A Vector-Fitting Formulation for Parameter Extraction of Lossy Microwave Filters

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***Abstract*—In this letter, a formulation based on the vector fitting is applied to extract the equivalent circuit model from the frequency response of lossy cross-coupled microwave filters. By approximating the lossy response with short-circuit admittance parameters in partial fractional expansion form, the proposed method can evaluate the unloaded quality factor of resonators and extract the transversal coupling matrix simultaneously. The methodology of the vector fitting can identify the poles and residues of the short-circuit admittance parameters even when the poles are on the complex plane. And the extracted transversal coupling matrix can further transform into the prescribed form corresponding to the physical layout. The proposed method can be used in the tuning process of filter designs where the extraction of a coupling matrix is essential. To verify the method, a cross-coupled quadruplet filter is used as an example.**

***Index Terms*—Bandpass filter (BPF), coupling matrix, vector fitting.**

I. INTRODUCTION

THE cross-coupled filters based on the model proposed in [1] and [2] have found wide applications in wireless communication systems since they can provide the generalized Chebyshev response which exhibits the optimal in-band response and selectivity. However, the tuning of the filters based on cross-coupled topologies is time-consuming. In order to tune the cross-coupled filters more efficiently, therefore, diagnosis methods are needed to guide the process of the filter tuning [3]–[5]. Since the model proposed in [1] and [2] can be expressed by a coupling matrix, most diagnosis methods in the literature are focused on extracting a coupling matrix from the simulated or measured response. By comparing the extracted coupling matrix to the desired coupling matrix, one can determine how to adjust the filter [3], [5].

Most parameter extraction methods are only valid for lossless filters since this is the assumption in their formulations. Thus, getting a coupling matrix from a lossy filter response is still an important research topic. Recently, a modified formulation of the Cauchy method which can extract the parameters of a lossless model from the response of a lossy bandpass filter (BPF) is proposed [6]. The formulation in [6] can generate characteristic polynomials suitable for the synthesis of a low pass prototype associated with the lossless model of the filter, which is not feasible in the formulation proposed in [5] and [7]. Strictly speaking, the methods in [5] would require lossless measured data and can not give a measure of how lossy a filter is.

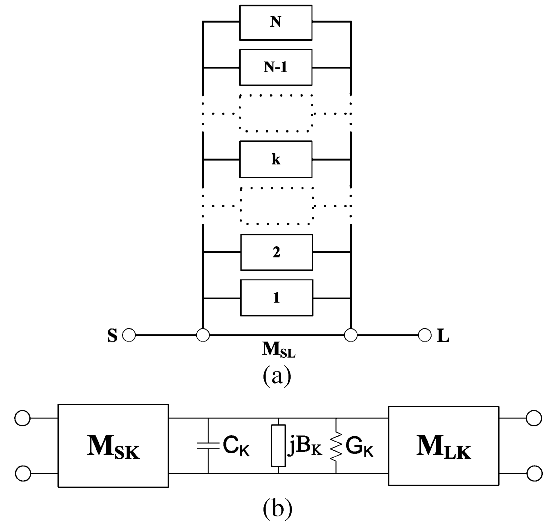

Fig. 1. Canonical transversal array. (a) N-resonator transversal array including direct source–load coupling $M_{\rm SL}$. (b) Equivalent circuit of the kth "low-pass resonator" in the transversal array.

In this letter, to take the loss of a filter into consideration, we propose to use the model in Fig. 1. The model in Fig. 1 was modified from the model first proposed in [8] for filter synthesis and known as transversal network. The only difference between the model used in [8] and here is that we added the conductance, G_k, in each branch of the transversal network to model the loss, as shown in Fig. 1(b). As the formulation in [8], the short-circuit admittance parameters, also know as Y-parameters, of the model in Fig. 1, can be expressed by a polynomial in partial fractional expansion form. Here, the introduction of the loss positions the poles of the Y-parameters on the complex plane instead of on the imaginary axis as in the lossless case. To effectively get the short-circuit admittance parameters in the partial fraction expansion form, the technique of vector fitting [9] is applied. The formulation based on the vector fitting can identify the positions of poles and calculate the residue of the Y-parameters. The poles of the Y-parameters contain the information of how lossy a filter is. Thus, the proposed method allows:

1) the evaluation of how lossy a filter is from the simulated or measured data;
2) the generation of the Y-parameters in the partial fraction expansion form, which is suitable for the synthesis of a low-pass prototype by the method in [8].

