

# Routing, ship size, and sailing frequency decision-making for a maritime hub-and-spoke container network

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## Abstract

This study formulates a two-objective model to determine the optimal liner routing, ship size, and sailing frequency for container carriers by minimizing shipping costs and inventory costs. First, shipping and inventory cost functions are formulated using an analytical method. Then, based on a trade-off between shipping costs and inventory costs, Pareto optimal solutions of the two-objective model are determined. Not only can the optimal ship size and sailing frequency be determined for any route, but also the routing decision on whether to route containers through a hub or directly to their destination can be made in objective value space. Finally, the theoretical findings are applied to a case study, with highly reasonable results. The results show that the optimal routing, ship size, and sailing frequency with respect to each level of inventory costs and shipping costs can be determined using the proposed model. The optimal routing decision tends to be shipping the cargo through a hub as the hub charge is decreased or its efficiency improved. In addition, the proposed model not only provides a tool to analyze the trade-off between shipping costs and inventory costs, but it also provides flexibility on the decision-making for container carriers.

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## 1. Introduction

The hub-and-spoke network has been a mainstay for a long time in airline passenger networks, airline freight networks, express delivery networks, Internet networks, and communication networks. In maritime transportation it has become popular for container carriers to provide their services as the global trade keeps expanding, and cargo traffic keeps growing. In a maritime hub-and-spoke network, major ports are usually selected as hub ports based on their location and the demands of freight shipping, while the other ports serve as feeder ports, i.e. spoke ports. Large mother ships are used on main lines to provide services among hub ports, while small feeder ships are used on feeder lines to provide the services between a hub port and its feeder ports. Main lines are usually shipping services between two continents or regions, such as Trans-Pacific Service, Trans-Atlantic Service, Asia–Europe Service, and

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Asia–Australia Service. On the other hand, feeder lines are mainly for collecting or distributing freight within a continent or region, such as China–Japan Feeder Service, Singapore–Surabaya Feeder Service and others.

Under a hub-and-spoke network, economies of flow can be realized by consolidating freight through a hub by using large ships. However, routing all freight through a hub is not necessarily appropriate for all situations. Although the average shipping cost per TEU<sup>1</sup> decreases on line-haul legs of hub-and-spoke networks, freight originating in feeder ports must be trans-shipped through a hub, and incur extra shipping distance, shipping time, port charges and loading/unloading charges. Furthermore, there possibly exist economies of flow for direct service if there is a large enough flow. Therefore, whether to route containers through a hub or directly to their destination depends on cargo flows, port charges and efficiency, locations of feeder ports, etc. The proper route, optimal ship size and sailing frequency can be decided by exploring those influencing factors. This study constructs an analytical model to address these issues.

Decision-making on routing, ship size, and sailing frequency are important issues for container carriers when planning their shipping services. The results of those decisions directly influence the operating effectiveness of container carriers and the quality of service provided to shippers. Since container carriers operate in an increasingly competitive and market-driven environment, they not only aim to lower their shipping costs, but also at to enhance their services in order to increase their competitiveness. Inventory costs related to the container shipping process are crucial factors affecting the quality of service provided to shippers. Container carriers generally enhance their shipping services by providing a high sailing-frequency service, using fast container ships, and planning shipping routes so as to shorten shipping time. The inventory costs due to freight waiting to be shipped in a loading port depend on the sailing frequency, whereas those due to freight being on a ship along the shipping route are related to ship speed and choice of shipping route. Inventory costs are borne by the shippers. However, Daganzo [1] suggested that all costs incurred by the cargoes from their origin to their destination should be taken into account regardless of who pays them. If the inventory costs are not being considered, an optimal decision-making will tend to transfer the burden of the operation from the container carrier to the shippers. As a result, the shippers may be less willing to have their containers being shipped by the carrier, meaning that, a better decision can be made if the inventory costs are taken into account. Therefore, in this study shipping costs and inventory costs are regarded as two major factors affecting shipping service decisions.

Inventory costs are commonly regarded as a major factor affecting shipping service decisions in the economic order quantity (EOQ) model in logistics literature. These studies usually determine the optimal shipping frequency by minimizing total shipping and inventory costs (e.g. [1,2]). In a maritime study, Jansson and Shneerson [3] developed a liner services cost model which shows that the optimal ship size is a function of both shipping company costs of service provision and user inventory costs and opportunity costs when goods are in transport. Pope and Talley [4] extended the work of Jansson and Shneerson [3]. They demonstrated that the optimal ship size is highly sensitive to the inventory management model selected, the treatment of stock-outs and safety stock, and the prevailing inventory management cost structure. These studies showed that inventory costs of shippers have been considered in maritime studies. In our study, only the inventory costs related to the container shipping process are taken into account. Furthermore, since shipping costs and inventory costs are borne respectively by the carriers and the shippers, the decision to minimize total shipping and inventory costs is system optimization. In reality, although container carriers do consider the inventory costs of the shippers as a decision factor, the weight placed on inventory costs is usually not equal to that of the shipping costs. Thus, this study regards shipping costs and inventory costs as two separate objectives, reflecting the conflicting interests of the carriers and shippers.

Many recent studies have shown the advantages of dealing with the multi-objective nature of transportation-planning issues. A two-objective approach generally produces better planning alternatives, as well as providing flexibility in the decision-making for container carriers. Moreover, single-objective decision-making by minimizing total shipping and inventory costs, or merely minimizing shipping costs can be regarded as a special case of the two-objective decision. A two-objective model also provides a tool to analyze the trade-off between shipping costs and inventory costs.

Good examples of multi-objective studies are the following works: on air service planning by Flynn and Ratick [5], on bus operations planning by Tzeng and Shiau [6], on airline flight planning by Teodorovic and Krmar-Nozic [7], on

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<sup>1</sup> TEU is the acronym of twenty-foot equivalent unit. It is a unit of container.

freight train planning by Fu and Wright [8], on transit network design by Israeli and Ceder [9], on air crew rostering by Teodorovic and Lucic [10], on passenger train services planning by Chang et al. [11], on airline network design by Hsu and Wen [12], and on train scheduling by Ghoseiri et al. [13]. Therefore, this study will formulate a two-objective model by minimizing shipping costs and inventory costs, to determine the optimal liner routing, ship size, and sailing frequency for container carriers.

A simple analytical approach is provided to determine the Pareto optimal solutions of the two-objective model. First, shipping and inventory cost functions are formulated using analytical method. Shipping costs include capital and operating costs, fuel cost, and port charges, whereas inventory costs include waiting time cost and shipping time cost. Then, because both costs are simple functions of the sailing frequency, a trade-off equation between shipping costs and inventory costs can be derived. Accordingly, Pareto optimal solutions of the two-objective model are determined in objective value space, and the optimal ship size and sailing frequency with respect to each level of inventory costs and shipping costs can be determined. Furthermore, suppose the routing decisions of a hub-and-spoke network system aim at minimizing total shipping costs and total inventory costs. Then, based on the trade-off equation, the decision to either route containers through a hub or directly to the destination can be further determined.

Previous researches on maritime shipping service planning, such as liner routing, fleet deployment, and ship scheduling, focused largely on general networks (e.g. [14–19]). A more recent review for ship routing and scheduling can be found in Christiansen et al. [20]. These studies concentrated mainly on the formulation of mathematical programming models and the development of solution algorithms, while flow economies or trans-shipment characteristics in hub-and-spoke networks were seldom considered, nor was the decision-making on ship size or sailing frequency. On the other hand, studies on ship size have focused mainly on the economies of ship size (e.g. [21–23]). These latter studies explored shipping economies from the view of ship supply and operation, rather than shippers' demand or the route of the cargo flow.

Until recently, researches on hub-and-spoke maritime networks have been few in number. However, as more container carriers have begun to use hub-and-spoke networks, so is there an increase in the number of researches that are being done on these networks. For example, Robinson [24] forecasted the future shipping service in Asian hub/feeder nets based on the trend of shipping development. Bendall and Stent [25] formulated a scheduling model for a high-speed containership service. They applied a short-sea hub-and-spoke network as an example. Mourão et al. [26] formulated a ship assignment model using constraints to deal with the characteristics of trans-shipment in hub-and-spoke networks. Hsieh and Chang [27] proposed a quadratic assignment IP (integer programming) model, based on the hub-and-spoke network model of O'Kelly [28,29] in air transportation. Their model introduced exogenous cost discount on main-line shipping to deal with flow economies. However, differing from previous studies, this study formulated flow-dependent cost functions to deal with flow economies in hub-and-spoke networks, and the extra shipping distance and time incurred when freight originating in feeder ports is trans-shipped.

A fundamental hub-and-spoke maritime network is considered in this study, as shown in Fig. 1. Freight shipping services are provided by a carrier between two continents or regions separated by a major ocean. In each region, one or several ports are selected as hub ports, and the others serve as feeder ports. This study explores a carrier's route decision-making process on whether freight between feeder ports on one feeder line in the region of origin (e.g. ports  $p_1$  and  $p_2$ ) and hub ports in the destination region (e.g. ports  $p_5$  and  $p_6$ ) should be shipped directly (i.e. shipped by the direct line,  $d : p_1, p_2, p_5, p_6, p_5, p_1$ ) or shipped through the local hub port at the region of origin (i.e. shipped via hub port,  $p_3$ , by routing the feeder line,  $s : p_1, p_2, p_3, p_1$ , and then the main line,  $h : p_3, p_4, p_5, p_6, p_5, p_4, p_3$ ).

