在變動成本函數下最佳化供應鏈排程

Optimize a Supply Chain Schedule under Variable Cost Functions

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摘要：在現今企業電子化的競爭環境中,利用供應鏈管理資訊系統處理客戶訂單和快速回應市場變動,已是必要且不可或缺的,然而,供應鏈管理資訊系統是否能有效地協助上、中、下游廠商安排最佳化的供應鏈排程,則完全取決於系統中供應鏈管理計量模式的有效性。尤其是當上、中、下游協同過程中,任何部份些微的儲運或生產活動的改變,均會牽動整個供應鏈的行程,因此,要能有效與迅速地安排最可行與最佳化的供應鏈排程,經理人除了溝通協調上的努力外,一個能切實反應真實儲運現況的供應鏈管理計量模式,更扮演著舉足輕重的角色。有鑑於目前市面上供應鏈管理資訊系統的計量模式,在處理成本函數時,不是將其簡化為固定的數字或線性的函數、就是以啓發式或針對特定目的的演繹法來處理非線性成本函數,前者不符實際產業的運作,後者則不易被撰寫於一般供應鏈管理資訊系統中,且所計算出來的解答也只是局部最佳解。因此,本研究在於提出一些易於處理各色各樣非線性成本函數的方法,以便能讓供應鏈管理計量模式在目前的資訊系統中,更能發揮其強大與有效地供應鏈排程最佳化能力。一個小型的實例運算,則被用來展現所提出方法的實用性與廣泛性。

關鍵詞：成本函數、儲運成本、供應鏈管理、計量模式

Abstract：In current e-business competitive climate, employing supply chain management (SCM) information systems to fulfill customer orders and quickly response the changing market is obligatory and crucial. However, the effectiveness of computerized SCM system heavily depends on the availability of SCM quantitative formulation. Particularly, a simple change within the supply chain can lead to the complicated dynamics along the whole supply channels. Consequently, to effectively and efficiently arranging the most feasible and promising supply chain schedule requires not only arduous managerial efforts, but also a practical SCM quantitative model. Unfortunately, current mathematical models embedded in prevailing SCM systems either use fixed values or linear functions to replace real-world cost functions or employ heuristics or special purpose algorithms to treat nonlinear cost functions. The former approach is out of the reality, while the latter is hard to be coded in general commercial SCM systems and can not guarantee the computed result is global optimal solution. Accordingly, this work attempts to present some easy-to-handle formulations of treating various types of cost functions such as nonlinear step cost functions, concave cost functions, and S-curve cost functions, and to make current computer-based SCM quantitative models more practical to enhance the execution of more elaborate coordination activities. A small example is employed to demonstrate the practicality and applicability of the proposed methods.

Keywords：Cost Functions, Logistics Costs, Supply Chain Management, Quantitative Model

1. Introduction

What is supply chain management (SCM)? Supply Chain Management is to effectively integrate supply chain processes across companies into a cohesive and high-performing business model which can quick response and the fulfillment of orders. Undoubtedly, without a computerized SCM system the coordination and collaboration among all partners within a supply chain cannot be executed. Since the effectiveness of a computerized SCM system strongly depends on the availability and effectiveness of mathematical formulation, a useful quantitative SCM model can help the management optimize logistics schedule and achieve reliable collaboration among partners. According to the definition by the Council of Supply Chain Management Professionals (CSCMP) (2006), an effective coordination and collaboration among supply chain partners is vital for a successful SCM.

Unfortunately, current SCM quantitative models frequently either use fixed values or linear functions to represent real-world cost functions (Anderson *et al*., 2004) or employ heuristics or special

purpose algorithms to treat nonlinear cost functions (Shapiro, 2005). The former approach is out of the real-life situation, while the latter is hard to be solved in prevailing commercial SCM systems and can not guarantee the computed result is global optimal solution. Since lack of available and effective mathematical formulation to linearize nonlinear terms will weaken the advantages and effectiveness of SCM system, this work aims to propose some methods to make current computer-based quantitative models more practical and solvable by prevailing linear programming (LP) packages. By doing so, the coordination and collaboration within supply chain partners can be carried out over the time subject to the optimal solution and sensitive analysis generated by a SCM system.

The rest of this work is organized below. Section 2 presents a generalized SCM quantitative model. Section 3 introduces logistics cost functions from the simple format to complicated format. Section 4 proposes theorems and methods of treating separable linear objective functions with continuous decision variables step by step. Section 5 describes the solution algorithm and numerical example to illustrate the practicality and applicability of the proposed methods. Section 6 summarizes the contribution and features of the proposed methods.

2. A Generalized SCM Quantitative Model

Although SCM has appeared as a hot management issue in last decade, its development can be traced back to the early twentieth century. As the turn of the twentieth century, economists (Shaw, 1915) considered distribution as the bridge between customer requirements and product availability, by which commodities move through the supply channel and determine the exchange process. If a mathematical model covering procurement, production, inventory, and distribution is considered the framework of quantitative SCM, which is the foundation of global logistics, then much of the pioneering work can be found in the late 1950's (Forrester, 1958; Hanssman, 1959). Although Hanssmann (1959) only considered single company logistics, his work was perhaps the earliest attempt to solving material procurement, goods production, inventory, and distribution by a quantitative model. In 1960s, a multi-echelon distribution network was proposed (Clark and Scarf, 1960; Drucker, 1969) which is perhaps an origin of an arborescent supply chain network, although it is not a complete supply chain network. The concept of global logistic management introduced by Ishii *et al*. (1988), Cohen *et al*. (1989), and Stevens (1989) in the late 1980's, culminated in the emergence of the modern SCM development. Since then, driven by global competitive pressures and information technology advances, numerous SCM quantitative models have been developed to coordinate and schedule the

entire supply chain activities. Plentiful references exist in the books (Handfield and Nichols, 2005; Ross, 2005; Tayur *et al.*, 2006) and the papers (Vidal and Goetschalckx, 2001; Sodhi, 2005; Holmström *et al.*, 2006; Tan *et al.*, 2006).

In industry, a real-life SCM quantitative model should consider the entire supply chain activities including material procurement, production, distribution, inventory, etc. in optimizing (1) The total and separable purchased amounts for each raw item from each raw material's original supplier to each upstream supplier; (2) The total and individual shipped amounts for each raw item from each raw material's original supplier to each upstream supplier; (3) The total and separable procurement amounts for each intermediate product (or part) from each upstream supplier to each middle-stream manufacturer; (4) The total and individual transported amounts for each intermediate product (or part) from each upstream supplier to each middle-stream manufacturer; (5) The total and separable manufactured amounts for each product in each manufacturer as well as each product required inventory in each manufacturer; (6) The total and separable outsourcing amounts for each product in each manufacturer; and (7) The total and individual delivered amounts for each product from each manufacturer to each down stream warehouse and from there to each end market (or customer).

Accordingly, a practical and generalized SCM quantitative model is widely formulated as follows:

Model 1

$$
\begin{aligned}
\text{Min} \quad & \sum_{t,b,s,i} \text{RM\_P\_Cost}_{tbsi} \times RM\_P\_QTY_{tbsi} + \sum_{t,b,s,i} \text{RM\_T\_Cost}_{tbsi} \times RM\_T\_QTY_{tbsi} + \sum_{t,s,i} \text{RM\_I\_Cost}_{tsi} \times RM\_I\_QTY_{tsi} + \\
& \sum_{t,s,j} \text{PSFG\_M\_Cost}_{tsj} \times PSFG\_M\_QTY_{tsj} + \sum_{t,s,j} \text{PSFG\_I\_Cost}_{tsj} \times PSFG\_I\_QTY_{tsj} + \sum_{t,s,f,j} \text{PSFG\_P\_Cost}_{tsfj} \times \\
& PSFG\_P\_QTY_{tsfj} + \sum_{t,s,f,j} \text{PSFG\_T\_Cost}_{tsfj} \times PSFG\_T\_QTY_{tsfj} + \sum_{t,f,j} \text{PSFG\_I\_Cost}_{tfj} \times PSFG\_I\_QTY'_{tfj} + \\
& \sum_{t,f,k} \text{G\_M\_Cost}_{tfk} \times G\_M\_QTY_{tfk} + \sum_{t,f,k} \text{G\_O\_Cost}_{tfk} \times G\_O\_QTY_{tfk} + \sum_{t,f,k} \text{G\_I\_Cost}_{tfk} \times G\_I\_QTY_{tfk} + \\
& \sum_{t,f,w,k} \text{G\_T\_Cost}_{tfwk} \times G\_T\_QTY_{tfwk} + \sum_{t,w,k} \text{G\_I\_Cost}_{twk} \times G\_I\_QTY'_{twk} + \sum_{t,w,c,k} \text{G\_T\_Cost}_{twck} \times G\_T\_QTY'_{twck}
\end{aligned}
$$

