Method for determining the fast axis and phase retardation of a wave plate

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Based on the heterodyne interferometric technique and the discrimination technique of using two light beams with different wavelengths, a novel method for identifying the fast axis of a wave plate and evaluating its phase retardation is presented. Some of the merits of the method, such as, a simple optical setup, high stability, better resolution, and easier operation, are presented, and the validity of the method is demonstrated.

1. INTRODUCTION

A wave plate is a commonly used optical element in an optical metrological system.[1–5] The retardation error of a wave plate significantly influences the measurement results. Furthermore, for accurate optical alignments it is necessary to identify the fast axis of the wave plate. Some papers[6,7] have reported how to identify the fast axis, but these studies were done qualitatively without high resolution. There are also some papers[7–14] on how to evaluate the phase retardation of a wave plate. Although the procedures reported have high resolution, they are performed under the condition that the fast axis be located accurately in advance at a special azimuth angle.

In this paper a novel method is presented that can identify the fast axis of a wave plate and at the same time evaluate its phase retardation. The method is based on the heterodyne interferometric technique and the discriminative technique of using two light beams with different wavelengths. The phase difference, rather than optical intensity, is measured electrically, thus giving better resolution. The method has some advantages such as a simple optical setup, high stability because of its common path configuration, and easier operation.

2. PRINCIPLE

The schematic diagram of this method is shown in Fig. 1. A linearly polarized light at wavelength of λ_1 or λ_2 passing through an electro-optic (EO) modulator is incident on a beam splitter BS and is divided into two parts: The reflected light and the transmitted light. The reflected light passes through an analyzer AN_r, then enters a photodetector D_r. If the amplitude of light arriving at D_r is E_r, then the intensity measured by D_r is $I_r = |E_r|^2$, where I_r is the reference signal. The transmitted light passes through a tested wave plate W and an analyzer AN_t and is detected by another photodetector D_t. If the amplitude of light arriving at D_t is E_t, then the intensity measured is $I_t = |E_t|^2$, where I_t is the test signal.

A. Intensities of the Reference Signal and the Test Signal

For convenience, the $+z$ axis is chosen to be along the light propagation direction and the y axis to be along the vertical direction. Let the incident light be linearly polarized in the x direction, the fast axis of the EO modulator under an applied electric field be 45° (the technique for locating the EO modulator is written in Appendix A), and the fast axis of the tested wave plate W be θ with respect to the x axis. The transmission axes of the two analyzers AN_r and AN_t are along the y axis. Then the Jones vector[15] of E_r is

$$\begin{pmatrix} Erx \\ Ery \end{pmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \exp(i\psi_x) & 0 \\ 0 & \exp(i\psi_y) \end{bmatrix} \times \begin{bmatrix} \cos\frac{\Gamma}{2} & i\sin\frac{\Gamma}{2} \\ i\sin\frac{\Gamma}{2} & \cos\frac{\Gamma}{2} \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{bmatrix} 0 \\ i\sin\frac{\Gamma}{2}\exp(i\psi_y) \end{bmatrix}, \quad (1)$$

the intensity of the reference signal is

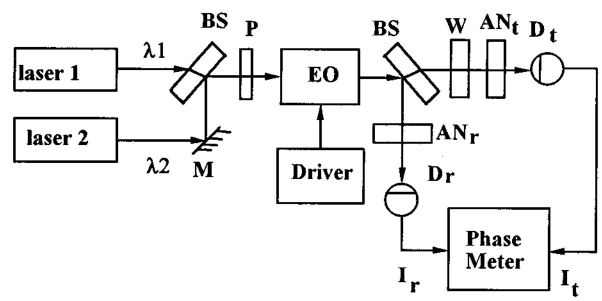

Fig. 1. Schematic diagram of the method: P, polarizer; BS, beam splitter; M, mirror; EO, electro-optic modulator; AN, analyzer; W, wave plate to be measured; D, photodetector.

