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Chaotic ranges of a unified chaotic system and its chaos for five periodic switch cases

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Abstract

In this paper, a unified chaotic system is studied in detail. Non-chaotic ranges within $\alpha \in [0,1]$ are found, where α is the constant parameter of the system. Chaotic range longer than $\alpha \in [0,1]$, $\alpha \in [-0.015, 1.152]$, is discovered, which is the extended chaotic range of unified chaotic system. Next, its chaos behaviors for five continuous periodic switch cases, $k\sin^2\omega T$, $m\sin\omega t$, $0 \sim 1$ triangular wave, $-1 \sim 1$ triangular wave, and $0 \sim 1$ sawtooth wave, are presented.

1. Introduction

In recent years, chaos, chaos control and chaos synchronization became very interesting problems and have been widely studied [1–8]. Over the last decade, chaos synchronization has received considerable attention [9–17] due to its potential application in many areas such as secure communication, information process, biological systems, and chemical reactions.

New chaotic systems have been constantly proposed in the last 30 years. In 1999, Chen found a new chaotic attractor, the Chen system [18], which is dual to the Lorenz system and has a similarly simple structure but displays even more sophisticated dynamical behavior. Here, duality is in the sense defined by Vaněček and Čelikovský [19]: for the linear part of the system, $A = [a_{ij}]_{3\times3}$, the Lorenz system satisfies the condition $a_{12}a_{21} > 0$ while the Chen system satisfies $a_{12}a_{21} < 0$. In 2002, Lü and Chen found another chaotic system, which satisfies the condition $a_{12}a_{21} = 0$ [20]. Very recently, Lü and Chen produced the third new chaotic system—unified chaotic system [21], which contains the Lorenz and Chen systems as two extremes and the Lü system as a special case. Recently, there are some results reported about the unified chaotic system [22–32].

In this paper, a unified chaotic system is studied in detail. Non-chaotic ranges within $\alpha \in [0,1]$ are found, where α is the constant parameter of the system. Chaotic range longer than $\alpha \in [0,1]$, $\alpha \in [-0.015, 1.152]$, is discovered, which is the extended chaotic range of unified chaotic system. Next, its chaos behaviors for five continuous periodic switch cases, $k\sin^2\omega t$, $m\sin\omega t$, $0 \sim 1$ triangular wave, $-1 \sim 1$ triangular wave and $0 \sim 1$ sawtooth wave are presented. This paper is organized as follows. In Section 2, differential equations of motion and the description of the unified chaotic system are introduced. In Section 3, non-chaotic ranges within $\alpha \in [0,1]$ are found, where α is the original constant parameter of

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the unified chaotic system. In Section 4, chaotic range longer than $\alpha \in [0,1]$, $\alpha \in [-0.015, 1.152]$, is discovered, which is the extended chaotic range of the unified chaotic system. In Section 5, the chaos of the unified chaotic system with $k\sin^2\omega t$ periodic switch are studied by Lyapunov exponents. The periodic time function $k\sin^2\omega t$ is used to replace the original constant parameter α of the unified chaotic system. In Sections 6–9, the chaos of the unified chaotic system with $m\sin\omega t$ switch, with $0 \sim 1$ triangular wave switch, with $-1 \sim 1$ triangular wave switch and with $0 \sim 1$ sawtooth wave switch are studied by Lyapunov exponents, respectively. In Section 10, conclusions are drawn.

2. The differential equations of motion and the description of the system

The unified chaotic system is described as follows:

$$\begin{aligned}\dot{x} &= (25\alpha + 10)(y - x),\\ \dot{y} &= (28 - 35\alpha)x + (29\alpha - 1)y - xz,\\ \dot{z} &= xy - \frac{8+\alpha}{3}z,\end{aligned} \tag{1}$$

where $\alpha \in [0,1]$. The system was said to be chaotic for any $\alpha \in [0,1]$ [24,32,33]. When $\alpha = 0$ the system is a classic Lorenz system, and when $\alpha = 1$ it becomes Chen system. Fig. 1 shows the Lyapunov exponents of the unified chaotic system. In three-dimensional space, the Lyapunov exponent spectra for a strange attractor, a two-torus, a limit cycle and a fixed point are described by $(+,0,-)$, $(0,0,-)$, $(0,-,-)$, $(-,-,-)$, respectively. In Figs. 2 and 3, we can see the phase portraits of the unified chaotic system with $\alpha = 0$ and $\alpha = 1$, respectively. Fig. 4 shows the phase portraits of case $(-,-,-)$ with $\alpha = -1$, and the phase portraits converge to a fixed point. Fig. 5 shows the phase portraits of case $(0,-,-)$ with $\alpha = 1.7$, and the phase portraits show a limit cycle.

3. The non-chaotic ranges within $\alpha \in [0,1]$ for the unified chaotic system

Unified chaotic system was told to be chaotic for any $\alpha \in [0,1]$ [33,34]. But we find that there exist some non-chaotic ranges within $\alpha \in [0,1]$. In order to confirm these ranges, three checking methods are used: Lyapunov exponents, phase portraits and power spectra. The Lyapunov exponent can be used to measure the sensitive dependence upon initial conditions of the states of the chaotic motion. It is the most reliable index for chaotic behavior. Different patterns of the solutions of the dynamical system, such as fixed point, periodic motion, quasi-periodic motion, and chaotic motion can be distinguished by them. Another valuable technique for the characterization of the solution of the system is the power

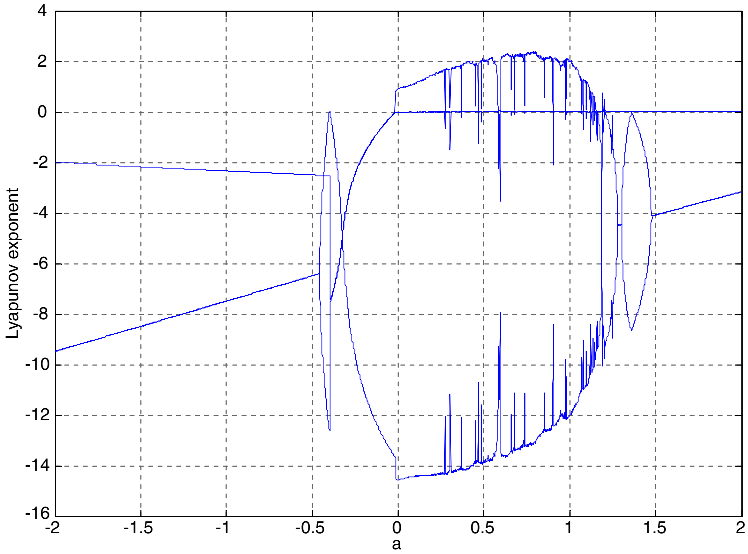

Fig. 1. The Lyapunov exponents of the unified chaotic system for α between 2 and -2.

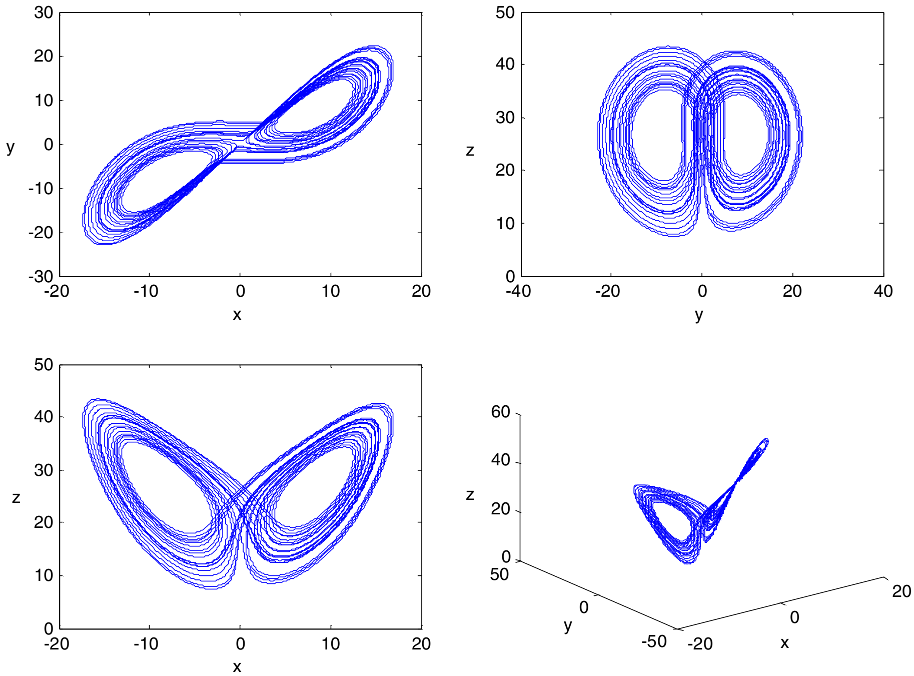

Fig. 2. The phase portraits of the unified chaotic system with $\alpha = 0$.

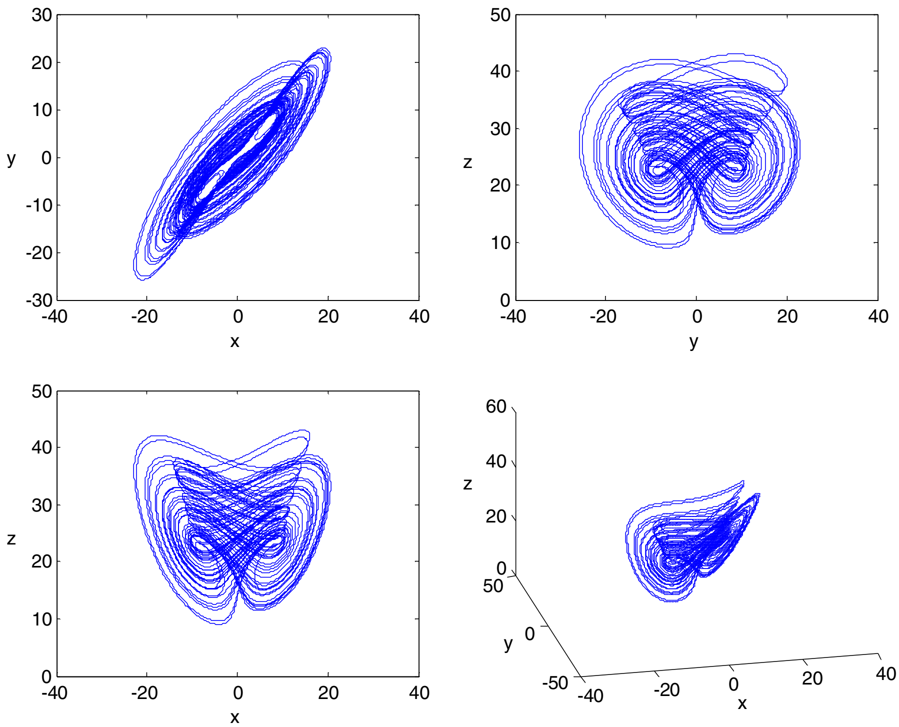

Fig. 3. The phase portraits of the unified chaotic system with $\alpha = 1$.

