Journal of Hydrology (2007) **335**, 406–418

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The use of sensitivity analysis in on-line aquifer parameter estimation

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Received 7 June 2006; received in revised form 20 October 2006; accepted 15 December 2006

KEYWORDS
Groundwater;
Pumping test;
Sensitivity analysis;
Parameter estimation model;
Leaky aquifer;
Unconfined aquifer

Summary Generally, a pumping test requires a lot of effort and expense to perform the test and the drawdown is measured and analyzed for determining the aquifer parameters. The estimated aquifer parameters obtained from graphical approaches may not be in good accuracy if the pumping time is too short to give a good visual fit to the type curve. Yet, the problems of long pumping time and required efforts can be significantly reduced if the drawdown data are measured and the parameters are simultaneously estimated on-line. However, the drawdown behavior of the leaky and unconfined aquifers in response to the pumping may have a time lag and the time to terminate the estimation may not be easily and quickly to decide when applying a parameter estimation model (PEM) on-line to analyze the parameters. This study uses the sensitivity analysis to explore the influence period of each aquifer parameter to the pumping drawdown and the influence period is used as a guide in terminating the estimation when applying the PEM for on-line parameter identification. In addition, the sensitivity analysis is also used to study the effects of different value of S_y and the distance between pumping well and observation well on the influence time of S_y during the pumping.

Introduction

Groundwater hydrologists often conduct pumping tests to obtain aquifer parameters, such as hydraulic conductivity and storage coefficient, which are necessary information for quantitative groundwater studies. Theis (1935) obtained the solution for unsteady groundwater flow toward a pumping well in a confined aquifer by analogy to the problem of heat conduction. Hantush and Jacob (1955) described non-steady radial flow to a well in a fully penetrating leaky aquifer under a constant pumping rate. In their model, the aquitard is overlain by an unconfined aquifer, and the main aquifer is underlain by an impermeable bed. Boulton (1954, 1963) developed the analytical solution by introducing the concept of delayed yield for unconfined formations. Neuman (1972, 1974) presented a solution that considers the effects of elastic storage and anisotropy of aquifers on drawdown behavior and recognized the existence of vertical flow components. Neuman's model can fit observed pumping

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doi:10.1016/j.jhydrol.2006.12.007

test data in many case studies and is very convenient to use in engineering practice. Moench (1997) presented a Laplace transform solution to a partially penetrating well of finite diameter in a slightly compressible water table aquifer. The solution, which accounts for the effects of well bore storage and skin, uses numerical inversion of Stehfest's algorithm to obtain the dimensionless drawdown in time domain.

In the past, the hydrogeological parameters were determined using graphical methods. Cooper and Jacob (1946) developed a method to approximate the Theis equation, together with a data analysis approach which does not require type-curve matching. Hantush (1964) developed a type-curve method for determining parameters of the leaky aquifer if the test period is long enough to reflect the influence of the leakage. Prickett (1965) described a systematic approach to determine the parameters, using a graphical procedure based on Boulton's type curves. Neuman (1975) also gave a graphical type-curve solution procedure to determine the hydraulic parameters in unconfined aquifer.

The aquifer parameters can also be obtained by parameter estimation model (PEM) which coupled an analytical solution or a numerical model in terms of aquifer drawdown along with a numerical approach such as nonlinear programming (e.g., Saleem, 1970), Marquardt algorithm (e.g., Chander et al., 1981), sensitivity matrix, (McElwee, 1980; Paschetto and McElwee, 1982), nonlinear least-squares and Newton's method (e.g., Yeh, 1987; Yeh and Han, 1989), nonlinear regression (e.g., Lebbe, 1999), and extended Kalman filter (e.g., Leng and Yeh, 2003; Yeh and Huang, 2005). Some commercial softwares, like AQTESOLV (Duffield, 2002), also use nonlinear weighted least-squares approach to match the time—displacement data obtained from an aquifer test with type curves or straight lines for parameter estimation. Alternatively, heuristic optimization approaches such as Simulated Annealing (e.g., Yeh et al., in press) was proposed to couple with an analytical solution for determining the best-fit parameters.

Recently, the sensitivity analysis is widely used in many fields. Cukier et al. (1973, 1975, 1978) as well as Schibly and Shuler (1973) developed a statistical approach for sensitivity analysis to nonlinear algebraic equations. Jiao and Rushton (1995) provided a sensitivity analysis of drawdown to parameters and drawdown's influence on parameter estimation for pumping tests in large-diameter wells. They concluded that the well storage reduces the sensitivities of drawdown to transmissivity and storativity. Kabala and Milly (1990) used sensitivity analysis for analyzing the effect of parameter uncertainty and soil heterogeneity on the transport of moisture in unsaturated porous media. Kabala (2001) proposed logarithmic sensitivity to analyze the pumping test on a well with wellbore storage and skin. In addition, Kabala et al. (2002) also studied the logarithmic sensitivity, plausible relative errors, and deterministic parameter correlations in a simple semi-analytic no-crossflow model of the transient flowmeter test (TFMT) that accounts for a thick skin around the wall. Vachaud and Chen (2002) analyzed a large-scale hydrologic model problem by sensitivity theory. Gooseff et al. (2005) performed sensitivity analysis of a conservative transient storage model and two different reactive solute transport models.

The pumping test was commonly performed for a long period of time when applying a graphical approach to analyze the measurement data in the past. Otherwise, the estimated result may not be in good accuracy if the pumping time is too short and the data points are too sparse to give a good visual fit to the type curve. In the leaky aquifer, the hydraulic head in the adjacent aquifer remains constant and that the two aquifers are in equilibrium at the beginning of the pumping. After pumping, the water is immediately withdrawn from the production aquifer and then the head difference between two aquifers induces a flow across the aquitard. Hence, the hydraulic parameters of the confining bed (aquitard) may not be accurately estimated if only first few drawdown data points are used.

Physically, the drawdown in an unconfined aquifer can be divided into three segments (Charbeneau, 2000). In the early stage, water is instantaneously released from storage by the compaction of the aquifer and the expansion of the water. In the second stage, the vertical gradient near the water table causes drainage of the porous matrix. The vertical hydraulic conductivity K_z starts to contribute to the pumping and the rate of decline in the hydraulic head slows or stops after a period of time. Finally, when the flow is essentially horizontal and most of the pumping is supplied by the specific yield, S_y. Therefore, the analysis of S_y requires sufficient long drawdown data fallen at the third section. In some cases, the effect of well bore storage can not be neglected because the diameter of pumping well is large. The water is withdrawn from the well at the start of pumping, and consequently the groundwater flows into the well due to the head difference between the well and the formation. An on-line PEM for identifying aquifer parameter can facilitate the applicability of the pumping test. A practical question involved when using on-line PEM is: when is a suitable time to terminate the estimation? The results of parameter estimation may be inaccurate if the parameter estimation is terminated before the character of aquifer parameters starts to affect the drawdown.