s. t.
$$RM\_I\_QTY_{t-1,si} + \sum_{b} RM\_T\_QTY_{tbsi} \geq PSFG\_M\_QTY_{tsj} \times \sum_{j_I} \text{BOM\_QTY}_{I \to j},$$

$$RM\_I\_QTY_{tsi} = RM\_I\_QTY_{t-1,si} + \sum_{b} RM\_T\_QTY_{tbsi} - PSFG\_M\_QTY_{tsj} \times \sum_{j_I} \text{BOM\_QTY}_{I \to j},$$

$$PSFG\_I\_QTY_{t-1,sj} + PSFG\_M\_QTY_{tsj} \geq \sum_{f} PSFG\_T\_QTY_{tsfj},$$

$$PSFG\_I\_QTY_{tsj} = PSFG\_I\_QTY_{t-1,sj} + PSFG\_M\_QTY_{tsj} - \sum_{f} PSFG\_T\_QTY_{tsfj},$$

$$PSFG\_I\_QTY'_{t-1,fj} + \sum_{s} PSFG\_T\_QTY_{tsfj} \geq G\_M\_QTY_{tfk} \times \sum_{k_J} \text{BOM\_QTY}_{J \to k},$$

$$PSFG\_I\_QTY'_{tfj}= PSFG\_I\_QTY'_{t-1,fj}+ \sum_{s} PSFG\_T\_QTY_{tsfj} -G\_M\_QTY_{tfk} \times \sum_{k_J} BOM\_QTY_{J\text{-}>k},$$

$$G\_I\_QTY_{t-1,fk}+ G\_M\_QTY_{tfk}+ G\_O\_QTY_{tfk} \geq \sum_{w} G\_T\_QTY_{tfwk},$$

$$G\_I\_QTY_{tfk}= G\_I\_QTY_{t-1,fk}+ G\_M\_QTY_{tfk}+ G\_O\_QTY_{tfk}- \sum_{w} G\_T\_QTY_{tfwk},$$

$$G\_I\_QTY'_{t-1,wk}+ \sum_{f} G\_T\_QTY_{tfwk} \geq \sum_{c} G\_T\_QTY'_{twck},$$

$$G\_I\_QTY'_{twk}= G\_I\_QTY'_{t-1,wk}+ \sum_{f} G\_T\_QTY_{tfwk}- \sum_{c} G\_T\_QTY'_{twck},$$

$$\sum_{w} G\_T\_QTY'_{twck} \geq G\_R\_QTY_{tck}, \quad \sum_{j} PSFG\_M\_QTY_{tsj} \leq M\_Cap_{ts}, \quad \sum_{k} G\_M\_QTY_{tfk} \leq M\_Cap_{tf},$$

$$\sum_{i} RM\_I\_QTY_{tsi} \leq RM\_I\_Limit_{ts}, \quad \sum_{j} PSFG\_I\_QTY_{tsj} \leq PSFG\_I\_Limit_{ts},$$

$$\sum_{j} PSFG\_I\_QTY'_{tfj} \leq PSFG\_I\_Limit_{tf}, \quad \sum_{k} G\_I\_QTY_{tfk} \leq G\_I\_Limit_{tf}, \quad \sum_{k} G\_I\_QTY'_{twk} \leq G\_I\_Limit_{tw},$$

where each capital letter denotes each variable's set, lower case letters denote realization of particular variables, italic letters denote decision variables, and notations and parameters are defined as follows：

T : the set of time, t : the t'th time period;

I : the set of raw materials (**RMs**), i : the i'th RM;

J : the set of parts or semi-finished goods (**PSFGs**), j : the j'th PSFG;

K : the set of finished goods (**FGs**), k : the k'th FG;

B : the set of beginning suppliers (**BSs**), b : the b'th BS;

S : the set of intermediate suppliers (**ISs**), s : the s'th IS;

F : the set of factories, f : the f'th factory;

W : the set of warehouses, w : the w'th warehouse; and

C : the set of end customers, c : the c'th end customer.

In a certain time period t, the following notations are used.

RM_P_Cost$_{tbsi}$: the cost of the s'th IS to purchase the i'th RM from the b'th BS,

RM_T_Cost$_{tbsi}$: the transportation cost of the ith RM from the b'th BS to the s'th IS,

RM_I_Cost$_{tsi}$: the inventory cost of the i'th RM in the s'th IS,

PSFG_M_Cost$_{tsj}$: the manufacturing cost of the j'th PSFG in the s'th IS,

PSFG_I_Cost$_{tsj}$: the inventory cost of the j'th PSFG in the s'th IS,

PSFG_P_Cost$_{tsfj}$: the cost of the f'th factory to buy the j'th PSFG from the s'th IS,

PSFG_T_Cost$_{tsfj}$: the transportation cost of the j'th part from the s'th IS to the f'th factory,

PSFG_I_Cost$_{tfj}$: the inventory cost of the j'th PSFG in the f'th factory,

G_M_Cost$_{tfk}$: the manufacturing cost of the k'th FG in the f'th factory,

G_O_Cost$_{tfk}$: the outsourcing cost of the k'th FG for the f'th factory,

G_I_Cost$_{tfk}$: the inventory cost of the k'th FG in the f'th factory,

G_T_Cost$_{tfwk}$: the transportation cost of the k'th FG from the f'th factory to the w'th warehouse,

G_I_Cost$_{twk}$: the inventory cost of the k'th goods in the w'th warehouse,

G_T_Cost$_{twck}$: the transportation cost of the k'th FG from the w'th warehouse to the c'th end customer,

G_R_QTY$_{tck}$: the requirement of the k' th FG from the c'th end customer's order,

RM_P_QTY$_{tbsi}$: the purchased quantity (QTY) of RM i between the b'th BS and the s'th IS,

RM_T_QTY$_{tbsi}$: the transported QTY of RM i from the b'th BS to the s'th IS,

RM_I_ QTY$_{tsi}$: the inventory QTY of the i'th RM in the s'th IS,

PSFG_M_QTY$_{tsj}$: the manufacturing QTY of the j'th PSFG in the s'th IS,

PSFG_I_QTY$_{tsj}$: the stock QTY of the j'th PSFG in the s'th IS,

PSFG_P_QTY$_{tsfj}$: the sold QTY of the j'th PSFG from the s'th IS to the f'th factory,

PSFG_T_QTY$_{tsfj}$: the shipped QTY of the j'th PSFG from the s'th IS to the f'th factory,

PSFG_I_QTY'$_{tfj}$: the inventory QTY of the j'th PSFG in the f'th factory,

G_M_QTY$_{tfk}$: the produced QTY of the k'th FG in the f'th factory,

G_O_QTY$_{tfk}$: the outsourcing QTY of the k'th FG for the f'th factory,

G_I_QTY$_{tfk}$: the inventory QTY of the k'th FG in the f'th factory,

G_T_ QTY$_{tfwk}$: the transported QTY of the k'th FG from the f'th factory to the w'th warehouse,

G_I_ QTY'$_{twk}$: the stock QTY of the k'th FG in the w'th warehouse,

G_T_QTY'$_{twck}$: the delivered QTY of the k'th FG from the w'th warehouse to the c'th end customer,

M_Cap$_{ts}$: the production capacity of the s'th IS for all PSFG,

M_ Cap$_{tf}$: the production capacity of the f'th factory for all FG,

RM_I_Limit$_{ts}$: the stock limit of the s'th IS for all RM,

PSFG_ I_Limit$_{ts}$: the stock limit of the s'th IS for all PSFG,

PSFG_I_Limit$_{tf}$: the stock limit of the f'th factory for all PSFG

G_I_Limit$_{tf}$: the stock limit of the f'th factory for all FG,

G_I_ Limit$_{tw}$: the stock limit of the w'th warehouse for all FG,

BOM_QTY$_{I->j}$: the bill of material (BOM) for producing each unit of the j'th PSFG from the set of RMs, and

BOM_QTY$_{J->k}$: the bill of material (BOM) for producing each unit of the k'th FG from the set of PSFGs.

3. Logistics Cost Functions

In real-world logistics operations, a logistics cost is not a constant but a function (Oum and Waters, 1996), and a logistics cost function in SCM generally comprises fixed costs, variable costs, and unpredicted costs (Slats *et al*., 1995; Shapiro, 2005).　Accordingly, a SCM model can be simply formulated below:

Model 2

Minimize Total_Cost = Fixed_Cost + Variable_Cost + Unpredicted_Cost
Subject to the constraints.

Essentially, fixed costs are stable and do not vary with amount ordered, transported, or produced; for example, the weekly salary of employees, the monthly rent on factories/offices, and the periodic maintenance expenses for equipment. Variable costs are not fixed and depend on the number of ordered, transported, or produced units.　Unpredicted costs are costs that arise unexpectedly, for example, costs associated with the late delivery of products, the breakdown of machines in production lines or the loss of electrical power at a plant.　Consequently, fixed costs are fixed values, while unpredicted costs can be estimated by probabilistic distribution. Building the above discussion, Model 2 can be reformulated as follows:

Model 3

Minimize Total_Cost = Fixed_Cost + Variable_Cost + $\displaystyle\sum_{n=1}^{N} P_n (\text{Unpredicted\_Cost}_n)$

Subject to the constraints,
where P_n is the probability of the n'th scenario's occurrence, Unpredicted_Cost$_n$ is the unexpected cost occurred at the n'th scenario, n = 1, 2, .., N, and $\displaystyle\sum_{n=1}^{N} P_n = 1$.

