Decentralized Stabilization of Neural Network Linearly Interconnected Systems via T-S Fuzzy Control

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The stabilization problem is considered in this study for a neural-network (NN) linearly interconnected system that consists of a number of NN models. First, a linear difference inclusion (LDI) state-space representation is established for the dynamics of each NN model. Then, based on the LDI state-space representation, a stability criterion in terms of Lyapunov's direct method is derived to guarantee the asymptotic stability of closed-loop NN linearly interconnected systems. Subsequently, according to this criterion and the decentralized control scheme, a set of Takagi-Sugeno (T-S) fuzzy controllers is synthesized to stabilize the NN linearly interconnected system. Finally, a numerical example with simulations is given to demonstrate the concepts discussed throughout this paper. [DOI: 10.1115/1.2234492]

1 Introduction

A number of large-scale systems (also called interconnected systems or composite systems) founded in the real world are composed of a set of small interconnected subsystems, such as electric power systems, nuclear reactors, aerospace systems, economic systems, chemical and petroleum industries, and different types of societal systems. The field of large-scale systems exists so widely, including the fundamental theory of modeling, optimization, and control or certain particular aspects and applications. In addition, large-scale systems analysis, design, and control theory have attained considerable maturity and sophistication and are receiving increasing attention from the theorists and practitioners due to their methodological interests and important real-life applications [1]. In the meanwhile, the stabilization problem of large-scale systems is also an important topic and has attracted lots of attention (see [2–6] and the references therein).

Fuzzy control has been fast developed in both academic and industrial communities in the past few years, and there have been many successful applications [7–13]. In spite of the success, there are still many basic issues that remain to be addressed further. Stability analysis and systematic design are certainly among the most important issues for fuzzy control systems. During the last decade, there have been significant research efforts on these issues (see [14–28]). For example, Cao et al. [21–23] derived some stability theorems for continuous-time fuzzy control systems in 1996. Akar and Özgüner [25–27] proposed decentralized techniques for the analysis and control of T-S fuzzy interconnected systems. Moreover, an LMI-based H^{∞} fuzzy control system design with a T-S framework was proposed by Hong and Langari in 2000 [28].

In the past few years, neural-network- (NN-) based modeling has become an active research field because of its unique merits in solving complex nonlinear system identification and control problems (see [29–31] and the references therein). Moreover, there are significant research efforts on analysis and synthesis of a class of discrete-time neural networks. For instance, Si and Michel [32] used the NN with nonlinear interconnections to implement an encoder, and they [33] applied the NN with multilevel threshold neurons to image processing. Neural networks are composed of simple elements operating in parallel. These elements are inspired by biological nervous systems. As a result, we can train a neural network to represent a particular function by adjusting the weights between elements. However, the sigmoid multilayer-perception network, which is essentially linear, except near the origin, cannot approximate an arbitrary continuous nonlinear state equation [31]. A lot of reports on the success of NN applications in control systems have appeared in literature. Despite several promising empirical results and its nonlinear mapping approximation property, the rigorous closed-loop stability results for systems using NN-based controllers are still difficult to establish. Therefore, an LDI state-space representation was introduced to deal with the stability analysis of NN systems (see [30,31], for examples). In this work, based on the LDI state-space representation and Lyapunov approach, a stability criterion is derived to guarantee the asymptotic stability of closed-loop NN linearly interconnected systems.

This paper is organized as follows. First, the NN linearly interconnected systems is presented. Then, an LDI state-space representation is established for the dynamics of each NN model. Next, a stability criterion with the guarantee of asymptotic stability is proposed. Subsequently, based on this criterion and the decentralized control scheme, a set of T-S fuzzy controllers is synthesized to stabilize the NN linearly interconnected system. Finally, a numerical example is given to illustrate the results, and the conclusions are drawn.

2 NN Linearly Interconnected Systems

Consider a neural-network (NN) interconnected system N that consists of L NN models. The lth $(l=1,2,\ldots,L)$ NN model N_l, shown in Fig. 1, has S_l layers with R_l^e $(e=1,2,\ldots,S_l)$ (for simplicity of notation, we use S instead of S_l in the remainder of this paper) neurons for each layer, in which $x_l(k) \sim x_l(k-p+1)$ are the state variables and $u_l(k) \sim u_l(k-q+1)$ (in the state-variable approach, the state variable p must be greater than or equal to input variable q (i.e., $p \geqslant q$)) are the input variables. In order to distinguish among these layers, the superscripts are used for identifying the layers. Specifically, we append the number of the layer as a superscript to the names for each of these variables. Thus, the weight matrix for the eth $(e=1,2,\ldots,S)$ layer is written as $\mathbf{W}_l^e$. Moreover, it is assumed that ν is the net input and all the output

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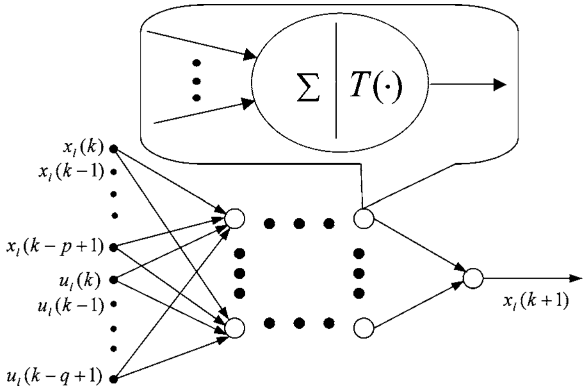

Fig. 1 The *l*th NN model

functions $T(\nu)$ of units in the lth NN model are described by the following sigmoid function:

$$T(\nu)=\delta\left(\frac{2}{1+\exp(-\nu/\tau)}-1\right) \tag{2.1}$$

where τ and δ are the positive parameters associated with the sigmoid function. Subsequently, the transfer function vector of the eth layer is defined as

$$\Psi_l^e(\nu)\equiv[T(\nu_1)\ T(\nu_2)\ \cdots\ T(\nu_{R_l^e})]^T$$

where $T(\nu_\zeta)(\zeta=1,2,\ldots,R_l^e)$ is the transfer function of the ζth neuron. Then the final output of the lth NN model can be inferred as follows:

$$x_l(k+1)=\Psi_l^S\{\mathbf{W}_l^S\Psi_l^{S-1}[\mathbf{W}_l^{S-1}\Psi_l^{S-2}(\cdots \cdots\Psi_l^2\{\mathbf{W}_l^2\Psi_l^1[\mathbf{W}_l^1Z_l(k)]\}\cdot\cdots\cdots)]\} \tag{2.2}$$

where $Z_l^T(k)=[x_l(k)\ x_l(k-1)\cdots x_l(k-p+1)\ u_l(k)\ u_l(k-1)\cdots u_l(k-q+1)]$.

3 Linear Difference Inclusion (LDI) State-Space Representation

In order to deal with the stability problem of the NN linearly interconnected system **N**, an LDI state-space representation is established for the dynamics of each NN model and described as [30,31,34]:

$$y(k+1)=A(a(k))y(k),\quad A(a(k))=\sum_{i=1}^{r}h_i(a(k))\bar{A}_i \tag{3.1}$$

where $y(k)=[y_1(k)\ y_2(k)\cdots y_m(k)]^T$ is the state vector in which m is a natural number. An $m\times m$ matrix $A(a(k))$ denotes the system matrix, $a(k)$ is a vector signifying the dependence of $h_i(\cdot)$ on its elements, r is a positive integer, and $\bar{A}_i$ $(i=1,2,\ldots,r)$ are constant matrices of dimension $m\times m$. Without loss of generality, we can use $h_i(k)$ instead of $h_i(a(k))$ in the remainder of this paper. Furthermore, it is assumed that $h_i(k)\geqslant 0$ and $\Sigma_{i=1}^r h_i(k)=1$.

