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Anticontrol of chaos of the fractional order modified van der Pol systems

Zheng-Ming Ge *, An-Ray Zhang

Department of Mechanical Engineering, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu 300, Taiwan, ROC

Abstract

Anticontrol of chaos of fractional order modified van der Pol systems is studied. Addition of a constant term and addition of $k|x|\sin x$ term where x is a state of the system are used to anticontrol the system effectively. By applying numerical results, phase portrait, Poincaré maps and bifurcation diagrams a variety of the phenomena of the chaotic motion can be presented. Finally, it can be find that chaos under these procedures exists in the fractional order systems of a modified van der Pol system.

Keywords: Chaos; Fractional order system; van der Pol equation; Modified van der Pol system; Anticontrol

1. Introduction

Anticontrol [1–7] and synchronization [8–16] of chaos have received great attention for many research activities in recent years. Anticontrol is an interesting, new and challenging phenomenon [17–19]. As a reverse process of suppressing or eliminating chaotic behaviors in order to reduce the complexity of an individual system or a coupled system, anticontrol of chaos aims at creating or enhancing the system complexity for some special applications. More precisely, anticontrolling chaos is to generate some chaotic behaviors from a given system, which is non-chaotic or even is stable originally. By fully exploiting the intrinsic non-linearity, this "control" technique provides another dimension for feedback systems design. Its potential applications can be easily found in many fields, including typically physics, biology, engineering, and medical as well as social sciences.

In addition, the topic of fractional calculus is enjoying growing interest not only among mathematicians, but also among physicists and engineers. In recent years, many scholars have devoted themselves to study the applications of the fractional order system to physics and engineering such as viscoelastic systems [20], dielectric polarization, and electromagnetic waves. More recently, there is a new trend to investigate the control [21] and dynamics [22–30] of the fractional order dynamical systems [31–34]. In [20] it has been shown that

* Corresponding author.
E-mail address: zmg@cc.nctu.edu.tw (Z.-M. Ge).

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non-linear chaotic systems can still behave chaotically when their models become fractional. In [32,33], it was found that chaos exists in a fractional order Chen system with order less than 3.

In this paper, anticontrol of chaos of modified van der Pol systems [35–38] in fractional order form are studied. This paper is organized as follows. In Section 2, a fractional derivative and its approximation are introduced. In Section 3, a modified van der Pol system and the corresponding fractional order system are presented. In Section 4, numerical simulations are given. In Section 5, conclusions are drawn.

2. A fractional derivative and its approximation

There are several definitions of fractional derivatives. The commonly used definition for a general fractional derivative is the Riemann–Liouville definition [39], which is given by

$$\frac{d^q f(t)}{dt^q} = \frac{1}{\Gamma(n-q)} \frac{d^n}{dt^n} \int_0^t \frac{f(\tau)}{(t-\tau)^{q-n+1}} d\tau, \tag{1}$$

where $\Gamma(\cdot)$ is the gamma function and n is an integer such that $n-1 < q < n$. This definition is different from the usual intuitive definition of derivative. Fortunately, the basic engineering tool for analyzing linear systems, the Laplace transform, is still applicable and works as one would expect:

$$L\left\{\frac{d^q f(t)}{dt^q}\right\} = s^q L\{f(t)\} - \sum_{k=0}^{n-1} s^k \left[\frac{d^{q-1-k} f(t)}{dt^{q-1-k}}\right]_{t=0}, \quad \text{for all } q, \tag{2}$$

where n is an integer such that $n-1 < q < n$. Upon considering the initial conditions to be zero, this formula reduces to the more expected form

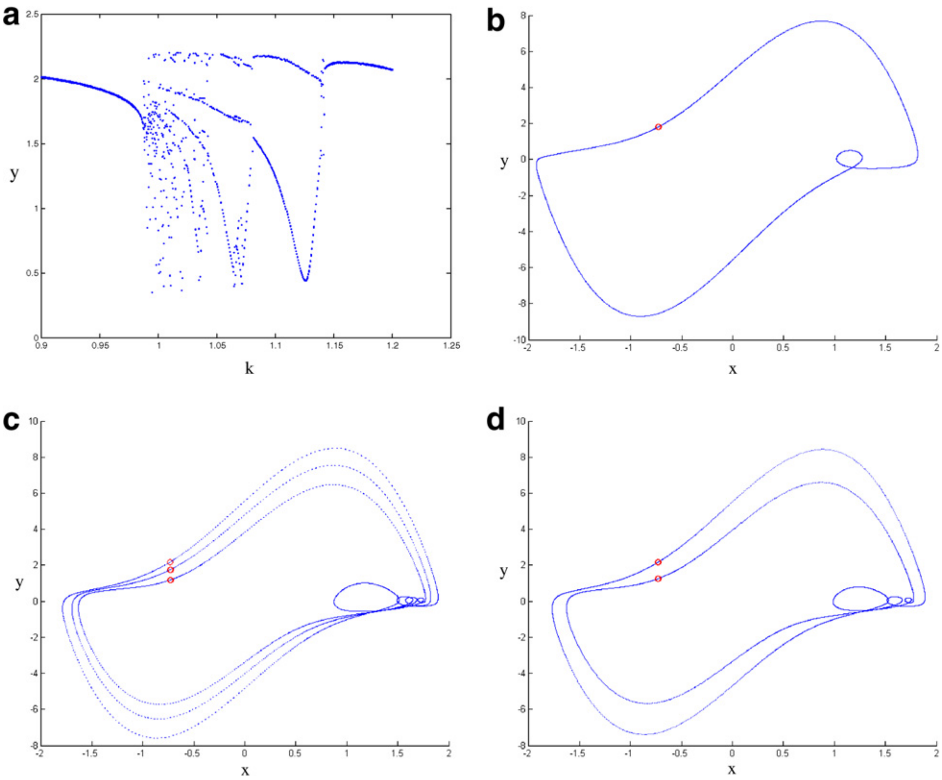

Fig. 1. (a) The bifurcation diagram for $\alpha = \beta = 0.9$. (b) The phase portrait for $\alpha = \beta = 0.9$, $k = 0$. (c) The phase portrait for $\alpha = \beta = 0.9$, $k = 1.05$. (d) The phase portrait for $\alpha = \beta = 0.9$, $k = 1.1$.

