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Vehicle routing problem with time-windows for perishable food delivery

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Abstract

This study has extended a vehicle routing problem, with time-windows (VRPTW), by considering the randomness of the perishable food delivery process, and constructing a SVRPTW model, to obtain optimal delivery routes, loads, fleet dispatching and departure times for delivering perishable food from a distribution center. Our objective was to minimize not only the fixed costs for dispatching vehicles, but also the transportation, inventory, energy and penalty costs for violating time-windows. We also discussed time-dependent travel and time-varying temperatures, during the day, modifying the objective functions as well as the constraints in the above mathematical programming models. Algorithms were developed to solve the proposed models; results indicated that inventory and energy costs can significantly influence total delivery costs. It was found that our proposed models yielded better results than the traditional VRPTW models.

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1. Introduction

Cold chain distribution is designed to keep temperaturesensitive food products in good condition from point of departure to final destination. Food products often deteriorate, due to extended travel times and frequent stops to serve customers, during the delivery process. It is, therefore, difficult to effectively manage cold chain distribution and ensure maximum freshness during hot or humid weather. Perishable food is delivered to retailers, using temperature-controlled vehicles; these vehicles have standard cold storage equipment and are usually more expensive, and consume more fuel, than regular vehicles. Due to changeable traffic conditions and the perishable nature of the food, travel time and food's preservation have inherently been characterized as unpredictable. In addition, perishable food usually has a short shelf life; thus, timely delivery of perishable food not only significantly affects

the delivery operator's costs, but also the revenues of retailers. Furthermore, the requirement to serving consumers with allowable delivery time-windows can increase the complexity of vehicle routing and scheduling problems for operators.

Perishable food deteriorates as a result of bacteria, light and air; the higher the temperature, the higher the rate of spoilage. In other words, the shelf life of perishable food depends on storage temperature; the lower the temperature, the longer the shelf life. It is critical that perishable food with a short shelf life, such as milk or lunch box items, be delivered in as timely a manner as possible, in order to reduce spoilage. Perhaps the loss in retailers' revenues ought to be transferred to distribution center operators as a penalty for delayed delivery. Outside temperatures can vary widely during the journey from the distribution center to the final destination, with different corresponding energy requirements for maintenance of proper temperatures. Thus, it is worth quantifying the changes in the quality of perishable food with corresponding time-dependent temperatures.

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Nowadays, many convenience store operators or larger retailers contract with distribution center operators to deliver perishable food within allowable delivery times, or time-windows. If vehicles arrive after a specified time-window, a penalty cost may be incurred. Therefore, a well designed delivery route will not only ensure delivery of the freshest food, but also satisfy customers' requirements in a cost-effective and timely manner. Fluctuating temperatures, at different times of day, can complicate the situation, however. Delivery time may be dependent on traffic conditions; travel time is longer during the rush hour in urban areas. Therefore, the consideration of time-dependent travel should be incorporated into determining optimal delivery routes, under time-window constraints. It also costs more to keep food from spoilage when a delivery vehicle is stuck in a traffic jam during hot spells. The impact of delays in delivering perishable food, is therefore, more critical than for general goods' distribution.

Vehicle routing problems (VRP), related to goods delivery, have been extensively examined (e.g., Belenguer, Benavent, & Martinez, 2005; Chu, 2005; Daganzo, 1987a, 1987b; Prindezis, Kiranoudis, & Marions-Kouris, 2003; Tarantilis, Ioannou, & Prastacos, 2005). Chu (2005) addressed the problem of routing a fixed number of trucks with limited capacity from a central warehouse to customers with known demand. Prindezis et al. (2003) presented an application service provider (ASP) to coordinate and disseminate tasks and related information for solving the VRP using appropriate metaheuristic techniques. Belenguer et al. (2005) developed the computer package RutaRep as a decision support system to automatically generate delivery routes in the meat industry. Moreover, Tarantilis et al. (2005) surveyed the research efforts on metaheuristics solution methodologies for the most widely studied version of the VRP, i.e., the Capacitated VRP. Some researchers have considered vehicle routing problems with time-window constraints, and have constructed penalty costs to reflect violation of these time-windows (e.g., Koskosidis, Powell, & Solomon, 1992; Sexton & Choi, 1986). These studies have mainly focused on determining optimal routes by minimizing total routing costs, including total distance and time costs and the cost of waiting, due to a vehicle's early arrival (Solomon & Desrosiers, 1988). Taniguchi and Shimamoto (2004) presented a dynamic vehicle routing and scheduling model that incorporates real time information using variable travel times. The results showed that the total cost is decreased by the proposed model based on variable travel times. Gendreau, Laporte, and Séguin (1996) reviewed the scientific literature on stochastic VRP. In the paper, the main problems are described within a broad classification scheme and the most important contributions are also summarized. Laporte, Louveaux, and Mercure (1992) further examined vehicle routing problems, using stochastic travel times, by formulating three stochastic programming models. Ahn and Shin (1991) considered temporal issues in routing problems and discussed vehicle routing problems with time-windows, under time-varying congestion. Considering time-varying and stochastic travel time, Fu (2002) focused on the dial-a-ride paratransit scheduling problems arising in paratransit service systems that are subject to tight service time constraints and time-varying, stochastic traffic congestion.

In another line of research, some studies formulated perishable food's inventory models and discussed optimal economic order quantity (EOQ) (e.g., Chakrabarty, Giri, & Chaudhuri, 1998; Giri & Chaudhuri, 1998; Hariga, 1996). Tarantilis and Kiranoudis (2001) developed a thresholdaccepting based algorithm to solve the heterogeneous fixed fleet vehicle routing problem. Tarantilis and Kiranoudis (2002) proposed an open multi-depot vehicle routing problem (OMDVRP) to deal with a real life distribution problem in Greece, in which the industry distributed fresh meat. Burfoot, Reavell, Wilkinson, and Duke (2004) aimed at estimating the energy savings that could be achieved using localised air delivery system based on experiments. Little has been done, however, to investigate inventory costs due to deterioration of perishable food and energy costs for cold storage vehicles, which are important issues in perishable food delivery.