To begin with, notice that the output, $\mathrm{T}(\nu)$, satisfies

$$\begin{aligned} g_1\nu\leqslant T(\nu)\leqslant g_2\nu,\quad \nu\geqslant 0\\ g_2\nu\leqslant T(\nu)\leqslant g_1\nu,\quad \nu<0 \end{aligned} \tag{3.2}$$

where g_1 and g_2 are the minimum and the maximum of the derivative of $\mathrm{T}(\nu)$, respectively.

Subsequently, the min-max matrix G_ζ^e is defined as follows:

$$G_\zeta^e=\mathrm{diag}(g_e(T(\nu_\zeta))),\quad e=1,2,\ldots,S;\quad \zeta=1,2,\ldots,R_l^e \tag{3.3}$$

According to the interpolation method and Eq. (2.2), we can obtain

$$\begin{aligned} x_l(k+1)&=\sum_{\zeta^s=1}^{2}h_{\zeta^s}(k)G_{R_l^s}^S\left[\mathbf{W}_l^S\left(\cdots\left\{\sum_{\zeta^2=1}^{2}h_{\zeta^2}(k)G_{R_l^2}^2\left[\mathbf{W}_l^2\left(\sum_{\zeta^1=1}^{2}h_{\zeta^1}(k)G_{R_l^1}^1[\mathbf{W}_l^1Z_l(k)]\right)\right]\right\}\cdots\right)\right]\\ &=\sum_{\zeta^s=1}^{2}\cdots\sum_{\zeta^2=1}^{2}\sum_{\zeta^1=1}^{2}h_{\zeta^s}(k)\cdots h_{\zeta^2}(k)h_{\zeta^1}(k)G_{R_l^s}^S\mathbf{W}_l^S\cdots G_{R_l^2}^2\mathbf{W}_l^2G_{R_l^1}^1\mathbf{W}_l^1Z_l(k)\\ &=\sum_{\Omega^e}h_{\Omega^e}(k)E_{\Omega^e}Z_l(k) \end{aligned} \tag{3.4}$$

where

$$\sum_{\zeta^e}h_{\zeta^e}(k)\equiv\sum_{q_1^e=1}^{2}\sum_{q_2^e=1}^{2}\cdots\sum_{q_{R_l^e}^e=1}^{2}h_{q_1^e}(k)h_{q_2^e}(k)\cdots h_{q_{R_l^e}^e}(k)$$

$$\text{for } e=1,2,\ldots,S;\quad h_{q_\zeta^e}(k)\in[0\ 1]$$

$$\sum_{q_\zeta^e=1}^{2}h_{q_\zeta^e}(k)=1\quad\text{for }\zeta=1,2,\ldots,R_l^e;$$

$$E_{\Omega^e}\equiv G_{R_l^s}^S\mathbf{W}_l^S\cdots G_{R_l^2}^2\mathbf{W}_l^2G_{R_l^1}^1\mathbf{W}_l^1$$

$$\sum_{\Omega^e}h_{\Omega^e}(k)\equiv\sum_{\zeta^s=1}^{2}\cdots\sum_{\zeta^2=1}^{2}\sum_{\zeta^1=1}^{2}h_{\zeta^s}(k)\cdots h_{\zeta^2}(k)h_{\zeta^1}(k)$$

Remark 1. According to Eq. (3.2), the sigmoid function $T(\nu)$ is bounded by $g_1\nu$ and $g_2\nu$. Based on the interpolation method, $T(\nu)$ can be represented as $T(\nu)=h_1(k)g_1\nu+h_2(k)g_2\nu$, where $h_1(k)$, $h_2(k)\geqslant 0$, and $h_1(k)+h_2(k)=1$. Therefore, r in Eq. (3.1) should be set to be 2 to derive Eq. (3.4) and $\Sigma_{q_\zeta^e=1}^2 h_{q_\zeta^e}(k)=h_1(k)+h_2(k)=1$.

Finally, based on Eq. (3.1), the dynamics of the lth NN model (3.4) is rewritten as the following LDI state-space representation:

$$X_l(k+1) = \sum_{i=1}^{r_l} h_{il}(k) E_{il} Z_l(k) \tag{3.5}$$

where r_l is a positive integer and E_{il} is a constant matrix with appropriate dimension associated with E_{Ω^e}. The LDI state-space representation (3.5) can be further rearranged as follows [30]:

$$X_l(k+1) = \sum_{i=1}^{r_l} h_{il}(k)[\bar{A}_{il} X_l(k) + \bar{B}_{il} U_l(k)] \tag{3.6}$$

where $X_l^T(k)=[x_l(k)\ x_l(k-1)\cdots x_l(k-p+1)]$, $U_l^T(k)=[u_l(k)\ u_l(k-1)\cdots u_l(k-q+1)]$, $\bar{A}_{il}$ and $\bar{B}_{il}$ are the partitions of E_{il} corresponding to the partition $Z_l^T(k)=[X_l^T(k)\ U_l^T(k)]$.

4 Decentralized Stabilization Via T-S Fuzzy Control

On the basis of the decentralized control scheme, a set of T-S fuzzy controllers is synthesized to stabilize the NN linearly interconnected system $\mathbf{N}$. The lth fuzzy controller is in the following form:

Rule j: IF $x_l(k)$ is M_{j1l} and$\cdots$and $x_l(k-p+1)$ is M_{jpl}

$$\text{THEN } U_l(k) = -F_{jl} X_l(k) \tag{4.1}$$

$j=1,2,\ldots,J_l$ and J_l is the number of IF-THEN rules of the fuzzy controller and $M_{j\mu l}(\mu=1,2,\ldots,p)$ are the fuzzy sets. Hence, the final output of this fuzzy controller is inferred as follows:

$$U_l(k) = -\frac{\sum_{j=1}^{J_l} w_{jl}(k) F_{jl} X_l(k)}{\sum_{j=1}^{J_l} w_{jl}(k)} = -\sum_{j=1}^{J_l} \bar{h}_{jl}(k) F_{jl} X_l(k) \tag{4.2}$$

with

$$w_{jl}(k) = \prod_{\mu=1}^{p} M_{j\mu l}(x_l(k-\mu+1)), \quad \bar{h}_{jl}(k) = \frac{w_{jl}(k)}{\sum_{j=1}^{J_l} w_{jl}(k)}$$

in which $M_{j\mu l}(x_l(k-\mu+1))$ is the grade of membership of $x_l(k-\mu+1)$ in $M_{j\mu l}$. In this study, it is also assumed that $w_{jl}(k) \geqslant 0$, $j=1,2,\ldots,J_l$; $l=1,2,\ldots,L$ and $\Sigma_{j=1}^{J_l} w_{jl}(k) > 0$ for all k. Therefore, $\bar{h}_{jl}(k) \geqslant 0$ and $\Sigma_{j=1}^{J_l} \bar{h}_{jl}(k)=1$ for all k. Substituting Eq. (4.2) into Eq. (3.6), we have

$$\begin{aligned} X_l(k+1) &= \sum_{i=1}^{r_l} \sum_{j=1}^{J_l} h_{il}(k) \bar{h}_{jl}(k) (\bar{A}_{il} - \bar{B}_{il} F_{jl}) X_l(k) \\ &= \sum_{i=1}^{r_l} \sum_{j=1}^{J_l} h_{il}(k) \bar{h}_{jl}(k) H_{ijl} X_l(k) \end{aligned} \tag{4.3}$$

where $H_{ijl}=\bar{A}_{il}-\bar{B}_{il}F_{jl}$.

Based on the above analysis and Eq. (4.3), the lth ($l=1,2,\ldots,L$) closed-loop subsystem with interconnections $\bar{N}_l$ can be described as follows:

$$\bar{N}_l: \begin{cases} X_l(k+1) = \sum_{i=1}^{r_l} \sum_{j=1}^{J_l} h_{il}(k) \bar{h}_{jl}(k) H_{ijl} X_l(k) + \phi_l(k) \\ \phi_l(k) = \sum_{\substack{n=1 \\ n \neq l}}^{L} C_{nl} X_n(k) \end{cases} \tag{4.4}$$

where C_{nl} is the interconnection matrix between the nth and lth NN models. Prior to the examination of asymptotic stability of the closed-loop NN linearly interconnected system $\bar{\mathbf{N}}$ that consist of L closed-loop subsystems described in Eq. (4.4), a useful concept is given below.

