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Decision Support

Extended VIKOR method in comparison with outranking methods

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Abstract

The VIKOR method was developed to solve MCDM problems with conflicting and noncommensurable (different units) criteria, assuming that compromising is acceptable for conflict resolution, the decision maker wants a solution that is the closest to the ideal, and the alternatives are evaluated according to all established criteria. This method focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria, and on proposing compromise solution (one or more). The VIKOR method is extended with a stability analysis determining the weight stability intervals and with trade-offs analysis. The extended VIKOR method is compared with three multicriteria decision making methods: TOPSIS, PROMETHEE, and ELECTRE. A numerical example illustrates an application of the VIKOR method, and the results by all four considered methods are compared.

Keywords: Multiple criteria analysis; Compromise; Extended VIKOR method

1. Introduction

Multicriteria optimization (MCO) is considered as the process of determining the best feasible solution according to established criteria which represent different effects. However, these criteria usually conflict with each other and there may be no solution satisfying all criteria simultaneously. Thus, the concept of Pareto optimality was introduced for a vector optimization problem (Pareto, 1896; Kuhn and Tucker, 1951; Zadeh, 1963). Pareto optimal solutions have the characteristic that, if one criterion is to be improved, at least one other criterion has to be made worse. In such cases, a system analyst can aid the decision making process by making a comprehensive analysis and by listing the important properties of the Pareto optimal (noninferior) solutions. However, in engineering and management practice there is a need to select a final solution to be implemented. An approach to determine a final solution as a compromise was introduced by Yu (1973), and other distance-based techniques have also been developed (Chen and Hwang, 1992). A comparison of three

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multicriteria methods, SMART (weighted sum), Centroid method, and PROMETHEE, was presented by Olson (2001), and a comparative study of MCDM methods is presented in (Triantaphyllou, 2000).

The VIKOR method was developed as a multicriteria decision making method to solve a discrete decision problem with noncommensurable and conflicting criteria (Opricovic and Tzeng, 2004). This method focuses on ranking and selecting from a set of alternatives, and determines compromise solutions for a problem with conflicting criteria, which can help the decision makers to reach a final decision. Here, the compromise solution is a feasible solution which is the closest to the ideal, and a compromise means an agreement established by mutual concessions. Another distance-based method, the TOPSIS method, determines a solution with the shortest distance from the ideal solution and the farthest distance from the negative-ideal solution, but it does not consider the relative importance of these distances (Hwang and Yoon, 1981; Yoon, 1987). A detailed comparison of TOPSIS and VIKOR is presented in the article by Opricovic and Tzeng (2004).

The extended VIKOR method is presented in Section 2. The background for this method, including aggregation, normalization, and DM's preference assessment is presented in Section 3, that in someway justifies the VIKOR method. In Section 4, the VIKOR method is compared with three MCDM methods, TOPSIS, PROMETHEE and ELECTRE, providing a contribution to the state of the art of MCDM. An illustrative example illustrates an application of VIKOR method in Section 5, and the results by VIKOR are compared with results by the other methods, providing a contribution to the practice of MCDM.

2. The VIKOR method

The VIKOR method was developed to solve the following problem:

$$\underset{j}{mco}\{(f_{ij}(A_j),\ j=1,\ldots,J),\ i=1,\ldots,n\}, \tag{1}$$

where J is the number of feasible alternatives; $A_j = \{x_1, x_2, \ldots\}$ is the jth alternative obtained (generated) with certain values of system variables x; f_{ij} is the value of the ith criterion function for the alternative A_j; n is the number of criteria; mco denotes the operator of a multicriteria decision making procedure for selecting the best (compromise) alternative in multicriteria sense. Alternatives can be generated and their feasibility can be tested by mathematical models (determining variables x), physical models, and/or by experiments on the existing system or other similar systems. Constraints are seen as high-priority objectives, which must be satisfied in the alternatives generating process.

The VIKOR algorithm is presented in this Section, extended with a stability analysis determining the weight stability intervals and with trade-offs analysis. Assuming that each alternative is evaluated according to all criteria, the compromise ranking could be performed by comparing the measure of closeness to the ideal solution F^* (the best values of criteria). The multicriteria merit for compromise ranking is developed from the L_p-*metric* used in compromise programming method (Yu, 1973; Zeleny, 1982).

The compromise ranking algorithm VIKOR has the following steps:

(a) Determine the best f_i^* and the worst f_i^- values of all criterion functions, $i = 1, 2, \ldots, n$;

$$f_i^* = \max_j f_{ij}, \quad f_i^- = \min_j f_{ij}, \text{ if the } i\text{-th function represents a benefit;}$$

$$f_i^* = \min_j f_{ij}, \quad f_i^- = \max_j f_{ij}, \text{ if the } i\text{-th function represents a cost.}$$

(b) Compute the values S_j and R_j, $j = 1, 2, \ldots, J$, by the relations

$$S_j = \sum_{i=1}^{n} w_i (f_i^* - f_{ij})/(f_i^* - f_i^-), \tag{2}$$

$$R_j = \max_i [w_i (f_i^* - f_{ij})/(f_i^* - f_i^-)], \tag{3}$$

where w_i are the weights of criteria, expressing the DM's preference as the relative importance of the criteria.

(c) Compute the values Q_j, $j = 1,2,\ldots,J$, by the relation

$$Q_j = v(S_j - S^*)/(S^- - S^*) + (1 - v)(R_j - R^*)/(R^- - R^*), \tag{4}$$

where $S^* = \min_j S_j$, $S^- = \max_j S_j$, $R^* = \min_j R_j$, $R^- = \max_j R_j$; and v is introduced as a weight for the strategy of maximum group utility, whereas $1 - v$ is the weight of the individual regret.

(d) Rank the alternatives, sorting by the values S, R and Q in decreasing order. The results are three ranking lists.

(e) Propose as a compromise solution the alternative ($A^{(1)}$) which is the best ranked by the measure Q (minimum) if the following two conditions are satisfied:

C1. *Acceptable advantage:*

$$Q(A^{(2)}) - Q(A^{(1)}) \geqslant DQ,$$

where $A^{(2)}$ is the alternative with second position in the ranking list by Q; $DQ = 1/(J - 1)$.

C2. *Acceptable stability in decision making:*

The alternative $A^{(1)}$ must also be the best ranked by S or/and R. This compromise solution is stable within a decision making process, which could be the strategy of maximum group utility (when $v > 0.5$ is needed), or "by consensus" $v \approx 0.5$, or "with veto" ($v < 0.5$). Here, v is the weight of decision making strategy of maximum group utility.

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of

- Alternatives $A^{(1)}$ and $A^{(2)}$ if only the condition C2 is not satisfied, or
- Alternatives $A^{(1)}, A^{(2)}, \ldots, A^{(M)}$ if the condition C1 is not satisfied; $A^{(M)}$ is determined by the relation $Q(A^{(M)}) - Q(A^{(1)}) < DQ$ for maximum M (the positions of these alternatives are "in closeness").

(f) Determine the weight stability interval $[w_i^L, w_i^U]$ for each (ith) criterion, separately, with the initial (given) values of weights. The compromise solution obtained with initial weights (w_i, $i = 1,\ldots,n$), will be replaced at the highest ranked position if the value of a weight is out of the stability interval. The stability interval is only relevant concerning one-dimensional weighting variations.