This study aims at providing a decision support using sensitivity analysis in terminating the estimation when applying the on-line PEM in determining the aquifer parameters. Three synthetic drawdown data sets, one for leaky aquifer (generated based on Hantush and Jacob's model), and two for unconfined aquifer (generated based on Neuman's model and Moench's models). A PEM based on Simulated Annealing algorithm is applied to identify the parameters in both leaky and unconfined aquifers on-line using the synthetic and real field time-drawdown data sets. In addition, AQTESOLV is employed to identify the parameters of unconfined aquifer considering the effect of well bore storage using the synthetic data set. The influence period obtained from the sensitivity analyses is used as an indication to terminate the on-line estimation because the drawdown already reflects the effects from the aquifer parameters. Finally, two sensitivity analyses for different S_y values and different distance between pumping well and observation well are performed to study their affects on the influence period of the S_y.

Drawdown of the pumping test in leaky and unconfined aquifers

The Hantush and Jacob's model describing the drawdown within a leaky aquifer in response to the pumping as a function of radial distance and time is (Hantush and Jacob, 1955)

$$s = \frac{Q}{4\pi T} \cdot W\left(u, \frac{r}{B}\right) \tag{1}$$

where s is drawdown, r is the distance between pumping well and observing well, u is dimensionless variable and it is defined as $r^2S/4Tt$, K' is the vertical conductivity of leaky confining bed, b' is thickness of aquitard, $r/B = L$ is called leakage coefficient and B is defined as $\sqrt{T/(K'/b')}$, Q is the pumping rate, and $W(u, r/B)$ is the leaky well function.

The leaky well function $W(u, r/B)$ can be expressed as

$$W\left(u, \frac{r}{B}\right) = \int_u^\infty \frac{1}{y} \cdot \exp\left[-y - \frac{(r/B)^2}{4y}\right] \cdot \mathrm{d}y \tag{2}$$

where y is a dummy variable.

The solution for the equation describing the groundwater flow system in an unconfined aquifer developed by Neuman (1974) is

$$s(r, z, t) = \frac{q}{4\pi T} \int_0^\infty 4yJ_0(y\beta^{1/2}) \left[u_0(y) + \sum_{n=1}^{\infty} u_n(y)\right] \mathrm{d}y \tag{3}$$

where $J_0(x)$ is the zero order Bessel function of the first kind, $\beta = K_z r^2/K_r b^2$ is a dimensionless parameter, y is a dummy variable, and

$$u_0(y) = \frac{\{1 - \exp\left[-t_s\beta\left(y^2 - r_0^2\right)\right]\} \cos h(r_0 z_D)}{[y^2 + (1+\sigma)r_0^2 - \left(y^2 - r_0^2\right)^2/\sigma] \cos h(r_0)} \times \frac{\sin h[r_0(1 - d_D)] - \sin h[r_0(1 - l_D)]}{(l_D - d_D) \sin h(r_0)} \tag{4}$$

$$u_n(y) = \frac{\{1 - \exp\left[-t_s\beta\left(y^2 + r_n^2\right)\right]\} \cos(r_n z_D)}{\left[y^2 - (1+\sigma)r_n^2 - \left(y^2 + r_n^2\right)^2/\sigma\right] \cos(r_n)} \times \frac{\sin[r_n(1 - d_D)] - \sin[r_n(1 - l_D)]}{(l_D - d_D) \sin(r_n)} \tag{5}$$

where $t_s = Tt/Sr^2$ represents the dimensionless time since pumping started, S equals $S_s \times b$, $z_D = z/b$ is the dimensionless elevation of observation point, $\sigma = S/S_y$ is a dimensionless parameter, $d_D = d/b$ denotes the dimensionless vertical distance between the top of perforation in the pumping well and the initial position of water table, and $l_D = l/b$ is the dimensionless vertical distance between the bottom of perforation in the pumping well and the initial position of water table. The term of r_0 and r_n are respectively the roots of the following two equations:

$$\sigma r_0 \sin h(r_0) - (y^2 - r_0^2) \cos h(r_0) = 0, \quad r_0^2 < y^2 \tag{6}$$

and

$$\sigma r_n \sin(r_n) + (y^2 + r_n^2) \cos(r_n) = 0, \quad (2n-1)(\pi/2) < r_n < n\pi \tag{7}$$

Moench (1997) derived a Laplace transform solution for transient flow to a partially penetrating large-diameter well in an unconfined aquifer. The dimensionless drawdown is

$$\bar{h}_D(r_D, z_D, p) = \frac{2E}{p(l_D - d_D)[1 + pW_D(A + S_w)]} \tag{8}$$

with

$$A = \frac{2}{(l_D - d_D)} \sum_{n=0}^{\infty} \frac{K_0(q_n)\{\sin[\varepsilon_n(1 - d_D)] - \sin[\varepsilon_n(1 - l_D)]\}^2}{\varepsilon_n q_n K_1(q_n)[\varepsilon_n + 0.5\sin(2\varepsilon_n)]} \tag{9}$$

$$E = 2\sum_{n=0}^{\infty} \frac{K_0(q_n r_D)\cos(\varepsilon_n z_D)[\sin(\varepsilon_n(1 - d_D)) - \sin(\varepsilon_n(1 - l_D))]}{q_n K_1(q_n)[\varepsilon_n + 0.5\sin(2\varepsilon_n)]} \tag{10}$$

where $W_D = \pi r_c^2/2\pi r_w^2 S_s(l - d)$, $\bar{q}_n = \sqrt{\left(\varepsilon_n^2\beta_w + p\right)}$, $\bar{q}_n r_D = \sqrt{\left(\varepsilon_n^2\beta + pr_D^2\right)}$, $\beta_w = r_w^2 K_z/b^2 K_r$, $\beta = K_z r^2/K_r b^2$, $S_w = K_r d_s/K_s r_w$, and $r_D = r/r_w$. Notice that r_w represents the well radius. The symbol ε_n is the root of

$$\varepsilon_n \tan(\varepsilon_n) = \frac{p}{(\sigma\beta_w + p/\gamma)} \tag{11}$$

where $\sigma = S/S_y$, $\gamma = \alpha_1 b S_y/K_z$, and α_1 is a fitting parameter for drainage from the unsaturated zone and has units of inverse time ($1/T$). A large value of α_1 effectively eliminates this parameter from the solution.

Sensitivity analysis of the aquifer parameters

The sensitivity is defined as a rate of change in one factor with respect to a change in another factor. The parametric sensitivity may be expressed as (McCuen, 1985)

$$S_{pi} = \frac{\partial O}{\partial P_i} = \frac{O(P_i + \Delta P_i; P_{j|j \neq i}) - O(P_1, P_2, \ldots, P_n)}{\Delta P_i} \tag{12}$$

where O is the output function of the system (i.e., the aquifer drawdown) and P_i is the ith input parameter of the system. However, the values of the parametric sensitivity for various parameters are useless for making comparison if the unit and/or the order of magnitude of the parameters are different. Thus, the normalized sensitivity is used and defined as (Kabala, 2001)

$$S_{i,t} = \frac{\partial O}{\partial P_i/P_i} = P_i \frac{\partial O}{\partial P_i} \tag{13}$$

where $S_{i,t}$ is the normalized sensitivity of ith input parameter at time t. Note that O is a function of P_i and t. The partial derivative of this equation may be approximated by a forward differencing formula as

$$\frac{\partial O}{\partial P_i} = \frac{O(P_i + \Delta P_i) - O(P_i)}{\Delta P_i} \tag{14}$$

The increment in the denominator may be approximated by the parameter value times a factor of 10^{-3}, i.e., $\Delta P_i = 10^{-3} P_i$. Eq. (13) measures the influence that the fractional change in the parameter, or its relative error, exerts on the output.