LEMMA 1 [21,35]. *For real matrices A and B with an appropriate dimension, we have*

$$A^T B + B^T A \leqslant \beta A^T A + \beta^{-1} B^T B$$

where β is a positive constant.

THEOREM 1. *The closed-loop neural-network linearly interconnected system $\bar{\mathbf{N}}$ is asymptotically stable, if there exist symmetric positive definite matrices P_l $(l=1,2,\ldots,L)$ and a positive constant β, and the feedback gains F_{jl}'s shown in Eq. (4.2) are chosen such that the following inequalities hold*:

$$\psi_{ijl} \equiv \lambda_M(Q_{ijl}) + \alpha_{ijl} < 0 \quad \text{for } i=1,2,\ldots,r_l; \quad j=1,2,\ldots,J_l; \quad l=1,2,\ldots,L \tag{4.5a}$$

$$\psi_{ijf} \equiv \lambda_M(Q_{ijfl}) + \alpha_{ijl} + \alpha_{ifl} < 0 \text{ for } i=1,2,\ldots,r_l; \quad j < f \leqslant J_l; \quad l=1,2,\ldots,L \tag{4.5b}$$

$$\psi_{ijdl} \equiv \lambda_M(Q_{ijdl}) + \alpha_{ijl} + \alpha_{djl} < 0 \text{ for } i < d \leqslant r_l; \quad j=1,2,\ldots,J_l; \quad l=1,2,\ldots,L \tag{4.5c}$$

$$\psi_{ijdfl} \equiv \lambda_M(Q_{ijdfl}) + \alpha_{ijl} + \alpha_{ifl} + \alpha_{djl} + \alpha_{dfl} < 0 \quad \text{for } i < d \leqslant r_l; \quad j < f \leqslant J_l; \quad l=1,2,\ldots,L \tag{4.5d}$$

where

$$Q_{ijl} = H_{ijl}^T P_l H_{ijl} - P_l, \quad \alpha_{ijl} = \sigma_{ijl} + \eta_l, \quad \text{for } i=1,2,\ldots,r_l; \quad j=1,2,\ldots,J_l; \quad l=1,2,\ldots,L \tag{4.6a}$$

$$Q_{ijfl} = H_{ijl}^T P_l H_{ifl} + H_{ifl}^T P_l H_{ijl} - 2P_l, \quad \text{for } i=1,2,\ldots,r_l; \quad j < f \leqslant J_l; \quad l=1,2,\ldots,L \tag{4.6b}$$

$$Q_{ijdl} = H_{ijl}^T P_l H_{djl} + H_{djl}^T P_l H_{ijl} - 2P_l, \quad \text{for } i < d \leqslant r_l; \quad j=1,2,\ldots,J_l; \quad l=1,2,\ldots,L \tag{4.6c}$$

$$Q_{ijdfl} = H_{ijl}^T P_l H_{dfl} + H_{dfl}^T P_l H_{ijl} + H_{ifl}^T P_l H_{djl} + H_{djl}^T P_l H_{ifl} - 4P_l, \quad \text{for } i < d \leqslant r_l; \quad j < f \leqslant J_l; \quad l=1,2,\ldots,L \tag{4.6d}$$

with

$$H_{ijl} = \bar{A}_{il} - \bar{B}_{il} F_{jl}, \quad \eta_l = \sum_{\substack{n=1 \\ n \neq l}}^{L} (L-1) \lambda_M(P_n) \|C_{ln}\|^2 \tag{4.7a}$$

$$\sigma_{ijl} = \lambda_M(\bar{Q}_{ijl}) + \beta^{-1}(L-1), \quad \bar{Q}_{ijl} = \beta H_{ijl}^T P_l \sum_{n=1}^{L} (C_{nl} C_{nl}^T) P_l H_{ijl} \tag{4.7b}$$

Moreover, $\lambda_M(A)$ denotes the maximum eigenvalue of the matrix A.

Proof. See the Appendix.

Remark 2. The common P_l can be solved via MATLAB LMI (linear matrix inequality) Toolbox. However, in many cases, even if a common P_l cannot be found, the system may still be asymptotically stabilized by using the design method of a piecewise smooth quadratic (PSQ) Lyapunov function approach proposed by Cao et al. [21–23]. It is easier to obtain a piecewise continuous Lyapunov function than a single Lyapunov function $V_l(t)$ for fuzzy rule-based systems.

Remark 3. Eq. (4.6a) implies that each closed-loop NN is stable, and, moreover, that all the H_{ijl} in Eq. (4.4) share a common Lyapunov matrix $P_l=P_l^T>0$, immediately implying that in Eq. (4.5a), $Q_{ijl}=Q_{ijl}^T<0$. Theorem 4.1 of Tanaka [30] implies that the

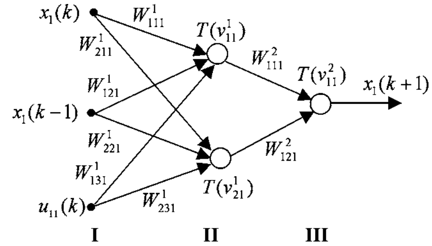

Fig. 2 The first NN model

Q_{ijfl} in Eq. (4.6*b*), Q_{ijdl} in Eq. (4.6*c*), and Q_{ijdfl} in Eq. (4.6*d*) are also symmetric negative definite, i.e.,

$$Q_{ijl} < 0 \text{ in Eq. (4.6a)} \Rightarrow \left\{ \begin{array}{l} Q_{ijfl} < 0 \text{ in Eq. (4.6b)} \\ Q_{ijdl} < 0 \text{ in Eq. (4.6c)} \\ Q_{ijdfl} < 0 \text{ in Eq. (4.6d)} \end{array} \right\} \tag{4.8}$$

Since

$$\sigma_{ijl} \to \beta^{-1}(L-1) \text{ as } C_{nl} \to 0 \tag{4.9}$$

where β is an arbitrarily large constant, we see $Q_{ijl}<0$ is sufficient for stability when linear coupling matrices C_{nl} are zero. So, with no coupling, the system is made up of L decoupled systems each having a quadratic Lyapunov function:

$$V_l(k) \equiv \frac{1}{2} x_l^T(k) P_l x_l(k), \quad l = 1, \ldots, L \tag{4.10}$$

with

$$\begin{aligned} &V_l(k+1) - V_l(k) \leqslant x_l^T(k) Q_l(x_l(k), u_l(k)) x_l(k) \\ &Q_l[x_l(k), u_l(k)] < 0, \quad \forall\, k, \quad l = 1, \ldots, L, \quad \text{if } C_{nl} = 0, \quad \forall\, n \end{aligned} \tag{4.11}$$

where one can create a "composite" Lyapunov function $V(k)$ that is the sum over all l of Eq. (4.10).

Then it is apparent that the inclusion of linear "disturbances" that are homogeneous and have sufficiently small Lipschitz constants (slopes) cannot cause the total system to become unstable. So, the existence of bounds on $\|C_{nl}\|$ that maintain global-exponential stability is obvious. The question then becomes how useful are the sufficient bounding conditions of Eq. (4.5). In fact, the sums in Eq. (4.7*a*) and Eq. (4.7*b*) make Eq. (4.5) look exactly like a block-diagonal-dominance condition. The relevant literature about block diagonal dominance and large-scale-linear-system stabilization is included [2–6]. Especially, an algorithm proposed by Edmunds [4] can be used to obtain block diagonal dominance in large-scale systems, enabling the use of simpler control structures. The application of such condition to "lightly-linearly-coupled" sigmoid multilayer perceptron neural network (sigmoid-MLPNN) models has been first pointed out in this paper.

