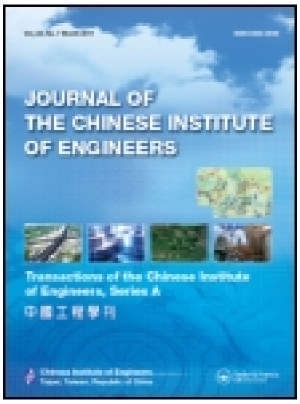

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A filter-based self-similar trace synthesizer

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A FILTER-BASED SELF-SIMILAR TRACE SYNTHESIZER

Chien Yao*, Kai-Lung Hua, Po-Ning Chen, Jin-Yuan Chen, and Tihao Chiang

ABSTRACT

Recent empirical studies have shown that modern computer network traffic is much more appropriately modelled by long-range dependent self-similar processes than traditional short-range dependent processes such as Poisson. Thus, if its self-similar nature is not considered in the synthesis of experimental network data, incorrect performance assessments for network systems may result. This raises the need of a self-similar trace synthesizing algorithm with long-range dependence. In this paper, we propose and examine the feasibility of a filter-based method for the synthesis of self-similar network traces. The proposed approach can alleviate the problems encountered by conventional synthesizers, such as *random midpoint displacement* and *Paxson's spectrum fitting*, which cannot generate self-similar traces on the fly and may give negative numbers. Additionally, the extended range of self-similarity of the filtered approach can be easily managed by the filter truncation window; therefore, a trace that faithfully matches the measured behavior of true network traffic, where the self-similar nature only lasts beyond a certain range but disappears as the considered aggregated window is much further extended, can be generated.

Key Words: self-similar processes, variance-time analysis, filter technique.

I. INTRODUCTION

Stationary random processes, according to their autocorrelation functions, can be classified as *short-range* dependent random processes or *long-range* dependent random processes. The former have summable autocorrelation functions, while the latter have non-summable autocorrelation functions. Simulations of short-range dependent random processes have attracted attention for years, and have found many applications, such as the traffic models of telecommunication systems (Bose, 2001). However, researchers recently found that the traffic in many modern communication media, such as the world wide web (Beran *et al*., 1995; Duffy *et al.*, 1994; Leland *et al*., 1994; Meier-Hellstern *et al*., 1991; Paxson and Floyd, 1995), and variable-bit-rate (VBR) video transmission (Garrett and Willinger, 1994), are significantly different from conventional shortrange dependent traffic models, and have the renowned self-similar nature.

In the literature, there have been several approaches proposed for the synthesis of long-rangedependent self-similar traffic. They include methods based on fast fractional Gaussian noise (Leland *et al*., 1994), the $M/G/\infty$ queue model (Krunz and Makowski, 1998), autoregressive processes (Beran, 1994), wavelet (Arby and Veitch, 1998), ···, etc. These synthesizers can be roughly divided into two categories: approaches derived from a "time-domain" aspect and ones developed from a "frequency-domain" standpoint. An example for the former is the randommidpoint displacement (RMD) algorithm proposed by Lau *et al*. (1995), while the spectrum fitting to the fractional Gaussian noise, as proposed by Paxon (1997), is a typical synthesizer for the latter type.

The procedure of the RMD algorithm is to recursively subdivide the present time intervals, and generate in each subdivision a new mid-point traffic data based on the end-point data obtained in the previous subdivision. This method can efficiently generate a well-approximated fractal Brownian motion

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(FBM) sequence. It however comes with the drawbacks that only the FBM traffic can be synthesized, and the desired amount of traffic has to be specified in advance.

Based on the power spectrum fitting to the fractional Gaussian noise (FGN), Paxson proposed a fast self-similar traffic generator using the inverse discrete-time Fourier transform (IDTFT), which is usually referred as the FFT method. By using an approximated form of the spectrum density of fractal Gaussian noises (FGN), a random sequence is formed in the frequency domain. An inverse Fourier transformation (IFFT) is then performed to transform the sequence from the frequency domain to the time domain. The FFT algorithm improves the RMD algorithm in speed. In particular, the FFT algorithm only takes half time of the RMD algorithm for the same sequence length. Again, its drawback is that the traffic sequence cannot be generated on the fly. In addition, the simplified form of the FGN spectrum causes the resultant degree of self-similarity to deviate from the target degree.

In applying the aforementioned approaches to the generation of self-similar traces, several problems can be encountered. First, the required length (i.e., amount) of traffic data must be previously determined; hence, when a longer traffic sequence is required, one has to drop the existing data, and re-generate a completely new trace of the required length. Secondly, the required traffic data must be generated in an *off-line* fashion before they can be put to use. This somewhat restricts their usage in situations where *on-the-fly* traffic synthesizers are needed. Thirdly, these traffic generators may produce negative numbers, which is an undesired value for, say, packet-train arrivals.

In this work, we propose a model that can produce long-range dependent sequences with adjustable levels of burstiness and correlation. When compared to the two known self-similar traffic generators—the RMD and the Paxson FFT, our model provides the additional advantages that the synthetic traffic can be generated on the fly, and is always non-negative. Although the variance-time analysis shows that the filter length W limits the valid aggregation size of selfsimilarity, this phenomenon turns out to match the measured behavior of true network traffic, where the self-similar nature only lasts beyond a practically manageable range, but disappears as the considered aggregated window is much further extended (Beran *et al*., 1995).

This paper is organized as follows. Section II briefs the necessary background of secondorder self-similar processes. Section III introduces the proposed filter-based self-similar trace synthesizer, and examines the degree of its self-similarity by variance-time analysis. The effect due to filter truncation and filter output rounding is subsequently investigated. Comparison between the use of the forward filter and that of the reverse filter is provided in Section IV. Section V concludes this work.

