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Physica C 267 (1996) 345-354

PHYSICA C

Thickness dependence of magnetic force and vortex creation in type-II superconducting thin film

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Received 30 May 1996

Abstract

We examined a single vortex line in an infinite type-II superconducting thin film with finite thickness. The thickness dependence of the magnetic force acting on a point dipole placed above a superconducting film is calculated as the negative of the gradient of the interaction energy, coupled to the London theory of superconductivity with electromagnetics. We also considered the situation of a single vortex creation in the thin film, caused by the field of the magnetic dipole tip itself. The thickness dependence of the critical position of the point dipole for creating the first vortex line in the thin film is calculated.

Keywords: Type-II superconductors; Thin films

1. Introduction

The phenomenon of superconducting levitation has always attracted wide attention. With the discovery of high T_c superconductors, the demonstration of levitation became even more impressive. The fact that the transition temperature is well above 77 K allows experiments to be performed in a petri dish partially filled with LN_2. High T_c superconductors have demonstrated tremendous potential for several fascinating applications such as magnetic bearing, magnetic levitation and other magnetic force related device [6]. Concerning the microscopic feature, the scanning tunneling microscope (STM) [7] has been applied on conventional low-temperature superconductor [8,9]. STM is demonstrated to resolve the Abrikosov flux lattice and the electronic fine structure of a single vortex. But it is not easy to gain direct access to the parameters of superconductivity by using STM tunneling. Recently the magnetic force microscopy (MFM) [10] has been proposed as a possible technique for investigating HTS samples, especially to observe the flux line structure below the transition temperature [11–13]. Most works [14–17] focus on calculating the magnetostatic interaction between a magnetic tip or a point dipole and a superconductor in the Meissner state. The result is suitable for type-I superconductors in the superconducting state and type-II superconductors when the field is below the lower critical field. The

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PII S0921-4534(96)00374-7

superconducting levitation force acting on a magnet over a semi-infinite type-II superconductor in both the Meissner and mixed states has recently been studied [18,19]. In the case of a superconductor with finite thickness, the thickness dependence of the levitation acting on the point dipole over a thin superconducting sheet in the Meissner state has been examined and calculated [16,19].

The purpose of this paper is to theoretically investigate the thickness dependence of the magnetic force exerted on a point dipole over a superconducting thin film in the mixed state with a single vortex line, based on energy considerations. The interaction energy between a point dipole and the screening current in the superconductor, U_{in} (or self-interaction energy [14–17,19,20]), may be written in the form

$$U_{in} = -\tfrac{1}{2}\boldsymbol{m} \cdot \boldsymbol{B}_{in}, \tag{1}$$

where $\boldsymbol{m}$ is the magnetic moment of the point dipole and $\boldsymbol{B}_{in}$ is the induced field caused by the screening current of the superconductor. The interaction energy between the point dipole and the vortex line, U_v, can be written in the form

$$U_v = -\boldsymbol{m} \cdot \boldsymbol{B}_v, \tag{2}$$

where $\boldsymbol{B}_v$ is the magnetic stray field of the vortex line. The magnetic force is expressed as the negative of the gradient of the interaction energy. The magnetic force acting on the magnetic point dipole can be calculated by the superposition of the magnetic field due to the vortex line and due to the screening current in the superconductor. We also show the thickness dependence of the critical position of the point dipole, at which the first vortex line is created in the thin film. The creation of the single vortex in the film is caused by the field of the magnetic point dipole itself as the point dipole is vertically lowered toward the surface of the film. We assume the single vortex line directed perpendicularly to the surface of the film. A certain limited case of the magnetic force can be treated analytically and is discussed.

2. Formulation and results

We consider a single vortex line embedded in an infinite superconducting thin film of finite thickness d and directed perpendicularly to its surface. The film lies in the x–y plane with the vortex located at distance r_0 from the origin of the coordinate system, and there is a magnetic point dipole with a moment $\boldsymbol{m}$ placed at a distance

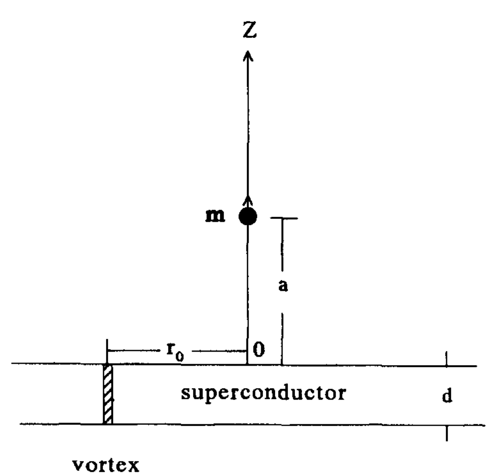


Fig. 1. The diagram of a magnetic point dipole ($\boldsymbol{m}$) placed above a superconducting plane and a single vortex line embedded in it.