5 Example

Our objective in this section is to synthesize a set of T-S fuzzy controllers such that the NN linearly interconnected system **N** which is composed of three NN models described as follows can be asymptotically stabilized.

Model 1 (N_1): The first NN model (without interconnection) is constructed by 3-2-1, shown in Fig. 2, with

$$\begin{aligned} &W^1_{111} = 1, \quad W^1_{211} = -1, \quad W^1_{121} = -0.5, \quad W^1_{221} = -0.6, \\ &W^1_{131} = 0.3, \quad W^1_{231} = -0.4, \quad W^2_{111} = 0.75, \quad W^2_{121} = 1 \end{aligned} \tag{5.1}$$

Moreover, all the transfer functions $\mathrm{T}(\nu)$ of units in the first NN model are described by the sigmoid function, shown in Eq. (1), with τ=0.75 and δ=1. From Fig. 2, we have

$$\nu^1_{\zeta 1} = W^1_{\zeta 11} x_1(k) + W^1_{\zeta 21} x_1(k-1) + W^1_{\zeta 31} u_{11}(k), \quad \zeta = 1,2 \tag{5.2}$$

$$\nu^2_{11} = W^2_{111} T(\nu^1_{11}) + W^2_{121} T(\nu^1_{21}) \tag{5.3}$$

$$x_1(k+1) = T(\nu^2_{11}) \tag{5.4}$$

According to Eq. (3.2), the minimum and the maximum of the derivative of the transfer function can be obtained as follows:

$$g_1 = 0, \quad g_2 = \tfrac{2}{3} \tag{5.5}$$

Therefore, based on the interpolation method, the transfer functions $T(\nu^1_{\zeta 1})$ and $\mathrm{T}(\nu^2_{11})$ can be represented by the following equations, respectively (The symbol $\nu^e_{\zeta l}$ denotes the net input of the ζth neuron of the eth layer in the lth NN model, and the indices e, ζ and l shown in $h^e_{\zeta\theta l}$ (θ=1,2) indicate the same thing):

$$T(\nu^1_{\zeta 1}) = (h^1_{\zeta 11}(k) g_1 + h^1_{\zeta 21}(k) g_2) \nu^1_{\zeta 1} \tag{5.6}$$

with $h^1_{\zeta 11}(k) \geqslant 0$, $h^1_{\zeta 21}(k) \geqslant 0$ and $h^1_{\zeta 11}(k) + h^1_{\zeta 21}(k) = 1$ for $\zeta = 1,2$,

$$T(\nu^2_{11}) = [h^2_{111}(k) g_1 + h^2_{121}(k) g_2] \nu^2_{11} \tag{5.7}$$

with $h^2_{111}(k) \geqslant 0$, $h^2_{121}(k) \geqslant 0$ and $h^2_{111}(k) + h^2_{121}(k) = 1$. From Eqs. (5.4) and (5.7), we have

$$x_1(k+1) = [h^2_{111}(k) g_1 + h^2_{121}(k) g_2] \nu^2_{11} = \sum_{\theta=1}^{2} h^2_{1\theta 1}(k) g_\theta \nu^2_{11} \tag{5.8}$$

Substituting Eqs. (5.3) and (5.6) into Eq. (5.8) yields

$$\begin{aligned} x_1(k+1) &= \sum_{\theta=1}^{2} h^2_{1\theta 1}(k) g_\theta \sum_{\zeta=1}^{2} W^2_{1\zeta 1} T(\nu^1_{\zeta 1}) \\ &= \sum_{\theta=1}^{2} h^2_{1\theta 1}(k) g_\theta \sum_{\zeta=1}^{2} W^2_{1\zeta 1} \{h^1_{\zeta 11}(k) g_1 + h^1_{\zeta 21}(k) g_2\} \nu^1_{\zeta 1} \\ &= \sum_{\theta=1}^{2} h^2_{1\theta 1}(k) g_\theta \sum_{p=1}^{2} \sum_{\xi=1}^{2} h^1_{1p1}(k) h^1_{2\xi 1}(k) \{g_p W^2_{111} \nu^1_{11} \\ &\quad + g_\xi W^2_{121} \nu^1_{21}\} \end{aligned} \tag{5.9}$$

By plugging Eq. (5.2) into Eq. (5.9), we obtain

$$\begin{aligned} x_1(k+1) = \sum_{\theta=1}^{2} \sum_{p=1}^{2} \sum_{\xi=1}^{2} & h^2_{1\theta 1}(k) h^1_{1p1}(k) h^1_{2\xi 1}(k) \{g_\theta [g_p W^2_{111} W^1_{111} \\ & + g_\xi W^2_{121} W^1_{211}] x_1(k) + g_\theta [g_p W^2_{111} W^1_{121} \\ & + g_\xi W^2_{121} W^1_{221}] x_1(k-1) \\ & + g_\theta [g_p W^2_{111} W^1_{131} + g_\xi W^2_{121} W^1_{231}] u_{11}(k)\} \end{aligned} \tag{5.10}$$

The matrix representation of Eq. (5.10) is

$$\begin{aligned} X_1(k+1) = \sum_{\theta=1}^{2} \sum_{p=1}^{2} \sum_{\xi=1}^{2} & h^2_{1\theta 1}(k) h^1_{1p1}(k) h^1_{2\xi 1}(k) \\ & \times \{A_{\theta p \xi} X_1(k) + B_{\theta p \xi} U_1(k)\} \end{aligned} \tag{5.11}$$

where

$$X_1^T(k)=[x_1(k)\ x_1(k-1)],\quad U_1(k)=u_{11}(k)$$

$$A_{\theta p\xi}=\begin{bmatrix} g_\theta(g_pW_{111}^2W_{111}^1+g_\xi W_{121}^2W_{211}^1) & g_\theta(g_pW_{111}^2W_{121}^1+g_\xi W_{121}^2W_{221}^1) \\ 1 & 0\end{bmatrix} \tag{5.12}$$

$$B_{\theta p\xi}=\begin{bmatrix} g_\theta(g_pW_{111}^2W_{131}^1+g_\xi W_{121}^2W_{231}^1) \\ 0\end{bmatrix}$$

Substituting Eqs. (5.1) and (5.5) into Eq. (5.12) yields

$$A_{111}=A_{112}=A_{121}=A_{122}=A_{211}=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}$$

$$A_{212}=\begin{bmatrix}-0.4444 & -0.2667\\ 1 & 0\end{bmatrix},\quad A_{221}=\begin{bmatrix}0.3333 & -0.1667\\ 1 & 0\end{bmatrix}$$

$$A_{222}=\begin{bmatrix}-0.1111 & -0.4333\\ 1 & 0\end{bmatrix} \tag{5.13}$$

$$B_{111}=B_{112}=B_{121}=B_{122}=B_{211}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$B_{212}=\begin{bmatrix}-0.1778\\ 0\end{bmatrix}$$

$$B_{221}=\begin{bmatrix}0.1000\\ 0\end{bmatrix},\quad B_{222}=\begin{bmatrix}-0.0778\\ 0\end{bmatrix}$$