In this study, perishable food were assumed to decrease in value, throughout their lifetime (Raafat, 1991), and had to be stored at chilled temperatures; the rate of deterioration, at any moment, being dependent on the temperature. The customers of the distribution centers are retailers selling perishable food to end-users. This study extended the vehicle routing problem to include time-windows (VRPTW), by considering the randomness of perishable food delivery process, and constructed a 'stochastic vehicle routing problem with time-windows' (SVRPTW) model, to obtain optimal delivery routes and vehicle loads, as well as fleet dispatching and departure times. Our objective was to minimize the fixed costs of dispatching vehicles, transportation, inventory, energy and penalties for violating timewindows. Among these costs, transportation costs are dependent on distance traveled, while inventory costs are related to the deterioration of perishable food; both of these costs can be characterized as stochastic. Energy costs arise from the energy consumption of the vehicles' cold storage equipment. To reflect the effects of time-dependent temperatures and travel, this study formulated a timedependent deterioration function for perishable food and calculated the probability of deterioration and the losses involved. We further constructed a penalty cost for violating time-windows, discussed time-dependent travel and time-varying temperatures during the day, and then modified the objective functions, as well as the constraints in the above mathematical programming problem. Algorithms were developed to solve the proposed models and compare the results.

The remainder of this paper is organized as follows. Section 2 formulates the vehicle routing problem with time-windows for perishable food delivery. Section 3 demonstrates development of the algorithms to solve the proposed models. A numerical example is provided in Section 4, to illustrate the application of the models, and the effects of changes in key parameters on the optimal solutions. In Section 5, we offer concluding remarks.

2. Perishable food distribution model

This study focused on the delivery of perishable, temperature-sensitive food, from a distribution center, using vehicles with frozen storage equipment, to various retailers, in a variety of locations, with delivery time-windows. This study extends traditional VRPTW by giving further consideration to the characteristics of perishable food delivery, which include stochastic travel speed due to traffic congestion, the perishable features of food within the distribution process, energy consumed by storage equipment and soft time-window constraints. Furthermore, considering the influences of time-dependent temperature and travel, on total delivery costs, we revised the VRPTW according to time-sensitive spoilage rates of perishable food. Due to these perishable features, the amount of food carried by the vehicle decreases, as spoilage increases: these are defined as undeliverable food, due to spoilage. "Delivery failure" is defined as the condition where the customer did not receive the ordered food within the appointed time-window; this situation usually meant higher costs for the operator, who must pay a penalty for the loss incurred by the customer. In this study, an *a priori* strategy was provided, in response to the spoilage characteristics of perishable food; that is, extra food, added to prevent delivery failure before departure of the vehicle from the distribution center, can be assessed and analyzed by the food spoilage rate, passage of time and temperature, during the delivery process.

Following the traditional VRP, this study defined a completely symmetric graph G = (V, A) with node set $V = \{v_0, v_1, \ldots, v_n\}$ and link set $A = \{(v_i, v_j) : v_i, v_j \in V, v_j\}$

 $i \neq j$ }. Let v_0 , v_i , and d_i represent the distribution center, the location and demand of customer i, i = 1, 2, ..., n, respectively, where n is the number of customers. Furthermore, let l denote the vehicle serving each route, l = 1, 2, ..., m, where m represents the total number of vehicles required for serving all customer demands and is a decision variable in the model. The transportation cost on link (v_i, v_j) for vehicle l is defined as c_{ij}^l and $c_{ij}^l = c_{ij}^l$.

2.1. Deterministic vehicle travel time

The total delivery costs for VRP, with hard time-windows, included fixed costs for dispatching vehicles, transportation costs, inventory costs and energy costs. The fixed costs for dispatching vehicles can be expressed as $\sum_{l=1}^{m} f^{l}$, where f^{l} is the fixed cost for dispatching vehicle l, l = 1, 2, ..., m. Transportation costs related to distance traveled were formulated as $\sum_{l=1}^{m} \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij}^{l} g_{ij}^{l}$, where g_{ij}^{l} is an indicator variable; and $g_{ij}^{l} = 1$ for vehicle l traveling via link (v_i, v_j) , otherwise, $g_{ij}^{l} = 0$.

Inventory costs arise from the delivery process of perishable food, with spoilage increasing as temperatures and time increase. We assumed that the vehicles' refrigeration equipment was able to ensure optimal temperatures, after which, we concluded that the loss of food was attributable to the time accumulated during the delivery process; this was dependent on vehicle travel time, and the frequency of opening the cargo hold, while unloading perishable food and serving customers. Fig. 1(a) and (b) show variations in the remaining food, as time elapsed, in vehicles departing from the distribution center, and in situations both with and without spoilage features, respectively. In these figures, y_i and u_i represent the vehicle arrival times, and duration, at customer *i*.

Considering the situation illustrated in Fig. 1(b), the total food lost was the sum of the loss resulting from the



Fig. 1. Variations in food remaining as time elapses, with and without considering perishable features.

vehicle travel time between two adjacent customers, labeled (2), and the loss due to opening the cargo hold at customer stops, labeled (3). The total loss is labeled (1) and the amount of food delivered to customers is labeled (4), in Fig. 1. Using the total load carried before departure as a basis for comparison, the amount of loss for the situation considering the spoilage feature, is larger than that without the spoilage feature, as shown in Fig. 1(a) and (b).

Growth of metabolites, during the delivery process, is characterized as a random variable (Chu, Cheng, & Lee, 1998), which influences the loss of food, as a function of delivery time. Let \tilde{b}_i represent the loss of inventory in the vehicle from the time of departure from customer (i - 1)to the time customer *i*, had been served, which includes vehicle travel time from customer (i - 1) to *i* and serving time at customer *i*. Since \tilde{b}_i depends on the deterioration function, it is also characterized as a random variable. The cost resulting from the above loss was defined as inventory cost, which also represents the penalty paid for carrying the extra food. Thus, the total expected inventory cost is formulated as

$$P\sum_{l=1}^{m}\sum_{i=1}^{n}z_{i}^{l}\bar{b}_{i},$$
(1)

where \bar{b}_i denotes the expected loss from departure from customer (i-1) to finishing serving customer *i*, *P* is the cost per item of food and z_i^l is an indicator; if vehicle *l* serves customer *i*, then $z_i^l = 1$; otherwise, $z_i^l = 0$.