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$$I_r = |E_r|^2 = \frac{1}{2}(1 - \cos\Gamma), \tag{2}$$

where Γ is the phase retardation introduced by the electro-optic modulator and ψ_x and ψ_y are the phase shifts for the x component and the y component, respectively, of the reflected beam from BS. From Eq. (2) it is obvious that the intensity of the reference beam is independent of the phase shifts corresponding to the reflection from BS. In contrast, the Jones vector of E_t is

$$\begin{pmatrix} E_{tx} \\ E_{ty} \end{pmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\frac{\delta}{2} + i\sin\frac{\delta}{2}\cos 2\theta & i\sin\frac{\delta}{2}\sin 2\theta \\ i\sin\frac{\delta}{2}\sin 2\theta & \cos\frac{\delta}{2} - i\sin\frac{\delta}{2}\cos 2\theta \end{bmatrix} \begin{bmatrix} \cos\frac{\Gamma}{2} & i\sin\frac{\Gamma}{2} \\ i\sin\frac{\Gamma}{2} & \cos\frac{\Gamma}{2} \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{bmatrix} 0 \\ i\sin 2\theta\sin\frac{\delta}{2}\cos\frac{\Gamma}{2} + i\cos\frac{\delta}{2}\sin\frac{\Gamma}{2} + \cos 2\theta\sin\frac{\delta}{2}\sin\frac{\Gamma}{2} \end{bmatrix}. \tag{3}$$

Then the intensity of the test signal is

$$I_t = \frac{1}{2}[1 - \sqrt{A^2 + B^2}\cos(\Gamma + \phi)]; \tag{4}$$

where δ is the phase retardation of W and

$$A = \cos^2 2\theta + \sin^2 2\theta\cos\delta, \tag{5}$$

$$B = \sin 2\theta\sin\delta. \tag{6}$$

In the above descriptions, ϕ is the phase difference between I_t and I_r. It can be any value between 0 and 2π with a period of 2π; here we arbitrarily define its value to be between $-\pi$ and π. Then, according to the value of δ and the signs of A and B, ϕ is given by the following expressions:

If $0° \leqslant |\delta| \leqslant 90°$, then

$$\phi = \tan^{-1}(B/A). \tag{7}$$

If $90° \leqslant |\delta| \leqslant 180°$ and if

$$A \geqslant 0, \quad \text{then} \quad \phi = \tan^{-1}(B/A); \tag{8}$$

$$A < 0 \text{ and } B \geqslant 0, \quad \text{then} \quad \phi = \pi + \tan^{-1}(B/A); \tag{9}$$

$$A < 0 \text{ and } B < 0, \quad \text{then} \quad \phi = -\pi + \tan^{-1}(B/A). \tag{10}$$

If Γ is constant, i.e., without the electro-optic modulator or with the electro-optic modulator off, then both I_r and I_t are constants, and it is difficult to evaluate the value of ϕ. If a sawtooth voltage signal with amplitude $V_{\lambda/2}$, the half-wave voltage of the EO modulator, is applied to the EO modulator, then $\Gamma = wt$, where w is the angular frequency of the sawtooth signal. Thus from Eqs. (2) and (4) it is seen that the two detected output signals are sinusoidal with a phase difference ϕ. These two sinusoidal signals are sent to a phase meter, and the phase difference ϕ can be measured.

B. Performing Procedures

From Eqs. (5)–(10) it is obvious that the phase difference ϕ between I_t and I_r is a function of θ and δ. The relation curves of ϕ versus θ for different δ that are from $-180°$ to 0° and from 0° to 180° with an interval of 30° are shown in Figs. 2(a) and 2(b), respectively. It can be seen in these figures that, despite the values of δ, ϕ equals zero when θ equals 0°, 90°, 180°, and 270°. These values occur when I_t and I_r are in phase. So this behavior can be used for identifying the azimuth angle when θ equals either 0° or 90°. Although the phase retardation of the wave plate at 180° or nearly 180° also has this behavior, it is difficult to identify the azimuth angle when θ equals either 0° or 90° by using only this behavior, because the curve of the phase difference is horizontal near $\theta = 0°$ and $\theta = 90°$. Since the phase retardation of a wave