II. PRELIMINARIES

Self-similar processes were first introduced by Mandelbrot and his co-workers in 1968 (Mandelbrot and Van Ness, 1968; Mandelbrot and Wallis, 1969; Mandelbrot, 1971). These processes thereafter found applications in many fields, such as astronomy, chemistry, economics, engineering, mathematics, physics, statistics, etc. Recently, measurement studies have shown that the actual traffic from computer networks is long-range dependent (Meier-Hellstern *et al*., 1991; Duffy *et al*., 1994; Leland *et al*., 1994; Beran e*et al*., 1995; Paxson and Floyd, 1995), and thus another new application for self-similar processes was initiated.

Assume a second-order stationary real-valued stochastic process $\boldsymbol{Y} \triangleq \{Y_i\}_{i \in I_1}$ with finite marginal mean μ and marginal variance σ^2, where $I_j \triangleq \{j, j+1, j+2, \cdots\}$. Denoted by $\boldsymbol{Y}^{(m)} \triangleq \{Y_i^{(m)}\}_{i \in I_1}$ the m-averaged process of $\boldsymbol{Y}$, where for $m, i \in I_1$,

$$Y_i^{(m)} \triangleq \frac{1}{m}\sum_{j=1}^{m} Y_{m(i-1)+j}\,.$$

Let the autocovariance and autocorrelation coefficient function of the m-averaged process $\boldsymbol{Y}^{(m)}$ be denoted by $C_m(k) \triangleq \mathrm{Cov}\{Y_i^{(m)}, Y_{i+k}^{(m)}\}$ and $\rho_m(k) \triangleq C_m(k)/C_m(0)$, respectively. Then, several variants of self-similarities can be defined as follows.

Definition 1. (Tsybakov and Georganas, 1998, Def. A) A second-order stationary process $\boldsymbol{Y}$ is called *exactly second-order self-similar* with parameter $H = 1 - (\beta/2)$, where $0 < \beta < 1$, if

$$\rho_1(k) = \frac{1}{2}[|k+1|^{2H} - 2|k|^{2H} + |k-1|^{2H}] \text{ for } k \in I_1.$$

Definition 2. (Tsybakov and Georganas, 1998, Def. D) A second-order stationary process $\boldsymbol{Y}$ is called *asymptotically second-order self-similar* with parameter $H = 1 - (\beta/2)$, where $0 < \beta < 1$, if

$$\lim_{m\to\infty} \rho_m(k) = \frac{1}{2}[|k+1|^{2H} - 2|k|^{2H} + |k-1|^{2H}] \quad \text{for } k \in I_1.$$

The parameter H in the above definitions is usually referred to as the Hurst parameter.

III. FILTER-BASED ASYMPTOTIC SELF-SIMILAR TRAFFIC SYNTHESIZER

In this section, we propose and prove that an asymptotic self-similar traffic can be theoretically synthesized through a filter technique with simple transfer function of infinite order. In its feasible realization, the filter of *infinite* order has to be truncated to a *finite* impulse response (FIR) filter. The resultant degradation due to filter truncation in asymptotic self-similar degree is subsequently examined.

1. Transfer Function in Self-Similar Traffic Synthesizer

Let $S_y(\omega)$ denote the power spectrum of discrete random process $\boldsymbol{Y}$ obtained by passing the random process $\boldsymbol{X}$ with power spectrum $S_x(\omega)$ through a filter with transfer function $H(\omega)$ as shown in Fig. 1. Elementary filtering theory immediately gives that $S_y(\omega) = |H(\omega)|^2 S_x(\omega)$. Accordingly, if $\boldsymbol{X}$ is i.i.d., and $|H(\omega)|^2$ well-approximates the power spectrum of an asymptotic self-similar traffic, then the filter output straightforwardly become self-similar, and can be obtained through $Y_n = X_n * h[n]$, where "*" denotes the convolution operator.

By Definition 2, the ultimate autocorrelation coefficient function of an asymptotic secondorder self-similar process with parameter H equals $\frac{1}{2}[|k+1|^{2H} - 2|k|^{2H} + |k-1|^{2H}]$ for $k \in I_1$, which gives a power spectrum $\sin(\pi H) \cdot \Gamma(2H+1) \cdot |1 - e^{-j\omega}|^2 \Sigma_{k=-\infty}^{\infty} |\omega + 2\pi k|^{-1-2H}$ for $-\pi \le \omega < \pi$, where $\Gamma(\cdot)$ is the Euler gamma function defined as $\Gamma(n) \triangleq \int_0^{\infty} t^{n-1} e^{-t} dt$. Since the asymptotic self-similar behavior of a process is only sensitive to those ω values around the origin (Paxson, 1997), we can replace the above infinite sum by its main term at $k = 0$, and yield $\sin(\pi H) \cdot \Gamma(2H+1) \cdot |1 - e^{-j\omega}|^2 \cdot |\omega|^{-1-2H}$ for $-\pi \le \omega < \pi$. We then observe that $|\omega|$ can be well-approximated by $|1 - e^{-j\omega}|$ when $|\omega|$ is small. As a consequence, our proposed filter output spectrum becomes $S_y(\omega) = |1 - e^{-j\omega}|^{1-2H}$ for $-\pi \le w < \pi$, where the coefficients, $\sin(\pi H) \cdot \Gamma(2H+1)$, are removed for analytical simplicity.