When vehicles stop to serve customers, the spoilage rate increases, with the rising temperatures, due to the opening of the cargo hold, with corresponding heat transfer from cold to warm. This study has assumed the loss of food due to opening the cargo hold resulted mainly from the time it was left open, related to the amount of customer demand; that is, duration increased with an increase in demand. Let $G(d_i)$ represent the probability that the food perished, due to an opened cargo hold, being a function of customer demand d_i , i = 1, 2, ..., n; and let d_0 be the total amount loaded into the vehicle in the distribution center. Assume the spoilage of food had not yet begun at the distribution center, thus $G(d_0) = 0$. Let f(y) denote the probability that the food had spoiled at time y and let $F(\cdot)$ be the cumulative probability density function of f(y). Furthermore, let y_i , u_i and L^l denote arrival time at customer *i*, service time required to serve customer *i* and the load of vehicle *l*, respectively. Then, the expected loss for serving customer *i*, b_i can be formulated as

$$\bar{b}_i = L^l \times \left[\int_{y_s^l}^{y_i + u_i} f(t) \, \mathrm{d}t + G(d_i) \right], \quad i = 1, 2, \dots, n,$$
 (2)

where y_s^l is the departure time from the distribution center of vehicle *l*. As shown in Eq. (2), the first term corresponds to vehicle travel time from the distribution center to the location of customer *i* plus time spent serving customer *i*, while the second term is related to the time duration of an opened cargo hold, which further depends on the demand volume of customer *i*. Let x_{0i}^l be the binary variable which represents the relationship between vehicle *l* and customer *i*; that is, for $x_{0i}^l = 1$, vehicle *l* is assigned to serve customer *i*, otherwise, $x_{0i}^l = 0$. Since loss does not exist in the distribution center, that is $F(y_s^l) = 0$, then $\int_{y_s^l}^{y_t+u_i} f(t) dt$ in Eq. (2) can be expressed in the form of a cumulative probability density function $F(\cdot)$ as

$$\begin{aligned} x_{0i}^{l}\bar{b}_{i} &= x_{0i}^{l}L^{l} \times [F(y_{i} - y_{s}^{l} + u_{i}) + G(d_{i})], \\ i &= 1, 2, \dots, n, \ l = 1, 2, \dots, m. \end{aligned}$$
(3)

Without loss of generality, the loss for serving customer i can also be modified in terms of the load of food after serving customer (i - 1) and time spent from the point of departure from customer (i - 1) to finishing serving customer i, which yields:

$$\begin{aligned} x_{(i-1)i}^{l}\bar{b}_{i} &= x_{(i-1)i}^{l}L_{(i-1)}^{l} \times [F(y_{i} - y_{s}^{l} + u_{i}) - F(y_{(i-1)} - y_{s}^{l} \\ &+ u_{(i-1)}) + G(d_{i})], \quad i = 2, 3, \dots, n, \\ l &= 1, 2, \dots, m, \end{aligned}$$
(4)

where $L_{(i-1)}^{l}$ is the load of vehicle *l* after serving customer (i-1) and $x_{(i-1)i}^{l}$ represents the relationship between vehicle *l* and customer *i*, i = 2, 3, ., n. The load of food in vehicle *l*, after serving customer *i* can be calculated as $L_{i}^{l} = L_{(i-1)}^{l} - \bar{b}_{i} - d_{i}$, which is constrained to be larger than or equal to zero, or else customer *i* cannot be assigned to vehicle *l*.

Regarding energy costs, thermal load originates from the sun's radiation heating the ground and warming the air; heat conduction results from the difference in temperature between the inside and outside of the cargo hold (MOEA-IDB, 2001). In practice, the thermal load, resulting from thermal convection by opening the cargo hold, can be calculated as follows (MOEA-IDB, 2001):

$$Q_s = (0.54V_l + 3.22)(T_{\rm O} - T_{\rm I}) \times \beta, \tag{5}$$

where Q_s is the thermal load per hour (kcal/h), V_l is the volume of the cargo hold, T_O and T_I represent outer and inner temperature and β is an indicator, reflecting the frequency of opening the cargo hold, respectively.

This study assumed homogenous vehicles, shipping the same kinds of food, so the volume and inner temperature of the cargo hold was the same for all vehicles. Symbol β represents the frequency of door openings and relates to the demand and spatial pattern of customers. This study also assumed that the operator had planned delivery routes in advance, so as to satisfy the time-constraints agreed to with customers. Therefore, the frequency of opening each vehicle's cargo hold can be represented by its expected value, $\overline{\beta}$. Then, Eq. (5) can be written as follows:

$$Q_s = \alpha_s \beta (T_{\rm O} - T_{\rm I}), \tag{6}$$

where α_s is a constant and equals (0.54 V_l + 3.22) in Eq. (5). Furthermore, this study assumed that the outside temperature was known; thus, the thermal load can be further simplified as a constant, i.e., the energy loss of opening the cargo hold, per hour, is fixed. For an operator with specific customers, if the demand pattern is fixed, then the energy cost of each vehicle, due to opening the cargo hold, is merely a function of total travel time and time serving customers. In practice, thermal conduction, due to the difference between the cargo hold's inside and outside temperatures can be estimated as

$$Q_{\rm T} = U \sqrt{A_{\rm I} A_{\rm O}} (T_{\rm O} - T_{\rm I}) (1 + \rho),$$
 (7)

where $Q_{\rm T}$ represents thermal load per hour (kcal/h), U denotes the conductivity of the cargo hold (kcal/h m² °C), while $A_{\rm I}$ and $A_{\rm O}$ represent the surface area of the inner and outer cargo hold, respectively, and ρ denotes the degree of inferior quality of the cargo hold. We have ignored the impact of the cargo hold on conductivity, the surface area and degree of inferior quality on the thermal load. Therefore, thermal load is mainly dependent on the temperature difference between the inside and the outside of the cargo hold. Thus, Eq. (7) is expressed as

$$Q_{\rm T} = \alpha_{\rm T} (T_{\rm O} - T_{\rm I}), \tag{8}$$

Similarly, the thermal load is a constant under the given outside temperature; total energy loss, due to thermal conduction during the delivery process, is then dependent on total travel time. The energy cost of the vehicle depends on energy loss, energy cost per kcal and total travel time. Total energy cost for all vehicles can be expressed as

$$q \sum_{l=1}^{m} [\alpha(y_{f}^{l} - y_{s}^{l})],$$
(9)

where y_l^l denotes arrival time at the distribution center after the delivery process for vehicle l, α is the thermal load, $\alpha = (\alpha_s \bar{\beta} + \alpha_T)(T_O - T_I)$ and q represents the energy cost per kcal. Let \bar{q} denote the energy cost per hour, $\bar{q} = q\alpha$, and then Eq. (9) can be simplified as $\bar{q} \sum_{l=1}^{m} (y_l^l - y_s^l)$.