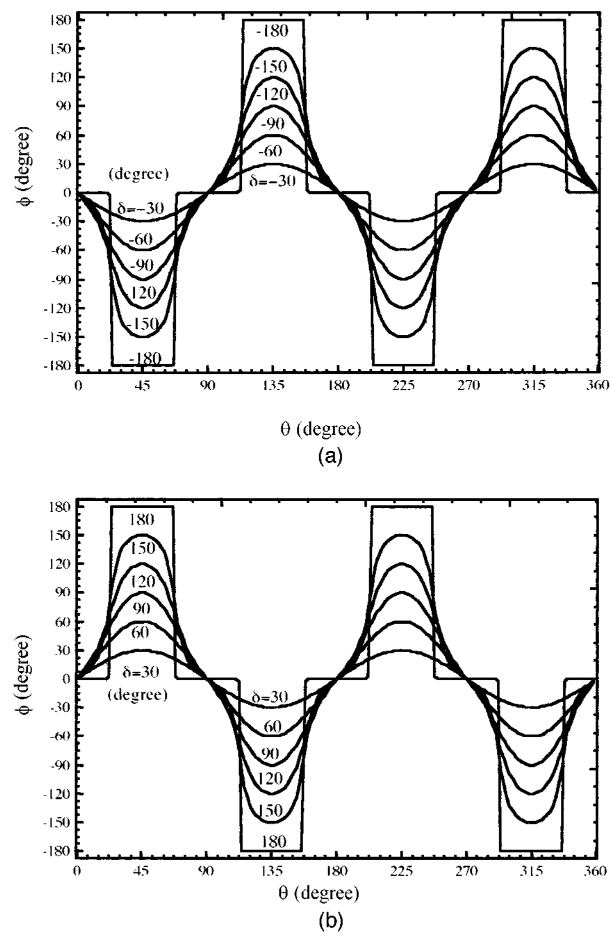

Fig. 2. Relation curves of ϕ versus θ for different δ: (a) $\delta = -180°$ to 0°, (b) $\delta = 0°$ to 180°, at 30° intervals.

plate depends on the wavelength, we can solve this problem by introducing another light source of wavelength λ_2. Then the phase retardation of the wave plate is changed and δ is no longer 180° or nearly 180°. Now the behavior can again be used to determine the azimuthal position at either $\theta = 0°$ or $\theta = 90°$. In contrast, ϕ has an extreme value when θ equals 45°, 135°, 225°, and 315°. For $\theta = 45°$ and 225°, $\delta = \phi$; and for $\theta = 135°$ and 315°, $\delta = -\phi$. Consequently, the phase retardation can be estimated. On the basis of the above descriptions, the procedures for determining the fast axis and the phase retardation of a wave plate, specified for wavelength λ_1, can be summarized as follows:

1. Rotate the wave plate W until I_t and I_r are in phase (i.e., $\phi = 0$) with light source of wavelength λ_1. Then rotate the wave plate back and forth slightly to check the sensitivity of ϕ versus θ. If it is sensitive, then keep the wave plate at the azimuth angle where ϕ equals zero. If it is insensitive, introduce another light source of wavelength λ_2 and rotate the wave plate until ϕ becomes zero. At this condition the fast axis of the wave plate is located at either $\theta = 0°$ or $\theta = 90°$. This represents the situation in which either the fast axis or the slow axis is along the x axis.
2. Next rotate the wave plate counterclockwise by 45°, and θ becomes either 45° or 135°. Under this arrangement light sources of wavelengths λ_1 and λ_2 are used separately, and ϕ_1 and ϕ_2, respectively, are obtained. Note that the phase retardation of a wave plate has a period of 360°; hence the corresponding ϕ_1 and ϕ_2 should be modified so that they are within the range 0° to 360°. The measurable range of the phase meter is between −180° and 180°; hence for the required modification to be obtained, the measured values of ϕ_1 and ϕ_2 must remain unchanged if they are positive; otherwise, 360° should be added to the data whenever it is negative.
3. Following the measurements of ϕ_1 and ϕ_2 in procedure 2, if the modified values of ϕ_1 and ϕ_2 are nearly satisfied with the following reference equation,

$$\phi_2 \cong \frac{\lambda_1}{\lambda_2}\,\phi_1, \tag{11}$$

then the fast axis is at 45° to the x axis (i.e., $\theta = 45°$), and the phase retardation to be measured is the original value ϕ_1. Otherwise, the slow axis is at 45° (i.e., $\theta = 135°$) to the x axis, and the phase retardation δ to be measured is the negative of the original value of ϕ_1.