a above the superconductor, as shown in Fig. 1. From London's and Maxwell's equations, the vector potential A can be expressed as

$$\partial \times \partial \times \boldsymbol{A} = -\mu_0 \boldsymbol{m} \times \partial[\delta(r)\delta(z-a)], \quad z>0, \tag{3}$$

$$\partial \times \partial \times \boldsymbol{A} + \frac{\boldsymbol{A}}{\lambda^2} = \frac{\phi_0}{2\pi\lambda^2} \frac{1}{|\boldsymbol{r}-\boldsymbol{r}_0|}\hat{\boldsymbol{\theta}}' \qquad -d<z<0, \tag{4}$$

$$\partial \times \partial \times \boldsymbol{A} = 0, \qquad z<-d. \tag{5}$$

where μ_0 is the vacuum permeability, λ is the penetration depth, ϕ_0 is the flux quantum $h/2e$, and $\hat{\boldsymbol{\theta}}'$ is the angular unit vector when the origin is located at the center of the vortex line.

By the superposition principle, the vector potential A is the superposition of the vector potential A_1 and A_2 due to the source of the magnetic dipole moment and the vortex line, respectively. We use the cylindrical coordinates (r, θ, z) with center at the origin, and with the boundary conditions for A (continuity and continuity of its normal derivative at the interfaces $z=0$ and $z=-d$). The solutions of A_1 and A_2 are obtained by expanding in the Bessel functions:

$$\begin{aligned} A_1 &= \frac{\mu_0 m}{4\pi}\left(\frac{r}{[r^2+(z-a^2)]^{3/2}} + \int_0^\infty \mathrm{d}k\, kC_1(k)\mathrm{e}^{-k(z+a)}J_1(kr)\right)\hat{\boldsymbol{\theta}}, \quad z>0, \\ &= \frac{\mu_0 m}{4\pi}\int_0^\infty \mathrm{d}k\, k\mathrm{e}^{-ka}\left[C_2(k)e^{(k^2+1/\lambda^2)^{1/2}z} + C_3(k)\mathrm{e}^{-(k^2+1/\lambda^2)^{1/2}z}\right]J_1(kr)\ \hat{\boldsymbol{\theta}}, \quad -d<z<0, \\ &= \frac{\mu_0 m}{4\pi}\int_0^\infty \mathrm{d}k\, kC_4(k)\mathrm{e}^{k(z-a)}J_1(kr)\ \hat{\boldsymbol{\theta}}, \quad z<-d, \end{aligned} \tag{6}$$

where the coefficient functions $C_1(k)$, $C_2(k)$, $C_3(k)$ and $C_4(k)$ are obtained through the boundary conditions

$$C_1(k) = \frac{1/\lambda^2}{(2k^2+1/\lambda^2) + 2k(k^2+1/\lambda^2)^{1/2}\coth(k^2+1/\lambda^2)^{1/2}d}, \tag{7}$$

$$C_2(k) = \frac{k\left[(k^2+1/\lambda^2)^{1/2}-k\right]\mathrm{e}^{-(k^2+1/\lambda^2)^{1/2}d}}{(2k^2+1/\lambda^2)\sinh(k^2+1/\lambda^2)^{1/2}d + 2k(k^2+1/\lambda^2)^{1/2}\cosh(k^2+1/\lambda^2)^{1/2}\ d}, \tag{8}$$

$$C_3(k) = \frac{k\left[(k^2+1/\lambda^2)^{1/2}+k\right]\mathrm{e}^{(k^2+1/\lambda^2)^{1/2}d}}{(2k^2+1/\lambda^2)\sinh(k^2+1/\lambda^2)^{1/2}\ d + 2k(k^2+1/\lambda^2)^{1/2}\cosh(k^2+1/\lambda^2)^{1/2}\ d}, \tag{9}$$

$$C_4(k) = \frac{2k(k^2+1/\lambda^2)^{1/2}\mathrm{e}^{kd}}{(2k^2+1/\lambda^2)\sinh(k^2+1/\lambda^2)^{1/2}d + 2k(k^2+1/\lambda^2)\cosh(k^2+1/\lambda^2)^{1/2}\ d}. \tag{10}$$

These coefficients coincide with the result of Ref. [16], for a thin superconducting sheet in the case of the Meissner state. Two extreme situations are considered below.