(g) Determine the trade-offs, $tr_{ik} = |(D_i w_k)/(D_k w_i)|$, $k \neq i$, $k = 1,\ldots,n$, where tr_{ik} is the number of units of the ith criterion evaluated the same as one unit of the kth criterion, and $D_i = f_i^* - f_i^-$, $\forall i$. The index i is given by the VIKOR user.

(h) The decision maker may give a new value of tr_{ik}, $k \neq i$, $k = 1,\ldots,n$ if he or she does not agree with computed values. Then, VIKOR performs a new ranking with new values of weights $w_k = |(D_k w_i tr_{ki})/D_i|$, $k \neq i$, $k = 1,\ldots,n$; $w_i = 1$ (or previous value). VIKOR normalizes weights, with the sum equal to 1. The trade-offs determined in step (g) could help the decision maker to assess new values, although that task is very difficult.

(i) The VIKOR algorithm ends if the new values are not given in step (h).

The results by the VIKOR method are rankings by S, R, and Q, proposed compromise solution (one or a set), weight stability intervals for a single criterion, and the trade-offs introduced by VIKOR.

The VIKOR method is an effective tool in multicriteria decision making, particularly in situations where the decision maker is not able, or does not know to express his/her preference at the beginning of system design. The obtained compromise solution could be accepted by the decision makers because it provides a maximum group utility of the "majority" (represented by min S, Eq. (2)), and a minimum individual regret of the "opponent" (represented by min R, Eq. (3)). The compromise solutions could be the base for negotiation, involving the decision makers' preference by criteria weights.

The VIKOR result depends on the ideal solution (influencing function Q), which stands only for the given set of alternatives. Inclusion (or exclusion) of an alternative could affect the VIKOR ranking of the new set of alternatives. Giving the best f_i^* and the worst f_i^- values, this effect could be avoided, but that would mean that a fixed ideal solution could be defined by the decision maker.

Matching MCDM methods with classes of problems would address the correct applications, and for this reason the VIKOR characteristics are matched with a class of problems as follows:

- Compromising is acceptable for conflict resolution.
- The decision maker (DM) is willing to approve solution that is the closest to the ideal.
- There exist a linear relationship between each criterion function and a decision maker's utility.
- The criteria are conflicting and noncommensurable (different units).
- The alternatives are evaluated according to all established criteria (performance matrix).
- The DM's preference is expressed by weights, given or simulated.
- The VIKOR method can be started without interactive participation of DM, but the DM is in charge of approving the final solution and his/her preference must be included.
- The proposed compromise solution (one or more) has an advantage rate.
- A stability analysis determines the weight stability intervals.

Some applications were made using the VIKOR method, with the results published in international journals (Opricovic and Tzeng, 2002; Tzeng et al., 2002).

Several fundamental issues of the VIKOR method are discussed in the next section.

3. VIKOR background

Development of the VIKOR [*vikor*] method started with the following form of L_p-*metric*

$$L_{p,j} = \left\{ \sum_{i=1}^{n} [w_i (f_i^* - f_{ij})/(f_i^* - f_i^-)]^p \right\}^{1/p}, \quad 1 \leqslant p \leqslant \infty; \ j = 1, 2, \ldots, J \tag{5}$$

The measure $L_{p,j}$ was introduced by Duckstein and Opricovic (1980) and it represents the distance of the alternative A_j to the ideal solution. The compromise solution $F^c = (f_1^c, \ldots, f_n^c)$ is a feasible solution that is the "closest" to the ideal F^*. Here, compromise means an agreement established by mutual concessions, represented by $\Delta f_i = f_i^* - f_i^c$, $i = 1, \ldots, n$.

3.1. Aggregation

Major approaches to decision making include multiattribute utility theory and outranking methods (Keeney and Raiffa, 1976; Sawaragi et al., 1985; Vincke, 1992). The fundamental assumption in utility theory is that the decision maker chooses the alternative for which the expected utility value is a maximum. However, the difficulty is that in many problems it is not possible to obtain a mathematical representation of the decision maker's utility function U, so many aggregating functions are introduced instead of a global utility function (Butler et al., 2001).

Yu (1973) introduced compromise solutions, based on the idea of finding a feasible solution that is as close as possible to an ideal point. Zeleny (1982) stated that alternatives that are closer to the ideal are preferred to those that are farther away. To be as close as possible to a perceived ideal is the rationale of human choice. As an aggregating function Yu (1973) introduced L_p-metric for a distance function, called the group regret for a decision, a regret that the ideal cannot be chosen. Here, L_1 is the sum of all individual regrets (disutility), and L_∞ is the maximal regret that an individual could have (Tchebycheff norm was explored by Steuer (1986)). Yu (1973, 1985) and Freimer and Yu (1976) indicated several properties of compromise solutions, and the role of parameter p. Scott and Antonsson (2000) considered parameter p as an additional parameter of a decision, introducing "trade-off strategy". The TOPSIS method determines a solution with the shortest distance (Euclidean) from the ideal solution and the farthest distance from the negative-ideal solution, but it does not consider the relative importance of these distances (Hwang and Yoon, 1981; Yoon, 1987).

Development of the VIKOR method started with the form (5) of L_p-*metric* as an aggregating function. Within the VIKOR method, $L_{1,j}$ (as S_j in Eq. (2)) and $L_{\infty,j}$ (as R_j in Eq. (3)) are used (as "merit functions") to formulate ranking. The solution obtained by $\min_j S_j$ is with a maximum group utility (majority rule), and the solution obtained by $\min_j R_j$ is with a minimum individual regret of the "opponent". The merit function Q aggregates S and R with weight v, as in Eq. (4).

Aggregating (compound) function should be used with extreme caution since that involves comparing potentially incomparable quantities (noncommensurable criteria).

3.2. Normalization

To add values of noncommensurable criteria, first we have to convert then into the same units. Normalization is used to eliminate the units of criterion functions, so that all the criteria are dimensionless. By "simple normalization" the normalized value is determined, dividing the value of criterion function by its maximum value. This is a simple scale transformation, transforming all criterion values in a linear (proportional) way, but the scales are not with equal lengths. Linear normalization used within VIKOR method, vector normalization used within TOPSIS method, and the normalization effects are discussed by Opricovic and Tzeng (2004).

Normalization involves trade-offs, as discussed in the following Section 3.3.

3.3. Preference

Weighting coefficients (weights w_i) are introduced to express the relative importance of different criteria. These weights have no clear economic meaning, but their use gives the opportunity for modelling the actual decision making.

The stability of the ranking results to changes in the criteria weights was considered by Mareschal (1988), who proposed a procedure for sensitivity analysis that defines stability intervals for the weights. The values of the weight of one criterion within the stability interval do not alter the results obtained with the initial set of weights, and all other weights have initial ratios. Wolters and Mareschal (1995) considered the determination of stability intervals for MCDM "additive methods" such as PROMETHEE. However, the VIKOR method does not belong to this class of methods, and it determines the weight stability intervals using the procedure as follows.