3. EXPERIMENTS AND RESULTS

To demonstrate the validity of this technique, a quarter-wave plate (WPQ-6328-4M) and a half-wave plate (WPQ-6328-2M) manufactured by Japan Sigma Koki Ltd., specified for a 632.8-nm wavelength, were tested. A He–Ne laser with a $\lambda_1 = 632.8$-nm and a He–Cd laser with $\lambda_2 = 441.6$-nm were used. An EO modulator (PC200/2) manufactured by Electro-Optics Developments Ltd., with half-wave voltage 170 V for 632.8-nm and 118.6 V for 441.6-nm wavelength, was used in this test. A sawtooth signal with frequency 2 kHz and amplitude $V_{\lambda/2}$, the half-wave voltage of EO modulator, was applied to the EO modulator. For a quarter-wave plate it was easy to identify the azimuthal position where θ equals either 0° or 90°, and $\phi_1 = 89.3°$ and $\phi_2 = 128.85°$ were obtained when light sources of λ_1 and λ_2, respectively, were used. Since ϕ_1 and ϕ_2 satisfied Eq. (11), the fast axis was at $\theta = 45°$, and its phase retardation was 89.3°. For the half-wave plate it was difficult to identify the azimuthal position where θ equals either 0° or 90°; the second light source of wavelength λ_2 was used to identify it. Then $\phi_1 = -179°$ and $\phi_2 = 102.3°$ were obtained when light sources of λ_1 and λ_2, respectively, were used. Because ϕ_1 is negative, it was modified to $\phi_{1m} = 181°$ according to procedure 2. Since the modified value ϕ_{1m} and ϕ_2 did not satisfy Eq. (11), the fast axis of the half-wave plate was at $\theta = 135°$, and its phase retardation was 179°.

To verify the correspondence between theory and experiments with this method, after the fast axes of these two wave-plates were determined, the phase differences ϕ were measured at different azimuth angles θ. The measurement results of the phase difference ϕ versus the azimuth angle θ are shown in Fig. 3. In this figure the solid curves and the dashed curves represent the theoretical values, and the symbols ○ and △ represent the measured values at wavelengths of 632.8 and 441.6 nm, respectively. The phase retardations at 632.8 nm can be read at $\theta = 45°$ to be 89.3° in Fig. 3(a) and 179° in Fig. 3(b). These results are in good accordance with the original

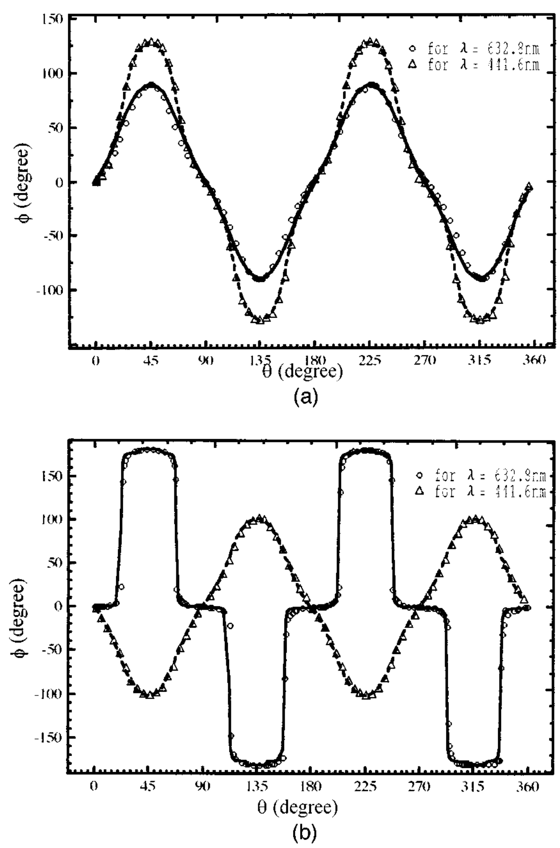

Fig. 3. Theoretical and experimental results for wave plates with 632.8-nm wavelength: (a) a quarter-wave plate, (b) a half-wave plate. Theoretical and experimental results with 441.6-nm wavelength are also included.

wave-plate specifications. The corresponding phase retardations at 441.6-nm wavelength are 128.85° and −102.3°.