The remainder of this paper is organized as follows. In Section 2, shipping and inventory cost functions are formulated for a multi-port calling route. Shipping costs include the container carriers' capital and operating costs, fuel cost, and the port charges on the route being served. The inventory costs include costs due to freight waiting to be shipped in a loading port and on a ship along the shipping route. Section 3 arrives at a relationship equation for exploring a trade-off between shipping costs and inventory costs by using different sizes of ship. Based on this, all feasible solutions and Pareto optimal solutions for the two-objective model are determined, and the optimal ship size and sailing frequency can be obtained, simultaneously. Section 4 then determines Pareto optimal solutions for routing containers, through a hub as well as directly to its destination. The routing decision between these two strategies can be made and illustrated in objective value space. Section 5 presents a case study that demonstrates the usefulness of the proposed model. The optimal routing, ship size, and sailing frequency with respect to each level of inventory costs and shipping costs are shown, and the effects of port charges and efficiency are also illustrated. Finally, concluding remarks are made in Section 6.

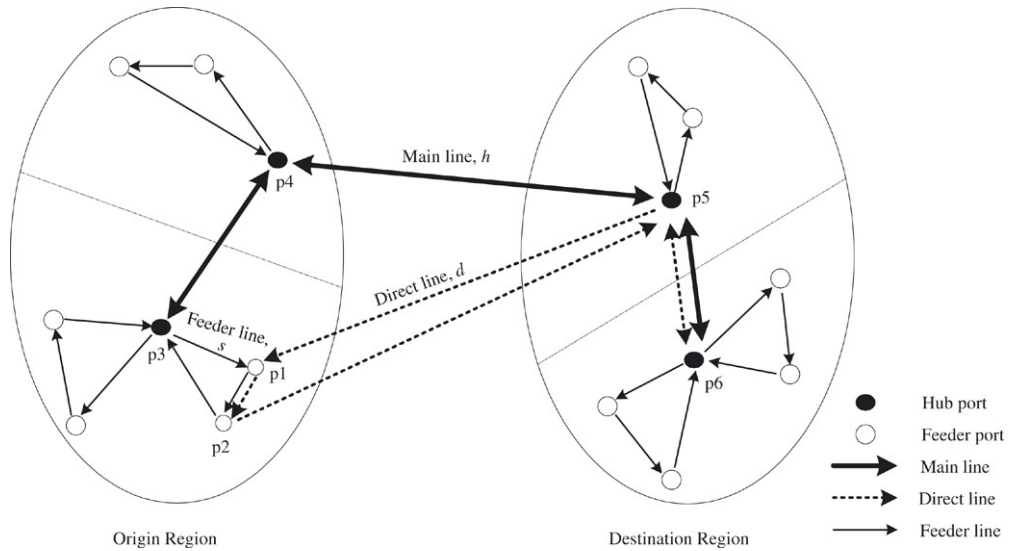


Fig. 1. The fundamental hub-and-spoke maritime network.

**2. Cost functions**

Shipping and inventory cost functions for a multi-port calling route are defined in this section based on Hsu and Hsieh [30]. In the mathematical model the following notation will be used.

- $m$  route
- $i, j, k, l$  port of call on route  $m$
- $f$  sailing frequency per season
- $t$  ship type
- $Q_{ij}^m$  flow from port  $i$  to port  $j$  on route  $m$  per season (TEU)
- $O_t$  average daily capital and operating costs for a ship of type  $t$  (US\$ per day)
- $D_i^m$  shipping distance between port  $i$  and port  $i + 1$  on route  $m$  (nautical mile)
- $F_t$  fuel cost at sea per nautical mile for a ship of type  $t$  (US\$ per nautical mile)
- $F_{it}$  fuel cost in port  $i$  by a ship of type  $t$  (US\$)
- $V_t$  average service speed for a ship of type  $t$  (nautical mile per day)
- $G_i$  average handling fee per TEU in port  $i$  (US\$ per TEU)
- $R_i$  average gross handling rate in port  $i$  (TEU per day)
- $W_i$  time a ship spends on the arrival and departure process in port  $i$  (day)
- $\alpha_{it}$  fixed portion of port  $i$  charge for a ship of type  $t$  (US\$)
- $\beta_{it}$  variable portion of port  $i$  charge for a ship of type  $t$  (US\$ per day)
- $T$  daily value of time per TEU (US\$ per TEU per day)
- $\delta_{ijk}^m = \begin{cases} 1, & \text{if path from port } i \text{ to } j \text{ contains a link between port } k \text{ and } k + 1; \\ 0, & \text{otherwise.} \end{cases}$

Shipping costs can be divided into three main categories: capital and operating costs, fuel costs and port charges. Capital and operating costs represent the total expenses paid for using the ship each day, including the cost of owning the ship, crew wages and meals, ship repair and maintenance, insurance, materials and supplies, and so on. Suppose carriers make the most of ship utilization, and the total shipping time per round voyage for a ship includes time spent in all ports and at sea. Then, the total capital and operating costs for shipping all container flow on route  $m$  per season are the product of the average daily capital and operating costs,  $O_t$ , the total shipping time per round voyage,  $\sum_i (W_i + \frac{D_i^m}{V_t}) + \frac{1}{f} \sum_i \sum_j (\frac{Q_{ij}^m + Q_{ji}^m}{R_i})$ , and the sailing frequency,  $f$ . That is  $f O_t \sum_i (W_i + \frac{D_i^m}{V_t}) + O_t \sum_i \sum_j (\frac{Q_{ij}^m + Q_{ji}^m}{R_i})$ .

Fuel costs are the expense of the fuel consumption by a ship sailing at sea and dwelling in port. Fuel costs increase with the ship size. Moreover, fuel costs at sea are proportional to the shipping distance, since a ship normally cruises at a constant speed at sea. Fuel costs in port are different from those at sea, for a ship must decelerate or accelerate

when entering or leaving a port, and the fuel consumption also depends on the distance a ship has to move into the port. That is, fuel costs for the same ship in different ports may vary. Then, the fuel costs for a season with a sailing frequency  $f$  on route  $m$  by a ship of type  $t$  are  $f \sum_i (F_t D_i^m + F_{it})$ .

Port charges are paid for a ship dwelling in port and can be divided into the charge for the ship and the stevedoring charge. The former is paid for servicing the ship, including pilotage, towage, line handling fee, and berth occupancy charge, etc. The latter is paid for cargo handling, including a container loading and unloading fee, equipment charge, and rent of container yard, etc. Then, the port charges for a season with a sailing frequency  $f$  on route  $m$  by a ship of type  $t$  are  $f \sum_i \alpha_{it} + \sum_i \sum_j [( \frac{\beta_{it}}{R_i} + G_i ) \cdot (Q_{ij}^m + Q_{ji}^m)]$ .

Therefore, the total shipping costs for a season with a sailing frequency  $f$  on route  $m$  by a ship of type  $t$ ,  $C_S^m$ , are the sum of the capital and operating costs, the fuel costs, and the port charges. That is

$$C_S^m = f \sum_i \left[ \alpha_{it} + O_t W_i + F_{it} + D_i^m \left( \frac{O_t}{V_t} + F_t \right) \right] + \sum_i \sum_j \left[ \left( G_i + \frac{\beta_{it}}{R_i} + \frac{O_t}{R_i} \right) (Q_{ij}^m + Q_{ji}^m) \right]. \quad (1)$$

Inventory costs represent the loss of opportunity cost or the loss of value because cargoes cannot be used or sold in the shipping process, and these costs are positively correlated with cargo volume, value of cargo, and length of storage time. In this study, only inventory costs related to the container shipping process are taken into account, and involve waiting time cost and shipping time cost. The waiting time cost is the cost related to the sailing frequency due to schedule delays, either from waiting in the loading port or at the place of production or origin. The higher the sailing frequency, the lower the waiting cost. Assume that the arrival process of containers at each loading port follows a uniform distribution, then the average waiting time per TEU in a loading port is one half of a shipping time cycle. Let  $T$  denote the daily value of time per TEU, and suppose one season equals approximately 13 weeks or 91 days, then the total waiting time cost per season for containers shipped on route  $m$  is  $\frac{91T}{2f} \sum_i \sum_j Q_{ij}^m$ . On the other hand, the shipping time cost is related to time while containers are shipped on a ship, and it increases with the shipping time. The shipping time cost for a season of containers shipped along route  $m$  by a ship of type  $t$  is  $T \sum_i \sum_j \sum_k Q_{ij}^m \delta_{ijk}^m (W_k + \frac{D_k^m}{V_t}) + \frac{T}{f} \sum_i \sum_j \sum_k \sum_l \frac{Q_{ij}^m \delta_{ijk}^m}{R_k} (Q_{kl}^m + Q_{lk}^m)$ .

