Strictly Nonblocking f-Cast $\text{Log}_d(N, m, p)$ Networks

Frank K. Hwang, Yang Wang, and Jinzhi Tan

***Abstract*—Necessary and sufficient conditions for $\text{Log}_d(N, m, p)$ network to be point-to-point strictly nonblocking are known. Recently, Kabacinski and Danilewicz obtained necessary and sufficient conditions for the $\text{Log}_2(N, 0, p)$ network to be broadcast strictly nonblocking. In this paper, we give necessary and sufficient conditions for $\text{Log}_d(N, m, p)$ to be f-cast strictly nonblocking for every f, thus covering the point-to-point case $(f = 1)$ and the broadcast case $(f = N)$ as special cases.**

***Index Terms*—f-cast, broadcast, strictly nonblocking.**

I. INTRODUCTION

LEA [1] first introduced the $\text{Log}_2(N, 0, p)$ network which has $N = 2^n$ inputs and outputs and $n + 2$ stages. The first (input) stage has N $1 \times p$ crossbars, the last (output) stage has N $p \times 1$ crossbars, and the inner n stages consist of p copies of an n-stage inverse banyan network $\text{BY}^{-1}(n, 0)$, where each input (output) crossbar is connected to every copy of $\text{BY}^{-1}(n, 0)$. (See Fig. 1 for an example of $\text{BY}^{-1}(4, 0)$.)

Shyy and Lea [2] extended this network to $\text{Log}_2(N, m, p)$ by replacing $\text{BY}^{-1}(n, 0)$ in the middle with m-extra-stage inverse banyan networks $\text{BY}^{-1}(n, m)$ where the connection pattern of the m extra stages is a mirror reflection of the first m stages of the inverse banyan network. (See Fig. 1 for an example of $\text{BY}^{-1}(4, 2)$, Fig. 2 for an example of $\text{Log}_2(8, 1, 3)$.) Note that $\text{Log}_2(N, n-1, p)$ is the Cantor network [3] with p copies of the Benes network [4] in the middle. The $\text{Log}_2(N, m, p)$ network can further be extended to the $\text{Log}_d(N, m, p)$ network by using d-ary crossbars. (See Fig. 3 for an example of $\text{Log}_3(27, 0, 2)$.)

Nonblocking networks are favorable in designing switching networks, since a conflict-free path is available for any pair of idle input and output. There are several kinds of nonblockingness. Strictly nonblocking means that any pair of idle input and output in a network can be connected regardless of the existing connections of other pairs in the network (all connecting paths must be link disjoint).

Hwang [5] extended a result of Shyy and Lea from binary to d-ary, as follows.

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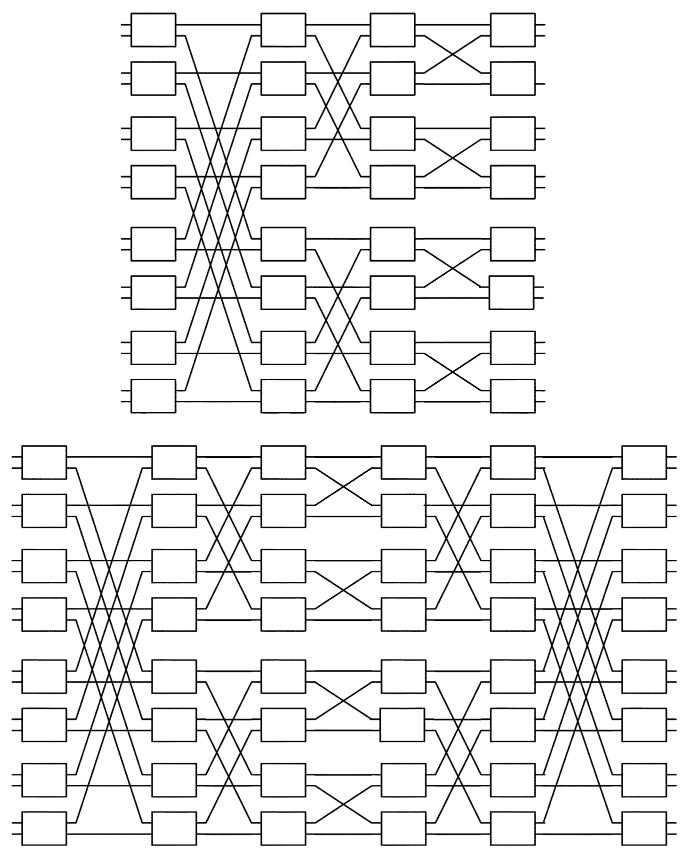

Fig. 1. $\text{BY}^{-1}(4, 0)$ and $\text{BY}^{-1}(4, 2)$.