Since this study aims to illustrate how various variable cost functions can be linearized by the presented methods, fixed and unpredicted costs are not included in Model 1.　The main reason is that adding fixed and unpredicted costs into this work only increase the reading complexity without giving extra contribution.　Therefore, to avoid complicating this work without changing the practicability and applicability of the proposed methods, this work highlights on variable costs.

In reality, vendors frequently offer quantity discounts to encourage buyers to order more; producers typically like to reduce the average production costs by mass production, and wholesalers attempt to minimize the average distribution cost by delivering huge quantities at once.　However, few quantitative SCM models highlight on dealing with a wide range of logistic cost functions (Oum

and Waters, 1996; Yu, 2006), and the unit cost used in existing quantitative SCM models is frequently a constant or a linear function as shown in Figs. 1a and b, respectively.   Figures 1a and b separately reveal that the average cost is fixed at a specific value or has a gradient that is independent of the quantity.   This assumption is impractical and out of the real-life situation.   Consequently, this section introduces some means to treat widespread occurred cost functions encountered by logistics managers.

3.1 Step cost functions

One of the most commonly used discount cost functions is a step cost function as shown in Fig. 2, in which Q_1, Q_2, and Q_3 are the incentive quantity levels and c_1, c_2, and c_3 are the discount costs (or prices). The step cost function first defined by Love (1979) is widely expressed as

$$\text{Cost}(Q) = \begin{cases} c_1 & \text{for } Q_1 < Q \le Q_2 \\ c_2 & \text{for } Q_2 < Q \le Q_3 \\ c_3 & \text{for } Q_3 < Q \le Q_4 \end{cases} \quad \text{where Cost(Q) is shown in Fig. 2.}$$

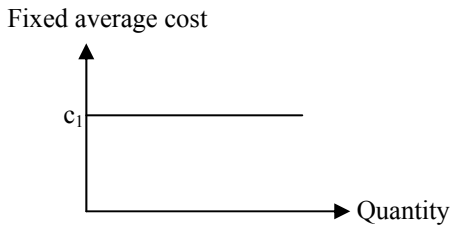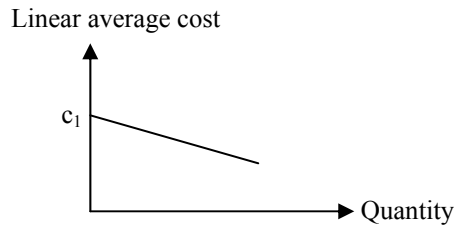Fixed average cost

c_1

Quantity

Fig. 1a   A constant cost

Linear average cost

c_1

Quantity

Fig. 1b   A linear cost function

Cost(Q) (Unit Cost/Price)

c_1=0.025
c_2=0.024
c_3=0.023

Quantity

Q_1=0 Q_2=1000 Q_3=2000 Q_4=3000   Q
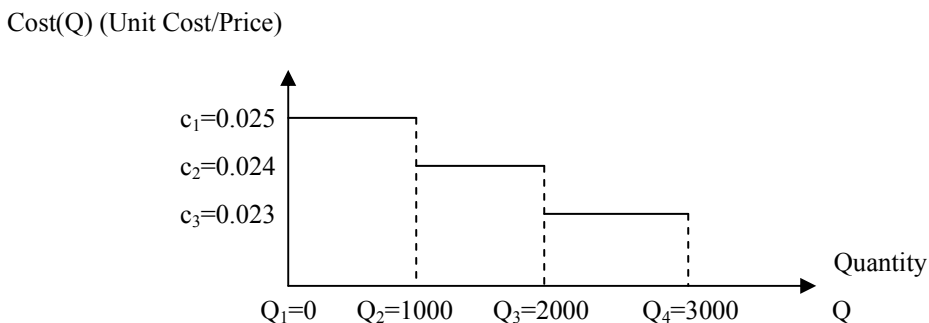
Fig. 2   A step cost function

3.2 Single change point cost functions

A quantity discount schedule with a single price breakpoint is widely applied to calculate average procurement, production, inventory and transportation costs, etc.   Figure 3a reveals that the starting average cost is 1.8 and declines when the required quantity increases, which implies that the slope before the quantity reaches 150 is different from that after the quantity surpasses 150.   Figure 3a display a convex cost function, while figure 3b shows a concave cost function.

3.3 Multiple change point cost functions

Another much broadly employed cost function is a discount quantity schedule with a multiple change-point function.   The function is similar to the one with a single change point function except in the number of change points, as depicted in Figs. 4a, b, and c.
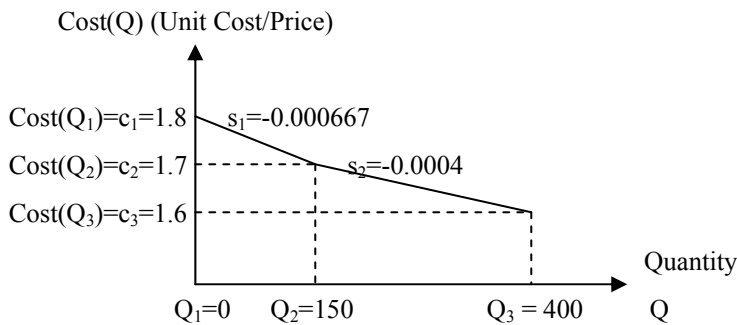

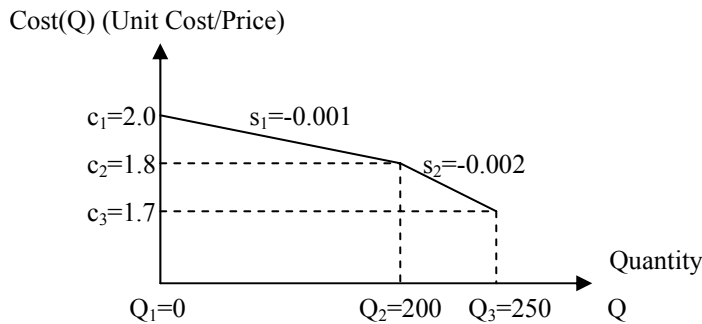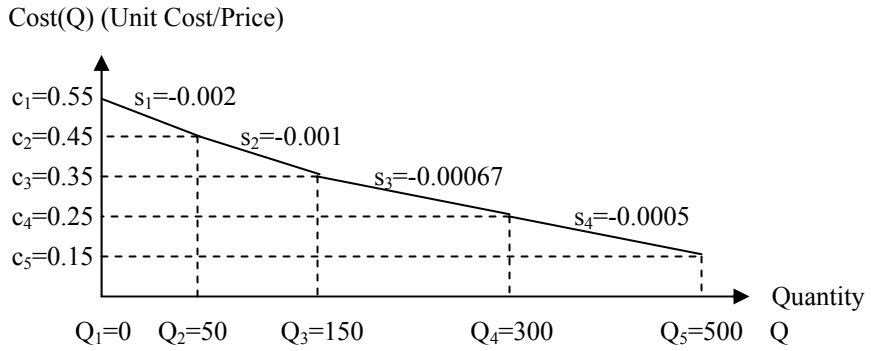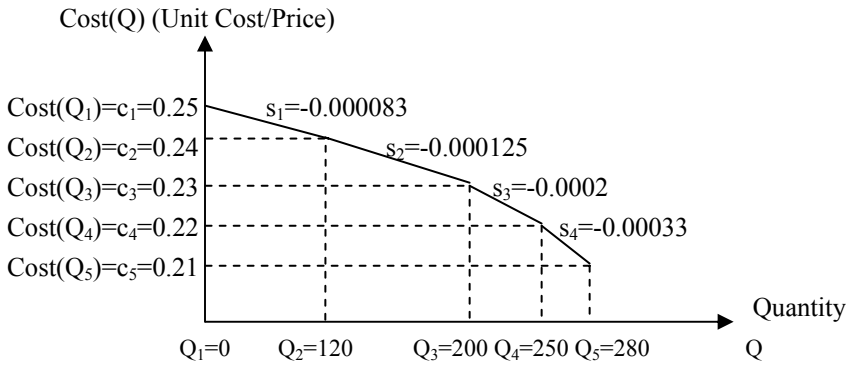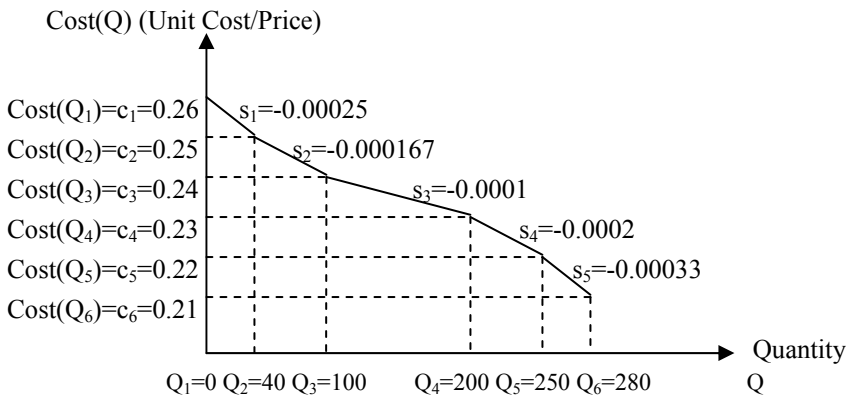**Fig. 3a   A convex single change-point cost function**