(i) The thickness of the film $d \ll \lambda$:

$$C_1(k) \to \frac{-1}{k\Lambda+1}, \qquad C_4(k) \to \frac{k\Lambda}{k\Lambda+1}, \qquad C_2+C_3 \to \frac{k}{k\Lambda+1}, \tag{11}$$

where $\Lambda = 2\lambda^2/d$ plays the role of an effective penetration depth.

(ii) For $d \to \infty$, $C_2(k)$ and $C_4(k) \to 0$, and:

$$C_1(k) \to \frac{\left(k^2 + 1/\lambda^2\right)^{1/2} - k}{\left(k^2 + 1/\lambda^2\right)^{1/2} + k}, \qquad C_3(k) \to \frac{2k}{\left(k^2 + 1/\lambda^2\right)^{1/2} + k}. \tag{12}$$

Now, the solution of A_2 is obtained and written as follows:

$$\begin{aligned}
A_2 &= \frac{\phi_0}{2\pi\lambda^2} \int_0^\infty \mathrm{d}k\, D_1(k) \mathrm{e}^{-kr} \Bigg[-\sum_{n=1}^{\infty} \left[J_{n+1}(kr) + J_{n-1}(kr) \right] J_0(kr_0) \sin n\theta\, \hat{r} \\
&\quad + \left(J_1(kr) J_0(kr_0) + \sum_{n=1}^{\infty} \left[J_{n+1}(kr) - J_{n-1}(kr) \right] J_n(kr_0) \cos n\theta \right) \hat{\theta} \Bigg], \quad z > 0, \\
A_2 &= \frac{\phi_0}{2\pi\lambda^2} \int_0^\infty \mathrm{d}k \left(\frac{1}{k^2 + 1/\lambda^2} + D_2(k) \mathrm{e}^{(k^2+1/\lambda^2)^{1/2} z} + D_3(k) \mathrm{e}^{-(k^2+1/\lambda^2)^{1/2} z} \right) \\
&\quad \times \Bigg[-\sum_{n=1}^{\infty} \left[J_{n+1}(kr) + J_{n-1}(kr) \right] J_0(kr_0) \sin n\theta\, \hat{r} \\
&\quad + \left(J_1(kr) J_0(kr_0) + \sum_{n=1}^{\infty} \left[J_{n+1}(kr) - J_{n-1}(kr) \right] J_n(kr_0) \cos n\theta \right) \hat{\theta} \Bigg], \quad -d < z < 0, \\
A_2 &= \frac{\phi_0}{2\pi\lambda^2} \int_0^\infty \mathrm{d}k\, D_4(k) \mathrm{e}^{kr} \Bigg[-\sum_{n=1}^{\infty} \left[J_{n+1}(kr) + J_{n-1}(kr) \right] J_0(kr_0) \sin n\theta\, \hat{r} \\
&\quad + \left(J_1(kr) J_0(kr_0) + \sum_{n=1}^{\infty} \left[J_{n+1}(kr) - J_{n-1}(kr) \right] J_n(kr_0) \cos n\theta \right) \hat{\theta} \Bigg], \quad z < -d.
\end{aligned} \tag{13}$$

where the coefficient functions $D_1(k)$, $D_2(k)$, $D_3(k)$ and $D_4(k)$ are obtained through the boundary conditions

$$D_1(k) = \left[1/\left(k^2 + 1/\lambda^2\right)^{1/2} \right] \times \frac{\left(k^2 + 1/\lambda^2\right)^{1/2} \sinh\left(k^2 + 1/\lambda^2\right)^{1/2} d + k \cosh\left(k^2 + 1/\lambda^2\right)^{1/2} d - k}{(2k^2 + 1/\lambda^2) \sinh\left(k^2 + 1/\lambda^2\right)^{1/2} d + 2k\left(k^2 + 1/\lambda^2\right)^{1/2} \cosh\left(k^2 + 1/\lambda^2\right)^{1/2} d}, \tag{14}$$

$$D_2(k) = \left[-k/2(k^2 + 1/\lambda^2) \right] \times \frac{\left[\left(k^2 + 1/\lambda^2\right)^{1/2} - k \right] + \left[\left(k^2 + 1/\lambda^2\right)^{1/2} + k \right] \mathrm{e}^{(k^2+1/k^2)^{1/2} d}}{(2k^2 + 1/\lambda^2) \sinh\left(k^2 + 1/\lambda^2\right)^{1/2} d + 2k\left(k^2 + 1/\lambda^2\right)^{1/2} \cosh\left(k^2 + 1/\lambda^2\right)^{1/2} d}, \tag{15}$$