Theorem 1: The sufficient condition for $\text{Log}_d(N, m, p)$ to be point-to-point strictly nonblocking is

$$p \geq \frac{2m(d-1)}{d} + d^{\lfloor (n-m-1)/2 \rfloor} + d^{\lceil (n-m-1)/2 \rceil} - 1.$$

A careful examination shows that the worst-case scenario assumed in the proof of the theorem can be realized. So the condition in *Theorem 1* is also necessary.

Lea mentioned that his argument for the point-to-point network can also apply to multicast traffic. Tscha and Lee [6] gave necessary and sufficient conditions for $\text{Log}_2(N, 0, p)$ to be multicast strictly nonblocking, but the result is really for wide-sense nonblocking, since a special routing algorithm is used. Kabacinski and Danilewicz [7] gave the following result.

Theorem 2: The necessary and sufficient conditions for $\text{Log}_2(N, 0, p)$ to be broadcast strictly nonblocking are $p > 2^{n-1} - 1$.

To clarify our terminology, f-cast means an input can request to be connected to at most f outputs. When f is unspecified, we use the general term multicast. When $f = N$, i.e, f is unconstrained, then we use the term broadcast.

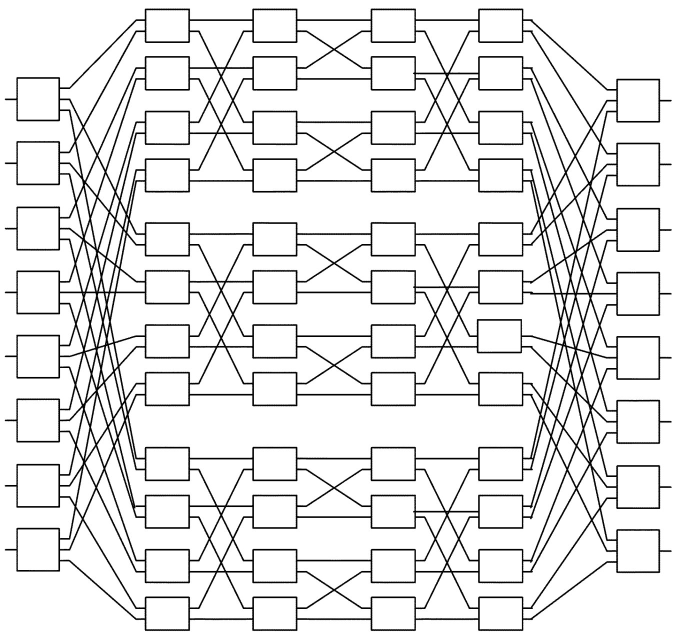

Fig. 2. $\mathrm{Log}_2(8,1,3)$.

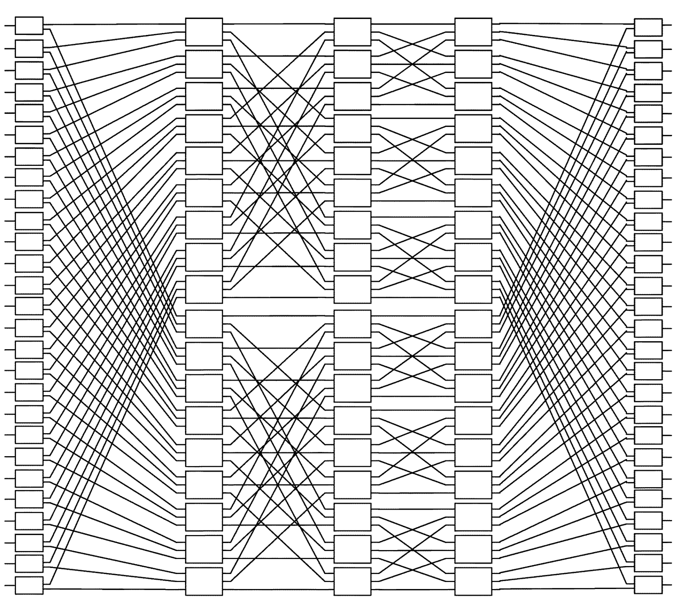

Fig. 3. $\mathrm{Log}_3(27,0,2)$.