The objective function of the parameter estimation model bases on Simulated Annealing algorithm

The aquifer parameters can be estimated for pumping test data based on Hantush and Jacob's model (1955) for a leaky aquifer and Neuman's model (1974) for an unconfined aquifer when minimizing the sum of square errors between the observed and predicted drawdowns. The objective function is defined as

$$\text{Minimize} \sum_{i=1}^{n} (O_{h_i} - P_{h_i})^2 \tag{15}$$

where n is the total time step and O_{h_i} and P_{h_i} are respectively the observed and predicted drawdowns at time step i. Based on Eq. (15), Simulated Annealing method can determine the best-fit aquifer parameters to the observed drawdown data.

Results and discussion

Sensitivity analysis of aquifer parameters

The synthetic time-drawdown data for a leaky aquifer listed in Table 1 was generated from Hantush and Jacob's model (1955). The pumping rate Q is 3000 m^3/day, the distance R between pumping well and observation well is 30 m, the transmissivity T is 1000 m^2/day, storage coefficient S is 10^{-4}, leakage coefficient L is 0.03. The observed pumping period ranges from 0.017 to 1000 min. The time-drawdown data and the normalized sensitivities are plotted in Figure 1. This figure indicates that the temporal distribution of each normalized sensitivity of the aquifer parameters reflects the temporal change of the drawdown in response to the relative change of each parameter. In other words, the non-zero periods in the normalized sensitivity curves imply that the aquifer parameters have influences on the drawdown at that time. In addition, this figure also indicates that all aquifer parameters have their own influence period to the drawdown. The influence period of parameter S increases from the start of pumping and decreases after 3 min. The drawdown is very sensitive to T except at the early period of the pumping and the normalized sensitivity is continuously increased through the end of the pumping. The parameter of leakage coefficient, L, appears to have influence on the drawdown from 1.5 min through the end of pumping. Such a phenomenon can be related to the physical behavior of the leaky aquifer. The normalized sensitivity of L keeps zero before 1.5 min, and it may ascribe to the fact that there is a time lag between the start of pumping and the response of the drawdown to the leakage effect. In contrast, the normalized sensitivities indicate that the parameters T and S have influence on the drawdown right at the beginning of pumping. In addition, the influence of S is larger than that of T at early pumping period. This result to some extend reflects the physical behavior of parameters T and S during the pumping.

Table 1 The synthetic drawdown data for the leaky aquifer

No.	Time (min)	Drawdown (m)
1	0.017	0.013
2	0.050	0.099
3	0.100	0.203
4	0.250	0.380
5	0.750	0.621
6	1.000	0.687
7	1.500	0.781
8	2.000	0.847
9	2.500	0.899
10	3.500	0.977
11	4.500	1.035
12	5.000	1.059
13	10.000	1.215
14	20.000	1.365
15	50.000	1.539
16	100.000	1.640
17	200.000	1.702
18	500.000	1.728
19	700.000	1.730
20	1000.000	1.730

Q = 3000 m^3/day, R = 30 m.

The time-drawdown data set 1 of an unconfined aquifer, generated by Neuman's model (1974), for pumping starting from 1 to 176,360 s (49 h) in an unconfined aquifer are listed in Table 2. The thickness of the aquifer, b, is 10 m, pumping rate Q is 3000 m^3/day, and the distance between the pumping well and observation well R is 10 m. The radial hydraulic conductivity K_r, vertical hydraulic conductivity K_z, storage coefficient S, and specific yield S_y are set to 1×10^{-3} m/s, 1×10^{-4} m/s, 1×10^{-4}, and 1×10^{-1}, respectively. The time-drawdown data and related normalized sensitivities are plotted in Figure 2. Similar to Figure 1, the temporal distribution of each normalized sensitivity reflects the temporal change of the drawdown in response to the relative change of each parameter, and all aquifer parameters affect the drawdown at different periods. The normalized sensitivity of parameter S starts from 1 to 10 s, K_z ranges from 1 to 1000 s, and S_y appears from 80 s to the end of pumping. The drawdown is most sensitive to the parameter K_r except at the early period of the pumping and the influence of K_r on the drawdown increases at the beginning and through the end of the pumping.

The normalized sensitivity of S begins with highest value and drops quickly after the start of the pumping. The normalized sensitivity of K_z reaches its highest value in a range between 10 and 1000 s, implying that the slow decline of the water table is attributed to the contribution of the K_z at the moderate pumping time. The drawdown stops increasing when the normalized sensitivity of K_z approaches its maximum. The temporal distribution of K_r's normalized sensitivity, displaying three segments during the pumping period, is similar to the drawdown curve. The second segment appears at 10 s and vanishes at 1000 s (16.67 min). Figure 2 shows that the drawdown increases in the third segment along with the decrease of K_z's normalized sensitivity, clearly indicating rapid decrease of vertical drainage. The sensitivity curve demonstrates that the aquifer parameter S_y does not have any contribution in response to the pumping at the beginning of the test and starts to react at about 80 s (1.33 min).

The time-drawdown data set 2 listed in Table 3 is generated by Moench's model (1997). The pumping starts from 0.6 to 600,000 s (16.67 hours). The thickness of the aquifer, b, is 10 m, pumping rate Q is 1000 m^3/day, and the distance between the pumping well and observation well R is 10 m. The parameters K_r, K_z, S, S_y, and well radius r_w, are set to 1×10^{-3} m/s, 1×10^{-4} m/s, 1×10^{-4}, 1×10^{-1}, and 1 m respectively. The time-drawdown data and related normalized sensitivities are plotted in Figure 3. The upper panel of Figure 3 shows the same plot without the normalized sensitivity of parameter K_r because the magnitude of K_r's normalized sensitivity is very large at the late time of pumping. Removing K_r's normalized sensitivity is helpful to recognize the small change of other parameter's normalized sensitivities at the early time of pumping. The normalized sensitivity of parameter r_w varies from 2 to 2000 s, S changes from 0.6 to 1000 s, K_z ranges from 100 to 10,000 s, and S_y appears from 100 s toward the end of pump-

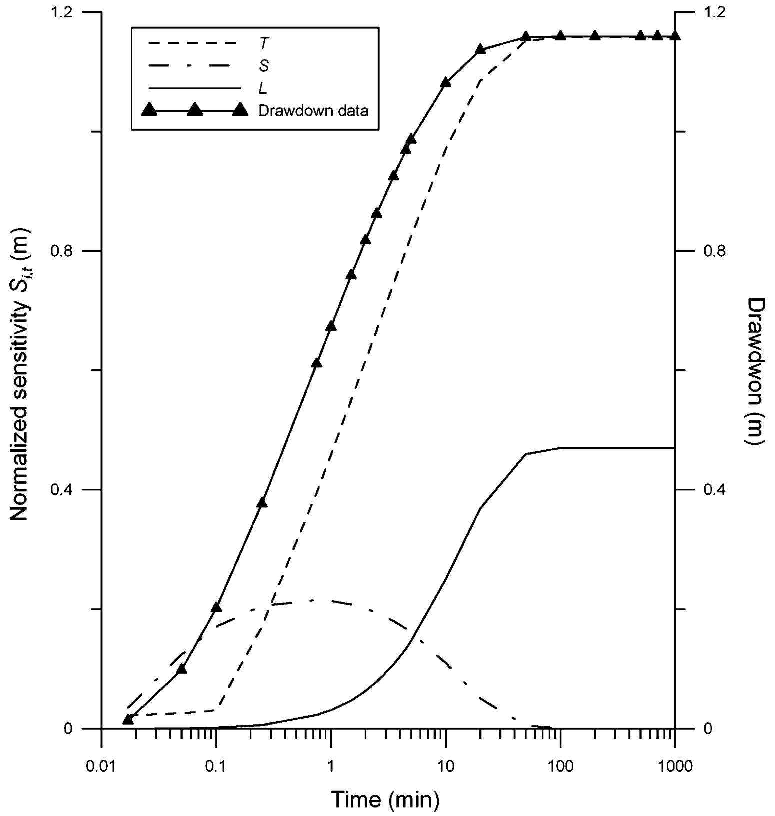

Figure 1 The time-drawdown data and the normalized sensitivities of the leaky aquifer parameters.