One may question whether such an extensive simplification to the target second-order selfsimilar spectrum may already remove its self-similar nature. However, it can be derived from Theorem 2.1(ii) in (Beran, 1994) and from the below equation,

$$\lim_{|\omega| \downarrow 0} \frac{S_y(\omega)}{|\omega|^{1-2H}} = \lim_{|\omega| \downarrow 0} \frac{|1 - e^{-j\omega}|^{1-2H}}{|\omega|^{1-2H}}$$

$$= \lim_{|\omega| \downarrow 0} \frac{(2|\sin(\omega/2)|)^{1-2H}}{|\omega|^{1-2H}} = 1 ,$$

that the autocorrelation function $C_1(k)$ of the filter output process $\boldsymbol{Y}$ with power spectrum $S_y(\omega) = |1 - e^{-j\omega}|^{1-2H}$ satisfies

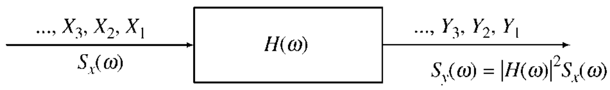

Fig. 1 Relation between the power spectral densities of the filter input and filter output random processes

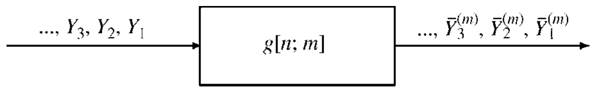

Fig. 2 The variance-equivalent *m*-averaged process

$$\lim_{k \to \infty} \frac{C_1(k)}{2\Gamma(2-2H)\sin(\pi H - \pi/2)k^{2H-2}} = 1 .$$

Thus, from Tsybakov and Georganas (1998, Thm. 3 (2)), the marginal variance $C_m(0)$ of the m-averaged process of the filter output process satisfies

$$\lim_{m \to \infty} \frac{C_m(0)}{C_1(0)m^{2H-2}} = \frac{2\Gamma(2-2H)\sin(\pi H - \pi/2)}{H(2H-1)} .$$

This implies that for a large number of m, $\log[C_m(0)/C_1(0)]$ behaves asymptotically as $(2H-2)\log(m) + \log[2\Gamma(2-2H)\sin(\pi H - \pi/2)/(H(2H-1))]$. Therefore, the filter output process is asymptotic self-similar with parameter H from the aspect of variance-time analysis, when the average window m is large.

A somewhat surprising result is that the designed filter output process $\boldsymbol{Y}$ is also quite "selfsimilar" for small m. In other words, $\boldsymbol{Y}$, in spite of its simple power spectrum formula, behaves close to an *exact* self-similar process from the aspect of variance-time analysis. This can be numerically verified as follows.

The self-similar nature of the filter output process at small m can be established by analyzing the marginal variance of its *variance-equivalent m-averaged process*. A *variance-equivalent m-average process* $\bar{Y}_1^{(m)}, \bar{Y}_2^{(m)}, \bar{Y}_3^{(m)}, \cdots$ of a random process $Y_1, Y_2, Y_3, \cdots$ is its output process through the filter $g[n;m] \triangleq (1/m) \cdot \mathbf{I}\{0 \le n < m\}$, where $\mathbf{I}\{\cdot\}$ is the indicator function that equals one if the event concerned is true, and zero, otherwise (cf. Fig. 2). It is named the *variance-equivalent m-averaged process* because its marginal variance is equal to that of the m-average process $\boldsymbol{Y}^{(m)}$.

The autocovariance function $\bar{C}_m(k)$ of the variance-equivalent m-averaged process can be given by:

$$\bar{C}_m(k) = E[\bar{Y}_{i+k}^{(m)} \bar{Y}_i^{(m)}]$$

$$= E[(\frac{Y_{(i+k)+1} \cdots Y_{(i+k)+m}}{m})(\frac{Y_{i+1} \cdots Y_{i+m}}{m})]$$

$$= \sum_{i=-\infty}^{\infty} \bar{C}_1(i) \cdot \pi(k-i) ,$$

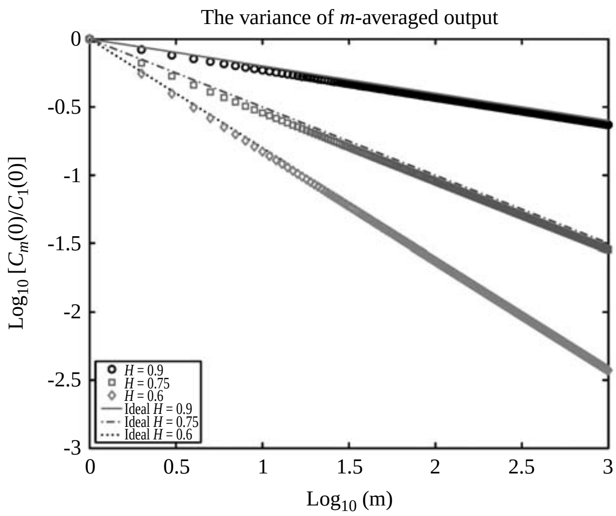

Fig. 3 The variance-time analysis of the filter output process

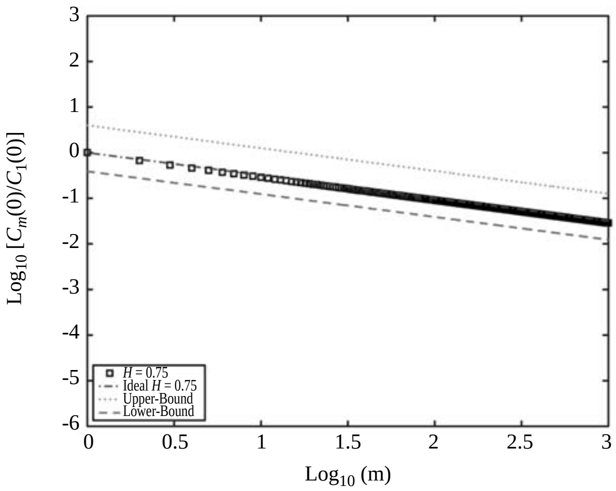

Fig. 4 The lower and the upper bounds of $\log[C_m(0) = C_1(0)]$

where

$$\pi(i) \triangleq \frac{m - |i|}{m^2} \mathbf{1}\{|i| \le m\}.$$

Thus, the power spectrum of the variance-equivalent m-averaged process is equal to