In this paper, we give necessary and sufficient conditions for $\mathrm{Log}_d(N,m,p)$ to be f-cast strictly nonblocking for all f, thus generalizing *Theorem 2* in three directions: 1) from binary network to d-ary network; 2) from no extra stage to m extra stages; and 3) from $f = N$ to general f. Our strategy is to deal with the $m = 0$ case first in Section II, and then extend the result to the general m case in Section III. We summarize our findings in Section IV.

II. $\mathrm{LOG}_d(N,0,p)$

Consider a request from input i to output o. Then the (i,o) channel graph is simply the path from i to o consisting of $n+1$ links $L_0, L_1, \ldots, L_n$. A path from $i' \neq i$ to $o' \neq o$ is called a j-intersecting path if it contains L_j. Note that a path can be both j-intersecting and j'-intersecting. An input is called a j-intersecting input if it can start a j-intersecting path. Note that a j-intersecting input is also a j'-intersecting input for $j < j' \leq n-1$. An input is j-marginal if it is j-intersecting but not $(j-1)$-intersecting. Similarly, an output is j-intersecting if it can end a j-intersecting path. A j-intersecting output is also a j'-intersecting output for $1 \leq j' < j$. An output is j-marginal if it is j-intersecting but not $(j+1)$-intersecting. We use the following definitions.

I_j	the set of j-intersecting inputs;
I'_j	the set of j-marginal inputs;
O_j	the set of j-intersecting outputs;
O'_j	the set of j-marginal outputs.

Let $|S|$ denote the cardinality of the set S. Note that in the $\mathrm{Log}_d(N,0,p)$ network, $I_j \subset I_{j+1}$, $O_{j+1} \subset O_j$, and

$$\begin{aligned} |I_j| &= d^j - 1, \quad |I'_j| = d^j - d^{j-1} \\ |O_j| &= d^{n-j} - 1, \quad |O'_j| = d^{n-j} - d^{n-j-1} \\ |I_j| &= d^j - 1 < d^{j+1} - d^j = |I'_{j+1}| \\ |O'_j| &= d^{n-j} - d^{n-j-1} \geq d^{n-j-1} - 1 = |O_{j+1}|. \end{aligned}$$

Take $\mathrm{Log}_2(32,0,3)$ as an example to explain the preceding definitions. Since $\mathrm{Log}_2(32,0,3)$ contains three identical copies of $\mathrm{BY}^{-1}(5,0)$ in the middle, we will only use one middle copy to illustrate the concepts. In Fig. 4, suppose the request to be connected is from input 0 to output 0. Then, $I_1 = \{1\}$, $I_2 = \{1,16,17\}$, $I_3 = \{1,8,9,16,17,24,25\}$, $I_4 = \{1,4,5,8,9,12,13,16,17,20,21,24,25,28,29\}$, $I'_1 = \{1\}$, $I'_2 = \{16,17\}$, $I'_3 = \{8,9,24,25\}$, $I'_4 = \{4,5,12,13,20,21,28,29\}$. $O_1 = \{1-15\}$, $O_2 = \{1-7\}$, $O_3 = \{1-3\}$, $O_4 = \{1\}$, $O'_1 = \{8-15\}$, $O'_2 = \{4-7\}$, $O'_3 = \{2-3\}$, $O'_4 = \{1\}$. Request $(\{1\},\{8\})$ generates a 1-intersecting path, request $(\{12\},\{1\})$ generates a 4-intersecting path.

Lemma 3: If $f \geq d^{n-2j}$, then $(|I'_j| + |I'_{j+1}|)f \geq |O_j|$.