$$L\left\{\frac{\mathrm{d}^q f(t)}{\mathrm{d}t^q}\right\} = s^q L\{f(t)\}. \tag{3}$$

An efficient method is to approximate fractional operators by using standard integer order operators. In [40–44], an effective algorithm is developed to approximate fractional order transfer functions. Basically the idea is to approximate the system behavior based on frequency domain arguments. By utilizing frequency domain techniques based on Bode diagrams, one can obtain a linear approximation of the fractional order integrator, the order of which depends on the desired bandwidth and discrepancy between the actual and the approximate magnitude Bode diagrams. In Table 1 of [45], approximations for $\frac{1}{s^q}$ with $q = 0.1$–0.9 in steps 0.1 are given, with errors of approximately 2 dB. These approximations are used in the following simulations.

3. A modified van der Pol system and the corresponding fractional order system

Firstly, a van der Pol oscillator driven by a periodic force is considered. The equation of motion can be written as

$$\ddot{x} + \varphi x + a\dot{x}(x^2 - 1) - b\sin\omega t = 0. \tag{4}$$

In Eq. (4), the linear term stands for a conservative harmonic force which determines the intrinsic oscillation frequency. The self-sustaining mechanism which is responsible for the perpetual oscillation rests on the non-linear term. Energy exchange with the external agent depends on the magnitude of displacement $|x|$ and on the sign of velocity $\dot{x}$. During a complete cycle of oscillation, the energy is dissipated if displacement $x(t)$ is large than one, and that energy is fed-in if $|x| < 1$. The time-dependent term stands for the external driving force with amplitude b and frequency ω. Eq. (4) can be rewritten as two first-order equations:

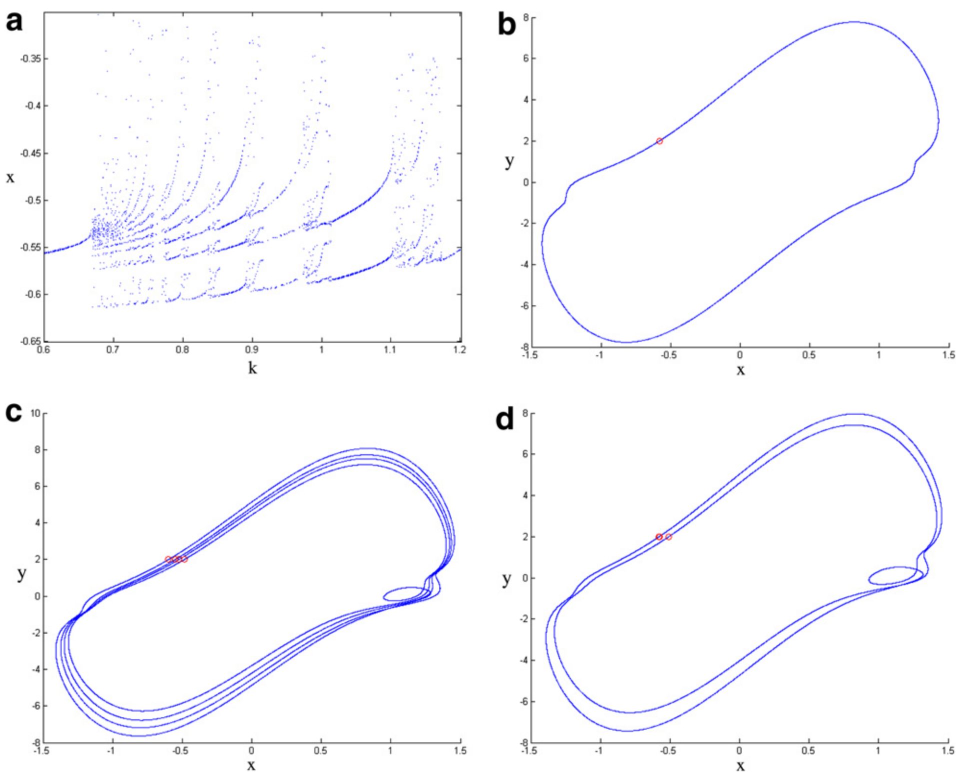

Fig. 2. (a) The bifurcation diagram for $\alpha = \beta = 0.8$. (b) The phase portrait for $\alpha = \beta = 0.8$, $k = 0$. (c) The phase portrait for $\alpha = \beta = 0.8$, $k = 0.87$. (d) The phase portrait for $\alpha = \beta = 0.8$, $k = 1.05$.