Then, the total inventory costs for a season of containers shipped along route  $m$  by a ship of type  $t$ ,  $C_I^m$ , can be expressed as the sum of the waiting time cost and the shipping time cost. That is

$$C_I^m = \frac{91T}{2f} \sum_i \sum_j Q_{ij}^m + T \sum_i \sum_j \sum_k Q_{ij}^m \delta_{ijk}^m \left( W_k + \frac{D_k^m}{V_t} \right) + \frac{T}{f} \sum_i \sum_j \sum_k \sum_l \frac{Q_{ij}^m \delta_{ijk}^m}{R_k} (Q_{kl}^m + Q_{lk}^m). \quad (2)$$

In the shipping and inventory cost functions above, ship type,  $t$ , and sailing frequency,  $f$ , are decision variables, while the others are exogenous variables. For any type of ship, shipping costs decrease while the inventory cost increases as the sailing frequency decreases. There exists a trade-off between shipping costs and inventory costs, which are linked by the sailing frequency. Furthermore, regarding alternative ship types, various combinations of sailing frequency and ship type will decide the carriers' shipping cost and/or inventory cost. Sailing frequency and ship type are correlated with each other. For example, for serving a certain volume of cargo, carriers may either use a larger ship with a lower sailing frequency, or use a smaller ship with a higher sailing frequency, resulting in different shipping costs and inventory costs.

In addition, the cost functions formulated herein also illustrate the shipping economies. On one hand, six ship-related variables,  $O_t$ ,  $F_t$ ,  $F_{it}$ ,  $V_t$ ,  $\alpha_{it}$ , and  $\beta_{it}$  being different values for different types of ship show the economies of ship size. On the other hand, the cost functions, being flow-related, show the economies of flow. To show the economies of flow clearly in the shipping cost function constructed herein for a multi-port calling route, let the port-related variables be identical in all ports of call and let the average shipping cost per TEU be the total shipping cost,  $C_S^m$ , divided by the total flow on route  $m$ ,  $\sum_i \sum_j Q_{ij}^m$ . Then, the average shipping cost per TEU can be reduced as

$$\frac{f \left[ n^m (\alpha_{it} + O_t W_i + F_{it}) + \left( \frac{O_t}{V_t} + F_t \right) \sum_i D_i^m \right]}{\sum_i \sum_j Q_{ij}^m} + 2 \left( G_i + \frac{\beta_{it} + O_t}{R_i} \right). \quad (3)$$

The economies of flow are shown in Eq. (3) in such a way that the average shipping cost per TEU decreases as the total flow,  $\sum_i \sum_j Q_{ij}^m$ , increases.

### 3. Decision on ship size and sailing frequency

Minimizing shipping costs and minimizing inventory costs are the two objectives of the proposed model. Since these two objectives conflict with each other, a fully optimal solution that simultaneously optimizes both objectives simply does not exist. Therefore, instead of a complete optimal solution, the Pareto optimality concept is introduced in this study. The Pareto optimality is the solution in which no objective can be reached without simultaneously worsening at least one of the remaining objectives [31].

A trade-off equation between the shipping cost and the inventory cost can be further derived from Eqs. (1) and (2) by simplifying the variables. Let  $\Phi_t^m$  and  $\Gamma_t^m$  denote the fixed components of shipping cost and inventory cost for ship type  $t$  on route  $m$ , respectively, and let  $\Lambda_t^m$  and  $\Psi_t^m$  denote the marginal shipping cost and the marginal inventory cost for increasing it with one more sailing frequency, respectively. Then,  $\Phi_t^m$ ,  $\Gamma_t^m$ ,  $\Lambda_t^m$ , and  $\Psi_t^m$  can be expressed as:

$$\Phi_t^m = \sum_i \sum_j \left[ \left( G_i + \frac{\beta_{it}}{R_i} + \frac{O_t}{R_i} \right) (Q_{ij}^m + Q_{ji}^m) \right], \tag{4}$$

$$\Gamma_t^m = T \sum_i \sum_j \sum_k Q_{ij}^m \delta_{ijk}^m \left( W_k + \frac{D_k^m}{V_t} \right), \tag{5}$$

$$\Lambda_t^m = \sum_i \left[ \alpha_{it} + O_t W_i + F_{it} + D_i^m \left( \frac{O_t}{V_t} + F_t \right) \right], \tag{6}$$

$$\Psi_t^m = \frac{91T}{2} \sum_i \sum_j Q_{ij}^m + T \sum_i \sum_j \sum_k \sum_l \frac{Q_{ij}^m \delta_{ijk}^m}{R_k} (Q_{kl}^m + Q_{lk}^m). \tag{7}$$

The shipping and inventory cost functions (Eqs. (1) and (2)) can be reduced as:

$$C_S^m = \Phi_t^m + \Lambda_t^m f, \tag{8}$$

$$C_I^m = \Gamma_t^m + \Psi_t^m (f)^{-1}. \tag{9}$$

Therefore, a trade-off equation between shipping cost and inventory cost can be derived from Eqs. (8) and (9). That is

$$(C_S^m - \Phi_t^m) (C_I^m - \Gamma_t^m) = \Lambda_t^m \Psi_t^m. \tag{10}$$

Eq. (10) indicates clearly that there exists a trade-off between shipping cost and inventory cost. When flow, ship-related variables, and port-related variables are held constant, the shipping cost for any type of ship decreases as the inventory cost increases, and the substitute rate between the two costs decreases. The shipping cost approaches  $\Phi_t^m$ , i.e.  $C_S^m \rightarrow \Phi_t^m$ , when the inventory cost approaches infinite, i.e.  $C_I^m \rightarrow \infty$ . Similarly, the inventory cost decreases as the shipping cost increases, and the substitute rate decreases. And  $C_I^m \rightarrow \Gamma_t^m$  when  $C_S^m \rightarrow \infty$ .

Eq. (10) is a hyperbolic function, an elementary geometry such as a circle, ellipse, or parabola. A hyperbolic function with center, vertex, focus, focal length and asymptote can be shown exactly in a two-dimensional space. Fig. 2 illustrates the hyperbolic function for any ship of type  $t$  in objective value space with a horizontal dimension representing the inventory cost and a vertical dimension representing the shipping cost, while center, vertex, focal length, horizontal asymptote and vertical asymptote of the hyperbolic function are  $(\Gamma_t^m, \Phi_t^m)$ ,  $(\Gamma_t^m + \sqrt{\Lambda_t^m \Psi_t^m}, \Phi_t^m + \sqrt{\Lambda_t^m \Psi_t^m})$ ,  $\sqrt{2\Lambda_t^m \Psi_t^m}$ ,  $C_S^m - \Phi_t^m = 0$ , and  $C_I^m - \Gamma_t^m = 0$ , respectively. The hyperbolic function illustrates not only the trade-off relationships between shipping cost and inventory cost, but also the solutions for the two-objective model for any ship of type  $t$ . However, since the cargo loaded on a ship cannot exceed its capacity on any voyage link, the sailing frequency for any ship of type  $t$  must be larger than or equal to the maximum link flow,  $\text{Max}_k \sum_i \sum_j \delta_{ijk}^m Q_{ij}^m$ , divided by the ship's capacity,  $U_t$ . That is  $f \geq \frac{\text{Max}_k \sum_i \sum_j \delta_{ijk}^m Q_{ij}^m}{U_t}$ . Moreover, let  $f_t^{m-\min}$  denote the minimum sailing frequency, i.e.  $f_t^{m-\min} = \frac{\text{Max}_k \sum_i \sum_j \delta_{ijk}^m Q_{ij}^m}{U_t}$ , and let  $\overline{C_{S,t}^m}$  and  $\overline{C_{I,t}^m}$  denote the minimum shipping cost and the

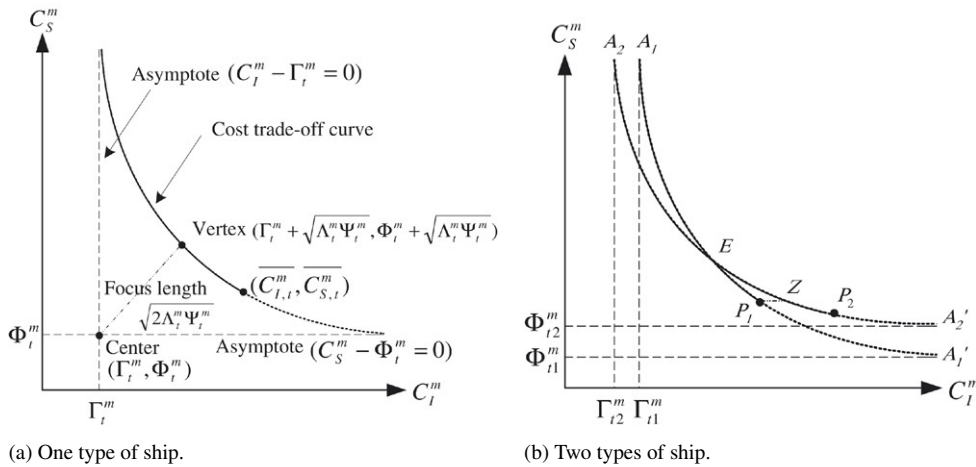


Fig. 2. Feasible solutions and Pareto optimal solutions.

maximum inventory cost when a ship is operating with the minimum sailing frequency, respectively. And let the points  $(\overline{C}_{I,t}^m, \overline{C}_{S,t}^m)$  in objective value space be the name for the minimum sailing frequency point. Then the feasible solutions of the two-objective model for any ship of type  $t$  are located at the hyperbolic function curve, and with the value of the shipping costs,  $C_S^m$ , higher than that of the minimum sailing frequency point,  $(\overline{C}_{I,t}^m, \overline{C}_{S,t}^m)$ , are shown by the solid curve in Fig. 2(a).