Proof:

$$\begin{aligned} \left(|I'_j| + |I'_{j+1}|\right) f - |O_j| &= (d^{j+1} - d^{j-1})f - (d^{n-j} - 1) \\ &\geq (d^{j+1} - d^{j-1})d^{n-2j} - d^{n-j} + 1 \\ &= d^{n-j-1}(d^2 - d - 1) + 1 > 0. \end{aligned}$$

■

Theorem 4: $\mathrm{Log}_d(N,0,p)$ is f-cast strictly nonblocking for $d^{n-2j} > f \geq d^{n-2j-2}$ if and only if

$$p > (d^j - 1)f + d^{n-j-1} - 1, \quad j = 0, 1, \ldots, \left\lfloor \frac{n-1}{2} \right\rfloor.$$

Proof: Suppose the current request involves x outputs. Since we can connect the x outputs independently one by one, we may assume $x = 1$. Let the current request be (i,o). Note that previous connections from i cannot block (i,o) since they can share links. Therefore, these connections will be ignored in the counting of intersecting paths.

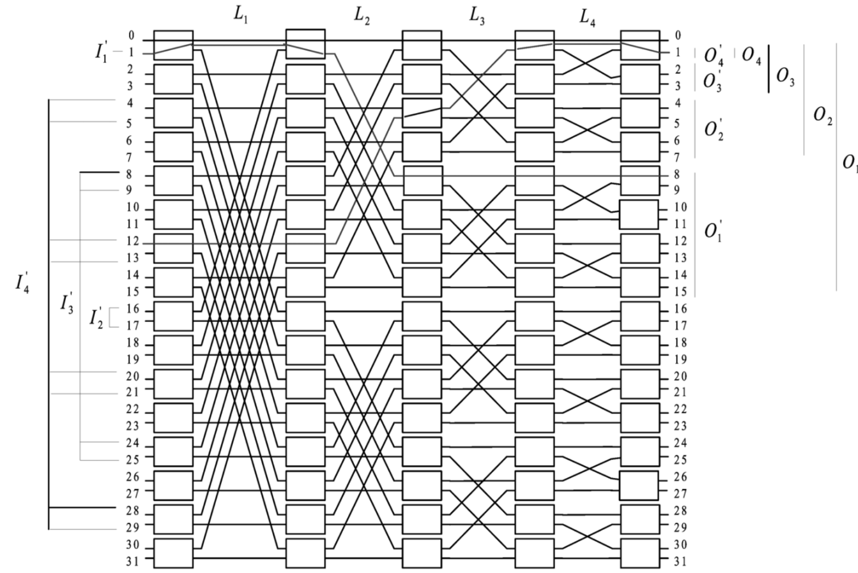

Fig. 4. $\mathrm{BY}^{-1}(5, 0)$.

Suppose $d^{n-2j} > f \geq d^{n-2j-2}$. The upper bound implies

$$f < d^{n-2k}, \quad \text{for } 1 \leq k \leq j.$$

Hence

$$|I'_k| f < (d^k - d^{k-1}) d^{n-2k} = d^{n-k} - d^{n-k-1} = |O'_k|$$

which implies that there are always enough outputs in O'_k to receive requests from I'_k. Thus the maximum number of k-intersecting paths is just $|I'_k| f$.

By *Lemma 3*, the lower bound implies

$$\left(\left|I'_{j+1}\right| + \left|I'_{j+2}\right|\right) f \geq |O_{j+1}|$$

i.e., the combined $(j+1)$-marginal and $(j+2)$-marginal inputs can use up all remaining outputs. Hence there is no need to count intersecting paths beyond L_{j+2}. Note that it does not matter whether I'_{j+1} alone can use up all of O_{j+1}, since in either case there is a total of $|O_{j+1}|$ paths intersecting L_{j+1} and L_{j+2}. Thus the total number of intersecting paths is

$$\begin{aligned}\sum_{k=1}^{j} |I'_k| f + |O_{j+1}| &= |I_j| f + |O_{j+1}| \\ &= (d^j - 1) f + d^{n-j-1} - 1.\end{aligned}$$

This maximum can be achieved since the proof assures the availability of inputs and outputs for all the intersecting paths counted (just make out a request frame according to the description given in the proof). ■

Again use $\mathrm{BY}^{-1}(5, 0)$ as an example. When $f = 8$, consider requests $(\{1\}, \{8-15\})$, $(\{16\}, \{1-7\})$, which will use 15 copies. When $f = 5$, consider requests $(\{1\}, \{8-12\})$, $(\{16\}, \{3-7\})$, $(\{17\}, \{1-2\})$, which will use 12 copies. When $f = 2$, consider requests $(\{1\}, \{8-9\})$, $(\{16\}, \{4-5\})$, $(\{17\}, \{6-7\})$, $(\{8\}, \{2-3\})$, $(\{9\}, \{1\})$, which will use 9 copies.