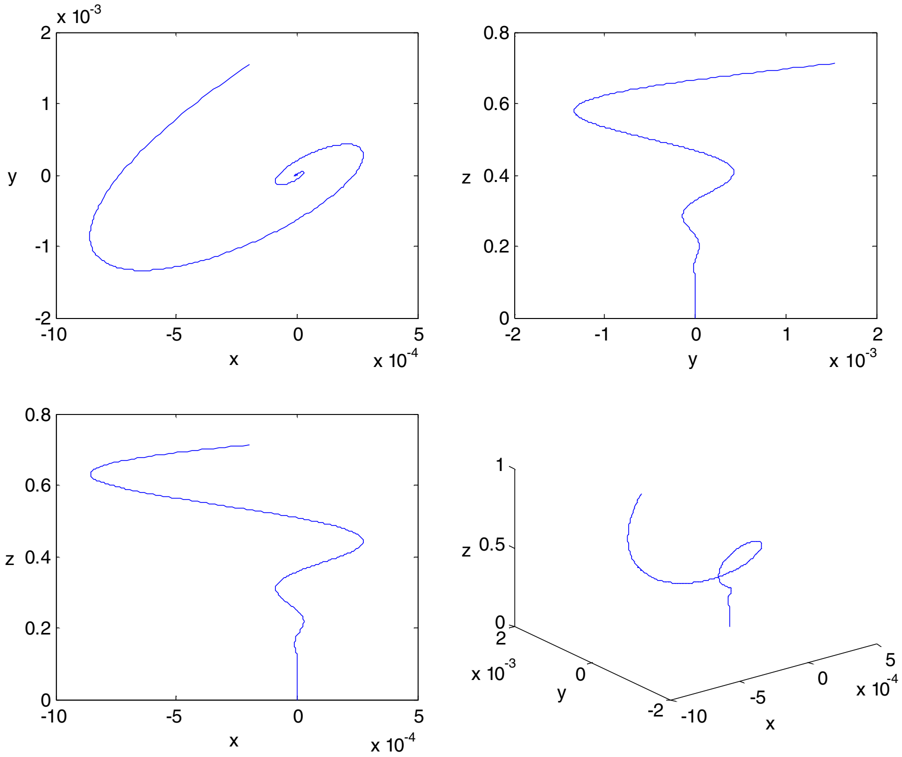

Fig. 4. The phase portraits of the unified chaotic system with $\alpha = -1$.

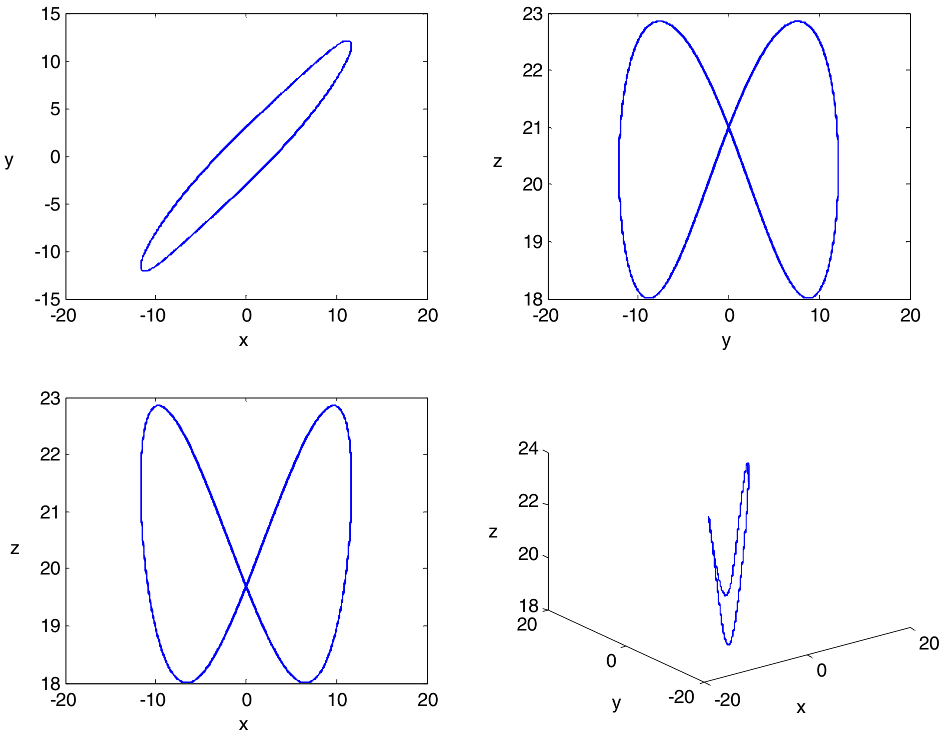

Fig. 5. The phase portraits of the unified chaotic system with $\alpha = 1.7$.

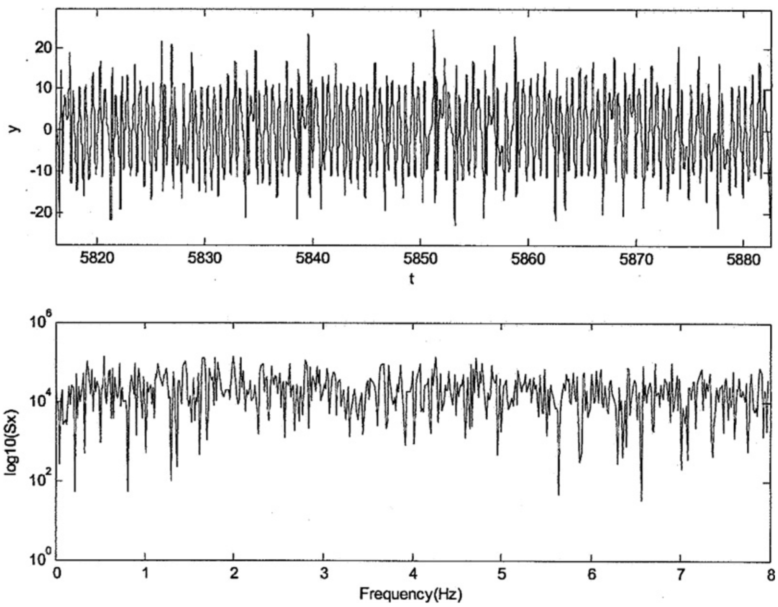

Fig. 6. The power spectrum of the unified chaotic system with $\alpha = 1$ (Chen system).

spectrum. It is often used to distinguish between the periodic, quasi-periodic and chaotic behaviors of a dynamical system. When a chaotic motion appears, the corresponding power spectrum is continuous and broadband. In Fig. 6, the power spectrum of the chaotic motion of the Chen system is shown. Although a broadband spectrum does not guarantee the sensitivity to initial conditions, it is still a reliable indicator of the chaos.

In Fig. 7, some non-chaotic ranges within $\alpha \in [0, 1]$ are observed. For example, we can easily observe between $\alpha = 0.575$ and $\alpha = 0.598$, the corresponding Lyapunov exponents are all less than 0. The three corresponding Lyapunov exponents are $(0, -, -)$ and the corresponding dynamic trajectory must be periodic motions. In Figs. 8(a) and 8(b), the phase portrait, the time history of state and the power spectrum of the non-chaotic motion ($\alpha = 0.583$) are shown,

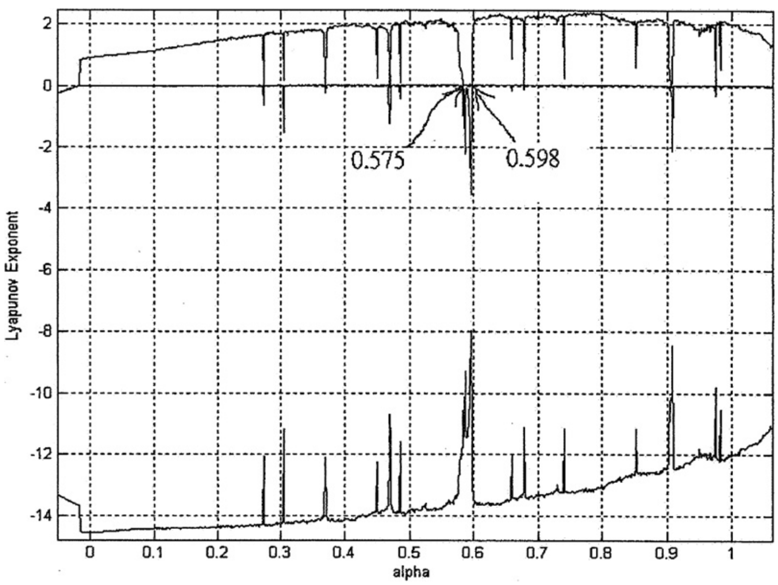

Fig. 7. The Lyapunov exponents of the unified chaotic system for α between -0.05 and 1.05.

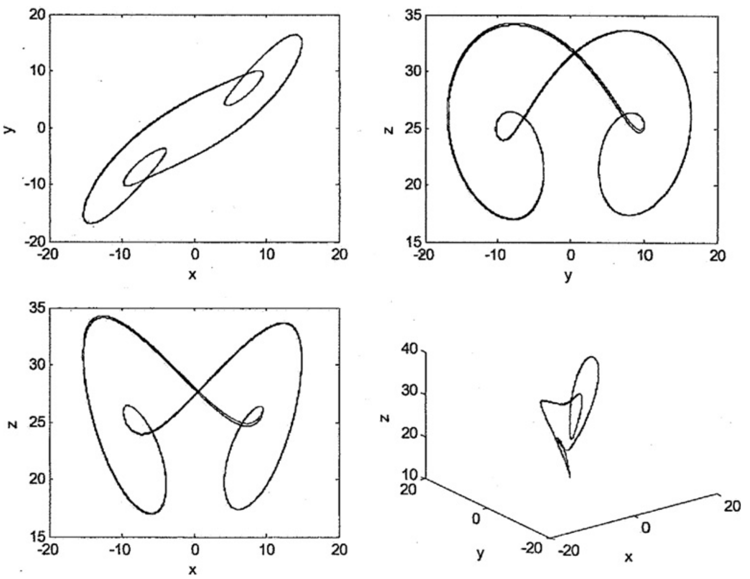

Fig. 8(a). The phase portraits of the unified chaotic system with $\alpha = 0.583$.