Table 2 The synthetic drawdown data set 1 for the unconfined aquifer

No.	Time (s)	Drawdown (m)	No.	Time (s)	Drawdown (m)	No.	Time (s)	Drawdown (m)
1	1	0.22	20	87	0.44	39	6000	0.76
2	2	0.31	21	120	0.44	40	8000	0.83
3	3	0.36	22	149	0.44	41	9354	0.86
4	4	0.38	23	176	0.44	42	11,429	0.91
5	5	0.40	24	212	0.45	43	14,925	0.98
6	6	0.41	25	272	0.45	44	18,235	1.03
7	7	0.41	26	332	0.46	45	22,274	1.09
8	8	0.42	27	393	0.46	46	25,882	1.13
9	9	0.42	28	472	0.47	47	32,696	1.19
10	10	0.42	29	600	0.48	48	41,295	1.25
11	11	0.43	30	792	0.49	49	47,195	1.29
12	12	0.43	31	967	0.50	50	59,224	1.35
13	13	0.43	32	1143	0.52	51	69,279	1.40
14	14	0.43	33	1350	0.53	52	81,302	1.44
15	15	0.43	34	1723	0.55	53	95,126	1.48
16	30	0.43	35	2154	0.58	54	118,168	1.54
17	44	0.43	36	2632	0.61	55	151,775	1.61
18	58	0.43	37	3215	0.64	56	176,360	1.65
19	74	0.43	38	4385	0.70			

ing. The drawdown is very sensitive to the parameter K_r after pumping for 300 s. The influence of K_r on the drawdown starts at about 60 s and increases through the end of the pumping.

The normalized sensitivity of parameter r_w starts at the beginning of the pumping, reflecting the phenomenon that the well bore storage contributes to the drawdown immediately after pumping. The normalized sensitivity of S is relative small compared with those of other parameters. The normalized sensitivity of K_z reaches its highest value in the range between 600 and 2000 s. Similar to Figure 2, the drawdown slowly increases when the normalized sensitivity of K_z approaches its maximum, indicating that the slow decline of the water table is attributed to the contri-

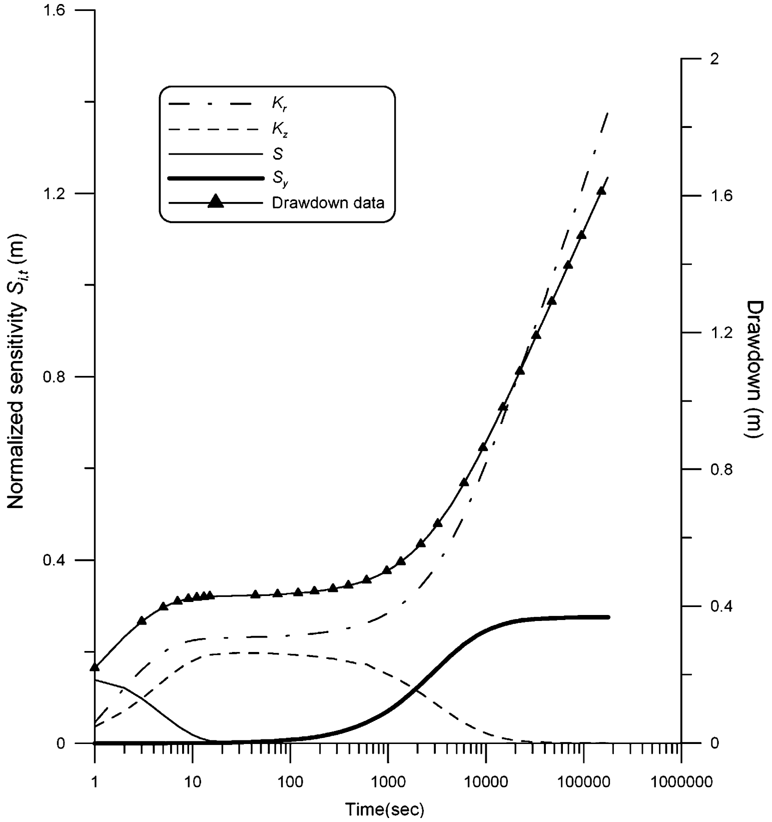

Figure 2 The time-drawdown data and the normalized sensitivities of the unconfined aquifer parameters (Neuman's model).

Table 3 The synthetic drawdown data set 2 for the unconfined aquifer

No.	Time (s)	Drawdown (m)	No.	Time (s)	Drawdown (m)	No.	Time (s)	Drawdown (m)
1	0.6	0.0001	17	24	0.0155	33	1138	0.2130
2	1	0.0003	18	27	0.0178	34	1722	0.2251
3	2	0.0008	19	31	0.0204	35	1977	0.2290
4	3	0.0014	20	36	0.0234	36	2992	0.2424
5	4	0.0020	21	41	0.0268	37	5970	0.2741
6	5	0.0028	22	47	0.0306	38	11,912	0.3189
7	6	0.0034	23	54	0.0348	39	18,029	0.3507
8	7	0.0040	24	63	0.0396	40	35,973	0.4088
9	8	0.0047	25	72	0.0449	41	62,514	0.4577
10	9	0.0055	26	82	0.0507	42	94,619	0.4950
11	10	0.0064	27	95	0.0572	43	124,732	0.5200
12	12	0.0075	28	125	0.0722	44	188,789	0.5578
13	14	0.0087	29	189	0.0993	45	328,078	0.6084
14	16	0.0101	30	249	0.1199	46	600,000	0.6638
15	18	0.0116	31	497	0.1720			
16	21	0.0134	32	655	0.1892			

bution of the K_z at the moderate pumping time. Figure 3 also shows that the effect of well bore storage is larger than that of K_r at early pumping period. This phenomenon indicates that the water is withdrawn from the well first after pumping and the groundwater flows into the well due to the head difference between the well and the aquifer. Certainly, the parameter S_y still does not have any contribution in response to the pumping at the beginning of the test and starts to react at about 100 s (1.67 min). Compared Figure 2 with Figure 3, the normalized sensitivities of parameters K_r, K_z, S, and S_y have similar temporal distributions but different magnitudes. In Moench's model, the effect of S is relative small, the influence periods of S and K_z are longer than that of Neuman's model, and the effect of r_w is larger than that of K_r at the beginning of pumping.