Setting $f = N - 1$ $(j = 0)$ and $f = 1$ $(j = \lfloor (n-1)/2 \rfloor)$, respectively, in *Theorem 4*, we obtain the following.

Corollary 5: $\mathrm{Log}_d(N, 0, p)$ is broadcast strictly nonblocking if and only if $p > d^{n-1} - 1$.

Corollary 6: $\mathrm{Log}_d(N, 0, p)$ is point-to-point strictly nonblocking if and only if $p > d^{\lfloor (n-1)/2 \rfloor} + d^{\lceil (n-1)/2 \rceil} - 2$.

These two results are, of course, known in the literature, in [5] and [7].

III. LOG$_d(N, m, p)$

We now study the general m extra stages case.

Theorem 7: $\mathrm{Log}_d(N, m, p)$ is f-cast strictly nonblocking if and only if the equation shown at the bottom of the page holds true.

$$p > \begin{cases} \frac{N-1}{d}, & \text{if } f \geq \frac{N-d}{d-1} \\ \frac{(d-1)(f+1)k}{d} + \frac{N-1-(d^k-1)(f+1)}{d^{k+1}}, & \text{if } \frac{N-d^k}{d^k-1} > f \geq \frac{N-d^{k+1}}{d^{k+1}-1}, 1 \leq k \leq m-1 \\ \frac{(d-1)(f+1)m}{d} + \frac{N-1-(d^m-1)(f+1)}{d^m}, & \text{if } \frac{N-d^m}{d^m-1} > f \geq \frac{N-d^{n-1}}{d^m-1} \\ \frac{(d-1)(f+1)m}{d} + d^{n-m-1} - 1, & \text{if } \frac{N-d^{n-1}}{d^m-1} > f \geq d^{n-m-2} \\ \frac{(d-1)(f+1)m}{d} + (d^j-1)f + d^{n-m-j-1} - 1, & \text{if } d^{n-m-2j} > f \geq d^{n-m-2j-2}, 1 \leq j \leq \lfloor \frac{n-m-1}{2} \rfloor \end{cases}$$

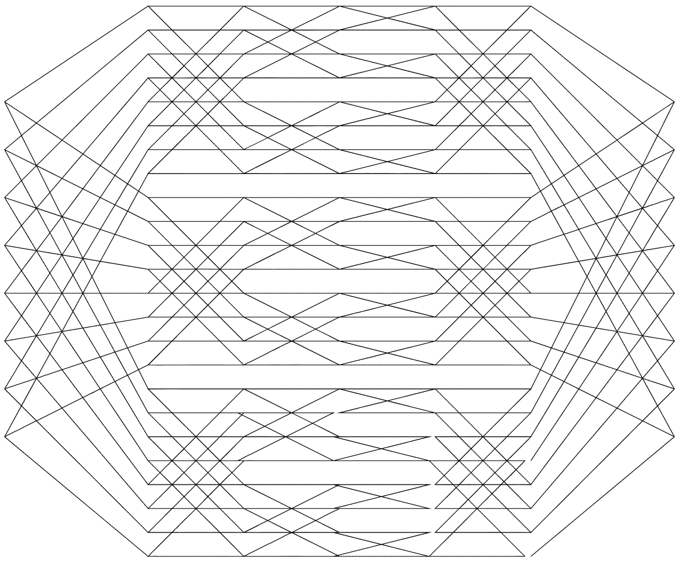

Fig. 5. Channel graph of $\text{Log}_2(8,1,3)$.

Proof: Again, we may assume that the current request is point-to-point and from input i to output o. For $1 \le j \le m$, the channel graph has d^j stage-j and d^j stage-$(n+m-j)$ links in every middle copy. Hence it takes d^j j-intersecting paths to block the channel graph of one middle copy (see Fig. 5). Using an argument of Shyy and Lea [2], each j-intersecting path blocks only $1/d^j$ portion of the channel graph of the middle copy, or blocks $1/d^j$ copy of the $\text{Log}_d(N,m,p)$ network. For $m+1 \le j \le n-1$, the channel graph of every middle copy has d^m links; hence each j-intersecting path blocks $1/d^m$ copy.