The Pareto optimal solutions of the two-objective model are the feasible solutions illustrated in Fig. 2(a) in the case that there is only one type of ship. Moreover, when there is more than one type of ship, the feasible solutions for each type of ship can be determined using a similar derivation. The Pareto optimal solutions for more than one type of ship are the closest solutions to origin (0,0) among all the feasible solutions, and are located along the envelope of all the curves in objective value space. For example, Fig. 2(b) illustrates the possible Pareto optimal solutions for two types of ship. In the case, let curves  $A_1 A_1'$  and  $A_2 A_2'$  denote the trade-off curves for two different types of ship, respectively, and let points  $P_1$  and  $P_2$  denote the minimum sailing frequency points for the two types of ship, respectively. Then, the Pareto optimal solutions are made up of three curves  $\overline{P_2 Z}$ ,  $\overline{P_1 E}$ , and  $\overline{E A_2'}$ .

#### 4. Routing decision

Suppose the routing decisions of a hub-and-spoke network system are made by minimizing two objectives, i.e. minimizing total shipping costs and minimizing total inventory costs, respectively. Then, the decision to either route containers through a hub or directly to the destination can be determined by comparing the Pareto optimal solutions for these two routing strategies. Therefore, with respect to any level of the total inventory costs, if the total shipping costs for routing containers directly, are lower than that for routing containers through a hub, then shipping directly is preferred; otherwise, shipping through a hub is preferred.

Since the Pareto optimal solutions for containers shipped from feeder ports on a feeder line in the region of origin to a hub in the destination region won't be affected by all other feeder lines, then the calculation of the shipping and inventory costs of all routes in the system is not necessary. As shown in Fig. 1, only costs on three lines, i.e. main line,  $h$ , direct line,  $d$ , and feeder line,  $s$ , are considered when the routing decisions are made.

Furthermore, since a hub has the advantage of cargo-consolidation, this study assumes that the cargo flow on a main line is large enough to realize economies of scale. Therefore, the main line can be served with the minimum shipping cost, no matter how large the inventory cost is. Let  $t^*$  denote the ship type with the lowest value of the minimum shipping cost on a main line. Then, the minimum shipping cost and the maximum inventory cost with respect to a ship of type  $t^*$  can be represented as  $C_{S,t^*}^h$  and  $C_{I,t^*}^h$ , respectively. Moreover, the minimum shipping cost is the shipping cost when a ship is operating with the minimum sailing frequency. That is,  $f = f_{t^*}^{h-\min} = \frac{\text{Max}_k \sum_i \sum_j \delta_{ijk}^h Q_{ij}^h}{U_{t^*}}$ .

Substituting it for  $f$  in Eqs. (1) and (2), respectively, then,  $\overline{C_{S,t}^h}$  and  $\overline{C_{I,t}^h}$  can be further expressed as:

$$\overline{C_{S,t}^h} = \frac{\text{Max}_k \sum_i \sum_j \delta_{ijk}^h Q_{ij}^h}{U_{t^*}} \sum_i \left[ \alpha_{it^*} + O_{t^*} W_i + F_{it^*} + D_i^h \left( \frac{O_{t^*}}{V_{t^*}} + F_{t^*} \right) \right] + \sum_i \sum_j \left[ \left( G_i + \frac{\beta_{it^*}}{R_i} + \frac{O_{t^*}}{R_i} \right) (Q_{ij}^h + Q_{ji}^h) \right], \tag{11}$$

$$\overline{C_{I,t}^h} = \frac{91TU_{t^*}}{2 \text{Max}_k \sum_i \sum_j \delta_{ijk}^h Q_{ij}^h} \sum_i \sum_j Q_{ij}^h + T \sum_i \sum_j \sum_k Q_{ij}^h \delta_{ijk}^h \left( W_k + \frac{D_k^h}{V_{t^*}} \right) + \frac{TU_{t^*}}{\text{Max}_k \sum_i \sum_j \delta_{ijk}^h Q_{ij}^h} \sum_i \sum_j \sum_k \sum_l \frac{Q_{ij}^h \delta_{ijk}^h}{R_k} (Q_{kl}^h + Q_{lk}^h). \tag{12}$$

Let  $TC_S$  and  $TC_I$  denote, respectively, the total shipping costs and the total inventory costs of the main line,  $h$ , the feeder line,  $s$ , and the direct line,  $d$ . When containers are shipped through a hub, there is no direct line,  $d$ . Then, if the main line is served with the minimum shipping cost, the total shipping costs,  $TC_S$ , and the total inventory costs,  $TC_I$ , for routing containers through a hub can be expressed as

$$TC_S = \overline{C_{S,t^*}^h} + C_S^s, \tag{13}$$

$$TC_I = \overline{C_{I,t^*}^h} + C_I^s, \tag{14}$$

where  $C_S^s$  and  $C_I^s$  in Eqs. (13) and (14) are the shipping and inventory costs for the feeder line,  $s$ , and there is a trade-off between these two costs. Thus, using a similar derivation in Section 3, a hyperbolic function showing the relationship between  $C_S^s$  and  $C_I^s$  for any ship of type  $t$  can be expressed as

$$C_I^s = \Gamma_t^s + \frac{A_t^s \cdot \Psi_t^s}{C_S^s - \Phi_t^s} \quad \text{for } C_S^s \geq \overline{C_{S,t}^s}. \tag{15}$$

Eq. (15) is Eq. (10) added by a capacity constraint, where the definitions of  $\Phi_t^s$ ,  $\Gamma_t^s$ ,  $A_t^s$ , and  $\Psi_t^s$  are the same as those in Eqs. (4)–(7).

Consequently, the relationship between  $TC_S$  and  $TC_I$  for routing containers through a hub can be obtained by substituting Eqs. (13) and (14) for  $C_S^s$  and  $C_I^s$  in Eq. (15). That is,

$$TC_I = \overline{C_{I,t^*}^h} + \Gamma_t^s + \frac{A_t^s \cdot \Psi_t^s}{TC_S - \overline{C_{S,t^*}^h} - \Phi_t^s} \quad \text{for } TC_S \geq \overline{C_{S,t^*}^h} + \overline{C_{S,t}^s}. \tag{16}$$

Eq. (16) shows that there is also a hyperbolic relationship between  $TC_S$  and  $TC_I$ . In fact, Eq. (16) is equal to Eq. (15) shifting to the right by a constant value  $\overline{C_{I,t^*}^h}$ , and then up by  $\overline{C_{S,t^*}^h}$  in the objective value space.

Similar to the Pareto optimal solutions for any route  $m$  being determined by Eq. (10) in Section 3, the Pareto optimal solutions of the feeder line can be determined by Eq. (15). Likewise, the Pareto optimal solutions for routing containers through a hub can be determined by Eq. (16). Moreover, the optimal ship size and sailing frequency for the main line and the feeder line can be further determined. Therefore, since the curve of Eq. (16) is equivalent to the curve of Eq. (15) shifting to the right and up, the Pareto optimal shipping cost for shipping through a hub is the Pareto optimal shipping cost of the feeder line added by a constant value,  $\overline{C_{S,t^*}^h}$ , and the Pareto optimal inventory cost is the Pareto optimal inventory cost of the feeder line added by a constant value,  $\overline{C_{I,t^*}^h}$ .

Next, the Pareto optimal solutions for routing containers directly to their destination are also derived. If the main line is served with the minimum shipping cost, the total shipping costs,  $TC_S$ , and the total inventory costs,  $TC_I$ , for routing containers directly can be expressed as

$$TC_S = \overline{C_{S,t^*}^h} + C_S^s + C_S^d, \tag{17}$$



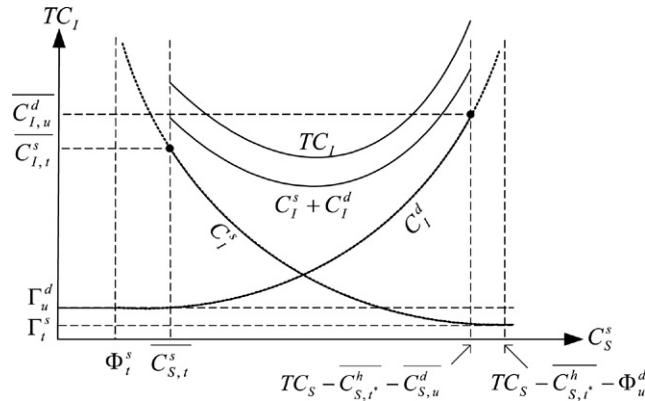


Fig. 3.  $TC_I$  versus  $C_S^s$  at a specific value of  $TC_S$  when routing containers directly to their destination.

$$TC_I = \overline{C_{I,t}^h} + C_I^s + C_I^d, \tag{18}$$

where  $C_S^s$  and  $C_I^s$  are the shipping and inventory costs of the feeder line,  $s$ , respectively, and  $C_S^d$  and  $C_I^d$  are the shipping and inventory costs of the direct line,  $d$ , respectively. In addition, the relationship between  $C_S^s$  and  $C_I^s$  for using any ship of type  $t$  on the feeder line can be expressed as

$$C_I^s = \Gamma_t^s + \frac{\Lambda_t^s \cdot \Psi_t^s}{C_S^s - \Phi_t^s} \quad \text{for } C_S^s \geq \overline{C_{S,t}^s}. \tag{19}$$

