STRUCTURAL ANALYSIS MODEL FOR MAT FOUNDATIONS

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ABSTRACT: A simplified structural analysis model for mat foundations with grid floor beams as stiffeners is presented here. In the model, the Winkler type of subgrade reaction spring is assumed for the whole area under the mat foundation, and the yield-line theory of slab is used to lump the Winkler springs under slabs to the corresponding locations under the adjacent floor beams of the slabs. Therefore, each floor beam is now supported by springs with a segmentally linearly varied spring constant. The total analysis model for the mat foundation is then simplified to be a grid beam system on an elastic foundation with a segmentally linearly varied spring constant, and is subjected to loadings from the columns of the building structure. Some numerical comparisons with the results by a sophisticated finite-element model are made in order to demonstrate the effectiveness and efficiency of the presented analysis model.

INTRODUCTION

There are two types of mat foundations (Bowles 1982). One is just a solid thick plate without apparent floor beams and the other is a thin plate with grid floor beams as stiffeners as shown in Fig. 1. The former is better in terms of rigidity, which is assumed in the conventional design method. However, the latter is more popular since it saves a large volume of concrete and the cells enclosed by grid floor beams can be used as the tanks of drinking water, fire protection water, and sanitary water.

The conventional design method completely neglects the effect of the moments and shears induced by differential elastic settlement of subsoil. Therefore, the conventional design method is sometimes conservative and may be unrealistic especially for the type of thin plate with grid floor beams, since the reaction of subsoil at the areas near the loading columns is larger due to the flexibility of floor beams. This means that the conventional method could overdesign the foundations. This situation of overdesign could be amplified when the offset from property limit for the part of the building structure above ground level is large and, therefore, the difference between the reactions of subsoil at the edges of the foundation and at the center of the foundation are huge. Unfortunately, this is the case for most building structures in urban areas due to architectural reasons which do not allow a building structure above ground to cover the whole area of a property. Therefore, some offsets from property limits for building structures above ground level are required as specified in the most architectural codes. However, due to the parking problem of cars in the city most buildings are built to have basements as large as possible in order to provide sufficient parking space. This situation will make some of the floor beams of a mat foundation act like cantilevers, and sometimes make the design of the floor beams impossible if the conventional design method is used. Even if the floor beams can be designed, it would result in uneconomic design.

This paper is aimed at presenting a structural analysis model for a mat foundation with grid floor beams as shown in Fig. 1. To set up the model, the Winkler spring is adopted to describe elastic deformation of the subsoil subjected to loadings.

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The spring constant is assumed to be constant for the entire area under the mat foundation. To simplify the analysis model, the yield line theory (Johansen 1962) for bottom slabs is employed to lump the subgrade reaction springs to the locations under grid floor beams. Now, the model becomes grid floor beams on an elastic foundation with loadings (moments and vertical forces from columns) applied at the intersections of floor beams. Also, the spring constant of the elastic foundation may vary linearly along the beams since points under the floor beams represent the condensation of different lengths of the subgrade reaction coefficient by the yield line theory of slab. Therefore, the stiffness matrix for the beam on an elastic foundation with piecewise linear variation of the spring constant also has to be derived.

To demonstrate the effectiveness and efficiency of the presented model, a simple example is used to compare the proposed model with a sophisticated finite-element model in which plate elements on a constant elastic foundation and beam elements are used. Also, a fictitious building structure is used as an example for the structural analysis of its mat foundation by employing the presented model. After extensive numerical analysis by the proposed model, some conclusions and discussions are also presented in this paper.

ANALYSIS MODEL

The plan view of a typical mat foundation of a building structure is shown in Fig. 1. To set up the analysis model of mat foundation, the Winkler spring is first assumed for the mat foundation structure, which is constant for the whole region under the mat foundation. The spring constant (subgrade reaction coefficient) is highly dependent on the dimensions, shapes, and depth of the embedment of the foundation, and the mechanical properties of subsoil (especially shear modulus). To obtain the spring constant for the structural model of the mat foundation, one can refer to the works by Terzaghi (1955), Vesic (1961, 1963), Bowles (1982), Horvath (1983) and Scott (1981).
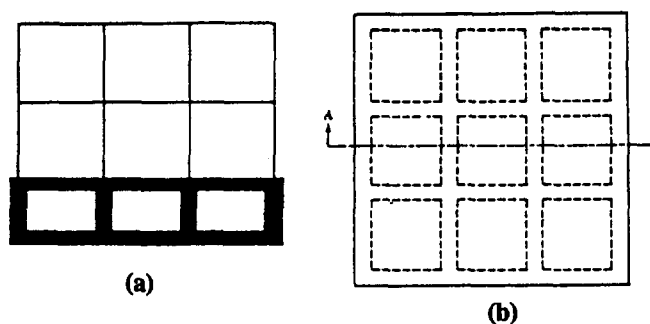