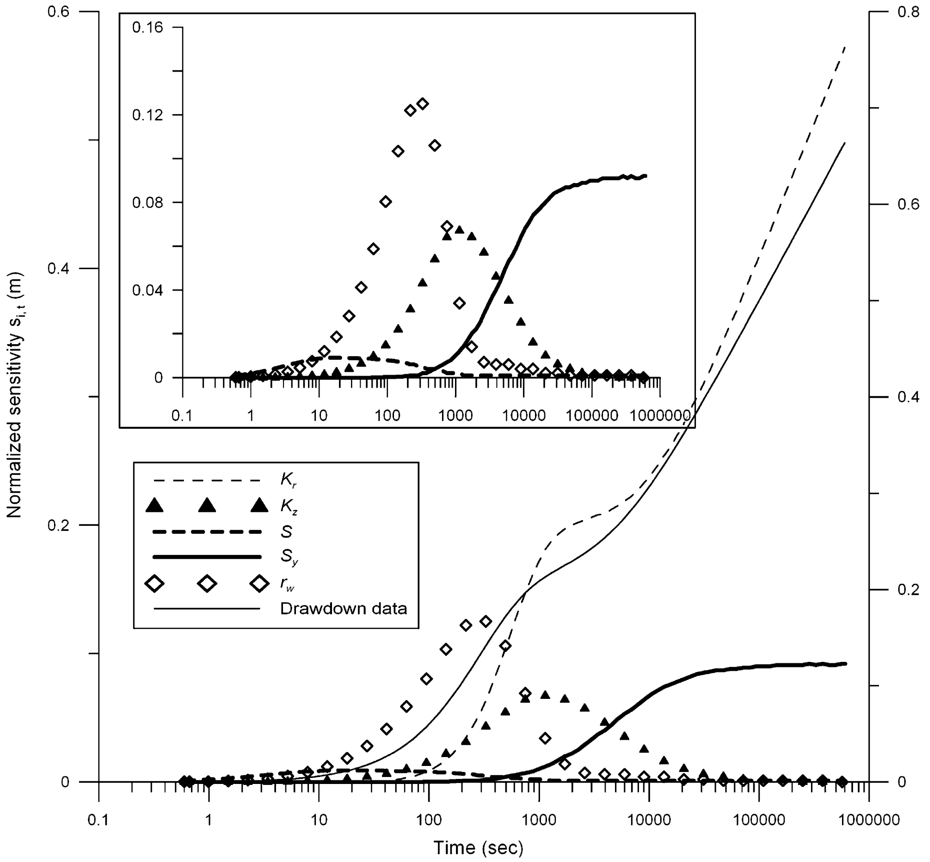

Figure 3 The normalized sensitivities of the unconfined aquifer parameters (Moench's model).

Aquifer parameter identification using on-line PEM

Table 4 lists the number of observations (drawdown data) used in the data analysis and the estimated parameters for a synthetic leaky aquifer case. The identification process starts with three observations (shown at the first column) since the number of unknown parameter is three. The target values of the parameters T, S and L are 1000 m^2/day, 10^{-4}, and 3×10^{-2}, respectively. The parameter estimation indicates that the parameters T and S are correctly identified even at the beginning of the pumping. The results of estimated parameter L using three, four, five, and six observation data points have the same order of magnitude as the target value, and the relative errors of estimated L are 63%, 16%, 8.7%, and 2%, respectively. The parameters are stably identified using more than seven observation data, i.e., after 1.5 min. These results indicate that the aquifer parameters are determined when the corresponding normalized sensitivities start to respond to the pumping. Moreover, the temporal curve of estimated L exhibited in Figure 4 shows fluctuation at first few steps and approaches a constant value after about 1.5 min. These results imply that the on-line PEM can successfully identify the parameters of leaky aquifer when the estimated parameter L starts to be stabilized.

Table 5 displays the field time-drawdown data and the estimated parameters for a leaky aquifer using different number of observations. The time-drawdown data measured from observation wells, as reported in Cooper, (1963) and cited by Lohman (1972, p. 31, Table 11), are selected for the data analysis. The distance between the pumping well and the observation well is 30.48 m. The pumping rate Q

Table 4 Number of observations used in the synthetic data analysis and the estimated parameters for a leaky aquifer

Number of observations	Time (min)	Estimated parameters		
		T (m^2/day)	$S \times 10^{-4}$	$L \times 10^{-2}$
3	0.10	1000.53	1.00	1.12
4	0.25	1000.32	1.00	2.52
5	0.75	1000.52	1.00	2.74
6	1.00	999.93	1.00	3.06
7	1.50	1000.02	1.00	3.00
8	2.00	999.96	1.00	3.03
9	2.50	999.98	1.00	3.01
10	3.50	999.99	1.00	3.00
11	4.50	999.99	1.00	3.01
12	5.00	999.95	1.00	3.01
13	10.00	1000.06	1.00	3.00
14	20.00	1000.02	1.00	3.00
15	50.00	1000.01	1.00	3.00
16	100.00	1000.02	1.00	3.00
17	200.00	1000.02	1.00	3.00
18	500.00	1000.04	1.00	3.00
19	700.00	1000.06	1.00	3.00
20	1000.00	1000.05	1.00	3.00

Target values: $T = 1000$ (m^2/day), $S = 10^{-4}$, and $L = 3 \times 10^{-2}$.

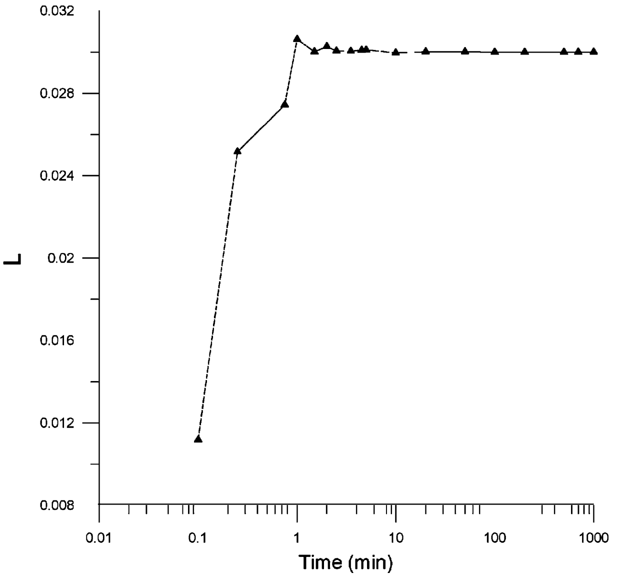

Figure 4 The estimated L versus time in the leaky aquifer case.

is 5450.98 m^2/day, the thickness of the aquifer is 30.48 m, and total pumping time is 1000 min (16.67 h). It is clear that the estimated values of parameters T and S do not fluctuate drastically when the number of observation using by on-line PEM is larger than 7, i.e. after 20 min. The estimated parameters T and S are 1203.80 m^2/day and 1.04×10^{-4}, respectively. Comparing with the estimated parameters calculated based on the total number of observations (1239.28 m^2/day for T and 9.80×10^{-5} for S), the relative errors of parameters T and S are both smaller than 5% when the number of observation is larger than 7. Similarly, the estimated values of parameter L remain almost the same when the number of the observation utilized by the on-line PEM is larger than 9. In this case, the on-line estimation can be terminated after 100 min. The on-line PEM saves tremendous 90% time and 3407 m^3 groundwater resources if compared with total pumping time and pumped water volume required by conventional graphical approaches. Note that small fluctuation in the estimated parameters at the late period of pumping and a longer parameter estimation time than that of the synthetic case may be attributed to aquifer heterogeneity and/or measurement errors in the observed drawdowns.