From the discussions above, the VRPTW for perishable food delivery can be formulated as follows:

$$\begin{array}{ll}
\underset{g_{ij}^{l},y_{i},y_{j}^{l},y_{f}^{l},b^{l},z_{i}^{l},m}{\min} & \sum_{l=1}^{m} f^{l} + \sum_{l=1}^{m} \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij}^{l} q_{ij}^{l} + P \sum_{l=1}^{m} \sum_{j=1}^{n} z_{j}^{l} \bar{b}_{j} \\
& + \bar{q} \sum_{l=1}^{m} (y_{f}^{l} - y_{s}^{l}),
\end{array} \tag{10a}$$

(m)

s.t.

m

 $\mathcal{Y}_{(i+1)}$

$$\sum_{l=1}^{n} z_{i}^{l} = \begin{cases} 1 & i = 1, \dots, n, \\ \sum_{i=0}^{n} g_{ij}^{l} = z_{j}^{l}, \quad j = 0, \dots, n, \quad l = 1, \dots, m, \end{cases}$$
(10b)

i = 0.

(10c)
$$y_i + u_i + t_{i(i+1)}^l - (1 - x_{i(i+1)}^l)M,$$

$$i = 1, \dots, n, \quad l = 1, \dots, m,$$
 (10d)
 $y_i \ge y_s^l + t_{0i}^l - (1 - x_{0i}^l)M, \quad i = 1, \dots, n,$

$$l = 1, \dots, m,$$
(10e)
$$v_{\ell}^{l} \ge v_{(l+1)} + u_{(l+1)} + t_{(l+1)0}^{l} - (1 - x_{(l+1)0}^{l})M.$$

$$i = 1, \dots, n, \quad l = 1, \dots, m,$$
 (10f)

$$r_i \leqslant y_i \leqslant s_i, \quad i = 1, \dots, n,$$

$$L^l = \sum_{i=1}^n z_i^l d_i + b^l \leqslant K^l, \quad l = 1, \dots, m,$$
(10g)

$$\begin{aligned} x_{0i}^{l}\bar{b}_{i} &= x_{0i}^{l}L^{l} \times [F(y_{i} - y_{s}^{l} + u_{i}) + G(d_{i})], \\ i &= 1, \dots, n, \quad l = 1, \dots, m. \end{aligned}$$
(10i)

Eq. (10a) is an objective function that minimizes the sum of fixed costs for dispatching vehicles, transportation costs, inventory costs and energy costs. Eqs. (10b)–(10h) are constraints as described in the VRPTW formulations. Eq. (10i) expresses the loss of food for serving customer *i* for vehicle *l*. The decision variables are g_{ij}^l , y_i , y_s^l , y_f^l , b^l , z_i^l and *m*.

That is, the operator can apply the model to optimally decide the vehicle delivery route, arrival time of each vehicle for serving each customer, departure time and return time of vehicles, the extra vehicle load, customers served by each vehicle, and the size of the delivery fleet. A trade-off relationship exists between transportation and inventory costs. That is, a larger fleet incurs a higher fixed cost for dispatching vehicles, but assigned customers and routing time are less for each vehicle within the fleet, thereby resulting in less inventory costs and less extra loads.

2.2. Stochastic vehicle travel time

The discussions in Section 2.1 dealt with deterministic travel time; however, vehicle travel speed may be affected by factors such as traffic volume, the weather and accidents, which are characterized as random in nature. Let \tilde{t}_{ij}^l denote travel time on the link (v_i, v_j) for vehicle *l*. This study adopted travel time, based on Lambert, Laporte, and Louveaux (1993). Let *A* and *A'* denote the sets of links without traffic congestion and with the probability of traffic congestion, respectively, $A \subseteq A'$. For every link $(v_i, v_j) \in A$, travel time on link (v_i, v_j) can be expressed as $t_{ij}^l = \beta_0 c_{ij}^l$, where β_0 is a parameter. Assume some links of *A'* have the probability *p* of being congested. For every link $(v_i, v_j) \in A'$, t_{ij}^l is equal to $\beta_1 c_{ij}^l$ with probability *p* and equal to $\beta_2 c_{ij}^l$ with probability (1 - p), where $\beta_2 \leqslant \beta_1$. Then, expected travel time on link (v_i, v_j) for vehicle *l*, \bar{t}_{ij}^l , is given by

$$\bar{t}_{ij}^{l} = \begin{cases} \beta_{0}c_{ij}^{l} & \text{if } (v_{i}, v_{j}) \in A, \\ [p\beta_{1} + (1-p)\beta_{2}]c_{ij}^{l} & \text{if } (v_{i}, v_{j}) \in A', \\ i = 1, \dots, n, \quad j = 1, \dots, n, \quad l = 1, \dots, m, \end{cases}$$
(11)

where c_{ij}^{l} represents travel costs on link (v_i, v_j) .

Because of the randomness of travel time on links, arrival time at each customer location is also characterized as a random variable. Since the real-time traffic conditions for every link is unknown before departure from the distribution center, arrival time at each customer is difficult to predict. Rather than trying to identify this uncertain travel

(10h)

time, we have employed the expected value of arrival time and revised Eqs. (10d)–(10f), which yield:

$$y_{(i+1)} \ge y_i + u_i + \bar{t}_{i(i+1)}^l - (1 - x_{i(i+1)}^l)M,$$

$$i = 1, \dots, n, \quad l = 1, \dots, m,$$

$$y_i \ge y_s^l + \bar{t}_{0i}^l - (1 - x_{0i}^l)M, \quad i = 1, \dots, n, \quad l = 1, \dots, m,$$

(13)

$$y_{f}^{l} \ge y_{(i+1)} + u_{(i+1)} + \overline{t}_{(i+1)0}^{l} - (1 - x_{(i+1)0}^{l})M,$$

$$i = 1, \dots, n, \quad l = 1, \dots, m.$$
(14)

Due to stochastic travel time, the time-window constraint of Eq. (10g) can be revised according to Lambert et al. (1993) as

$$(r_{i} - y_{s}^{l}) \left[p \frac{\beta_{1}}{\beta_{2}} + (1 - p) \right] + y_{s}^{l} \leqslant \bar{y}_{i}$$

$$\leqslant (s_{i} - y_{s}^{l}) \left[p + (1 - p) \frac{\beta_{2}}{\beta_{1}} \right] + y_{s}^{l}, \quad i = 1, \dots, n.$$
(15)

Since $\frac{\beta_1}{\beta_2} > 1$, the left hand side of Eq. (15) is larger than the lower bound of time-window, r_i ; the right hand side of Eq. (15) is also smaller than the upper bound of time window, s_i . Therefore, the duration of the time-window, considering stochastic travel time, is narrower than when no consideration was given. Moreover, the equations involved with arrival time and travel time have been revised accordingly; i.e., the loss of food during delivery process as Eq. (10i); and the inventory and energy cost in the objective function as Eq. (10a).