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II. Applying Vector Fitting to Parameters Extraction

Following the formulation in [8], one can determine the two-port short-circuit admittance matrix $[Y_N]$ for the parallel-connected transverse array in Fig. 1 as

$$[Y_N] = \begin{bmatrix} y_{11}(s) & y_{12}(s) \\ y_{21}(s) & y_{22}(s) \end{bmatrix} = j\begin{bmatrix} 0 & M_{SL} \\ M_{SL} & 0 \end{bmatrix} + \sum_{k=1}^{N} \frac{1}{(sC_k + jB_k + G_k)} \times \begin{bmatrix} M_{Sk}^2 & M_{SK}M_{LK} \\ M_{SK}M_{LK} & M_{Lk}^2 \end{bmatrix}. \quad (1)$$

Thus, if we can approximate the measured or simulated Y-parameters by polynomials, $y_{ij,\mathrm{appx}}(s)$, in the following form:

$$[Y_{\mathrm{appx}}] = \begin{bmatrix} y_{11,\mathrm{appx}}(s) & y_{12,\mathrm{appx}}(s) \\ y_{21,\mathrm{appx}}(s) & y_{22,\mathrm{appx}}(s) \end{bmatrix} = j\begin{bmatrix} 0 & K_0 \\ K_0 & 0 \end{bmatrix} + \sum_{k=1}^{N} \frac{1}{(s - j\lambda_k)} \begin{bmatrix} r_{11k} & r_{12k} \\ r_{21k} & r_{22k} \end{bmatrix}. \quad (2)$$

Note that the residues r_{11k}, r_{12k}, r_{21k}, and r_{22k} are real numbers while the λ_k is a complex number in general in (2). By comparing the first column of the (1) and (2), we can obtain $M_{SL} = K_0$, $C_k = 1$, $M_{kk} = B_k = -\mathrm{Re}[\lambda_k]$, $G_k = \mathrm{Im}[\lambda_k]$, $M_{Sk} = \sqrt{r_{11k}}$, $M_{Lk} = r_{21k}/\sqrt{r_{11k}}$. Once the M_{kk}, M_{SK}, M_{LK}, M_{SL} are determined, the transversal matrix is formed [8]. To transform the transversal coupling matrix into another coupling matrix with the prescribed coupling route corresponding to the physical structure, the methodology in [10] is used in this letter.

To obtain approximated Y-parameters of simulated or measured data in the form of (2), the technique of vector fitting is applied. The vector fitting technique is a general methodology for the fitting of measured or calculated frequency domain response with rational function approximation [9]. Instead of directly fitting the data into a ratio of two polynomials, the methodology generates a polynomial in partial fractional expansion form. The source code can be obtained from the authors of the paper [9], but it can not be directly used to determine the polynomials that fit the model in Fig. 1. The reason is that the formulation in [9] is in the bandpass frequency domain s, $s = j2\pi f$, and the generated polynomials can not fit into the model in Fig. 1 even after bandpass-to-lowpass frequency transformation. To apply the formulation of vector fitting to fit the model in Fig. 1, the formulation in [9] is followed except that the symbol s stands for normalized frequency $s = j\Omega$. For the need of parameter extraction, one must calibrate the position of reference planes of the input and output ports [4], and then fit the $y_{11,\mathrm{appx}}(s)$ and $y_{21,\mathrm{appx}}(s)$ simultaneously. Thus, stack the $y_{11,\mathrm{appx}}(s)$ and $y_{21,\mathrm{appx}}(s)$ to form a vector $\overline{y}(s)$, as shown in the following equation:

$$\overline{y}(s) = \begin{bmatrix} y_{11,\mathrm{appx}}(s) \\ y_{21,\mathrm{appx}}(s) \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{N} \frac{r_{11k}}{(s-j\lambda_k)} \\ jK_0 + \sum_{k=1}^{N} \frac{r_{21k}}{(s-j\lambda_k)} \end{bmatrix}. \quad (3)$$

The procedure in [9] is followed to identify the poles. As mentioned in [9], final positions of poles are determined through iterative calculations and not sensitive to the starting positions of poles. It is worth noting that the complex poles come in pairs in [9]; however, for the case here, it is not necessary for the poles to be in complex pairs since they are in the normalized frequency domain. After identifying the poles, one can identify the corresponding residues and obtain the polynomials in (3).