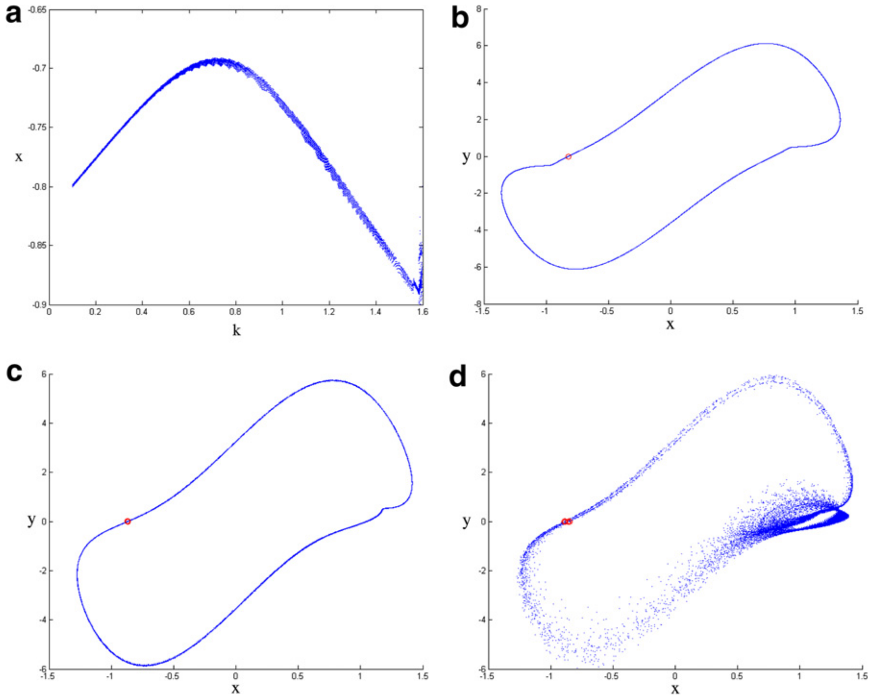

Fig. 3. (a) The bifurcation diagram for $\alpha = \beta = 0.7$. (b) The phase portrait for $\alpha = \beta = 0.7$, $k = 0$. (c) The phase portrait for $\alpha = \beta = 0.7$, $k = 1.5$. (d) The phase portrait for $\alpha = \beta = 0.7$, $k = 1.6$.

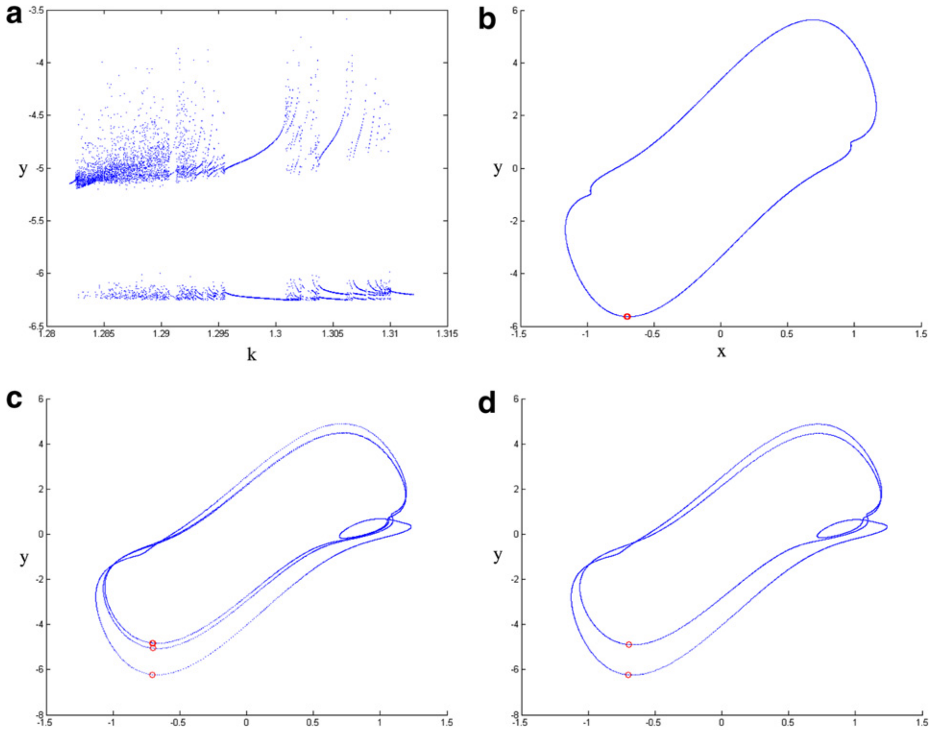

Fig. 4. (a) The bifurcation diagram for $\alpha = \beta = 0.6$. (b) The phase portrait for $\alpha = \beta = 0.6$, $k = 0$. (c) The phase portrait for $\alpha = \beta = 0.6$, $k = 1.291$. (d) The phase portrait for $\alpha = \beta = 0.6$, $k = 1.298$.

$$\begin{cases} \dot{x} = y, \\ \dot{y} = -\varphi x + a(1 - x^2)y + b\sin\omega t. \end{cases} \tag{5}$$

The modified van der Pol system and its fractional order system studied in this paper are

$$\begin{cases} \frac{d^\alpha x}{dt^\alpha} = y, \\ \frac{d^\beta y}{dt^\beta} = -x + a(1 - x^2)y + bz, \\ \dot{z} = w, \\ \dot{w} = -cz - dz^3, \end{cases} \tag{6}$$

where α, β are integer numbers and fractional numbers, respectively.

System (6) can be separated into two parts:

$$\begin{cases} \frac{d^\alpha x}{dt^\alpha} = y, \\ \frac{d^\beta y}{dt^\beta} = -x + a(1 - x^2)y + bz \end{cases} \tag{7}$$

and

$$\begin{cases} \dot{z} = w, \\ \dot{w} = -cz - dz^3. \end{cases} \tag{8}$$

In Eq. (5) changing the integral order derivatives to the fractional order derivations and replacing $\sin\omega t$ by z which is the periodic time function solution of the non-linear oscillator (8), we obtain system (7). In Eq. (8) if $d = 0$, z is a sinusoidal function of time. Now $d \neq 0$, z is a periodic motion of time but not a sinusoidal function of time. As a result, system (7) can be considered as a non-autonomous system with two states, while system (6) consisting of Eqs. (7) and (8) can be considered as an autonomous system with four states. When $\alpha = \beta = 1$, Eq. (6) is the modified van der Pol system.