The weight for the ith criterion function f_i may be increased or decreased from its initial value w_i, and this modified weight may be expressed as $w_i' = \lambda w_i$. Then in order to have the modified weights normalized, so that $\sum_{k=1}^{n} w_k' = 1$, other weights are modified, keeping initial ratios: $w_k' = \varphi w_k$, $k \neq i$, $k = 1, \ldots, n$. The function $\varphi(\lambda)$ is obtained from the equation $\lambda w_i + \varphi \sum_{k \neq i} w_k = 1$ in the following form $\varphi = (1 - \lambda w_i)/(1 - w_i)$. The parameter λ may be varied in the following interval $0 \leqslant \lambda \leqslant 1/w_i$. Applying the VIKOR method with different values of the parameter λ (searching), the interval $\lambda_1 \leqslant \lambda \leqslant \lambda_2$ can be obtained for the same compromise solution (obtained with initial weights). This interval we call the "stability interval". The weight stability interval for the ith criterion is

$$w_i^L \leqslant w_i' \leqslant w_i^U, \quad \text{where } w_i^L = \lambda_1 w_i, \text{ and } w_i^U = \lambda_2 w_i.$$

Then the weight stability intervals are determined for each criterion function, $i = 1, \ldots, n$, with the same (given) initial values of weights. The compromise solution obtained with initial weights (w_i, $i = 1, \ldots, n$), will be replaced at the highest ranked position if the value of a weight is out of the stability interval. Note however that the stability interval is only relevant concerning one-dimensional weighting variations.

Trade-offs assessment is one of the most difficult issues in MCDM and many methods have been developed to alleviate this problem. The VIKOR method introduces trade-offs in connection with the linear normalization used in Eqs. (2) and (3), assuming the decision maker (DM) is willing to approve these trade-offs. The weights w_i and dimension conversion coefficients $1/|f_i^* - f_i^-|$ in Eqs. (2) and (3) involve an assumption that all values $|D_i|/w_i$, $i = 1, \ldots, n$, where $D_i = f_i^* - f_i^-$, have the same "global utility", or $|D_i/w_i|\vec{i} \approx |D_k/w_k|\vec{k}$ where $\approx$ indicates indifference within the VIKOR method, whereas $\vec{i}$ and $\vec{k}$ represent units. The trade-offs $tr_{ik} = |(D_i w_k)/(D_k w_i)|$, $k \neq i$, $k = 1, \ldots, n$ are determined, where tr_{ik} is the number of units $\vec{i}$ of the ith criterion evaluated as same as one unit $\vec{k}$ of the kth criterion. This means that there exists indifference between tr units $\vec{i}$ and one unit $\vec{k}$. The VIKOR user gives the index i. The DM may give a new value of tr_{ik}, $k \neq i$, $k = 1, \ldots, n$, if he or she does not agree with the computed values. Then, VIKOR performs a new ranking with new values of weights $w_k = |(D_k w_i tr_{ki})/D_i|$, $k \neq i$, $k = 1, \ldots, n$; and $w_i = 1$ (or previous value). VIKOR normalizes weights, with the sum equal to 1.

4. Comparing VIKOR with other MCDM methods

Here the VIKOR method is compared with three different MCDM methods, TOPSIS, PROMETHEE and ELECTRE. These methods are selected as appropriate to point out the VIKOR background. The focus is on aggregating function and decision maker's preference.

4.1. VIKOR and TOPSIS

The VIKOR method uses an aggregating function Q in (4), representing "closeness to the ideal". The TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) method determines a solution with the shortest distance from the ideal solution and the farthest distance from the negative-ideal solution (Chen and Hwang, 1992; Tzeng et al., 1994). TOPSIS introduces an aggregating function $C_j^* = D_j^- / (D_j^* + D_j^-)$, where D_j^* is the distance from the ideal, and D_j^- is the distance from the negative-ideal. According to the formulation of C_j^* (ranking index), the alternative A_j is better than A_m if $C_j^* > C_m^*$ or $D_j^-/(D_j^* + D_j^-) > D_m^-/(D_m^* + D_m^-)$, which can hold if

1. $D_j^* < D_m^*$ and $D_j^- > D_m^-$ or
2. $D_j^* > D_m^*$, $D_j^- > D_m^-$, and $D_j^* < D_m^* D_j^- / D_m^-$.

Let A_m be an alternative with $D_m^* = D_m^-$ and $C_m^* = 0.5$, then all alternatives A_j with $D_j^* > D_m^*$ and $D_j^- > D_j^*$ are better ranked than A_m, although A_m is closer to the ideal A^*. This indicates that a solution by TOPSIS is not always the closest to the ideal. The relative importance of distances D_j^* and D_j^- was not considered, although it could be of major concern in decision making. A detailed comparison of TOPSIS and VIKOR is presented in the article by Opricovic and Tzeng (2004).

4.2. VIKOR and PROMETHEE

The PROMETHEE (Preference Ranking Organization METHod for Enrichment Evaluations) method introduces "net preference flow" as an aggregating (utility) function (Brans et al., 1984).

The net preference flow is formulated as follows:

$$\Phi_j = \Phi_j^+ - \Phi_j^-, \quad j = 1, \ldots, J,$$

where $\Phi_j^+ = \sum_{m=1}^{J} \Pi(A_j, A_m)$, a positive flow

$$\Phi_j^- = \sum_{m=1}^{J} \Pi(A_m, A_j), \text{ a negative flow,}$$

$$\Pi(A_j, A_m) = \sum_{i=1}^{n} w_i P_i(A_j, A_m),$$

$$P_i(A_j, A_m) = P(|f_i(A_j) - f_i(A_m)|) \text{ if } A_j \succ A_m \text{ (better), otherwise } P_i(A_j, A_m) = 0.$$

Six possible types of preference function P are proposed for comparing alternatives; 5 are linear or stepwise linear and one has Gaussian shape. A decision maker can use one of these 6 types of preference functions $P_i(A_j, A_m)$.

A comparative analysis of VIKOR and PROMETHEE shows that PROMETHEE and decision "by S_j" in VIKOR have the same MCDM foundation ("group utility" by summing). Since the VIKOR method assumes existing of linear relationship between each criterion function and a decision maker's utility, let us assume the PROMETHEE use linear preference function. In this case, there is a linear relationship between Φ in PROMETHEE and S (Eq. (2)) in VIKOR

$$\Phi_j = -JS_j + c \quad \left(\text{where } c = \sum_{j=1}^{J} S_j\right). \tag{6}$$

The derivation is presented in Appendix A.

Alternative A_j is better than A_m according to Φ if $\Phi_j > \Phi_m$, and according to S if $S_j < S_m$. From Eq. (6) we may conclude that ranking results by PROMETHEE are the same as ranking "by S_j" in VIKOR, when PROMETHEE uses linear preference function (type III).

The PROMETHEE method offers 6 types of preference (utility) function, while the VIKOR method introduces linear normalization. A result by PROMETHEE is based on the maximum of group utility, whereas the VIKOR method integrates maximum group utility and minimal individual regret.

4.3. VIKOR and ELECTRE

The ELECTRE methods (I, II, III, IV) have been developed based on Roy's philosophy of decision aid discussed for instance in (Roy, 1996). The methods ELECTRE II, III and IV are designed for ranking problems. The ELECTRE II and III are used when it is possible and desirable to quantify the relative importance of criteria and ELECTRE IV when this quantification is not possible. The ELECTRE II is founded on the concepts of concordance and discordance. The ELECTRE III was originally developed by Roy (on the traces of ELECTRE II) to incorporate the fuzzy nature of decision making, by using thresholds of indifference and preference. We chose the ELECTRE II as appropriate one to compare with VIKOR in order to point out the VIKOR background.