$$D_3(k) = \left[-k/2(k^2 + 1/\lambda^2) \right] \times \frac{\left[\left(k^2 + 1/\lambda^2\right)^{1/2} + k \right] + \left[\left(k^2 + 1/\lambda^2\right)^{1/2} - k \right] \mathrm{e}^{-(k^2+1/\lambda^2)^{1/2} d}}{(2k^2 + 1/\lambda^2) \sinh\left(k^2 + 1/\lambda^2\right)^{1/2} d + 2k\left(k^2 + 1/\lambda^2\right)^{1/2} \cosh\left(k^2 + 1/\lambda^2\right)^{1/2} d}, \tag{16}$$

$$D_4(k) = \left[\mathrm{e}^{kd}/\left(k^2 + 1/\lambda^2\right)^{1/2} \right] \times \frac{\left(k^2 + 1/\lambda^2\right)^{1/2} \sinh\left(k^2 + 1/\lambda^2\right)^{1/2} d + k \cosh\left(k^2 + 1/\lambda^2\right)^{1/2} d - k}{(2k^2 + 1/\lambda^2) \sinh\left(k^2 + 1/\lambda^2\right)^{1/2} d + 2k\left(k^2 + 1/\lambda^2\right)^{1/2} \cosh\left(k^2 + 1/\lambda^2\right)^{1/2} d}. \tag{17}$$

Two extreme situations are considered below.

(i) If $d \ll \lambda$, then

$$D_1(k),\ D_4(k) \to \frac{\lambda^2}{k\Lambda+1}, \qquad D_2(k)+D_3(k)+\frac{1}{k^2+1/\lambda^2} \to \frac{\lambda^2}{k\Lambda+1}. \tag{18}$$

(ii) If $d \to \infty$, then $D_3(k)$ and $D_4(k) \to 0$, and

$$D_1(k) \to \frac{1}{(k^2+1/\lambda^2)^{1/2}\left[(k^2+1/\lambda^2)^{1/2}+k\right]}, \qquad D_2(k) \to \frac{-k}{(k^2+1/\lambda^2)\left[(k^2+1/\lambda^2)^{1/2}+k\right]}. \tag{19}$$

The results of Eqs. (11) and (18) for a sufficiently thin film were obtained in our earlier paper [20].

Now let us go back to the general situation. The magnetic induction can be calculated by taking the curl of $\boldsymbol{A}$, i.e., $\boldsymbol{B} = \partial \times \boldsymbol{A}$. The magnetic force acting on the point dipole can then be obtained from the interaction energy through $\boldsymbol{F} = -\partial U_{\text{int}}$. The interaction energy [20] is

$$U_{\text{int}} = -\tfrac{1}{2}\boldsymbol{m}\cdot\boldsymbol{B}_{\text{in}}(0,0,a) - \boldsymbol{m}\cdot\boldsymbol{B}_{\text{v}}(0,0,a). \tag{20}$$

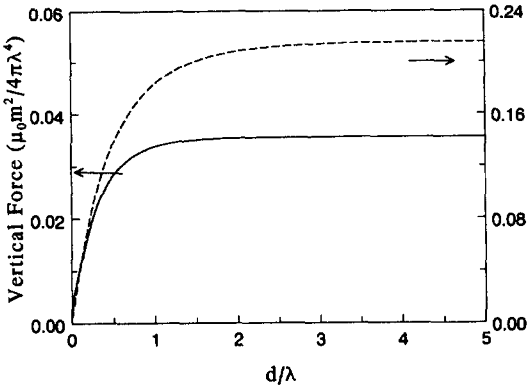
The first term represents the self-interaction energy, caused by the screening current, and the second term represents the interaction energy between the point dipole and the vortex line. The force components acting on the point dipole we get as follows:

$$\begin{aligned} F_z &= -\frac{\partial U_{\text{int}}}{\partial a} \\ &= \frac{\mu_0 m^2}{4\pi\lambda^2}\int_0^\infty \mathrm{d}k \frac{k^3 \mathrm{e}^{-2ka}}{(2k^2+1/\lambda^2)+2k(k^2+1/\lambda^2)^{1/2}\coth(k^2+1/\lambda^2)^{1/2}\,d} - \frac{m\phi_0}{2\pi\lambda^2}\int_0^\infty \mathrm{d}k \\ &\quad\times \frac{k^2\mathrm{e}^{-ka}\left[(k^2+1/\lambda^2)^{1/2}\sinh(k^2+1/\lambda^2)^{1/2}\,d + k\cosh(k^2+1/\lambda^2)^{1/2}\,d - k\right]J_0(kr_0)}{(k^2+1/\lambda^2)^{1/2}\left[(2k^2+1/\lambda^2)\sinh(k^2+1/\lambda^2)^{1/2}\,d + 2k(k^2+1/\lambda^2)^{1/2}\cosh(k^2+1/\lambda^2)^{1/2}\,d\right]}, \end{aligned} \tag{21}$$