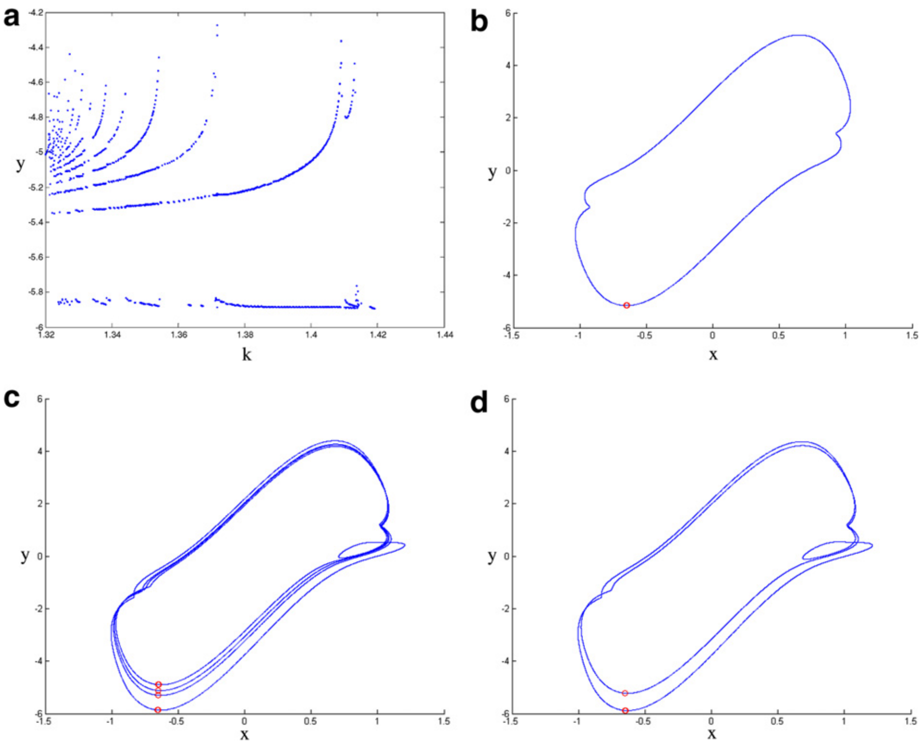

Fig. 5. (a) The bifurcation diagram for $\alpha = \beta = 0.5$. (b) The phase portrait for $\alpha = \beta = 0.5$, $k = 0$. (c) The phase portrait for $\alpha = \beta = 0.5$, $k = 1.35$. (d) The phase portrait for $\alpha = \beta = 0.5$, $k = 1.38$.

4. Numerical simulations

Anticontrol of chaos is making a non-chaotic dynamical system chaotic. This means that the regular behavior will be destroyed and replaced by chaotic behavior.

We will show that the anticontrols by means of addition of a constant term k or addition of a non-linear term $k|x|\sin x$ are effective. The results are shown by numerical simulations, such as phase portrait, Poincaré maps and bifurcation diagrams.

Case 1: We add a constant term k in the second equation of system (6) and it become:

$$\begin{cases} \frac{d^\alpha x}{dt^\alpha} = y, \\ \frac{d^\beta y}{dt^\beta} = -x + a(1-x^2)y + bz + k, \\ \dot{z} = w, \\ \dot{w} = -cz - dz^3. \end{cases} \tag{9}$$

In system (9), the parameter b is adjusted to achieve periodic motion for different α and β when $k = 0$. a, c, d are fixed and they are chosen as $a = 5$, $c = 0.01$, $d = 0.001$.

1. Let $\alpha = \beta = 0.9$, $b = 2.5$, and $k \in [0.9, 1.2]$. Fig. 1(a) shows the bifurcation diagram of the 1.8 order system. Fig. 1(b)–(d) are the phase portraits with $k = 0, 1.05, 1.1$.
2. Let $\alpha = \beta = 0.8$, $b = 1.5$, and $k \in [0.6, 1.2]$. Fig. 2(a) shows the bifurcation diagram of the 1.6 order system. Fig. 2(b)–(d) are the phase portraits with $k = 0, 0.87, 1.05$.
3. Let $\alpha = \beta = 0.7$, $b = 1.3$, and $k \in [0.1, 1.6]$. Fig. 3(a) shows the bifurcation diagram of the 1.4 order system. Fig. 3(b)–(d) are the phase portraits with $k = 0, 1.5, 1.6$.
4. Let $\alpha = \beta = 0.6$, $b = 1$, and $k \in [1.282, 1.312]$. Fig. 4(a) shows the bifurcation diagram of the 1.2 order system. Fig. 4(b)–(d) are the phase portraits with $k = 0, 1.291, 1.298$.
5. Let $\alpha = \beta = 0.5$, $b = 1$, and $k \in [1.32, 1.42]$. Fig. 5(a) shows the bifurcation diagram of the 1.0 order system. Fig. 5(b)–(d) are the phase portraits with $k = 0, 1.35, 1.38$.
6. Let $\alpha = \beta = 0.4$, $b = 1.5$, and $k \in [1.88, 1.94]$. Fig. 6(a) shows the bifurcation diagram of the 0.8 order system. Fig. 6(b)–(d) are the phase portraits with $k = 0, 1.915, 1.93$.
7. Let $\alpha = \beta = 0.3$, $b = 1.5$, and $k \in [1.89, 1.98]$. Fig. 7(a) shows the bifurcation diagram of the 0.6 order system. Fig. 7(b)–(d) are the phase portraits with $k = 0, 1.9, 1.94$.

Case 2: We add a non-linear term $k|x|\sin x$ in the second equation of system (6) and it become:

$$\begin{cases} \frac{d^\alpha x}{dt^\alpha} = y, \\ \frac{d^\beta y}{dt^\beta} = -x + a(1-x^2)y + bz + k|x|\sin x, \\ \dot{z} = w, \\ \dot{w} = -cz - dz^3. \end{cases} \tag{10}$$

In system (10), the parameter b is also adjusted to achieve periodic motion for different α and β when $k = 0$. a, c, d are fixed as Case 1.