The ELECTRE II (ELimination and (Et) Choice Translating REality) method is an approach to multicriteria decision aid, based on the outranking relation (Roy and Bertier, 1972), and introducing "concordance" and "discordance". It provides good pairwise comparisons.

The concordance condition for alternatives A_j and A_m is formulated as

$$\sum_{i\in I^+,I^=} w_i \Big/ \sum_{i\in I} w_i \geqslant q \quad \text{and} \quad \sum_{i\in I^+} w_i > \sum_{i\in I^-} w_i,$$

where $I^+(A_j,A_m) = \{i : f_i(A_j) \succ f_i(A_m)\}$; $I^-(A_j,A_m) = \{i : f_i(A_j) \prec f_i(A_m)\}$; $I^=(A_j,A_m) = \{i : f_i(A_j) = f_i(A_m)\}$. The parameter q is the minimal level of concordance for alternative A_j to outranks A_m, $A_j \succ A_m$. The concordance index represent the strength of arguments favouring the statement A_j outranks A_m.

The discordance condition for alternatives A_j and A_m is formulated as

$$(1/C) \times \max_{i\in I^-} |s_i(A_j) - s_i(A_m)| \leqslant r,$$

where s_i is the "surrogate" ith criterion function; C is for scaling; and the parameter r is the maximum level of discordance compatible with the assertion A_j outranks A_m. The parameters $q, r \in [0,1]$ although for a real application interesting intervals are $0.5 \leqslant q \leqslant 1$ and $0 \leqslant r \leqslant 0.5$.

To some extent ELECTRE and VIKOR are based on similar principles as:

(a) Consideration of a certain global measure (concordance and group utility).
(b) The opposition of the other criteria—the "minority"—is not too strong (nondiscordance).

A comparative analysis of VIKOR and ELECTRE shows that, under certain assumptions, discordance condition and decision "by R_j" in VIKOR have the same MCDM foundation (minimum individual regret). For complete ranking let us introduce here an aggregating (global) discordance index as follows:

$$d_j = \max_m \left\{ (1/C) \times \max_{i\in I^-} |s_i(A_j) - s_i(A_m)| \right\}.$$

Introducing the function s_i and constant C as follows:

$$s_i(A_j) = w_i(f_i(A_j) - f_i^-)/D_i \quad \text{and} \quad C = \max_i w_i$$

the following relation is derived in Appendix B:

$$d_j = R_j/C.$$

This relation confirms that the decision results by R and by discordance are based on individual regret, and the ranking results by R and by d are the same.

The decision "by S_j" in VIKOR has some MCDM characteristics similar to concordance, leading to maximum of group utility or strength of agreement (using summation, see Appendix B). In order to illustrate the similarity between merit S and concordance let us introduce an aggregating (global) concordance index as follows:

$$c_j = \sum_{m \neq j}^{J} c_{jm}/(J-1) \quad \text{where } c_{jm} = \sum_{i \in I^+, I^=} w_i.$$

Ranking results by S_j and by c_j, $j = 1, \ldots, J$, could be very similar since they are based on the similar decision foundation (S on global utility and c on global strength). There is no mathematical relation between c_j and S_j, although in many cases it is close to $c_j = 1 - S_j$ (see Tables 2 and 8).

The compromise solution by the VIKOR method provides a balance between a maximum group utility of the majority, obtained by measure S that represents concordance (agreement), and a minimum of individual regret of the opponent, obtained by measure R that represents discordance (disagreement).

5. Illustrative example

5.1. Hydropower system on the Drina River

Previous studies of hydropower potential for the Drina River, in the former Yugoslavia, have selected potential dam sites for reservoirs to provide hydropower. In addition, comprehensive analysis was required to resolve conflicting technical, social and environmental features. Even if the topographic surveys confirm that the required reservoir capacity is available, a hydrological solution may conflict with environmental, social, and cultural features.

The VIKOR method was applied to evaluate alternative hydropower systems on the Drina River. The alternatives were generated by varying two system parameters, dam site and dam height. The following six alternatives were selected for multicriteria optimization:

A_1 Hydropower system (HPS) Gorazde, one reservoir, normal level at 375 m.a.s.l;
A_2 HPS Gorazde 383;
A_3 Cascade HPS: Gorazde 352, Sadba 362, Ustikolina 373, Paunci 384;
A_4 Cascade HPS: Gorazde 375, Paunci 384;
A_5 Cascade HPS: Gorazde 362, Ustikolina 373, Paunci 384;
A_6 Cascade HPS: Sadba 362, Ustikolina 373, Paunci 384.

The systems consist of from one (A_1 and A_2) to four reservoirs (A_3). The dam site Gorazde is at river km 298, Sadba at km 301 (upstream), Ustikolina at km 307, and Paunci at km 315. The dams within a system with more than one reservoir form a cascade. The designed reservoir systems are evaluated according to the following criteria:

f_1 Profit (10^6 Dinar, Yugoslav currency);
f_2 Costs (10^6 Dinar);
f_3 Total energy produced (GW hour/year);
f_4 Peak energy produced (GW hour/year);
f_5 Number of homes to be relocated;
f_6 Area flooded by reservoirs (ha);
f_7 Number of villages to displace (even partially);
f_8 Environmental protection (grades 1–5).

The values of criterion functions are obtained by a comprehensive study of this reservoir system on Drina river, and the results are presented in Table 1. The multicriteria optimization task is to maximize the criterion functions f_1, f_3, f_4, and f_8, and to minimize functions f_2, f_5, f_6, and f_7.

5.2. Results by VIKOR

Alternatives are ranked using the VIKOR method with the data from Table 1 and four sets of weight values. The obtained results are presented in Table 2. The equal criteria weights, unnormalized values $W1 = \{w_i = 1,\ \forall i\}$, represent indifference of the decision maker. The criteria weights $W2 = \{w_i = 2, i = 1,2,3,4;\ w_i = 1, i = 5,6,7,8\}$ express an economic preference. The weights $W3 = \{w_i = 1, i = 1,2,3,4;\ w_i = 2, i = 5,6,7,8\}$ express preferences for social attributes and environment, and $W4 = \{w_i = 1, i = 1,2,3,4;\ w_i = 3.2, i = 5,6,7,8\}$ emphasizes more social criteria. All these weights were proposed in order to analyse the preference stability of the compromise solution. Here the weight $v = 0.5$.

The ranking results in Table 2 indicate that alternative A_5 is the best ranked, with good advantage, for the weight sets $W1$, $W2$, and $W5$. With the weights $W3$, and $W4$ the compromise sets are obtained $\{A_5, A_3, A_6\}$, $\{A_3, A_5, A_6\}$, respectively. In these cases the first ranked alternative has no advantage to be a single solution. If the weights of social criteria are increased, such as $W4$, the alternative A_3 moves to the first place.

The weight stability intervals in Table 3 (for $W1$) show the stability of alternative A_5 as the highest ranked for small weight values, although it will loose the first place if some of the criteria is relatively highly preferred.