The identification results with different number of observation using on-line PEM for the synthetic unconfined aquifer data set 1 are listed in Table 6. The identification process starts with four observation data points because the number of unknown parameter is four. The target values of the parameters K_r, K_z, S, and S_y are 1×10^{-3} m/s, 1×10^{-4} m/s, 1×10^{-4}, and 1×10^{-1}, respectively. This table only lists the results when the number of observations is less than 20 because the estimated parameters are almost the same as the target values when the number of observation is larger than 20. Figure 2 shows that the normalized sensitivities of parameters K_r, K_z, and S have immediate response to the pumping and the parameter S_y has a time lag in response to the pumping. The identification results also reflect this phenomenon. The estimated S_y ranges from 4.44×10^{-2} to

Table 5 The field time-drawdown data and the estimated parameters for a leaky aquifer using different number of observations

Number of observations	Time (min)	Drawdown (m)
1	0.2	0.536
2	0.5	0.838
3	1	1.094
4	2	1.298
5	5	1.609
6	10	1.798
7	20	1.972
8	50	2.109
9	100	2.167
10	200	2.195
11	500	2.198
12	1000	2.198

Estimated results using on-line PEM

Number of observations	Estimated values		
	T (m^2/day)	$S \times 10^{-4}$	$L \times 10^{-2}$
4	1060.40	1.12	15.70
5	1182.30	1.05	1.61
6	1182.70	1.04	6.76
7	1203.80	1.03	5.85
8	1211.33	1.02	5.61
9	1222.18	1.00	5.32
10	1232.32	0.99	5.09
11	1236.93	0.98	4.99
12	1239.28	0.98	4.93

Table 6 Number of observations used in the data analysis and the estimated parameters based on the synthetic data set 1

Number of observations	Time (s)	Estimated parameters			
		K_r (m/s) $\times 10^{-3}$	K_z (m/s) $\times 10^{-4}$	$S \times 10^{-4}$	$S_y \times 10^{-1}$
4	4	0.997	1.006	1.000	0.612
5	5	1.000	0.999	1.000	0.616
6	6	1.000	0.999	1.000	1.190
7	7	0.997	1.010	1.000	0.444
8	8	1.000	0.998	1.000	1.570
9	9	0.999	1.000	1.000	0.933
10	10	1.000	1.000	1.000	0.972
11	11	0.998	1.000	1.000	0.712
12	12	1.000	0.995	1.000	2.010
13	13	1.000	0.997	1.000	1.140
14	14	1.000	0.998	1.000	1.220
15	15	0.998	0.998	1.000	0.816
16	30	1.000	1.000	1.000	1.040
17	44	0.998	0.997	1.000	0.987
18	58	1.000	1.000	1.000	1.010
19	74	1.000	1.000	1.000	0.993
20	87	1.000	1.000	1.000	1.000

Target values: $K_r = 1 \times 10^{-3}$ (m/s), $K_z = 1 \times 10^{-4}$ (m/s), $S = 1 \times 10^{-4}$, and $S_y = 1 \times 10^{-1}$.

2.01×10^{-1} and the largest relative errors are 101% when using 12 observation data. The identification results of S_y did not approach the target value until the number of observation is over 20, i.e., about 80 s. Therefore, the on-line PEM may not obtain accurate results of S_y if the time-drawdown data is too short to cover the response period of S_y. Similar to Figure 4, the curve of estimated S_y versus time displayed in Figure 5 shows dramatic fluctuation in the early period and converges to a constant value after about 80 s. Figure 2 demonstrates that the on-line PEM can successfully identify the aquifer parameters when S_y just starts to affect the drawdown. Therefore, the on-line estimation based on Neuman's model can be terminated once the identified parameters become stable.

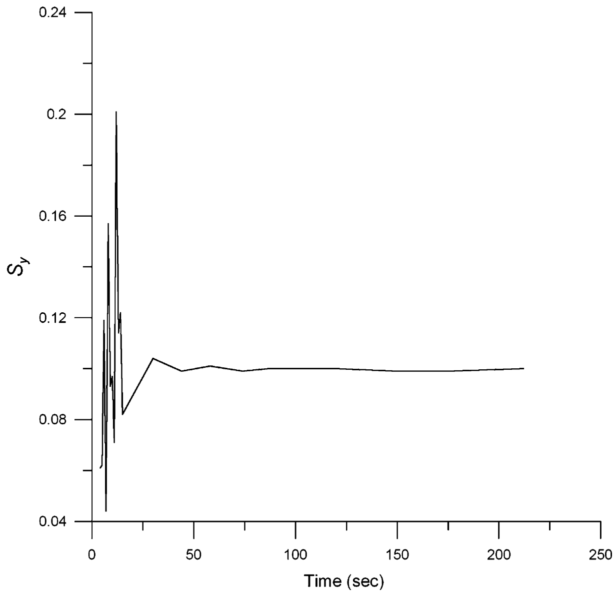

Figure 5 The estimated S_y versus time using the synthetic data set 1.

Similar to Table 6, the identification results for the synthetic data set 2 are listed in Table 7. The target values of the parameters K_r, K_z, S, S_y, and r_w are 1×10^{-3} m/s, 1×10^{-4} m/s, 1×10^{-4}, 1×10^{-1}, and 1 m, respectively. The estimated parameters are all the same as the target values when the number of observation is larger than 30. The parameters K_r, K_z, S, and r_w are accurately determined at first few seconds. The estimated S_y ranges from 1.00×10^{-2} to 2.91×10^{-1} and did not approach the target value until the pumping time is more than 125 s. The curve of estimated S_y versus time displayed in Figure 6 also shows dramatic fluctuation at the early time and converges to a fix value after about 125 s. Hence, the on-line estimation can be terminated even based on Moench's model.

Table 8 shows the estimated parameters when using different number of observations obtained from the field pumping test at an unconfined aquifer. The site of Cape Cod, Massachusetts (Moench et al., 2000) is selected for the study. The aquifer was composed of unconsolidated glacial outwash sediments that were deposited during the recession, 14,000—15,000 years before present, of the late Wisconsinan continental ice sheet. The depth of the pumping well was 24.4 m below the land surface. The top and bottom of the screen were located 4.0 and 18.3 m, respectively, below the initial water table, which was approximately 5.8 m below land surface. The aquifer saturated thickness was about 48.8 m. The well F507-080 was pumped at an average rate 1.21 m^3/min for 72 hours. The data set of the observation well F505-032 is selected in this case. The distance between pumping well and observation well is 7.28 m. From Table 8, the estimated K_r ranges from 2.20×10^{-4} m/s to 1.97×10^{-3} m/s, the estimated K_z ranges from 1.0×10^{-6} m/s to 2.25×10^{-4} m/s, the estimated S ranges from 3.45×10^{-3} to 7.29×10^{-3}, and the estimated S_y ranges from 0.016 to 0.3. This table demonstrates that

Table 7 Number of observations used in the data analysis and the estimated parameters based on the synthetic data set 2