Fig. 3b   A non-convex single change cost function

Cost(Q) (Unit Cost/Price)

$c_1=0.55$ $s_1=-0.002$
$c_2=0.45$ $s_2=-0.001$
$c_3=0.35$ $s_3=-0.00067$
$c_4=0.25$ $s_4=-0.0005$
$c_5=0.15$

Quantity
$Q_1=0$ $Q_2=50$ $Q_3=150$ $Q_4=300$ $Q_5=500$ Q

Fig. 4a   A convex type of multiple change point function

Cost(Q) (Unit Cost/Price)

$Cost(Q_1)=c_1=0.25$ $s_1=-0.000083$
$Cost(Q_2)=c_2=0.24$ $s_2=-0.000125$
$Cost(Q_3)=c_3=0.23$ $s_3=-0.0002$
$Cost(Q_4)=c_4=0.22$ $s_4=-0.00033$
$Cost(Q_5)=c_5=0.21$

Quantity
$Q_1=0$ $Q_2=120$ $Q_3=200$ $Q_4=250$ $Q_5=280$ Q

Fig. 4b   A concave type of multiple change point function

Cost(Q) (Unit Cost/Price)

$Cost(Q_1)=c_1=0.26$ $s_1=-0.00025$
$Cost(Q_2)=c_2=0.25$ $s_2=-0.000167$
$Cost(Q_3)=c_3=0.24$ $s_3=-0.0001$
$Cost(Q_4)=c_4=0.23$ $s_4=-0.0002$
$Cost(Q_5)=c_5=0.22$ $s_5=-0.00033$
$Cost(Q_6)=c_6=0.21$

Quantity
$Q_1=0$ $Q_2=40$ $Q_3=100$ $Q_4=200$ $Q_5=250$ $Q_6=280$ Q

Fig. 4c   A convex-concave type of multiple change point function

4. Proposed Theorems and Methods

To treat a step cost, as depicted in Fig. 2, the following theorem is introduced.

__Theorem 1__　A program expressed as

{Minimize $Q(c_1u_1+c_2u_2+c_3u_3)$ subject to $Q_k+M(u_k-1)\leq Q\leq Q_{k+1}+M(1-u_k)$ for k=1,2,3, $u_1+u_2+u_3=1$, where u_1, u_2, u_3 are 0-1 variables used to indicate which cost level will be adopted based on the required quantity}　(1)

can be linearized as

$$\{\text{Minimize } c_1z_1+c_2z_2+c_3z_3$$
$$\text{subject to } Q_k+M(u_k-1)\leq Q\leq Q_{k+1}+M(1-u_k) \text{ for k=1,2,3,}$$
$$z_k\geq Q+M(u_k-1), z_k \geq 0, \text{ for k=1,2,3,}$$
$$u_1+u_2+u_3=1, u_1, u_2, u_3 \in \{0,1\},$$

where M is a big constant and can be set as

$$M = \text{Max}\{Q_1,Q_2,Q_3,Q_4\} \}$$　(2)

__Proof__　To treat the product terms u_1Q, u_2Q, and u_3Q in Program (1), the program (2) first uses z_1, z_2, and z_3 to replace them respectively. Consider the following Lemma.

Lemma 1

PP1: Minimize u_kQ subject to u_k is an 0-1 variable, $Q \geq 0$.

is equivalent to

PP2: Minimize $z_k = u_kQ$

$$\text{subject to } z_k\geq Q+M(u_k-1), z_k \geq 0,$$　(3)

where M is a big value and can be set as the upper bound of Q, $Q \geq 0$,

and $u_k \in \{0\text{-}1\}$.

Lemma 1 can be verified as follows:

(i) if at optimal solution $u_k = 1$, then equations of (3) result in $z_k = Q$;

(ii) if at optimal solution $u_k = 0$, then equations of (3) result in $z_k = 0$.

Based on the Lemma 1, the program (2) can then be examined as follows:

(i) In case of the cost level c_1 is selected

At the optimal solution, $u_1+u_2+u_3=1$ will force $u_1=1$, $u_2=u_3=0$, which results in $Q_1 \le Q \le Q_2$.

  (ii) In case of the cost level c_2 is selected

At the optimal solution, $u_1+u_2+u_3=1$ will force $u_2=1$, $u_1=u_3=0$, which results in $Q_2 \le Q \le Q_3$.

 (iii) In case of the cost level c_3 is selected

At the optimal solution, $u_1+u_2+u_3=1$ will force $u_3=1$, $u_1=u_2=0$, which results in $Q_3 \le Q \le Q_4$.

The theorem is then verified.

Take Fig. 2 as an instance, the model of optimizing this step cost function is formulated as

$$\text{Minimize } 0.025z_1+0.024z_2+0.023z_3$$
$$\text{subject to } Q_k+M(u_k-1) \le Q \le Q_{k+1}+M(1-u_k) \text{ for k =1, 2, 3,}$$
$$z_k \ge Q+M(u_k-1), z_k \ge 0 \text{ for k =1, 2, 3, } u_1+u_2+u_3=1, u_1, u_2, u_3 \in \{0,1\},$$

where $Q_1=0$, $Q_2=1000$, $Q_3=2000$, $Q_4=3000$, and $M = \text{Max}\{Q_1,Q_2,Q_3,Q_4\} = 3000$.

After inserting the constraint $Q \ge 1500$ for the illustrative purpose and then executing this program in the LINGO or Excel, the obtained solution is ($u_2=1$, $u_1= u_3= z_1 = z_3 =0$, $z_2 = Q = 1500$, the objective value is 36, and the average cost Cost(Q) is 0.024). Building on Theorem 1 and the above demonstration, Method 1 capable of representing a general step cost function is presented below:

Method 1 A SCM model of optimizing a step cost function Cost(Q), depicted in Fig. 5, can be formulated as follows:

$$\text{Minimize Cost(Q)} = \sum_{k=1}^{n-1} c_k z_k$$

Cost(Q) (Unit Cost/Price)
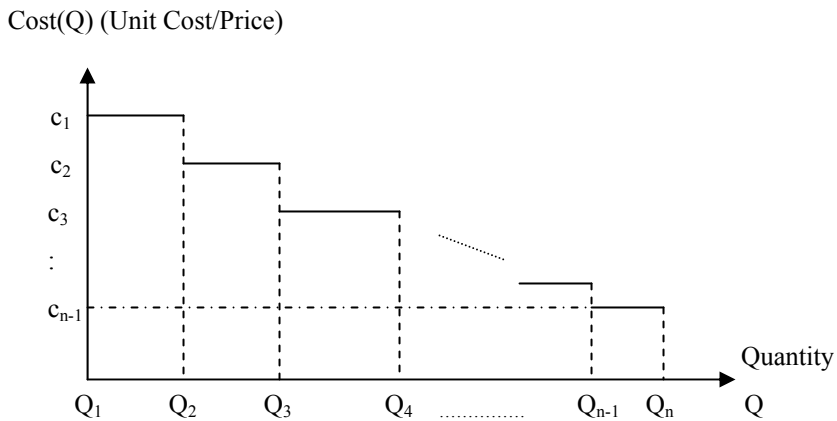
Fig. 5   A generalized step function

$$\text{subject to } \sum_{k=1}^{n-1} u_k = 1, u_k \in \{0,1\},$$

$$Q_k + M(u_k - 1) \leq Q \leq Q_{k+1} + M(1 - u_k) \text{ for } k = 1, 2, .., n-1,$$

$$z_k \geq Q + M(u_k - 1), z_k \geq 0 \text{ for } k = 1, 2, .., n-1,$$

where M is a big constant and set as $M = \text{Max}\{Q_1, Q_2, \ldots, Q_n\}$, and z_k stands for $u_k Q$.

To treating single point change functions of Figs. 3a and b, the following lemmas of 2 - 4 are introduced. In following Lemmas 2-4, Q is ordered quantity and Q_1 and Q_2 are the incentive quantity levels.