Table 1
Performance matrix

Criteria				Alternatives					
	Name	Unit	Extrem	A_1	A_2	A_3	A_4	A_5	A_6
f_1	Profit	10^6 Din	Max	4184.3	5211.9	5021.3	5566.1	5060.5	4317.9
f_2	Costs	10^6 Din	Min	2914.0	3630.0	3920.5	3957.9	3293.5	2925.9
f_3	Total energy produced	GW hour	Max	407.2	501.7	504.0	559.5	514.1	432.8
f_4	Peak energy produced	GW hour	Max	251.0	308.3	278.6	335.3	284.2	239.3
f_5	Homes to be relocated	Num.	Min	195	282	12	167	69	12
f_6	Reservoirs area	ha	Min	244	346	56	268	90	55
f_7	Villages to displace	Num.	Min	15	21	3	16	7	3
f_8	Environmental protect.	Grade	Max	2.41	1.41	4.42	3.36	4.04	4.36

Table 2
Ranking by VIKOR

Weights			A_1	A_2	A_3	A_4	A_5	A_6
$W1$	Equal $w_i = 1$, $\forall i$	Q_j	0.991	1.0	0.473	0.670	**0.0**	0.578
		S_j	0.692	0.7	0.29	0.423	0.28	0.346
		R_j	0.125	0.125	0.121	0.125	0.067	0.125
$W2$	Economics $w_i = 2$, $i \leqslant 4$	Q_j	1.0	0.533	0.552	0.563	**0.0**	0.686
		S_j	0.701	0.6	0.386	0.365	0.317	0.459
		R_j	0.167	0.114	0.161	0.167	0.089	0.167
$W3$	Social $w_i = 2$, $i \geqslant 5$	Q_j	0.684	1.0	**0.147**	0.554	**0.041**	**0.191**
		S_j	0.683	0.8	0.193	0.48	0.243	0.232
		R_j	0.113	0.167	0.08	0.122	0.044	0.083
$W4$	"More social" $w_i = 3.2$, $i \geqslant 5$	Q_j	0.668	1.0	**0.051**	0.588	**0.058**	**0.078**
		S_j	0.678	0.857	0.138	0.513	0.222	0.167
		R_j	0.129	0.190	0.057	0.139	0.042	0.060
$W5$	From Table 5	Q_j	0.991	0.966	0.477	0.629	**0.0**	0.503
		S_j	0.69	0.664	0.331	0.424	0.301	0.383
		R_j	0.152	0.153	0.143	0.149	0.073	0.137

Table 3
Weight stability intervals $[w^L, w^U]$

	Weights $W1$			Weights $W4$		
	Initial	w^L	w^U	Initial	w^L	w^U
w_1	0.125	0.0	0.185	0.06	0.0	0.38
w_2	0.125	0.1	0.199	0.06	0.0	0.06
w_3	0.125	0.0	0.195	0.06	0.0	0.24
w_4	0.125	0.0	0.162	0.06	0.03	0.162
w_5	0.125	0.0	0.184	0.19	0.1	1.0
w_6	0.125	0.0	0.187	0.19	0.0	0.92
w_7	0.125	0.0	0.184	0.19	0.177	1.0
w_8	0.125	0.0	0.186	0.19	0.0	1.0

The alternative A_5 is a real compromise. The first position of alternative A_3 is stable with higher values of weights for criteria, f_5, f_6, f_7, and f_8 ("social" criteria), but only for a small value of w_2 for cost (see results for $W4$ in Table 3).

The trade-offs values determined by VIKOR are presented in Table 4, showing how many 10^6 Din are evaluated as one unit of kth criterion, for example, the tr_{25} (for $W1$) shows that one home (average) is 3.87 10^6 Din, whereas for $W4$ it is 12.37 10^6 Din.

The trade-offs values obtained by VIKOR match most economic trade-offs that existed in the region, and only tr_{28} seems too high.

The new trade-offs values were given by the decision maker, as presented in Table 5, and VIKOR determined the new weights. The ranking list by VIKOR is $A_5, A_3, A_6, A_4, A_2, A_1$ and the compromise solution with these new weights is alternative A_5.

Factor analysis (computing means of variables, standard deviations, sums of cross-products of deviations, correlation coefficients, eigenvalues and eigenvectors, performing a principal component solution and orthogonal rotation of a factor matrix) indicates two factors. Each factor underlies four criteria, the first one for f_5, f_6, f_7, and f_8, and the second one for f_1, f_2, f_3, and f_4. These two factors could be called the social factor and the economic factor, respectively. Local residents in many cases oppose hydropower systems due to the social factor.

5.3. Results by TOPSIS, PROMETHEE, and ELECTRE

This numerical experiment was done in order to illustrate the comparison of MCDM methods presented in Section 4. The input data are from Section 5.1, and additional data for the MCDM methods were given according to the statements in Section 4.

Table 4
Trade-offs by VIKOR

Weights		tr_{2k}, $k = 1,\ldots,n$ (10^6 Din/$\vec{k}$)							
		$\vec{1}$	$\vec{2}$	$\vec{3}$	$\vec{4}$	$\vec{5}$	$\vec{6}$	$\vec{7}$	$\vec{8}$
$W1$	$w_i = 1$, $\forall i$	0.76	1	6.85	10.87	3.87	3.59	57.99	346.8
$W4$	$w_i = 3.2$, $i \geqslant 5$	0.76	1	6.85	10.87	12.37	11.48	185.6	1109.8

Table 5
New trade-offs and new weights

	$\vec{1}$	$\vec{2}$	$\vec{3}$	$\vec{4}$	$\vec{5}$	$\vec{6}$	$\vec{7}$	$\vec{8}$
tr_{2k}, $k = 1,\ldots,n$	0.66	1	7	10	4	2	60	100
New weights	0.130	0.149	0.152	0.137	0.154	0.083	0.154	0.043
New weights ($w_2 = 1$)	0.87	1	1.02	0.92	1.03	0.56	1.03	0.29

Alternatives A_3 and A_5 are ranked the highest by TOPSIS, and they are very close to each other (Table 6). The results by TOPSIS using vector normalization are different from the results by VIKOR with weights $W1$ and $W2$. Alternative A_5 is ranked the highest by VIKOR, whereas TOPSIS ranks A_3 highest. In Section 4.1, it was stated that the solution by TOPSIS is not always the closest to the ideal. This is the case with the results using vector normalization with weights $W2$, where the top ranked is A_3 (by C_j^*), although the alternative closest to the ideal is A_5 (by D_j^*).

The PROMETHEE method was applied using preference function P with linear shape (type III)

$$P_i(A_j, A_m) = \begin{cases} 0 & \text{if } \Delta f_i \leqslant 0, \\ \Delta f_i/\rho_i & \text{if } 0 < \Delta f_i \leqslant \rho_i, \\ 1 & \text{if } \Delta f_i > \rho_i, \end{cases}$$

where ρ_i is the parameter introduced by PROMETHEE; $\Delta f_i = |f_i(A_j) - f_i(A_m)|$ only if $A_j \succ A_m$ (better), otherwise set $\Delta f_i = 0$. For this experiment it is $\rho_i = |f_i^* - f_i^-|$. The results by PROMETHEE with weights $W1 = \{w_i = 1,\ \forall i\}$ are presented in Table 7. The alternatives are ranked in the following order: $A_5, A_3, A_6, A_4, A_1, A_2$, which is the same as the ranking "by S_j" in VIKOR in Table 2. The numerical results in Table 7 (by PROMETHEE) and S_j by VIKOR in Table 2 confirm Eq. (6), in this example it has the following form:

$$\Phi_j = -6S_j + 2.731.$$

The numerical results by PROMETHE (Φ in Table 7) and by VIKOR (S in Table 2) are consistent with their common foundations discussed in Section 4.2.