$$S_y(\omega)\frac{\sin^2(m\omega/2)}{m^2\sin^2(\omega/2)}$$

and the variance of the m-averaged process of $\boldsymbol{Y}$ is given by:

$$C_m(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} S_y(\omega)\frac{\sin^2(m\omega/2)}{m^2\sin^2(\omega/2)}d\omega$$

$$= \frac{2^{2-2H}}{\pi}\int_0^{\pi/2} \frac{\sin^2(m\omega)}{m^2\sin^{2H+1}(\omega)}d\omega\,,$$

which immediately gives:

$$\log\frac{C_m(0)}{C_1(0)} = \log\frac{\displaystyle\int_0^{\pi/2}\frac{\sin^2(m\omega)}{m^2\sin^{2H+1}(\omega)}d\omega}{\displaystyle\int_0^{\pi/2}\sin^{1-2H}(\omega)d\omega}$$

$$= \log\frac{2\Gamma(1.5-H)\displaystyle\int_0^{\pi/2}\frac{\sin^2(m\omega)}{m^2\sin^{2H+1}(\omega)}d\omega}{\Gamma(1-H)\sqrt{\pi}}\,.$$

Based on the above formula, we depict the relation between $\log[C_m(0)/C_1(0)]$ and $\log(m)$ in Fig. 3, and observe a perfect self-similarity from the aspect of variance-time analysis even for very small m.

In fact, we can analytically obtain lower and upper bounds that hold for every m for $\log[C_m(0)/C_1(0)]$ through two inequalities

$$\int_0^{\pi/2}\frac{\sin^2(m\omega)}{m^2\sin^{2H+1}(\omega)}d\omega \ge m^{2H-2}\frac{(2/\pi)^{2H}}{2(1-H)}$$

and

$$\int_0^{\pi/2}\frac{\sin^2(m\omega)}{m^2\sin^{2H+1}(\omega)}d\omega$$

$$\le m^{2H-2}\frac{(1+2H\pi)[2^{-2H}-(1-H)]\pi^2}{8H^2(2H-1)(1-H)}$$

and they again confirm the almost perfect self-similarity of the filter output process (cf. Fig. 4).

After the verification of self-similarity of the filter output process, it remains to design a filter whose output spectrum due to an i.i.d. input of unity power spectrum equals $S_y(\omega)$, or specifically, $|H(\omega)|^2 = |1 - e^{-j\omega}|^{1-2H}$. First, we note that the z-transforms, $X(z)$ and $Y(z)$, of the filter input and output can be characterized by $(1 - z^{-1})^{-a}X(z) = Y(z)$, where $a \triangleq (2H - 1)/2$. By Taylor's expansion, we obtain:

$$(1-z)^{-a} = 1 + \frac{a}{1!}z + \frac{a(a+1)}{2!}z^2 + \cdots$$

$$= \sum_{n=0}^{\infty}\frac{\Gamma(n+a)}{\Gamma(n+1)\Gamma(a)}z^n\,.$$

Therefore, the outputs $y[1]$, $y[2]$, $y[3]$ $\cdots$ can be obtained through

$$y[n] = \sum_{k=0}^{\infty}\frac{\Gamma(k+a)}{\Gamma(k+1)\Gamma(a)}x[n-k] = \sum_{k=0}^{\infty}h[k]\cdot x[n-k]\,,$$

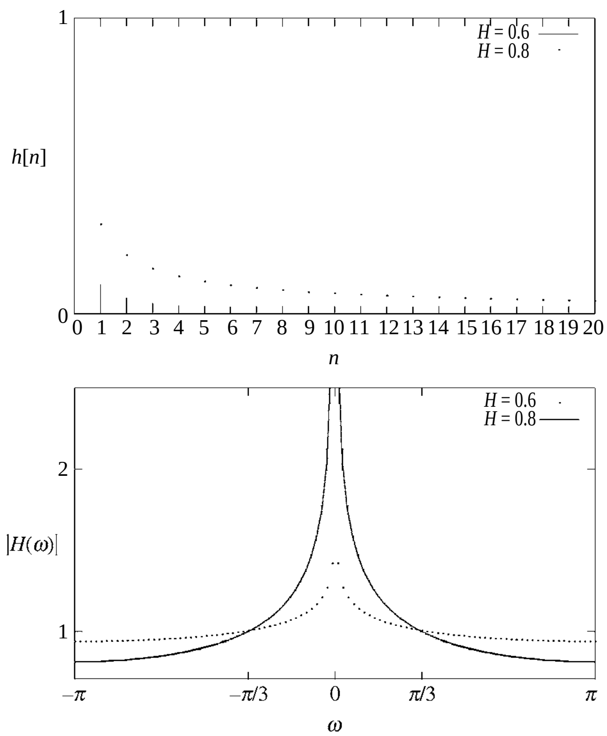

Fig. 5 The filter impulse response $h[n]$ and its frequency response $|H(\omega)|$

where

$$h[n] \triangleq \frac{\Gamma(n+a)}{\Gamma(n+1)\Gamma(a)} = \frac{\Gamma(n+H-0.5)}{\Gamma(n+1)\Gamma(H-0.5)} \quad \text{for } k \geq 0.$$

The filter impulse response $h[n]$, as well as its frequency response $|H(\omega)| = |1 - e^{-j\omega}|^{-(2H-1)/2} = (2|\sin(\omega/2)|)^{-(2H-1)/2}$, is plotted in Fig. 5.