2.3. Relaxation from hard time-windows to soft timewindows

The time-window constraints, discussed in Section 2.1, were "hard" constraints, which cannot be violated. These hard time-window constraints increase the complexity of determining optimal delivery routing, however. In contrast, in the case of soft time-windows, constraints can be violated, but with a penalty cost. When a vehicle arrives early, or with an acceptable delay, the food can still be delivered,



Fig. 2(a). The relationship between arrival time, time-windows and penalty cost.

with a penalty cost. The relationship between penalty cost and arrival time can be seen in Fig. 2(a).

Let R and S denote the earliest acceptable time for early arrival and the latest acceptable time for late arrival, $R \leq r$ and $S \ge s$, respectively. As shown in Fig. 2(a), the acceptable periods for early arrival and delay are [R, r) and (s, S], respectively; within each range, there are different penalties. When arrival time is beyond [R, S], customers may refuse to receive the food, and a large M, representing a huge penalty, has been introduced to avoid this occurrence. When early arrival lies within [R, r], the operator must decide whether to immediately serve the customer or wait until time r. In practice, the increased cost resulting from waiting until the beginning of the time-window is very low; this is because the difference between the earliest acceptable time, R, and the beginning of the time-window, r, is usually relatively small. Therefore, this study assumed the operator would rather wait and serve on time, since the increased cost is negligible; consequently, R is approximated to r, similar to hard time-window constraints.

To avoid double counting, the penalty cost for violating the time-window was considered as the revenue lost due to late delivery. The probability, that perishable food can be sold, depends on the time between purchasing and expiration date; this probability decreases, at an increasing rate, as the time of purchase nears the expiration date. Thus, customer revenue may be reduced due to late delivery. The penalty cost, due to violating the upper bounds of the time-window, specified by customer *i*, s_i , can be formulated as $\eta(y_i - s_i)^v \times P \times d_i$, where v and η represent parameters, and $v \ge 1$. Substituting λ for $\eta \times P$, the penalty cost can be simplified as $\lambda d_i(y_i - s_i)^v$, and is given by

$$Q_{i}(y_{i}) = \begin{cases} M, & y_{i} < r_{i} \\ 0, & r_{i} \leq y_{i} \leq s_{i} \\ \lambda d_{i}(y_{i} - s_{i})^{v}, & s_{i} < y_{i} \leq S_{i} \\ M, & y_{i} > S_{i} \end{cases}, \quad i = 1, \dots, n,$$
(16)

where $Q_i(y_i)$ represents the penalty cost of customer *i*, and is a function of vehicle arrival time at customer *i*, y_i . The



Fig. 2(b). The revised relationship between arrival time, time-windows and penalty cost.

revised relationship between arrival time, time-window and the penalty cost is illustrated in Fig. 2(b).

The vehicle routing problem with soft time-window constraints (VRPSTW) discussed above can be formulated as follows:

$$\begin{array}{ll} \min_{g_{ij}^{l}, y_{i}, y_{s}^{l}, y_{f}^{l}, b^{l}, z_{i}^{l}, m} & \sum_{l=1}^{m} f^{l} + \sum_{l=1}^{m} \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij}^{l} q_{ij}^{l} + P \sum_{l=1}^{m} \sum_{j=1}^{n} z_{j}^{l} \bar{b}_{j} \\ & + q \sum_{l=1}^{m} [\alpha(y_{f}^{l} - y_{s}^{l})] + \lambda \sum_{i=1}^{n} d_{i} [(y_{i} - s_{i})^{+}]^{\nu}, \end{array}$$

$$(17)$$

s.t. (10b)–(10f), (10h) and (10i),

$$r_i \leqslant y_i \leqslant S_i, \quad i = 1, \dots, n, \tag{18}$$

where $(y_i - s_i)^+ = \max\{0, (y_i - s_i)\}.$

2.4. Time-dependent temperatures and vehicle travel time

This study further relaxes the assumption of a constant temperature, discussed in Section 2.1. Let H(y), $\Delta H(y)$ and H_0 denote the temperature at time y, the difference in temperature between the outer and inner cargo hold at time y and the optimal inner temperature, required to keep the food fresh, respectively, where $\Delta H(y) = H(y) - H_0$. The spoilage rate, due to an opened cargo hold to serve customer *i* can, thus, be revised as $G'(d_i) = g(d_i)\Delta H(y)$, where $g(d_i)$ is the average rate of spoilage per unit temperature difference and is a function of the demand of customer *i*, d_i .

As stated, the total energy cost arises from the thermal load, due to an opened cargo hold and travel time; both of these influences depend on the difference between outside and inside temperatures, as shown in Eqs. (6) and (8), respectively. Then, the energy cost of vehicle *l*, during one routing period, from distribution center departure to return, can be formulated as $q \int_{y_s^l}^{y_f^t} \alpha' \Delta H(y) \, dy$, where α' denotes the energy loss per hour per unit temperature difference. The energy cost per hour, under one unit temperature difference, can be further expressed by energy loss per hour under one unit temperature difference, α' , and energy cost per kcal, q, that is, $\bar{q}' = q\alpha'$. Moreover, the total energy cost of *m* vehicles can be expressed as $\bar{q}' \sum_{l=1}^{m} \left[\int_{y_s^l}^{y_f^l} \Delta H(y) \, \mathrm{d}y \right]$. To show how time-dependent traffic can affect travel time on a link, this study considered travel time on link (v_i, v_j) as a function of entering time on link $(v_i, v_j), y'_i$, that is, $t^l_{ij}(y'_i)$. If traffic is heavy on link (v_i, v_j) at time y'_i , more time is spent navigating that link.

3. Algorithm

The VRP inherently belongs to the NP-hard problem (Golden & Assad, 1988) and a VRP with a hard time-window is more complex than the simple VRP (Solomon, 1987). This study adopted a heuristic method, which extended the "Time-Oriented Nearest-Neighbor Heuristic" by Solomon (1983). The heuristics for the VRP with a hard time-window consist of the following steps:

- Step 1. Input basic data, such as demand, supply parameters and network, G = (V, A).
- Step 2. Denote the distribution center as the beginning of a route.
- Step 3. Determine the customer closest to the last customer added to the route.
- Step 4. Repeat Step 3 until the vehicle is filled to capacity.
- Step 5. Assign another vehicle and repeat Step 2 until all customers have been served.