And, the relationship between  $C_S^d$  and  $C_I^d$  for using any ship of type  $u$  on the direct line can be expressed as

$$C_I^d = \Gamma_u^d + \frac{\Lambda_u^d \cdot \Psi_u^d}{C_S^d - \Phi_u^d} \quad \text{for } C_S^d \geq \overline{C_{S,u}^d}. \tag{20}$$

By substituting Eqs. (19) and (20) for  $C_I^s$  and  $C_I^d$  in Eq. (18), and substituting  $TC_S - \overline{C_{S,t}^h} - C_S^s$  for  $C_S^d$ , then the relationship between  $TC_S$  and  $TC_I$  for routing containers directly can be obtained. That is,

$$TC_I = \overline{C_{I,t}^h} + \Gamma_t^s + \frac{\Lambda_t^s \cdot \Psi_t^s}{C_S^s - \Phi_t^s} + \Gamma_u^d + \frac{\Lambda_u^d \cdot \Psi_u^d}{TC_S - \overline{C_{S,t}^h} - C_S^s - \Phi_u^d}$$

for  $\overline{C_{S,t}^s} \leq C_S^s \leq TC_S - \overline{C_{S,t}^h} - \overline{C_{S,u}^d}$  and  $TC_S \geq \overline{C_{S,t}^h} + \overline{C_{S,t}^s} + \overline{C_{S,u}^d}$ . (21)

A one-to-many relationship between  $TC_S$  and  $TC_I$  for routing containers directly is shown in Eq. (21). In particular, when  $TC_S$  is held constant, the inventory cost of the feeder line,  $\Gamma_t^s + \frac{\Lambda_t^s \cdot \Psi_t^s}{C_S^s - \Phi_t^s}$ , decreases as  $C_S^s$  increases, yet the inventory cost of the direct line,  $\Gamma_u^d + \frac{\Lambda_u^d \cdot \Psi_u^d}{TC_S - \overline{C_{S,t}^h} - C_S^s - \Phi_u^d}$ , increases as  $C_S^s$  increases. Then, the value of  $TC_I$  first decreases and then increases as  $C_S^s$  increases, as shown in Fig. 3. Appendix A presents a more rigid proof for deriving the conclusion.

The lowest value of  $TC_I$  in Fig. 3 is the optimal inventory costs with respect to the specific value of  $TC_S$  and can be obtained by solving the first derivative of the function  $TC_I$  equaled zero with respect to  $C_S^s$ . Moreover, the lowest value of  $TC_I$  with respect to any value of  $TC_S$  can be obtained, as illustrated in Fig. 4. When the value of  $TC_S$  equals to  $\overline{C_{S,t}^h} + \overline{C_{S,t}^s} + \overline{C_{S,u}^d}$ , then the lowest value of  $TC_I$  equals to  $\overline{C_{I,t}^h} + \overline{C_{I,t}^s} + \overline{C_{I,u}^d}$ ; as the value of  $TC_S$  increases, the lowest value of  $TC_I$  decreases; when the value of  $TC_S$  is approaching infinite, the lowest value of  $TC_I$  is approaching  $\overline{C_{I,t}^h} + \Gamma_t^s + \Gamma_u^d$ . Appendix B presents a more rigid proof for deriving the conclusion.

The curve in Fig. 4 also illustrates the Pareto optimal solutions for using ships of type  $t$  and  $u$  on the feeder line and direct line, respectively. Consequently, the Pareto optimal solutions for shipping containers directly can be determined by comparing the Pareto optimal solutions of each type of ship on either the direct line or the feeder line. Furthermore,

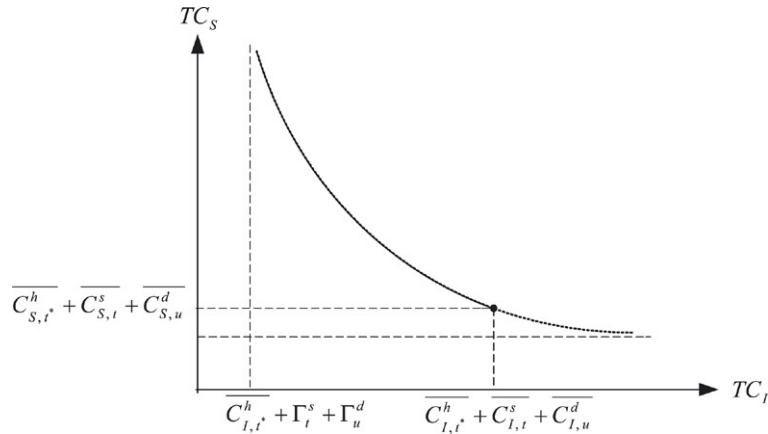


Fig. 4. Pareto optimal solutions for routing containers directly when ships of type  $t$  and  $u$  are used on a feeder line and direct line, respectively.

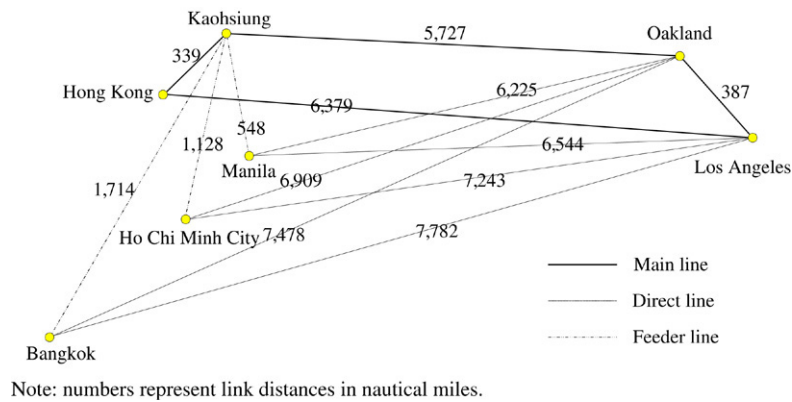


Fig. 5. The marine networks in the case study.

the optimal ship size and sailing frequency with respect to each level of total inventory costs and total shipping costs can be obtained at the same time.

### 5. Case study

To demonstrate the application and the results of the proposed model, this study uses a container carrier which operates the liner services with hub-and-spoke networks as an example. Trans-Pacific Service operates between the Far East and the U.S. West Coast, and it has only one main line and three feeder lines, which should simplify the analysis. Kaohsiung and Hong Kong are used as hub ports in the Far East, while Oakland and Los Angeles are used as hub ports on the U.S. West Coast. In the Far East, Bangkok, Ho Chi Minh City, and Manila are, respectively, the feeder ports of Thailand, Vietnam, and the Philippines, from which containers are shipped through the hub port, Kaohsiung. The objective of this case study is to apply the proposed model in order to illustrate the analyses of the routing, ship size, and sailing frequency decisions. The routing decisions are focused on whether containers should be shipped from these feeder ports to the U.S. West Coast via Kaohsiung or should be shipped directly to the U.S. West Coast.

The marine networks of the case study including main line, direct lines, and feeder lines are shown in Fig. 5, in which the distances are indicated in nautical miles. In this case study, routing decisions are made each time for only one of Thailand, Vietnam, and the Philippines while cargoes from the other two regions are assumed to be trans-shipped through the hub to their destination using feeder and main lines. Moreover, as cargoes between the hub port, Kaohsiung, and the three feeder ports are taken into consideration, there are still feeder lines to do local shipping when cargoes from any feeder port are shipped directly to the U.S. West Coast.

Table 1  
The values of parameters for each type of ship

Types of ship <sup>a</sup>	T1	T2	T3	T4	T5	T6
Capacity, $U_i$ (TEU)	1,164	1,810	2,728	3,428	4,211	5,652
Service speed, $V_i$ (nautical miles per day)	448.8	504.0	492.0	496.8	600.0	600.0
Daily capital and operating costs, $O_i$ (US\$) <sup>b</sup>	21,289	21,940	22,865	23,571	24,360	25,813
Fuel cost per nautical mile, $F_i$ (US\$) <sup>b</sup>	13.65	15.51	20.81	24.32	23.57	29.89
Fuel cost in port, $F_{it}$ (US\$) <sup>b</sup>	68.23	77.62	104.05	121.59	117.84	149.44

<sup>a</sup> Source: <http://www.evergreen-marine.com>.

<sup>b</sup> Source: Wang [35].

There are six types of ship used by the container carrier, and  $T_i$  ( $i = 1 \dots 6$ ) denotes the types of ship from  $i = 1$ , the smallest, to  $i = 6$ , the largest. Table 1 shows the capacity, service speed, daily capital and operating costs, fuel cost per nautical mile, and fuel cost when in port for each type of ship. In addition, since differences among ports for these variables do not significantly affect the decision results, to simplify the analysis and illustrate the major effects of the crucial factors, the value of the fixed port charge,  $\alpha_{it}$ , the variable port charge,  $\beta_{it}$ , and the average cargo handling charge,  $G_i$ , in port are supposed to be identical in all ports and are estimated by the port charges of Kaohsiung Harbor [32]. The average gross handling rate,  $R_i$ , and the port arrival and departure time,  $W_i$ , are estimated from the related data of Kaohsiung. Furthermore, two-way flows are estimated from the data in published government reports [33,34]. The market shares of the container carrier are estimated to be about 15%.