Two problems will be encountered when one wishes to synthesize a self-similar network packet-arrival traffic in terms of the proposed filter system. First, it is of infeasibly infinite length. Secondly, the filter outputs are in general non-integer-values even if the filter inputs are integer-values. Modifications such as filter truncation to finite length and rounding to the nearest integers are therefore necessary. We will numerically examine the impact on self-similarity due to filter truncation and output rounding in the later subsections.

2. Impact on Self-Similarity Due to Filter Truncation

Define $h[k; W] \triangleq h[k] \cdot \mathbf{1}\{0 \leq k < W\}$. Then, the impact of the truncation window size W on the degree of self-similarity of the filter output process can be characterized through the derivation of the marginal variance $C_m(0; W)$ of the respective m-averaged

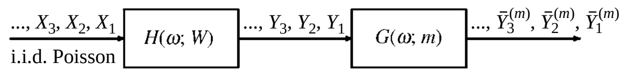

Fig. 6 The *variance-equivalent m-averaged process* of the truncated filter output process

filter output process, as illustrated in Fig. 6. Again, we derive $C_m(0; W)$ through the help of the technique of the variance-equivalent m-average process.

Let $G(\omega; m)$ be the transfer function of the filter $g[n; m]$, and let $L(\omega; W; m) \triangleq H(\omega; W)G(\omega; m)$. Then,

$$\ell[n; W, m] = \sum_{i=0}^{n} g[i; m] \times h[n-i; W]$$

$$= \frac{1}{m} \sum_{i=\max\{0,\, n-W+1\}}^{\min\{n,\, m-1\}} h[n-i] \,.$$

By letting $S_y(\omega; W)$ be the truncated counterpart of $S_y(\omega)$, we obtain:

$$C_m(0; W)$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} S_y(\omega; W)d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} [\sum_{n=0}^{\infty} \ell[n]e^{-jn\omega}]\,[\sum_{n=0}^{\infty} \ell[n]e^{jn\omega}]d\omega$$

$$= \sum_{n=0}^{\infty} |\ell[n]|^2$$

$$= \frac{1}{m^2}\{\sum_{l=0}^{W-m} (\sum_{n=l}^{l+m-1} h[n])^2$$

$$+ \sum_{l=0}^{m-2} [(\sum_{n=0}^{l} h[n])^2 + (\sum_{m=W-1-l}^{W-1} h[n])^2]\} \,.$$

Based on the above formula, we numerically depict $\log_{10}[C_m(0; W)/C_1(0; W)]$ versus $\log_{10}(m)$ in Figs. 7 and 8, and observe that there are two apparently different self-similar behaviors for different m values. The resultant degree of self-similarity is close to the target one when $m \leq W$, but the slope of the variance-time curve quickly turns to a non-self-similar value, -1, once $m > W$. This result indicates that the degree of self-similarity of the network trace synthesized through truncated $h[n]$ can be well controlled by adjusting the truncation window W.

3. Impact on Self-Similarity Due to Output Rounding

In this subsection, we further empirically examine the output rounding effect on self-similarity. Table I lists the resultant Hurst parameter of the trace synthesized according to the system in Fig. 9. It indicates that the rounding-to-the-nearest-integer operation at the