$$\begin{aligned} F_{r_0} &= -\frac{\partial U_{\text{int}}}{\partial r_0} \\ &= -\frac{m\phi_0}{2\pi\lambda^2}\int_0^\infty \mathrm{d}k \frac{k^2\mathrm{e}^{-ka}\left[(k^2+1/\lambda^2)^{1/2}\sinh(k^2+1/\lambda^2)^{1/2}\,d + k\cosh(k^2+1/\lambda^2)^{1/2}d - k\right]J_1(kr_0)}{(k^2+1/\lambda^2)^{1/2}\left[(2k^2+1/\lambda^2)\sinh(k^2+1/\lambda^2)^{1/2}\,d + 2k(k^2+1/\lambda^2)^{1/2}\cosh(k^2+1/\lambda^2)^{1/2}d\right]}, \end{aligned} \tag{22}$$

where F_z is the force in the vertical direction (i.e. z-direction) and F_{r_0} is the force in the lateral direction. The m^2 term in Eq. (21) is the component of the vertical force acting on the point dipole when the superconductor is in the Meissner state, and the $m\phi_0$ term is the component of the vertical force contributed by the vortex line. In this system, the interaction force between the point dipole and the screening current is repulsive, and the interaction force between the point dipole and the vortex line is attractive. The force due to the flux line reduces the strength of the vertical levitation force on the point dipole and may contribute to the lateral force.

We consider a simple case to examine the vertical force, if a point dipole is placed just over the vortex line, where $r_0 = 0$, then $J_0(kr_0) = 1$ and $J_1(kr_0) = 0$. Fig. 2 presents the thickness (in units of the penetration depth λ) dependence of the two components of the (dimensionless) vertical force as a given distance $a/\lambda = 1$. This figure shows the vertical force acting on the point dipole includes two parts: one (solid line and in units of $\mu_0 m^2/4\pi\lambda^4$) is due to the screening field (Meissner effect), and the other (dashed line and in units of $m\phi_0/2\pi\lambda^3$) is due to the vortex line. These two forces are oppositely directed. Both forces show a significant thickness dependence. The forces monotonically increase with increasing thickness of film and then reach



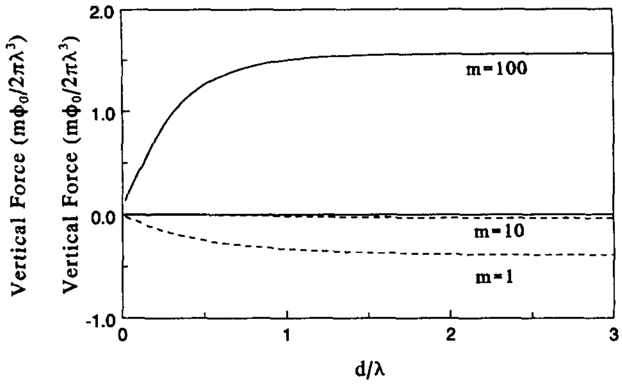


Fig. 2. Thickness dependence of the vertical force components due to the screening current (solid line) and due to the vortex (dashed line) at distance $a/\lambda = 1$. The force acting on the point dipole caused by the screening current is repulsive, the force caused by the vortex is attractive.

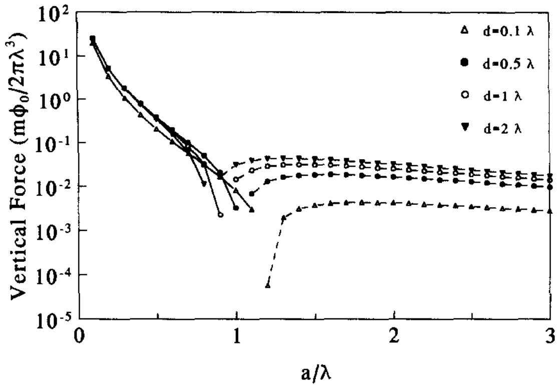
Fig. 3. Thickness dependence of the vertical force F_z for various strengths of the magnetic moment normalized in units of $\phi_0 \lambda/\mu_0$ at distance $a/\lambda = 1$. Solid and dashed lines mean the repulsive and attractive forces acting on the point dipole, respectively.