The ELECTRE method was applied using parameters $q = 0.6$, $r = 0.5$, and the "surrogate" function $s_i(A_j) = C(f_i(A_j) - f_i^-)/D_i$ (here $C = \max_i w_i$).

Table 6
Ranking by TOPSIS

Weights	Norm.	Ranking						
$W1$ $w_i = 1, \forall i$	Vector	C_j^*	$A_3(0.88)$	$A_6(0.85)$	$A_5(0.80)$	$A_4(0.40)$	$A_1(0.34)$	$A_2(0.12)$
		D_j^*	$A_3(0.02)$	$A_6(0.03)$	$A_5(0.03)$	$A_4(0.09)$	$A_1(0.10)$	$A_2(0.14)$
	Linear	C_j^*	$A_5(0.70)$	$A_3(0.64)$	$A_6(0.59)$	$A_4(0.55)$	$A_2(0.36)$	$A_1(0.36)$
		D_j^*	$A_5(0.11)$	$A_3(0.16)$	$A_6(0.20)$	$A_4(0.20)$	$A_1(0.27)$	$A_2(0.27)$
$W2$ $w_i = 2, i \leqslant 4$	Vector	C_j^*	$A_3(0.78)$	$A_5(0.77)$	$A_6(0.74)$	$A_4(0.45)$	$A_1(0.34)$	$A_2(0.22)$
		D_j^*	$A_5(0.02)$	$A_3(0.03)$	$A_6(0.03)$	$A_4(0.06)$	$A_1(0.07)$	$A_2(0.09)$
	Linear	C_j^*	$A_5(0.65)$	$A_4(0.60)$	$A_3(0.53)$	$A_2(0.48)$	$A_6(0.47)$	$A_1(0.37)$
		D_j^*	$A_5(0.14)$	$A_4(0.20)$	$A_3(0.21)$	$A_2(0.22)$	$A_6(0.26)$	$A_1(0.30)$
$W4$ $w_i = 3.2, i \geqslant 5$	Vector	C_j^*	$A_3(0.96)$	$A_6(0.95)$	$A_5(0.81)$	$A_4(0.38)$	$A_1(0.33)$	$A_2(0.04)$
		D_j^*	$A_3(0.01)$	$A_6(0.01)$	$A_5(0.04)$	$A_4(0.14)$	$A_1(0.14)$	$A_2(0.21)$
	Linear	C_j^*	$A_3(0.84)$	$A_6(0.80)$	$A_5(0.80)$	$A_4(0.45)$	$A_1(0.34)$	$A_2(0.16)$
		D_j^*	$A_3(0.07)$	$A_5(0.08)$	$A_6(0.09)$	$A_4(0.24)$	$A_1(0.27)$	$A_2(0.38)$

Table 7
Results by PROMETHEE

	Preference index $\Pi(A_j, A_m)$						Φ^+
	A_1	A_2	A_3	A_4	A_5	A_6	
A_1	–	0.253	0.120	0.142	0.045	0.017	0.578
A_2	0.245	–	0.091	0.039	0.045	0.227	0.647
A_3	0.523	0.501	–	0.302	0.085	0.176	1.587
A_4	0.412	0.317	0.169	–	0.149	0.342	1.389
A_5	0.458	0.465	0.094	0.292	–	0.192	1.502
A_6	0.363	0.582	0.119	0.419	0.126	–	1.610
Φ^-	2.002	2.119	0.594	1.194	0.451	0.954	
Φ	−1.424	−1.471	0.993	0.195	1.051	0.656	

Table 8
Results by ELECTRE

		Concordance (c_{jm}) and discordance (d_{jm}) index												c_j	d_j
		A_1		A_2		A_3		A_4		A_5		A_6			
		c_{jm}	d_{jm}	c_{jm}	d_{jm}	c_{jm}	d_{jm}	c_{jm}	d_{jm}	c_{jm}	d_{jm}	c_{jm}	d_{jm}		
$W1$	A_1	–	–	0.625	0.744	0.125	0.678	0.375	1.0	0.125	0.702	0.250	0.678	0.3	1.0
	A_2	0.375	0.686	–	–	0.375	1.0	0.125	0.648	0.250	0.880	0.375	1.0	0.3	1.0
	A_3	0.875	0.964	**0.625**	**0.309**	–	–	0.625	0.591	0.5	0.601	0.75	0.953	0.675	0.964
	A_4	0.625	1.0	**0.875**	**0.314**	0.375	0.728	–	–	0.375	0.636	0.375	0.989	0.525	1.0
	A_5	**0.875**	**0.363**	**0.750**	**0.251**	0.5	0.222	0.625	0.532	–	–	0.375	0.352	0.625	0.532
	A_6	**0.750**	**0.122**	0.625	0.719	0.5	0.509	0.625	1.0	0.625	0.537	–	–	0.625	1.0
$W2$	A_1	–	–	0.5	0.744	0.167	0.334	0.333	1.0	0.167	0.702	0.333	0.324	0.3	1.0
	A_2	0.5	0.686	–	–	0.5	0.5	0.167	0.324	0.333	0.434	0.5	0.49	0.4	0.686
	A_3	0.833	0.964	0.5	0.309	–	–	0.5	0.591	0.333	0.6	0.75	0.953	0.583	0.964
	A_4	0.667	1.0	**0.833**	**0.314**	0.5	0.361	–	–	0.5	0.636	0.5	0.989	0.6	1.0
	A_5	**0.833**	**0.363**	**0.667**	**0.251**	**0.667**	**0.063**	0.5	0.532	–	–	0.5	0.352	0.633	0.532
	A_6	**0.667**	**0.122**	0.5	0.719	0.417	0.509	0.5	1.0	0.5	0.537	–	–	0.517	1.0
$W4$	A_1	–	–	**0.821**	**0.179**	0.060	0.941	0.440	0.560	0.060	0.941	0.119	0.881	0.3	0.678
	A_2	0.179	0.350	–	–	0.179	1.0	0.060	0.648	0.119	0.880	0.179	1.0	0.143	1.0
	A_3	**0.940**	**0.301**	**0.821**	**0.097**	–	–	**0.821**	**0.185**	**0.762**	**0.188**	**0.750**	**0.298**	0.819	0.301
	A_4	0.560	0.312	**0.940**	**0.098**	0.179	0.728	–	–	0.179	0.612	0.179	0.732	0.407	0.732
	A_5	**0.940**	**0.114**	**0.881**	**0.078**	0.238	0.222	**0.821**	**0.166**	–	–	0.179	0.222	0.612	0.222
	A_6	**0.881**	**0.038**	**0.821**	**0.225**	**0.631**	**0.128**	**0.821**	**0.312**	**0.821**	**0.146**	–	–	0.795	0.312