FIG. 1. Typical Mat Foundation with Grid Floor Beams: (a) Elevation A-A; (b) Plan View
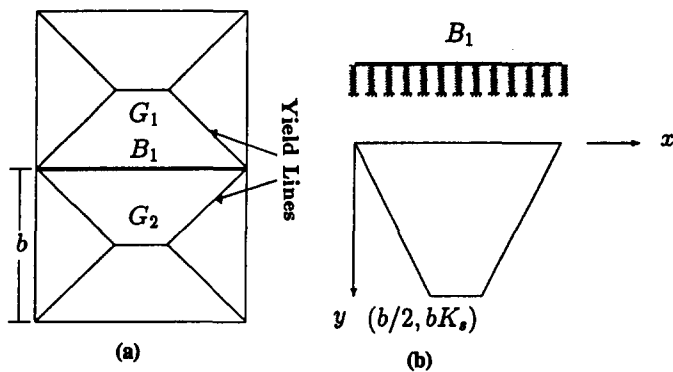
FIG. 2. (a) Yield Line Theory; (b) Variation of Spring Constant

After the subgrade reaction coefficient representing the subsoil has been obtained, the yield line theory for slabs is employed to lump the springs under the slabs to the corresponding locations under floor beams as shown in Fig. 2. In Fig. 2(a) the subgrade reaction springs under the areas G_1 and G_2 are condensed to the location under floor beam B_1. Therefore, the spring constant under the floor beams may vary linearly or keep constant for some portion of the floor beams as shown in Fig. 2(b). This kind of variation of the spring constant is called segmentally linear variation here. After the aforementioned simplification, the structural analysis model of the mat foundation in Fig. 1 becomes a grid structure resting on an elastic foundation with piecewise linear variation of the spring constant, and is subjected to the loadings (moments and vertical forces from columns) at the intersections of floor beams.

For the mat foundation grid model, one has to generate the stiffness matrix for each individual beam first before assembling the global stiffness matrix for the grid model. If one assumes the shapes of slabs adjacent to a floor beam can be rectangular, triangular, or trapezoidal, the beam at most has five segments with different slopes of linear variations of spring constants. For example, beam B_1 in Fig. 2 has three segments with different slopes of linear variations of spring constants. This is the consequence of using the yield line theory to condense the subgrade reaction coefficient under slabs. For each segment of the beam on the linear elastic foundation, the governing equation can be expressed as follows:

$$EI \frac{d^4Y(x)}{dx^4} + K(x)Y(x) = 0 \tag{1}$$

where EI = cross-sectional rigidity of the segment of the beam; and $K(x)$ = linearly varied spring constant. In (1), the external loadings can only be applied at the intersections of floor beams.

If $K(x)$ = constant in (1), the solution for (1) can be simply obtained as follows:

$$Y(x) = e^{\lambda x}(C_1 \cos \lambda x + C_2 \sin \lambda x)$$

$$+ e^{-\lambda x}(C_3 \cos \lambda x + C_4 \sin \lambda x) \tag{2}$$

where $\lambda^4 = K/4EI$; and C_1, C_2, C_3, and C_4 = unknown coefficients and can be determined by the boundary conditions of the beam segment.

For $K(x)$ which varies linearly, the solution can be obtained by the change of variables technique. The linearly varied spring constant is rewritten as

$$K(x) = \frac{K_b - K_a}{b} x + K_a = mx + K_a \tag{3}$$

where K_b and K_a = spring coefficients at both ends of the segment; and b = length of the segment. If $m > 0$, let variable $\xi = mx/K_b + K_a/K_b$. After some mathematical manipulations, (1) becomes

$$\frac{d^4Y(\xi)}{d\xi^4} + \alpha \xi Y(\xi) = 0 \tag{4}$$

where $\alpha = K_b/EI[(K_b/m)]^4$. The general solution for (4) is composed of four infinite polynomial series as follows (Hetenyi 1946):