4. DISCUSSION

It is described in the beginning of Subsection 2.A that the precondition of this method is that the fast axis of the EO modulator be located accurately at 45° to the x axis. In the following, we discuss the influence of this accuracy on the determination of the fast axis and on the measurement of the phase retardation of the tested wave plate. If there is an azimuthal angular error ϵ in locating the fast axis of the EO modulator, then the Jones matrix of the fast axis can be expressed as

$$\mathrm{EO} = \begin{bmatrix} \cos\frac{\Gamma}{2} - i\sin 2\epsilon \sin\frac{\Gamma}{2} & i\cos 2\epsilon \sin\frac{\Gamma}{2} \\ i\cos 2\epsilon \sin\frac{\Gamma}{2} & \cos\frac{\Gamma}{2} + i\sin 2\epsilon\sin\frac{\Gamma}{2} \end{bmatrix}. \tag{12}$$

With similar derivations from Eqs. (1)–(4), the intensities of the reference signal and the test signal are

$$I_r = \frac{\cos 2\epsilon}{2}(1 - \cos wt), \tag{13}$$

$$I_t = \frac{1}{2}(\gamma + \beta) - \frac{1}{2}[(\gamma - \beta\cos 2\alpha)^2 + (\beta \sin 2\alpha)^2]^{1/2}\cos(wt + \phi'), \tag{14}$$

respectively, where

$$\alpha = \tan^{-1}\left(\frac{\cos 2\epsilon \cos(\delta/2)}{\sin 2\theta \sin(\delta/2)}\right),$$

$$\beta = [\sin 2\theta \sin(\delta/2)]^2 + [\cos 2\epsilon \cos(\delta/2)]^2,$$

$$\gamma = \cos^2(2\theta - 2\epsilon)\sin^2(\delta/2),$$

and

$$\phi' = \tan^{-1}\left\{\frac{\cos 2\epsilon \sin 2\theta \sin\delta}{[\cos^2(2\theta - 2\epsilon) - \sin^2 2\theta]\sin^2(\delta/2) + \cos^2 2\epsilon \cos^2(\delta/2)}\right\}.$$

According to Eqs. (13) and (14), the relation curves of ϕ versus θ for different δ when $\epsilon = 5°$ and $-5°$ are shown in Figs. 4(a) and 4(b) respectively, in which the corresponding curves shown in Fig. 2 are added for comparison. In these figures, the solid and the dashed curves represent the results without and with azimuth angular error ϵ, respectively. Here, positive ϵ means an error in the counterclockwise direction. It is seen that the behavior that δ equals zero when $\theta = 0°$, 90°, 180°, or 270° remains unchanged, so δ has no influence on identifying the fast axis of the tested wave plate. In contrast, the measurement error of the phase retardation is