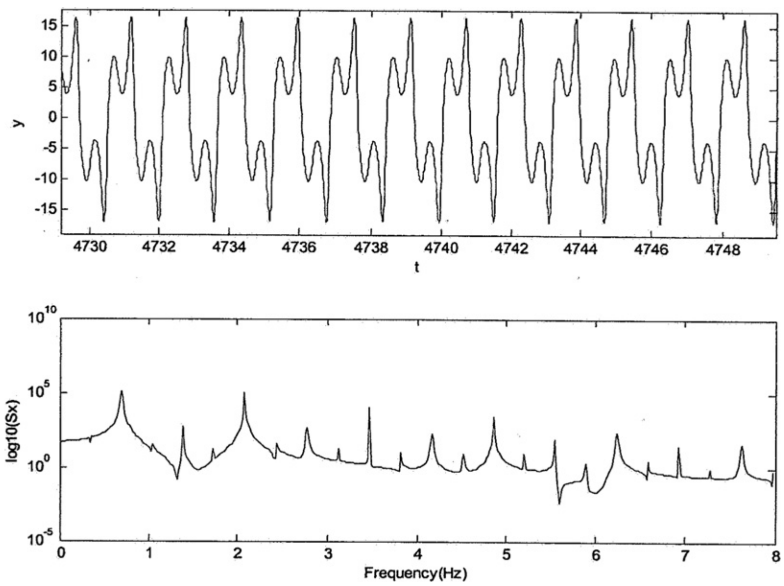

Fig. 8(b). The time history of state y and the power spectrum of the unified chaotic system with $\alpha = 0.583$.

respectively. Certified by these three different methods, many non-chaotic ranges and points of the unified chaotic system within $\alpha \in [0,1]$ are presented in Fig. 9.

4. The chaotic ranges outside of $\alpha \in [0,1]$

We try to find the chaotic ranges outside of $\alpha \in [0,1]$. In Fig. 1, it is shown that outside of $\alpha \in [0,1]$, there still exists some small ranges of which the corresponding largest Lyapunov exponents are greater than 0. In Figs. 10(a) and 10(b), the part of largest Lyapunov exponents for α larger than 1 and that for α smaller than 0 are magnified, respectively. We

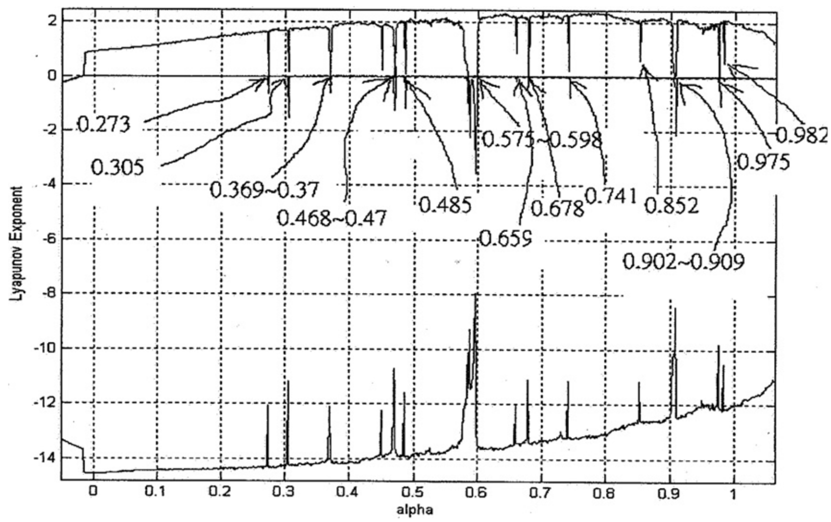

Fig. 9. Many non-chaotic ranges and points of the unified chaotic system within $\alpha \in [0, 1]$.

finally obtain that the chaotic range of α is $\alpha \in [-0.015, 1.152]$ certified by the above three checking methods. The extended chaotic range for the unified system is obtained.

5. The unified chaotic system with $k\sin^2\omega t$ periodic switch

In Ref. [32], a non-autonomous unified chaotic system periodically switching between the Lorenz and Chen system is given. In following sections, we extend the periodic switch to various forms. Five different periodic switches, $k\sin^2\omega t$, $m\sin\omega t$ two different triangular waves, and sawtooth wave, are used to replace α which is the original constant parameter of unified chaotic system. Many interesting results will be found.

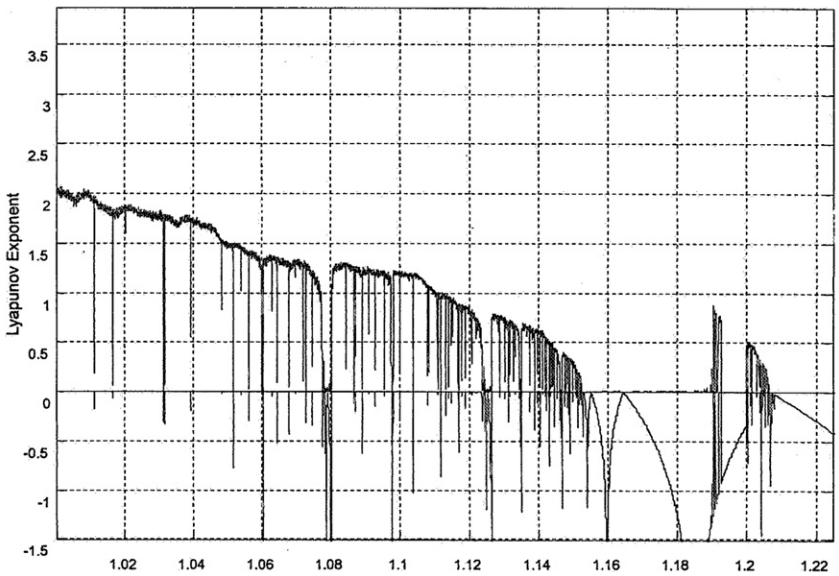

Fig. 10(a). The magnified part of the largest Lyapunov exponents for $\alpha > 1$ of the unified chaotic system.

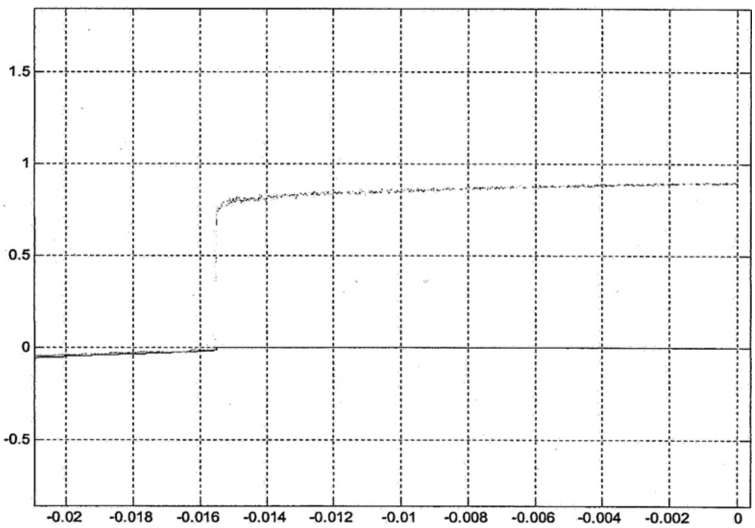

Fig. 10(b). The magnified part of the largest Lyapunov exponents for $\alpha < 0$ of the unified chaotic system.

$k\sin^2\omega t$ is used in place of α so that α is a continuous periodic switch between 0 and where k is the amplitude. The system is described as follows:

$$\begin{aligned}
\dot{x} &= \big(25(k\sin^2\omega t) + 10\big)(y - x),\\
\dot{y} &= \big(28 - 35(k\sin^2\omega t)\big)x + \big(29(k\sin^2\omega t) - 1\big)y - xz,\\
\dot{z} &= xy - \frac{8 + (k\sin^2\omega t)}{3}z.
\end{aligned} \tag{2}$$

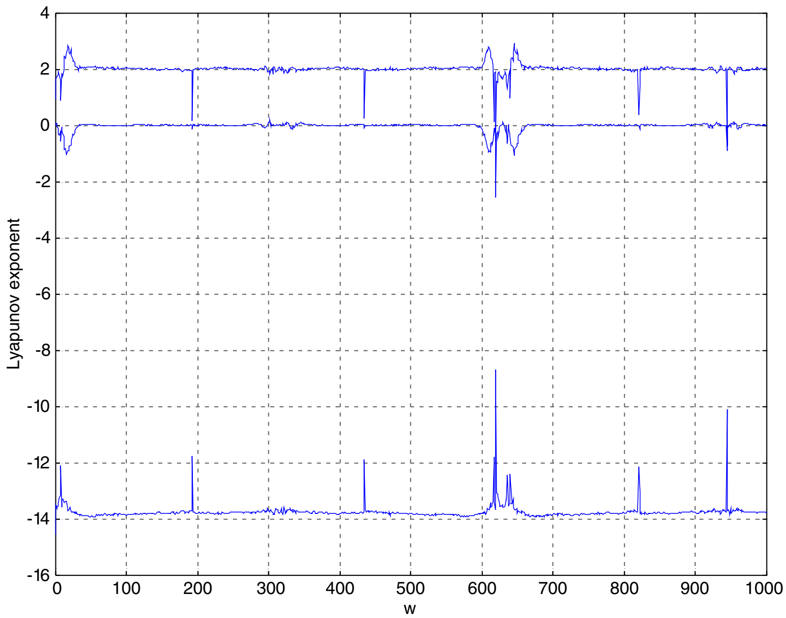

Fig. 11. The Lyapunov exponents of the unified chaotic system with $\alpha = \sin^2\omega t$.