III. Example

To illustrate the proposed method, a cross-coupled quadruplet filter is given as an example. The layout of the filter is the same as [11, Fig. 4] excluding the S/L coupling controlling line. The center frequency and fractional bandwidth of the filter are 2.4 GHz and 3.75%, respectively. To demonstrate the ability of extracting the coupling matrix from a lossy filter response, the conductor loss is included. The simulation was performed using Sonnet [12] and the result is shown in Fig. 2. The method in [4] is used to calibrate the position of the reference plane. The following bandpass-to-lowpass frequency transformation is adopted:

$$s = j\Omega = j(f_0/\Delta f)(f/f_0 - f_0/f) \quad (4)$$

where Δf and f_0 are bandwidth and center frequency of the filter, respectively. By using the proposed method, the extracted normalized transversal coupling matrix M_1 is calculated to be (5), shown at the bottom of the page, along with other parameters $G_1 = 0.1472$, $G_2 = 0.1427$, $G_3 = 0.1539$, and $G_4 = 0.1501$. Then, transform the coupling matrix M_1 into the coupling matrix M_2 which corresponds to the coupling route of the cross-coupled quadruplet by the method in [10]. The matrix M_2 is (6), shown at the bottom of the page.

$$M_1 = \begin{bmatrix} 0 & 0.3409 & 0.5920 & 0.3704 & 0.6124 & 0 \\ 0.3409 & 1.3887 & 0 & 0 & 0 & -0.3629 \\ 0.5920 & 0 & 0.8265 & 0 & 0 & 0.5941 \\ 0.3704 & 0 & 0 & -1.0926 & 0 & 0.3886 \\ 0.6124 & 0 & 0 & 0 & -0.4793 & -0.6192 \\ 0 & -0.3629 & 0.5941 & 0.3886 & -0.6192 & 0 \end{bmatrix} \quad (5)$$

$$M_2 = \begin{bmatrix} 0 & 1.0016 & 0 & 0 & 0 & 0 \\ 1.0016 & 0.1178 & 0.8401 & -0.0016 & -0.1496 & 0 \\ 0 & 0.8401 & 0.1592 & 0.7409 & -0.0016 & 0 \\ 0 & -0.0016 & 0.7409 & 0.1592 & 0.8401 & 0 \\ 0 & -0.1496 & -0.0016 & 0.8401 & 0.1178 & 1.0016 \\ 0 & 0 & 0 & 0 & 1.0016 & 0 \end{bmatrix} \quad (6)$$

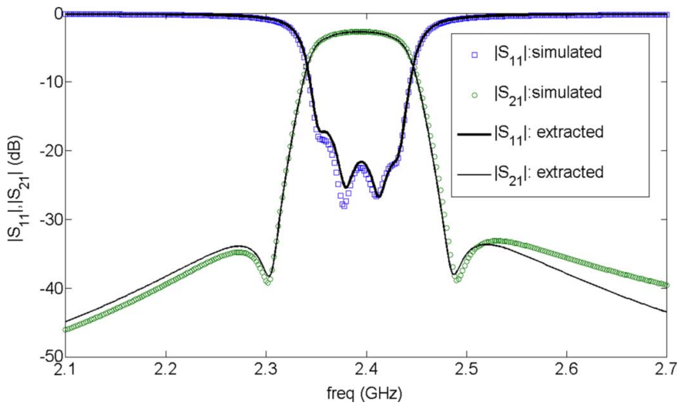

Fig. 2. Simulated and extracted results of the cross-coupled quadruplet filter under consideration.

With the approximation that the unloaded quality factor of each resonator is the same, we can get a measure of how lossy the filter is by calculating the average value of $G_k s$. In this case, the average value of $G_k s$ is $G_{\text{loss}} = (G_1 + G_2 + G_3 + G_4)/4 = 0.1476$. From the quantity G_{loss}, we can evaluate the unloaded quality factor, Q_u, by $Q_u = \Delta f/(G_{\text{loss}} f_0)$. In this case, Q_u is equal to 180.67. A normalized coupling matrix [M] is related to the responses of $S_{11}(\Omega)$ and $S_{21}(\Omega)$ via the following equations [13]:

$$S_{11} = 1 + 2j[A^{-1}]_{1,1} \tag{7}$$

$$S_{21} = -2j[A^{-1}]_{N+2,1}. \tag{8}$$

Here, $A = \Omega[U_0] + [M] - j[G], \Omega = (f_0/\Delta f)(f/f_0 - f_0/f)$, $[U_0]$ is similar to the $(N+2) \times (N+2)$ identity matrix except that $[U_0]_{1,1} = [U_0]_{N+2,N+2} = 0$, and $[G]$ is the diagonal matrix $[G] = \text{diag}\{1, G_{\text{loss}}, \ldots, G_{\text{loss}}, 1\}$. Substituting the extracted coupling matrix M_2 with $G_{\text{loss}} = 0.1476$ into the (7) and (8), one can obtain the response shown in Fig. 2.