The details for Step 3 are described as follows:

Assume customer i is the first customer added to the route and customer (i+1) represents the next customer added to the route. Two constraints must be satisfied before the closest customer can be determined: (1) the time-window constraint, specified by customer (i + 1); (2) the food remaining in excess of the demand of customer (i+1). In this study, the factors determining whether the closest customer can be added to the route included the demand of customer (i + 1), $d_{(i+1)}$, travel costs from customer *i* to customer (i+1), $h_{i(i+1)}$, time duration between finishing serving customer i to arrival at customer (i + 1), $\Delta y_{i(i+1)}$, and duration from the end of the time-window of customer (i + 1) to the earliest service time for customer (i+1), $a_{i(i+1)}$. Among these factors, $h_{i(i+1)}$ was classified as the spatial distance factor, while $\Delta y_{i(i+1)}$ and $a_{i(i+1)}$ are time distance factors. The definitions of $\Delta y_{i(i+1)}$ and $a_{i(i+1)}$ are $\Delta y_{i(i+1)} = y_{(i+1)} - (y_i + u_i)$ and $a_{i(i+1)} = s_{(i+1)} - (y_i + u_i + u_i)$ $t_{i(i+1)}$), respectively, where $y_{(i+1)}$ represents arrival time at customer (i + 1). Then, the cost function determining the closest customer can be formulated as

$$C_{i(i+1)} = \delta_1 h_{i(i+1)} + \delta_2 \Delta y_{i(i+1)} + \delta_3 a_{i(i+1)} + \delta_4 d_{(i+1)}, \tag{19}$$

where $C_{i(i+1)}$ is the cost function for customer (i + 1), while δ_1 , δ_2 , δ_3 and δ_4 express the weights of these influences, and represent the marginal cost of the objective function in the constructed model with respect to the addition of one unit of $h_{i(i+1)}$, $\Delta y_{i(i+1)}$, $a_{i(i+1)}$ and $d_{(i+1)}$, respectively. The customer with the smallest $C_{i(i+1)}$ is closest. Note that $\delta_1 + \delta_2 + \delta_3 + \delta_4 = 1$ and $\delta_1 \ge 0$, $\delta_2 \ge 0$, $\delta_3 \ge 0$, $\delta_4 \le 0$. The weights of δ_1 , δ_2 , δ_3 and δ_4 can be further evaluated as:

(1) Travel cost from customer *i* to (i + 1), $h_{i(i+1)}$. An increase of one unit of $h_{i(i+1)}$ means customers *i* and (i + 1) are one more distance apart, and the addition of customer (i + 1) will further increase the total delivery cost by δ_1 units.

(2) Time duration from finishing serving customer *i* to arrival at customer (i + 1), $\Delta y_{i(i+1)}$. The costs incurred by one more unit of $\Delta y_{i(i+1)}$ can be divided into energy and inventory costs. Suppose there is an increase of one unit of $\Delta y_{i(i+1)}$, meaning that customer (i + 1) and *i* are one more time-distance apart; the energy costs are increased by δ units. Moreover, the increased inventory cost is brought about by the loss of food due to one additional unit of travel time. Let

 μ represent the average life of the perishable food: the average spoilage rate is $1/\mu$. And, let L_i^l denote the food remaining after serving customer *i* using vehicle *l*. The increased inventory costs, due to an increase of one unit in travel time can be formulated as $p \times L_i^l/\mu$. One procedure in the algorithm is the addition of customers into the routes, one after the other, implying that the remainder of food, after serving customer *i* is known before construction of the route has been completed. Let ϕ_i be the total amount of food delivered after serving customer *i*, then the remainder of food becomes $K^l - \phi_i$. In sum, the total delivery costs are increased by $\alpha + p(K^l - \phi_i)/\mu$ due to one additional unit of Δy_{ij} .

(3) The time duration from the end of the time-window of customer (i + 1) to the earliest service time for customer (i + 1), $a_{i(i+1)}$. Factor $a_{i(i+1)}$ represents the influence resulting from the order of the customers' time-windows on the delivery routes. Because of the order of the time-windows, adding customer (i + 1) may mean that other customers cannot join the route. Only if $a_{i(i+1)} \ge 0$, will the time-window of customer (i + 1) be satisfied and customer (i + 1) will be considered to be added into the route.

(4) Demand of customer (i + 1), $d_{(i+1)}$. Once customer (i + 1) has been added into the route, the load carried by the vehicle is increased by one additional unit of $d_{(i+1)}$ and the inventory cost, per unit, reduced accordingly. The decrease in total delivery cost due to the increase of one unit, $d_{(i+1)}$, can be estimated by the purchasing cost per unit of food, and the average loss of food per hour, due to serving customer (i + 1).

The modifications in Step 2 of the proposed heuristics were necessary to solve the VRP models with soft time-windows, time-dependent temperatures and vehicle travel times. To reflect the impact of soft time-window customer constraints in determining the delivery routes, the time duration from the end of the time-window of customer (i+1) to the earliest service time for customer (i+1), $a'_{i(i+1)}$, was modified as $a'_{i(i+1)} = S_{(i+1)} - (y_i + u_i + t^l_{i(i+1)}) \ge 0$, where $S_{(i+1)}$ denoted the end of soft time-window of customer (i + 1). The condition $a'_{i(i+1)} < 0$ holds for models with soft time-windows, and the penalty cost, due to adding customer (i + 1) into the route, with a violation of the timewindow, is $\lambda d_{(i+1)} a_{i(i+1)}^{\prime \nu}$, as shown in Eq. (16). The marginal cost of the objective function in the constructed model, with respect to the additional unit of $a'_{i(i+1)}$, can then be formulated as $\lambda d_{(i+1)} a_{i(i+1)}^{\prime \nu-1}$. In practice, the operator will try to avoid an enormous penalty cost; $\lambda d_{(i+1)} a_{i(i+1)}^{\prime \nu}$ was assumed to be ten percent, or less, of the fixed $\cot f$. In response to time-dependent temperatures and travel time, related parameters and variables of the cost function in Step 2 must be revised accordingly, i.e., using time-dependent penalty cost, $\alpha' \Delta H(y)$, and time-dependent travel speed, $\tau \times \frac{t'_{ij}}{t'_{i}(y'_{i})}$