When $k = 1$, the system is the same as that in Ref. [32]. The Lyapunov exponent is shown in Fig. 11. It is easily observed that there exists chaos in this system for almost all $\omega \in [0, 1000]$. Next, we increase the amplitude of $\sin^2 \omega t$. Figs. 12 and 13 are Lyapunov exponent diagrams for $k = 2$ and for $k = 4$, respectively. In Fig. 12, there exists chaos for a considerable part of $\omega \in [0, 1000]$. But when k increases to 4, there exists chaos for only a small part of $\omega \in [0, 1000]$, as shown in Fig. 13. The chaotic phenomena decrease as the amplitude k increases.

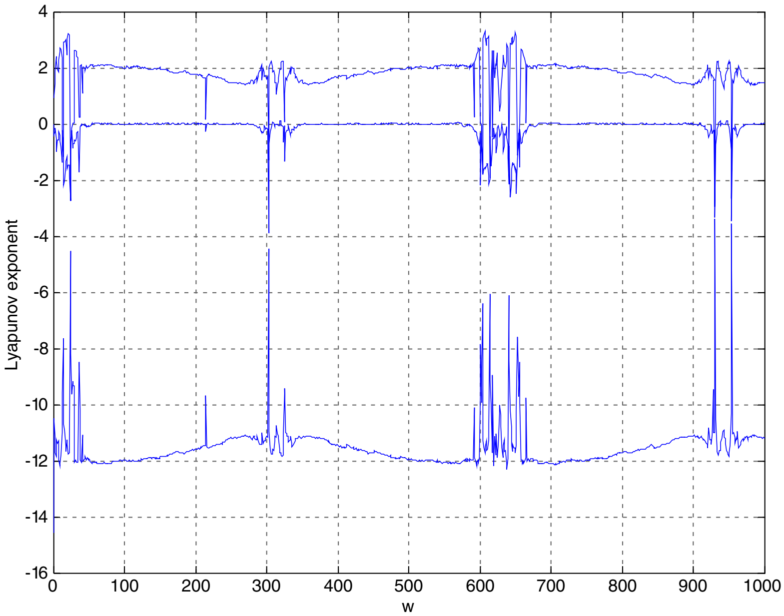

Fig. 12. The Lyapunov exponent of the unified chaotic system with $\alpha = 2\sin^2 \omega t$.

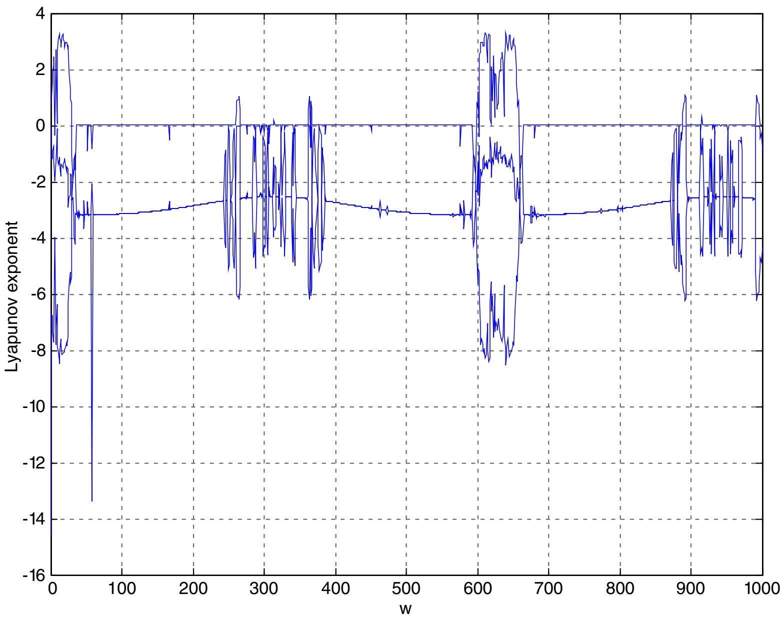

Fig. 13. The Lyapunov exponent of the unified chaotic system with $\alpha = 4\sin^2 \omega t$.

6. The unified chaotic system with $m\sin\omega t$ periodic switch

In this section, we extend α to negative values. A continuous periodic switch between $[-1 \sim 1]$ is used. Consider the following system:

$$\begin{aligned} \dot{x} &= (25(m\sin\omega t) + 10)(y - x), \\ \dot{y} &= (28 - 35(m\sin\omega t))x + (29(m\sin\omega t) - 1)y - xz, \\ \dot{z} &= xy - \frac{8 + (m\sin\omega t)}{3}z. \end{aligned} \tag{3}$$

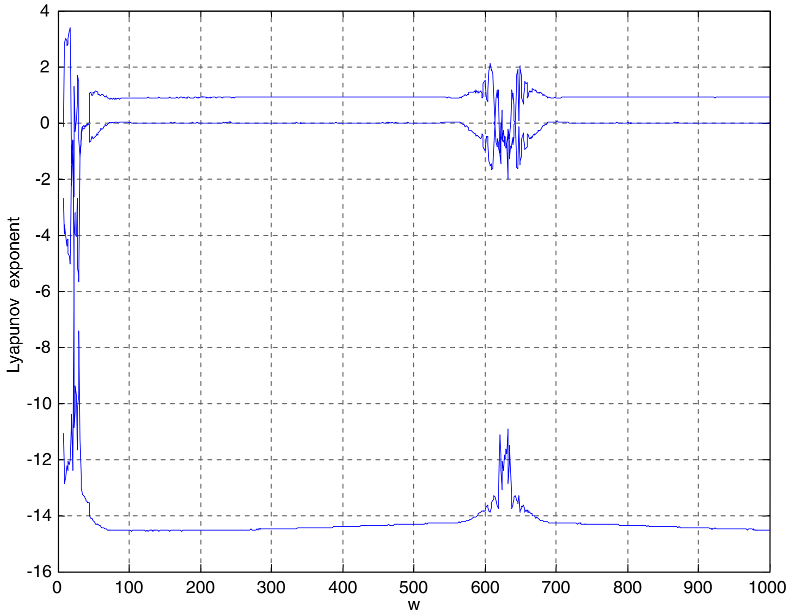

Fig. 14. The Lyapunov exponents of the unified chaotic system with $\alpha = \sin\omega t$.

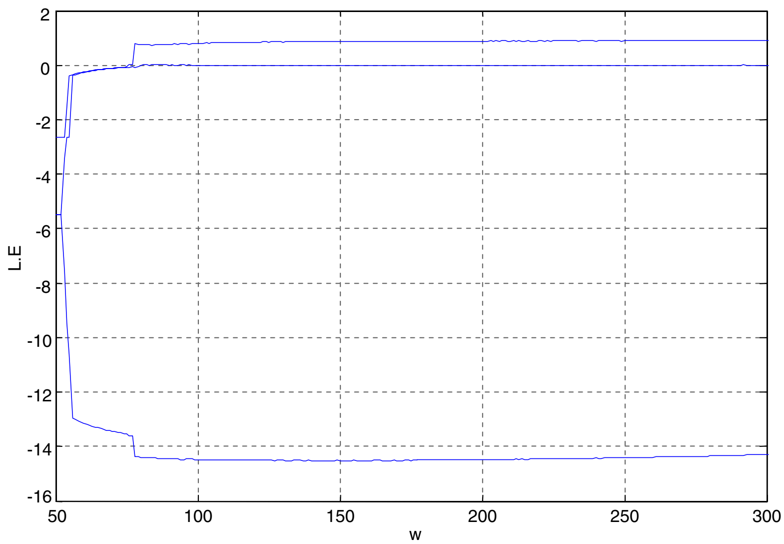

Fig. 15(a). The Lyapunov exponents of the unified chaotic system with $\alpha = 2\sin\omega t$.

$m\sin\omega t$ is used in place of α so that it is a continuous periodic switch between -1 and 1, where m is the amplitude. Firstly, take $m = 1$. Then amplitude m is gradually increased to 2, 4, 6, 8, and 10. In Fig. 14, we can easily observe that when $m = 1$ there exists chaos for almost all $\omega \in [0, 1000]$. As shown in Figs. 15(a)–15(e), when the amplitude m increases, chaotic phenomena in the system decrease.

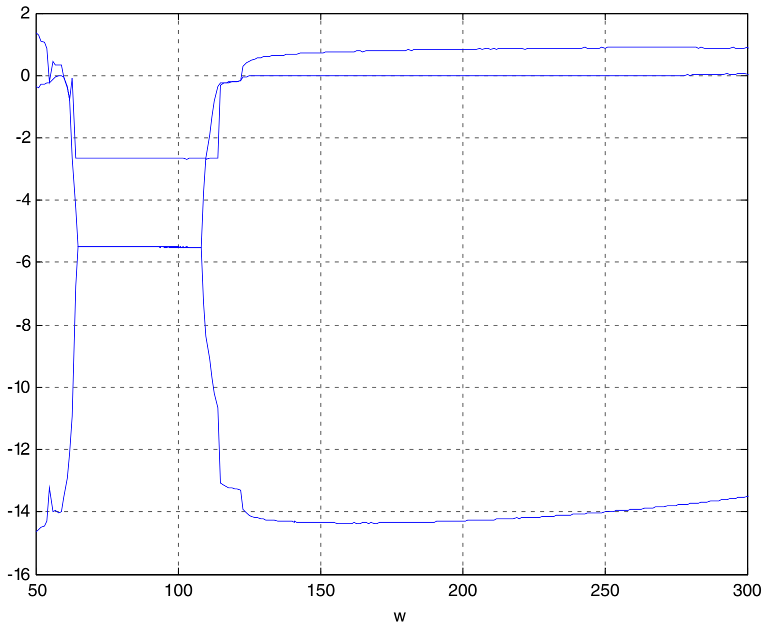

Fig. 15(b). The Lyapunov exponents of the unified chaotic system with $\alpha = 4\sin\omega t$.