$$Y(\xi) = C_1 Y_1(\xi) + C_2 Y_2(\xi) + C_3 Y_3(\xi) + C_4 Y_4(\xi) \tag{5a}$$

where

$$Y_1(\xi) = 1 - \frac{\alpha}{5!} \xi^5 + \frac{6\alpha^2}{10!} \xi^{10} - \frac{6 \cdot 11\alpha^3}{15!} \xi^{15}$$

$$+ \frac{6 \cdot 11 \cdot 16\alpha^4}{20!} \xi^{20} - \cdots \tag{5b}$$

$$Y_2(\xi) = \xi - \frac{2\alpha}{6!} \xi^6 + \frac{2 \cdot 7\alpha^2}{11!} \xi^{11} - \frac{2 \cdot 7 \cdot 12\alpha^3}{16!} \xi^{16}$$

$$+ \frac{2 \cdot 7 \cdot 12 \cdot 17\alpha^4}{21!} \xi^{21} - \cdots \tag{5c}$$

$$Y_3(\xi) = \frac{1}{2!} \xi^2 - \frac{3\alpha}{7!} \xi^7 + \frac{3 \cdot 8\alpha^2}{12!} \xi^{12} - \frac{3 \cdot 8 \cdot 13\alpha^3}{17!} \xi^{17}$$

$$+ \frac{3 \cdot 8 \cdot 13 \cdot 18\alpha^4}{22!} \xi^{22} - \cdots \tag{5d}$$

$$Y_4(\xi) = \frac{1}{3!} \xi^3 - \frac{4\alpha}{8!} \xi^8 + \frac{4 \cdot 9\alpha^2}{13!} \xi^{13} - \frac{4 \cdot 9 \cdot 14\alpha^3}{18!} \xi^{18}$$

$$+ \frac{4 \cdot 9 \cdot 14 \cdot 19\alpha^4}{23!} \xi^{23} - \cdots \tag{5e}$$

and the unknown coefficients C_1, C_2, C_3, C_4 can be determined by the boundary conditions of the beam segment.

For the case of $m < 0$, the solution for (1) is the same as that in (5) except when redefining $\xi = mx/K_a + 1$ and $\alpha = (K_a/EI)(K_a/m)^4$.

The accuracy of truncating the infinite series in (5b)–(5e) is dependent on the value of α, since, according to the definition in (4), ξ is always less than 1. A large α can occur only when m, the slope of variation of the spring constant as defined in (3), is approaching zero. Fortunately as $m \rightarrow 0$, one can just use the solution for constant $K(x)$ in (2) without losing accuracy. After some extensive numerical investigations, it is concluded that one just needs to control the α value within the limit of 10,000 for using the solution in (5a)–(5d) by truncating the terms after the fifth term in the infinite series, otherwise one must employ the solution of (2).

After the solution for each segment of beam has been obtained, the stiffness matrices for beams can be generated by employing the continuity conditions of displacement, rotation, moment, and shear at joints between segments and the boundary conditions at both ends of the beams. Now, the global stiffness matrix of the total model can be assembled and the total model is subjected to loadings from columns at the intersections of the floor beams.

NUMERICAL ANALYSIS

Two numerical examples are presented in order to demonstrate the effectiveness and efficiency of the proposed analysis model for the mat foundation of a thin plate with grid floor beams. The dimensions and nodal numbers of the first example are indicated in Fig. 3, and the foundation is subjected to a 1,224 t of vertical force at node 5 only. The cross section of the floor beams is assumed to be 3-m high and 0.6-m wide and Young's modulus of concrete is 3.0×10^6 t/m^2. The level of

the foundation is assumed to be at 10 m below ground level, and the shear modulus and Poisson's ratio are assumed to be 1,830 t/m^2 and 0.45 for subsoil, respectively. Terzaghi's (1955) method is used to calculate the subgrade reaction coefficient for the analysis model.

To investigate the accuracy of the proposed model, the analysis results of the model are compared to that of the finite-element model in which the commercial computer program ANSYS 50 is used. In the finite-element model, beam elements and plate elements on an elastic foundation are used to model the floor beams, slabs, and the subgrade reaction coefficient. After some comparison study, one can find that each beam in Fig. 3 has to be modeled by eight identical beam elements and each slab has to be modeled by 64 identical plate elements in order to obtain good results by the finite-element method. Three kinds of thickness (0.5, 0.6, and 0.7 m) of slab, which are commonly used in design practice for a mat foundation with grid floor beams, are chosen in the finite-element model in order to see the effect of thickness of slab on the overall structural behavior of the mat foundation.