1. Let $\alpha = \beta = 0.9$, $b = 2.5$, and $k \in [0.2, 0.4]$. Fig. 8(a) shows the bifurcation diagram of the 1.8 order system. Fig. 8(b)–(d) are the phase portraits with $k = 0, 0.35, 0.4$.
2. Let $\alpha = \beta = 0.8$, $b = 2.5$, and $k \in [1.0, 1.3]$. Fig. 9(a) shows the bifurcation diagram of the 1.6 order system. Fig. 9(b)–(d) are the phase portraits with $k = 0, 1.03, 1.2$.
3. Let $\alpha = \beta = 0.7$, $b = 0.5$, and $k \in [1.19, 1.24]$. Fig. 10(a) shows the bifurcation diagram of the 1.4 order system. Fig. 10(b)–(d) are the phase portraits with $k = 0, 1.205, 1.215$.
4. Let $\alpha = \beta = 0.6$, $b = 1$, and $k \in [0.8, 1.3]$. Fig. 11(a) shows the bifurcation diagram of the 1.2 order system. Fig. 11(b)–(d) are the phase portraits with $k = 0, 0.95, 1.05$.

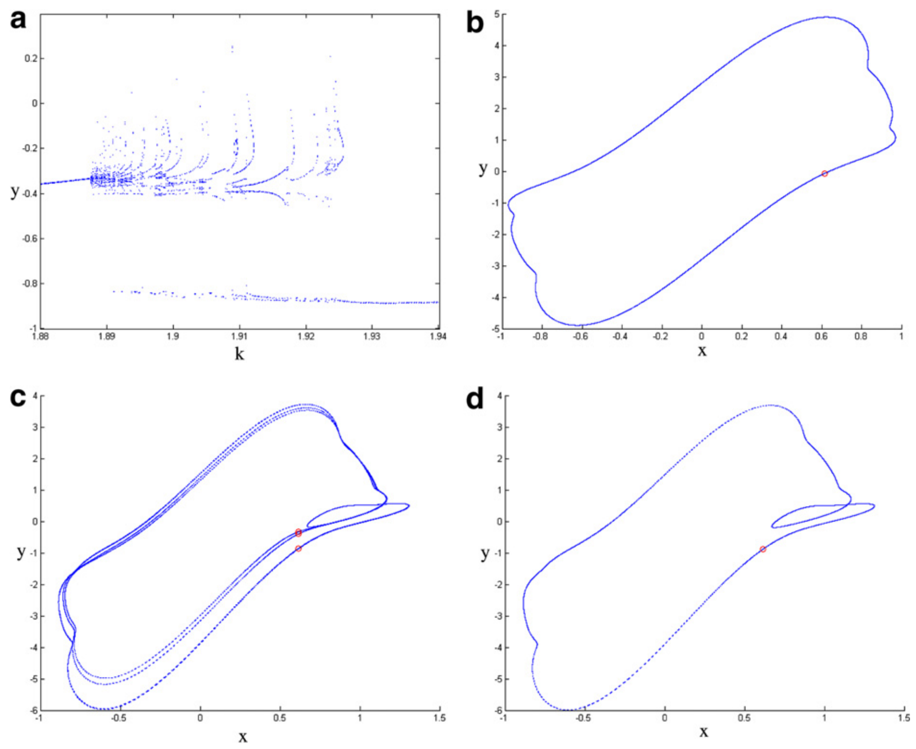

Fig. 6. (a) The bifurcation diagram for $\alpha = \beta = 0.4$. (b) The phase portrait for $\alpha = \beta = 0.4$, $k = 0$. (c) The phase portrait for $\alpha = \beta = 0.4$, $k = 1.915$. (d) The phase portrait for $\alpha = \beta = 0.4$, $k = 1.93$.

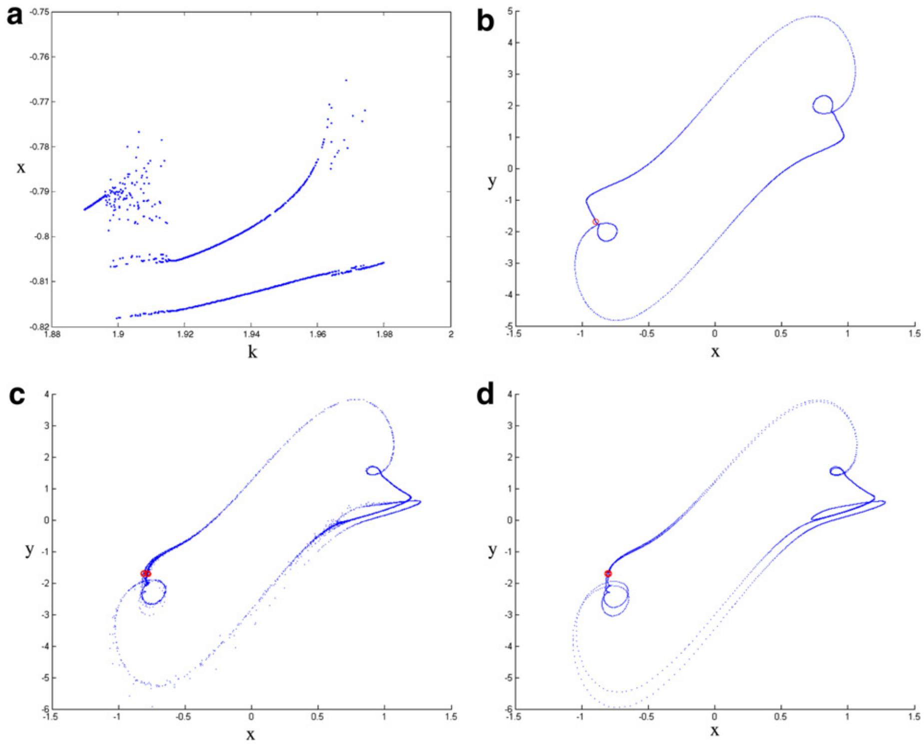

Fig. 7. (a) The bifurcation diagram for $\alpha = \beta = 0.3$. (b) The phase portrait for $\alpha = \beta = 0.3$, $k = 0$. (c) The phase portrait for $\alpha = \beta = 0.3$, $k = 1.9$. (d) The phase portrait for $\alpha = \beta = 0.3$, $k = 1.94$.