IV. Conclusion

The transversal network including a conductance in each branch has been proposed to model a lossy filter. The technique of vector fitting has been applied to obtain the approximated short-circuit admittance parameters which correspond to the transversal network. With the help of the vector fitting, the poles and residues of short-circuit admittance parameters can easily be identified. Using the poles and residues, a transversal coupling matrix corresponding to the measured or simulated response was constructed. The transversal matrix can be transformed to the prescribed form corresponding to the physical layout. In addition, the loss term can be extracted from the positions of poles. The proposed method is useful in computer-aided filter tuning, where the extraction of the coupling matrix is essential. A cross-coupled quadruplet filter was used as an example to verify the theory, and the extracted response fits well with the simulated result.

References

[1] A. Atia and Williams, “New type of waveguide bandpass filters for satellite transponders,” *COMSAT Tech. Rev.*, vol. 1, no. 1, pp. 21–43, 1971.

[2] S. Amari, U. Rosenberg, and J. Bornemann, “Adaptive synthesis and design of resonator filters with source/load-multiresonator coupling,” *IEEE Trans. Microw. Theory Tech.*, vol. 50, no. 8, pp. 1969–1978, Aug. 2002.

[3] P. Harscher, R. Vahldieck, and S. Amari, “Automated filter tuning using generalized low-pass prototype networks and gradient-based parameter extraction,” *IEEE Trans. Microw. Theory Tech.*, vol. 49, no. 12, pp. 2532–2538, Dec. 2001.

[4] H.-T. Hsu, Z. Zhang, K. A. Zaki, and A. E. Atia, “Parameter extraction for symmetric coupled-resonator filters,” in *IEEE MTT-S Int. Dig.*, June 2002, vol. 3, pp. 1445–1448.

[5] A. Garia-Lamperez, S. Llorente-Romano, M. Salazar-Palma, and T. K. Sarkar, “Efficient electromagnetic optimization of microwave filters and multiplexers using rational models,” *IEEE Trans. Microw. Theory Tech.*, vol. 52, no. 2, pp. 508–521, Feb. 2004.

[6] G. Macchiarella and D. Trina, “A formulation of the Cauchy method suitable for the synthesis of lossless circuit models of microwave filters from lossy measurements,” *IEEE Microw. Wireless Compon. Lett.*, vol. 16, no. 5, pp. 243–245, May 2006.

[7] A. García-Lampérez, T. K. Sarkar, and M. Salazar-Palma, “Generation of accurate rational models of lossy systems using the cauchy method,” *IEEE Microw. Wireless Compon. Lett.*, vol. 14, no. 10, pp. 490–492, Oct. 2004.

[8] R. J. Cameron, “Advanced coupling matrix synthesis techniques for microwave filters,” *IEEE Trans. Microw. Theory Tech.*, vol. 51, no. 1, pp. 1–10, Jan. 2003.

[9] B. Gustavsen and A. Semlyen, “Rational approximation of frequency domain responses by vector fitting,” *IEEE Trans. Power Delivery*, vol. 14, no. 3, pp. 1052–1061, Jul. 1999.

[10] P. Kozakowski, A. Lamecki, P. Sypek, and M. Mrozowski, “Eigenvalue approach to synthesis of prototype filters with source/load coupling,” *IEEE Microw. Wireless Compon. Lett.*, vol. 15, no. 2, pp. 98–100, Feb. 2005.

[11] C.-K. Liao and C.-Y. Chang, “Design of micorstrip quadruplet filters with source-load coupling,” *IEEE Trans. Microw. Theory Tech*, vol. 53, no. 7, pp. 2302–2308, Jul. 2005.

[12] *“Em User’s Manual,”* Sonnet Software, Inc., Liverpool, NY, 2004.

[13] A. Garia-Lamperez, S. Llorente-Romano, M. Salazar-Palma, M. J. Padilla-Cruz, and I. H. Carpintero, “Synthesis of cross-coupled lossy resonator filters with multiple input/output couplings by gradient optimization,” in *Proc. IEEE Antennas Propag. Soc. Int. Symp.*, Jun. 2003, vol. 2, pp. 52–55.