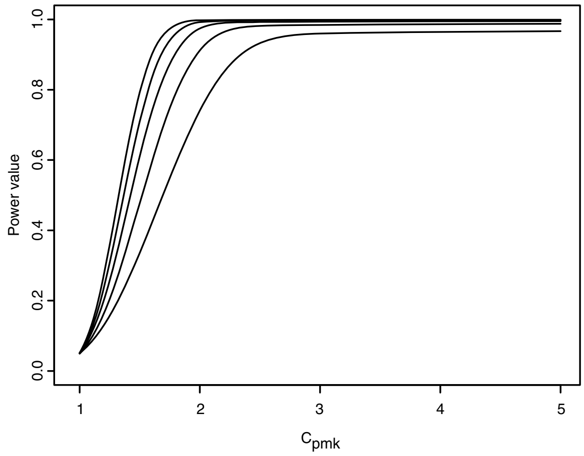

Fig. 3b. Power curves for $C = 1.00$ and $n = 10(5)30$ (from bottom to top in the plot) with $\alpha = 0.05$.

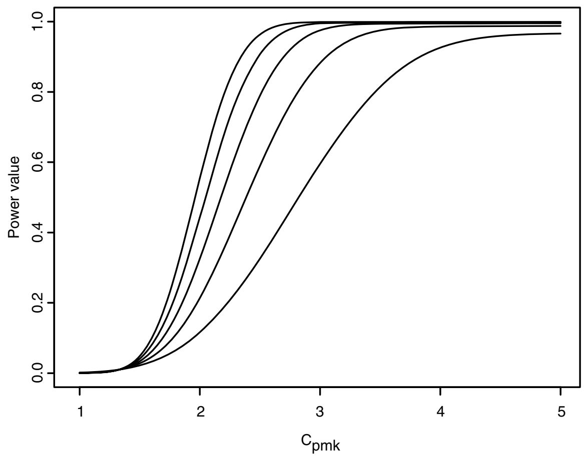

Fig. 3c. Power curves for $C = 1.33$ and $n = 10(5)30$ (from bottom to top in the plot) with $\alpha = 0.01$.

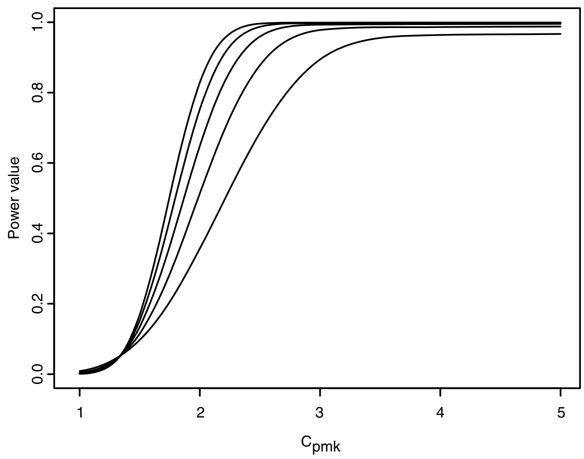

Fig. 3d. Power curves for $C = 1.33$ and $n = 10(5)30$ (from bottom to top in the plot) with $\alpha = 0.05$.

data consist of 105 observations arranged in seven subgroups of 15 each. The plot of the individual values in the series is depicted in Fig. 4. The increasing trend of the individual values due to tool wear appears to be linear in nature. Also, the values of the lid height of each component is influenced by the amount of tool wear at that instant, which is likely to be dependent on the condition of the tool when previous component was processed. Now, the goal is to maintain some minimum level of capability at all times and to monitor/manage this processes under the influence of tool wear problem. When the measure of process capability comes closer to the minimum acceptable level, the processing should be stopped and the tool should be replaced.

Suppose for this particular process under consideration to be capable, the process index C_{pmk} must reach at least a certain level C, say, 1. Thus, applying the proposed capability measure for dynamic, the practitioners can monitor the process by calculating the measure of C_{pmk}. The proposed testing procedure for a process involving tool wear is similar to those used in monitoring a process with control chart. In this case we can obtain the critical value of $\widehat{C}_{pmk}$ is 1.55 by checking Table 1 under the given values of risk $\alpha = 0.05$, sample size $n = 15$ and minimum capability requirement $C = 1.00$. While the estimated process capability drops below the critical value of $\widehat{C}_{pmk}$, the practitioner should stop the process and reset the tool because there is an evidence to consider that the process is nearing the end of its ability to produce agreeable product. As regards the values of $\widehat{C}_{pmk}$ greater than 1.55 the process is considered capable and is allowed to continue. Based on the data listed in Table 2, the calculated $\widehat{C}_{pmk}$ for dynamic process at each time period are summarized in Table 3. Fig. 5 plots the measure of process capability $\widehat{C}_{pmk}$ for dynamic process at each time period over a