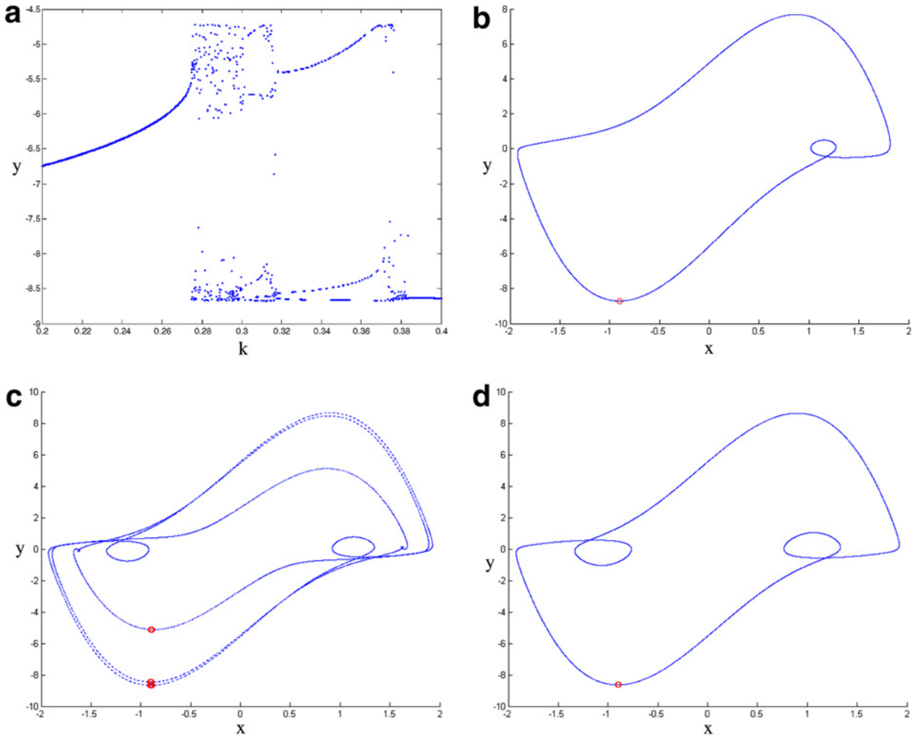

Fig. 8. (a) The bifurcation diagram for $\alpha = \beta = 0.9$. (b) The phase portrait for $\alpha = \beta = 0.9$, $k = 0$. (c) The phase portrait for $\alpha = \beta = 0.9$, $k = 0.35$. (d) The phase portrait for $\alpha = \beta = 0.9$, $k = 0.4$.

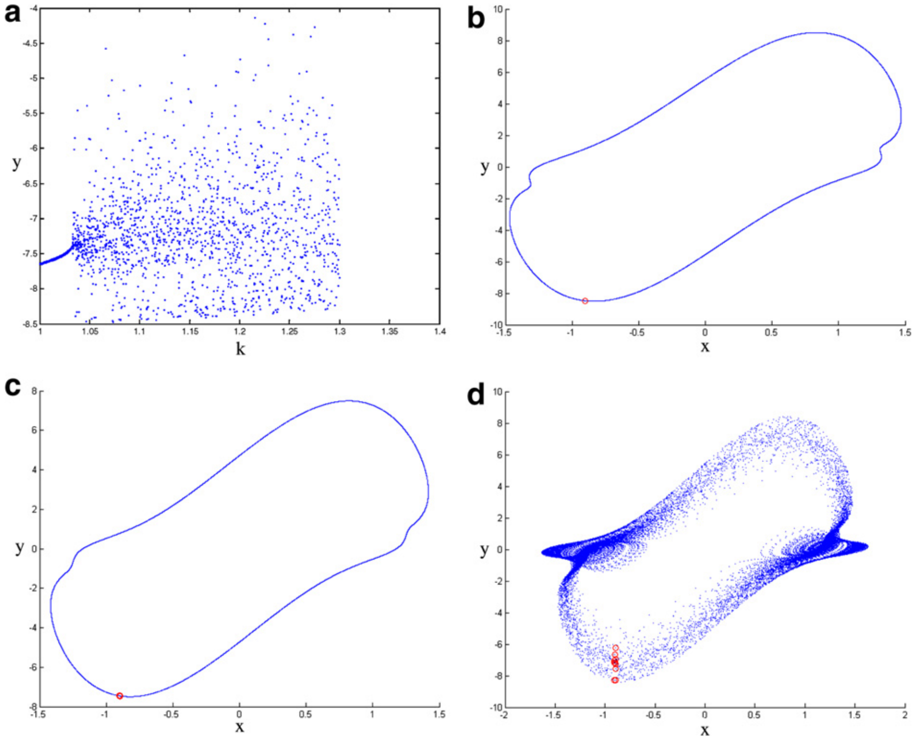

Fig. 9. (a) The bifurcation diagram for $\alpha = \beta = 0.8$. (b) The phase portrait for $\alpha = \beta = 0.8$, $k = 0$. (c) The phase portrait for $\alpha = \beta = 0.8$, $k = 1.03$. (d) The phase portrait for $\alpha = \beta = 0.8$, $k = 1.2$.