### 5.1. Determining the optimal ship size and sailing frequency

The study uses the computer software Mathematica 4.0 to solve the case-study problem, which applies the two-objective model proposed in previous sections. The Pareto optimal solutions of the model for any line can be determined by calculating the trade-off equations and comparing the feasible solutions among all types of ship, while the optimal ship size and sailing frequency with respect to different levels of inventory costs and shipping costs are determined simultaneously. Using the main line as an example, when all flows from the feeder ports are trans-shipped through the hub port of Kaohsiung, the Pareto optimal solutions are determined as:

$$C_S^h = \begin{cases} 2.215 * 10^7 + \frac{5.598 * 10^{14}}{C_I^h - 5.090 * 10^7} & \text{for } 5.090 * 10^7 < C_I^h \leq 7.675 * 10^7, \\ 2.270 * 10^7 + \frac{6.347 * 10^{14}}{C_I^h - 5.090 * 10^7} & \text{for } 8.098 * 10^7 \leq C_I^h \leq 8.560 * 10^7. \end{cases} \quad (22)$$

As shown by the solid lines in Fig. 6, the Pareto optimal solutions in an objective value space are made up of all feasible solutions of ships of type T5 and part of the feasible solutions of ships of type T6. Moreover, the optimal sailing frequency with respect to ships of types T5 and T6 are, respectively, from 1.95 to  $\infty$  and from 1.46 to 1.68 per week. The shipping costs are lower and the inventory costs are higher as the lines move to the right. As a result, using a ship of type T6 is preferred if the inventory costs are not constrained, and using a ship of type T5 is preferred if the inventory costs are constrained to be lower than  $8.098 * 10^7$  US\$.

In addition, consider that in reality the sailing frequency for the liner service is generally weekly, and that a sailing frequency less than every two weeks is commonly regarded as inadequate. The study further determines the optimal ship size and sailing frequency that could be possibly chosen by container carriers in reality. The discrete points in Fig. 6 depict those practical results.

### 5.2. Routing decision

The routing decision can be made using the model proposed in Section 4. Not only can the Pareto optimal solutions for trans-shipment and direct shipping be obtained, but also the routing decision, optimal ship size, and sailing frequency can be determined. In this example shipping services for cargoes from Thailand, the Philippines, and Vietnam to the U.S. West Coast are discussed, and discussions are carried out for any region in case cargoes from the other two regions are being trans-shipped.

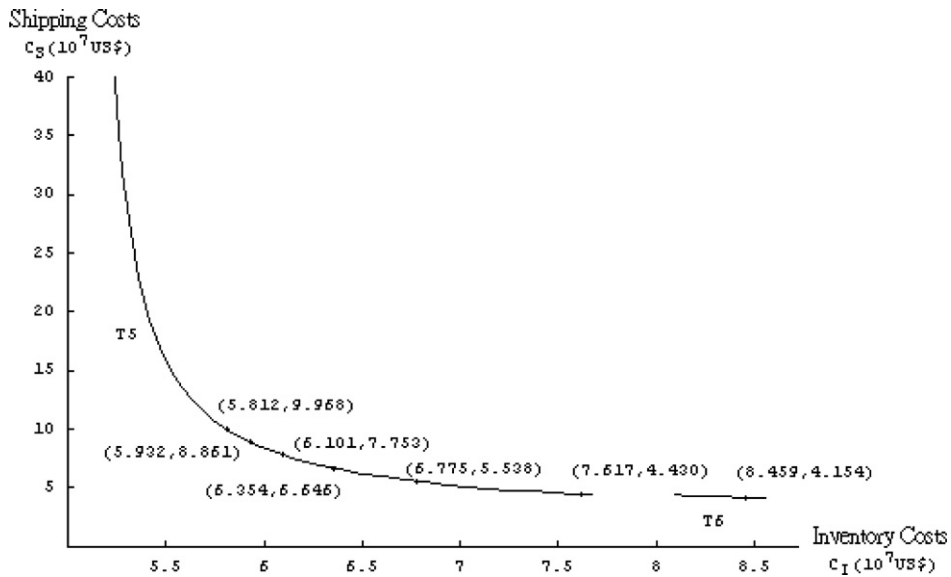
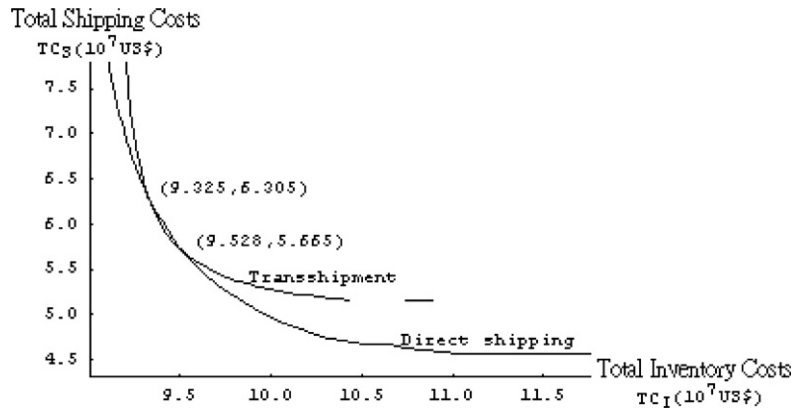


Fig. 6. Pareto optimal solutions and practical solutions for the main line when all flows are trans-shipped.

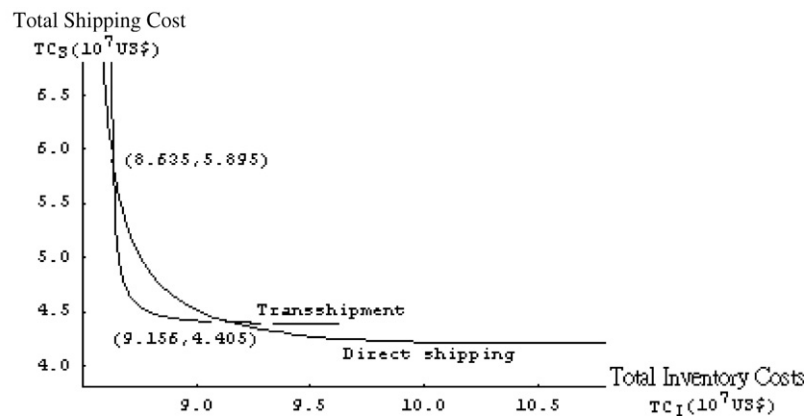
Fig. 7(a) shows the Pareto optimal solutions for trans-shipment and direct shipping for Thailand. In this figure, the two curves representing the two types of shipping, cross each other at two points, i.e.  $(9.325 \times 10^7, 6.305 \times 10^7)$  and  $(9.528 \times 10^7, 5.665 \times 10^7)$ . This shows that trans-shipment is preferred for the range of total inventory costs between  $9.325 \times 10^7$  and  $9.528 \times 10^7$  US\$, while direct shipping is preferred for those less than  $9.325 \times 10^7$  US\$ and larger than  $9.528 \times 10^7$  US\$. Fig. 7(b) and 7(c) show the Pareto optimal solutions for both trans-shipment and direct shipping for the Philippines and for Vietnam, respectively. Likewise, in each figure, the two curves for trans-shipment and direct shipping cross each other at two points. Again, trans-shipment is preferred for the range between two intersection points, and direct shipping is preferred for those outside the range. However, the magnitudes of the trans-shipment ranges are different for the three regions, and especially the range in Fig. 7(a) is very small. It shows that cargoes from Thailand should probably be shipped directly and that cargoes from the Philippines and Vietnam are likely to be trans-shipped. Variations in the results are due to the differences in cargo flow, shipping distance, and feeder port location among these three regions. As shown by the data, Thailand has the largest flow, the Philippines has the median flow, and Vietnam has the smallest flow. Thailand has the longest shipping distance, Vietnam has the median shipping distance, and the Philippines has the shortest distance. Therefore, the results imply that direct shipping is preferred if the cargo flow is large and/or shipping distance is long; otherwise, trans-shipment is preferred.

In reality, the cargoes from Thailand are more likely to be shipped directly than the cargoes from the Philippines and Vietnam. Currently, major container carriers who operate the Trans-Pacific Services, such as Evergreen, Orient Overseas Container Line, American President Line, Maersk Sealand, etc., trans-ship their cargoes from the Philippines and Vietnam via a hub port in the Far East to the U.S. West Coast. However, cargoes from Thailand are not always trans-shipped via a hub port. For example, Evergreen and American President Line ship their cargoes from Thailand through a hub, but Orient Overseas Container Line and Maersk Sealand ship their cargoes from Thailand directly to hub ports in the U.S. West Coast. The results of this case study are reasonable and are in accordance with the real-world routing decisions of current carriers.