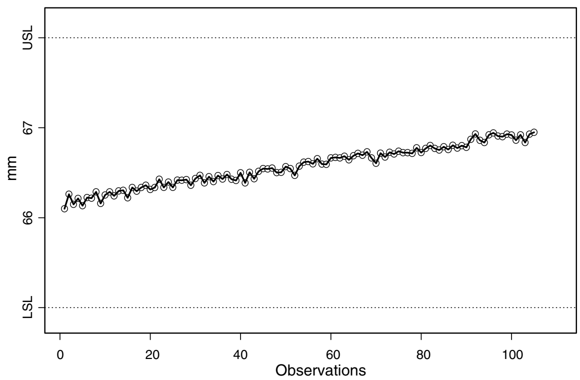

Fig. 4. Plot of the original data.

Table 2
The collected 7 subgroups each of 15 observations (unit: mm)

i	Time period t						
	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	$t = 7$
1	66.100	66.335	66.470	66.542	66.670	66.722	66.872
2	66.261	66.295	66.387	66.551	66.665	66.722	66.931
3	66.147	66.335	66.456	66.501	66.684	66.715	66.860
4	66.214	66.361	66.402	66.504	66.644	66.777	66.836
5	66.133	66.314	66.468	66.568	66.689	66.724	66.922
6	66.223	66.335	66.430	66.546	66.715	66.770	66.943
7	66.216	66.428	66.480	66.470	66.695	66.803	66.907
8	66.288	66.337	66.428	66.572	66.732	66.770	66.900
9	66.159	66.397	66.413	66.618	66.665	66.753	66.929
10	66.252	66.337	66.499	66.625	66.606	66.789	66.919
11	66.288	66.418	66.387	66.599	66.717	66.758	66.862
12	66.242	66.416	66.504	66.656	66.675	66.805	66.922
13	66.297	66.423	66.432	66.596	66.727	66.774	66.836
14	66.304	66.361	66.516	66.594	66.708	66.800	66.929
15	66.221	66.435	66.546	66.665	66.739	66.781	66.950

Table 3
The estimated C_{pmk} for dynamic process at each time period

Time	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	$t = 7$
$\widehat{C}_{pmk}$	1.657	3.464	7.111	9.644	3.306	2.194	1.278

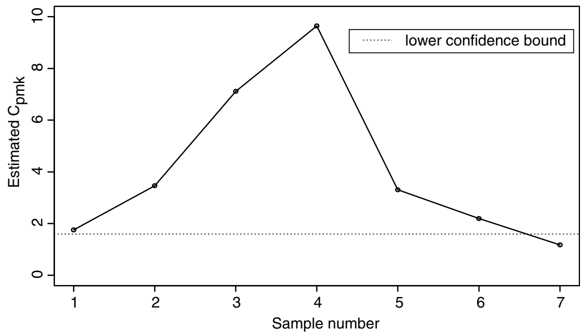

Fig. 5. Capability plot for dynamic process at each time period.

single cycle of the process. It is observed that the estimated $\widehat{C}_{pmk}$ reaches maximum at time period $t = 4$ and then drops below the line of critical values 1.55 at time period $t = 7$. Therefore, based on these results obtained we would suggest that the process should be stopped and the tool should be replaced at time period $t = 7$ to avoid produce unacceptable components.

6. Conclusions

In most manufacturing industries, a tool replacement policy is essential in order to minimize the fraction defective and the manufacturing cost. Therefore, an effective tool management policy is essential for the manufacturing industry in order to meet customer's requirements. Capability indices are effective methods for quantifying process performance and for conveying critical information regarding the suitability of a manufacturing process to meet the required quality standards. The index C_{pmk} combines the merits of the two indices C_{pk}, and C_{pm} to provide numerical measures on process performance. In this paper, we have applied the process capability index C_{pmk} to determine the optimal tool replacement time under tool wear condition. Under the assumption of normality, the sampling distribution of the estimated C_{pmk} is a mixture of the chi-square and the non-central chi-square distributions. We implemented the derived results to develop an effective procedure to assess process capability at each time period over a process cycle, and to calculate the critical values for various sample sizes. Practitioners can use the proposed procedure to determine whether their process meets the preset capability requirement, and to make reliable decisions in determining the optimal time for tool replacements.

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