Number of observations	Time (s)	Estimated parameters				
		K_r (m/s) $\times 10^{-3}$	K_z (m/s) $\times 10^{-4}$	$S \times 10^{-4}$	$S_y \times 10^{-1}$	r_w (m)
8	7	1.66	1.63	1.67	0.10	1.01
9	8	1.00	1.00	1.00	0.53	1.00
10	9	1.00	1.00	1.00	2.91	1.00
11	10	1.00	1.00	1.00	0.20	1.00
12	12	1.00	1.00	1.00	0.67	1.00
13	14	1.00	1.00	1.00	1.17	1.00
14	16	1.00	1.00	1.00	0.48	1.00
15	18	1.00	1.00	1.00	0.56	1.00
16	21	1.00	1.00	1.00	0.69	1.00
17	24	1.00	1.00	1.00	0.65	1.00
18	27	1.00	1.00	1.00	0.77	1.00
19	31	1.00	1.00	1.00	0.85	1.00
20	36	1.00	1.00	1.00	1.23	1.00
21	41	1.01	1.01	1.01	0.59	1.00
22	47	1.00	1.00	1.00	0.96	1.00
23	54	1.00	1.00	1.00	1.03	1.00
24	63	1.00	1.00	1.00	0.90	1.00
25	72	1.00	1.00	1.00	1.00	1.00
26	82	1.00	1.00	1.00	0.99	1.00
27	95	1.00	1.00	1.00	1.03	1.00
28	125	1.00	1.00	1.00	1.00	1.00
29	189	1.00	1.00	1.00	1.00	1.00
30	249	1.00	1.00	1.00	1.00	1.00

Target values: $K_r = 1 \times 10^{-3}$ (m/s), $K_z = 1 \times 10^{-4}$ (m/s), $S = 1 \times 10^{-4}$, $S_y = 1 \times 10^{-1}$, and $r_w = 1$ m.

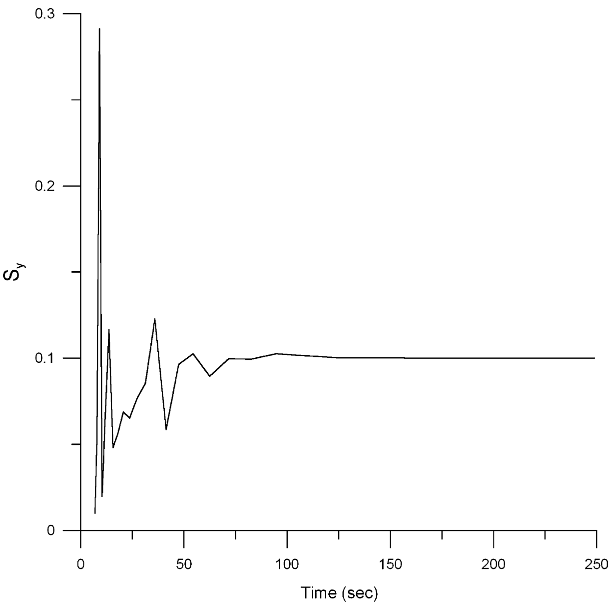

Figure 6 The estimated S_y versus time using the synthetic data set 2.

the ranges of estimated K_r and S are small as compared with those of the K_z and S_y. Such results may attribute to the fact that the parameters K_r and S have significant influence on the drawdown as the pumping starts and thus can be estimated using only few observations. In contrast, the influence periods of parameters K_z and S_y have some time lags after the start of pumping and the estimated results fluctuate significantly at the early period of the pumping. Note that the estimated parameter S_y keeps the highest value (0.3) at early pumping period then dramatically decreases to a small value (0.016) after 20 min (18 observations). This result implies that S_y does not affect the estimation for other parameters before that time, i.e., the variation of parameter S_y does not significantly change the estimation result. Figure 7 displays the estimated S_y versus pumping time (different number of observations). In addition, the value of S_y versus logarithmic time is also shown in the upper panel of the figure. The estimated S_y keeps almost constant at first 20 min, then goes down rapidly and reaches a minimal at 100 min. After that the estimated S_y gradually increases and becomes flat after 1000 minutes (16.67 h), implying that the on-line estimation can be terminated at that time. In this case, the on-line PEM can save 77% pumping time if the test is terminated and 4041.4 m^3 groundwater resources if compared with total pumping time and pumped water volume required by conventional graphical approaches.

The tests of other impacts to the influence period of the parameter S_y

The parameter S_y has the longest time lag in response to the drawdown than other parameters as indicated in Figures 1 and 2. The on-line PEM can correctly identify the aquifer parameters only when the parameters start to influence the drawdown. In the unconfined aquifer case, the S_y was assigned to 0.1 where the reasonable value is 0.01–0.3 (Batu, 1998). The normalized sensitivities reflect the sensi-

Table 8 The estimated parameters for an unconfined aquifer (Cape Cod site) using different number of observations

Number of observations	Time (min)	Estimated parameters			
		$K_r \times 10^{-3}$ (m/s)	$K_z \times 10^{-5}$ (m/s)	$S \times 10^{-3}$	$S_y \times 10^{-1}$
5	0.15	0.65	0.10	7.18	3.00
6	0.22	0.73	1.05	7.29	3.00
7	0.32	0.91	0.98	7.45	3.00
8	0.47	0.88	1.18	7.17	3.00
9	0.68	0.96	1.19	7.16	3.00
10	1.00	0.51	1.89	5.61	3.00
11	1.47	0.32	2.15	4.28	3.00
12	2.15	0.22	2.23	3.32	3.00
13	3.17	0.24	2.41	3.52	2.91
14	4.75	0.30	2.78	4.31	3.00
15	6.75	0.26	2.60	3.83	3.00
16	10.10	0.25	2.20	3.64	2.95
17	14.90	0.44	2.34	5.64	3.00
18	21.90	1.01	1.51	8.14	1.49
19	31.90	1.39	0.74	7.00	0.56
20	46.90	1.74	0.48	6.21	0.27
21	67.90	1.92	0.39	5.65	0.18
22	99.90	1.96	0.37	5.52	0.16
23	151.00	1.97	0.36	5.49	0.16
24	221.00	1.92	0.38	5.63	0.18
25	325.00	1.82	0.42	5.98	0.24
26	492.00	1.70	0.46	6.39	0.33
27	675.00	1.60	0.49	6.71	0.43
28	1050.00	1.54	0.51	6.94	0.52
29	1470.00	1.50	0.52	7.10	0.59
30	2190.00	1.47	0.53	7.19	0.64
31	3100.00	1.46	0.54	7.26	0.68
32	4330.00	1.45	0.54	7.29	0.69