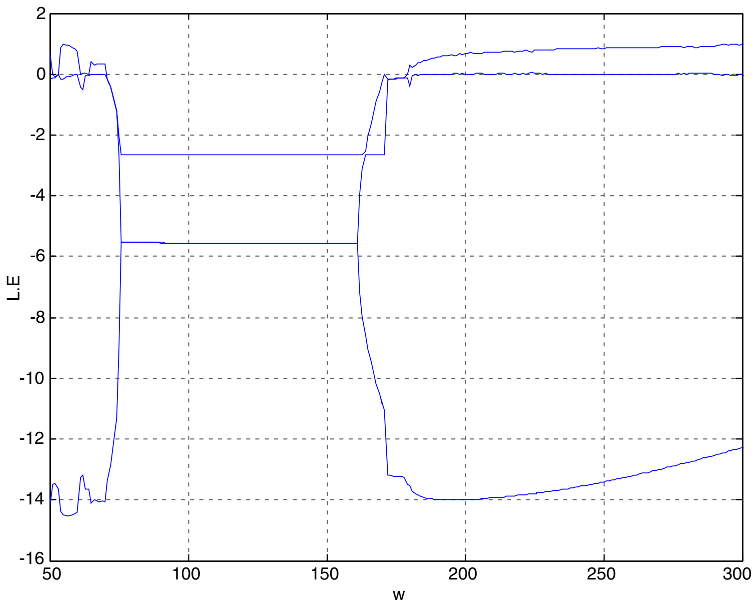

Fig. 15(c). The Lyapunov exponents of the unified chaotic system with $\alpha = 6\sin\omega t$.

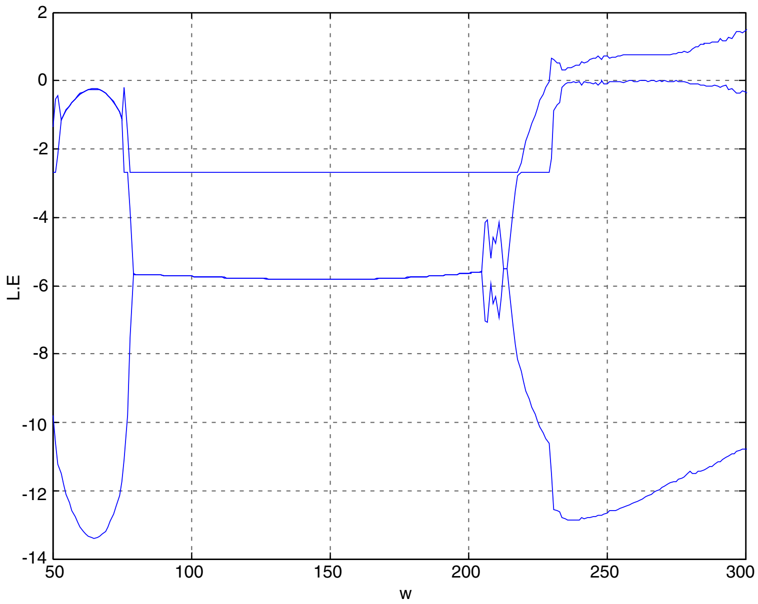

Fig. 15(d). The Lyapunov exponents of the unified chaotic system with $\alpha = 8\sin\omega t$.

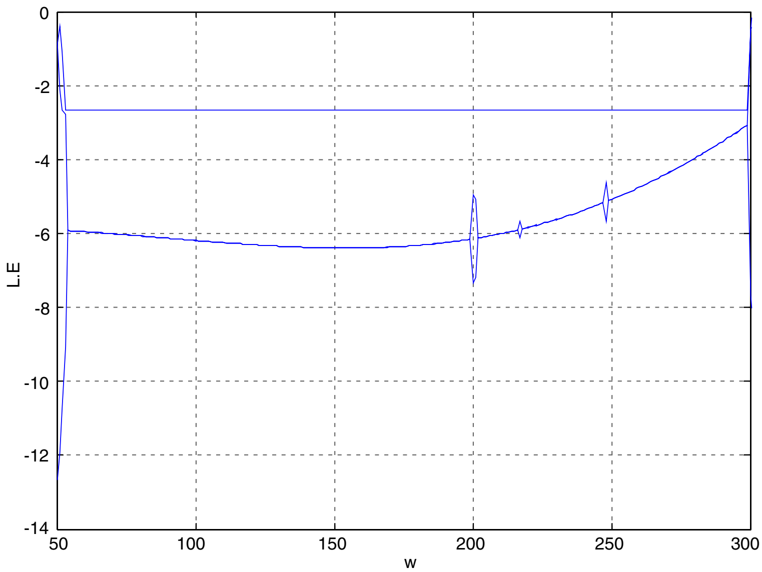

Fig. 15(e). The Lyapunov exponents of the unified chaotic system with $\alpha = 10\sin\omega t$.

7. The unified chaotic system with $0 \sim 1$ triangular wave switch

In this section, a triangular wave is used to substitute α so that it is a periodic switch between 0 and 1. Consider the following triangular function of time t:

$$f(t) = \begin{cases} \frac{2k}{L}t & \text{if } 0 < t < \frac{L}{2}, \\ \frac{2k}{L}(L-t) & \text{if } \frac{L}{2} < t < L, \end{cases} \tag{4}$$

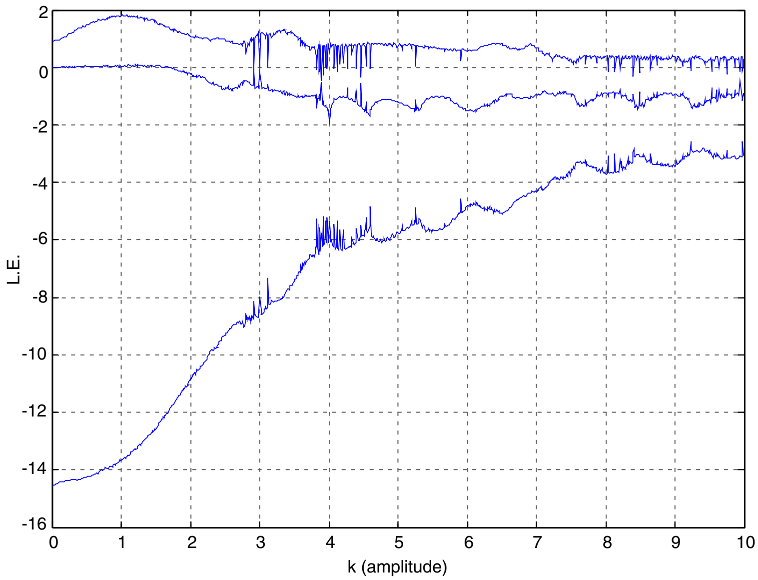

Fig. 16(a). The Lyapunov exponents of the unified chaotic system with triangular wave (4), period $L = \pi$.

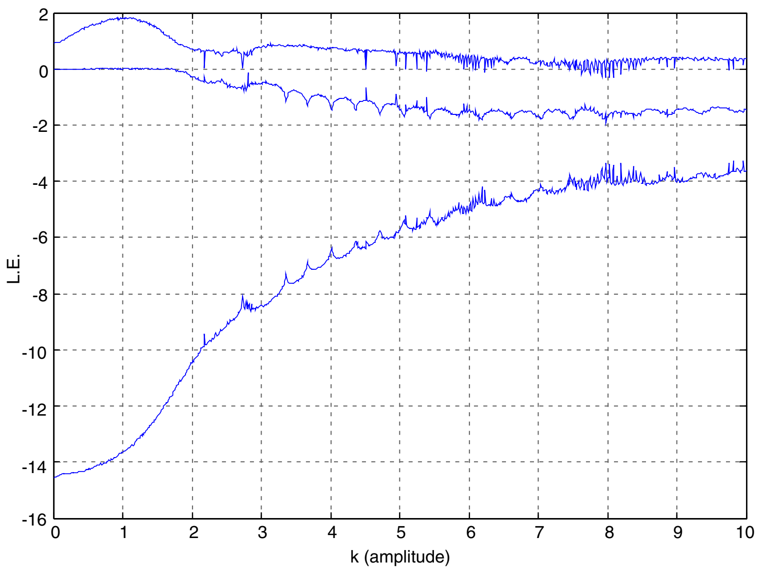

Fig. 16(b). The Lyapunov exponents of the unified chaotic system with triangular wave (4), period $L = 2\pi$.

where k and L are the amplitude and the period of the triangular wave, respectively. Two methods, fixed period L and fixed amplitude k, are used. $f(t)$ is used in place of α and the system is shown as follows:

$$\begin{aligned}\dot{x} &= (25f(t) + 10)(y - x),\\ \dot{y} &= (28 - 35f(t))x + (29f(t) - 1)y - xz,\\ \dot{z} &= xy - \frac{8 + f(t)}{3}z.\end{aligned} \tag{5}$$

Firstly, the period L is fixed, and the amplitude k is gradually increased from 1 to 10. Fig. 16(a) shows the Lyapunov exponent of the unified chaotic system with $\alpha = f(t)$, period $L = \pi$. We can easily find that the larger the amplitude k is, the smaller the largest Lyapunov exponent is. So we can say that the chaotic phenomena decrease as the amplitude k increases. Figs. 16(b) and 16(c) show the Lyapunov exponents of the unified chaotic system with $\alpha = f(t)$, period $L = 2\pi$ and with $\alpha = f(t)$, $L = 3\pi$, respectively. The increase of L changes the largest Lyapunov exponent slightly.

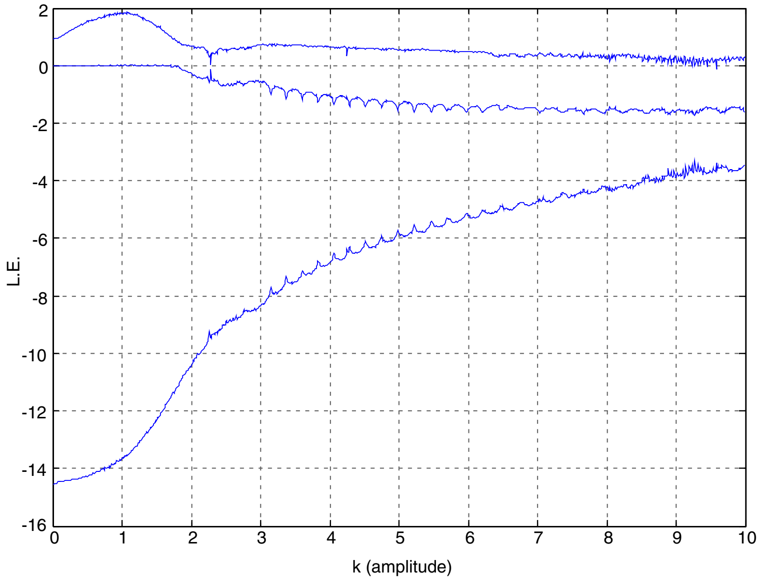

Fig. 16(c). The Lyapunov exponents of the unified chaotic system with triangular wave (4), period $L = 3\pi$.