Lemma 2 Based on Proposition 1 in the literature (Li and Yu, 1999, 2000), any separable linear functions Cost(Q), shown in Figs. 3a and 3b where s_k (k = 1, 2) are the slopes of line segments between Q_k and Q_{k+1}, can be represented by

$$\text{Cost}(Q) = \text{Cost}(Q_1) + s_1(Q - Q_1) + ((s_2 - s_1) \div 2))(|Q - Q_2| + Q - Q_2) \tag{4}$$

where $Q \geq 0$, $Q_2 \geq Q_1$.

Take Figs. 3a and b as illustration, by using Equation (4) we have

$$\text{Cost}(Q) = 1.8 - 0.000667Q + (0.000267 \div 2)(|Q-150| + Q-150) \text{ (in Fig. 3a)} \tag{5}$$

$$\text{Cost}(Q) = 2.0 - 0.001Q + (-0.001 \div 2)(|Q-200| + Q-200) \text{ (in Fig. 3b)} \tag{6}$$

where 0.000267 and -0.001 were calculated by the formula of $(s_2 - s_1)$ as shown in Equation (4).   When $s_2 > s_1$, the break point is convex point as depicted in Fig. 3(a). Conversely, if $s_2 < s_1$, the break point is concave point as depicted in Fig. 3(b).

To linearize absolute terms with the positive coefficient in Equation (5), Lemma 3 is introduced.

Lemma 3

PP3:   Minimize $C = (s \div 2)(|Q-g| + Q-g)$ subject to Q, g, s ≥ 0,

is equivalent to

PP4:   Minimize $C' = s(Q-g+d)$ subject to $Q - g + d \geq 0$, Q, g, d, s ≥ 0.

This lemma can be confirmed as follows:

 (i) if $Q - g \geq 0$ then at the optimal solution d = 0, which results in $C' = s(Q-g) = C$;

(ii) if $Q - g < 0$ then at the optimal solution d = g – Q, which results in $C' = 0 = C$.

Take Fig. 3a as a demonstration. The model of optimizing Equation (5) is programmed as:

Minimize Cost(Q)=1.8-0.000667Q+0.000267(Q-150+d)=-0.0004Q+0.000267d+1.75995

subject to $Q - 150 + d \geq 0$, Q, d ≥ 0.

To linearize absolute terms with the negative coefficient in Equation (6), Lemma 4 is introduced.

Lemma 4

<u>PP5</u>:　Minimize $C=(s \div 2)(|Q-g|+Q-g)$ subject to $Q, g \geq 0, s < 0$,

is equivalent to

<u>PP6</u>:　Minimize $C' = s(Q-w+gu-g)$ subject to $Q + \overline{Q}(u-1) \leq w$, $Q, w \geq 0$, $s < 0$, $u \in \{0,1\}$, and $\overline{Q}$ is the upper bound of Q (Take Fig. 4b as instance, $\overline{Q} = 250$).

Lemma 4 can be examined as follows:

(i)　if $Q - g \geq 0$ then at the optimal solution u=0 and w=0, which results in $C'=s(Q-g)= C$;

(ii)　if $Q - g < 0$ then at the optimal solution u=1 and w=Q, which results in $C'=0 =C$.

Based on Lemma 4, the model of optimizing Equation (6) can be programmed as

Minimize $Cost(Q) = 2–0.001Q–0.001(Q-w+200u-200) = -0.002Q+0.001w-0.2u+2.2$

subject to $Q + 250(u-1) \leq w$, $Q, w \geq 0$, $u \in \{0,1\}$, and $Q \leq 250$.

Building on Lemmas 2 - 4, Remark 1 and Methods 2 and 3 are presented as the following:

Remark 1　A single change cost function Cost(Q) where $s_2 > s_1$ as displayed in Fig. 3a is a convex function, while Cost(Q) as displayed in Fig. 3b is a concave function where $s_2 < s_1$.

Method 2　A SCM model of optimizing a convex type of single change cost function, depicted in Fig. 3a, can be formulated as follows:

Minimize Cost(Q)

subject to　　$Cost(Q) = Cost(Q_1)+ s_1(Q-Q_1)+(s_2-s_1)(Q-Q_2+d_2)$, $Q-Q_1+d \geq 0$, and $Q \geq 0$,

where $s_2 > s_1$, $Q_1 \geq Q_2$.

Method 3　A SCM model of optimizing a concave type of single change cost function, depicted in Fig. 3b, can be formulated as follows:

Minimize Cost(Q)

subject to　　$Cost(Q) = Cost(Q_1)+ s_1(Q-Q_1)+(s_2-s_1)(Q-w+uQ_2-Q_2)$,

$w \geq Q+\overline{Q}(u-1)$, $Q, w \geq 0$, $u \in \{0-1\}$, $0 \leq Q \leq \overline{Q}$,

where $s_2 < s_1$, and $\overline{Q}$ is the upper bound of Q.

To treating multiple change point functions, Remark 2, Method 4, and Theorem 2 are presented as follows.

Remark 2　Based on Proposition 1 in Li and Yu (1999, 2000), a separable linear function Cost(Q), as

depicted in Figs. 4a, 4b, or 4c where $c_k = \text{Cost}(Q_k)$, Q_k are change points, and s_k are slopes of line segments between Q_k and Q_{k+1}, can be expressed as

$$\text{Cost}(Q) = \text{Cost}(Q_1) + s_1(Q-Q_1) + \sum_{k=2}^{n-1} \frac{s_k - s_{k-1}}{2}(|Q-Q_k| + Q-Q_k) \tag{7}$$

where　$Q \geq 0$, $0 < Q_1 < Q_2 < ... < Q_n$, and $s_{k+1} > s_k$ for $k = 1, 2, …, n-1$.

To treat Equation (7) with all positive coefficients, Method 4 is introduced.

Method 4　Based on Proposition 4 in Li and Yu (2000), a SCM model of optimizing Equation (7) with all positive coefficients can be programmed as follows:

Minimize Cost(Q)

$$\text{Subject to } \text{Cost}(Q) = \text{Cost}(Q_1) + s_1(Q-Q_1) + \sum_{k=2}^{n-1} ((s_k - s_{k-1})(Q-Q_k + \sum_{j=1}^{k-1} d_j))$$

$$Q + d_1 + d_2 + ... + d_{n-1} \geq Q_{n-1}, Q_{k+1} - Q_k \geq d_k, \text{ and } d_k \geq 0 \text{ for } k = 1, 2, …, n-1.$$

where $Q \geq 0$, $0 < Q_1 < Q_2 < ... < Q_n$, and $s_{k+1} > s_k$ for all k.

Consider the following illustration.

Example 1

Minimize Cost(Q)

subject to $Q \leq 500$,

where Cost(Q) is depicted in Fig. 4a.

By employing Method 4, Example 1 is linearized as follows:

Minimize　Cost(Q)=0.55-0.002Q+0.001(Q-50+d_1)+0.00033(Q-150+d_1+d_2)+
0.00017(Q-300+d_1+d_2+d_3) = -0.0005Q +0.0015d_1+0.0005d_2+0.00017d_3+0.39999

subject to Q+d_1+d_2+d_3 ≥ 300, 0≤d_1≤150, 0≤d_2≤100, 0≤d_3≤50, Q ≥ 0 and Q ≤ 500.

After executing this program in the LINGO, the obtained solution is ($d_1 = d_2 = d_3 = 0$, Q = 500, and the average cost Cost(Q) = 0.15).　As noted, one merit of Method 4 is that it contains only one inequality constraint and all the other constraints are bounded constraints, which could speed the computational efficiency.　Although Li and Yu's methods (1999, 2000) is quite promising to linearize a convex function, their methods either has to use k 0-1 variables to linearize a concave function with k line segment (Li and Yu, 1999) or cannot directly treat a concave function (Li and Yu, 2000).　As a result, to treat Equation (7) with all negative coefficients, Theorem 2 is introduced to lessen the computational burden on treating the concave cost function.

Theorem 2   A cost(Q) shown in Fig. 6 can be expressed as

$$\text{Cost}(Q_k)+s_k(Q\text{-}Q_k)\text{-}M\,(\sum_{j=1}^{r}b_{kj}+\sum_{j=1}^{r}(-2b_{kj}+1)u_j)\leq\text{Cost}(Q)$$

$$\text{Cost}(Q)\leq\text{Cost}(Q_k)+s_k(Q\text{-}Q_k)+M\,(\sum_{j=1}^{r}b_{kj}+\sum_{j=1}^{r}(-2b_{kj}+1)u_j) \tag{8}$$

$$Q_k\text{-}M\,(\sum_{j=1}^{r}b_{kj}+\sum_{j=1}^{r}(-2b_{kj}+1)u_j)\leq Q\leq Q_{k+1}+M\,(\sum_{j=1}^{r}b_{kj}+\sum_{j=1}^{r}(-2b_{kj}+1)u_j)$$

where Q_k are change points, k=1, 2, …, m-1, s_k are slopes of line segments between Q_k and Q_{k+1}, r is the smallest value for $2^r \geq$ m-1, b_{kj} (j=1, 2, …,r) are assigned parameters which are derived by $\sum_{j=1}^{r}2^{j-1}b_{kj}=k-1$, u_j are zero-one variables, and M is a large constant.