Next, by renumbering the matrices, the first NN model (5.11) can be rewritten as the following LDI state-space representation:

$$X_1(k+1)=\sum_{i=1}^{4} h_{i1}(k)\{\bar{A}_{i1}X_1(k)+\bar{B}_{i1}U_1(k)\} \tag{5.14}$$

where

$$\bar{A}_{11}=A_{111}=A_{112}=A_{121}=A_{122}=A_{211},\quad \bar{A}_{21}=A_{212}$$

$$\bar{A}_{31}=A_{221},\quad \bar{A}_{41}=A_{222}$$

$$\bar{B}_{11}=B_{111}=B_{112}=B_{121}=B_{122}=B_{211},\quad \bar{B}_{21}=B_{212}$$

$$\bar{B}_{31}=B_{221},\quad \bar{B}_{41}=B_{222} \tag{5.15}$$

$$\begin{aligned} h_{11}(k)&=h_{111}^2(k)h_{111}^1(k)h_{211}^1(k)+h_{111}^2(k)h_{111}^1(k)h_{221}^1(k)\\ &\quad+h_{111}^2(k)h_{121}^1(k)h_{211}^1(k)+h_{111}^2(k)h_{121}^1(k)h_{221}^1(k)\\ &\quad+h_{121}^2(k)h_{111}^1(k)h_{211}^1(k)\end{aligned}$$

$$h_{21}(k)=h_{121}^2(k)h_{111}^1(k)h_{221}^1(k),\quad h_{31}(k)=h_{121}^2(k)h_{121}^1(k)h_{211}^1(k)$$

$$h_{41}(k)=h_{121}^2(k)h_{121}^1(k)h_{221}^1(k)$$

Model 2 (N_2): The second NN model (without interconnection) is constructed by 3-3-1 with

$$W_{112}^1=0.5,\quad W_{212}^1=0.5,\quad W_{312}^1=0.25,\quad W_{122}^1=0.4$$

$$W_{132}^1=0.25,\quad W_{232}^1=0.8,\quad W_{332}^1=-0.25$$

$$W_{222}^1=0.35,\quad W_{322}^1=0.5,\quad W_{112}^2=0.25,\quad W_{122}^2=-0.75 \tag{5.16}$$

$$W_{132}^2=1,\quad \tau=0.7 \text{ and } \delta=1$$

Using the same procedures as those in the first NN model, we obtain the following LDI state-space representation:

$$X_2(k+1)=\sum_{i=1}^{8} h_{i2}(k)\{\bar{A}_{i2}X_2(k)+\bar{B}_{i2}U_2(k)\} \tag{5.17}$$

where

$$\bar{A}_{12}=A_{1p\xi t}=A_{2111}=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix},\quad p,\ \xi,\ t=1,2$$

$$\bar{A}_{22}=A_{2112}=\begin{bmatrix}0.1276 & 0.2551\\ 1 & 0\end{bmatrix}$$

$$\bar{A}_{32}=A_{2121}=\begin{bmatrix}-0.1913 & -0.1339\\ 1 & 0\end{bmatrix}$$

$$\bar{A}_{42}=A_{2122}=\begin{bmatrix}-0.0638 & 0.1212\\ 1 & 0\end{bmatrix}$$

$$\bar{A}_{52}=A_{2211}=\begin{bmatrix}0.0638 & 0.0510\\ 1 & 0\end{bmatrix}$$

$$\bar{A}_{62}=A_{2212}=\begin{bmatrix}0.1913 & 0.3061\\ 1 & 0\end{bmatrix}$$

$$\bar{A}_{72}=A_{2221}=\begin{bmatrix}-0.1276 & -0.0829\\ 1 & 0\end{bmatrix}$$

$$\bar{A}_{82}=A_{2222}=\begin{bmatrix}0 & 0.1722\\ 1 & 0\end{bmatrix}$$

$$\bar{B}_{12}=B_{1p\xi t}=B_{2111}=\begin{bmatrix}0\\ 0\end{bmatrix},\quad p,\ \xi,\ t=1,2$$

$$\bar{B}_{22}=B_{2112}=\begin{bmatrix}-0.1276\\ 0\end{bmatrix},\quad \bar{B}_{32}=B_{2121}=\begin{bmatrix}-0.3061\\ 0\end{bmatrix}$$

$$\bar{B}_{42}=B_{2122}=\begin{bmatrix}-0.4337\\ 0\end{bmatrix},\quad \bar{B}_{52}=B_{2211}=\begin{bmatrix}0.0319\\ 0\end{bmatrix} \tag{5.18}$$

$$\bar{B}_{62}=B_{2212}=\begin{bmatrix}-0.0957\\ 0\end{bmatrix}$$

$$\bar{B}_{72}=B_{2221}=\begin{bmatrix}-0.2742\\ 0\end{bmatrix},\quad \bar{B}_{82}=B_{2222}=\begin{bmatrix}-0.4018\\ 0\end{bmatrix}$$

$$h_{12}(k) = h^2_{112}(k)h^1_{112}(k)h^1_{212}(k)h^1_{312}(k) + h^2_{112}(k)h^1_{112}(k)h^1_{212}(k)h^1_{322}(k) + h^2_{112}(k)h^1_{112}(k)h^1_{222}(k)h^1_{312}(k) + h^2_{112}(k)h^1_{112}(k)h^1_{222}(k)h^1_{322}(k) + h^2_{112}(k)h^1_{122}(k)h^1_{212}(k)h^1_{312}(k) + h^2_{112}(k)h^1_{122}(k)h^1_{212}(k)h^1_{322}(k) + h^2_{112}(k)h^1_{122}(k)h^1_{222}(k)h^1_{312}(k) + h^2_{112}(k)h^1_{122}(k)h^1_{222}(k)h^1_{322}(k) + h^2_{122}(k)h^1_{112}(k)h^1_{212}(k)h^1_{312}(k)$$

$$h_{22}(k) = h^2_{122}(k)h^1_{112}(k)h^1_{212}(k)h^1_{322}(k)$$

$$h_{32}(k) = h^2_{122}(k)h^1_{112}(k)h^1_{222}(k)h^1_{312}(k)$$

$$h_{42}(k) = h^2_{122}(k)h^1_{112}(k)h^1_{222}(k)h^1_{322}(k)$$

$$h_{52}(k) = h^2_{122}(k)h^1_{122}(k)h^1_{212}(k)h^1_{312}(k)$$

$$h_{62}(k) = h^2_{122}(k)h^1_{122}(k)h^1_{212}(k)h^1_{322}(k)$$

$$h_{72}(k) = h^2_{122}(k)h^1_{122}(k)h^1_{222}(k)h^1_{312}(k)$$

$$h_{82}(k) = h^2_{122}(k)h^1_{122}(k)h^1_{222}(k)h^1_{322}(k)$$

Model 3 (N_3): The third NN model (without interconnection) is constructed by 3-2-1 with

$$W^1_{113} = -0.5, \quad W^1_{213} = 0.25, \quad W^1_{123} = 1$$
$$W^1_{223} = 0.2, \quad W^1_{133} = -0.5, \quad W^1_{233} = 0.75 \qquad (5.19)$$
$$W^2_{113} = 0.5, \quad W^2_{123} = -1, \quad \tau = 0.6 \text{ and } \delta = 1$$

In a similar fashion, we have the following LDI state-space representation:

$$X_3(k+1) = \sum_{i=1}^{4} h_{i3}(k)\{\bar{A}_{i3}(k)X_3(k) + \bar{B}_{i3}U_3(k)\} \qquad (5.20)$$

where

$$\bar{A}_{13} = A_{111} = A_{112} = A_{121} = A_{122} = A_{211} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$
$$\bar{A}_{23} = A_{212} = \begin{bmatrix} -0.1736 & -0.1389 \\ 1 & 0 \end{bmatrix}$$
$$\bar{A}_{33} = A_{221} = \begin{bmatrix} -0.1736 & 0.3472 \\ 1 & 0 \end{bmatrix}$$
$$\bar{A}_{43} = A_{222} = \begin{bmatrix} -0.3472 & 0.2083 \\ 1 & 0 \end{bmatrix}$$
$$\bar{B}_{13} = B_{111} = B_{112} = B_{121} = B_{122} = B_{211} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\bar{B}_{23} = B_{212} = \begin{bmatrix} -0.5208 \\ 0 \end{bmatrix} \qquad (5.21)$$
$$\bar{B}_{33} = B_{221} = \begin{bmatrix} -0.1736 \\ 0 \end{bmatrix}, \quad \bar{B}_{43} = B_{222} = \begin{bmatrix} -0.6944 \\ 0 \end{bmatrix}$$

$$h_{13}(k) = h^2_{113}(k)h^1_{113}(k)h^1_{213}(k) + h^2_{113}(k)h^1_{113}(k)h^1_{223}(k) + h^2_{113}(k)h^1_{123}(k)h^1_{213}(k) + h^2_{113}(k)h^1_{123}(k)h^1_{223}(k) + h^2_{123}(k)h^1_{113}(k)h^1_{213}(k)$$