4. Numerical example

This section presents an application of the proposed models, using a numerical example. A rectangular grid net-

work was used in this study, with one depot at coordinate (0,0), representing the distribution center, which dispatches vehicles to deliver lunch box items to local retail customers. The study covered an area of fifty square kilometers and comprised a random extraction of the characteristics of fifty customers, which included locations, time-window constraints and demand; customers' time-windows were randomly generated between 6:20-11:00 a.m. and customer demand ranged between ten items to items making up onequarter of vehicle capacity. Identical customer time-window duration, i.e., one half hour, was assumed, in order to simplify the problem, while service times for customers were demand-dependent, i.e., $u_i = \frac{d_i}{20}$ (min). The life cycles of the lunch box items, and the required temperature to preserve these items, were 24 h and 18 °C, respectively. For simplification, the perishable rate of the lunch box items was estimated by the reciprocal of the life cycle, which was a constant, under the appropriate temperature; in addition, the cumulative probability function, $F(\cdot)$, can be simplified as $\frac{\Delta y}{1440}$, where the denominator represents the life cycles of the lunch box items (min). We further assumed the spoilage of food, due to the opened cargo hold, was double that due to vehicle travel time: i.e., $G(d_i) = \frac{u_i}{1440} = \frac{d_i}{28,800}$. Base values for the parameters in the total delivery cost function, and time-window constraints, were estimated by interviewing the distribution center operator, as listed in Table 1.

For comparisons, the traditional VRPTW, giving no consideration to either energy or inventory costs, was applied here, in order to determine optimal delivery routes, where three sets of weights, i.e., $(\delta_1, \delta_2, \delta_3) = (0.5, 0.5, 0)$, (0.4, 0.4, 0.2), (0.3, 0.3, 0.4) were employed to explore changes to these optimal solutions, due to variations in key parameters. Table 2 shows the results for a basic model with deterministic and stochastic travel time, respectively, together with models, both with and without consideration being given to energy and inventory costs, using various values of δ_3 . The total inventory cost was divided into: (1) inventory cost due to vehicle travel time; (2) inventory cost due to opening of the cargo hold as shown in Table 2.

l	a	bl	e	1		

Initial values of parameters

Symbol	Definition	Initial value
τ	Average travel speed (km/h)	30
f^{t}	Fixed cost for dispatching vehicles (NT \$/vehicle)	750
Р	Purchasing cost per item (NT \$/item)	50
\bar{q}	Energy cost per hour (NT \$/h)	30
K^l	Vehicle capacity (items)	300
$F(\cdot)$	Cumulative probability density function of $f(y)$	$\frac{\Delta y}{1440}$
$G(d_i)$	The probability that the food is perished due to an opened carriage	$\frac{d_i}{28,800}$
р	The probability that the link of A' is congested	0.5
β_0		1/100
β_1		1/100
β_2		1/120
μ	Life cycle of lunch box (h)	24

Table 2 Results from basic models with hard time-windows (unit: NT \$)

		Parameter	Inventory cost		Energy cost	Transportation cost	Fixed costs for dispatching vehicles	Total delivery cost
			(1)	(2)				
Deterministic travel time	With considering the loss of food and	$\delta_3 = 0$	3600	500	549	1530	9000	15,180
	energy cost	$\delta_3 = 0.15$	4100	550	556	1557	8250	14,963
		$\delta_3 = 0.3$	5050	450	568	1639	7500	15,206
	Without considering the loss of food and energy cost	$(\delta_1, \delta_2, \delta_3) = (0.5, 0.5, 0)$	3550	550	585	1597	9000	15,281
		$(\delta_1, \delta_2, \delta_3) = (0.4, 0.4, 0.2)$	4350	650	624	1397	8250	15,270
		$(\delta_1, \delta_2, \delta_3) = (0.3, 0.3, 0.4)$	5550	650	512	1557	7500	15,769
Stochastic travel time		$\delta_3 = 0.15$	5350	500	708	1750	9000	17,308
Percentage of t total delivery	he y cost		33	%	4%	10%	53%	100%

Table 2, compares different costs, using percentage of total delivery cost. Total inventory cost accounts for the highest percentage, i.e., 33%, with inventory cost due to routing time with time-dependent vehicle travel time and inventory cost due to opening the cargo hold, accounting for approximately 29% and 4%, respectively. The percentage total, of both inventory and energy costs, was 37%. Parameter δ_3 represents the weight that the operator placed on customers' time-window constraints during the delivery route design, regarding sequence of service. With a larger value of δ_3 , there were less vehicles required, as well as a lower fixed cost for dispatching vehicles. On the other hand, without considering the order of time-windows, i.e., $\delta_3 = 0$, more vehicles would have to be dispatched; however, inventory costs, due to vehicle routing time, was higher for models with a higher value of δ_3 than for those with a smaller value of δ_3 , which shows that a trade-off relationship exists between inventory costs and total costs for dispatching vehicles. An appropriate value of δ_3 may not only reflect the impact of time-window constraints on the service sequence of customers, but may also result in the lowest total delivery costs. As shown in Table 2, $\delta_3 = 0.15$ and $(\delta_1, \delta_2, \delta_3) = (0.4, 0.4, 0.2)$ yielded the lowest total delivery costs for the deterministic models, with and without consideration being given to energy and inventory costs, respectively.

As for the traditional VRPTW with no consideration for energy and inventory costs, transportation and fixed costs, for dispatching vehicles, were the most influential factors in total delivery costs. However, the revised results, after applying the proposed model in this study, i.e., Eqs. (1) and (9), show that total energy and inventory costs account for a significant percentage of the total costs, i.e., 37%, which implies that the delivery route, using the traditional VRPTW, is not optimal, since neither energy nor inventory costs, critical to delivery of perishable food, were considered. Furthermore, the average total delivery costs, using models with no consideration given to energy and inventory costs was NT \$15,440, which is greater than for those models which took these costs into consideration, i.e., NT \$15,116. Further comparison of models with deterministic and stochastic travel time, where energy and inventory costs were both considered, showed that total delivery costs were higher for the model with stochastic vehicle travel time, than for the model with deterministic vehicle travel time. This finding implies that time-window constraints are more rigid, with respect to stochastic travel time, with more vehicles required to satisfy customers' demand. In addition, the inventory, energy and transportation costs, for models using stochastic travel time, were higher than for those using deterministic travel time.