For simplicity of presentation, only the results for beam 4_5, in which the numbers are referred to nodal numbers in Fig. 3, are compared in the paper. Fig. 4 shows the comparisons of the displacements of the beams in the proposed model with that in the finite-element model. In the finite-element model, one can see Figs. 3 and 4 that three different thicknesses of slab are used. On examining these figures, one can observe that the proposed model gives pretty good results for the displacement of beams and the effect of thickness of slab on the structural behavior is not important. Again, one can also observe from Fig. 5 that the results of moment by the proposed model agree with those by the sophisticated finite-element model. After examining the comparison in the example and other numerical comparisons, one can easily conclude that the
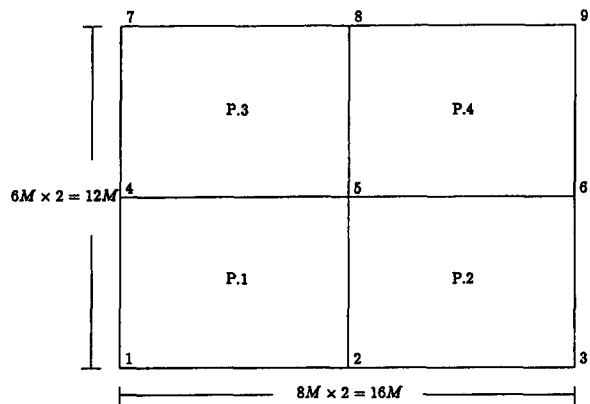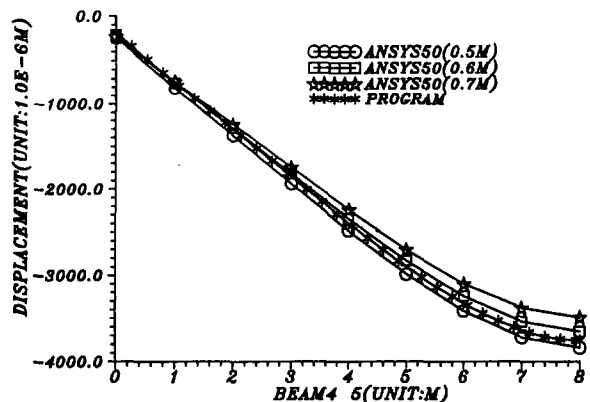
proposed model can give quite accurate results and is very efficient.

Also, one can compare the total moment of the three moments at the critical cross sections of the beams 1_2, 4_5, and 7_8 with the corresponding total moment by conventional analysis method, which assumes the subgrade reaction is uniformly distributed. The total moment by the proposed method is 1,470 t/m and the corresponding total moment by the conventional method is 2,330 t/m. This means that the conventional analysis method gives a very conservative result and will make the design uneconomic.

The second example presented is the foundation for a fictitious building. The building is 20 stories above ground level and two stories below ground level, with a 56 × 48 m foundation base. To calculate the loadings applied at the nodal points of the mat foundation, a uniform 1.0 t/m^2 loading is assumed for the floors above the ground and a uniform 1.5 t/m^2 loading for the ground and basement floors. The depth of embedment of the foundation is 10 m, and all other basic data are the same as those used in the first example. In the example, the average subgrade reaction on each slab of the foundation is also calculated, which will be useful for the design of the slab. To calculate the average reaction, one just needs to uniformly redistribute the spring reactions under the beams around the slab. The average reactions on slabs are shown in Table 1. In the table, only a quarter number of slabs for the foundation is shown, since the foundation and loadings are symmetric with respect to both the x and y directions. From Table 1, one can see that the reaction on the slab ranges from 6.2 t/m^2 to 26 t/m^2 in contrast to a uniformly distributed 13.3 t/m^2 in the conventional analysis method.