$$h_{23}(k) = h^2_{123}(k)h^1_{113}(k)h^1_{223}(k)$$

$$h_{33}(k) = h^2_{123}(k)h^1_{123}(k)h^1_{213}(k)$$

$$h_{43}(k) = h^2_{123}(k)h^1_{123}(k)h^1_{223}(k)$$

Moreover, the interconnection matrices among three NN models are given in the following:

$$C_{21} = \begin{bmatrix} 0.13 & -0.12 \\ 0 & 0 \end{bmatrix}, \quad C_{31} = \begin{bmatrix} -0.12 & -0.1 \\ 0 & 0 \end{bmatrix}$$
$$C_{12} = \begin{bmatrix} 0.1 & -0.15 \\ 0 & 0 \end{bmatrix}, \quad C_{32} = \begin{bmatrix} 0.12 & 0.1 \\ 0 & 0 \end{bmatrix} \qquad (5.22)$$
$$C_{13} = \begin{bmatrix} 0.16 & -0.13 \\ 0 & 0 \end{bmatrix}, \quad C_{23} = \begin{bmatrix} -0.15 & 0.12 \\ 0 & 0 \end{bmatrix}$$

Therefore, based on Eqs. (5.14), (5.17), (5.20), and (5.22), the NN linearly interconnected system can be represented as follows:

$$N: \begin{cases} X_1(k+1) = \sum_{i=1}^{4} h_{i1}(k)\{\bar{A}_{i1}X_1(k) + \bar{B}_{i1}U_1(k)\} + \phi_1(k) \\ X_2(k+1) = \sum_{i=1}^{8} h_{i2}(k)\{\bar{A}_{i2}X_2(k) + \bar{B}_{i2}U_2(k)\} + \phi_2(k) \\ X_3(k+1) = \sum_{i=1}^{4} h_{i3}(k)\{\bar{A}_{i3}X_3(k) + \bar{B}_{i3}U_3(k)\} + \phi_3(k) \\ \phi_l(k) = \sum_{\substack{n=1 \\ n\neq l}}^{3} C_{nl}X_n(k) \end{cases} \qquad (5.23)$$

in which the matrices $\bar{A}_{il}$ and $\bar{B}_{il}$, $i=1,2,\ldots,r_l$; $l=1,2,3$ are illustrated in Eqs. (5.15), (5.18), and (5.21). In order to stabilize the NN linearly interconnected system (5.23), three T-S fuzzy controllers are synthesized as follows.

Fuzzy controller of model 1:

$$\text{Rule 1: IF } x_1(k) \text{ is } M_{111} \quad \text{THEN } U_1(k) = -F_{11}X_1(k)$$
$$\text{Rule 2: IF } x_1(k) \text{ is } M_{211} \quad \text{THEN } U_1(k) = -F_{21}X_1(k) \qquad (5.24)$$

and the membership functions for Rule 1 and Rule 2 are

$$M_{111}(x_1(k)) = 0 \quad \text{when } x_1(k) \geq 1$$
$$M_{111}(x_1(k)) = \frac{-x_1(k)+1}{2} \quad \text{when } -1 \leq x_1(k) \leq 1$$
$$M_{111}(x_1(k)) = 1 \quad \text{when } x_1(k) \leq -1$$
$$M_{211}(x_1(k)) = 1 - M_{111}(x_1(k))$$

Fuzzy controller of model 2:

$$\text{Rule 1: IF } x_2(k) \text{ is } M_{112} \text{ THEN } U_2(k) = -F_{12}X_2(k)$$
$$\text{Rule 2: IF } x_2(k) \text{ is } M_{212} \text{ THEN } U_2(k) = -F_{22}X_2(k) \qquad (5.25)$$

and the membership functions for Rule 1 and Rule 2 are

$$M_{112}(x_2(k)) = \frac{1}{1+\exp(-x_2(k)/0.5)}$$

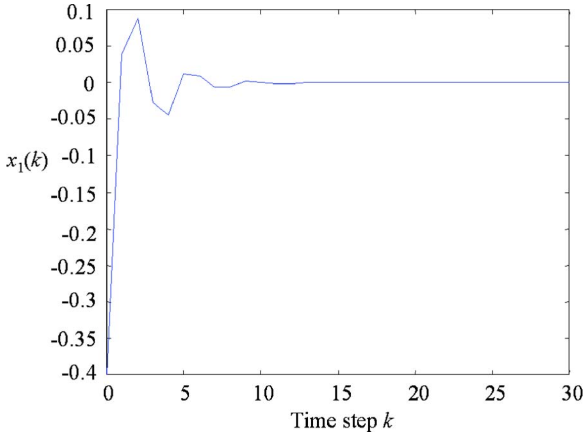

Fig. 3 The state $x_1(k)$ of subsystem 1

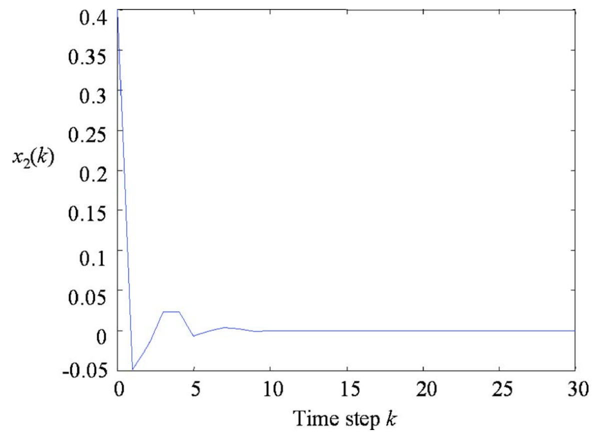

Fig. 4 The state $x_2(k)$ of subsystem 2

$$M_{212}(x_2(k)) = 1 - M_{112}(x_2(k))$$

Fuzzy controller of model 3:

$$\text{Rule 1: IF } x_3(k) \text{ is } M_{113} \text{ THEN } U_3(k) = -F_{13}X_3(k)$$
$$\text{Rule 2: IF } x_3(k) \text{ is } M_{213} \text{ THEN } U_3(k) = -F_{23}X_3(k) \tag{5.26}$$

and the membership functions for Rule 1 and Rule 2 are

$$M_{113}(x_3(k)) = \exp(-8(x_3(k) - 0.5)^2)$$

$$M_{213}(x_3(k)) = 1 - M_{113}(x_3(k))$$

To meet the inequalities (4.5a), the matrices Q_{ijl}'s in Eq. (4.6) must be chosen to be negative definite. Hence, we can obtain the following positive definite matrices P_l $(l=1,2,3)$ and the feedback gains F_{jl}'s via LMI (linear matrix inequality) optimization techniques such that all the matrices are negative definite:

$$P_1 = \begin{bmatrix} 76.5477 & -0.0361 \\ -0.0361 & 39.3816 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 73.6939 & 1.2549 \\ 1.2549 & 38.7814 \end{bmatrix}$$
$$P_3 = \begin{bmatrix} 65.0683 & 0.9759 \\ 0.9759 & 35.7440 \end{bmatrix} \tag{5.27}$$

$$F_{11} = [0.75\ 0.5], \quad F_{21} = [0.6\ 0.5], \quad F_{12} = [-0.5\ \ -0.3]$$
$$F_{22} = [0.2\ \ -0.25], \quad F_{13} = [0.3\ 0.2], \quad F_{23} = [0.5\ 0.25] \tag{5.28}$$

Next, substituting Eqs. (5.15), (5.18), (5.21), (5.22), (5.27), and (5.28) into Eqs. (4.5a), (4.5b), (4.5c), and (4.5d) with $\beta = \frac{1}{5}$, we have that all the matrices of ψ_{ijl}'s, ψ_{ijfl}'s, ψ_{ijdl}'s, and ψ_{ijdfl}'s are negative definite.