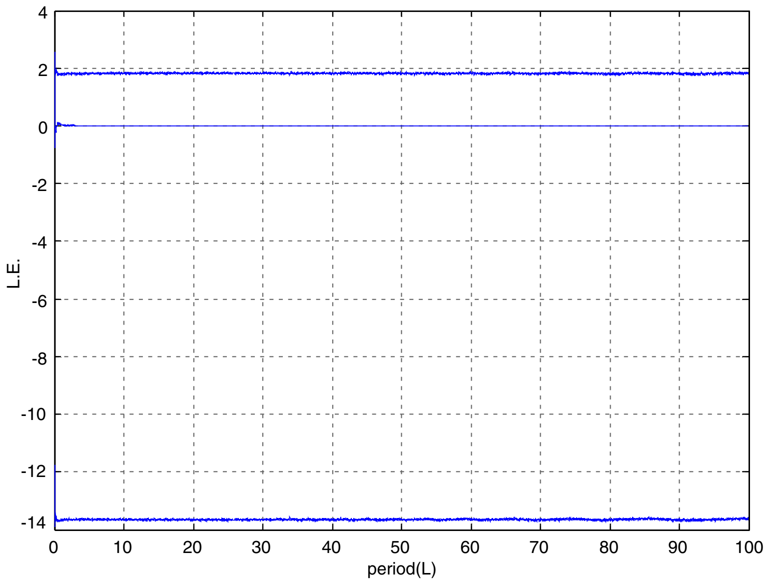

Fig. 17(a). The Lyapunov exponents of the unified chaotic system with triangular wave (4), $k = 1$.

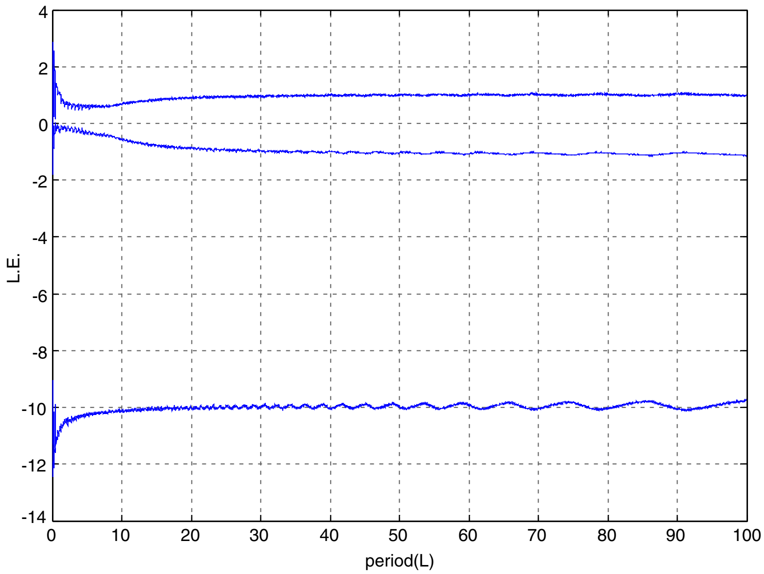

Fig. 17(b). The Lyapunov exponents of the unified chaotic system with triangular wave (4), $k = 2$.

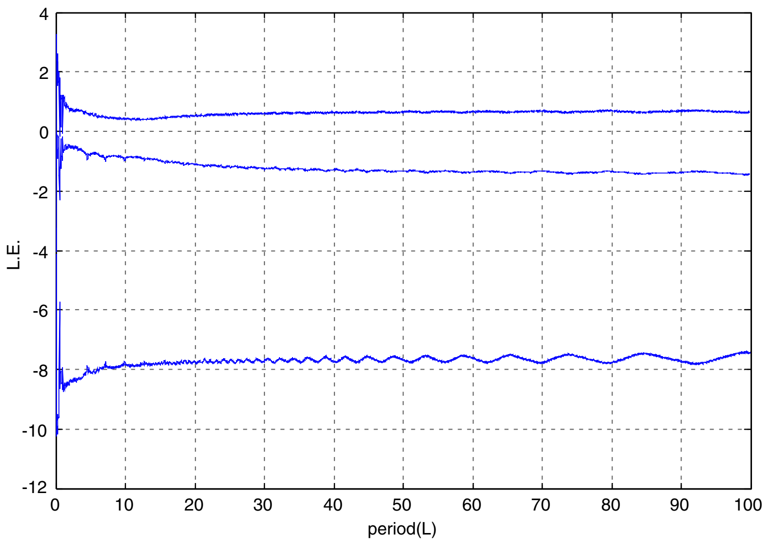

Fig. 17(c). The Lyapunov exponents of the unified chaotic system with triangular wave (4), $k = 3$.

Next, the amplitude is fixed, and the period gradually increases from 1 to 100. Fig. 17(a) shows the Lyapunov exponents of the unified chaotic system with $\alpha = f(t)$, amplitude $k = 1$. The largest value of Lyapunov exponent are around 2, which are almost constant no matter what the period is. Figs. 17(b)–17(d) show the Lyapunov exponents of the unified chaotic system for $\alpha = f(t)$ with amplitude $k = 2$, 3, 10, respectively. The largest Lyapunov exponent becomes smaller and smaller as the fixed amplitude k increases. Fig. 18 shows the bifurcation diagram of the unified chaotic system with $\alpha = f(t)$, amplitude $k = 1$. It appears that the system is chaotic for all $\omega \in [0, 1000]$ where $\omega = 2\pi/L$.

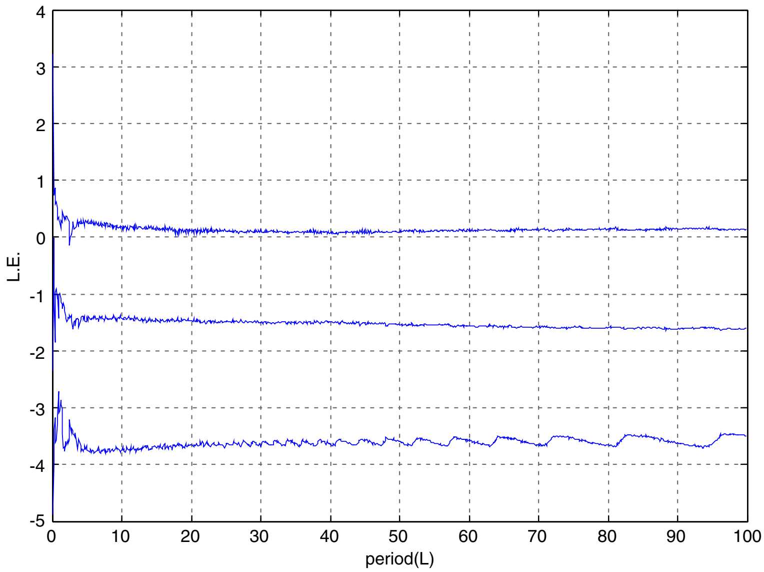

Fig. 17(d). The Lyapunov exponents of the unified chaotic system with triangular wave (4), $k = 10$.

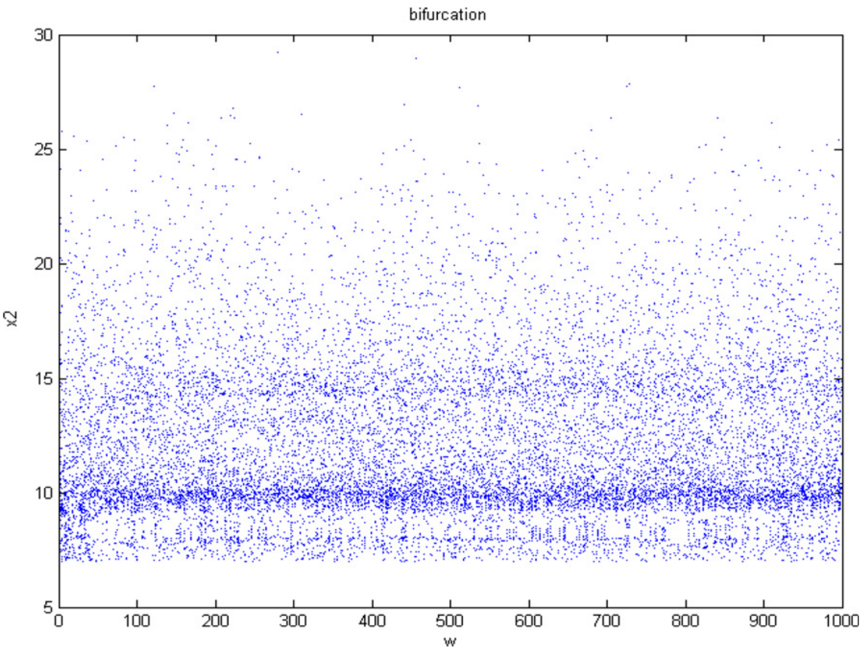

Fig. 18. The bifurcation diagram of the unified chaotic system with triangular wave (4), $k = 1$, $\omega = 2\pi/L$.

8. The unified chaotic system with $-1 \sim 1$ triangular wave switch

In this section, we extend α to negative values. A triangular periodic switch between -1 and 1 is used. Consider the following function of time t:

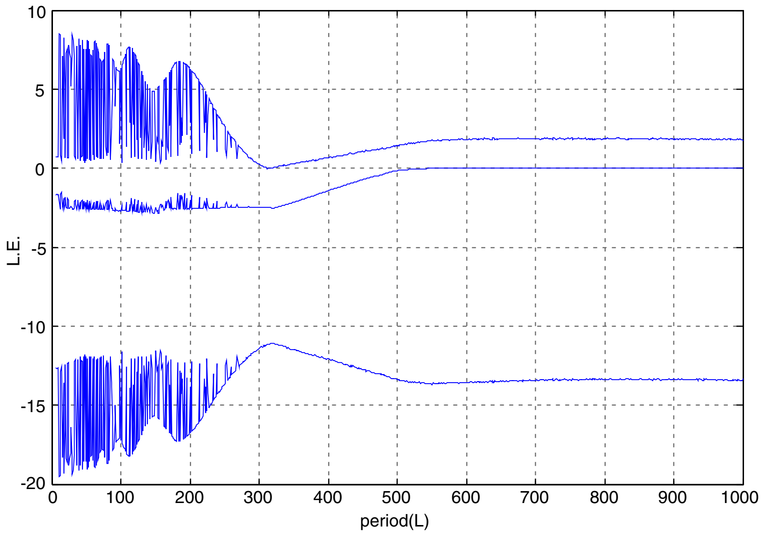

Fig. 19(a). The Lyapunov exponents of the unified chaotic system with triangular wave (6), $k = 1$.