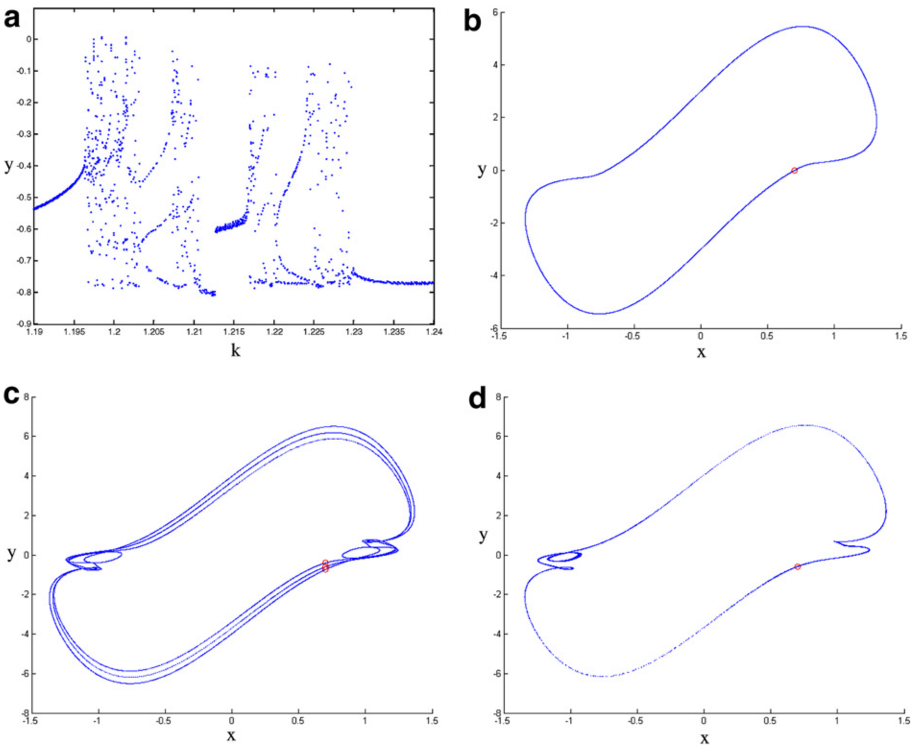

Fig. 10. (a) The bifurcation diagram for $\alpha = \beta = 0.7$. (b) The phase portrait for $\alpha = \beta = 0.7$, $k = 0$. (c) The phase portrait for $\alpha = \beta = 0.7$, $k = 1.205$. (d) The phase portrait for $\alpha = \beta = 0.7$, $k = 1.215$.

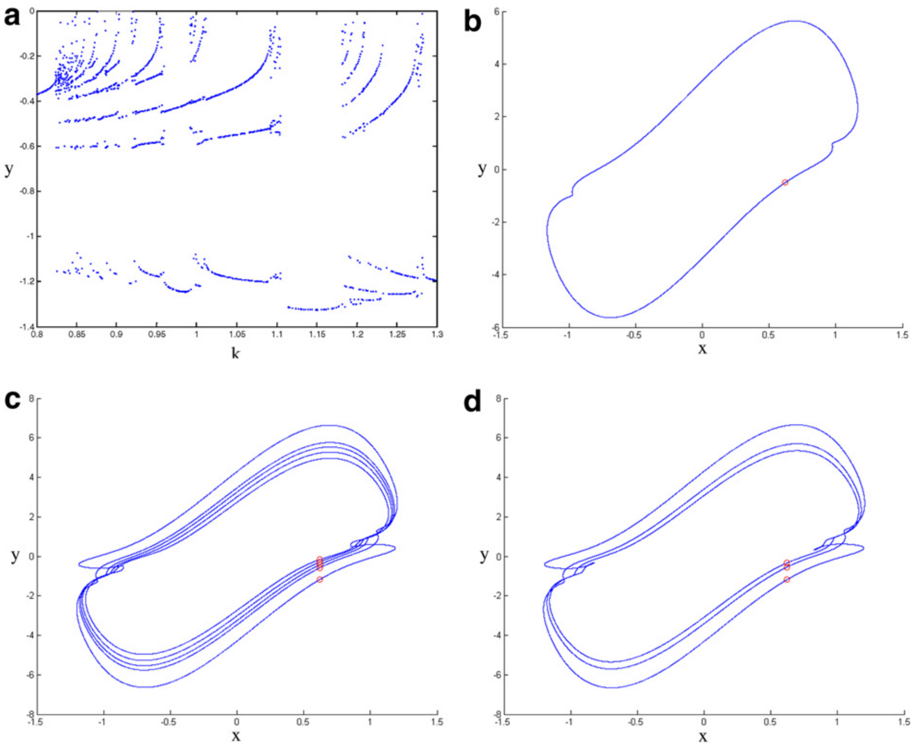

Fig. 11. (a) The bifurcation diagram for $\alpha = \beta = 0.6$. (b) The phase portrait for $\alpha = \beta = 0.6$, $k = 0$. (c) The phase portrait for $\alpha = \beta = 0.6$, $k = 0.95$. (d) The phase portrait for $\alpha = \beta = 0.6$, $k = 1.05$.