Therefore, based on Theorem 1, the T-S fuzzy controllers described in Eqs. (5.24)–(5.26) and (5.28) can asymptotically stabilize the NN linearly interconnected system (5.23). The simulation results of each closed-loop subsystem $\bar{N}_l$ $(l=1,2,3)$ are illustrated in Figs. 3–5 with initial conditions, $x_1(0)=-0.4$, $x_2(0)=0.4$, and $x_3(0)=0.3$.

6 Conclusions

The stabilization problem is considered in this study for a neural-network (NN) linearly interconnected system that consists of a number of NN models. In order to deal with the stability problem of NN linearly interconnected systems, an LDI state-space representation is first established for the dynamics of each NN model. Then, based on the LDI state-space representation and Lyapunov approach, a stability criterion is derived to guarantee the asymptotic stability of closed-loop NN linearly interconnected systems. Subsequently, based on this criterion and the decentralized control scheme, a set of T-S fuzzy controllers is synthesized to stabilize the NN linearly interconnected system. Finally, a numerical example with simulations is given to demonstrate the results.

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Appendix: Proof of Theorem 1

Let the Lyapunov function [27] for the closed-loop NN linearly interconnected system $\bar{\mathbf{N}}$ be defined as

$$V(k) = \sum_{l=1}^{L} X_l^T(k) P_l X_l(k) \tag{A1}$$

We then evaluate the backward difference of $V(k)$ on the trajectories of Eq. (4.4) to get

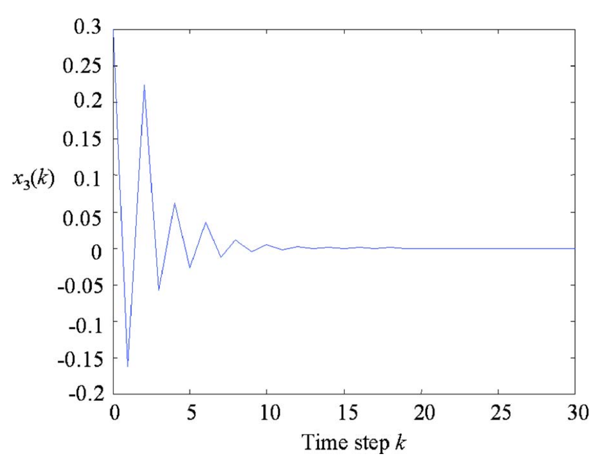

Fig. 5 The state $x_3(k)$ of subsystem 3

$$\Delta V(k) = V(k+1) - V(k)$$
$$= \sum_{l=1}^{L}\left[\left(\sum_{i=1}^{r_l}\sum_{j=1}^{J_l} h_{il}(k)\bar{h}_{jl}(k)H_{ijl}X_l(k) + \phi_l(k)\right)^T \times P_l\left(\sum_{d=1}^{r_l}\sum_{f=1}^{J_l} h_{dl}(k)\bar{h}_{fl}(k)H_{dfl}X_l(k) + \phi_l(k)\right) - X_l^T(k)P_lX_l(k)\right]$$
$$= D_1 + D_2 + D_3 + D_4 + D_5 + D_6 \tag{A2}$$

where

$$D_1 \equiv \sum_{l=1}^{L}\sum_{i=d=1}^{r_l}\sum_{j=f=1}^{J_l} h_{il}^2(k)\bar{h}_{jl}^2(k)X_l^T(k)(H_{ijl}^TP_lH_{ijl} - P_l)X_l(k)$$
$$\leqslant \sum_{l=1}^{L}\sum_{i=1}^{r_l}\sum_{j=1}^{J_l} h_{il}^2(k)\bar{h}_{jl}^2(k)\lambda_M(Q_{ijl})X_l^T(k)X_l(k) \tag{A3}$$

$$D_2 \equiv \sum_{l=1}^{L}\sum_{i=d=1}^{r_l}\sum_{\substack{j=1\\ j\neq f}}^{J_l} h_{il}^2(k)\bar{h}_{jl}(k)\bar{h}_{fl}(k)X_l^T(k)(H_{ijl}^TP_lH_{ifl} - P_l)X_l(k)$$
$$= \sum_{l=1}^{L}\sum_{i=1}^{r_l}\sum_{j<f}^{J_l} h_{il}^2(k)\bar{h}_{jl}(k)\bar{h}_{fl}(k)X_l^T(k)[H_{ijl}^TP_lH_{ifl} + H_{ifl}^TP_lH_{ijl} - 2P_l]X_l(k)$$
$$\leqslant \sum_{l=1}^{L}\sum_{i=1}^{r_l}\sum_{j<f}^{J_l} h_{il}^2(k)\bar{h}_{jl}(k)\bar{h}_{fl}(k)\lambda_M(Q_{ijfl})X_l^T(k)X_l(k) \tag{A4}$$

$$D_3 \equiv \sum_{l=1}^{L}\sum_{\substack{i=1\\ i\neq d}}^{r_l}\sum_{j=f=1}^{J_l} h_{il}(k)h_{dl}(k)\bar{h}_{jl}^2(k)X_l^T(k)(H_{ijl}^TP_lH_{djl} - P_l)X_l(k)$$
$$= \sum_{l=1}^{L}\sum_{i<d}^{r_l}\sum_{j=1}^{J_l} h_{il}(k)h_{dl}(k)\bar{h}_{jl}^2(k)X_l^T(k)(H_{ijl}^TP_lH_{djl} + H_{djl}^TP_lH_{ijl} - 2P_l)X_l(k)$$
$$\leqslant \sum_{l=1}^{L}\sum_{i<d}^{r_l}\sum_{j=1}^{J_l} h_{il}(k)h_{dl}(k)\bar{h}_{jl}^2(k)\lambda_M(Q_{ijdl})X_l^T(k)X_l(k) \tag{A5}$$

$$D_4 \equiv \sum_{l=1}^{L}\sum_{\substack{i=1\\ i\neq d}}^{r_l}\sum_{\substack{j=1\\ j\neq f}}^{J_l} h_{il}(k)h_{dl}(k)\bar{h}_{jl}(k)\bar{h}_{fl}(k)X_l^T(k)(H_{ijl}^TP_lH_{dfl} - P_l)X_l(k)$$
$$= \sum_{l=1}^{L}\sum_{i<d}^{r_l}\sum_{j<f}^{J_l} h_{il}(k)h_{dl}(k)\bar{h}_{jl}(k)\bar{h}_{fl}(k)X_l^T(k)(H_{ijl}^TP_lH_{dfl} + H_{dfl}^TP_lH_{ijl} + H_{ifl}^TP_lH_{djl} + H_{djl}^TP_lH_{ifl} - 4P_l)X_l(k)$$
$$\leqslant \sum_{l=1}^{L}\sum_{i<d}^{r_l}\sum_{j<f}^{J_l} h_{il}(k)h_{dl}(k)\bar{h}_{jl}(k)\bar{h}_{fl}(k)\lambda_M(Q_{ijdfl})X_l^T(k)X_l(k) \tag{A6}$$