Since the subgrade reaction spring cannot resist tensile stress, uplift may occur at some part of the foundation during a rainstorm. For this example, the foundation is located at 10 m below the ground level, thus a uniform 10 t/m^2 buoyancy is also considered. The 10 t/m^2 uplift stress is greater than some reactions in Table 1. To cancel the tensile stress caused by buoyancy, one can release the tensile stress using the same
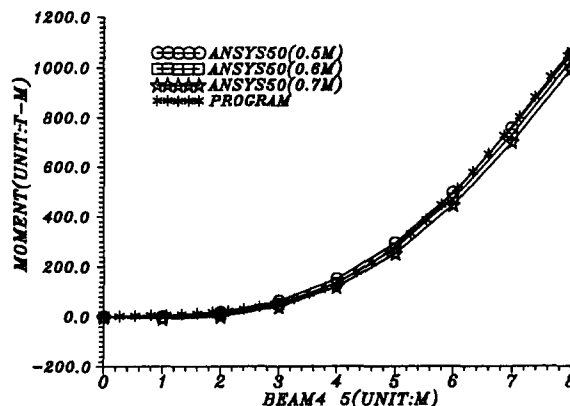

FIG. 3. Dimensions and Nodal Numbers of First Example

FIG. 4. Displacement of Beam 4_5

FIG. 5. Moment Diagram of Beam 4_5

TABLE 1. Subgrade Reactions on Slabs

Equivalent Uniform Subgrade Reactions							
Slab (1)	Reaction (t/m^2) (2)	Slab (3)	Reaction (t/m^2) (4)	Slab (5)	Reaction (t/m^2) (6)	Slab (7)	Reaction (t/m^2) (8)
P.1	6.851	P.8	7.768	P.15	9.785	P.22	11.02
P.2	6.218	P.9	10.23	P.16	16.73	P.23	20.76
P.3	6.358	P.10	11.70	P.17	20.38	P.24	25.72
P.4	6.385	P.11	11.88	P.18	20.58	P.25	25.98

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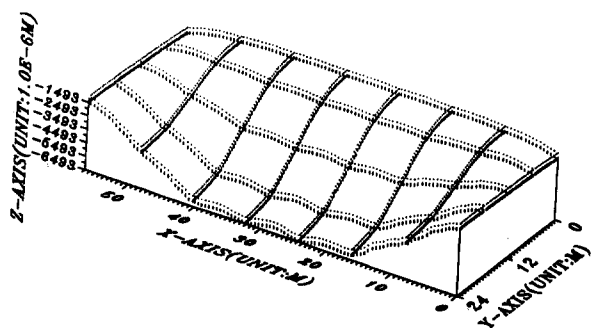
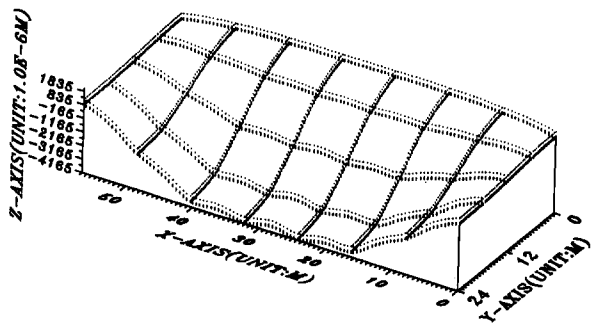**FIG. 6. Displacement of Mat Foundation for Case without 10 t/m² Uplift**

FIG. 7. Displacement of Mat Foundation for Case with 10 t/m² Uplift

structural model and recalculate the distribution of subgrade reaction until there is no negative reaction.

Figs. 6 and 7 show the deformations of the mat foundation for the cases without and with 10 t/m² buoyancy, respectively. From Fig. 7, one can observe that uplift does occur. This indicates that the case with buoyancy may govern the design of some part of the mat foundation.

CONCLUDING REMARKS

After extensive numerical investigations, some conclusions regarding the proposed model can be drawn as follows.

The proposed analysis model can be easily implemented as a computer program, and the preparation of input data is much easier than that in the finite-element method. For example, one just needs to give the subgrade reaction coefficient and the program will automatically generate the stiffness matrix for each beam on an elastic foundation with piecewise linear variation of the spring constant.

Through extensive comparisons, the results by the proposed model are quite close to those by the sophisticated finite-element analysis. This would validate the usage of the proposed model.

Most importantly, the proposed model can dramatically slash the computational cost for the structural analysis of mat foundation with grid floor beams, and give accurate results for design purposes. For example, the central processing time (CPU) time on a 486-DX50 personal computer is 6 s for the analysis of the first example, while it takes 100 s using the commercial software ANSYS 50 on the same machine. Generally, the more complicated the mat foundation is, the more the savings of computational cost will be. Also, the memory space needed in the computer by using the proposed model will be much less than that by using the finite-element model.

ACKNOWLEDGMENT

APPENDIX. REFERENCES

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