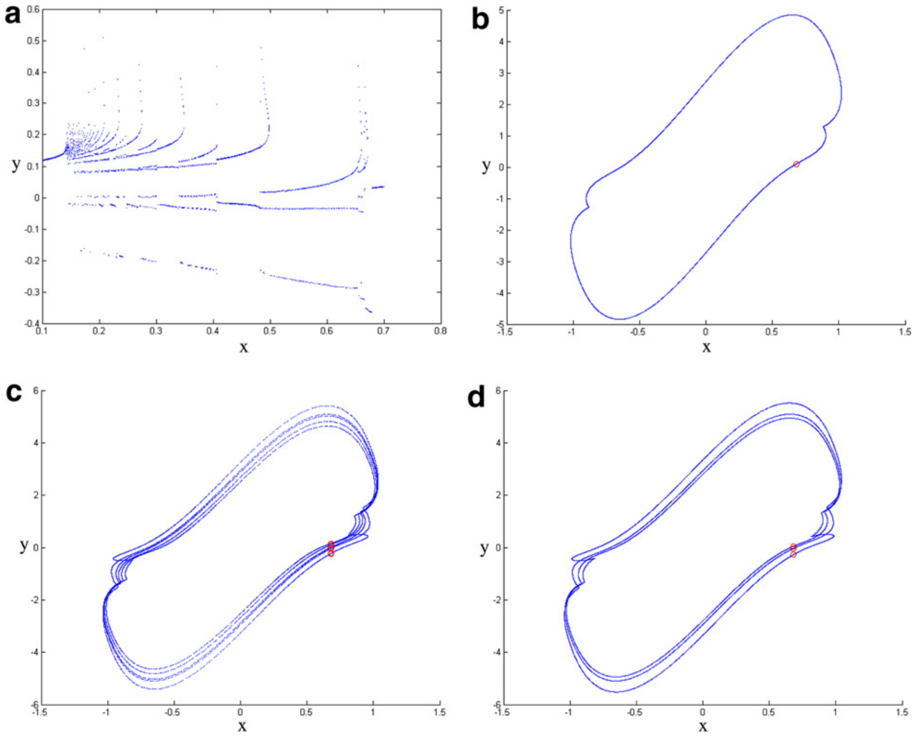

Fig. 12. (a) The bifurcation diagram for $\alpha = \beta = 0.5$. (b) The phase portrait for $\alpha = \beta = 0.5$, $k = 0$. (c) The phase portrait for $\alpha = \beta = 0.5$, $k = 0.4$. (d) The phase portrait for $\alpha = \beta = 0.5$, $k = 0.6$.

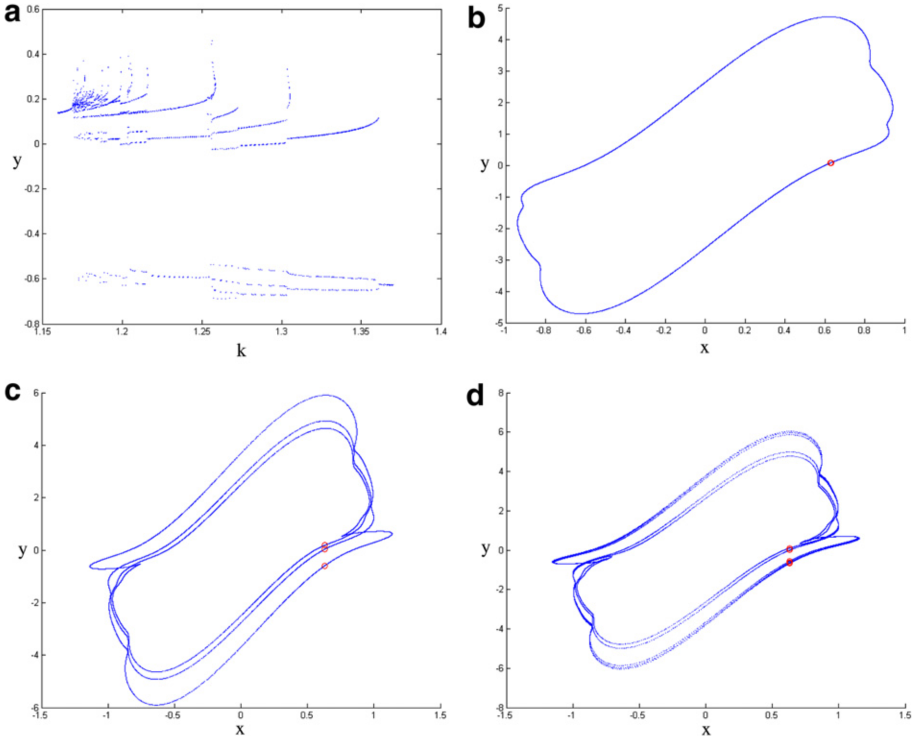

Fig. 13. (a) The bifurcation diagram for $\alpha = \beta = 0.4$. (b) The phase portrait for $\alpha = \beta = 0.4$, $k = 0$. (c) The phase portrait for $\alpha = \beta = 0.4$, $k = 1.25$. (d) The phase portrait for $\alpha = \beta = 0.4$, $k = 1.3$.

5. Let $\alpha = \beta = 0.5$, $b = 0.5$, and $k \in [0.1, 0.7]$. Fig. 12(a) shows the bifurcation diagram of the 1.0 order system. Fig. 12(b)–(d) are the phase portraits with $k = 0, 0.4, 0.6$.
6. Let $\alpha = \beta = 0.4$, $b = 1$, and $k \in [1.16, 1.37]$. Fig. 13(a) shows the bifurcation diagram of the 0.8 order system. Fig. 13(b)–(d) are the phase portraits with $k = 0, 1.25, 1.3$.

5. Conclusions

Anticontrol of chaos in the fractional order systems of a modified van der Pol system are studied in the paper. An efficient way to transform a non-chaotic dynamical system into a chaotic one is easily made by addition of a constant term or by addition of $k|x|\sin x$ term where x is a state variable of the system. It is found that chaos exists in the fractional order systems with order from 1.8 down to 0.6 for the addition of constant term, and from 1.8 down to 0.8 for the addition of $k|x|\sin x$ term.

Acknowledgments

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