Proof

If $u_j = b_{pj}$ for j = 1, 2, …, r and $1\leq p\leq$m-1 (which means the p'th cost level will be adopted based on the required quantity Q), then $\sum_{j=1}^{r}b_{pj}+\sum_{j=1}^{r}(-2b_{pj}+1)u_j= 0$ and $\sum_{j=1}^{r}b_{kj}+\sum_{j=1}^{r}(-2b_{kj}+1)u_j> 0$ for all k except for k=p (i.e., k =1, 2, …, p-1, p+1, …, m-1). Consequently, all equations in (8) can be replaced as follows:

$$\text{Cost}(Q_p)+s_p(Q\text{-}Q_p)\leq\text{Cost}(Q) \tag{9}$$
$$\text{Cost}(Q)\leq\text{Cost}(Q_p)+s_p(Q\text{-}Q_p) \tag{10}$$
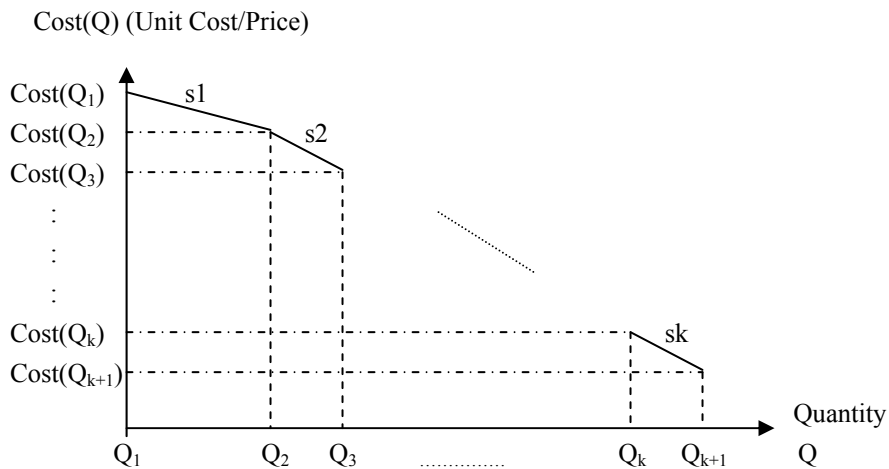
Fig. 6   A general concave type of multiple change point function

$$Q_p \leq Q \leq Q_{p+1} \tag{11}$$

$$\text{Cost}(Q_k)+s_k(Q-Q_k)-M \leq \text{Cost}(Q) \text{ for all k except for k = p} \tag{12}$$

$$\text{Cost}(Q) \leq \text{Cost}(Q_k)+s_k(Q-Q_k)+M \text{ for all k except for k = p} \tag{13}$$

$$Q_k \leq Q \leq Q_{k+1} \text{ for all k except for k = p} \tag{14}$$

where M is a large constant.

Since only the p'th cost level is adopted based on the required quantity Q, constraints (12)-(14) are inactive. Hence, (9)-(11) force $\text{Cost}(Q)= \text{Cost}(Q_p)+s_p(Q-Q_p)$ and $Q_p \leq Q \leq Q_{p+1}$. In doing so, **Theorem 2 is then proved.**

Consider the following illustration.

Example 2

> Minimize Cost(Q)
>
> subject to Q ≤ 260,

where Cost(Q) is depicted in Fig. 4b.

Figure 4b displays that the break points of Cost(Q) are 0, 120, 200, 250, and 280. Hence, referring to Theorem 2, $r = 2$ is the smallest value for $2^r \geq$ m-1 = 4, and each b_{kj} is derived by $\sum_{j=1}^{r} 2^{j-1} b_{kj} = k - 1$ as listed in Table 1.

By employing Theorem 2, Example 2 is linearized as follows:

> Minimize Cost(Q)
>
> subject to Q ≤ 260,

$0.25\text{-}0.000083Q\text{-}M(u_1+u_2) \leq \text{Cost}(Q) \leq 0.25\text{-}0.000083Q+M(u_1+u_2),$

$0.24\text{-}0.000125(Q\text{-}120)\text{-}M(1\text{-}u_1+u_2) \leq \text{Cost}(Q) \leq 0.24\text{-}0.000125(Q\text{-}120)+M(1\text{-}u_1+u_2),$

$0.23\text{-}0.0002(Q\text{-}200)\text{-}M(1+u_1\text{-}u_2) \leq \text{Cost}(Q) \leq 0.23\text{-}0.0002(Q\text{-}200)+M(1+u_1\text{-}u_2),$

$0.22\text{-}0.00033(Q\text{-}250)\text{-}M(2\text{-}u_1\text{-}u_2) \leq \text{Cost}(Q) \leq 0.22\text{-}0.00033(Q\text{-}250)+M(2\text{-}u_1\text{-}u_2),$

Table 1　The derived values of b_{kj}

the k'th line segment	j = 1	j = 2
k = 1	0	0
k = 2	1	0
k = 3	0	1
k = 4	1	1

$$-M(u_1+u_2) \leq Q \leq 120+M(u_1+u_2),$$

$$120-M(1-u_1+u_2) \leq Q \leq 200+M(1-u_1+u_2),$$

$$200- M(1+u_1-u_2) \leq Q \leq 250+ M(1+u_1-u_2),$$

$$250-M(2-u_1-u_2) \leq Q \leq 280+M(2-u_1-u_2),$$

where u_1 and u_2 are 0-1 variables, and M is a big number.

After running this program in LINGO, the obtained solution is ($u_1 = u_2 = 1$, Q = 260, and the average cost Cost(Q) = 0.2167). The comparison between Li and Yu's linearization techniques and the proposed method is summarized in Table 2, which displays that to linearize a multiple change point concave function the proposed method needs much less number of 0-1 variables than Li and Yu's method (1999) does.

As discussed in Introduction Section, the heuristic approach is also widely used to treat SCM problems with nonlinear cost functions (Shapiro, 2005). Actually, the heuristic algorithm and the mathematical model (i.e., the proposed method) have specific pros and cons, and serve different purposes. In reality, the heuristic is to generate a feasible solution to a problem at every iteration step, retain the best feasible solution between iteration steps, and the solution hold at the termination of the algorithm is deemed as the best solution.   Therefore, the heuristic algorithm is appropriate to solve NP-complete (or called NP-hard) problem because its purpose is to find a best feasible solution when finishing the pre-set iteration procedures.   In contrast, the proposed method is a mathematical model used to find a global optimal solution for a deterministic problem.   Nevertheless, the heuristic algorithm is unsuitable to optimize a SCM problem with variable cost functions shown in Figs. 2-4, while the presented methods can linearize various types of cost functions as demonstrated in the above. Moreover, building on Method 4 and Theorem 2, a SCM model with a convex-concave type cost function as depicted in Fig 4c can also be transformed into a linear mixed integer programming program that is solvable to obtain a global optimal solution by many commercialized LP packages and SCM systems.

Table 2   Comparison between Li and Yu's methods and the proposed method

	Li and Yu's method (1999)	Li and Yu's method (2000)	Proposed method
Number of extra 0-1 variables	m	Cannot directly treat this type of function	$\log_2(m)$
Number of extra continuous variables	m		1

m: number of separable line segments

5. Solution Algorithm and Numerical Example

Notably, when unit costs are not constants, the Cost(Q) × Q (such as RM_P_Cost$_{tbsi}$ × *RM_P_QTY$_{tbsi}$*, RM_ T_Cost$_{tbsi}$ × *RM_T_QTY$_{tbsi}$*, etc.) in the objective function of Model 1 will be a product term. Accordingly, Remark 3 is introduced as follows:

Remark 3 After the necessary variables transformation (Charnes and Cooper, 1962), the objective function with product terms in the proposed SCM quantitative model can be straightly solved by a linear programming.

Consider the following illustration.

Example 3

Minimize Cost(Q) × Q   (15)

subject to Cost(Q) = -0.002Q+0.001w-0.2u+2.2,

Q + 250 (u-1) ≤ w, 0≤ Q ≤ 250, w ≥ 0, u is a zero-one variable,

where Cost(Q) is depicted in Fig. 3b.

The objective function of Cost(Q) × Q involves three product terms of Q×Q, Q×w, and Q×u. Based on Remark 2 and using the variable transformation, Q×Q, Q×w, and Q×u can be replaced by Q', w', and u', respectively. In doing so, Q×Q + 250 (u-1) ×Q ≤ w×Q and 0≤ Q×Q ≤ 250Q are incurred. As a result, Program (15) can be reformulated as follows:

Minimize Cost(Q)'   (16)

subject to Cost(Q)' = -0.002Q' +0.001w' - 0.2u'+ 2.2Q   ,

Q+250(u-1)≤w, 0≤ Q ≤ 250, w ≥ 0, u is a zero-one variable,

Q'+250u'-250Q ≤ w', 0 ≤ Q' ≤ 250Q,

where Cost(Q)', Q', w', and u' denote Cost(Q) × Q, Q×Q, Q×w, and Q×u, respectively.