Let I be the set of all inputs except i, and O all outputs except o. Notice that an intersecting path originated from I_j, $j = 1, 2, \ldots, m$, can arrive at every output of O. Similarly, any intersecting path to O_{n+m-j}, $j = 1, 2, \ldots, m$, can start from any input of I. Furthermore, requests from I'_j, or to O'_{n+m-j}, $j = 1, 2, \ldots, m$, can block $1/d^j$ copy, as mentioned above. In the worst case, requests from I'_j or to O'_{n+m-j} will take priority over connections from I'_{j+1}, or to $O'_{n+m-j-1}$ for $j = 1, 2, \ldots, m-1$, since they have stronger blocking ability. In the $\text{Log}_d(N,m,p)$ network, we have $I_j = I$, for $n \le j \le m+n-1$, and $O_j = O$, for $1 \le j \le m$

$$|I_j| = \begin{cases} d^j - 1, & \text{if } j = 1, 2, \ldots, n-1 \\ d^n - 1, & \text{if } j = n, n+1, \ldots, m+n-1 \end{cases}$$
$$|O_j| = \begin{cases} d^n - 1, & \text{if } j = 1, 2, \ldots, m \\ d^{n+m-j} - 1, & \text{if } j = m+1, \ldots, n+m-1. \end{cases}$$

Note that

$$\begin{aligned} |I_j| f + |O_{n+m-j}| &= (d^j - 1) f + d^j - 1 \\ &= (d^j - 1)(f+1), \quad \text{for } j = 1, 2, \ldots, m. \end{aligned}$$

We proceed by counting intersecting paths in the order L_1 and L_{n+m-1}, L_2 and $L_{n+m-2}, \ldots, L_m$ and L_n, $L_{m+1}, L_{m+2}, \ldots, L_{n-1}$, and stop whenever the counted intersecting paths have used up the remaining outputs. The proof is partitioned into cases depending on when the remaining outputs are used up.

1) $f \ge (N-d)/(d-1)$.
Then $|I_1| f + |O_{n+m-1}| = (d-1)(f+1) \ge N-1$, which means the f-cast requests from I_1 and to O_{n+m-1} can use up O, and the number of blocked copies is $(N-1)/d$.

2) $(N-d^k)/(d^k-1) > f \ge (N-d^{k+1})/(d^{k+1}-1), 1 \le k \le m-1$.
The upper bound and the lower bound are equivalent to

$$\begin{aligned} |I_k| f + |O_{n+m-k}| &< |O| \\ |I_{k+1}| f + |O_{n+m-k-1}| &\ge |O|. \end{aligned}$$

Thus, I'_j and O'_{n+m-j}, $j = 1, \ldots, k$, all together will block

$$\sum_{j=1}^{k} |I'_j| f \frac{1}{d^j} + \sum_{j=1}^{k} |O'_{n+m-j}| \frac{1}{d^j} = \frac{d-1}{d}(f+1)k$$

copies. The remaining outputs will be used up by requests from $|I'_{k+1}|$ or to $|O'_{n+m-k-1}|$, each such intersecting path blocks $1/d^{k+1}$ copy. Therefore, the total number of blocked copies is

$$\frac{(d-1)(f+1)}{d}k + \frac{N-1-(d^k-1)(f+1)}{d^{k+1}}.$$

3) $(N-d^m)/(d^m-1) > f \ge (N-d^{n-1})/(d^m-1)$.
The upper bound and the lower bound are equivalent to

$$\begin{aligned} |I_m| f + |O_n| &< |O| \\ |I_m| f &\ge |O| - |O_{m+1}| = |O'_m|. \end{aligned}$$

Hence, the requests from I_m and to O_n cannot use up O, but the requests from I_m can overflow from O'_m to O_{m+1}. Further

$$\begin{aligned} |I_{m+1}| f + |O_n| &\ge (d^{m+1} - 1)\frac{N - d^{n-1}}{d^m - 1} + d^m - 1 \\ &> d(N - d^{n-1}) + d^m - 1 \\ &\ge d^n + d^m - 1 \\ &> N - 1 = |O|. \end{aligned}$$