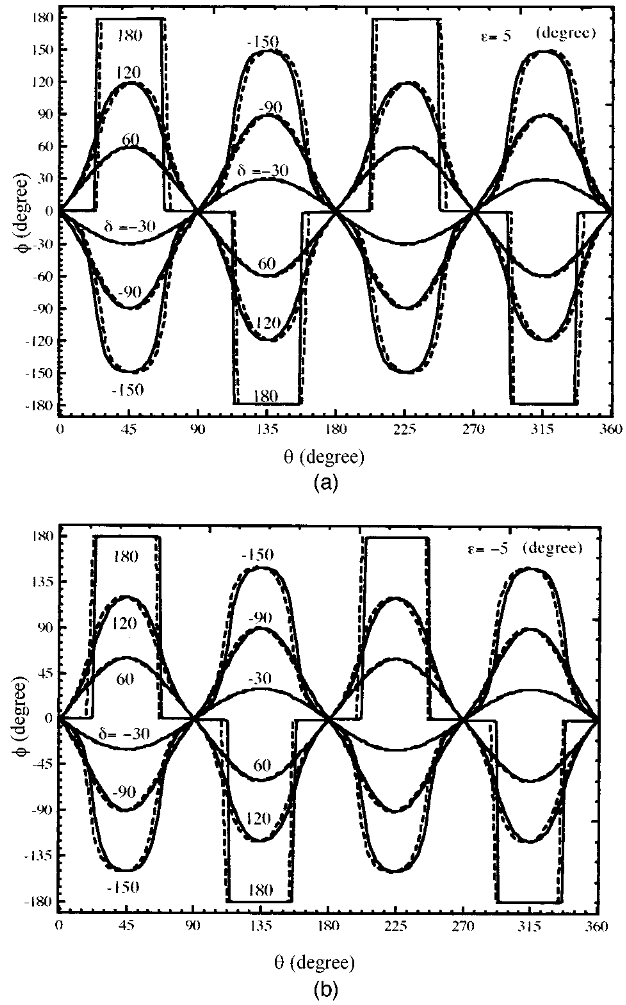

Fig. 4. Comparison of the relation curves of ϕ versus θ for different δ as (a) $\epsilon = 5°$, (b) $\epsilon = -5°$. Solid and dashed curves represent results without and with azimuth angular error ϵ, respectively.

$$\Delta\delta = \tan^{-1}\left(\frac{\sin\delta}{\cos 2\epsilon\cos\delta}\right) - \delta. \tag{15}$$

With $|\epsilon| = 2°$ and 5°, the measurement errors of different wave plates are calculated and shown in Fig. 5. From these curves it is clear that even when $|\epsilon|$ is as large as 5°,

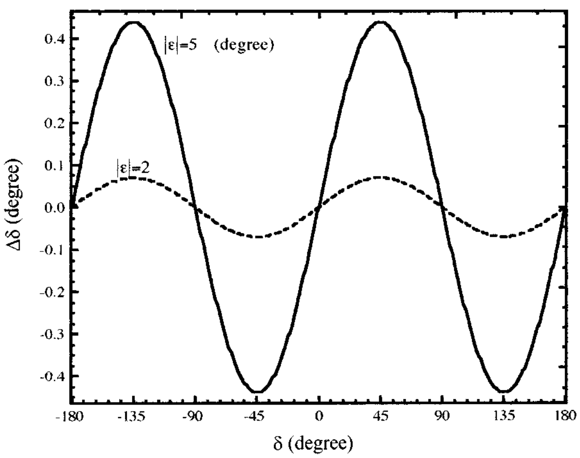

Fig. 5. Measurement errors for wave plates as $|\epsilon| = 2°$ and $5°$.

the measurement error is still small. The maximum error occurs at the wave plates with phase retardations $-135°$, $-45°$, $45°$, and $135°$. However, the maximum error is always less than $0.45°$.

This method is suitable for wave plates with phase retardation between 0° and 360°; its resolution depends on the phase retardation of the wave plate and the resolution of the phase meter. Our experiments show that this method provides a resolution of better than 0.1° when a phase meter with 0.01° resolution is employed to measure a quarter-wave plate.

5. CONCLUSION

A novel method for identifying the fast axis of a wave plate and evaluating its phase retardation was presented. The method is based on the heterodyne interferometric technique and the discriminative technique of using two light beams with different wavelengths. Some of its merits are a simple optical setup, high stability, better resolution, and easier operation. The performance of this system was demonstrated.