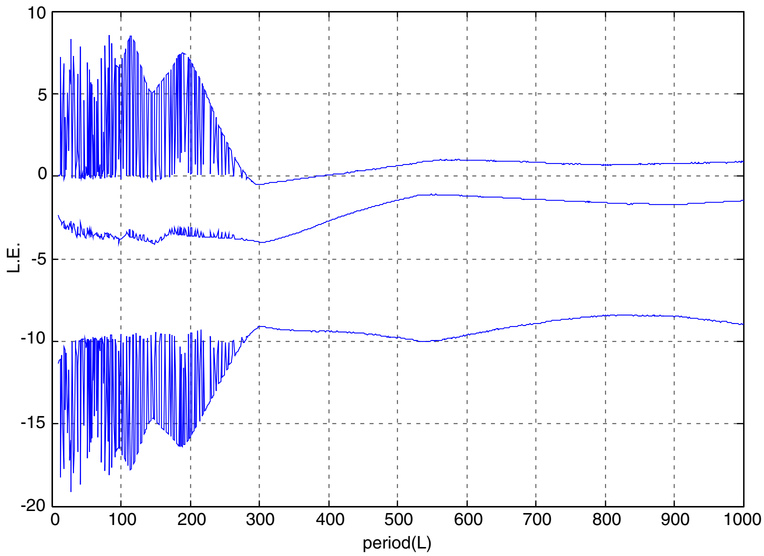

Fig. 19(b). The Lyapunov exponents of the unified chaotic system with triangular wave (6), $k = 2$.

$$g(t) = \begin{cases} \frac{2k}{L}t & \text{if } 0 < t < \frac{L}{2}, \\ \frac{2k}{L}(L - t) & \text{if } \frac{L}{2} < t < \frac{3}{2}L, \\ \frac{2k}{L}(t - 2L) & \text{if } \frac{3}{2}L < t < 2L, \end{cases} \tag{6}$$

where k and L are the amplitude and the period of the triangular wave, respectively. $g(t)$ is used in place of α. The system is shown as follows:

$$\begin{aligned}\dot{x} &= (25g(t) + 10)(y - x),\\ \dot{y} &= (28 - 35g(t))x + (29g(t) - 1)y - xz,\\ \dot{z} &= xy - \frac{8 + g(t)}{3}z.\end{aligned} \quad (7)$$

The amplitude k is fixed at 1, 2, 4, 6, 8, 10, and the period L gradually increases from 1 to 1000. Fig. 19(a) shows the Lyapunov exponents of the unified chaotic system with $\alpha = g(t)$, amplitude $k = 1$. It is evident that when period L is smaller than 300, the Lyapunov exponent vibrates seriously. It becomes smooth when period L is larger than 300. Figs.

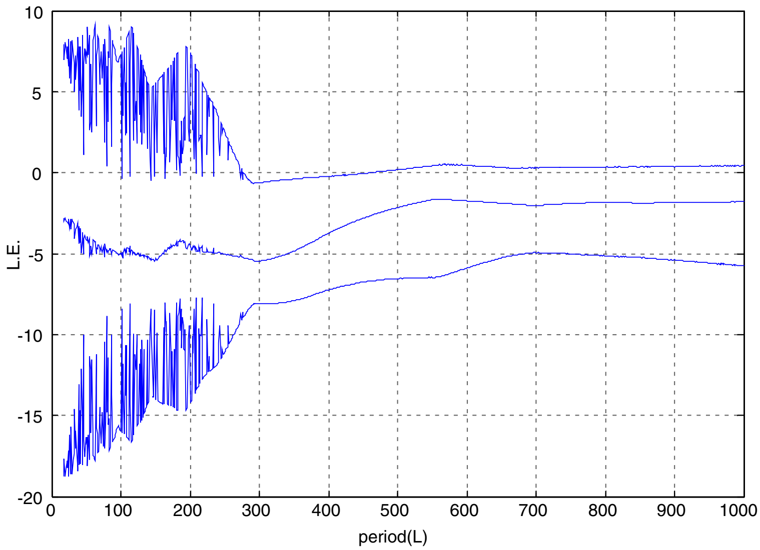

Fig. 19(c). The Lyapunov exponents of the unified chaotic system with triangular wave (6), $k = 4$.

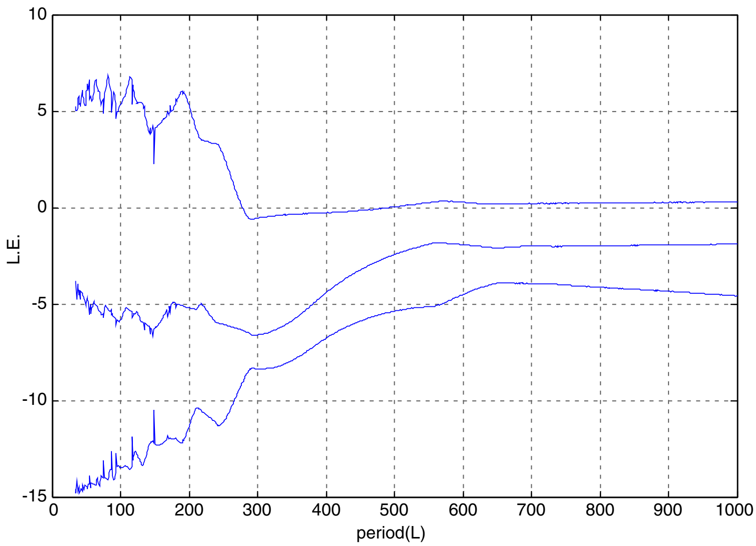

Fig. 19(d). The Lyapunov exponents of the unified chaotic system with triangular wave (6), $k = 6$.

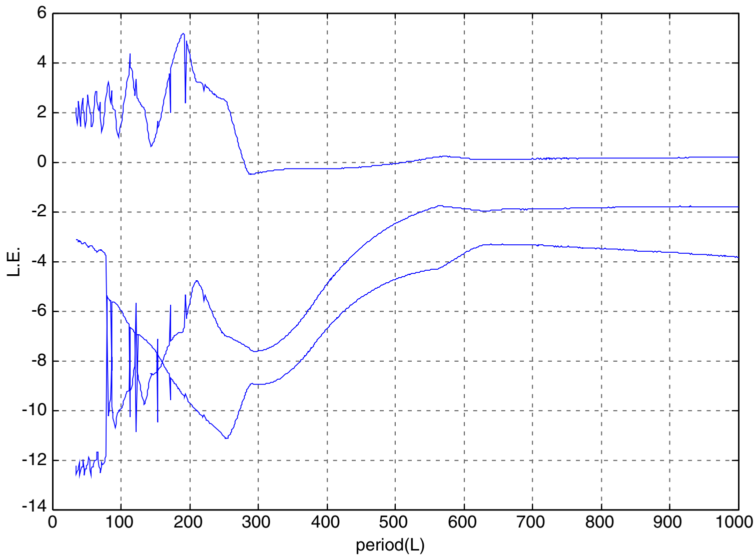

Fig. 19(e). The Lyapunov exponents of the unified chaotic system with triangular wave (6), $k = 8$.

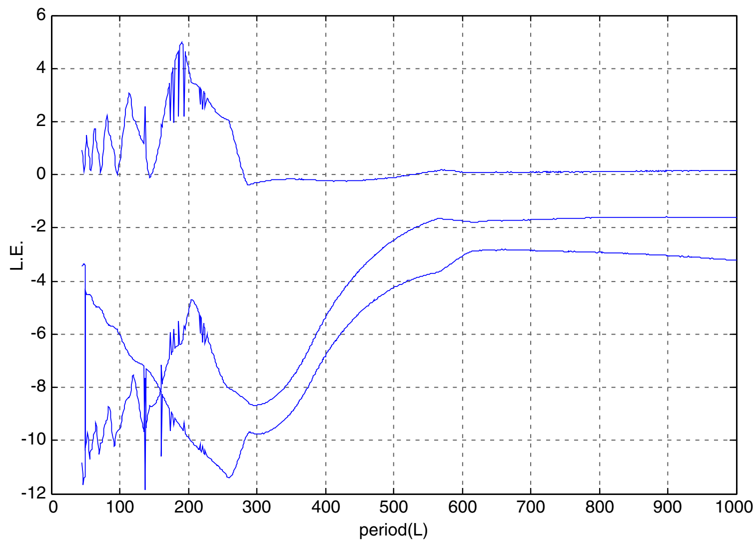

Fig. 19(f). Lyapunov exponent of unified chaotic system with triangular wave (6), $k = 10$.

19(b)–19(f) show the Lyapunov exponents of the unified chaotic system for $\alpha = g(t)$ with amplitude $k = 2, 4, 6, 8, 10$, respectively. The largest Lyapunov exponent becomes smaller as the fixed amplitude k increases. It is found that when the fixed amplitude k is greater than 6, the front parts of the Lyapunov exponents no longer vibrate seriously.