Hence there is no need to count beyond stage $m+1$. Thus the total number of blocked copies is

$$\begin{aligned} &\sum_{j=1}^{m} |I'_j| f \frac{1}{d^j} + \sum_{j=1}^{m} |O'_{n+m-j}| \frac{1}{d^j} + (|O| - |I_m| f - |O_n|)\frac{1}{d^m} \\ &\quad = \frac{(d-1)(f+1)}{d}m + \frac{N-1-(d^m-1)(f+1)}{d^m}. \end{aligned}$$

4) $(N-d^{n-1})/(d^m-1) > f \ge d^{n-m-2}$.
The upper bound is equivalent to

$$|I_m| f < |O'_m|.$$

Hence, requests from I_m do not overflow to O_{m+1}. The lower bound implies that, by setting $n = n+m$ and $j = m+1$ in *Lemma 3*

$$\left(|I'_{m+1}| + |I'_{m+2}|\right) f \ge |O_{m+1}|$$

i.e., all outputs available at step $m+1$ will be used up by the next step. Again, it does not matter whether they are

$$p > \begin{cases} \frac{N-1}{d}, & \text{if } f \geq \frac{N-d}{d-1} \\ \frac{(d-1)(f+1)k}{d} + \frac{N-1-(d^k-1)(f+1)}{d^{k+1}}, & \text{if } \frac{N-d^k}{d^k-1} > f \geq \frac{N-d^{k+1}}{d^{k+1}-1}, 1 \leq k \leq n-2 \\ \frac{(d-1)(f+1)(n-1)}{d}, & \text{if } \frac{N-d^{n-1}}{d^{n-1}-1} > f \end{cases}$$

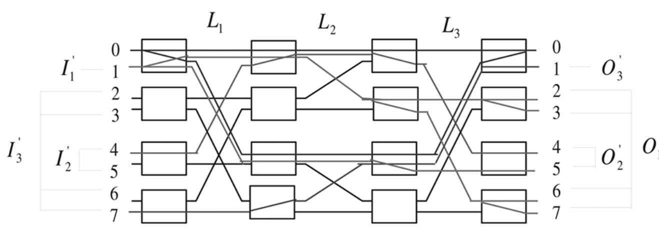

Fig. 6. $\mathrm{BY}^{-1}(3,1)$.

used up at step $m+1$. Thus the total number of blocked copies is

$$\sum_{j=1}^{m} |I_j'| f \frac{1}{d^j} + \sum_{j=1}^{m} |O_{n+m-j}'| \frac{1}{d^j} + |O_{m+1} - O_n| \frac{1}{d^m} = \frac{d-1}{d}(f+1)m + d^{n-m-1} - 1.$$

5) $d^{n-m-2j} > f \geq d^{n-m-2j-2}, 1 \leq j \leq \lfloor (n-m-1)/2 \rfloor$.

The middle $n-m$ stages consist of d^m copies of $\mathrm{BY}_d^{-1}(n-m, 0)$, which we will refer to as the reduced inverse banyan networks. The upper bound of f says that the outputs of these reduced inverse banyan networks are intact, i.e., none of them is used by requests from I_m. Hence, we can apply *Theorem 4* with n replaced by $n-m$ everywhere. Note that for stage j in a reduced inverse banyan network

$$|I_j| = d^{m+j} - d^m = d^m(d^j - 1)$$
$$|O_{n-j}| = d^{n+m-j} - d^m = d^m(d^{n-j} - 1)$$

which are d^m times of a normal $\mathrm{BY}_d^{-1}(n-m, 0)$. However, to block a copy of $\mathrm{BY}_d^{-1}(n, m)$ takes the blocking of d^m copies of the reduced inverse banyan networks. So the net effect of blocking in the reduced inverse banyan network is same as in the normal $\mathrm{BY}_d^{-1}(n-m, 0)$, and *Theorem 4* applies.

Again, the worst case described above can be achieved, since the description in the proof assures the availability of the inputs and outputs counted in the intersecting paths. ∎

Note that in the first three cases, outputs in O are used up. In the last two cases, O_k' may not be used up by I_k, but are not available for $I_{k+1}', I_{k+2}', \ldots, I_{n-1}'$.