The results by ELECTRE II method are presented in Table 8 for three sets of weight values: "equal" unnormalized values $W1 = \{w_i = 1, \forall i\}$, "economic" $W2 = \{w_i = 2, i = 1,2,3,4;\ w_i = 1, i = 5,6,7,8\}$, and "more social" $W4 = \{w_i = 1, i = 1,2,3,4;\ w_i = 3.2, i = 5,6,7,8\}$ (as in Table 2). An alternative A_j, in the jth row, outranks A_m, in the mth column, if concordance and discordance conditions are both satisfied (bold face in Table 8). The numerical results with "equal" weights $W1$ determine the following outranking $A_3 \succ A_2$, $A_4 \succ A_2$, $A_5 \succ \{A_1, A_2\}$, and $A_6 \succ A_1$, satisfying concordance and discordance condition. According to concordance condition: $A_1 \succ A_2$, $A_3 \succ \{A_1, A_2, A_4, A_6\}$, $A_4 \succ \{A_1, A_2\}$, $A_5 \succ \{A_1, A_2, A_4\}$, $A_6 \succ \{A_1, A_2, A_4, A_5\}$. This partial outrankings point out A_5,A_3, A_6 as good alternatives, without complete ranking. With "economic" weights $W2$ there exit outranking: $A_4 \succ A_2$, $A_5 \succ \{A_1, A_2, A_3\}$, $A_6 \succ A_1$, and A_5 seems the best option. And with "social" weights $W4$ there exit: $A_1 \succ A_2$, $A_3 \succ \{A_1, A_2, A_4, A_5, A_6\}$, $A_4 \succ A_2$, $A_5 \succ \{A_1, A_2, A_4\}$, $A_6 \succ \{A_1, A_2, A_3, A_4, A_5\}$. In this case partial ranking by ELECTRE II is: (A_3, A_6), A_5, (A_1, A_4), A_2; and ranking by VIKOR is: $A_3 \approx A_5 \approx A_6$, A_4, A_1, A_2 (by Q in Table 2).

The numerical results for c_j and d_j in Table 8, and for S_j and R_j in Table 2, are consistent with the discussion in Section 4.3.

5.4. Discussion and proposed solution

The results indicate the set $\{A_3, A_5, A_6\}$ as good alternatives. The alternatives ranked highest by VIKOR are A_5 and A_3, of which alternative A_5 is closer to the ideal according to the "economic" criteria f_1,f_2,f_3,f_4. Alternative A_3 has the additional "defect" in that it is more expensive, although it would be preferred from the social point of view. As an alternative for a final solution, alternative A_5 could be considered the best compromise. A comparison of alternatives A_5 and A_3 is presented in Table 9, where d_{ij} denotes a normalized distance of jth alternative to the ideal F^* according to ith criterion.

Alternatives A_3 and A_5 are top ranked by TOPSIS, and they are very close to each other. Some results by TOPSIS are different from the results by VIKOR, and the solution by TOPSIS is not always the closest to the ideal. For certain weights, the alternative ranked highest by TOPSIS is A_3, whereas the closest to the ideal is A_5.

Ranking by PROMETHEE gives the same results as ranking "by S_j" in VIKOR. For the linear preference function, a linear relation holds between net preference flow, introduced by PROMETHEE, and measure S introduced by VIKOR in Eq. (2).

Table 9
Comparison of alternatives A_5 and A_3

Criteria				Comparison				
	Name	Unit	Extrem	A_5	A_3	d_{i5}	d_{i3}	$A_5 > A_3$?
f_1	Profit	10^6 Din	Max	5060.5	5021.3	0.366	0.394	$>$
f_2	Costs	10^6 Din	Min	3293.5	3920.5	0.364	0.964	$\gg$
f_3	Total energy produced	GW hour	Max	514.1	504.0	0.298	0.364	$\gg$
f_4	Peak energy produced	GW hour	Max	284.2	278.6	0.532	0.591	$>$
f_5	Homes to relocate	Num.	Min	69	12	0.211	0.0	$<$
f_6	Reservoirs area	Ha	Min	90	56	0.120	0.003	$<$
f_7	Villages to displace	Num.	Min	7	3	0.222	0.0	$<$
f_8	Environmental protect.	Grade	Max	4.04	4.42	0.126	0.0	$<$

The results by ELECTRE II show outranking $A_5 \succ A_3$ for "economic" weights, and $A_3 \succ A_5$ "social" weights.

It may be concluded that three alternatives $\{A_3, A_5, A_6\}$ are indicated as good solutions. Alternatives A_5 and A_6 are similar three-reservoir systems, where two of the reservoirs are the same. Alternative A_3 is a system of four small reservoirs. The decision makers for the Drina project prefer alternative A_5, which could be developed in two phases. The first phase develops the system of two reservoirs, and the second phase adds the third reservoir, with a different dam site that could be analyzed later (alternatives A_5 and A_6).

6. Conclusions

The VIKOR method focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. It determines a compromise solution that could be accepted by the decision makers because it provides a maximum group utility for the "majority", and a minimum of individual regret for the "opponent". The extended VIKOR method determines the weight stability intervals and trade-offs.

The VIKOR method is based on an aggregating function representing "closeness to the ideal", using linear normalization. The TOPSIS method introduces two reference points, using vector normalization, but it does not consider the relative importance of the distances from these points. Ranking by PROMETHEE, with a linear preference function, gives the same results as ranking by VIKOR, with measure S representing "group utility". Results by ELECTRE II, with linear "surrogate" criterion functions, are relatively similar to the results by VIKOR. The similar results PROMETHE-VIKOR (based on S-measure) and VIKOR-ELECTRE are consistent with the discussion in Section 4.

To decide which method to apply, matching methods with classes of appropriate problems are needed. The validation procedures have to be developed, and application feasibility should be explored. The conceptual and operational validation of the application of a method in real world problems is needed. Researchers are challenged to provide a guide for choosing the method that is both theoretically well founded and practically operational to solve actual problems.

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Appendix A

The assumption in this paper is that the PROMETHEE method uses preference function P with linear shape (type III)

$$P_i(A_j, A_m) = \begin{cases} 0 & \text{if } \Delta f_i \leqslant 0, \\ \Delta f_i/\rho_i & \text{if } 0 < \Delta f_i \leqslant \rho_i, \\ 1 & \text{if } \Delta f_i > \rho_i, \end{cases} \quad \text{(A.1)}$$

where ρ_i is the parameter introduced by PROMETHEE; $\Delta f_i = |f_i(A_j) - f_i(A_m)|$ only if $A_j \succ A_m$ (better), otherwise set $\Delta f_i = 0$. For this comparison it is $\rho_i = |f_i^* - f_i^-|$. The relation (A.1) could be written as

$$P_{ijm} = (f_{ij} - f_{im})/D_i \quad \text{for } i \in I, \text{ where } I = \{i : f_{ij} \geqslant f_{im}\},$$

$$\Pi_{jm} = \sum_{i \in I} w_i P_{ijm},$$

$$\Phi_j = \sum_{m=1}^{J} \Pi_{jm} - \Pi_{mj},$$

$$\Phi_j = \sum_{m=1}^{J} \left[\sum_{i \in I} w_i (f_{ij} - f_{im})/D_i - \sum_{i \in I^-} w_i (f_{im} - f_{ij})/D_i \right], \text{ or}$$

$$\Phi_j = \sum_{m=1}^{J} \left[\sum_{i \in I} w_i f_{ij}/D_i + \sum_{i \in I^-} w_i f_{ij}/D_i - \sum_{i \in I} w_i f_{im}/D_i - \sum_{i \in I^-} w_i f_{im}/D_i \right].$$

Due to $|I \cup I^-| = n$, $\Phi_j = -\sum_{m=1}^{J} \sum_{i=1}^{n} w_i (f_i^* - f_{ij})/D_i + \sum_{m=1}^{J} \sum_{i=1}^{n} w_i (f_i^* - f_{im})/D_i$.
Since $S_j = \sum_{i=1}^{n} w_i (f_i^* - f_{ij})/D_i$, finally

$$\Phi_j = -JS_j + c, \quad \text{where } c = \sum_{m=1}^{J} S_m. \quad \text{(A.2)}$$