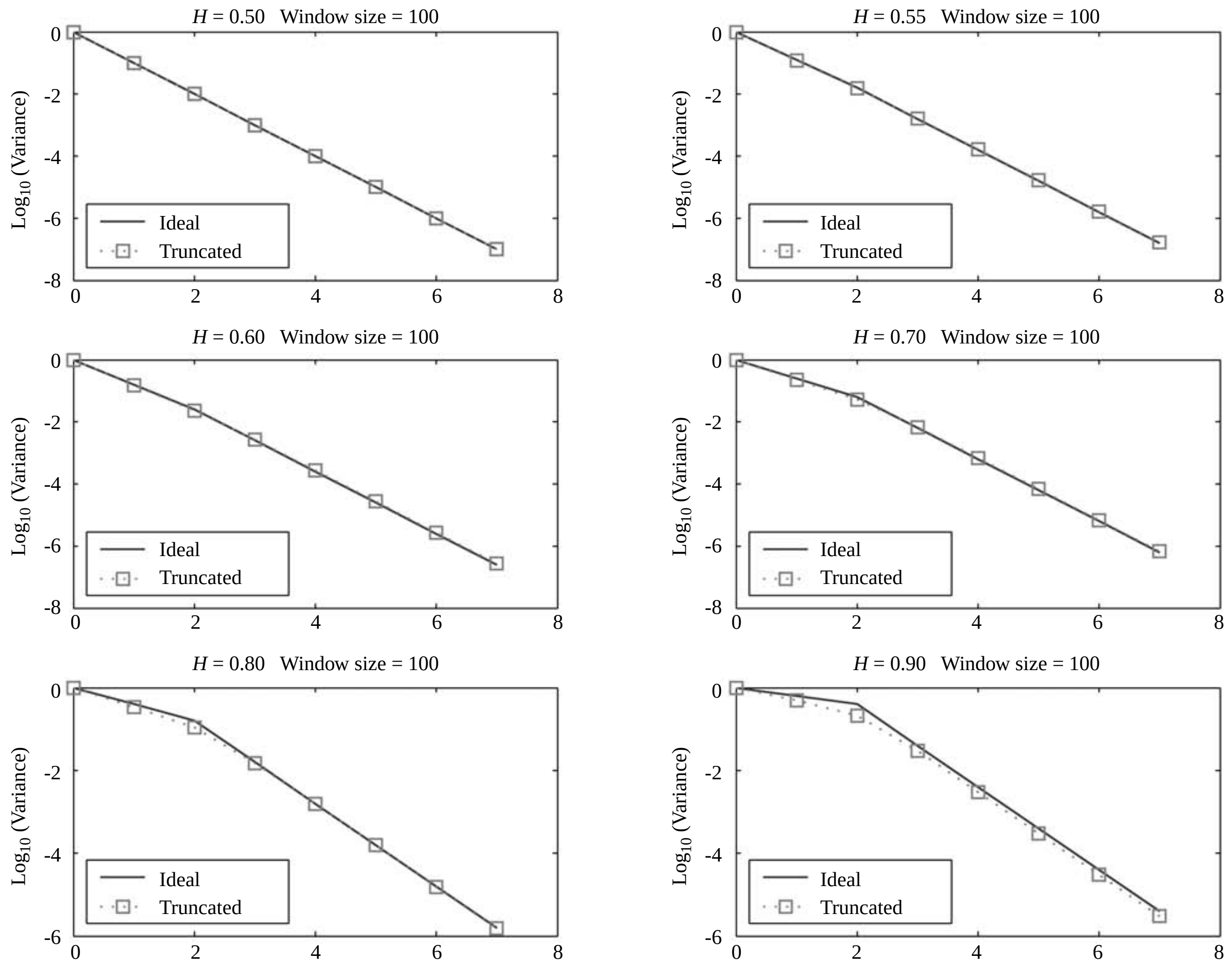

Fig. 7 Variance-time analysis for the truncated-filter output with truncation window $W = 10^2$. The slope of the solid line is equal to $2H - 2$ for $m \le W$, and -1 for $m > W$. The label of the horizontal axis for each subfigure is $\log_{10}(m)$

filter output will have "unstable" impact on the degree of self-similarity of the output trace. Our simulations suggest that such an unstable impact can be neglected if the ratio of the maximal rounding error (i.e., 0.5) against the input mean λ is made less than 5%.

IV. THE REVERSE FILTER VERSUS THE FORWARD FILTER

It can be easily seen that the z-transforms, $X(z)$ and $Y(z)$, of the filter input and output can be re-characterized by $(1 - z^{-1})^a Y(z) = X(z)$. Again, by Taylor's expansion,

$$(1 - z^{-1})^a = 1 + \frac{-a}{1!} z^{-1} + \frac{-a(1-a)}{2!} z^{-2} + \cdots$$

$$= 1 - a \sum_{n=1}^{\infty} \frac{\Gamma(n-a)}{\Gamma(n+1)\Gamma(1-a)} z^{-n} .$$

Hence, the outputs $y[1], y[2], y[3] \cdots$ can be also obtained through an infinite impulse response (IIR) filter as:

$$y[n] = x[n] + a \sum_{k=1}^{\infty} \frac{\Gamma(k-a)}{\Gamma(k+1)\Gamma(1-a)} y[n-k]$$

$$= x[n] + \sum_{k=1}^{\infty} h'[k] \cdot y[n-k] ,$$

where

$$h'[n] \triangleq \frac{a \cdot \Gamma(n-a)}{\Gamma(n+1)\Gamma(1-a)} = \frac{(H-0.5) \cdot \Gamma(n-H+0.5)}{\Gamma(1.5-H)\Gamma(n+1)} \quad \text{for } k \ge 1.$$

We refer $h[\cdot]$ as the *forward filter* and $h'[\cdot]$ as the *reverse filter*, since the latter has a feedback or reverse path. Both $h[\cdot]$ system and $h'[\cdot]$ system can generate a *true* self-similar process in response to, say, an i.i.d. Poisson input; however, unlike the forward filter, the reverse filter gives an infinite impulse response filter (IIR) even if a finite truncation on

Table 1 Comparison between the resultant hurst parameters of the traces synthesized by the filter-based algorithm and the targeted ideal hurst parameters

Window size= 10000		
Ideal H	V-T (λ = 1)	V-T (λ = 10)
0.5001	0.4898783	0.5064982
0.55	0.5504289	0.5344366
0.6	0.6413529	0.5641452
0.7	0.4775099	0.7013537
0.8	0.5399816	0.7799114
0.9	0.5958403	0.8716414