Noteworthy, since u is a zero-one variable in Programs (15) and (16), the product term Q×u cannot be simply represented by u' without other constraints. Accordingly, based on Lemma 1, Program (16) is then re-programmed as follows:

Minimize Cost(Q)'   (17)

subject to Cost(Q)' = -0.002Q' +0.001w' - 0.2u'+ 2.2Q,

Q+250(u-1)≤w, 0≤ Q ≤ 250, w ≥ 0, u is a zero-one variable,

Q'+250u'-250Q ≤ w', 0 ≤ Q' ≤ 250Q, u' ≥ Q + M (u-1), u' ≥ 0,

where M can be specified as 250 (the upper bound of Q).

The constraint $Q \geq 165$ is added into Example 3 for the illustration. After executing the program (17) in LINGO or Excel, the computed solution is ($u = w = 0$, $Q = 165$, the average cost of Cost(Q) = 1.87, and the total cost of Cost(Q)×Q = 280.5) which is the optimal solution as obtained in Program (15). Consider another demonstration.

Example 4

$$\text{Minimize Cost}(Q) \times Q \tag{18}$$

subject to Cost(Q) =$-0.0005Q +0.0015d_1 +0.0005d_2+0.00017d_3+0.39999$,

$Q+d_1+d_2+d_3 \geq 300$, $0 \leq d_1 \leq 150$, $0 \leq d_2 \leq 100$, $0 \leq d_3 \leq 50$, $0 \leq Q \leq 500$,

where Cost(Q) is depicted in Fig. 4a.

Suppose the average cost of Cost(Q) is a multiple change cost function as depicted in Fig. 4a. By using the variable transformation, $Q \times Q$, $Q \times d_1$, $Q \times d_2$, and $Q \times d_3$ can be replaced by Q', d'_1, d'_2, and d'_3, respectively. In doing so, $Q'+d'_1+d'_2+d'_3 \geq 300Q$, $0 \leq d'_1 \leq 150Q$, $0 \leq d'_2 \leq 100Q$, $0 \leq d'_3 \leq 50Q$, and $0 \leq Q' \leq 500Q$ are incurred. Due to no 0-1 variable existing in Model (16), therefore, Example 4 can be straightly reformulated as

$$\text{Minimize Cost}(Q)' \tag{19}$$

subject to Cost(Q)'= $-0.0005Q' +0.0015d'_1 +0.0005d'_2+0.00017d'_3+0.39999Q$,

$Q+d_1+d_2+d_3 \geq 300$, $0 \leq d_1 \leq 150$, $0 \leq d_2 \leq 100$, $0 \leq d_3 \leq 50$, $0 \leq Q \leq 500$,

$Q'+d'_1+d'_2+d'_3 \geq 300Q$, $0 \leq d'_1 \leq 150 \times Q$, $0 \leq d'_2 \leq 100 \times Q$, $0 \leq d'_3 \leq 50 \times Q$, $0 \leq Q' \leq 500Q$.

For the illustrative purpose, the constraint $Q \geq 265$ is added into Example 4. After executing the program (19) in LINGO, the computed solution is ($d_1 = d_2 = 0$, $d_3 = 35$, $Q = 265$, the average cost Cost(Q) = 0.27344, the total cost Cost(Q) × Q = 39.747345) which is the optimal solution as found in Program (18).

Based on the above illustration, a solution algorithm is described as follows to solving a general SCM problem.

Solution Algorithm

Step 1. Use Model 1 to formulate a generalized SCM problem.

Step 2. Use Methods 1 - 4 and Theorem 2 to treat various types of cost functions such as nonlinear step functions, nonlinear concave functions, and nonlinear S-curve functions.

Step 3. Transform product terms into single variables. Since the rule of variables transformation is simple and distinct, a computer program is easily coded to handle variables transformation.

Step 4. Solve the problem by available commercialized packages such as LINGO or EXCEL.

Step 5. Execute sensitive analysis and coordination and negotiation among supply chain partners. Thereafter, the most feasible and promising supply chain plan is scheduled.

Example 5

In the consumer electronics industry, semi-products j_1 and j_2 are composed of (three units i_1, one unit i_2, and two units i_3) and (two units i_2 and two units i_4), respectively.   The finished product k_1 consists of one j_1 and two j_2 units.   The bill of material (BOM) and the entire supply chain of the product k_1 are displayed in Figs. 6 and 7, separately. According to customer orders from (c_1, c_2, c_3, c_4), they require the k_1 goods (300, 355, 320, 340), (360, 370, 350, 280) and (350, 375, 275, 360) at times t_2, t_3, and t_4, respectively.

According to procurement contracts in this example, the purchasing costs between BSs and ISs and between ISs and factories involve transportation costs.   To simplify this illustration, assume all raw materials i_1, i_2, i_3, and i_4 hold the same unit purchasing and transportation cost. Unit purchasing &
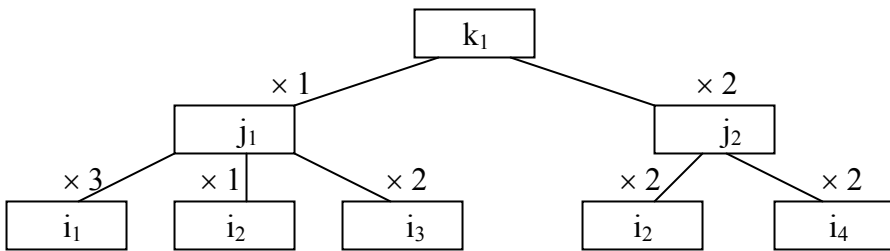
Fig. 7   The bill of material (BOM) of the product k_1
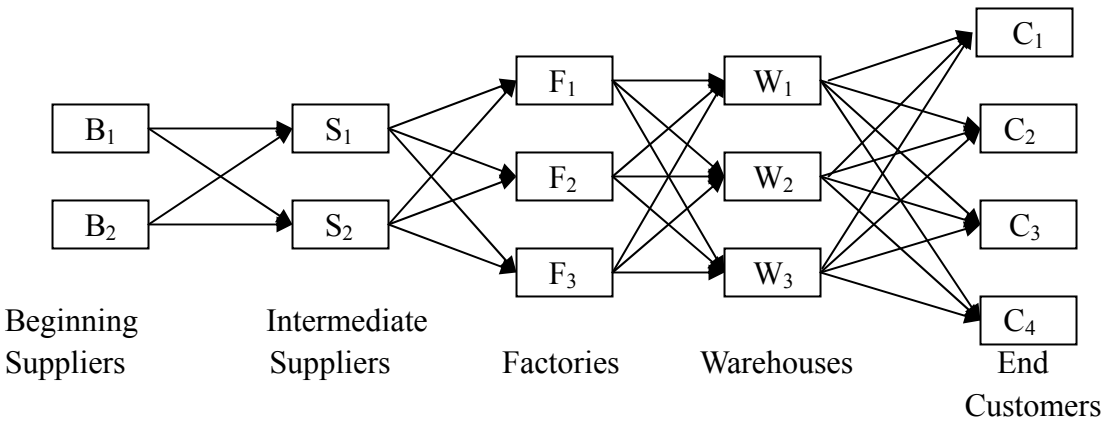
Fig. 8   The entire supply chain of the goods k_1

transportation costs for (b_1 to s_1, b_2 to s_1, b_1 to s_2, b_2 to s_2) are (0.023, 0.02, 0.02, Cost(Q)), (0.024, 0.021, 0.021, Cost(Q)), (0.025, 0.022, 0.022, Cost(Q)), and (0.024, 0.021, 0.021, Cost(Q)) for all raw materials at the t_1, t_2, t_3 and t_4 time separately, where the Cost(Q) is depicted in Fig. 2.   Assume all semi products have the same purchasing and transportation cost.   Unit procurement and transportation costs for (s_1 to f_1, s_1 to f_2, s_1 to f_3, s_2 to f_1, s_2 to f_2, s_2 to f_3) at the t_1, t_2, t_3 and t_4 time are (0.2, 0.5, 0.1, 0.3, 0.2, 0.4), (0.15, 0.45, 0.15, 0.35, 0.25, 0.45), (0.1, 0.45, 0.2, 0.25, 0.2, 0.35), and (0.15, 0.4, 0.1, 0.3, 0.15, 0.3), respectively.