Let us take Fig. 6 as an example to see the concept of overflow. In the figure, we have $N = 8$, $m = 1$, $n = 3$. Setting $f = 5$, then it is the third case, that is, requests from I_1 and to O_3 cannot use up O, but requests from I_1 can overflow from O_1' to O_2. In the figure, input 1 generates requests to outputs $(\{2, 3, 5, 6, 7\})$, which include one output of O_2.

Again, setting $f = N-1$ and $f = 1$, respectively, we obtain the following.

Corollary 8: $\mathrm{Log}_d(N, m, p)$ is broadcast strictly nonblocking if and only if $p > d^{n-1} - 1$.

Corollary 9: $\mathrm{Log}_d(N, m, p)$ is point-to-point strictly nonblocking if and only if

$$p > \frac{2m(d-1)}{d} + d^{\lfloor (n-m-1)/2 \rfloor} + d^{\lceil (n-m-1)/2 \rceil} - 2.$$

Setting $m = n-1$, we obtain the following.

Corollary 10: The d-ary Cantor network is f-cast strictly nonblocking if and only if the equation shown at the top of the page holds true.

IV. Conclusion

Recently, Kabacinski and Danilewicz gave necessary and sufficient conditions for $\mathrm{Log}_2(N, 0, p)$ to be broadcast strictly nonblocking. We extended it to $\mathrm{Log}_d(N, m, p)$. Further, we obtained the surprising result that the conditions are independent of m (*Corollary 8*).

Bassalygo and Pinsker [8] proved that a strictly nonblocking broadcast network contains at least $O(N^2)$ crosspoints, not fewer than those of an $N \times N$ crossbar. Thus the only hope is to construct efficient f-cast strictly nonblocking networks.

We gave necessary and sufficient conditions for $\mathrm{Log}_d(N, m, p)$ to be f-cast strictly nonblocking for every f, thus containing the point-to-point $(f = 1)$ and broadcast $(f = N)$ as special cases. Note that the number of copies of $\mathrm{BY}^{-1}(n, m)$ in the middle decreases rapidly with f. For example, the number is d^{n-1} for $f = N$, and is the minimum integer larger than

$$\frac{2m(d-1)}{d} + d^{\lfloor (n-m-1)/2 \rfloor} + d^{\lceil (n-m-1)/2 \rceil} - 1$$

for $f = 1$. In particular, we obtain necessary and sufficient conditions for the Cantor network, i.e., $\mathrm{Log}_d(N, n-1, p)$, to be f-cast strictly nonblocking. Though we get the strictly nonblocking condition as above, it only guarantees the existence of a path. How to find it efficiently is still an issue, and we will take it for future research.

References

[1] C.-T. Lea, "Multi-$\log_2 N$ networks and their applications in high-speed electronic and photonic switching systems," *IEEE Trans. Commun.*, vol. 38, no. 10, pp. 1740–1749, Oct. 1990.

[2] D.-J. Shyy and C.-T. Lea, "$\log_2(N, m, p)$ strictly nonblocking networks," *IEEE Trans. Commun.*, vol. 39, no. 10, pp. 1502–1510, Oct. 1991.

[3] D. G. Cantor, "On non-blocking switching networks," *Networks*, vol. 1, pp. 367–377, 1971–1972.

[4] V. E. Beneš, "Mathematical theory of connecting networks and telephone traffic," in *Mathematics in Science and Engineering*. New York: Academic, 1965, vol. 17.

[5] F. Hwang, "Choosing the best $\log_d(N, m, P)$ strictly nonblocking networks," *IEEE Trans. Commun.*, vol. 46, no. 4, pp. 454–455, Apr. 1998.

[6] Y. Tscha and K. Lee, "Yet another result on multi-$\log_2 N$ networks," *IEEE Trans. Commun.*, vol. 47, no. 9, pp. 1425–1431, Sep. 1999.

[7] W. Kabacinski and G. Danilewicz, "Wide-sense and strict-sense nonblocking operation of multicast multi-$\log_2 N$ switching networks," *IEEE Trans. Commun.*, vol. 50, no. 6, pp. 1025–1036, Jun. 2002.

[8] L. A. Bassalygo and M. S. Pinsker, "Asymptotically optimal networks for generalized rearrangeable switching and generalized switching without rearrangements," in *Problemy Peredači Informacii*, 1980, vol. 16, pp. 94–98.

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