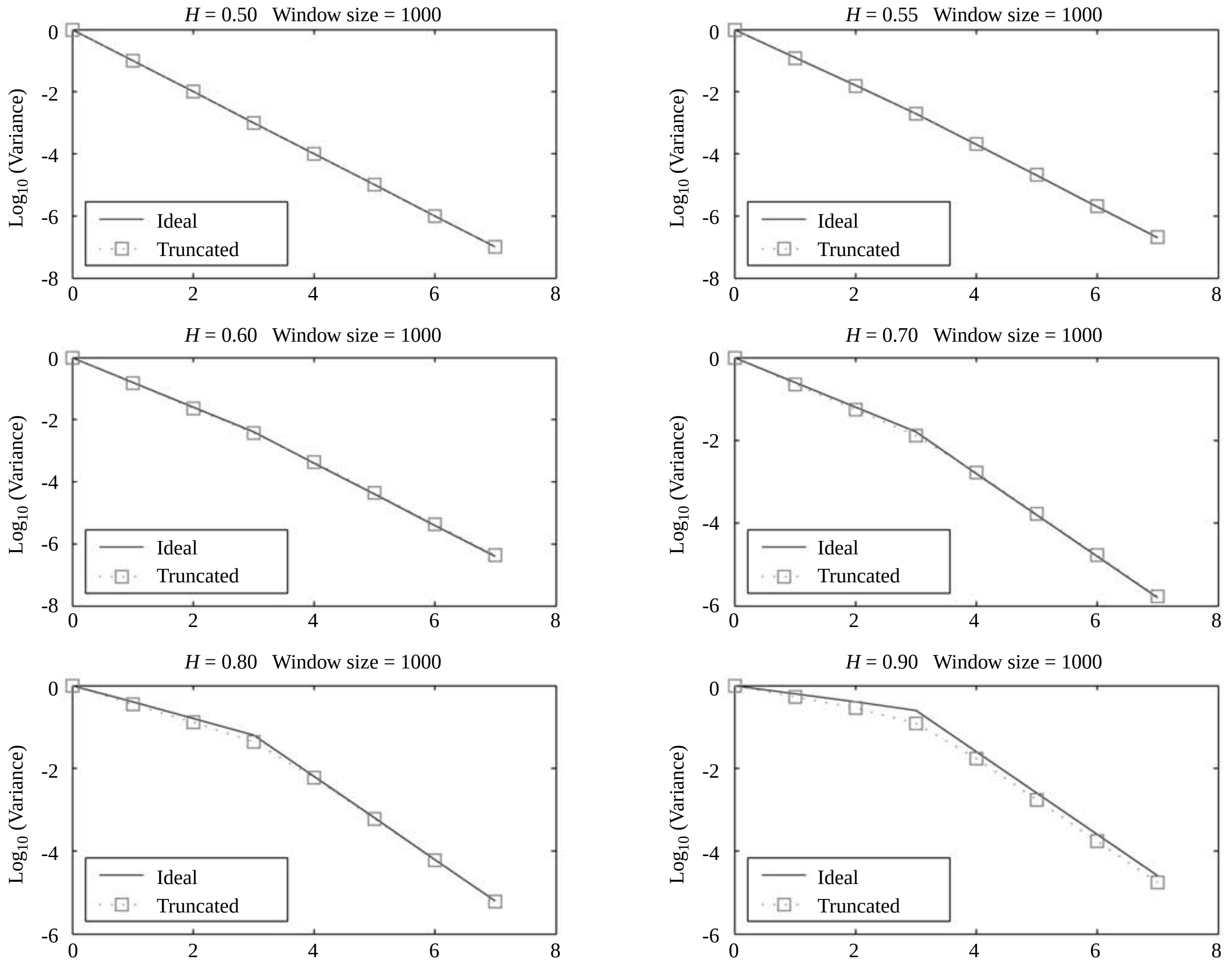

Fig. 8 Variance-time analysis for the truncated-filter output with truncation window $W = 10^3$. The slope of the solid line is equal to $2H - 2$ for $m \le W$, and -1 for $m > W$. The label of the horizontal axis for each subfigure if $\log_{10}(m)$

$h'[\cdot]$ is applied. This may give a false impression that the reverse system equipped with an infinite impulse response (IIR) filter of *finite* number of coefficients can synthesize a more self-similar trace than the forward system with truncated forward filter of the same computational complexity (or more specifically, the same truncation window). Our simulations, however, indicate that the effective ranges of both filters are actually similar (cf. Fig. 10).

V. CONCLUDING REMARKS

In this paper, a new model is proposed for the synthesis of self-similar traffic based on the filter

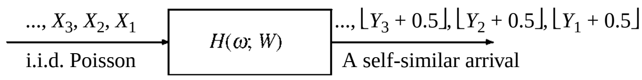

Fig. 9 The proposed asymptotic self-similar traffic synthesizer. $H(\omega; W)$ represents a truncated version of $H(\omega)$ with truncation window W. The quantity $\lfloor Y_i + 0.5 \rfloor$ equals the closest integer to Y_i