To simplify the calculation, over the time, unit transportation costs from f_1 to w_1, f_1 to w_2, f_1 to w_3, f_2 to w_1, f_2 to w_2, f_2 to w_3, f_3 to w_1, f_3 to w_2, and f_3 to w_3 are 0.6, 0.4, 0.3, 0.3, 0.5, 0.4, 0.55, 0.4, and 0.4, separately.   Unit transportation costs from each w_1, w_2, and w_3 to each c_1, c_2, c_3, and c_4 are 0.04, 0.03, 0.03, 0.04, 0.02, 0.03, 0.04, 0.03, 0.04, 0.03, 0.02, and 0.03 separately, for all time. Average production costs for k_1 in (f_1, f_2, f_3) are (1.75, 1.8, G_M_Cost), (1.77, 1.79, G_M_Cost), (1.76, 1.78, G_M_Cost), and (1.75, 1.8, G_M_Cost) at t_1, t_2, t_3 and t_4 time, respectively, where G_M_Cost is specified in Fig. 3a. Average outsourcing costs of k_1 for f_1, f_2, and f_3 are all same as specified in Fig. 3b over the entire period. Unit production costs for (j_1, j_2) in s_1 and s_2 are (1.7, 1.75) and (1.75, 1.65) for all time.

Inventory capacities of (raw materials, semi-products), (semi-products, finished goods) and (finished goods) for all suppliers, factories, and warehouses separately are (50, 50), (150, 150) and (30) for all period. The maximal capacity of each factory to produce k is no more than 250 and manufacturing capacities in s_1 and s_2 are not restricted in this example. The total outsourcing amount of k_1 cannot exceed 600 units for each factory. Unit inventory costs for (i_1, i_2, i_3, i_4, j_1, j_2) in s_1 and s_2 are separately (RM_I_Cost, 0.3, 0.2, 0.3, 0.4, 0.45) and (RM_I_Cost, 0.3, 0.3, 0.2, 0.43, 0.4) for all time, where RM_I_Cost is specified in Fig. 4c. Unit inventory costs for (j_1, j_2, k_1) in f_1, f_2 and f_3 separately are (0.2, PSFG_I_Cost, 0.6) (0.3, PSFG_I_Cost, 0.55) and (0.3, PSFG_I_Cost, 0.5) for all time where PSFG_I_Cost is displayed in Fig. 4a. Unit inventory costs for k_1 in w_1, w_2, and w_3 are respectively 0.3, 0.35, and 0.35 over the time.

After finished Steps 1 – 4 in Solution Algorithm, the computed supply chain schedule by running LINGO is summarized in Table 3.   By employing the presented techniques, logistical managers along intersecting supply channels can negotiate and mutually cooperate to optimize supply chain schedule throughout the entire supply chain (since the solution generated by the global optimal solution can conveniently perform sensitivity analysis).   In Step 5, by exploiting the computer's powerful capability of computing and exchanging information, the management can adjust and coordinate the entire supply chain schedule until partners within the supply chain have committed the schedule.

Table 3 The generated solution for Example 5

T = 1	ISs	Factories	Warehouses	End customers
BSs	RM_T_QTY$_{1211}$=1200 RM_T_QTY$_{1212}$=2400 RM_T_QTY$_{1213}$=800 RM_T_QTY$_{1214}$=1600			
ISs	PSFG_M_QTY$_{111}$=400 PSFG_M_QTY$_{112}$=800 PSFG_M_QTY$_{113}$=267 PSFG_M_QTY$_{114}$=533	PSFG_T_QTY$_{1111}$=200, PSFG_T_QTY$_{1131}$=200, PSFG_T_QTY$_{1112}$=400, PSFG_T_QTY$_{1132}$=400,		
Factories		G_M_QTY$_{111}$=G_M_QTY$_{131}$= 200,G_I_QTY$_{111}$=50,G_I_ QTY$_{131}$=150	G_T_QTY$_{1131}$=150 G_T_QTY$_{1311}$=50	

T = 2	ISs	Factories	Warehouses	End customers
BSs	RM_T_QTY$_{1211}$=1485 RM_T_QTY$_{1212}$=2970 RM_T_QTY$_{1213}$=990 RM_T_QTY$_{1214}$=1980			
ISs	PSFG_M_QTY$_{211}$=495 PSFG_M_QTY$_{212}$=990 PSFG_M_QTY$_{213}$=330 PSFG_M_QTY$_{214}$=660	PSFG_T_QTY$_{2111}$=200, PSFG_T_QTY$_{2112}$=400, PSFG_T_QTY$_{2121}$=45, PSFG_T_QTY$_{2122}$=90, PSFG_T_QTY$_{2131}$=250, PSFG_T_QTY$_{2132}$=500		
Factories		G_M_QTY$_{211}$=G_O_QTY$_{221}$= 200, G_M_QTY$_{221}$=45, G_M_QTY$_{231}$=250, G_O_QTY$_{211}$= 40, G_O_QTY$_{231}$=180	G_T_QTY$_{2131}$=290 G_T_QTY$_{2211}$=245 G_T_QTY$_{2321}$=580	
Warehouses				G_T_QTY'$_{2111}$=295,G_T_QTY'$_{2211}$= 5, G_T_QTY'$_{2221}$=355,G _T_QTY'$_{2231}$=220, G_T_QTY' $_{2331}$=100, G_T_QTY'$_{2341}$=340

T = 3	ISs	Factories	Warehouses	End customers
BSs	RM_T_QTY$_{3211}$=2010 RM_T_QTY$_{3212}$=4020 RM_T_QTY$_{3213}$=1340 RM_T_QTY$_{3214}$=2680			
ISs	PSFG_M_QTY$_{311}$=670 PSFG_M_QTY$_{312}$=1340 PSFG_M_QTY$_{313}$=477 PSFG_M_QTY$_{314}$=893	PSFG_T_QTY$_{3111}$=200, PSFG_T_QTY$_{3112}$=400, PSFG_T_QTY$_{3131}$=250, PSFG_T_QTY$_{3132}$=500, PSFG_T_QTY$_{3121}$= 220, PSFG_T_QTY$_{3122}$=440		
Factories		G_M_QTY$_{311}$=G_O_QTY$_{321}$ =200, G_M_QTY$_{321}$=220, G_M_QTY$_{331}$=250, G_O_QTY $_{311}$=280, G_O_QTY$_{331}$=210	G_T_QTY$_{3131}$=480 G_T_QTY$_{3211}$=420 G_T_QTY$_{3321}$=460	
Warehouses				G_T_QTY'$_{3131}$=350,G_T_QTY'$_{3141}$ =70,G_T_QTY'$_{3211}$=90, G_T_QTY'$_{3221}$=370,G_T_QTY'$_{3311}$= 270, G_T_QTY'$_{3341}$=210

T = 4	ISs	Factories	Warehouses	End customers
ISs		PSFG_T_QTY$_{4112}$=400, PSFG_T_QTY$_{4121}$=220, PSFG_T_QTY$_{4122}$=440, PSFG_T_QTY$_{4131}$=250, PSFG_T_QTY$_{4132}$=500		
Factories		G_M_QTY$_{411}$= G_O_QTY$_{421}$=200, G_M_QTY$_{421}$=220, G_M_QTY$_{431}$=250, G_O_QTY$_{411}$=280, G_O_QTY$_{431}$=210	G_T_QTY$_{4131}$=480 G_T_QTY$_{4211}$=420 G_T_QTY$_{4331}$=460	
Warehouses				G_T_QTY'$_{4121}$=375, G_T_QTY'$_{4131}$ =45,G_T_QTY'$_{4311}$=350, G_T_QTY'$_{4331}$=230,G_T_QTY'$_{4341}$= 360

6. Concluding Remarks

Although the importance of computer based quantitative models for coordinating supply chain activities is undoubted, their successful power depends heavily on the effectiveness and availability of mathematical formulation. Therefore, the establishment of a mathematical model to interpreting practical supply chain behavior and thus helping enterprises perform complex, laborious, and even trivial analysis has been a challenging and difficult task for a long time (Holmstrom and Hameri, 1999). Although Lemmas 2-4 presented by Li and Yu (1999, 2000) can be efficiently employed to tackle a convex cost function, their methods have some deficiencies in treating a concave function. Consequently, compared to current SCM quantitative models, the proposed methods have the following features:

(1) The proposed techniques can straightforward treat widespread logistic cost functions. For example, Method 1 is presented to treat a step function with Theorem 1 and Lemma 1, Methods 2 and 3 are presented to treat a single change point cost function with Lemmas 2-4, and Method 4 is proposed to treat a concave type of multiple change point function.

(2) Logistical managers can easily arrange and adjusting an entire SCM schedule via negotiation and cooperation with supply chain partners, since the yielded solution is optimal and sensitivity analysis can be performed conveniently.

(3) The presented model can be easily coded in current computer program to aid the management in handling SCM, because of the simple and distinct methods developed herein.

(4) The formulated SCM model can also be executed using many available commercialized packages, such as LINGO and Excel.

(5) The proposed model offers available-to-promise and handles the fulfillment of orders with the total cost minimization, due to synchronization of the capacitated supply, production, outsourcing, inventory, and distribution along intersecting supply chains.

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