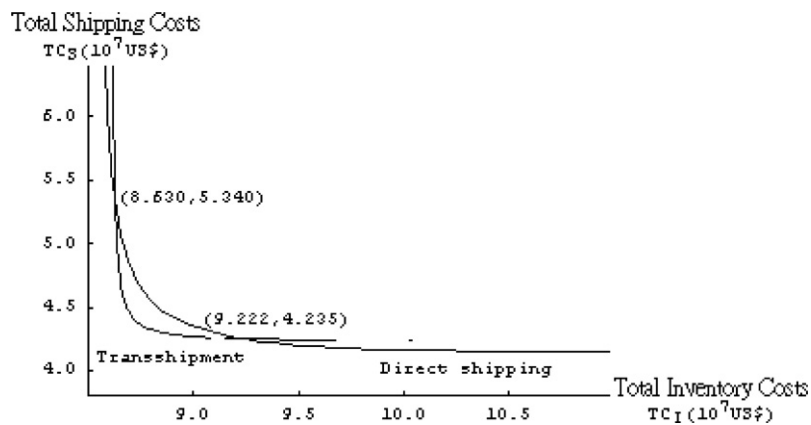
Furthermore, Table 2 takes the Philippines as an example to show the routing decisions and the optimal ship size and sailing frequency of each line. In Table 2, most of the optimal ship types are ships with higher speed, i.e. ships of types T5, T6, and T2. Ships with lower speed, i.e. ships of types T1, T3, and T4, are seldom the optimal ship types, or the range of those ships when they are the optimal ship type is very small. This implies that the optimal ship tends to be the ship with a higher speed. Next, the optimal ship type of the direct lines is the larger ship, i.e. ships of types T5 and T6, while the optimal ship type of the feeder lines includes five types of ship, i.e. ships of types T1, T2, T3, T5, and T6. It implies that the optimal ship size tends to be large as the shipping distance increases. In addition, if the range of T6 being the optimal ship type of direct lines is compared among the three regions, then it shows



(a) Thailand.



(b) The Philippines.



(c) Vietnam.

Fig. 7. Pareto optimal solutions for both trans-shipment and direct shipping for three regions.

that the range increases with the cargo flow. This implies that the optimal ship size tends to be the large ship as the cargo flow increases. However, in reality the cargo flows of three direct lines are not large enough to push container carriers to provide their services using ships of type T6, because the weekly sailing frequencies are less than every two weeks.

Table 2

The routing decision and the optimal ship size and sailing frequency of each line for the Philippines

Total shipping costs (10 <sup>7</sup> US\$)	Total inventory costs (10 <sup>7</sup> US\$)	Routing decision	Main line		Direct line		Feeder line	
			Ship type	Weekly sailing frequency	Ship type	Weekly sailing frequency	Ship type	Weekly sailing frequency
5.895–∞	8.506–8.635	Direct shipping	T6	1.33	T5	1.33–∞	T5	3.33–∞
4.659–5.895	8.635–8.707	Trans-shipment	T6	1.46	–	–	T5	2.74–4.26
4.407–4.659	8.707–9.131						T2	0.57–3.08
4.405–4.407	9.131–9.156						T3	0.46–0.49
4.270–4.405	9.156–9.506	Direct shipping	T6	1.33	T5	0.17–0.26	T2	0.45–0.69
4.260–4.270	9.506–9.550				T5	0.16–0.17	T1	0.43–0.45
4.252–4.260	9.550–9.589				T5	0.16–0.16	T2	0.41–0.43
4.244–4.252	9.589–9.661				T5	0.15–0.16	T3	0.37–0.39
4.240–4.244	9.661–9.681				T5	0.15–0.15	T5	0.37–0.37
4.238–4.240	9.681–9.810				T6	0.13–0.13	T1	0.36–0.37
4.225–4.238	9.810–9.897				T6	0.12–0.13	T2	0.34–0.36
4.217–4.225	9.897–9.998				T6	0.11–0.12	T3	0.30–0.32
4.212–4.217	9.998–10.789				T6	0.11–0.11	T5	0.29–0.30

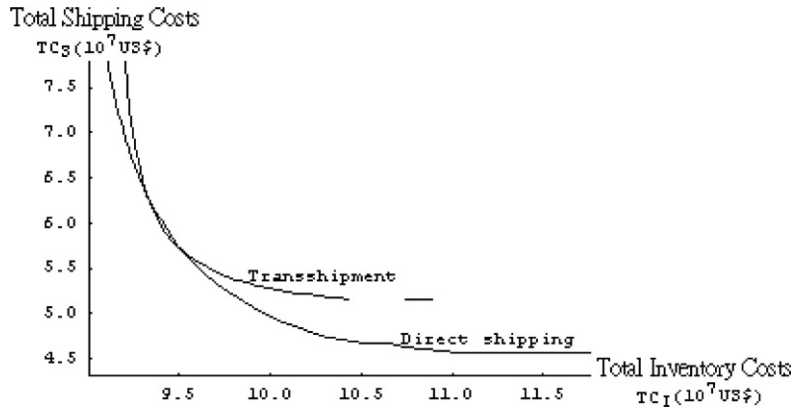
### 5.3. Hub port charges and efficiency

In this subsection, a sensitivity analysis on the effects of hub port charges and efficiency is made and discussed. Port charges and efficiency in a hub port are key factors that influence the routing decision. The authority of a hub port may lower port charges or improve operating efficiency to attract more carriers to trans-ship their containers through the port. Moreover, adopting different charge discounts for different routes is also a strategy. For example, in this case study, cargoes from the Philippines and Vietnam tend to be trans-shipped through Kaohsiung, but cargoes from Thailand tend to be shipped directly to the U.S. West Coast. If the Kaohsiung harbor authority would provide a charge discount to containers from Thailand, it might attract more carriers to trans-ship more containers originating from Thailand through Kaohsiung. Fig. 8(a) and 8(b) show the routing decision of Thailand under two scenarios where the port charges in Kaohsiung are the same and reduced by 50%, respectively. A comparison of the two figures shows that as the port charges decrease, the range of trans-shipment being the optimal route increases, and consequently the routing decision tends to be shipping cargoes through Kaohsiung. In addition, Fig. 8(c) shows the routing decision if the cargo handling rate would be improved up to 140 TEU per hour and the average arrival and departure process time in Kaohsiung port would be reduced to 1 hour. It is obvious that the range of trans-shipment being the optimal route increases and so the routing decision tends to be the trans-shipment strategy.

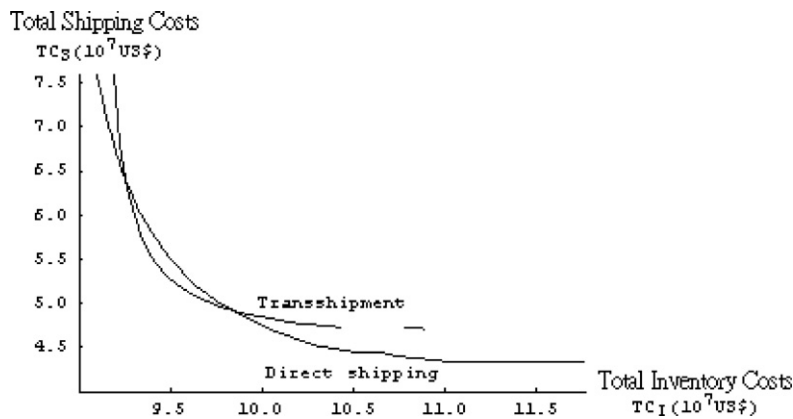
The policies of decreasing port charges and improving port efficiency are different. It is easy for a port authority to lower either the charges on a ship or charges on the cargo handling and thereby reduce the relative shipping costs for cargo trans-shipped via the port. For example, Pusan Port, the largest port in South Korea, has adopted the strategy of lowering its charges to attract trans-shipment cargoes from the Northeast of China, and it is shown that this strategy is successful. It not only markedly increases the container throughput in Pusan, but it also makes Pusan Port the most important hub port in Northeast Asia. On the other hand, improving port efficiency usually requires the lowering of the cargo handling time or ship dwelling time in the hub port. This will lower the inventory costs for the shippers and the shipping costs for the carriers at the same time. However, it is not easy to achieve as it usually requires an upgrading of the port facilities and/or using different management strategies. Nevertheless, Kaohsiung Port currently leases part of its container terminals to ocean carriers and stevedoring companies. This strategy has indeed improved the port's efficiency, because private companies will use every possible method to earn money.

## 6. Conclusions

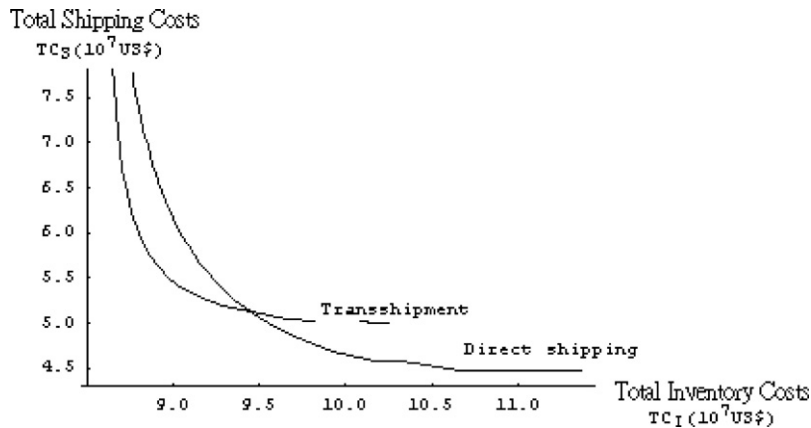
This study developed a two-objective model to decide ship routing, ship size, and sailing frequency on a maritime hub-and-spoke network. Different from previous studies which focus on making single-objective decisions by



(a) Current charges.