APPENDIX A

In our experiments the fast axis of the EO modulator is located at $\theta = 45°$ by means of the following procedures:

1. The EO modulator is inserted between polarizer P and analyzer A as shown in Fig. 6, and we let the transmission axes of P and A be 0° and 90°, respectively, relative to the x axis. If the fast axis of the EO modulator is at η to the x axis and a sawtooth signal with angular frequency w and amplitude $V_{\lambda/2}$ is applied to it, then the intensity detected by photodetector D can be expressed as

$$I = \frac{\sin^2 2\eta}{2}(1 - \cos wt), \tag{A1}$$

based on the derivations of Jones calculus. The signal is an ac signal with amplitude $(\sin^2 2\eta)/2$. It is obvious that the amplitude equals zero, and the signal becomes a dc signal when $\eta = 0°$ or $90°$ by rotation of the EO modulator until a dc signal appears in the oscilloscope. Then the fast axis of the EO modulator is located at either $\theta = 0°$ or $90°$.

2. The optical setup is modified, as shown in Fig. 7. The azimuth angle of the fast axis of the EO modulator is unchanged, and the transmission axes of the polarizer and the analyzers are located at 45° relative to the x axis. The incident beam is diffracted by acousto-optic modulator AOM and is divided into two beams of frequencies f_0 and $f_0 + f_s$, where f_0 is the optical frequency and f_s is the driven frequency applied to AOM. The beam of frequency $f_0 + f_s$ is reflected by mirror M to polarization beam splitter PBS. Then x- and y-polarization components enter photodetectors D_1 and D_2, respectively. The beam of frequency f_0 passes through the EO modulator and also enters PBS. A sawtooth signal with frequency f and amplitude $V_{\lambda/2}$ is applied to the EO modulator; thus the Jones vector of the light beam after leaving the EO modulator is

$$E_0 = \begin{Bmatrix} \exp\left[i2\pi\left(f_0 \pm \frac{f}{2}\right)t\right] \\ \exp\left[i2\pi\left(f_0 \mp \frac{f}{2}\right)t\right] \end{Bmatrix}, \tag{A2}$$

where the upper signs are taken if the fast axis of the EO is at $\theta = 0°$. The beam is then divided by PBS, and the y- and the x-polarization components enter D_1 and D_2, re-

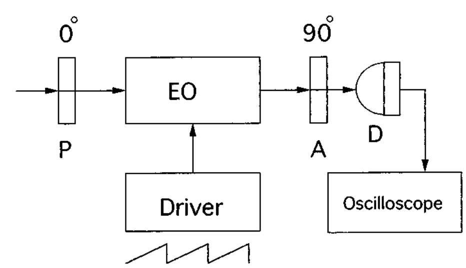

Fig. 6. Optical setup for locating the fast axis of the EO modulator at either $\theta = 0°$ or $\theta = 90°$: P, polarizer; EO, electro-optic modulator; A, analyzer; D, photodetector.

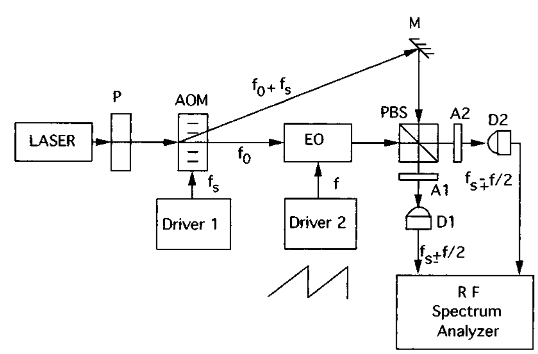

Fig. 7. Optical setup for identifying the fast axis of EO modulator: P, polarizer; AOM, acoustic-optic modulator; M, mirror; EO, electro-optic modulator; PBS, polarization beam splitter; A's, analyzer; D's, photodetectors.

spectively. Consequently, D_1 and D_2 detect the interference signals with beat frequencies at f_1 and f_2, and they can be expressed as

$$f_1 = f_s \pm f/2, \tag{A3}$$

and

$$f_2 = f_s \mp f/2, \tag{A4}$$

respectively. These two signals are sent to the RF spectrum analyzer for comparison. If $f_1 < f_2$, the fast axis of the EO modulator is along the x axis (i.e., $\theta = 0°$). Otherwise, the fast axis of the EO modulator is along the y axis (i.e., $\theta = 90°$). After determining the fast axis of the EO modulator, we use a high-resolution rotation stage to rotate the fast axis of the EO modulator to $\theta = 45°$.

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