saturation. It is seen that the force, caused by the Meissner effect, has about 95% of the saturation when the thickness is equal to the penetration depth, and the force caused by the vortex line has about 85% of the saturation when the thickness is equal to the penetration depth. Fig. 3 shows the curves which represent the (dimensionless) vertical force F_z (in units of $m\phi_0/2\pi\lambda^3$) which acts on the point dipole versus thickness (in units of the penetration depth λ) for different strengths of the magnetic moment $m = 1$, 10 and 100 $\phi_0\lambda/\mu_0$ with a distance $a/\lambda = 1$. The positive value (solid line) means the force acting on the point dipole is repulsive. The negative value (dashed line) means the force acting on the point dipole is attractive. It can be seen that for smaller thickness the force acting on the point dipole is repulsive and becomes attractive for larger thickness at a strength of the magnetic moment of the point dipole $m = 10\ \phi_0\lambda/\mu_0$. Fig. 4 shows the (dimensionless) vertical force F_z (in units of $m\phi_0/2\pi\lambda^3$) as a function of vertical distance a (in units of the penetration depth λ) for $m = 10\ \phi_0\lambda/\mu_0$ with a film thickness of 0.1, 0.5, 1 and 2 λ. It can be seen that depending on the thickness of the thin film, there is a change of sign of the vertical force. The vertical force acting on the point dipole is repulsive (solid line) under a certain distance, approaches zero, and then becomes attractive (dashed line) with increasing separation between the dipole and the thin film. It is clear that the vertical force acting on the point dipole is either attractive or repulsive depending on the strength of the magnetic moment of the point dipole, the distance between point dipole and vortex line, and the thickness of the superconducting thin film. For the case of a point dipole placed just over the vortex, i.e., $r_0 = 0$, two limiting cases of Eq. (21) are given by

$$F_z = \frac{3\mu_0 m^2}{32\pi a^4}\left[1 - 4\frac{\lambda}{a}\coth\left(\frac{d}{\lambda}\right)\right] - \frac{m\phi_0}{\pi a^3}\left(1 - 3\frac{\lambda}{a}\,\frac{\cosh(d/\lambda)+1}{\sinh(d/\lambda)}\right), \quad \text{for } a \gg \lambda, \tag{23}$$

$$F_z = \frac{\mu_0 m^2}{64\pi a^2\lambda^2}\left(1 - \frac{1}{(1+d/a)^2}\right) - \frac{m\phi_0}{4\pi a\lambda^3}\left(1 - \frac{1}{(1+d/a)}\right), \quad \text{for } a \ll \lambda. \tag{24}$$

It is useful to know the lateral force acting between the point dipole and the vortex line since it might be possible for the vortex line to become depinned when the point dipole tip is scanned over it. As mentioned earlier, the interaction force between the point dipole and the vortex line is attractive. Fig. 5 shows the thickness (in units of the penetration depth λ) dependence of the (dimensionless) lateral force (in units of $m\phi_0/2\pi\lambda^3$) for a point dipole at a fixed distance $a/\lambda = 1$ and $r_0/\lambda = 1$. It is seen that the lateral force has qualitatively the same behavior as the above calculation of vertical force. Fig. 6 shows the (dimensionless) lateral force (in units



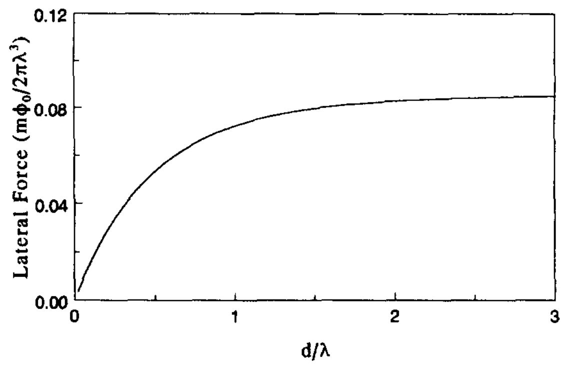


Fig. 4. The vertical force F_z as a function of the vertical distance a for various thicknesses of the thin film at a given magnetic moment $m = 10\ \phi_0 \lambda / \mu_0$. Solid and dashed lines mean the repulsive and attractive forces acting on the point dipole, respectively.