(b) 50% off current charges.



(c) Improving port efficiency.

Fig. 8. The influence of port charges and efficiency in Kaohsiung on the routing decision.

minimizing total shipping and inventory costs or merely minimizing shipping cost, this study minimized shipping costs and inventory costs separately, to show that carriers aim not only to lower their shipping costs but also to enhance their services in order to increase their competitiveness. Therefore, the proposed model can provide flexibility on decision-making and produce better planning alternatives for carriers.

The shipping costs include capital and operating costs, fuel cost, and port charges, while the inventory costs include cargo waiting time cost and shipping time cost. In contrast with previous literature, in this study these two cost functions were formulated as flow-dependent and varying with the ship size so as to demonstrate shipping economies. Moreover, this study derived a relationship equation for exploring the trade-off between shipping costs and inventory costs. Based on this, Pareto optimal solutions of the two-objective model were determined. As a result, not only the optimal ship size and sailing frequency can be obtained for any route, but also the routing decision on whether to route containers through a hub or directly to its destination can be made in objective value space.

The two-objective model was applied to a simplified version of hub-and-spoke network that included one main line and three feeder lines. The results showed that decision-making on either routing cargoes directly or through a hub can be determined for any region, and that the optimal ship size and sailing frequency are determined simultaneously. The results also showed that cargoes from Thailand to the U.S. West Coast are likely to be shipped directly, and that cargoes from the Philippines and Vietnam to the U.S. West Coast are likely to be trans-shipped through a hub. This is reasonable and in accordance with the real world routing decisions of current carriers. The results of the sensitivity analysis showed that the routing decision tends to be shipping containers through a hub as hub port charges decrease or the efficiency improves.

In summary, this study demonstrated that the proposed model could be used to determine the optimal routing, ship size, and sailing frequency with respect to each level of inventory costs and shipping costs. It also illustrated the effects of port charges and efficiency on the optimal decisions. In addition, the proposed model not only provides flexibility for carriers on decision-making, but also provides a tool to analyze the trade-off between shipping costs and inventory costs. Future studies may extend or apply the two-objective model proposed in this study on some practical topics, such as the potential market and shipping economies of ultra large ships and/or high-speed ships, alliance or competition strategies between carriers, or to a carriers’ decision-making process on hub ports.

**Appendix A**

When the value of  $TC_S$  is held constant, the second partial derivative of the function  $TC_I$  with respect to  $C_S^s$  can be derived. That is

$$\frac{\partial^2(TC_I)}{\partial(C_S^s)^2} = \frac{2A_t^s \Psi_t^s}{(C_S^s - \Phi_t^s)^3} + \frac{2A_u^d \Psi_u^d}{(TC_S - \overline{C_{S,t}^h} - C_S^s - \Phi_u^d)^3}. \tag{A.1}$$

Because  $A_t^s, \Psi_t^s, A_u^d, \Psi_u^d > 0, C_S^s > \Phi_t^s$ , and  $TC_S - \overline{C_{S,t}^h} - C_S^s - \Phi_u^d = C_S^d - \Phi_u^d > 0$ , thus  $\frac{\partial^2(TC_I)}{\partial(C_S^s)^2} > 0$ . Therefore,  $TC_I$  is a convex function, and the value of  $TC_I$  first decreases and then increases as  $C_S^s$  increases, as shown in Fig. 3.

**Appendix B**

$TC_S$  and  $TC_I$  denote, respectively, the total shipping costs and the total inventory costs of main line,  $h$ , feeder line,  $s$ , and direct line,  $d$ . When the main line is served with the minimum shipping cost, the total shipping costs and the total inventory costs for routing containers directly can be expressed, respectively, as

$$TC_S = \overline{C_{S,t}^h} + C_S^s + C_S^d, \tag{B.1}$$

$$TC_I = \overline{C_{I,t}^h} + C_I^s + C_I^d, \tag{B.2}$$

where  $C_S^s$  and  $C_I^s$  are the shipping and inventory costs of feeder line,  $s$ , respectively, and  $C_S^d$  and  $C_I^d$  are the shipping and inventory costs of direct line,  $d$ , respectively.

The relationship between  $C_S^s$  and  $C_I^s$  for using any ship of type  $t$  on the feeder line is

$$C_I^s = \Gamma_t^s + \frac{A_t^s \cdot \Psi_t^s}{C_S^s - \Phi_t^s} \quad \text{for } C_S^s \geq \overline{C_{S,t}^h}. \tag{B.3}$$

Eq. (B.3) is a curve similar as the solid curve shown in Fig. 2(a). When the value of  $C_S^s$  equals the minimum value of the shipping costs,  $\overline{C_{S,t}^h}$ , the value of  $C_I^s$  must be equal to the maximum value of the inventory costs,  $\overline{C_{I,t}^h}$ ; when the



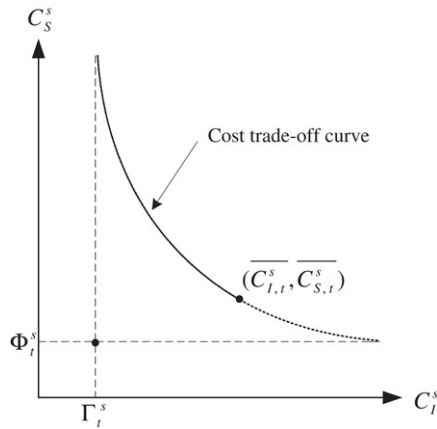


Fig. B.1. The relationship between  $C_S^s$  and  $C_I^s$  for using any ship of type  $t$ .

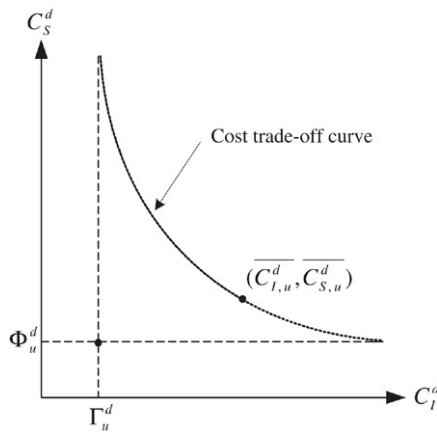


Fig. B.2. The relationship between  $C_S^d$  and  $C_I^d$  for using any ship of type  $u$ .

value of  $C_S^s$  increases, the value of  $C_I^s$  decreases; when the value of  $C_S^s$  approaches infinite, the value of  $C_I^s$  approaches  $\Gamma_t^s$ , as shown in Fig. B.1.

Likewise, the relationship between  $C_S^d$  and  $C_I^d$  for using any ship of type  $u$  on the direct line is

$$C_I^d = \Gamma_u^d + \frac{A_u^d \cdot \Psi_u^d}{C_S^d - \Phi_u^d} \quad \text{for } C_S^d \geq \overline{C_{S,u}^d}. \tag{B.4}$$

Eq. (B.4) is also a curve similar as the solid curve shown in Fig. 2(a). When the value of  $C_S^d$  equals the minimum value of the shipping costs,  $\overline{C_{S,u}^d}$ , the value of  $C_I^d$  must be equal to the maximum value of the inventory costs,  $\overline{C_{I,u}^d}$ ; when the value of  $C_S^d$  increases, the value of  $C_I^d$  decreases; when the value of  $C_S^d$  approaches infinite, the value of  $C_I^d$  approaches  $\Gamma_u^d$ , as shown in Fig. B.2.

The sum of  $\overline{C_{S,t^*}^h} + \overline{C_{S,t}^s} + \overline{C_{S,t}^d}$  is the minimum value of the total shipping costs,  $TC_S$ . As  $TC_S$  equals  $\overline{C_{S,t^*}^h} + \overline{C_{S,t}^s} + \overline{C_{S,t}^d}$ , the shipping costs on the feeder line and on the direct line must be  $\overline{C_{S,t}^s}$  and  $\overline{C_{S,u}^d}$ , respectively. Accordingly, the inventory costs on the feeder line and on the direct line must be  $\overline{C_{I,t}^s}$  and  $\overline{C_{I,u}^d}$ , respectively, as shown above. Thus, as  $TC_S$  equals  $\overline{C_{S,t^*}^h} + \overline{C_{S,t}^s} + \overline{C_{S,t}^d}$ , then  $TC_I$  equals  $\overline{C_{I,t^*}^h} + \overline{C_{I,t}^s} + \overline{C_{I,u}^d}$ . Furthermore, since  $TC_S = \overline{C_{S,t^*}^h} + C_S^s + C_S^d$ , the value of  $TC_S$  being increased means that the value of  $C_S^s$  increases and/or  $C_S^d$  increases. Again as shown above, when  $C_S^s$  increases, the value of  $C_I^s$  decreases, and when  $C_S^d$  increases, the value of

$C_I^d$  decreases. Therefore, because  $TC_I$  is equal to the sum of  $\overline{C_{I,t}^h}$ ,  $C_I^s$ , and  $C_I^d$ , the total inventory costs,  $TC_I$ , must decrease when the total shipping costs,  $TC_S$ , increases, as shown in Fig. 4.

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