9. The unified chaotic system with $0 \sim 1$ sawtooth wave switch

In this section, a sawtooth wave is used to substitute α which is a periodic switch between 0 and 1 expressed as

$$q(t) = \frac{k}{L}t \ \ (0 < t < L) \quad \text{and} \quad q(t+L) = q(t), \tag{8}$$

where k and L are the amplitude and the period of the sawtooth wave, respectively. $q(t)$ is used in place of α and the system is shown as follows:

$$\begin{aligned} \dot{x} &= (25q(t) + 10)(y - x), \\ \dot{y} &= (28 - 35q(t))x + (29q(t) - 1)y - xz, \\ \dot{z} &= xy - \frac{8 + q(t)}{3}z. \end{aligned} \tag{9}$$

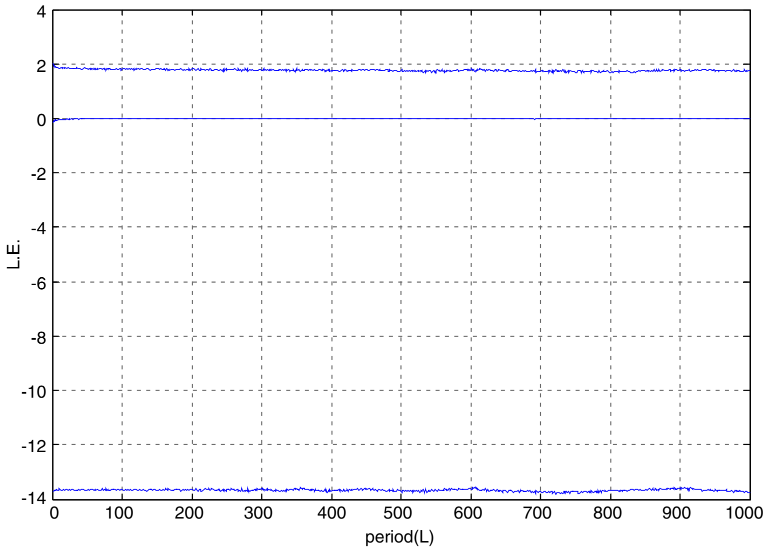

Fig. 20(a). The Lyapunov exponents of the unified chaotic system with sawtooth wave switch, $k = 1$.

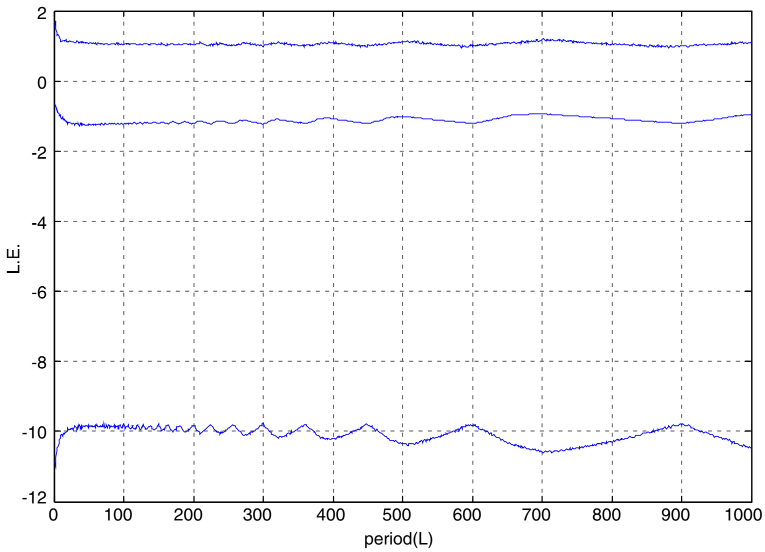

Fig. 20(b). The Lyapunov exponents of the unified chaotic system with sawtooth wave switch, $k = 2$.

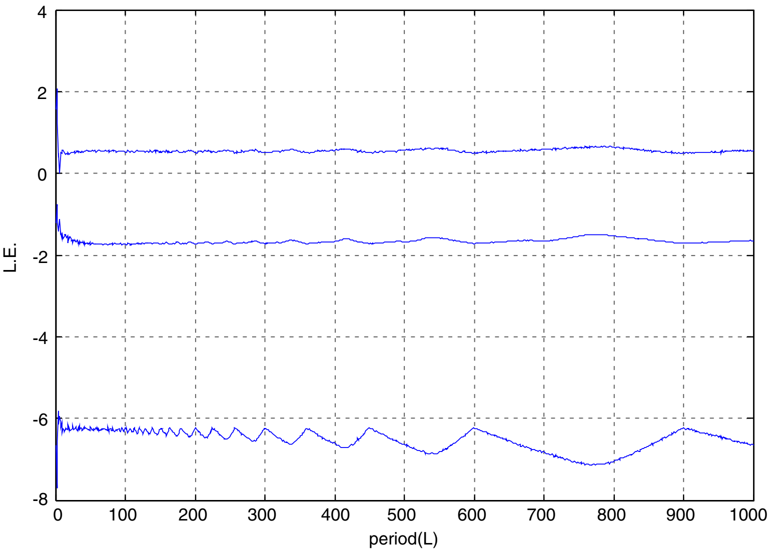

Fig. 20(c). The Lyapunov exponents of the unified chaotic system with sawtooth wave switch, $k = 4$.

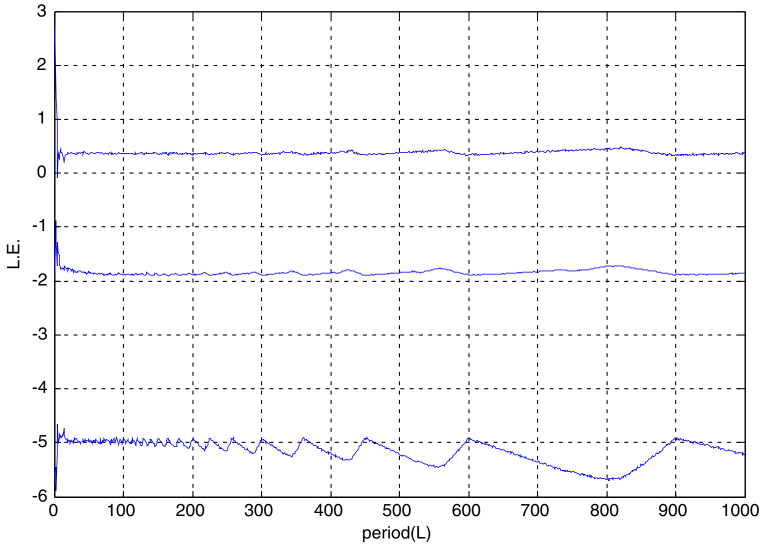

Fig. 20(d). The Lyapunov exponents of the unified chaotic system with sawtooth wave switch, $k = 6$

The amplitude is fixed at 1, 2, 4, 6, 8, 10, and the period gradually increases from 1 to 1000. Fig. 20(a) shows the Lyapunov exponent of unified chaotic system with $\alpha = q(t)$, amplitude $k = 1$. The largest value of Lyapunov exponent is around 2, which is almost constant no matter what the period is. Figs. 20(b)–20(f) show the Lyapunov exponents of the unified chaotic system with $\alpha = q(t)$, for amplitude $k = 2$, 4, 6, 8, 10, respectively. The largest Lyapunov exponents become smaller as the fixed amplitude k increases. As shown in Figs. 20(b)–20(f), the largest Lyapunov exponent is almost always greater than 0 when the fixed amplitude k increases from 1 to 10. It means that even k increase to 10, chaos almost always exists in the system.

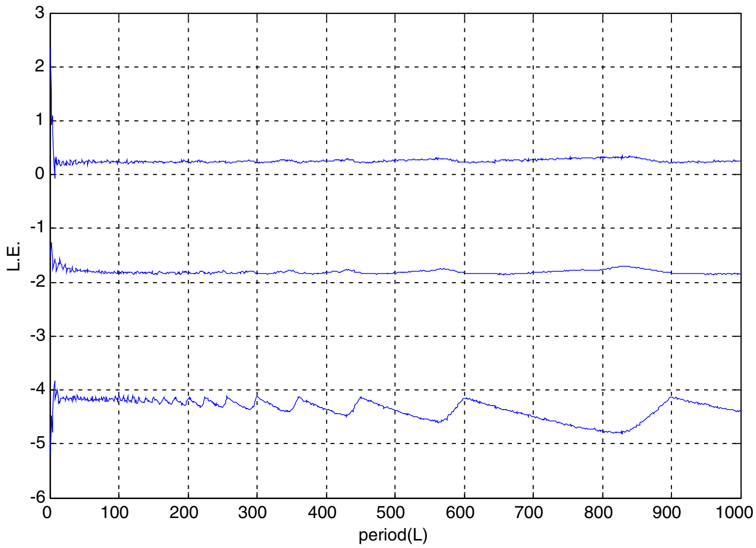

Fig. 20(e). The Lyapunov exponents of the unified chaotic system with sawtooth wave switch, $k = 8$.

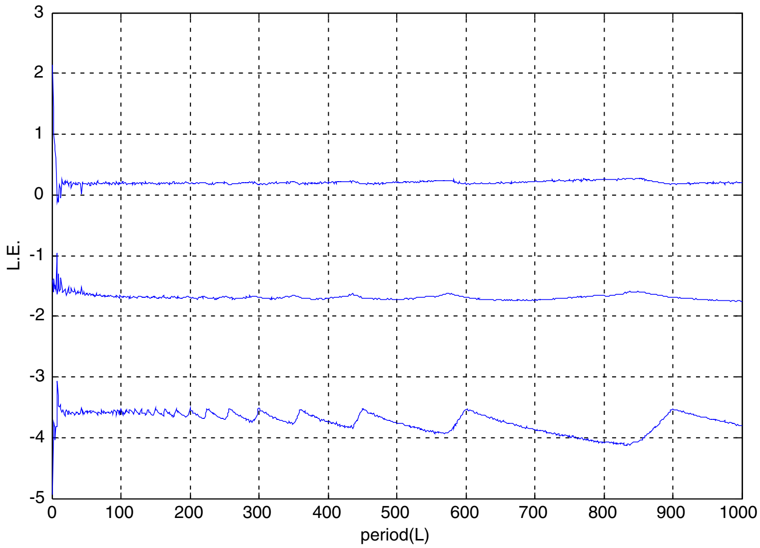

Fig. 20(f). The Lyapunov exponents of the unified chaotic system with sawtooth wave switch, $k = 10$.

10. Conclusions

In this paper, a unified chaotic system is studied in detail. The unified chaotic system is instructed to be chaotic for any $\alpha \in [0, 1]$, where α is the constant parameter of the system. But we find that there exists many non-chaotic ranges and points within $\alpha \in [0, 1]$. Chaotic range longer than $\alpha \in [0, 1]$, $\alpha \in [-0.015, 1.152]$, is discovered, which is the extended chaotic range for unified system.

Chaos in the unified chaotic system with five periodic switches are studied by the Lyapunov exponents method. Five periodic functions of time, $k \sin^2 \omega t$, $m \sin \omega t$ two triangular waves and sawtooth wave, are used to replace α, respectively. Many interesting results are found. Different switching functions cause different behaviors of the system. The common trend is that the larger the switching range is, the smaller the largest Lyapunov exponent will be.

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