Fig. 5. Thickness dependence of the lateral force F_{r_0} for $a/\lambda = 1$ and $r_0/\lambda = 1$.

of $m\phi_0/2\pi\lambda^3$) as a function of vertical distance a (in units of the penetration depth λ) with a film thickness of 0.1, 0.5, 1 and 2 λ. In the limit of $r_0 \ll \lambda$, we find the limiting behavior of Eq. (22)

$$F_{r0} = -\frac{3m\phi_0}{2\pi a^4} r_0 \left[1 - 4\frac{\lambda}{a}\left(\frac{\cosh(d/\lambda) + 1}{\sinh(d/\lambda)}\right)\right], \quad \text{for } a \gg \lambda, \tag{26}$$

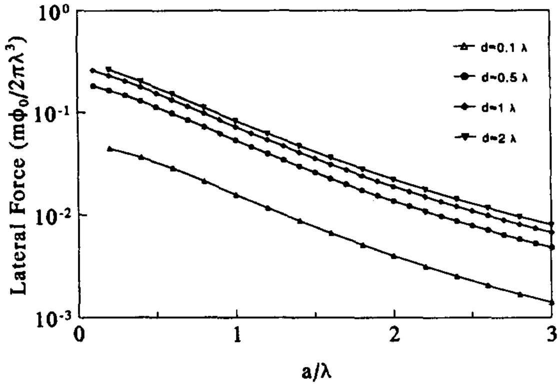
$$F_{r0} = \frac{m\phi_0}{8\pi a^2 \lambda^2} r_0 \left(1 - \frac{1}{(1 + d/a)^2}\right), \quad \text{for } a \ll \lambda. \tag{27}$$

It should be noted that the above results are obtained under the assumption that there is no vortex line created in the superconducting thin film when a single vortex line is probed by the magnetic point dipole tip. This probed vortex line may be created by an external magnetic field, with the field strength close to the lower critical field H_{c1} and only one vortex line existing in the superconductor. A natural question would arise. Do any extra vortex lines form in the thin film when the point dipole is approaching the film surface? Of course, it is possible for a vortex to be formed in the thin film in the field of the point dipole. If the vortex line, which produced by the magnetic point dipole, is created in the thin film, this will allow us to theoretically find the condition of creation of the first vortex line in the thin film, if initially no vortex line was locked in it. To simplify the analysis of vortex creation in the thin film, we make the assumption that the vortex line is created perpendicularly through the thin film when the point dipole is vertically lowered toward it. We use the critical condition that the system free energy before and after the creation of the first vortex line in the thin film is equal for the system to decide the position of the point dipole.

The free energy of the system is determined by the formula

$$U = \frac{1}{2\mu_0}\int B^2\, \mathrm{d}v_1 + \frac{1}{2\mu_0}\int \left(B^2 + \lambda^2 |\partial \times B|^2\right) \mathrm{d}v_2. \tag{28}$$

The integral $\int \mathrm{d}v_1$ is taken over the whole space except the space of the superconducting thin film and the point dipole. If the sample is in the Meissner state, the integral $\int \mathrm{d}v_2$ is taken over the sample volume. But if a single vortex line exists in the superconducting thin film, the integral $\int \mathrm{d}v_2$ is taken over the sample volume except for the core region of the line. We transform Eq. (28) into a surface integral, using the London equation and



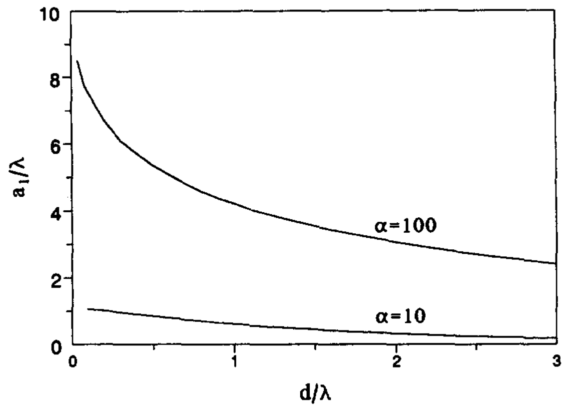


Fig. 6. The lateral force F_{r_0} as a function of the vertical distance a for various thicknesses of the thin film.

Fig. 7. Thickness dependence of the critical position a_1, which is the position of the point dipole for creating the first vortex line in the superconducting thin film, for two different values of α and the parameter $\xi/\lambda = 10^{-2}$.

divergence theorem [21,22]. We obtain the following expressions for the free energy of the system before, U_0, and after, U_1, the creation of the first vortex line, respectively.