Appendix B

The *discordance condition* for alternatives A_j and A_m is formulated as

$$(1/C) \times \max_{i \in I^-} |s_i(A_j) - s_i(A_m)| \leqslant r. \quad \text{(B.1)}$$

Here the function s_i could have the following form:

$$s_i(A_j) = w_i(f_i(A_j) - f_i^-)/D_i \quad \text{and } C = \max_i w_i.$$

The discordance index in (B.1) could be written as

$$d_{jm} = (1/C) \max_{i \in I^-} |w_i (f_{ij} - f_{im})/D_i|$$

or since $I^-(A_j, A_m) = \{i : f_{ij} < f_{im}\}$

$$d_{jm} = (1/C) \max_i [w_i (f_{im} - f_{ij})/D_i].$$

The discordance condition provides pairwise comparisons, although it does not provide complete ranking. For complete ranking let us introduce here an aggregating discordance index as follows:

$$d_j = \max_m d_{jm} = \max_m (1/C) \max_i [w_i (f_{im} - f_{ij})/D_i], \text{ or}$$

$$d_j = \max_m (1/C) \max_i [w_i (f_i^* - f_{ij})/D_i - w_i (f_i^* - f_{im})/D_i], \quad \text{or}$$

$$d_j = (1/C) \max_i [w_i (f_i^* - f_{ij})/D_i - \min_m w_i (f_i^* - f_{im})/D_i].$$

Since $R_j = \max_i [w_i (f_i^* - f_{ij})/D_i]$, and $\min_m w_i (f_i^* - f_{im})/D_i = 0$, finally it is

$$d_j = R_j/C \quad \left(C = \max_i w_i \right). \quad \text{(B.2)}$$

The decision results by R and by discordance are based on minimizing individual regret.

The *concordance condition* for alternatives A_j and A_m is formulated as

$$\sum_{i\in I^+,I^=} w_i \Big/ \sum_{i\in I} w_i \geqslant q \quad \text{and} \quad \sum_{i\in I^+} w_i > \sum_{i\in I^-} w_i, \tag{B.3}$$

where $I^+(A_j, A_m) = \{i : f_i(A_j) \succ f_i(A_m)\}$; $I^-(A_j, A_m) = \{i : f_i(A_j) \prec f_i(A_m)\}$; $I^=(A_j, A_m) = \{i : f_i(A_j) = f_i(A_m)\}$. For a special case with equal weights, $w_i = 1/n$, the relations (B.3) have the following form:

$$|I^+ \cup I^=| \geqslant qn \quad \text{and} \quad |I^+| > (n - |I^=|)/2,$$

where $|I|$ denotes a cardinal number.

Ranking by S in VIKOR is based on $S_j < S_m$

$$\sum_{i=1}^{n} w_i(f_i^* - f_{ij})/D_i < \sum_{i=1}^{n} w_i(f_i^* - f_{im})/D_i, \text{ or}$$
$$\sum_{i=1}^{n} w_i f_{ij}/D_i > \sum_{i=1}^{n} w_i f_{im}/D_i.$$

There is no mathematical relationship between concordance index and merit S.

References

Brans, J.P., Mareschal, B., Vincke, Ph., 1984. PROMETHEE: A new family of outranking methods in multicriteria analysis. In: Brans, J.P. (Ed.), Operational Research '84. North-Holland, New York, pp. 477–490.

Butler, J., Morrice, D.J., Mullarkey, P.W., 2001. A multiple attribute utility theory approach to ranking and selection. Management Science 47 (6), 800–816.

Chen, S.J., Hwang, C.L., 1992. Fuzzy Multiple Attribute Decision Making: Methods and Applications. Springer-Verlag, Berlin.

Duckstein, L., Opricovic, S., 1980. Multiobjective optimization in river basin development. Water Resources Research 16 (1), 14–20.

Freimer, M., Yu, P.L., 1976. Some new results on compromise solutions for group decision problems. Management Science 22, 688–693.

Hwang, C.L., Yoon, K., 1981. Multiple Attribute Decision Making. Lecture Notes in Economics and Mathematical Systems, vol. 186. Springer-Verlag, Berlin.

Keeney, R., Raiffa, H., 1976. Decisions with Multiple Objectives—Preferences and Value Tradeoffs. John Wiley & Sons, New York.

Kuhn, H.W., Tucker, A.W., 1951. Nonlinear Programming. In: Neyman, J. (Ed.), Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability. University of California Press, pp. 481–492.

Mareschal, B., 1988. Weight stability intervals in multicriteria decision aid. European Journal of Operational Research 33 (1), 54–64.

Olson, D.L., 2001. Comparison of three multicriteria methods to predict know outcomes. European Journal of Operational Research 130 (3), 576–587.

Opricovic, S., Tzeng, G.H., 2002. Multicriteria planning of post-earthquake sustainable reconstruction. The Journal of Computer-Aided Civil and Infrastructure Engineering 17 (3), 211–220.

Opricovic, S., Tzeng, G.H., 2004. The Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. European Journal of Operational Research 156 (2), 445–455.

Pareto, V., 1896. Cours d'Economie Politique. Droz, Geneva.

Roy, B., 1996. Multicriteria Methodology for Decision Aiding. Kluwer, Dordrecht, The Netherlands.

Roy, B., Bertier, B., 1972. La metode ELECTRE II. Sixieme Conference internationale de rechearche operationelle. Dublin.

Sawaragi, Y., Nakayama, H., Tanino, T., 1985. Theory of Multiobjective Optimization. Academic Press, New York.

Scott, M.J., Antonsson, E.K., 2000. Using indifference points in engineering decisions. In: Proceedings of ASME Design Engineering Technical Conferences, Baltimore, USA.

Steuer, R.E., 1986. Multiple Criteria Optimization: Theory, Computation and Application. John Wiley & Sons, New York.

Triantaphyllou, E., 2000. Multi-Criteria Decision Making Methods: A Comparative Study. Kluwer Academic Publishers, Dordrecht.

Tzeng, G.H., Shiau, T.A., Teng, J.J., 1994. Multiobjective decision-making approach to energy supply mix decisions in Taiwan. Energy Sources 16 (3), 301–316.

Tzeng, G.H., Tsaur, S.H., Laiw, Y.D., Opricovic, S., 2002. Multicriteria analysis of environmental quality in Taipei: Public preferences and improvement strategies. Journal of Environmental Management 65 (2), 109–120.

Vincke, P., 1992. Multicriteria Decision-Aid. John Wiley & Sons, New York.

Wolters, W.T.M., Mareschal, B., 1995. Novel types of sensitivity analysis for additive MCDM methods. European Journal of Operational Research 81 (2), 281–290.

Yoon, K., 1987. A reconciliation among discrete compromise solutions. Journal of Operational Research Society 38 (3), 272–286.

Yu, P.L., 1973. A class of solutions for group decision problems. Management Science 19 (8), 936–946.

Yu, P.L., 1985. Multiple-criteria Decision Making: Concepts, Techniques, and Extensions. Plenum Publishing Corporation, New York.
Zadeh, L.A., 1963. Optimality and non-scalar-valued performance criteria. IEEE Transactions on Automatic Control 8 (1), 59–60.
Zeleny, M., 1982. Multiple Criteria Decision Making. McGraw-Hill, New York.