Table 3 shows the results of revised models with soft time-windows and time-dependent temperature and travel. As shown in Table 3, penalty costs arose, due to violations of time-window constraints, which were relaxed here, as soft time-windows. Fixed costs for dispatching vehicles, using the model with soft time-windows were less than for the model which considered hard time-windows for $\delta_3 = 0.15$, shown in Table 2, respectively; this indicates that the release of hard time-window constraints resulted in a smaller number of vehicles being required to serve customers. However, a delay in delivery may result in higher inventory costs than when delivery is achieved within time-window constraints. The models, which considered time-dependent travel and temperature, captured the impact of variations in these functions, at different times, on optimal decisions; therefore, total delivery costs as well as inventory, energy and transportation costs were lower in these models, than in those which did not consider timedependent travel and temperatures. Table 3 also shows the service sequence of vehicle routing and cost components related to each vehicle for models with soft time-windows and time-dependent temperature and travel time, where the parentheses denote customer demand, and asterisks show that customer time-windows have been violated,

Table 3	
Results from revised models with soft time-windows and time-dependent temperature and travel (unit:	NT \$)

		Inventory cost		Energy cost	Penalty cost	Transportation cost	Fixed costs for dispatching vehicles	Total delivery cost	
		(1)	(2)						
Rev	Revised model ^a		450	619	103	1834	7500	15,453	
Rev	vised model ^b	4500	500	538	178	1426	6750	13,713	
Ro	ute sequences								
1	$0 \rightarrow 35 \rightarrow 40 \rightarrow 1 \rightarrow 30^{*} \rightarrow 24 \rightarrow 50$ (16)(26) (56) (32) (14) (42) $\rightarrow 10 \rightarrow 49 \rightarrow 46 \rightarrow 23 \rightarrow 0$ (21) (24) (13) (35)	600	50	67	16	184	750	1651	
2	$\begin{array}{c} (27) (27) (27) (29) (27) \\ 0 \to 32 \to 5 \to 42 \to 28 \to 25 \to \\ (54) (43) (27) (49) (27) \\ 16 \to 0 \\ (74) \end{array}$	750	50	63	0	168	750	1781	
3	$\begin{array}{c} 0 \rightarrow 7 \rightarrow 0 \\ (31) \end{array}$	50	0	14	0	82	750	896	
4	$\begin{array}{c} 0 \to 15 \to 11^* \to 6^* \to 12^* \to 13 \to \\ (58) \ (63) \ (74) \ (46) \ (22) \\ 36 \to 0 \\ (18) \end{array}$	300	100	49	33	138	750	1337	
5	$\begin{array}{c} 0 \to 43 \to 20 \to 14^* \to 27 \to 37 \to \\ (65) \ (60) \ (41) \ (36) \ (58) \\ 26 \to 0 \\ (23) \end{array}$	600	100	64	0.4	115	750	1629	
6	$\begin{array}{c} 0 \to 33 \to 31 \to 38 \to 3 \to 47 \to 2 \to \\ (16) \ (17) \ (61) \ (31) \ (66) \ (32) \\ 39 \to 29^* \to 0 \\ (22) \ (42) \end{array}$	550	50	69	52	196	750	1615	
7	$\begin{array}{l} (12) & (12) \\ 0 \rightarrow 48 \rightarrow 21 \rightarrow 4^* \rightarrow 41^* \rightarrow 44 \rightarrow \\ (36) & (50) & (25) & (39) & (21) \\ 17^* \rightarrow 19^* \rightarrow 0 \\ (24) & (35) \end{array}$	950	50	97	76	253	750	2100	
8	$\begin{array}{c} (24) \ (53) \\ 0 \rightarrow 18 \rightarrow 34 \rightarrow 0 \\ (54) \ (51) \end{array}$	200	0	48	0	105	750	1103	
9	(67)(67) $0 \to 9 \to 45 \to 22 \to 8 \to 0$ (67)(72)(61)(71)	500	100	66	0	186	750	1602	

Note: Parentheses denote customer demand and customers with asterisks show that the time-window specified by that customer is violated.

^a Model with soft time-windows.

^b Model with soft time-windows and time-dependent temperature and travel time.

respectively. Ten customers were identified as having had their time-window constraints violated, with the operator having to pay a small penalty, i.e., NT \$178; this shows that most customers can put up with time-window violations. The VPRSTW is capable of finding solutions in cases where a hard time-window formulation would fail. Problems resulting from tight time-windows and a small fleet may not be able to satisfy all customers, while a small fleet may not be able to satisfy all customers on time. In this case, the VRPSTW would yield a solution where some of the customers would not be serviced on time. Naturally, this solution is not feasible for hard time-window models, but the operator would, at least, have a solution at hand. This solution can be either accepted as is, or can be improved by adjusting the appropriate time-windows, to produce routes to service more, or all of the customers, on time. The VRSPTW solution can provide ample information on the customers to whom the schedule is infeasible; the "trouble maker" can be easily identified from the routes at hand (Koskosidis et al., 1992).

5. Conclusions

This study has focused on determining the optimal delivery routing, loads and departure times of vehicles, as well as the required number of vehicles for delivering perishable food to many customers, from a distribution center. Features related to delivery of perishable food were considered, such as the time-window constraints of customers and the stochastic characteristics of travel time and food's preservation. Models, using stochastic vehicle routing problems with time-windows, for perishable food, were constructed using mathematical programming methods. Time-dependent temperatures and travel time, and soft time-windows with penalty costs, were further discussed and the objective functions, as well as the constraints, in the mathematical programming models were modified, accordingly.

The results showed that the sum of inventory and energy costs constitutes a significant percentage of total costs, which cannot be ignored. We also discovered a trade-off relationship between the fixed costs of dispatching vehicles and inventory costs, showing that delivery, using a smaller number of vehicles, may result in lower fixed costs, but higher inventory costs. The models we have proposed. which take the energy and inventory costs, related to deliverv of perishable food, into consideration, vielded better results for deciding optimal delivery routes than the traditional VRPTW. These results also showed that, when no consideration was given to the order effects of time-windows on total delivery costs, the operator had to dispatch more vehicles to satisfy customer time-windows; an appropriate setting of parameters may reflect not only the impact of time-window constraints on customer service sequencing, but may also result in the lowest delivery costs. The results from the models using stochastic travel times implied that time-window constraints were more difficult to satisfy than models using deterministic travel times, requiring more vehicles to be dispatched, in order to satisfy customers' needs. The models with soft time-windows also yielded a smaller vehicle requirement than those incorporating hard time-windows; a delay in delivery, however, may result in higher inventory and penalty costs. Models considering time-dependent travel and temperatures were shown to result in lower total delivery costs, as well as lower inventory, energy and transportation costs, than those which did not consider these factors.

In summary, this study has shown how crucial characteristics, related to the delivery of perishable food may be considered in formulating vehicle routing solutions, with time-window constraints. The proposed models provide effective tools, which may enable operators to make effective delivery decisions, under time-varying temperatures and time-dependent travel, by assessing the impact of random delivery times, food's spoilage and time-windows on vehicle routing and the resultant costs.

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