$$U_0 = U_{\mathrm{in}}(a_1), \tag{29}$$

$$U_1 = U_{\mathrm{in}}(a_1) + U_{\mathrm{v}}(a_1) + U_{\mathrm{self(v)}} \tag{30}$$

where U_{in} is the self-interaction energy of the point dipole, U_{v} is the interaction energy between the point dipole and the vortex line, $U_{\mathrm{self(v)}}$ is the self-energy of the vortex line, and a_1, the critical position at which the first vortex is created in the thin film. The critical position of the point dipole a_1 can be found by setting $U_0 = U_1$. We find that a_1 may be determined by the relation

$$\begin{aligned}
&\int_0^\infty \mathrm{d}t\, \frac{t e^{(a_1/\lambda)t}\left[(t^2+1)^{1/2}\sinh(t^2+1)^{1/2}(d/\lambda) + t\cosh(t^2+1)^{1/2} d/\lambda - t\right]}{(t^2+1)^{1/2}\left[(2t^2+1)\sinh(t^2+1)^{1/2}\, d/\lambda + 2t(t^2+1)^{1/2}\cosh(t^2+1)^{1/2}\, d/\lambda\right]} \\
&= \frac{1}{2\alpha} K_0\left(\frac{\xi}{\lambda}\right)\frac{d}{\lambda} + \frac{1}{\alpha}\int_0^\infty \mathrm{d}t \\
&\quad\times \frac{\left[(t^2+1)^{1/2}\sinh(t^2+1)^{1/2}\, d/\lambda + t\cosh(t^2+1)^{1/2}\, d/\lambda - t\right] J_0(\xi/\lambda\, t)}{(t^2+1)^{3/2}\left[(2t^2+1)\sinh(t^2+1)^{1/2}\, d/\lambda + 2t(t^2+1)^{1/2}\cosh(t^2+1)^{1/2}\, d/\lambda\right]}.
\end{aligned} \tag{31}$$

where $\alpha = \mu_0 m/\phi_0 \lambda$ reflects the normalized strength of the magnetic moment, ξ is the coherence length, K_0 is the zero-order modified Bessel function and J_0 is the zero-order Bessel function. Fig. 7 shows the thickness (in units of the penetration depth λ) dependence of the critical position a_1 (in units of the penetration depth λ) for $\alpha = 10$ and 100 as the parameter $\xi/\lambda = 10^{-2}$. The height of the critical position of the point dipole decreases with increasing thickness of the thin film. The stronger the field a magnetic dipole moment has, the higher the height of the point dipole at which the first vortex line is created in the film we expected.

3. Conclusion and discussion

We have calculated the vector potential produced by the induced screening current and a single vortex line, and the thickness dependence of the magnetic force acting on a point dipole placed above a type-II superconducting thin film with a single vortex line. We have determined the sign of the vertical force acting on the point dipole depending on the strength of the magnetic moment of the point dipole, the distance between the point dipole and the vortex, and the thickness of the thin film. The lateral force exerted on a point dipole by a single vortex line has also been calculated. How does the lateral force compare with the local pinning force? However, if the vortex becomes depinned while a certain height of the point dipole above the film is being scanned, then the attraction force between the point dipole and the vortex line may cause the vortex to move closer to the bottom of the point dipole. The local pinning force for a single vortex line may be estimated by the difference in lateral force for vortex with and without pinning. The results given above allow a more complete description of the magnetic force acting on the point dipole while the superconductor is in either the Meissner or mixed state. The result for the vertical force due to the superconductor in the Meissner state, i.e., the first term in Eq. (21), has been obtained in Refs. [16,19]. Our main results are further generalized to include angular dependence. The formulation is readily extended to the case with more vortices. In particular, we have examined a certain limiting case of the magnetic force. Not only the analytical results, but physically important situations such as the well studied limit $a/\lambda \gg 1$ are readily apparent, which could be important for studying pairing symmetry in superconductors [19]. As concerns a MFM study of sufficiently superconducting thin films and semi-infinite superconductors, that can be easily carried out by using our results.

The vertical as well as the lateral force show a common feature of dependence on thickness. The forces increase monotonically with increasing thickness d and are saturated when the thickness is much larger than the penetration depth. The force has qualitatively the same behavior for various a/λ values.

Finally, the creation of the vortex in the thin film caused by the field of the point dipole as it is lowered vertically down to the surface of the film is studied. With regard to a single quantum vortex creation in the thin film, the dependence of the critical position of the point dipole for creating the first vortex line on the thickness of the thin film has been obtained. The height of the critical position of the point dipole decreases with increasing thickness of the film, but decreases with decreasing strength of the magnetic moment of the point dipole tip. We emphasize that the discussion of the creation of a vortex in the thin film in this paper holds only for a perfect crystal grain or a perfect thin film. However, the presence of the grain boundaries and other imperfections in high-T_c superconducting materials will lead to a more complex creation of a vortex line in it. Eq. (21) implied a step-jump in the vertical levitation force after the creation of the first vortex line, so the force changes discontinuously. Such a force change originating from the creation of the vortex in the superconducting thin film, may be detected by the MFM method through the vertical levitation force measurement.

Acknowledgments

We wish to acknowledge the financial support by National Science Council of Taiwan through the grant number NSC 85-2112-M009-029-PH.

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