$$D_5 \equiv \sum_{l=1}^{L}\sum_{i=1}^{r_l}\sum_{j=1}^{J_l} h_{il}(k)\bar{h}_{jl}(k)X_l^T(k)H_{ijl}^TP_l\phi_l(k) + \sum_{l=1}^{L}\sum_{i=1}^{r_l}\sum_{j=1}^{J_l} h_{il}(k)\bar{h}_{jl}(k)\phi_l^T(k)P_lH_{ijl}X_l(k)$$
$$= \sum_{l=1}^{L}\sum_{i=1}^{r_l}\sum_{j=1}^{J_l}\sum_{\substack{n=1\\ n\neq l}}^{L} h_{il}(k)\bar{h}_{jl}(k)\{X_l^T(k)H_{ijl}^TP_lC_{nl}X_n(k) + X_n^T(k)C_{nl}^TP_lH_{ijl}X_l(k)\} \leqslant \sum_{l=1}^{L}\sum_{i=1}^{r_l}\sum_{j=1}^{J_l}\sum_{\substack{n=1\\ n\neq l}}^{L} h_{il}(k)\bar{h}_{jl}(k)$$
$$\times\{\beta X_l^T(k)H_{ijl}^TP_lC_{nl}C_{nl}^TP_lH_{ijl}X_l(k) + \beta^{-1}X_n^T(k)X_n(k)\} \quad \text{(from Lemma 1)}$$
$$= \sum_{l=1}^{L}\sum_{i=1}^{r_l}\sum_{j=1}^{J_l}\sum_{n=1}^{L} h_{il}(k)\bar{h}_{jl}(k)X_l^T(k)\beta H_{ijl}^TP_lC_{nl}C_{nl}^TP_lH_{ijl}X_l(k) + \sum_{l=1}^{L}\sum_{i=1}^{r_l}\sum_{j=1}^{J_l}\sum_{n=1}^{L} h_{il}(k)\bar{h}_{jl}(k)\beta^{-1}\frac{L-1}{L}X_n^T(k)X_n(k)$$
$$\leqslant \sum_{l=1}^{L}\sum_{i=1}^{r_l}\sum_{j=1}^{J_l} h_{il}(k)\bar{h}_{jl}(k)\left(\lambda_M(\bar{Q}_{ijl}) + \sum_{n=1}^{L}\beta^{-1}\frac{L-1}{L}\right)X_l^T(k)X_l(k)$$
$$= \sum_{l=1}^{L}\sum_{i=1}^{r_l}\sum_{j=1}^{J_l} h_{il}(k)\bar{h}_{jl}(k)\sigma_{ijl}X_l^T(k)X_l(k) \tag{A7}$$

with

$$\bar{Q}_{ijl} \equiv \beta H_{ijl}^TP_l\sum_{n=1}^{L}(C_{nl}C_{nl}^T)P_lH_{ijl}$$

$$\sigma_{ijl} \equiv \lambda_M(\bar{Q}_{ijl}) + \sum_{n=1}^{L}\beta^{-1}\frac{L-1}{L} = \lambda_M(\bar{Q}_{ijl}) + \beta^{-1}(L-1)$$

$$D_6 \equiv \sum_{l=1}^{L}\phi_l^T(k)P_l\phi_l(k)$$
$$= \sum_{l=1}^{L}\left\{\sum_{\substack{n=1\\ n\neq l}}^{L}[C_{nl}X_n(k)]^TP_l\sum_{\substack{n=1\\ n\neq l}}^{L}[C_{nl}X_n(k)]\right\}$$
$$\leqslant \sum_{l=1}^{L}\sum_{\substack{n=1\\ n\neq l}}^{L}[(L-1)\lambda_M(P_l)\|C_{nl}X_n(k)\|^2]$$
$$= \sum_{l=1}^{L}\sum_{\substack{n=1\\ n\neq l}}^{L}[(L-1)\lambda_M(P_n)\|C_{ln}X_l(k)\|^2]$$
$$\leqslant \sum_{l=1}^{L}\eta_l\|X_l(k)\|^2 \text{ with } \eta_l \equiv \sum_{\substack{n=1\\ n\neq l}}^{L}(L-1)\lambda_M(P_n)\|C_{ln}\|^2 \tag{A8}$$

Substituting Eqs. (A3)–(A8) into Eq. (A2) yields

$$
\begin{aligned}
\Delta V(k) \leqslant \sum_{l=1}^{L} \Bigg\{ & \sum_{i=1}^{r_l} \sum_{j=1}^{J_l} h_{il}^2(k)\bar{h}_{jl}^2(k)\lambda_M(Q_{ijl}) \\
& + \sum_{i=1}^{r_l} \sum_{j<f}^{J_l} h_{il}^2(k)\bar{h}_{jl}(k)\bar{h}_{fl}(k)\lambda_M(Q_{ijfl}) \\
& + \sum_{i<d}^{r_l} \sum_{j=1}^{J_l} h_{il}(k)h_{dl}(k)\bar{h}_{jl}^2(k)\lambda_M(Q_{ijdl}) \\
& + \sum_{i<d}^{r_l} \sum_{j<f}^{J_l} h_{il}(k)h_{dl}(k)\bar{h}_{jl}(k)\bar{h}_{fl}(k)\lambda_M(Q_{ijdfl}) \\
& + \sum_{d=1}^{r_l} \sum_{f=1}^{J_l} \sum_{i=1}^{r_l} \sum_{j=1}^{J_l} h_{dl}(k)\bar{h}_{fl}(k)h_{il}(k)\bar{h}_{jl}(k)\alpha_{ijl} \Bigg\} \\
& \times \|X_l(k)\|^2 \text{ with } \alpha_{ijl} \equiv \sigma_{ijl} + \eta_l \\
= \sum_{l=1}^{L} \Bigg\{ & \sum_{i=1}^{r_l} \sum_{j=1}^{J_l} h_{il}^2(k)\bar{h}_{jl}^2(k)\lambda_M(Q_{ijl}) \\
& + \sum_{i=1}^{r_l} \sum_{j<f}^{J_l} h_{il}^2(k)\bar{h}_{jl}(k)\bar{h}_{fl}(k)\lambda_M(Q_{ijfl}) \\
& + \sum_{i<d}^{r_l} \sum_{j=1}^{J_l} h_{il}(k)h_{dl}(k)\bar{h}_{jl}^2(k)\lambda_M(Q_{ijdl}) \\
& + \sum_{i<d}^{r_l} \sum_{j<f}^{J_l} h_{il}(k)h_{dl}(k)\bar{h}_{jl}(k)\bar{h}_{fl}(k)\lambda_M(Q_{ijdfl}) \\
& + \sum_{i=d=1}^{r_l} \sum_{j=f=1}^{J_l} h_{il}^2(k)\bar{h}_{jl}^2(k)\alpha_{ijl} + \sum_{i=d=1}^{r_l} \sum_{j<f}^{J_l} h_{il}^2(k)\bar{h}_{jl}(k)\bar{h}_{fl}(k) \\
& \times (\alpha_{ijl} + \alpha_{ifl}) + \sum_{i<d}^{r_l} \sum_{j=f=1}^{J_l} h_{il}(k)h_{dl}(k)\bar{h}_{jl}^2(k)(\alpha_{ijl} + \alpha_{djl}) \\
& + \sum_{i<d}^{r_l} \sum_{j<f}^{J_l} h_{il}(k)h_{dl}(k)\bar{h}_{jl}(k)\bar{h}_{fl}(k)(\alpha_{ijl} + \alpha_{ifl} + \alpha_{djl} + \alpha_{dfl}) \Bigg\} \\
& \times \|X_l(k)\|^2 = \sum_{l=1}^{L} \Bigg\{ \sum_{i=1}^{r_l} \sum_{j=1}^{J_l} h_{il}^2(k)\bar{h}_{jl}^2(k)\psi_{ijl} \\
& + \sum_{i=1}^{r_l} \sum_{j<f}^{J_l} h_{il}^2(k)\bar{h}_{jl}(k)\bar{h}_{fl}(k)\psi_{ijfl} \\
& + \sum_{i<d}^{r_l} \sum_{j=1}^{J_l} h_{il}(k)h_{dl}(k)\bar{h}_{jl}^2(k)\psi_{ijdl} \\
& + \sum_{i<d}^{r_l} \sum_{j<f}^{J_l} h_{il}(k)h_{dl}(k)\bar{h}_{jl}(k)\bar{h}_{fl}(k)\psi_{ijdfl} \Bigg\} \|X_l(k)\|^2 \qquad \text{(A9)}
\end{aligned}
$$

where ψ_{ijl}, ψ_{ijfl}, ψ_{ijdl}, and ψ_{ijdfl} are defined in Eqs. (4.5a), (4.5b), (4.5c), and (4.5d). Based on Eq. (4.5), we have $\Delta V(k) < 0$ and the proof of Theorem 1 is thereby completed.

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