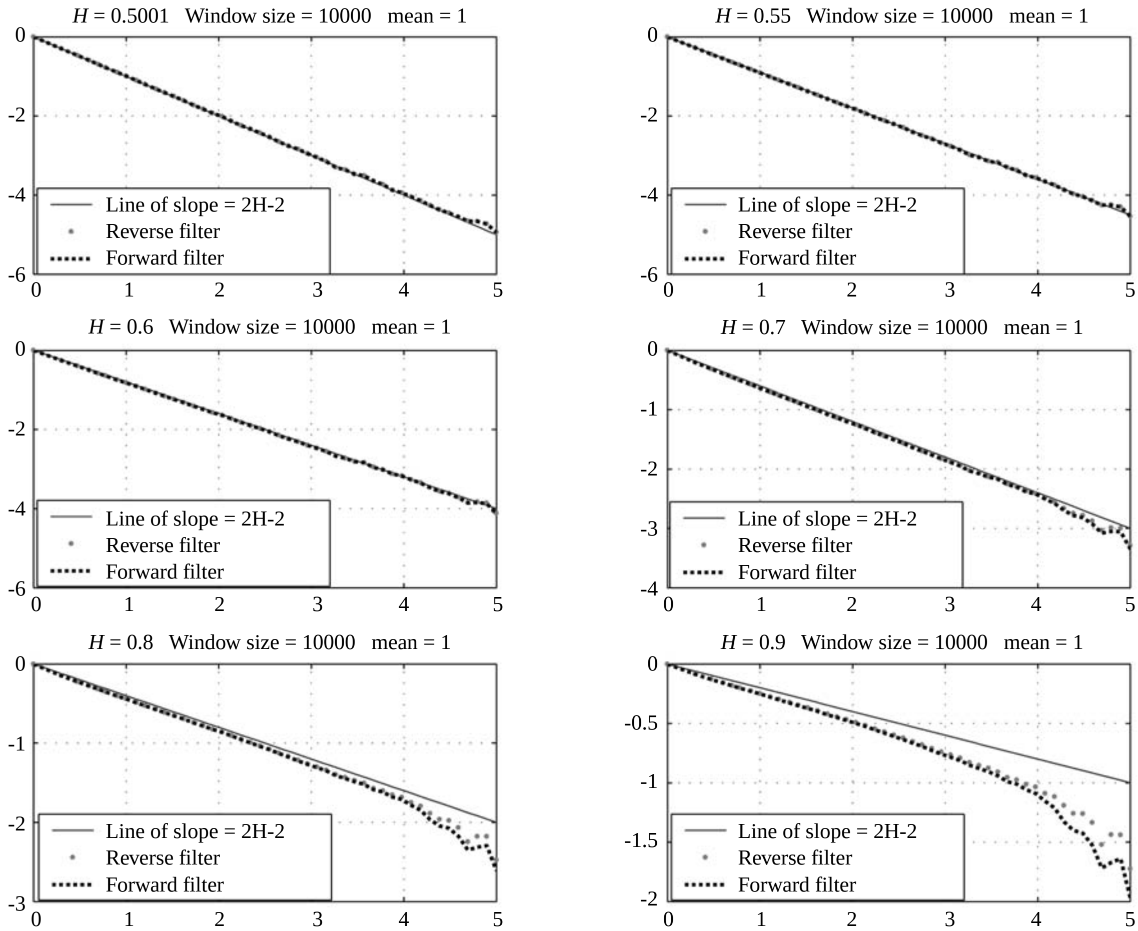

Fig. 10 Variance-time plots ($\log_{10}$ scale) for the two filter-based synthetic arrivals with truncation window 10^4 and mean rate 1

technique. The synthesized trace can be made long-range dependent with adjustable levels of burstiness and correlation. Only three parameters need to be specified in our model: H is the targeted self-similar parameter that controls the burstiness and correlation of the synthetic traffic, λ defines the mean of the synthesized traffic, and W determines not only the length of the filter (which in turns determines the algorithmic complexity) but also the valid aggregation size of self-similar nature from the aspect of variance-time analysis.

When being compared to the two known self-similar traffic synthesizers—*random midpoint displacement and Paxson's spectrum fitting*, our model provides the advantages that the synthetic traffic can be generated on the fly, and is always non-negative. The algorithmic complexity of Paxon's spectrum fitting was shown to be less than the random midpoint displacement, and is given by $(n/2)\log_2(n + 2)$, where n is the length of the synthetic trace. The complexity of our model, however, is also dependent on W, and is equal to $n \times W$. Hence, when the valid aggregation size of self-similar nature is specified, the complexity of our model only grows linearly with the trace size.

NOMENCLATURE

$C_m(k)$	autocovariance function of $\boldsymbol{Y}^{(m)}$
$C_m(k; W)$	autocovariance function of $\boldsymbol{Y}^{(m)}$ due to truncated filter
$\bar{C}_m(k)$	autocovariance function of $\bar{\boldsymbol{Y}}^{(m)}$

$G(\omega; m)$	transfer function of $g[n; m]$
$g[n; m]$	filter for the generation of variance-equivalent m-averaged process
H	self-similar or Hurst parameter
$H(\omega)$	transfer function of $h[n]$
$h[n]$	impulse response of a forward filter
$h'[n]$	impulse response of a reverse filter
$h[k; W]$	impulse response of a filter with truncation window W
$I_j \triangleq \{j, j+1, j+2, \cdots\}$	integer set starting from j
$S_x(\omega)$	power spectrum of $\boldsymbol{X}$
$S_y(\omega)$	power spectrum of $\boldsymbol{Y}$
$S_y(\omega; W)$	power spectrum of output due to truncated filter
$X(z)$	filter input in z-transform domain
$\boldsymbol{X} \triangleq \{X_i\}_{i \in I_1}$	second-order stationary real-valued stochastic process
$\boldsymbol{X}^{(m)} \triangleq \{X_i^{(m)}\}_{i \in I_1}$	m-averaged process $\boldsymbol{X}^{(m)}$ of $\boldsymbol{X}$
$x[n]$	filter input in time domain
$Y(z)$	filter output in z-transform domain
$\boldsymbol{Y} \triangleq \{Y_i\}_{i \in I_1}$	output process due to filter input $\boldsymbol{X}$
$\bar{\boldsymbol{Y}}^{(m)} \triangleq \{\bar{Y}_1^{(m)}\}_{i \in I_1}$	variance-equivalent m-averaged process
$y[n]$	filter output in time domain
$\beta \triangleq 2(1-H)$	negative slope of variance-time analysis
$\Gamma(n) \triangleq \int_0^\infty t^{n-1}e^{-t}dt$	Euler gamma function
$\rho_m \triangleq C_m(k)/C_m(0)$	aucocorrelation coefficient function of $\boldsymbol{Y}^{(m)}$

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