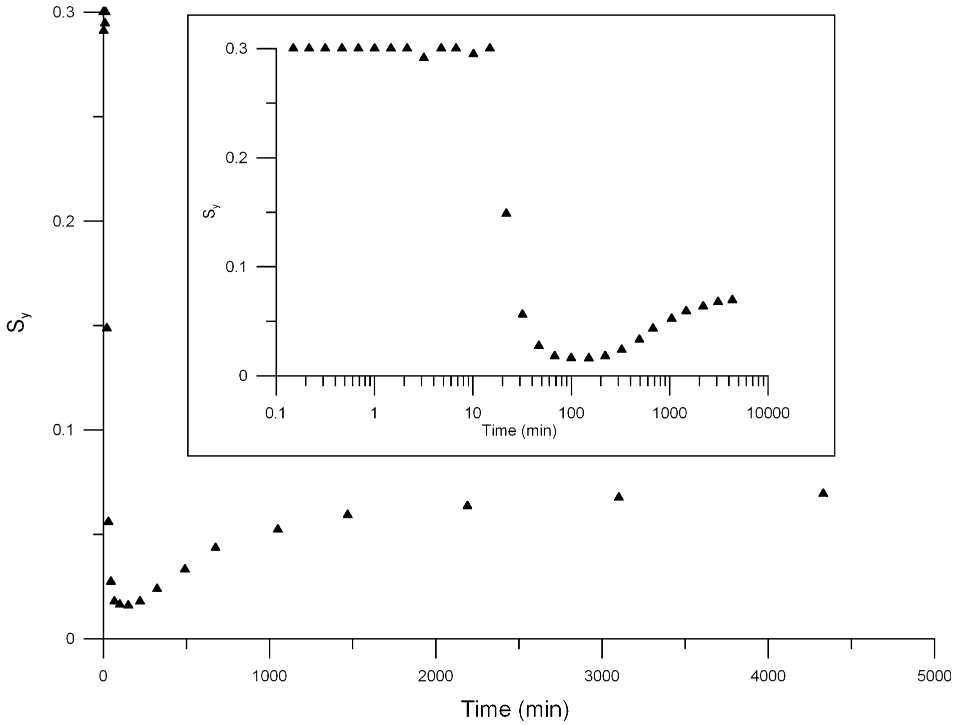

Figure 7 The estimated S_y versus time in the field unconfined aquifer.

tivity of the drawdown in response to the relative change of each parameter at different time. Thus, it is interesting to examine the temporal distribution of normalized sensitivity for different value of S_y. Moreover, the distance between pumping well and observation well, R, is another problem deserved attention because the drawdown in response to the pumping becomes smaller when the distance from the pumping well goes farther. For investigating the effect of various value of S_y or R on the on-line parameter estimation, two tests are performed. The first test assigns three different values of S_y including two extreme values, i.e., 0.01 and 0.3, while the other parameters are kept the same as those given in previous unconfined aquifer case. The second test examines the effect of distance on the normalized sensitivity when the observation well is located at 10, 30, or 50 m from the pumping well.

The normalized sensitivity of S_y versus time for the first test is demonstrated in Figure 8. The influence period starts slightly later when the S_y value gets larger. The S_y starts to influence the drawdown at 5 and 100 s when the value of S_y is 0.01 and 0.3, respectively, indicating that the time lag of the S_y may not be larger than 2 min in these two extreme cases. Figure 8 indicates that the largest normalized sensitivities are about the same in those cases because of the normalization of S_y. The results of the second test shown in Figure 9 indicate that a longer distance from the well has a slower response time. The shortest response time is about 10 seconds and the latest one is about 100 s. Comparing with the total pumping time of 176,360 s (2.04 days), the differences of the estimated parameters in these three cases may be negligible. In addition, the sensitivity analysis may be performed along with the on-line parameter estimation and provide a double check in terminating the pumping.

Concluding remarks

The sensitivity analysis is used to investigate the influence period of aquifer parameters in both leaky and unconfined aquifers. The influences of parameters L and S_y on the drawdown are shown to have time lag in response to pumping in the leaky and unconfined aquifers, respectively. An on-line parameter estimation model is applied to estimate the parameters based on the data obtained from synthetic and field pumping tests for both leaky and unconfined aquifers. The results indicate that the on-line estimation can be terminated when the parameters are stabilized and their corresponding normalized sensitivities start to react the pumping. In the synthetic cases, the termination time of the on-line estimation is consistent with the influence period of the parameter which has longest time lag from the beginning of the pumping. This phenomenon indicates that the on-line estimation can be terminated if all identified parameters tend to be stabilized, i.e., the drawdown already reacts to the effect of aquifer parameters. In the field cases, the results indicate that the on-line parameter estimation model can save 90% pumping time in the leaky aquifer and 77% pumping time in the unconfined aquifer. Note that the small fluctuation in the estimated parameters at the late period of pumping and a longer on-line estimation time than that of the synthetic case occur. These results may be mainly caused by aquifer heterogeneity and/or measurement errors in the observed drawdown data. Finally, different values of the specific yield and distance between pumping well and observation well do not significantly affect the influence time of specific yield during the pumping. These results may provide a useful reference for on-line aquifer parameter estimation.

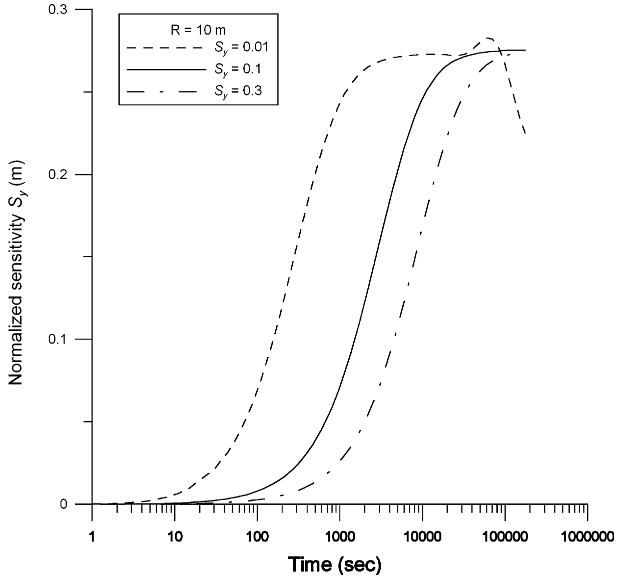

Figure 8 The normalized sensitivity of S_y for S_y = 0.01, 0.1, or 0.3 and R = 10 m.

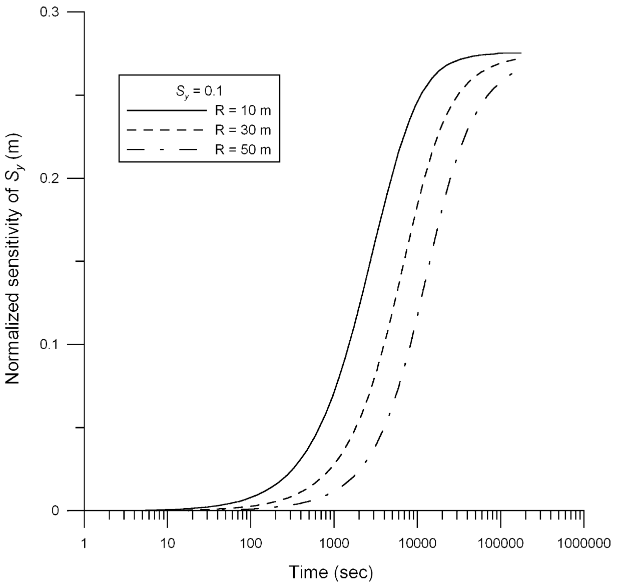

Figure 9 The normalized sensitivity of S_y for S_y = 0.1 and R = 10, 30, or 50 m.

Acknowledgements

This study was partly supported by the Taiwan National Science Council under the grant NSC94-2211-E-009-015. The authors thank two anonymous reviewers for constructive comments and suggested revisions.

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